UNIVERSITY OF CAPE COAST

ALGEBRA TEACHING KNOWLEDGE OF BASIC SCHOOL MATHEMATICS TEACHERS

WILLIAMS OSEI

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BY

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Thesis submitted to the Department of Mathematics and I.C.T. Education of the Faculty of Science and Technology Education, College of Education Studies, University of Cape Coast, in partial fulfilment of the requirements for the award of Master of Philosophy Degree in Mathematics Education.

AUGUST 2020

DECLARATION

Candidate's Declaration

I hereby declare that this thesis is the result of my own original research and that no part of it has been presented for another degree in this university or elsewhere.

Candidate's Signature: …………………………………… Date: ………………...

Name: Williams Osei

Supervisor's Declaration

I hereby declare that the preparation and presentation of the thesis were supervised in accordance with the guidelines on supervision of thesis laid down by the University of Cape Coast.

Supervisor's Signature: …………………………. Date: ……………......

Name: Mr. Benjamin Yao Sokpe

ABSTRACT

This study is situated on the fact that, teachers' knowledge of algebra for teaching affects students' algebra knowledge, hence their general performance in mathematics. In view of this, the algebra teaching knowledge level of basic school mathematics teachers should be monitored to positively affect pupils' mathematics performance. The study focused on basic school mathematics teachers who obtained their teaching certificates through Distance Education Programmes and those who had theirs from Colleges of Education. The re-conceptualized KAT framework was used to bring to light the algebra teaching knowledge levels of basic school mathematics teachers and also checked if their algebra teaching knowledge improve as their years of teaching experience increases. The study employed an explanatory sequential mixed method research design and used 203 basic school mathematics teachers. The study brought to view that, basic school mathematics teachers from the two groups have fairly satisfactory algebra knowledge for teaching mathematics and needs to be improved. It also revealed that, although they all have fairly satisfactory algebra knowledge for teaching, that of those who obtained their teaching certificates from Colleges of Education is relatively higher than those from Distance Education programmes. The study finally brought to view that, the algebra teaching knowledge of the two groups do not improve significantly as their years of teaching experience increases. In reference to the outcome of the study, it is recommended that, there should be in-service training for basic school mathematics teachers within the study area to enable them improve on their algebra teaching knowledge as their years of teaching experiences increase.

KEY WORDS

Basic school education

Teaching experience

Content knowledge

Pedagogical knowledge

Pedagogical content knowledge

Knowledge of algebra for teaching (KAT)

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DEDICATION

To my parents: Mr. and Mrs. Frimpong, my siblings: Bright, Emmanuel, Stephen, Priscilla and Gertrude and my dear Maud

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CHAPTER ONE

INTRODUCTION

Mathematics is indeed one of the most fascinating and useful subjects of study humans have created. The importance of mathematics in our everyday lives cannot be downplayed. Nearly every part of human life involves mathematics. Mathematics is the bedrock of science and technology (Haruna, 2014). Mathematics is the gate and key of the sciences, neglect of mathematics works injury to all knowledge (Bacon, 1994). This makes it clear that, mathematics forms the foundation on which science and technology rest. The absence of mathematics brings science and technology which forms the basis of national development to a naught. Life without mathematics is almost an impossibility (Githua & Mwangi, 2003). Those who understand and can mathematize will have significantly enhanced opportunities and options for shaping their future (National Council of Teachers of Mathematics, 2000).

Algebra is of vital importance since it functions as a gatekeeper for later mathematics courses (Ferrini-Mundy, McCrory, Senk & Marcus, 2005). It is the gatekeeper of the foundation of mathematics (National Council of Teachers of Mathematics, 2000). In other words, algebra has significant and pivotal roles in mathematics as a whole. It plays the foundational role in mathematics (Yarkwah, 2017). This implies that, mathematics falls greatly on algebra. A good foundation in algebra will positively affect the performance of students in mathematics and the opposite is also true. Algebra has applications in almost all the other areas of mathematics (Yarkwah, 2017). The knowledge acquired in algebra affects mathematics performance in general. This makes it difficult for one to excel in mathematics, when he or she has a weak foundation in algebra.

The evidence that Algebra forms the basis for many contents and concepts in mathematics is clearly seen in areas such as Sets, Length and Area, Shapes and Space, Relations, Algebraic Expressions, Linear Equations and Inequalities, Areas and Volume, Angles etc. looking at the Junior High School Mathematics Curriculum. The Junior High School mathematics curriculum allows Algebraic Expressions to be taught in Form 1, Form 2 and Form 3 (CRDD, 2007). This rings a bell on the crucial advantages for basic school pupils to get control over algebra contents and its applications in other basic school mathematics contents. Teachers' good repertoire of knowledge in algebra has the potential of affecting students' achievement in mathematics (Yarkwah, 2017), and as consequence affects science and technology and hence national development.

This study is situated on the fact that, the knowledge of teachers greatly affects students' learning (the knowledge they acquire) and hence their performance. (Eisenberg, 1977; Wilmot, 2009; Yarkwah, 2017). This establishes that, teachers' algebra knowledge affects students' algebra learning (the algebra knowledge they acquire), hence their performance in mathematics. Research has also established that, students' algebra knowledge affects significantly their general mathematical knowledge hence their general performance in mathematics, this is because mathematics falls significantly on algebra. (Yarkwah, 2017). In reference to this assertion, one can say that, the algebra teaching knowledge of mathematics teachers affects the general performance of students in mathematics. In view of this, the algebra teaching knowledge level of basic school teachers should be monitored to positively affect students' algebra knowledge, hence their performance in mathematics at the basic school level.

The study was to explore and bring to light the algebra teaching knowledge levels of basic school mathematics teachers who obtained their teaching certificates through Distance Education programmes and their counterparts who obtained theirs from Colleges of Education. It was also to check if they have the algebra teaching knowledge it takes to effectively teach algebra and its related contents at the basic school level to positively affect pupils' general mathematics performance.

Background to the Study

Basic Education as the foundation of higher levels of education prepares and makes pupils ready to enter into higher levels of education. Students with strong foundation from the basic school levels tend to excel during their various higher levels of education. Basic school teachers have significant roles in ensuring that, the foundation built for basic school pupils are firm and strong in their various fields of study, mathematics is no exception and a good foundation in algebra will influence positively the building of firm foundation for basic school pupils in mathematics. As a result of the advantages of mathematics and the sensitive nature of algebra in mathematics, the algebra teaching knowledge of basic school teachers should not be overlooked. This is because algebra forms the foundation of mathematics (Wilmot, 2008).

Over the years, basic school pupils have demonstrated weak control over algebra related questions in the Basic Education Certificates Examination. This has contributed significantly to the poor performance in the Basic Education Certificates Examination for about a decade now and confirms the assertion that, students' performance in algebra is particularly worrisome (Blume, 2000; RAND Mathematics Study Panel, 2003). For instance, the chief examiners' reports for mathematics in the Basic Education Certificate Examination (2010, 2011, 2012, 2013, 2014, 2015, 2017 & 2018) constantly indicated that, a good number of basic school pupils demonstrated weak control over algebra and its related items in the mathematics examination. The following comments were made on pupils' weaknesses regarding algebra. Basic school pupils have difficulty in:

a. working with variables, especially where variables are more than one.
b. arithmetic operations involving positive and negative signs.
c. removal of brackets, especially brackets within brackets and brackets which have exponents.
d. solving word problems involving fractions
e. arranging fractions in ascending order of magnitude
f. arithmetic computations
g. applying BODMAS correctly
h. manipulation of variables and numbers
i. simplifying and handling Algebraic Expressions

In tracking the possible causes of basic school pupils' inability to handle algebra and its related contents, it is very important to investigate if their mathematics teachers have the knowledge it takes to teach algebra and algebra related contents at the basic school level. This is because the algebra teaching knowledge of the mathematics teachers affect the algebra knowledge of pupils, hence their general performance in mathematics, since algebra forms the foundation of mathematics. Although many factors affect a teacher's effectiveness, teacher knowledge (subject matter knowledge, in this case, algebra content knowledge) is one of the key factors that influence classroom atmosphere and student achievement (Fennema & Franke, 1992). An investigation into the algebra content knowledge of basic school mathematics teachers will help to identify the kind of algebra knowledge they transmit to these young pupils. The knowledge teachers possess is what they communicate to their students and basic school mathematics teachers are no exception. Research also shows that students record the most gains when assigned effective teachers in term of content knowledge (Ogar, 2006).

Agreeing to Ball, Hill, and Bass (2005), the quality of mathematics teaching depends on teachers' mathematical content knowledge; and, alarmingly, many teachers lack firm mathematical understanding and skill. If the teacher has a strong foundation in subject matter and pedagogy, it will affect the pupils positively in their performance in mathematics and help in building strong foundations for their future. We therefore need to pay attention to the knowledge of mathematics teachers at the basic school level especially with their knowledge for teaching algebra which affects the general mathematics performances of basic school pupils. Several researches show that, teachers' knowledge greatly affects or influences how they teach (see for example Ambrose, 2004; An, Kulm & Wu, 2004; Hill & Ball, 2004; Stipek, Givvin, Salmon, & MacGyvers, 2001). Evidence available from researches suggest that teachers' intellectual resources significantly affect students' learning experiences (Odumosu**,** Olusesan & Abel, 2016) and this has made educators to focus on the knowledge of the subject matter because researches suggest that teachers of mathematics lack essential content knowledge for teaching the subject (Ma, 1999; Olfos, Goldrine & Estrella, 2014). Again, teachers' knowledge greatly influences how and what they teach (Yarkwah, 2017). It is out of the knowledge a teacher has that he or she communicates knowledge (teaches). It is very difficult and almost impossible for a teacher to communicate knowledge he or she does not possess to students and this greatly affects the students negatively. Ball, Thames and Phelps (2008) have made it clear that, there may be nothing more foundational to teacher competency than how well teachers know the subjects they teach. Ma (1999) affirms with the argument that, a profound understanding of fundamental mathematics provides a necessary base for successful mathematics teaching. Teachers' knowledge greatly affects students' performance (Wilmot, 2008).

The main sources of basic school mathematics teachers in the country for the last two decades include Colleges of Education, Distance Education programmes and the various universities that undertake professional programmes and courses in Basic Education. Basic school teachers possess different qualifications and may be trained in one of the above stated channels or programmes (Abudu & Mensah, 2016). However, Colleges of Education and Distance Education Programmes have become the major sources of basic school mathematics teachers for about two decades now. Before 1999, Basic school teachers were mainly from the Colleges of Education apart from the various universities in the country which also produced teachers with degrees in Basic Education (Osei & Mintah, 2014). Distance Education became one of the main alternative sources of basic school mathematics from early 2000s (Osei & Mintah, 2014). The first set of Basic School teachers that was produced from the Distance Education programme in the country were from the University of Education, Winneba, which included basic school mathematics teachers. Since the year 2000, Distance Education programmes in the various universities have become a major source of basic school teachers of which mathematics teachers are no exception in Ghana (Osei & Mintah, 2014).

There are different groups of mathematics teachers at the basic school level, based on where they obtained their training (teaching certificates) as basic school mathematics teachers. These groups may include, those from the Distance Education programmes across the country, Colleges of Education and those from the various universities that offer degree programmess in Basic Education. Teachers who obtained their teaching certificates through Distance Education programmes and those who had theirs from Colleges of Education have dominated at the Basic School level within the last two decades. It is also important to know that, teachers from these two major sources are awarded the same level of certificates after their training processes (Diploma in Basic Education).

This research considered basic school mathematics teachers who obtained their teaching certificates from Colleges of Education and those who had theirs from Distance Education programmes. For the purpose of this study, the researcher's attention was focused on basic school teachers who specialized in mathematics and have become mathematics teachers at the basic school level in the Asunafo North Municipality and Asunafo South District.

In 2017 the researcher had the opportunity to help prepare some Distance Education mathematics students from some higher institutions that offer Distance Education programmes for their end of semester mathematics examination. The researcher being a mathematics educator observed that, some prospective basic school mathematics teachers from the Distance Education program do memorize solutions to mathematics past questions without understanding. Others also have problems with basics concepts in algebra, for instance expanding brackets, working with fractions and expressions with exponents etc.

A probe into this observation by referring to some algebra related written scripts of some sampled students from some Distance Education programmes and to some team leaders' reports after marking some of the scripts in question, the following challenges were observed with respect to some Distance Education students who have already become basic school mathematics teachers.

1. Manipulation of numbers and variables
2. Expanding of brackets; application of distributive properties
3. Working with fractions
4. Change of subjects etc.

Also, a probe into the chief examiners' reports of their counterparts who are from Colleges of Education on some algebra related courses they took as prospective basic school mathematics teachers was also necessary. The chief examiners' reports of Colleges of Education on some algebra related courses between 2013 and 2018 revealed the following as challenges faced by a number of prospective basic school mathematics teachers.

1. Difficulty in handling expressions with exponents
2. Weak control over expressions involving fractions
3. Weak control over equations involving logarithms
4. Weak control over expressions involving indices
5. Difficulty in expanding brackets as expected

The issue in question is that, although Distance Education and Colleges of Education students successfully go through all the courses required especially those related to algebra during their training process as prospective basic school mathematics teachers, could it be that, a significant number of them do not really understand conceptually some contents related to algebra? If so, what would be the effect of such situations on their students, looking at the pivotal role algebra plays in mathematics as a whole. There is the need to be sure that, basic school mathematics teachers are not communicating weak foundation and control over algebra related contents to the pupils. This is a very critical issue because, the knowledge of teachers greatly affects students' performance and basic school pupils are no exception (Eisenberg, 1977; Wilmot, 2009; Yarkwah, 2017).

Again, it should be noted that, if algebra as the gatekeeper to the foundation of mathematics is not well communicated by teachers to basic school pupils, the outcome will be devastating. This is due to the fact that, a weak foundation in algebra would be built in the minds of basic school pupils, which is likely to affect their general performance in mathematics at the basic school level.

A careful examination of these sets of challenges suggest that, the basic school pupils and their prospective teachers from either the Distance Education Programmes and those from Colleges of Education, have similar challenges or difficulties with respect to algebra and algebra related contents. A body of research indicates that teachers' knowledge influences the quality of their teaching as well as student learning (Hoover, Mosvold, Ball, & Lai, 2016). In view of this, the unanswered question is that, could the reality be the case that, since the prospective basic school mathematics teachers from Colleges of Education and those from Distance Education programs have some difficulties in demonstrating adequate control over algebra and algebra related contents as suspected? they transmit that same inability to the basic school pupils?

There appears to be general consensus that mathematics teachers need to know the content in ways that surpass the knowledge of educated people outside the teaching profession (Ball, Thames & Phelps, 2008). More research is however needed in order to investigate the types of knowledge needed for teaching particular mathematical topics at particular levels (Hoover et al., 2016).

The fact is that, the two groups of basic school mathematics teachers in focus with respect to this study go through different curricula during their training processes, which may suggest that, there is no common grounds for a similar assessment. In view of this, it was necessary that, the assessment and investigation be done based on what these basic school mathematics teachers teach on the field as in-service mathematics teachers (contents enshrined in the basic school mathematics curriculum) not necessarily what they should know (Content courses they were taught whiles receiving their training as prospective basic school mathematics teachers). Prospective basic school mathematics teachers at either the Colleges of Education or Distance Education programmes do take a number of content courses meant to enrich themselves to have control over everyday problems they may face on the field and not necessarily to teach the basic school pupils. This is what puts the mathematics teacher above the students he or she teaches in terms of content knowledge. In view of this, what the teacher should teach or teaches is a subset of what he should know to enrich himself. The two groups of basic school teachers mathematics were purely assessed on what they teach as basic school mathematics teachers. Hence, the assessment carried out in this study was based on algebra contents in the basic school mathematics curriculum. This brought the two groups of basic school mathematics teachers to the same grounds which enabled a smooth and efficient assessment.

Moreover, these two groups are awarded the same value of certificates and are expected to teach mathematics at the basic school level, hence it is not out of place to assess their output as mathematics teachers at the basic school level. These two groups are trained as basic school mathematics teachers and are deployed to teach at the various Basic schools and so are expected to give a similar output. This is because the basic school pupils they teach are assessed using one common examination at the end of their Basic School Education. It is therefore not out of place to assess their algebra knowledge on the same grounds.

Statement of the Problem

The role of algebra in mathematics is so sensitive that, proper attention needs to be given to its study and understanding (Ball, 2003a). Algebra as a foundational course, serves as a concierge, posing varying opportunities for entry into advanced mathematics courses for ground work for college and for the world of work (Silver, 1997). Algebra plays a key role in building the mathematical foundation of young people who desire to pursue mathematics at the higher levels of education. Research tells us that success in algebra is a factor in many other important student outcomes. Emerging researches suggest that, students who start an algebra curriculum in the early grades tend to do better in the subject in secondary school level (Knuth, Stephens, Blanton & Gardiner, 2016). The knowledge of teachers greatly affects students' performance (Eisenberg, 1977; Wilmot, 2009; Yarkwah, 2017). In effect, it is what teachers have that they pass on to their students. If a teacher has a weak foundation in a subject matter, he or she will pass same or similar knowledge to his or her students and nothing higher than his or her own knowledge can be communicated. In this regard, as mathematics educators, we must consider the types of mathematical knowledge basic school teachers need to provide all students with reasonable prospects to learn algebra (Yarkwah, 2017). We can never dispute the fact that teachers need a profound understanding of the mathematics they teach to help students to perform as expected on the field (CBMS, 2001).

Research is packed with facts that teachers' content knowledge is often thin and insufficient to provide instruction for students in today's classrooms (Ball, 1988a, 2003b; Ball & Bass, 2000; Ma, 1999; Stacey, et al, 2001). This generally affects the overall performances of students in these subjects where these problems exist and mathematics is no exception.

Over the years, basic school pupils have demonstrated weak control over algebra related contents in the Basic Education Certificate Examination (B.E.C.E.). West African Examination Council Chief Examiners' reports (2010, 2011, 2012, 2013, 2014, 2015, 2017 & 2018). A great number of the pupils who sat for the Basic Education Certificate Examination for the years listed demonstrated weak foundation in the content areas related to algebra and it affected their general mathematics performance. The reports added that, basic school pupils have difficulty when working with variables, arithmetic operations involving signs (positive +), (negative -), removal or expanding of brackets, solving word problems involving fractions, arranging fractions in ascending or descending order of magnitude, poor arithmetic computation, failure to apply BODMAS correctly. These specific issues with algebra related contents may have contributed significantly to the poor performance of pupils in basic school mathematics over the years in the Basic Education Certificate Examination (B. E. C.E.). This assertion is made based on the sensitive nature of algebra in mathematics.

Amidst all these poor performances over the years, a critical look has not been taken to observe or assess the subject matter knowledge of the teachers who teach these young pupils at the basic school level with respect to algebra and its related contents. Could it be that, these basic school pupils are unable to demonstrate control over algebra related contents because their teachers have similar challenges? If a teacher has a weak foundation in a subject matter, it is almost impossible to communicate knowledge to build strong foundation in his or her students. Teachers' good repertoire of knowledge in algebra has the potential of affecting students' achievement in mathematics (Yarkwah, 2017). According to (CBMS) as cited in (Yarkwah, 2017), if learners are estimated to develop mathematical ability and apply mathematics in real world situations, then for the instructors or teachers no less can be expected of them. Again, it is important to know that, teachers' scores can be statistically significant prognosticators of how much students learn (Yarkwah, 2017).

The broader picture of how weak basic school pupils handle algebra and its related contents in the Basic Education Certificate Examination (B. E. C. E.) has made it an urgent issue to investigate if basic school mathematics teachers have the algebra knowledge it takes to effectively communicate algebra contents to positively affect the general performance of pupils in mathematics. This is because the algebra teaching knowledge of teachers affect students' performance in mathematics as discussed. It is almost an impossibility for a teacher to communicate knowledge he or she does not possess to effectively affect students' general performance in mathematics.

Adapting the Knowledge of Algebra for Teaching (KAT) project, Black (2008) and Yarkwah (2017) instruments, adequate information would be obtained about the level of knowledge Mathematics teachers at the basic school level (Products of Distance Education Programme and Colleges of Education) possess in

Algebra, the differences in their algebra knowledge, if any and also if their algebra knowledge improves with respect to teaching experiences.

Purpose of the study

Knowing clearly the effects of teachers' knowledge on students in the mathematics classroom and their performance in mathematics in general and also the significant role algebra plays in building the mathematical foundation of students, it is important to be aware of the level of algebra knowledge possessed by basic school mathematics teachers, taking into consideration those from two of the major sources; Distance Education programmes and those from Colleges of Education. In view of this, the purpose of this study was to find out:

1. The level of algebra knowledge possessed by basic school mathematics teachers who obtained their teaching certificates from either Distance Education programme or Colleges of Education.
2. The difference in algebra knowledge between basic school mathematics teachers who obtained their teaching certificates from Distance Education programmes and those had theirs from Colleges of Education.
3. If the algebra knowledge of basic school mathematics teachers who obtained their teaching certificates from Distance Education programmes and those who had theirs from Colleges of Education improve as their years of teaching experience increase.
4. What factors may account for the differences in the algebra knowledge of basic school mathematics teachers who obtained their teaching certificates

from Distance Education programmes and those who had theirs from Colleges of Education, if any.

Research Questions

The following research questions guided the study.

1. What level of Algebra knowledge for teaching do basic school mathematics teachers who obtained their teaching certificates from Colleges of Education and those from Distance Education possess?
2. What factors account for the difference in algebra knowledge between basic school mathematics teachers who obtained their teaching certificates through Distance Education Programmes and those from Colleges of Education? If any

Research Hypotheses

The following research hypotheses guided the study:

H_{01}: There is no significant difference between the algebra teaching knowledge of basic school mathematic teachers who obtained their teaching certificates from Colleges of Education and those who had theirs through Distance Education programmes.

H_{02}: There is no significant difference between the algebra teaching knowledge of basic school mathematics teachers who obtained their teaching certificates from Colleges of Education and their years of teaching experience.

H_{03}: There is no significant difference between the algebra teaching knowledge of basic school mathematics teachers who obtained their teaching certificates through Distance Education programmes and their years of teaching experience

Significance of the Study

Identifying the role of basic school mathematics teachers in basic school pupils' state of algebra knowledge which manifests in the Basic Education Certificate Examination has been a challenge over the years. It is uncertain whether the poor control many basic school pupils exhibit in the B. E. C. E. is influenced by basic school mathematics teachers' knowledge for teaching algebra. This research will bring to view the state of the algebra knowledge basic school mathematics teachers possess and communicate to the basic school pupils. The research will help us to know if basic school mathematics teachers have the required content knowledge for teaching algebra to influence students' mathematical performance positively.

In view of this, the various higher institutions that are involved in the training of basic school mathematics teachers can make the necessary reorganization and adjustment to improve the subject matter knowledge of their prospective basic school mathematics teachers if there is the need. This will go a long way to affect the knowledge of basic school teachers to effect positive changes in students' performance in algebra and its related contents, and hence performance in mathematics in general.

Again, the results of this study will inform the stakeholders of Distance Education programmes and Colleges of Education across the country on the kind of basic school mathematics teachers the programmes have produced for the country in almost two decades. This will help in bringing out programmes and interventions to curb existing problems, if any.

Moreover, the instrument used in the study can serve as a guide for assessing teachers' knowledge in other specific mathematical domains. This will help in the general improvement of mathematical knowledge across the country to positively affect mathematical performance of students and teachers.

Delimitation

This research was designed to assess teachers' knowledge for teaching algebra at the basic school level in Ghana using the re-conceptualized KAT framework. The study employed the participation of basic school mathematics teachers who obtained their teaching certificates through Distance Education programmes and those who had theirs through Colleges of Education.

Firstly, the domain of knowledge used by this study was algebra. This implies that, all the other domains of mathematics were not considered. The instrument contained only items from algebra and algebra related contents.

Again, the study used only basic school mathematics teachers who obtained their teaching certificates from Colleges of Education and those from Distance Education programmes. The research did not consider all other cohorts that are teaching mathematics at the basic school level, for instance basic school mathematics teachers who graduated from the various universities through the regular programmes. This was due to the fact that, the two groups employed are on the same rank in terms of certification (Diploma in Basic Education).

Furthermore, the study used basic school mathematics teachers who teach mathematics within the Asunafo North Municipality and Asuanfo South District under the Ahafo Region of Ghana. This implies that, all basic school mathematics teachers who teach outside the study area were not considered. This was done because of time and financial constraints.

Finally, the teacher related factors that were used in the research are teachers' knowledge for teaching algebra at the basic school level, years of teaching experience and how they obtained their professional training as basic school mathematics teachers. This implies that, all other teacher related factors were not considered.

Limitations

The major limitation of the study was in relation to the use of a teacher-made achievement test in assessing the algebra teaching knowledge of colleague basic school mathematics teachers. Teachers naturally do not want to be examined, hence may not put up their maximum effort to reflect their actual algebra knowledge as expected, especially when they are being assessed by their colleagues. This may have affected the outcome of the study.

Also, the few number of basic school mathematics teachers used because of time constraint. The number of basic school mathematics teachers who have obtained their teaching certificates through Distance Education programmes and also those who have had theirs from Colleges of Education large and generalizing based on the outcome of the few teachers who participated in the study can be misleading and so has to be done with caution.

Definition of Terms

With respect to this study, the following terms are defined as follows.

1. Basic School Education: Junior High School

2. Teaching Experience: the number of years that a particular Basic School mathematics teacher has taught mathematics at the Basic School level.
3. Content knowledge: the knowledge obtained in content-specific courses.
4. Pedagogical Knowledge: the knowledge of how best knowledge is communicated and practiced
5. Pedagogical Content Knowledge: the combination of Content knowledge and Pedagogical Knowledge
6. KAT: Knowledge of Algebra for Teaching

Organisation of the Study

The entire study is made up of five chapters.

The Introduction (Chapter 1) takes care of the Background of the Study, Statement of Problem, Purpose of the Study, Research Questions and Hypotheses, Significance of the Study, Delimitations, Limitations, Definition of Terms and the Organisation of the Study.

The second chapter (Literature Review) takes a critical look at the literature relevant to the study. The review is broken down into the following sub-headings; Theoretical Framework, Teacher Subject Matter knowledge, Algebra in Basic School Mathematics, Basic Conceptual issues in Algebra, Procedural and Conceptual Knowledge of Algebra, Pedagogical Content Knowledge and finally Years of Teaching Experience and Students Achievement.

Chapter Three (Research Methods) looks at the Research Design, Population, Sampling Procedure, Data Collection Instruments, Data Collection Procedures and finally Data Processing and Analysis.

The fourth chapter (Results and Discussion) presents the statistical analysis and discussions concerning the study. It makes available tables and diagrams to clearly show the outcome of the study.

Finally, Chapter Five presents the Summary of the entire study, the key Findings, Conclusions and Recommendations for practice and for further research.

CHAPTER TWO

LITERATURE REVIEW

The sensitive nature and contribution of teachers' knowledge in students' performance and the significant role algebra plays in mathematics as a whole as its foundation has made it a necessity to be given attention. Over the years, basic school pupils have demonstrated weak control over algebra related contents. This has become a general problem in basic school mathematics education. Researchers have tried to fathom the causes of the poor performances of basic school pupils in the Basic Education Certificate Examination in relation to algebra.

This study seeks to find out if basic school mathematics teachers who obtained their teaching certificates from Distance Education Programmes and those from Colleges of Education have the algebra teaching knowledge it takes to successfully teach algebra and its related contents to positively affect students' general performance in mathematics at the basic school level. The outcome of the study will make clear the role basic school mathematics teachers who obtained their teaching certificates from either Distance Education programme or Colleges of Education have played and are playing in pupils' inability to demonstrate control over algebra and its related contents at the basic school level. Research makes it clear that, a teacher cannot give or pass on to his or her students what he or she does not possess. In view of this, a teacher who has a weak foundation in algebra will eventually pass on to his or her students a similar or worse knowledge in algebra hence affecting the general performance of students.

In this chapter, the theoretical framework that guides the study has been presented. The subthemes under the theoretical framework have also been explained to fit the core objective of investigating into the algebra teaching knowledge of basic school mathematics teachers who obtained their teaching certificates through Distance Education programmes in some universities within the country and those from Colleges of Education. The chapter also looked at a survey of concepts, history and cohorts that are related to the subject in questions. In relation to the above and for the purpose of this study, the literature review has been discussed in details under sub-themes.

Theoretical Framework

An in-depth analysis of literature and the work of the researchers in the Knowledge of Algebra for Teaching (KAT) Project theorized that, the comprehensive Knowledge for teaching Algebra is composed of three types of knowledge. These include,

1. Knowledge of School Algebra
2. Advanced Algebra Knowledge
3. Algebra Teaching Knowledge

The Researchers of the Knowledge of Algebra for Teaching (KAT) project made it clear that, the three main types of knowledge for teaching algebra as stated above are not hierarchical and does not exists in continuum with well-defined borders. Instead, their borders are blurry and intertwined in numerous ways (Wilmot, 2019). In order to achieve a more domain specific goal, Wilmot (2019) conceptualized that, the three main types of knowledge be called; School Algebra

Knowledge instead of Knowledge of School Algebra, Advanced Algebra Knowledge instead of Advanced Knowledge, and Algebra Teaching Knowledge instead of Teaching Knowledge.

Moreover, the integration of the three major types of knowledge produce an ultimate knowledge (Pedagogical Content Knowledge in Algebra) which enhances teachers' ability to properly communicate algebra contents to students. In order for an effective and efficient communication of algebra knowledge to basic school pupils, basic school mathematics teachers should have the Pedagogical Content Knowledge in Algebra. This is achieved as a result of uniting the three major knowledge types stated above.

Subsequently, a reconceptualization of the three major types of algebra knowledge was done in Wilmot, Yarkwah & Abreh (2018). They argued that, since the integration of the three types of knowledge produce another ultimate knowledge called Pedagogical Content Knowledge in Algebra, the combination of any two types knowledge should also produce another form of knowledge and that, the combination of any two cannot be said not to exits or be in a vacuum. They then conceptualized that, the combination of School Algebra Knowledge and Advanced Algebra Knowledge produce another form of knowledge called Profound Knowledge of Algebra, School Algebra Knowledge and Algebra Teaching Knowledge produce School Algebra Teaching Knowledge and the coming together of Advanced Algebra Knowledge and Algebra Teaching Knowledge produce Advanced Algebra Teaching Knowledge. Their study also agreed that the combination of the three types of knowledge produce Pedagogical Content

Knowledge in Algebra and mathematics teachers are expected to possess this ultimate knowledge to enable easy and effective communication of algebra knowledge to their students in the mathematics classroom.

Underneath is a schematic diagram of the expanded KAT framework, which guided this study. The reconceptualized framework shows the three major types of domain specific knowledge and the types of knowledge they produce when integrated as conceptualized by Wilmot, Yarkwah & Abreh (2018).

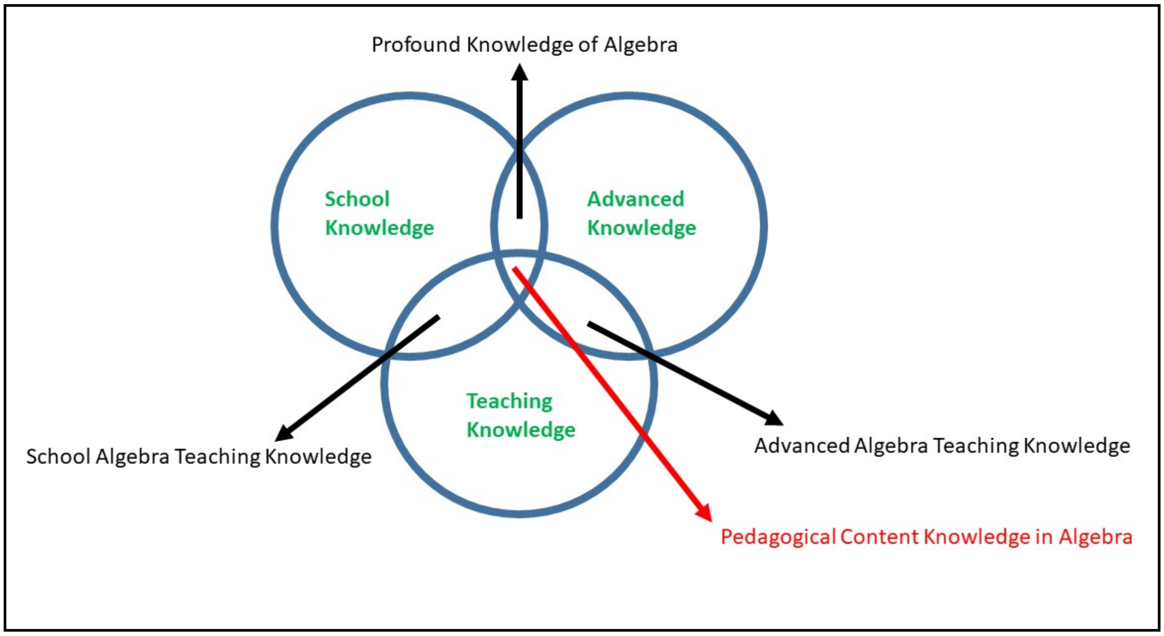


Figure 1: Expanded framework of domain specific teacher knowledge for teaching algebra, Willmot, Yarkwah & Abreh (2018).

Interpretation of the Theoretical Framework

A careful analysis of Figure 1 brings to view that, each of the three major algebra knowledge types has its own complete circle and the intersection of these major knowledge types produce specific algebra knowledge. Below is a list of how the other knowledge types are produced out of the three major algebra knowledge types;

Note; Let,

SAK denotes, School Algebra Knowledge

AAK denotes, Advanced Algebra Knowledge

ATK denotes, Algebra Teaching Knowledge

Therefore,

SAK + AAK = Profound Knowledge of Algebra

SAK + ATK = School Algebra Teaching Knowledge

AAK + ATK = Advanced Algebra Teaching Knowledge

SAK + AAK + ATK = Pedagogical Content Knowledge in Algebra

Knowledge of School Algebra

Knowledge of School Algebra, which is also known as "School Knowledge" is defined as the knowledge of mathematics in the intended curriculum, here in our context, the mathematics curriculum of Junior High Schools. This is the content of school algebra that mathematics teachers are expected to help students discover or learn in their algebra class (Wilmot, 2008). It sounds realistic, however, to conjecture that for teachers to impact students learning, the teachers themselves need to understand the content of school algebra since students at that level are expected to learn such (Yarkwah, 2017). This content is what is mostly found in textbooks and other resources for teaching Basic School Mathematics. Teachers' ability to demonstrate control over these contents will significantly affect the Junior High School pupils in the area of algebra and subsequently mathematics as a whole.

In the United States, ideas about knowledge such as this are described in booklets such as the National Council of Teachers of Mathematics (NCTM)'s Principles and Standards for School Mathematics (NCTM, 2000) while the precise grade-level algebra content is defined in the various states' standards, textbooks and other instructional materials or resources used in the schools (Yarkwah, 2017). The Knowledge of Algebra for Teaching (KAT Project) restricted this type of knowledge by reviewing content standards of ten different states in the United States (Wilmot 2008). In Ghana, the content of the Knowledge of School Algebra is built in the mathematics Syllabus of the Basic Schools and it is taken by the Basic School pupils. The KAT project considers this type of knowledge as vital because in their view algebra teachers would find it difficult to influence student learning unless they comprehend the grade-level algebra content they are to teach (Wilmot, 2019)

Advanced Algebra Knowledge of Mathematics

The Advanced Knowledge of Mathematics which is also referred to as "Advanced Knowledge" is defined in the Knowledge of Algebra for Teaching (KAT) Project as other knowledge which gives a teacher perspective trajectory and growth of mathematical ideas beyond school algebra (Ferrini-Mundy, Senk & McCrory, 2005, p. 1) as cited in Yarkwah (2017). Areas that incorporate the Advanced Algebra Knowledge of Mathematics include; sets of numbers, operations on numbers and Algebraic Expressions etc. (Wilmot, 2019).

Again, knowing alternate definitions, extensions and generalizations of familiar theorems, and a wide diversity of the uses of school mathematics are also features of an advanced standpoint of mathematics (Ferrini-Mundy et al. 2005). This type of knowledge is also referred to as the applied algebra. It is the Application of the algebra contents in other topics. Hence, it can be maintained that having an advanced viewpoint of mathematics gives teachers a deeper understanding (Profound) of algebra. The relevance of this type of algebra knowledge is clear because, it makes it possible for a teacher to make appropriate networks across topics, eliminating difficulties while retaining integrity and unzipping of content of school algebra to learners whereas unloading the complexity of mathematics content to make that content more understandable (Yarkwah, 2017). Moreover, it is hoped that any Basic School Mathematics Teacher who owns this type of knowledge would hold a quite a respectable knowledge of the path of the content of school mathematics.

Teaching Knowledge

Teaching Knowledge is the last major type of knowledge in the Knowledge of Algebra for Teaching KAT Framework. According to the KAT Framework by Ferrini-Mundy, McCrory, Senk and Marcus (2005, p. 2) as cited in Wilmot (2019), the teaching knowledge of Algebra is the knowledge that is precise to teaching algebra that may not be taught in advanced mathematics courses. It comprises such things as what makes a particular concept problematic to learn and what misconceptions lead to precise mathematical inaccuracies. It also contains mathematics required to identify mathematical goals, within and across lessons, to choose among algebraic tasks or texts, to select what to highlight with curricular paths in mind and to ratify other tasks of teaching.

Also, this type of algebra knowledge deals with how the content knowledge of algebra is communicated to the Basic School pupils in the context of this study. It may look at how the teacher imparts the algebraic knowledge he has to the Basic School pupils or the students of the mathematics classroom. The Mathematics Teacher must first have the content knowledge or the subject matter to be able to communicate effectively to the Basic School Pupil in the mathematics classroom for effective teaching and learning.

Furthermore, this is the type of knowledge teachers possess and they apply or employ in teaching the subject matter of school algebra (Yarkwah, 2017). Again, the KAT Project mentioned that, the teaching knowledge of algebra may fall into the kind of pedagogical content knowledge or purely mathematical content applied to teaching (Ferri-Mundy *et al*., 2005). This type of knowledge distinguishes an Algebra Teacher from a Mathematician or an Engineer (Yarkwah, 2017).

Relationship between the three major types of algebra knowledge

The KAT project by Ferrini-Mundy, McCrory, Senk and Marcus (2005) hypothesized that, the three major types of algebra knowledge, that is; School Knowledge, Advanced Knowledge and Teaching knowledge are not hierarchical in nature. They also made it clear that, the three types of knowledge do not exist in continuum with well-defined independent boundaries, instead, their borders are fuzzy and they are intertwined with each other in many different ways.

Consequently, it was affirmed through the re-conceptualization that, the interwoven nature of the three types of algebra knowledge produce different kinds of algebra knowledge. When any two or all the three types of knowledge intersect, it results in the production of another kind of algebra knowledge which is an advanced form of knowledge as compared to the basic knowledge that came together to produce that particular knowledge in question. The new types of advanced knowledge that are produced as a result of the intersection of two or all the three basic types of algebra knowledge are to help the basic school mathematics teacher who obtained his teaching certificate from either the Distance Education Programmes or those from College of Education to address important issues that would arise in the classroom in the course of their teaching.

Advanced Knowledge produced from the Intersection of the three types algebra knowledge

The intersection of all the three types of algebra knowledge and any two produce four types of advanced knowledge according to the KAT re-conceptualization by Yarkwah (2017). Blow are the four advanced knowledge produced as a result of the intersection of the three main types of algebra knowledge.

Profound Knowledge of School Algebra

The Profound Knowledge of School Algebra comes to play when the School Algebra Knowledge and Advanced Algebra knowledge intersect. Since the Profound Knowledge of School Algebra is the combination of School Algebra

Knowledge and Advanced Algebra knowledge, it is an advanced form of the two main types of algebra knowledge that comes together to produce it.

Again, this type of knowledge is fundamentally the teacher's ability to hold alternative definitions, extensions and generalization of familiar theorems, content that comes before school algebra as well as those that advance it (Yarkwah, 2017). It also looks at the application of a comprehensive range of Basic School Algebra in our context. Teachers with this type of knowledge demonstrate deep understanding of school algebra and are more likely to communicate algebra contents with understanding.

Advanced Algebra Teaching Knowledge

The Advanced Algebra Teaching Knowledge comes to play when the Advanced Algebra Knowledge and Algebra Teaching Knowledge intersect. Since the Advanced Algebra Teaching Knowledge is the combination of Advanced Algebra Knowledge and Algebra Teaching Knowledge, it is an advanced form of the two main types of algebra knowledge that comes together to produce it.

The Advanced Algebra Teaching Knowledge is the type of knowledge that empowers instructors of mathematics to make appropriate connections across various topics in advanced algebra courses (Yarkwah, 2017). Basic School Mathematics Teachers who have this type of knowledge are able to engage in appropriate pruning and breakdown when it becomes necessary for them to teach anything that falls within Advanced Algebra. Again, teachers who possess this knowledge type are people who have been able to integrate the Advanced Algebra Knowledge and Algebra Teaching Knowledge they have.

School Algebra Teaching Knowledge

The School Algebra Teaching Knowledge comes to play when the School Algebra Knowledge and Teaching Algebra knowledge intersect. Since the School Algebra Teaching Knowledge is the combination of School Algebra Knowledge and Teaching Algebra knowledge, it is an advanced form of the two main types of algebra knowledge that comes together to produce it.

Teachers who have this type of advanced knowledge are said to have a good trajectory of School Algebra. To a large extent, this type of knowledge allows Basic School Mathematics teachers to teach algebra in an unsolidified manner which is a way heighten understanding of heterogeneous group of students (Yarkwah, 2017). It also helps Mathematics teachers with that type of knowledge to make connections across the various algebra related topics; remove complication that may ensue while maintaining some level of reliability and unpack complexity to make the content being taught more comprehensive (Yarkwah, 2017).

Pedagogical Content Knowledge in Algebra

Pedagogical Content Knowledge in Algebra comes into play when the three types of Algebra knowledge (Knowledge of School Algebra, Advanced Algebra Knowledge of Mathematics and Algebra Teaching Knowledge) conceptualized by Ferri-Mundy et al., 2005 intersect. This is an advanced form of algebra knowledge that enables Basic School Mathematics Teachers to employ the first three advanced knowledge that emerged when any two of the main algebra knowledge intersect. It ensures effective communication of algebra contents to Pupils in the mathematics classroom. This is due to the fact that; it brings to play the three types of algebra knowledge required to communicate algebra effectively to students. Basic School Mathematics Teachers who possess this type of advanced knowledge can contribute greatly to students' performance in algebra and in mathematics as a whole. This type of Pedagogical knowledge is domain specific with respect to algebra.

The initial conceptualization of the Pedagogical Content Knowledge was by Shulman (1986b). He defined it as involving a complex combination of some form of content and pedagogy. Even though Shulman did a great work, his conceptualization of Pedagogical Content Knowledge was general but not domain specific. This is because, it does not specifically communicate how a specific mathematics content should be communicated effectively, but illustrates how on a general level, any content can be communicated. In view of this, there was the need for domain specific module to ensure and enable effective communication of mathematical contents such as algebra.

Importance of the Theoretical Framework to the Study

The pivot of this research is to investigate into the Algebra teaching knowledge of Basic School Mathematics Teachers who obtained their teaching certificates from the Distance Education Programmes and those from Colleges of Education. It is also to ascertain if there are differences in their algebra teaching knowledge based on the re-conceptualized KAT framework by Wilmot *et al*. (2018) and if their algebra teaching knowledge increases with respect to their years of teaching experience as Basic School Mathematics Teachers.

In the act of investigating the algebra teaching knowledge, it is important to dissect algebra knowledge as a whole to find out its composition. The theoretical

Framework has provided in details the various knowledge types that come together in the formation of Algebra Teaching Knowledge as a whole as discussed above. The Theoretical Framework is important because it has provided in details all the aspects of Algebra Knowledge that should be looked at for effective assessment of the algebra knowledge of Basic School Mathematics teachers. The re-conceptualization of the KAT Framework Wilmot *et al.* (2018) has made it easy in identifying the various knowledge components of algebra teaching knowledge that emerge when any of the basic algebra knowledge come together.

It is also important to note that, when assessing the algebra teaching knowledge of basic school mathematics teachers, content knowledge, curriculum and pedagogical knowledge are key factors. The theoretical framework has conceptualized these important items which make it relevant to this particular study. For the purpose of this study, Pedagogical Content Knowledge is of special interest in the knowledge base of teaching because it represents the blending of content and pedagogy into and understanding of how concepts are presented to the learner (Yarkwah, 2017).

Again, before the advents of the KAT project and the later re-conceptualization of the KAT Framework. Shulman's conceptualization of Pedagogical Content Knowledge was not domain specific, hence made it difficult to effectively assess specific mathematics subject matter such as algebra. The advent of the KAT framework and the re-conceptualization which conceptualized the other possible knowledge that emerge whenever any of the major knowledge come together has made it easy for a domain specific assessment of some specific subject matter such as algebra teaching knowledge of teachers. In view of this, the theoretical framework is relevant to the study.

Also, KAT framework and the re-conceptualized KAT framework has educated me the researcher on some important issues. Below are some of the issues I have been enlightened on; Mathematics teachers should not only know about the content they are teaching but should also know about the relationship that exist between the content and other topics, they should also know the advanced knowledge of the contents they are teaching and how they will communicate the content knowledge to the Basic School Pupil for effective teaching and learning.

Finally, the advent of the KAT framework and the later re-conceptualization has made it possible for a domain specific assessment of other topics in mathematics at the Basic School level and beyond and even other subjects. The KAT framework and its re-conceptualization is useful to this study because it allows for both quantitative and qualitative assessment of teacher knowledge; measurable types of teacher knowledge which eradicates the reliance on proxy measures of teacher knowledge. As a researcher, I have been enlightened to take up other tasks in other topics where there are problems to help improve mathematics Education.

Teachers Subject Matter knowledge

One of the key issues in teaching and learning is the subject matter knowledge of the teacher. Many prominent scholars, mathematicians, and policy makers agree that a teacher's mathematical knowledge is a key element of his or her effectiveness as a teacher (e.g., Askey, 1999; Ball & Bass, 2000b; CBMS, 2001;

Hill, Rowan, & Ball, 2005; Milgram, 2004). Subject matter refers to what one knows about what he or she teaches, see for instance (Cochran, DeRuiter & King, 1993). In our case, and for the purpose of this research, subject matter knowledge of the mathematics teacher is what he or she knows (knowledge possessed) about algebra (algebra contents). Teachers' subject matter knowledge has been analyzed approached more qualitatively, emphasizing knowledge and understanding of facts, concepts, and principles and the ways in which they are as well as knowledge about the discipline (Ball, 1988; 1991; Kennedy, 1991; Leinhardt & Smith 1985, Shulman, 1986; Tamir, 1987; Wilson, Shulman, & Richert 1987). In fact, the subject matter knowledge is the knowledge about what (Content) the teacher communicates for students to grasp or keep in their minds.

In recent years, discussion on teachers' content knowledge (TCK) and teacher's pedagogy knowledge (TPK) has attracted increasing attention from several agents of change in education industry. Research regarding teachers' knowledge is as important to scholastic reform today as it was four decades ago (Ball, Lubienski & Mewborn, 2001). Teachers should be knowledgeable in the content areas for which they are responsible to teach. This must include a deep understanding of the mathematics they are teaching (NCTM, 2000). In order for a teacher to teach very well he or she needs to know about the subject matter in both width and depth to a degree unlikely to be found amongst those beginning teacher training course (Simon, 1993). Teachers can only teach within the knowledge they have. They cannot communicate knowledge they do not possess. It is a common believe that every good teacher must learn more mathematics and that the higher the level of mathematics a teacher knows the better teacher he/she becomes (American Council of Education, 1999). Teachers are reflective thoughtful characters and that teaching is a multifaceted, cognitively demanding process involving problem solving and decision making (Clarke & Peterson, 1986). It is very true that, the subject matter knowledge of teachers are very essential, however, Thompson and Thompson (1996) stated that, mathematical content knowledge is essential for effective teaching; nevertheless, study revealed that teachers require further than just a strong knowledge of the content in order to teach mathematics. This suggests that, there is more to acquiring just the concepts and the subject matter. Mewborn (2003) also brought to view that, while teachers are said to have some level of appropriate knowledge of mathematics, unfortunately, these teachers lack a conceptual understanding of the mathematics they are to teach. He further said that, mathematics teachers have a strong procedural knowledge, but lack conceptual knowledge of mathematics. In other words, Mewborn communicated that, a large number of mathematics teachers have a strong command of the procedural knowledge but lack a conceptual understanding of the ideas that underpin the procedures. This implies that, a number of mathematics teachers find it difficult to conceptualize the procedures for students understanding in the mathematics classrooms.

Again, Mathematics teachers who have strong foundation in subject matter knowledge of mathematics specifically algebra are able to solve problems using a variety of methods, adapting to different contexts (see for instance, Black 2008). They are also able to identify errors and misconceptions of students on the mathematical concepts in question. Basic school mathematics teachers need a strong and deep mastery over the subject matter of knowledge of basic school algebra to adequately help their students to develop strong foundation in algebra and mathematics as a whole. Mathematics teachers who have weak foundation in algebra may find it difficult impossible to effectively identify and correct students' errors and misconceptions in the mathematics classroom.

In summary, basic school mathematics teachers need to have adequate knowledge of the subject matter with respect to algebra both in depth and in breadth to enable them to effectively communicate the algebra knowledge needed at the Basic School level to the Basic School Pupils. The in-depth Content knowledge expected from mathematics teachers cannot be less than whatever is expected of their students (see for instance; McCrory *et. al*., 2012). The Knowledge of basic school mathematics teachers should be deeper than what students are to grasp, in effect, teachers are to demonstrate control over the content knowledge than their students. That is what puts the teacher above his or her students with respect to teaching and learning. Teachers are supposed to demonstrate high level of content knowledge of what they teach. Although, having a deep control over the subject matter or content of a particular mathematical subject is not of itself complete to effectively teach mathematics, however, it forms the foundation to enable a mathematics teacher teach effectively if he or she has an adequate knowledge of pedagogy.

Algebra in Basic School Mathematics

Algebra refers to the interplay between numbers, variables and operations. In algebra, symbols (variables) are used to represent quantities in expressions and equations. In other words, mathematizing with unknown numbers is all that algebra is about. The use of variables (symbols) to represent quantities (numbers) and the introduction of operations to show the relationships between numbers and variables occur almost in every aspect of mathematics. Research tells us that success in algebra is a factor in many other important student outcomes. Emerging research suggests that students who start an algebra curriculum in the early grades take to the subject better in secondary school (Knuth *et al*, 2016). This explains the sensitive nature of algebra as a foundational area in mathematics as a whole. Scholars say algebra is the linchpin to success in mathematics because of its foundational role in all areas of mathematics (NCTM, 2000; National Mathematics Advisory Panel, 2008; RAND Mathematics Study Panel, 2003). Algebra seems to be the foundational area in mathematics. Teachers' good repertoire of knowledge in algebra has the potential of affecting students' achievement in mathematics (Yarkwah, 2017). Algebra is of particular importance since it functions as a gatekeeper for later mathematics courses (Ferrini-Mundy *et al*., 2005). Students who do not get the opportunity to develop a firm and solid foundation in algebra at the basic school level are likely to perform poorly in mathematics at the higher levels of education or may not appreciate mathematics. Algebra has become an academic passport for passage into virtually every avenue of the job market and every street of schooling (Knuth *et al.*, 2016).

A careful look at the Junior High School Mathematics Curriculum suggests that, stakeholders of mathematics, basic school Curriculum Experts and the Ghana Education Service (GES) want the basic school pupils to develop a strong foundation in algebra to affect their general mathematics performances and for future mathematics. This is due to the fact that, topics such as algebraic expressions, linear equations and inequalities and change of subject which helps students to improve on their algebra knowledge are spread throughout the Junior High School mathematics curriculum. For instance, Algebraic Expressions as a topic is taught in all the three classes that make up the Junior High School (Form 1,2 &3). It is expected that, the basic school pupils after going through these algebra content topics in their mathematics classrooms will develop at least the basic algebra knowledge and skill needed at their level and also prepare them for future mathematics at higher institutions. Much attention has been given to algebra because of its foundational role, its applications and its implications in Basic school mathematics and further mathematics.

The basic school mathematics curriculum gives room for the basic school pupils to acquire the basic knowledge of algebra with the help of their respective mathematics instructors. In view of this, mathematics teachers at the various basic schools should have what it takes (required knowledge) to adequately help these young ones to develop strong foundations in algebra which would affect their entire mathematical performances.

Conceptual Understanding Issues in Algebra

In making sure that, algebra concepts are grasped by students, in our case, basic school pupils, Knuth *et al*., (2016) proposes three core understandings issues critical to success in algebra accompanied by classroom activities that can help students build these understandings:

a) The ability to use variables to represent unknowns or varying quantities is critical to success in algebra.

b) A core understanding critical to student success in algebra is that the equal sign represents a relation between two equivalent quantities.

c) Student success in algebra also requires an ability to detect and generate patterns and to generalize those patterns symbolically.

In a similar manner, Joffrion (2005) suggested that, in helping students to understand or grasp algebra concepts, special attention should be given to; Concept of Equality, Concept of Variable and Modeling equations from verbal representations

1. **Concept of equality**.

A number of students have either misconceptions or little knowledge about the meaning of the equal sign. Many students are of the view that, the equal sign only precedes an answer to a question, and that, its only duty is to separate the question from the answer. Joffrion (2005) stated clearly that, in algebra, students must see the equals sign as relational, denoting either side has equal value. Based on the above, it is prudent for Basic School Pupil to grasp the concept of equal sign such that, whenever they come into contact, they would recognize and understand that it represents quantitative sameness (Saenz-Ludlow & Walgumuth, 1998). A clear understanding of the concept of equality may enhance students' ability in solving equations. For instance, The Balance Model for teaching about equality as proposed by Vlassis (2002) brought into view that, in any equation, since the equal sign represents quantitative sameness, whatever is added to or subtracted from one side must be added or subtracted from the other side too. This is one of the effective ways students can handle and demonstrate strong control on equations which also form part of Basic School Algebra.

2. **Concept of variable.**

Students need to have a developed concept of the meaning of variable. This understanding should be rooted in experiences with patterns and generalizations (Joffrion, 2005). Variable takes on many different meanings in the study of algebra and therefore the concept is difficult for students and even sometimes teachers of which basic School mathematics teacher are no exception. They should be treated as tools for expressing relationships. Research suggests that it may be helpful for students to verbally express a generalization before attempting to represent it using symbols (Schoenfeld & Arcavi, 1988).

On many occasions, misconceptions about variables are seen among students who are learning to use them. For instance, the variable *x* has been mistaken for the multiplication symbol by many students (Martinez, 2002). Again, there has been instances where students think that, when a variable let's say 'a' is changed to 'b' in the same equation, it changes the answer of the equation. This is a clear picture that, a number of students have problems with the conceptual understanding of variables with respect to algebra. For instance, Wagner (1977) as cited in Joffrion (2005), presented students with two equations, identical except for different letters were used as the single variable. The researcher received a variety of responses when she asked participants if different solutions would be obtained from solving both equations. Many students believed it was impossible to determine whether or not they were the same until both had been solved.

3. **Modeling equations from verbal representations**

One of the issues that has caught attention these days in algebra is students' ability to model equations from verbal representations. Modeling equations from problem situations that incorporates problem solving is a problem for many students and sometimes even teachers themselves. Translating from verbal relational statements to symbolic equations, or from English to "math," causes students of all ages a great deal of confusion. (Joffrion, 2005). Lodholz (1990) observed that writing equations from word problems is often a skill taught in contrived situations or in isolation. Children may translate English sentences to mathematical expressions, simply moving from left to right. "Three less than a number" is interpreted by many students as "3 – x" since the words "less than" (which mean to subtract, they have always been told) follow the 3. Teachers must be aware of these misconceptions and address them in instruction (Lodholz, 1990). Before students learn to represent algebraic situations symbolically, they should have opportunities to discuss them in easily understood, everyday language, thus developing their conceptual understanding (Kieran & Chalouh, 1993).

Basic school mathematics teachers can only assist their pupils to get control over the above issues if they themselves have control over them. It is therefore important that we assess to see if basic school teachers have control over these mathematical concepts that they are supposed to teach in the mathematics classroom.

Procedural and Conceptual Knowledge of Algebra

Lately, the teaching and learning process has shifted its focus towards a balance between procedural and conceptual understanding of mathematics. Procedural mathematics understanding is knowledge that focuses on skills and step-by-step procedures without explicit reference to mathematical ideas (Hope, 2006). Anderson (1989) defines procedural knowledge as "organization of conceptual knowledge into action units" (p. 24), without conceptual knowledge, this definition of procedural knowledge is useless. Based on Anderson's definition of procedural knowledge, without conceptual knowledge, procedural knowledge is useless and meaningless. This is because, it would be implemented without understanding and mathematizing would be like a puzzle of numbers and operations without understanding. Procedural knowledge answers the question (How?); the procedure to be followed without necessarily understanding (Why?) the conceptual understanding of what is being done. Ordinary procedural skills often fail to provide readily applicable methods to solve mathematics problems. Conceptual mathematics understanding is knowledge that involves a thorough understanding of underlying and foundational concepts behind the algorithms performed in mathematics (Hope, 2006). It answers the question (WHY?); the details of whatever is being done. Thus, it involves a situation where students are able to recreate formulas and proofs without the rote process. Moreover, students are allowed to make choices and apply their understanding through active engagement (Boaler, 2000). Students must have an understanding of both if they are to understand mathematics in depth (Wilkins, 2000). Students learning a topic like algebra are facing problems with topics related to concepts and not with those involving algorithms and procedures (Ghazali & Zakaria, 2011).

To successfully complete an algebra problem, students must develop both procedural and conceptual understanding (Mary & Heather, 2006). In effect, the balance between procedural and conceptual knowledge of algebra is a necessity for both basic school mathematics teachers and pupils. Basic school pupils need to develop both the conceptual and procedural understanding of algebra to obtain firm and strong foundation for future mathematics. When the objective of studying mathematics is solely to pass examinations, one may tend to focus more on procedural knowledge at the expense of conceptual knowledge of algebra and mathematics. This does not help in assisting students to build a strong foundation in Basic School Mathematics which affects their general mathematics performances and future mathematics.

According to Lim (2002), procedural understanding can aid in getting control over conceptual understanding. This is possible if Basic School mathematics teachers would go an extra mile to let the basic school pupils to know 'why' after knowing 'how'. Until today, mathematics education researchers are continuing studies to understand the balance between the two understandings.

Some researchers agree that both are important and that integrating both of them is important to increase students' understanding (Mary & Heather, 2006). It is very difficult and sometimes impossible for teachers who do not understand the concepts themselves to assist basic school pupils to gain conceptual understanding and even sometimes procedural understanding. In view of this, teachers should be assessed to know if they have mastery or control over algebra content conceptually and procedurally.

There has been some debate and different researchers have argued over the relationship between conceptual and procedural knowledge and which type of understanding develops first as students encounter new mathematics in the mathematics classroom (see for instance; Gelman & Williams, 1998; Siegler, 1991; Siegler & Crowley, 1994). Rittle-Johnson, Siegler, and Alibali (2001) proposed a mediating viewpoint, that, in fact, the two types of knowledge are not necessarily distinct, but rather opposite ends of a continuum and improvements in one type of understanding typically result from or result in improvements in the other type. Their research asserts that, the process of development of concept and procedure is iterative and closely intertwined such that, they cannot be distinctively separated. In a similar research, Star (2002) reiterated Rittle-Johnson, Siegler, and Alibali's (2001) point that conceptual and procedural knowledge are not distinct entities. The researcher gave three examples of student solutions to a complicated single-variable equation. Each student was able to solve the equation successfully, but the degree of efficiency and sophistication of the solutions varied. The strategies and procedures employed by the students were very distinct and clearly reflected varying levels of conceptual understanding as manifested by their procedures. In order to succeed in algebra, students need to develop both conceptual understanding of numbers and relationships and procedural skills in using them efficiently. With the completion of the Third International Mathematics and Science Study (TIMSS), much attention has come to procedural teaching and conceptual teaching (Joffrion, 2005).

Pedagogical Content Knowledge (PCK)

Pedagogical Content Knowledge **(**PCK) is expertise that demonstrates a combined knowledge of pedagogy and disciplinary subject matter (Bond-Robinson, 2005). Pedagogical Content knowledge (PCK) as interwoven pedagogy and subject matter knowledge is necessary for good disciplinary teaching (Shulman 1986). This kind of knowledge is described as knowing the ways of representing and formulating the subject matter that make it comprehends to others as well as understanding what makes the learning of specific topics easy or difficult (Ball, 1988; Even & Markovits, in press; Lampert, 1986; Shulman, 1986, 1987; Tamir, 1987; Wilson et al., 1987). Teachers' pedagogical content knowledge is influenced by their subject -matter knowledge (Even, 1993). In other words, whiles the content knowledge is about 'what', the pedagogical knowledge is about 'how'. The addition of these two knowledge types would produce pedagogical content knowledge which deals with 'the item to be communicated (what) and the medium by which the subject is communicated (how).

According to Shulman (1986), mathematical content knowledge and pedagogical content knowledge are integrated parts of effective mathematics instruction. The balance of these type of knowledge makes effective teaching and learning in the mathematics classroom. In order to build mathematical concepts in the minds of students or to help them grasp a concept, pedagogical knowledge and mathematical content knowledge are needed. This implies that, teachers who do not understand the concepts themselves may have difficulty or not be able to transmit these mathematical concepts to their students. Teachers' professional knowledge may be considered the single most important characteristic in instruction (Enkrolt, Buschhuter & Borowski, 2018). The manner in which teachers relate their subject matter (what they know about what they teach) to their pedagogical knowledge (what they know about teaching) is very key in Mathematics Education. Subject matter knowledge is a part of the process of pedagogical reasoning and the two knowledge are seen as parts of pedagogical Content knowledge (Cochran, DeRuiter & King, 1993). According to Kahan, Cooper and Bethea's (2003) review, a number of researchers frequently conclude that students would learn more mathematics if their teachers knew more mathematics which I agree. This is because, the knowledge transmitted from the mathematics teacher to the students in the mathematics classroom is no different from what the teacher possesses. It is what the teacher has that he or she transmits to the students and Basic School Mathematics teachers and pupils are no exception.

Researchers have focused on many aspects of teaching, but more often than not little attention has been given to how teachers need to understand the subjects they teach. Several factors may affect the teaching process of mathematics in the mathematics classroom, however, the role of teachers in the process is a key factor that affects the learning process significantly. It is a different thing knowing mathematics (Content) and knowing how to teach mathematics (Pedagogy). The blend of these two knowledge increases the possibility of a successful mathematics classroom. One must understand conceptually what to teach (Content knowledge) to enable him decide how to teach what he or she knows (Pedagogical Knowledge). Fennema and Franke (1992) argued that if a teacher has conceptual understanding of mathematics, it would influence classroom instruction positively. Teachers' solid knowledge is very essential as well as procedural rules.

Years of Teaching Experience and Students' Achievement

One of the indicative variables for teacher competence is teachers' years of teaching experience (Darling-Harmmond, 2000). Teachers who have taught mathematics for so long are expected to improve based on the experiences on the field. There should be a constant growth daily as the teacher explores and meets different problems, sets of people and new concepts. Research makes it clear that, teachers learn through their teaching experiences (Klecker, 2002; Rosenholtz, 1986). Leikin, (2006) said it best in my view, the main source of teachers' expertise is their interactions with students and learning materials. This affirms what I said early on that, through interaction with students, teachers become aware of new solutions to problems, new properties of concepts and objects of mathematics and this helps them to communicate well their mathematical understanding to students. Teachers gain more experience when they make conscious effort to know better than their students, to know the material well enough, and to predict students' possible difficulties, answers and questions (Leikin, 2006). These experiences are acquired on the field when teaching as in-service Basic School Mathematics Teachers and it comes with time. It is not achieved overnight.

Research has established that, teachers with more years of teaching experience are more effective than inexperienced teachers; especially those with less than three years of teaching experience (Klecker, 2002; Rosenholtz, 1986). Teaching experience has a positive and significant effect on students' achievement (Farooq and Shalizad, 2006). The role teaching experience play in teaching and learn and students' achievement is very significant and the experience teachers acquire comes on the field as in-service Basic School Mathematics Teachers.

Klecker (2002) and Bodenhausen 1988) found teaching experience as one of the determinants of students' achievement. There was an examination of the relationship between teachers' years of teaching experience and students' mathematics achievement. The research took into consideration students' scores on eighth grade National Assessment of Education progress (NAEP) mathematics test and the teaching experience of mathematics teachers measured on five categories: 2 years and below, 3-5 years, 6-10 years, 11-24years and 25years and above. The result showed that, students of teachers with higher teaching experience scored higher marks or performed well as compared to students with teachers of low teaching experiences (Klecker, 2002).

In as much as research agrees that, the competence of teachers increase with respect to their years of teaching experience, there has been some misconceptions about new teachers who enter the field of teaching for the first time. Teaching is one of the few professions in which the professionals are assumed to exhibit excellence the first year on the job (Klecker, 2002). It is not to say that teachers who enter the field of teaching are not to exhibit any level of excellence in their work, it is to affirm that, they will always become better teachers on the job or field. This is primary due to the exposure they get as in-service Mathematics Teachers.

Summary of Literature Review

Algebra has a very key role in mathematics in general. A strong foundation in algebra would affect the general performance of students in mathematics. Teachers have a key role in students learning and in building strong mathematical foundations for the future. Teachers who have weak foundation in algebra would find it difficult or impossible to communicate well algebra contents to students for them to understand. This would breed or result in students with weak foundation in algebra, hence affecting their general mathematical performances. In view of this, Basic School Mathematics teachers' knowledge for teaching algebra at the Basic School level should be checked or monitored. This would help stakeholders of Mathematics Education to help improve their knowledge if there are gaps to affect students' mathematical performances.

CHAPTER THREE

RESEARCH METHODS

The pivot of this research was to investigate the algebra teaching knowledge levels of basic school mathematics teachers who obtained their teaching certificates through Distance Education Programmes and those who obtained theirs from Colleges of Education based on the re-conceptualized Knowledge of Algebra for Teaching (KAT) framework by Wilmot *et al.* (2018). The study was also to find out if there are differences in the algebra teaching knowledge of basic school mathematics teachers who obtained their teaching certificates through Distance Education Programmes and those from Colleges of Education and finally if their algebra teaching knowledge improves as their years of teaching experiences as basic school mathematics teachers increase.

This chapter describes the methodology that was employed in the research. It looks at the following; Research Design, Population, Sample and Sampling Procedure, Instrumentation, Data collection and Analysis procedures.

Research Design

The research focused on investigating basic school mathematics teachers' knowledge for teaching algebra. In order for the researcher to investigate and get a clearer picture of the algebra teaching knowledge of the basic school mathematics teachers in question and also get detailed interpretations to the broader picture of their algebra teaching knowledge levels, Explanatory Sequential mixed method research design was used. This was due to the fact that, both quantitative and qualitative data were needed to bring to view the level of algebra teaching knowledge of basic school mathematics teachers who obtained their teaching certificates from Colleges of Education and those who had theirs through Distance Education programmes and also furnish the researcher with details of the outcome of the teacher-made achievement test on Knowledge of Algebra for Teaching.

Explanatory sequential mixed methods design is also called a two-phase model (Creswell & Plano Clark, 2011). The explanatory sequential mixed methods design allowed the researcher to collect quantitative and qualitative information sequentially in two phases. The quantitative data was collected first and analyzed, then the qualitative data was later collected and also examined. The quantitative data gave a broader or general picture of the outcome of the research in focus, whiles the qualitative data was collected to elaborate or explain the broader picture (quantitative results). It provided the researcher with details about the outcome of the quantitative data. In effect, the qualitative data was collected to give possible reasons and details to the results obtained from the first phase of the study. Figure 2 is a diagrammatic representation of how data was collected and analysed.

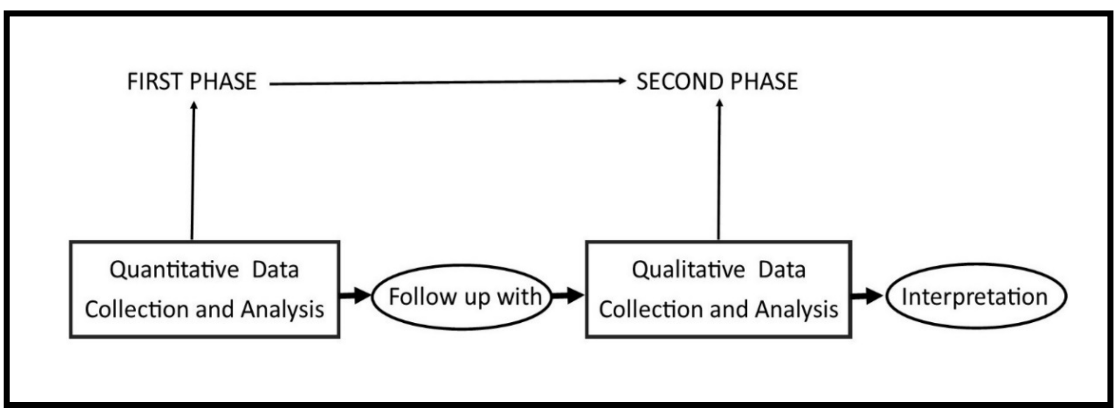


Figure 2: Research Design: Flow of the Study (Creswell & Plano Clark, 2011)

According to Tashakkori and Teddle (1998), the Explanatory Sequential mixed design is suited to studies in which the researcher wants qualitative results to explain significant, non-significant or surprising quantitative results. The approach is more useful when the research problem is more quantitatively oriented, the researcher has the time and ability to conduct the study in two phases, and the research develops new questions that arise from the quantitative results (Creswell & Plano Clark, 2011). Creswell & Plano Clark further added that, the design is also purposeful in identifying quantitative participant characteristics to guide the sampling for the qualitative phase of the study. The researcher has to identify the quantitative findings that need additional exploration and use these findings to steer or direct the qualitative phase (Creswell & Plano Clark, 2007).

The explanatory sequential mixed methods research design was relevant to this particular study based on the research objectives. This is because while some required quantitative data for their achievements, others required qualitative data. A careful look at some mixed methods research designs brought to view that, Explanatory sequential research design was appropriate based on the nature of the research questions and hypotheses used in the study. For instance, the first Research question and all the three research hypotheses required quantitative data through a teacher-made achievement test. The second research question required a qualitative data through interview by the researcher. However, the keen issue that made the entire study fit explanatory sequential research design was that, the second research question probed the bigger picture provided by the first research question and the research hypotheses. Again, since the second research was a probing research question (giving details to an existing issue), it was appropriate that the researcher collects the two sets of data sequentially instead of simultaneously, this makes the study fit the explanatory sequential mixed method design.

The advantages of explanatory sequential mixed method design include its strong quantitative orientation, the two phase structure that allows for a probe into the findings of the quantitative results and the link to emergent approaches where the second phase can be designed as a result of the outcomes of the first phase (Creswell & Plano Clark, 2011). Also, its weakness may include; time consumption and financial demands, since data is collected twice. The researcher planned the entire research taking into consideration the available time to help address the time consumption that comes with the selected research design.

Population

The target population for this study was all basic school mathematics teachers who obtained their teaching certificates from either Distance Education Programmes or from Colleges of Education and are teaching mathematics at the various basic schools within the Asunafo North Municipality and Asunafo South District in the Ahafo Region. According to the Programme Based Budget Estimates for the Asunafo North Municipality and Asunafo South District for 2018, there are at least 297 basic school mathematics teachers in the study area. The number of basic school mathematics teachers who obtained their teaching certificate from Collages of Education and those who obtained theirs through Distance Education programmes are 141 and 130 respectively. The study areas were selected by the researcher based on familiarity and flexibility in data collection. It was also because of the great number of basic schools and for convenience in data collection due to the present location of the researcher.

Again, since the instrument for the first phase of the data collection was a teacher made achievement test on teachers' knowledge on algebra for teaching, and teachers are usually not willing to be quizzed, the researcher had to select an area where he is well known and can persuade the basic school teachers to participate in the study. This contributed to the selection of the Asunafo North Municipality and Asunafo South District in the Ahafo Region.

Sampling Procedure

The focus of the study was on two major groups of basic school mathematics teachers, that is, basic school mathematics teachers who obtained their teaching certificates from Colleges of Education and those who obtained theirs from Distance Education programmes. The study employed stratified sampling technique. The two major groups were considered as strata and random sampling technique was used to select from each subgroup proportional to the stratum size. The samples selected from the two groups were combined to form the sample for the study. This was to aid the selection of a good number of basic school teachers from each group to fairly represent each stratum. A simple random sampling technique was used subsequently to select from the two strata.

The lists of all the basic school mathematics teachers in the Asunafo North Municipality and Asunafo South District were obtained and computer-generated random numbers were employed to select 107 and 98 teachers from the stratum of basic school mathematics teachers who obtained their teaching certificates from

Collages of Education and those from Distance Education Programmes respectively, based on Krejcie and Morgan's sample determination table as cited by the Research Advisors (2006). This gave all the members of each stratum the same probability to be selected and to be used for the research. It helped to avoid any bias from the researcher that may affect the outcome of the study.

The second phase of the study employed purposive sampling technique to select some participants whose results were of interest to the researcher. These included those who performed very well, those who performed averagely and finally those who performed poorly on the test. The purpose of the second phase was to probe the outcome of the first phase (quantitative results).

The researcher selected the following number of basic school mathematics teachers for the second phase of the study, this was based on their performance in the teacher-made achievement test of the first phase of the study. From the basic school mathematics teachers who obtained their teaching certificates from Distance Education Programmes, six (6) of those who performed poorly, four (4) of those who performed averagely and five (5) of those who performed very well were selected for the second phase of the study. Their test scores categorizations were (29% - 46%) poor, (51% - 57%) average and (66% - 71%) very well. Also, from the basic school mathematics teachers who obtained their teaching certificates from Colleges of Education, seven (7) of those who performed poorly, five (5) of those who performed averagely and six (6) of those who performed very well were selected for the second phase of the study. Their test scores categorizations were (37% - 49%) poor, (51% -57%) average and (74% -89%) very well. In total, thirtythree (33) Basic School mathematics teachers were selected to participate in the second phase of the study of which fifteen (15) were products of Distance Education programmes and eighteen (18) were products of Colleges of Education which is acceptable (Guest, Bunce & Johnson, 2006).

Data Collection Instruments

The study employed two research instruments for the study. These were; teacher-made achievement test and an unstructured interview. The two research instruments were developed by the researcher. The instruments inquired on the algebra teaching knowledge of basic school mathematics teachers and probed for some details to their performances. The contents of the instrument were based on the basic school mathematics curriculum. This is due to the fact that, the curriculum contains what basic school mathematics teachers teach at the basic school level, hence the teachers themselves should demonstrate adequate control over both content and pedagogy to enable effective transmission of mathematical knowledge in the mathematics classroom.

Teacher-made Achievement test

The study adapted a teacher made achievement test instrument from the Knowledge of Algebra for Teaching (KAT) Project, Black, (2008) and Yarkwah, (2017) for the first phase of the study. The researcher adjusted the above listed instruments in subject matter and pedagogy to fit the level where the research was conducted and the two groups employed for the study; basic school mathematics teachers who obtained their teaching certificates from Distance Education programme and those who had theirs from Colleges of Education.

The teacher-made achievement test of the study was made up of 35 items on teachers' knowledge for teaching algebra at the basic school level. The instrument was divided into five parts. The first part of the instrument (Part I) was made up of four items seeking for respondents' details which enabled easy classification of the instrument into the following headings; Years of teaching experience, how respondents obtained their respective teaching certificates and the type of teaching certification they have as at the time when the research was conducted.

The second part of the teacher made achievement test (Part II) was made up of 25 multiple choice type questions on knowledge for teaching algebra which required that respondents circle the correct answer to each question. The multiple-choice type questions were carefully selected to cater for the various knowledge types for teaching algebra under the theoretical framework used for the research.

The third part of the teacher made achievement test (Part III) was made up of 9 questions on knowledge for teaching Algebra. The questions required short responses from respondents. The final part of the teacher made achievement test was made up of a question on knowledge for teaching Algebra. The last item required the respondents to assess the responses of three basic school pupils to a mathematics question. Respondents were supposed to mark and make their comments based on their observations. They were also expected to comment on the correctness of each response from each of the three basic school pupils as basic school mathematics teachers.

Interview

An unstructured opened-ended interview questions were used to probe the outcome of the teacher made achievement test (Quantitative results) for the second phase of the study. Unstructured open-ended interview questions were appropriate because, the researcher asked the selected respondents questions based on previous responses and comment(s) made by each respondent about some particular questions. The questions were based on individual marked test scripts. The interview probed the general picture that was given by the first phase of the study (Quantitative Results). This brought to view details and possible reasons behind the general picture the qualitative data revealed after the analyses. The interview was not for every respondent who participated in the first phase of the study. The researcher selected those whose performances were of interest after the various analyses of the data obtained from the first phase of the study. The selection was based on excellent, average and poor performances of the basic school mathematics teachers on the teacher-made achievement test. The second phase of the study probed for the reasons behind each respondent's performance. The interview brought to view a clear understanding of the bigger picture provided by the teacher made achievement test in the first phase of the study.

The advantages of the instrument were that, the items were spread across the types of algebra knowledge for teaching and also the Bloom's taxonomy. This made it possible to test almost every aspect of teacher knowledge for effective teaching. It also covered a wide range of issues in the Junior High School mathematics syllabus. The weakness of the instrument was with regards to the duration available for the probe during the second phase of the study.

Validity

The content validity of the instruments was established by presenting the test and its scheme to two Mathematics Education Lecturers in the Department of Mathematics and I.C.T. Education for inspection to ensure that the type of knowledge hypothesized in the re-conceptualized KAT framework are satisfactorily covered and well structured. Again, the researcher and his supervisor had a meeting to deliberate on each item on the instrument to ensure content validity.

The instrument was first tested by the researcher in collaboration with one teaching assistant from the Department of Mathematics and I.C.T. Education and eight basic school mathematics teachers from the Asunafo South District in the Ahafo Region. This exercise enabled additional adjustments to be done including the time allocation for answering of the items on the instrument to obtain the final instrument for the pilot study.

Pilot Testing

The instrument was field tested after it was improved based on the outcome of the first test, advice from professionals to meet standard and later approved by professionals including my supervisor from the Department of Mathematics and I.C.T. Education in the University of Cape Coast. The test was administered to the selected basic school mathematics teachers in the Asutifi South District. In all 64 basic school mathematics teachers from either Colleges of Education or Distance Education programmes participated in the pilot test.

Reliability

At the end of the pilot testing exercise of the instrument on 64 Basic School mathematics teachers, the total scores obtained by each of the teachers ranged between 0 and 27 out of 35 items. Although, 120 minutes was allocated to the answering of the pilot test instrument, it took them approximately 90 minutes to complete the 35 items on the instrument. A test re-test was conducted and the reliability was calculated on the scores of 30 of the respondents and the correlation was 0.801. Again, the reliability of the pilot test was calculated using the KR-20 formula and Cronbach's alpha was found to be 0.798. These estimates agree with Nunnally and Bernstein (1994) and Vaske 's (2008) recommendation that reliability coefficients in the .65 –.80 range are 'adequate' and acceptable. The difficulty and discrimination indices were also calculated to affirm the validity of the instrument. The pilot test enabled the researcher to effect few changes including the duration for answering the 35 items on the instrument. The reliability coefficient of the final instrument was 0.808.

Data Collection Procedure

The pivot of this research was to investigate the Algebra teaching knowledge of basic school mathematics teachers who obtained their teaching certificates from Colleges of Education and Distance Education programmes. In order to bring to view the Algebra teaching knowledge of these basic school mathematics teachers, teacher-made achievement test was used to obtain information on the algebra knowledge levels, whether there is a significant difference between their knowledge levels and also if their knowledge levels increase with respect to their years of teaching experience.

Acquisition of letters from supervisor and the Department of Mathematics and I.C.T. Education to request for ethical clearance from the Institutional Review Board of the University of Cape Coast to enable the data collection was undertaken successfully. The instruments used for the study were administered to respondents in their respective basic schools. The administration of the questionnaire (Teacher-made achievement test on knowledge for teaching algebra) was done in the months of November and December 2019. The researcher first sought permission from the Headmasters of various basic schools where respondents teach. The researcher then met respondents in their respective schools and explained to them the purpose of the study, the duration involved in answering the items on the questionnaire, the measures to ensure privacy of the data collected from them and the potential benefits of partaking in the study for their consent. There were few basic school mathematics teachers who decided not to partake in the study and they were allowed to opt out. Some respondents agreed to answer the questionnaires right after the meeting, others also scheduled different times for the administration of the test.

The researcher collected data from the various basic schools in the Asunafo North Municipality and Asunafo South District with the help of three final year university students. The researcher together with the three assistants used the list of the sampled basic school mathematics teachers to visit their various schools for the administration of the test instrument. Throughout the data collection process, either the researcher himself or one of the three assistants was present with each of the respondents answering the items on the teacher made achievement test. In order not to influence and also to reduce tension on the part of the respondents, we stayed a distant away from where the respondents were answering the items. Respondents were given 90 minutes to respond to all the items on the instrument.

After the analysis of the quantitative data, there was the need to probe for explanations on some specific results of interest. This was due to the fact that, there was a significant difference between the algebra teaching knowledge of basic school mathematics teachers who obtained their teaching certificates from Distance Education Programmes and those who obtained theirs from Colleges of Education. Also, some respondents had difficulty in handling some items on the instrument which made it necessary for further questioning. Some specific basic school mathematics teachers were selected for the second phase of the study. These included those who performed very well, averagely and poorly on the test. The researcher approached these selected individuals to probe for some answers and explanations on issues regarding their performances. Some further clarifications regarding their experiences when they were receiving training as students and as in-service basic school mathematics teachers were also sought for to make things clearer.

Confidentiality was a key factor in the process of data collection. A good number of them opened up for discussions because of assured confidentiality of any data collected from them. For the purpose of privacy, the personal information of the respondents such as names, names of the schools they teach and ages were not recorded. The researcher used identification codes that only the researcher understands for each respondent who participated in the study. This ensured strict confidentiality and also helped the researcher to identify the selected respondents for the second phase of the study which did not involve all the participants of the first phase. The codes also minimized the fear of being exposed which is usually associated with responding questionnaires of research.

Data Processing and Analysis

The process of making a raw data meaningful and relevant for decision making may include ordering and shaping of data generated from the study to produce knowledge (Howard & Sharp, 1983), as well as decreasing, organizing bulky data collected, and analyzing it to produce findings.

Data Analysis in relation to First Research Question

The first research question that guided the study was "What level of Algebra knowledge for teaching do basic school mathematics teachers who obtained their teaching certificates from Colleges of Education and those from Distance Education possess?" The research question was answered using the achievement test scores of both the respondents from Colleges of Education as well as those from Distance Education programmes. The focus of this research question was to look at the level of the algebra knowledge basic school mathematics teachers from Collages of Education and Distance Education possess. The analysis with respect to this research question was descriptive statistics, using means, standard deviations, frequencies and percentages. In addition, bar charts were used to give a clearer view of the outcome of the knowledge levels of these Basic School Mathematics Teachers.

Analysis related to the First Research Hypothesis

The first research hypothesis that guided the study was, "There is no significant difference between the algebra teaching knowledge of basic school mathematics teachers who obtained their teaching certificates from Colleges of Education and those through Distance Education Programmes". Since the hypothesis sought to find out whether there is a significant difference between the algebra teaching knowledge of basic school mathematics teachers from Colleges of Education and those from Distance Education, the independent sample t-test was employed in the analysis of the data obtained from the two groups. The analysis was done based on the scores of the teachers on the teacher made achievement test on knowledge for teaching algebra. This analysis was done at the 5% level of significance. The independent samples t-test was chosen because it makes it possible to relate the mean scores on some continues variables, in this case test scores of Basic School Mathematics Teachers, for two different groups (Basic School Mathematics Teachers from Distance Education programmes and those from Colleges of Education).

Analysis related to the Second Research Hypothesis

The second research hypothesis that guided the study was, "there is no significant difference between the algebra teaching knowledge of Basic School Mathematics teachers who obtained their teaching certificates from Colleges of Education and their years of teaching Experience". This research hypothesis was analyzed using the data obtained from the teacher made achievement test on knowledge for teaching Algebra of Mathematics Teachers from Colleges of Education. Since the teachers' knowledge was compared across three year groups, the appropriate statistical tool was the Analysis of Variance (ANOVA). It is appropriate because it makes it possible to compare three or more means simultaneously. ANOVA employs the F-distribution which shows whether a difference exits among the means, however, it cannot disclose where the difference actually lies if there is any. In view of this, if the F-distribution shows that, there is a difference between the means, a follow up test such as the turkey test is required to locate where exactly the difference exists.

Analysis related to the Third Research Hypothesis

The third research hypothesis that guided the study was, "there is no significant difference between the algebra teaching knowledge of Basic School Mathematics Teachers who obtained their teaching certificates from Distance Education Programmes and their years of teaching Experience". This research hypothesis was analyzed using the data obtained from the teacher made achievement test on knowledge for teaching Algebra of Mathematics Teachers from the various Distance Education Programmes across the country. Since the teachers' knowledge was compared across three year groups, the appropriate statistical tool was the Analysis of Variance (ANOVA). It is appropriate because it makes it possible to compare three or more means simultaneously.

CHAPTER FOUR

RESULTS AND DISCUSSION

The primary focus of this study was to measure the algebra teaching knowledge of basic school mathematics teachers who obtained their teaching certificates from Distance Education Programmes and those who obtained theirs from Colleges of Education.

The study explored and also compared the algebra teaching knowledge of basic school mathematics teachers trained by the Distance Education Programmes and those trained by the Colleges of Education. It also probed to find out some possible factors that may account for the differences in their algebra teaching knowledge. Finally, the study went further to ascertain if the knowledge of algebra for teaching of the two groups of basic school mathematics teachers improve as their years of teaching experiences increase.

The study sampled 205 basic school mathematics teachers who obtained their teaching certificates from either Colleges of Education or Distance Education programmes for the first phase of the study. However, 203 of them made themselves available for the study, of which 106 and 97 were from Colleges of Education and Distance Education Programmes respectively. Also, the researcher purposively selected 33 out of the 203 basic school mathematics teachers to participate in the second phase of the study. These basic school mathematics teachers were selected for the second phase of the study because their results were of interest and could give further details and explanation to the outcome of the first phase of the study. The purposive selection was done based on their performances in the teacher-made test in knowledge of algebra for teaching.

The findings of the first and second phases of the study are discussed concurrently based on the research questions and hypotheses with respect to the re-conceptualized KAT Framework. The findings related to each research question or hypothesis are fully discussed before moving to the next research question or hypothesis. The first phase forms the basis for any claims made and the second phase throws more light and brings out some possible details to the findings.

Research Question One

The first research question that guided the study was, "What level of Algebra knowledge for teaching do basic school mathematics teachers who obtained their teaching certificates from Colleges of Education and those from Distance Education possess?"

To answer this research question, data obtained from the teacher-made achievement test on algebra knowledge for teaching was used. It considered the test scores of the two categories of basic school mathematics teachers.

Table 1 shows the distribution of scores obtained by all the basic school mathematics teachers who participated in the study on the teacher-made achievement test. The test scores were put into classes.

Table 1: Distribution of test scores and their respective frequencies and percentages

Class	Frequency	Percentage
10 – 19	0	0
20 – 29	6	2.96
30 – 39	17	8.37
40 – 49	41	20.20
50 – 59	72	35.47
60 – 69	55	27.09
70 – 79	9	4.43
80 – 89	3	1.48
90 – 99	0	0
Total	203	100

Source: Field survey (2019)

Table 1 shows the frequencies and percentages associated with each class of test scores. A cursory look at Table 1 indicates that, 64 basic school mathematics teachers representing 31.53% of a total of 203 basic school mathematics teachers performed poorly on the test, scoring below 50% mark. This is worrying, looking at the effect of teacher knowledge on students' performance. Basic school mathematics teachers are supposed to demonstrate adequate control over the contents they teach to enable them communicate effectively to the pupils they teach to enhance their performance. Since the teacher-made achievement test was purely based on what these basic school mathematics teachers teach at the basic school level, it was expected that, every single mathematics teacher who participated in the study do well on the test. Examining carefully the influence these 64 (31.53%) basic school mathematics teachers will exert on the understanding of the basic school pupils they teach, it is obvious that, a good number of basic school pupils may have difficulty in understanding some basic algebra concepts. This is very likely to affect their general mathematics performance looking at the pivotal role algebra plays in mathematics.

Analysis of results also brought to view that, 127 basic school mathematics teachers representing 62.56% performed averagely on the test. These teachers demonstrated average performance with scores ranging from 50% - 69%. Relatively, these teachers performed well as compered to those who fell below the 50% mark. However, as mathematics teachers, and taking into consideration the effect of teacher knowledge on students' performance, much was expected from them. It was expected that, they would perform excellently looking at the scope of the items on the teacher-made achievement test. Teachers who demonstrate high knowledge on what they teach are likely to communicate effectively to enhance students' performance. This is due to the fact that, it is out of the knowledge a teacher possess that he teaches.

Moreover, 12 basic school teachers representing 5.91% performed very well on the test. Their test scores ranged from 70% - 89%. These basic school mathematics teachers demonstrated relatively high knowledge in the subject matter of basic school algebra. Their performance suggests that, they are most likely to communicate effectively their high knowledge in algebra and its related contents to basic school pupils to positively affect pupils' general performance in mathematics.

Collectively, there was the need for the assessment of the general performance of all the basic school mathematics teachers who participated in the study. Descriptive statistics of the test scores of the basic school mathematics teachers who participated in the study are shown in Table 2.

Table 2: Descriptive statistics of the total score of all respondents on knowledge for teaching algebra at the basic school level.

Respondents	N	Range	Min.	Max.	Mean	Std. Error Mean	Std. Deviation
Total Scores	203	60	29	89	60.93	1.438	13.018

Source: Field survey (2019)

Table 2 indicates that, the basic school mathematics teachers who participated in the study have fairly satisfactory (average) knowledge for teaching algebra in their respective basic schools. This claim was based on their mean score of 60.93% with a standard deviation of 13.018. This indicates that, collectively, basic school mathematics teachers demonstrated relatively adequate knowledge on algebra and algebra related items. However, since their knowledge and the control they demonstrate on algebra related items affect the general performance of basic school pupils in mathematics, much was expected from them. A relatively higher mean will be an evidence that, they have exceptional knowledge and control over the content they teach at the basic school level. Even though, they had a mean score higher than 50%, taking into account their status as mathematics teachers, much more is to be desired.

From Table 2, the minimum and maximum scores recorded were 29% and 89% respectively. The range of 60% showed a very wide gap in teacher knowledge between those who performed well and those who performed poorly on the test. It explains the high standard deviation of 13.018. This is worrying because teachers who scored within the minimum class range (20% - 29%) and those who scored within the maximum class range (80% - 89%) teach the same class of basic school pupils who write the same examination at the end of their basic school education and these pupils are expected to give the same output. The wide difference between the performances of some of the basic school mathematics teachers in the test on knowledge of algebra for teaching may put some basic school pupils at a disadvantage, hence affecting their general mathematics performance looking at the fundamental role algebra plays in mathematics and the effect of teacher knowledge on students' performance.

A careful look at some specific items on the teacher-made achievement test and the responses obtained from the basic school mathematics teachers revealed some issues that demand attention. Some basic school mathematics teachers who participated in the study had difficulty handling some items on the teacher-made achievement test. The issues identified in the various responses to specific items on the teacher-made achievement test are of great concern because, basic school mathematics teachers were expected to demonstrate excellent control on these items to enable them communicate effectively the mathematics knowledge they possess to their respective pupils. For instance, item number 2 which was given as;

The expression $2a[(a + 3b) + 4(2a - b)]$ can be simplified as ………

A. $18a^2 + 14ab$

B. $18a^2 - 2ab$

C. $2a^2 + 6ab + 8a - 4b$

D. $18a^2 + 4ab$

Figure 3 shows a diagrammatic representation of how item 2 on the teacher-made achievement test was handled by basic school mathematics teachers.

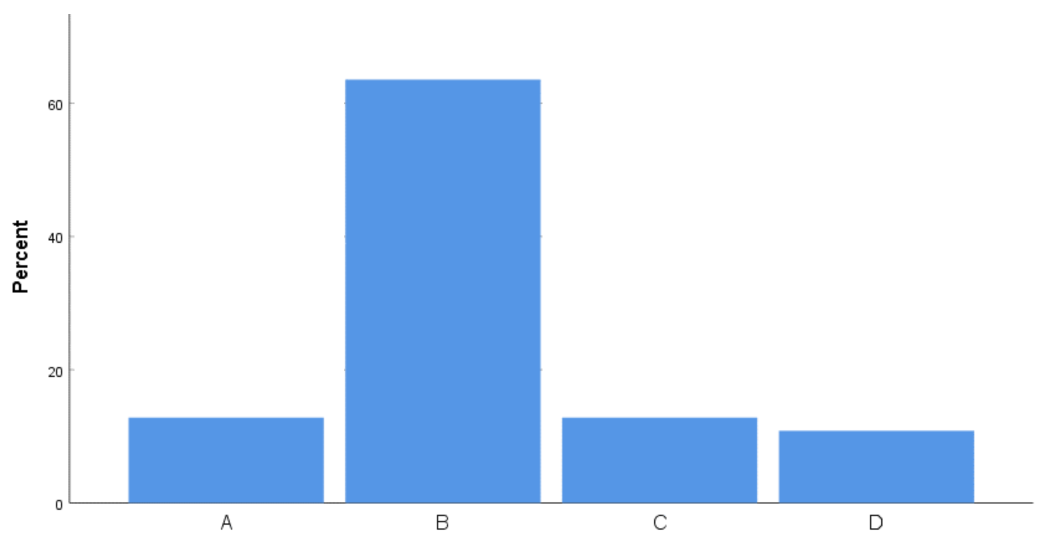


Figure 3: Responses of basic school mathematics teachers to item 2 on the teacher-made achievement test

Figure 3 gives a pictorial view of the percentages of basic school mathematics teachers who chose option A, B, C or D as their response to item 2 on the teacher-made achievement test. According to item 2, it can be observed that, majority (63.5%) of the basic school mathematics teachers got the answer correct by choosing option B as their response to the item. It is however disturbing that, 36.5% of the basic school mathematics teachers had this item wrong. A careful look at the percentages associated with options (A, B, C, and D) reveal the following about basic school mathematics teachers. The 12.8% of basic school mathematics teachers who selected option A as their response to item 2, had challenges in multiplying 4 by (-b). They presented 4b instead of (-4b) as the product of 4 and (-b). This led them to obtain option A. $18a^2 + 14ab$ as their final answer to item 2. Again, 26 basic school mathematics teachers representing 12.8% also chose option C as their response to item 2 on the teacher-made achievement test. These basic school mathematics teachers did not multiply 2a by 4 (2a - b). They only multiplied the 2a by (a + 3b) and neglected 4 (2a – b). This was what led them to arrive at option C as their response to item 2. The last group of basic school teachers who chose option D as their response to item 2 also had challenges in expanding the bracket 4 (2a - b). They also did not multiply the 4 by (- b), hence ended up getting option D. $18a^2 + 4ab$ as their answer.

The above issues are of great concern looking at the effect of teacher knowledge on pupils' performance. Basic school mathematics teachers are supposed to demonstrate adequate control over the contents they teach to enable them to communicate effectively the mathematical contents and concepts required to basic school pupils to positively enhance their performance. Research is replete with the fact that the teacher is the most important factor that influences students' achievement (see for instance, Begle, 1972; Hanushek, 1972;Eisenberg, 1977; Harbison & Hanushek, 1992; Shulman & Quinlan, 1996; Mullens, Murnane & Willett, 1996; Rowan, Chiang & Miller, 1997; Wilmot, 2009; Yara, 2009) In view of this, basic school teachers who have difficulty in some contents they teach may contribute in building weak algebra foundation for basic school pupils.

Again, another item that attracted attention was item number 29 which was given as;

"What are the values of ***c***, if ***c + d = 10*** and ***c*** is less than ***d***?" (Note: *d ≤ 10 and are integers*) also brought out some issues of concern

Item 29 was one of the items most basic school mathematics teachers were unable to handle properly. Majority of them were not able to demonstrate control over this particular item. Figure 4 shows the statistics on how item 29 was handled by the basic school mathematics teachers.

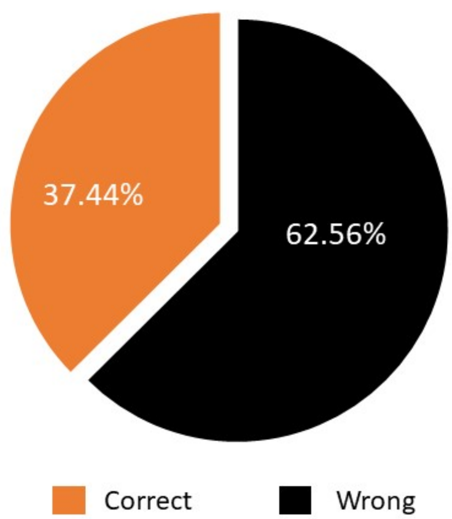


Figure 4: Statistics of basic school mathematics teachers' response on item 29

Figure 4 indicates that, only 37.44% of the 203 basic school mathematics teachers were able to get item 29 correct, whereas 62.56% of them had it wrong. This indicates that, a great number of them did not demonstrate adequate control over this particular question. Based on how poorly most participants handled item

29, a probe into what might have accounted for such performance was done during the second phase of the study.

In the second phase of the study, 17 out of 33 respondents said that, they are not used to questions of this nature and that if they knew a formula or a routine way of solving it, it would not have been challenging as observed. For instance, one respondent made this statement, “is there a formula for finding these values? I think that will make it easier” The above situation suggests that, quite a good number of the basic school mathematics teachers prefer routine mathematics questions and that may be the nature of questions they also give to their pupils. This implies that, they do not give to the basic school pupils questions that deviate from the norm to elicit deeper understanding of algebra contents and other contents in basic school mathematics in their respective classrooms. This perhaps may be affecting their control over similar items. It is very necessary for basic school teachers to also employ non-routine questions to push the pupils to think for deeper understanding of algebra and other mathematics contents.

Even though, teachers on the collective level showed evidence of relatively satisfactory knowledge in algebra for teaching, teachers’ performance on some of the items leave much to be desired. For example, item 24. The item was given as: Find the value of x in the equation $4^{2x-1} = \frac{1}{16}$.

A. $-\frac{1}{2}$

B. $\frac{5}{8}$

C. $\frac{1}{2}$

D. $3\frac{1}{2}$

Table 3 shows the statistics on respondents' choice of options for item 24.

Table 3: Responses of Basic School Mathematics Teachers to question 24

Option	Frequency	Percent
A	127	62.56
B	28	13.79
C	29	14.29
D	19	9.36
Total	203	100.0

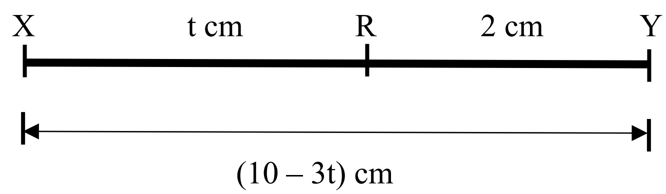
Source: Field survey (2019)

In Table 3, the four multiple choice options A, B, C and D are associated with their respective percentage of teachers who chose that option as their responds to question 24. The purpose of the item was to assess teachers' *school knowledge* in relation to algebra. The school knowledge is the knowledge of algebra in the intended curriculum of which teachers are expected to help students discover in their mathematics classrooms. The correct option to item 24 is "A". Although most of the teachers (62.56%) had this item correct, the number of teachers who got this particular item wrong (37.44%) is quite disturbing taking into consideration the effect of teacher knowledge on students' performance.

Since the item focused on what these basic school mathematics teachers teach the basic school pupils, it was expected that every basic school mathematics teacher who participated in the study do well on this item. It is expected that, teachers themselves understand the content of school algebra so that they can communicate the content knowledge to the basic school pupils to positively affect their general mathematics performance.

Another item of interest on the teacher-made achievement test was question 30 and was structured as follows:

Write down an expression for the length *XY* and proceed to find the value of *t*.



A diagrammatic representation of how item 30 was answered by the basic school mathematics teachers is shown in Figure 5

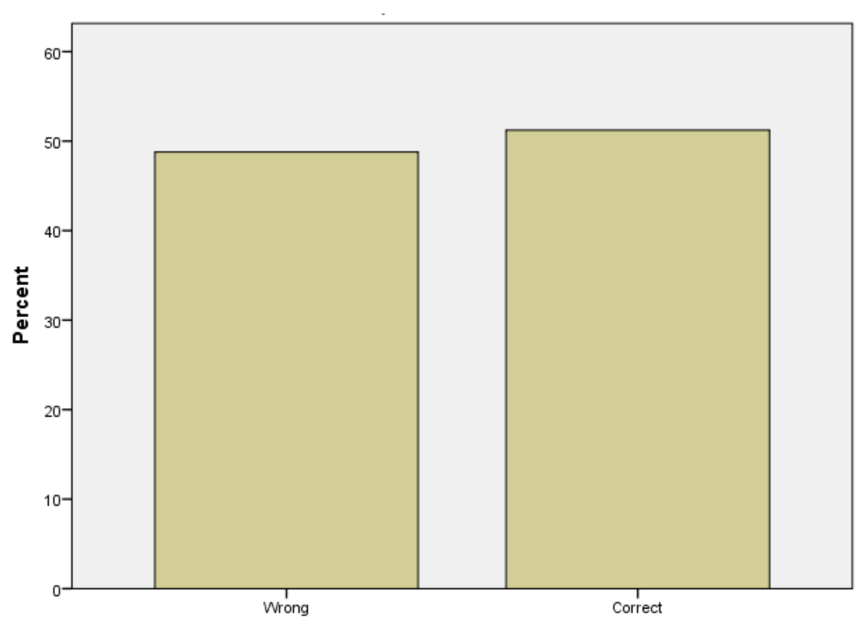


Figure 5: Response of Basic School Mathematics Teachers to question 30

The purpose of question 30 was to assess teachers' ability to handle other mathematical knowledge and also apply the knowledge acquired in algebra in other scopes of basic school mathematics. The item looked at how the basic school mathematics teachers would build on the concept of length and also apply the algebra knowledge they have acquired to solve questions that are outside algebra. The item also looked at the demonstration of control over contents beyond school knowledge. It was quite disturbing seeing almost half of the respondents get this item wrong. According to Figure 5, 104 (51.23%) of the basic school mathematics teachers had the item correct whiles 99 (48.77%) of them had it wrong. The above outcome shows that, a great number of basic school teachers have some challenges applying algebra in other scope of basic school mathematics and this is more likely to affect the pupils they teach. This is due to the fact that, teachers teach out of the knowledge they possess.

Moreover, item 22 on the teacher-made achievement test was to probe how the basic school teachers will alley the fears of pupils on how a particular question is supposed to be solved. It was also to check how basic school mathematics teachers can identify some of the mistakes basic school pupils usually make when handling algebra related items. Item 22 was posed as follows:

Gertrude's solution to an equation is shown below

Given: $n + 8(n + 20) = 200$

Step 1: $n + 8n + 20 = 200$

Step 2: $9n + 20 = 200$

Step 3: $9n = 200 - 20$

Step 4: $9n = 180$

Step 5: $\frac{9n}{9} = \frac{180}{9}$

Step 6: $n = 20$

Which of the statements about Gertrude's solution is true?

A. Gertrude's solution is correct

B. Gertrude's mistake started from step 1

C. Gertrude's mistake started from step 2

D. Gertrude's mistake started from step 3

The responses of the basic school mathematics teachers for item 22 are organized in Figure 6

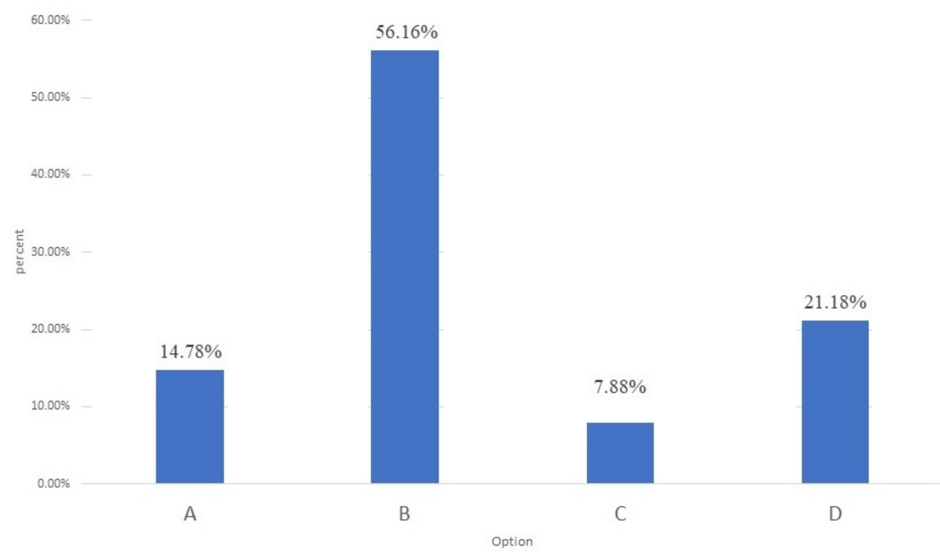


Figure 6: Responses of basic school mathematics teachers to question 22

The basic school mathematics teachers were to carefully assess the procedure used by Gertrude, a pupil of basic school and indicate where they think she made a mistake or indicate if she is correct. As many as 89 (43.84%) basic school mathematics teachers could not identify where the mistake was. Only 114 (56.16%) of them chose the correct answer to the question which is option "B".

It is worrying to have 43.84% of the basic school mathematics teachers get item 22 wrong. They were unable to identify where the basic school pupil made a mistake in addressing the problem. Mathematics teachers are supposed to demonstrate adequate control over the subject matter they teach and also identify where students make mistakes and correct them.

Finally, item 35 which tested the *Pedagogical Content Knowledge Algebra* of basic school mathematics teachers was also of interest. It was to probe the procedural and conceptual understanding of basic school mathematics teachers on quadratic equations and to bring to view some of the common mistakes basic school pupils make when handling such items. This was how the item was structured:

Item 35: Maud, Priscilla and Gertrude were asked by their mathematics teacher to solve for $\boldsymbol{x}$ in the equation $\boldsymbol{2x^2 = 6x}$ **.**

Their respective solutions are shown as follows. Carefully examine their responses by marking and writing your observations. State clearly if any of the solutions are correct.

Maud's Solution

$2x^2 = 6x$

=» $x^2 = 3x$ Step 1 ------ She divided both sides by 2

=» $x = 3$ Step 2 ------ She divided both sides by x

Therefore, the value of $\boldsymbol{x}$ is 3

Comment on the correctness of Maud's solution

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Priscilla's Solution

$2x^2 = 6x$

$2x = 6$ Step 1 ------ She divided both sides by x

$x = 3$ Step 2 ------ She divided both sides by 2

Therefore, the value of *x* is 3

Comment on the correctness of Priscilla's solution

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Gertrude's Solution

$2x^2 = 6x$

=» $x^2 = 3x$ Step 1 ------ She divided both sides by 2

=» $x^2 - 3x = 0$ Step 2 ------ She grouped like terms

=» $x(x - 3) = 0$ Step 3 ------ She factorized x out

=» $x = 0$

and

=» $x - 3 = 0$ =» $x = 3$

Therefore, the values of *x* are 0 and 3

Comment on the correctness of Gertrude's solution

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The respondents were supposed to examine the responses of the three basic school pupils, identify where there are mistakes and comment on the correctness of each of the solutions presented. Participants' responses to item 35 are shown in Table 4

Table 4: Responses of Basic School Mathematics Teachers to question 35

	Frequency	Percent
Correct	131	64.5
Wrong	72	35.5
Total	203	100

Source: Field survey (2019)

Table 4 shows that, 131 basic school mathematics teachers representing 64.5% got the item correct, while 72 of them representing 35.5% got it wrong. It was quite encouraging to see majority (64.5%) of the basic school mathematics teachers do well on this particular item. However, the 35.5% who had the item wrong is too high looking at the effect of teacher knowledge on students' performance and also the pivotal role algebra plays in mathematics. Those who performed well on this item showed evidence of both conceptual and procedural understanding of the item in question. However, it is still discouraging to notice that, 36.6% could not do well on this item, because they are expected to have control over similar items to enable them communicate the knowledge to the basic school pupils in their mathematics classrooms to positively affect students' performance.

Generally, majority of the basic school mathematics teachers from Colleges of Education and those from Distance Education programmes showed evidence of fairly satisfactory (mean = 60.93%) algebra knowledge for teaching at the various basic school levels they teach. A good number of basic school mathematics teachers (68.49%) performed well on the test. What was a bit worrying was with respect to the 31.53% of the mathematics teachers who performed poorly on the teacher made achievement test on algebra knowledge for teaching at the basic school level. Although majority of basic school mathematics teachers passed the test on algebra knowledge for teaching, much more was expected, looking at the effect of teachers' knowledge on pupils' performance.

Teachers who performed poorly may have difficulty explaining some conceptual and procedural issues relating to algebra to basic school pupils in their mathematics classrooms (Thompson & Thompson, 1996) and (Mewborn, 2003). This will go a long way to affect students' algebra understanding and hence students' general mathematical performance.

Research Hypothesis One

The first research hypothesis that guided the study was, "There is no significant difference between the algebra teaching knowledge of basic school mathematic teachers who obtained their teaching certificates from Colleges of

Education and those who obtained theirs through Distance Education programmes."

To answer this research hypothesis, the achievement test scores of both basic school teachers from Distance Education programmes and those from Colleges of Education were used. The independent sample t-test was employed in the analysis of the data obtained from the two groups of basic school mathematics teachers in the teacher-made achievement test on knowledge of algebra for teaching. The analysis was done at 5% level of significance.

The descriptive statistics on the test scores of the two categories of basic school mathematics teachers are displayed in Table 5.

Table 5: Descriptive statistics of test scores of basic school mathematics teachers based on programmes

Programme	N	Range	Min.	Max.	Mean	Std. Deviation
Distance Education Teachers	97	42	29	71	54.74	10.244
Colleges of Education Teachers	106	52	37	89	64.67	13.152

Source: Field survey (2019)

The results in Table 5 show that, out of the 203 basic school mathematics teachers who participated in the study, 97 and 106 obtained their teaching certificates through Distance Education Programmes and from Colleges of Education respectively. The table shows that, both categories of basic school mathematics teachers have average algebra knowledge for teaching at the basic school level, however, that of those who obtained their teaching certificates from Colleges of Education is relatively higher. The mean score of the basic school mathematics teachers from Colleges of Education was 64.67% with standard deviation of 13.152 whiles those from Distance Education was 64.67% with standard deviation of 10.244. Even though there is difference in the mean scores of the two groups of basic school teachers, the independent samples t-test will show if the difference is statistically significant. Basic school mathematics teachers from Colleges of Education had scores ranging from 37% - 89% and those from Distance Education had scores ranging from 29% - 71%. The above figures relating to the range of the test scores bring to light that, the difference in algebra knowledge between teachers who obtained their teaching certificates from Collages of Education is relatively wider than those who obtained their teaching certificates from Distance Education.

In order to find out whether there is a statistically significant difference between the performances of the two categories of basic school mathematics teachers, an independent samples t-test was conducted. The summary statistics are shown in Table 6

Table 6: Results of Independent Samples t-test on test scores of basic school mathematics teachers from Distance Education and those from Colleges of Education

	Levene's Test for Equality of Variances		t-test for Equality of Means		
	F	Sig.	T	Df	Sig. (2 tailed)
Equal variances assumed	3.484	.066	-3.591	201	.001

Table 6 contains the results of the independent samples t-test conducted to compare the mean scores of basic school mathematics teachers who obtained their teaching certificates through Distance Education Programmes and those who had theirs from Colleges of Education. The test was conducted to check if there is a significant difference between their algebra teaching knowledge. The results revealed that, there is a statistically significant difference between the algebra teaching knowledge of the two categories of basic school mathematics teachers.

Basic school mathematics teachers who obtained their teaching certificates through Distance Education programmes had Mean =54.74 and Standard Deviation = 10.244 and their counterparts from the Colleges of Education had Mean = 64.68 and Standard Devaition = 13.170. The P-Value was .001 and since it is less than $\alpha = 0.05$, we reject the null hypothesis and conclude that, there is a statistically significant difference between the algebra teaching knowledge of basic school mathematics teachers who obtained their teaching certificates from Colleges of Education and those from Distance Education programmes. The statistically significant difference is in favour of basic school mathematics teachers who obtained their teaching certificates from Colleges of Education.

The statistically significant difference observed in the mean scores of the two categories of teachers suggests that, basic school mathematics teachers who obtained their teaching certificates from Colleges of Education have relatively higher algebra knowledge for teaching as compared to those who obtained theirs through Distance Education Programmes. This suggests that, basic school mathematics teachers who obtained their teaching certificates from Colleges of Education demonstrated higher level of algebra content and pedagogical knowledge to effectively enhance their teaching as compared to their counterparts from the Distance Education Programmes.

A probe into the difference that exists between the algebra teaching knowledge of the two categories of basic school mathematics teachers was done during the second phase of the study. It revealed some issues that may have contributed to the respective performances associated with the two groups of basic school mathematics teachers. The researcher collected further information on the algebra courses these basic school mathematics teachers took when they were undergoing their respective training to become mathematics teachers.

The information obtained about the modules that were used for teaching the two categories of basic school mathematics teachers revealed that, those who obtained their teaching certificates through Distance Education took more algebra courses than those from Colleges of Education. The algebra related courses that were taken by those from Distance Education were Algebra and Trigonometry, Abstract Algebra and Vector Algebra whiles those who obtained their teaching certificates from Colleges of Education took two algebra related courses; namely, Numbers and Basic Algebra and Further Algebra.

A careful look at the algebra courses that were taken by the two groups of basic mathematics teachers and their contents bring to view the differences that exist between what they learned during their training as prospective basic school mathematics teachers. Some of the algebra courses that were taken by the basic school mathematics teachers from Distance Education have little connection with what they are expected to teach at the basic school level as compared to that of the Collages of Education. For instance, Abstract Algebra and Vector Algebra may build on what a teacher should know (content courses to build on personal competence over his or her students) instead of what a teacher should or is expected to teach (enshrined in the curricula of basic schools). Again, some basic school mathematics teachers who obtained their teaching certificates through Distance Education revealed that, more attention was given to the trigonometry aspect of the course, Algebra and Trigonometry, instead of the algebra content. These issues as revealed in the second phase may partially contribute to the relatively low performance of basic school mathematics teachers from Distance Education programmes.

The results of the independent samples t-test revealed a statistically significant difference between the algebra teaching knowledge of basic school mathematics teachers who obtained their teaching certificates from Colleges of Education and those from Distance Education Programmes in favour of those from

Colleges of Education. This implies that, basic school mathematics teacher who obtained their teaching certificates from Colleges of Education actually demonstrated higher algebra teaching knowledge than those who obtained their teaching certificates through Distance Education Programmes.

Research Hypothesis Two

The second research hypothesis that guided the study was, "There is no significant difference between the algebra teaching knowledge of basic school mathematics teachers who obtained their teaching certificates from Colleges of Education and their years of teaching experience."

This research hypothesis was answered using the scores obtained from the teacher-made achievement test on knowledge of algebra for teaching and the information obtained about the years of teaching experiences of basic school mathematics teachers who obtained their teaching certificates from Colleges of Education. Teaching experience was categorized into three main groups, namely 5 years and below, 6 – 10 years and above 10 years. Since the teachers' knowledge was compared across three teaching experience year groups, the appropriate statistical tool was the Analysis of Variance (ANOVA). The hypothesis was tested at 0.05 level of significance.

Table 7 shows the distribution of test scores of basic school mathematics teachers who obtained their teaching from Colleges of Education based on their years of teaching experience.

Table 7: Descriptive Statistics on test scores of teachers who obtained their teaching certificates from Colleges of Education based on teaching experience

Teaching Experience	N	Mean	Std. Deviation	Min.	Max.
5 years and below	35	60.97	12.077	37	86
6 - 10 years	38	66.44	14.427	40	89
Above 10 years	33	70.06	11.846	51	89
Total	106				

Source: Field survey (2019)

A cursory look at Table 7 reveals that, all teachers in the three categories of teaching experience had mean scores above 60%. This indicates that, they demonstrated relatively substantial algebra teaching knowledge needed for teaching mathematics at the basic school level. The high standard deviations recorded for the various categories of basic school mathematics teachers ranging from 11.846 for those who have taught for more than 10 years to 14.427 for 6 – 10 years indicate wide spread of scores for all categories as seen in the minimum and maximum test scores.

The face values of the various mean scores of the respective groups show an increasing performance as years of teaching experiences increase. Figure 7 shows a mean plot of the three categories of basic school mathematics teachers based on years of teaching experience.

Figure 7: Mean plot of the categories of basic school mathematics teachers who obtained their teaching certificates from Colleges of Education

Figure 7 shows an increase in mean score as years of teaching experience increases. It also suggests that, there is a difference in the mean scores of the three categories of mathematics teachers. However, there is the need to ascertain if the difference in means are significant. The Analysis of Variance gives a clearer picture if the differences in the mean scores are significant.

Test of homogeneity is a necessity to enable the ANOVA test. This was to check if the variability of scores for each category is similar. This assumption was tested using the Levene test for equality of variance. The test result of the Levene test for equality of variances is shown in Table 8.

Table 8: Test of Homogeneity of variance in scores of basic school mathematics teachers who obtained their teaching certificates from Colleges of Education

Levene Statistic	df1	df2	Sig.
1.044	2	103	.360

Table 8 shows the output of the test of homogeneity of variance. The significant value of 0.360 is greater than α = 0.05. This indicates that the test is not significant, which also implies that, the variances of the scores of the three categories of basic school mathematics teachers (based on years of teaching experience) who obtained their teaching certificates from Colleges of Education are approximately the same. In view of this, the assumption of equality of variances was satisfied hence warranted the ANOVA test.

The output of the ANOVA test on the scores obtained by basic school mathematics teachers who obtained their teaching certificates from Colleges of Education taking into consideration their years of teaching experiences is displayed in Table 9.

Table 9: ANOVA test results for the three categories of graduates of Colleges of Education based on years teaching experience

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	.066	2	.033	1.941	.149
Within Groups	1.751	103	.017		
Total	1.817	105			

Source: Field survey (2019)

Table 9 shows that the sig. value =.149 is greater than α = 0.05, which implies that, there is no significant difference across the three categories (based on years of teaching experience) of basic school mathematics teachers who obtained their teaching certificates from Colleges of Education. The implication of the result is that, the algebra teaching knowledge of basic school mathematics teachers who have spent less than 5 years on the field is not different from those who have spent 6 -10 years and above 10 years on the field. It also suggests that, the algebra teaching knowledge of basic school mathematics teachers who obtained their teaching certificates from Colleges of Education do not increase significantly as their years of teaching experience increases.

The finding is in contradiction to the claims made that, one of the indicative variables for teacher competence is teachers' years of teaching experience (Darling-Harmmond, 2000). Again, it is also in conflict with what research has established that, teachers with more years of teaching experience are more effective than inexperienced teachers; especially those with less than three years of teaching experience (Klecker, 2002; Rosenholtz, 1986). In Klecker (2002), it was revealed that, students of teachers with higher teaching years of experience scored higher marks or performed well as compared to students with teachers of low teaching experiences. This is possible if the mathematics teachers with more years of teaching experience are demonstrating high knowledge in algebra than those with few years of teaching experience, which is not the case in the context of this study.

The second phase of the study brought to view that, out of the 18 basic school mathematics teachers who were sampled from those who obtained their teaching certificates from Colleges of Education, only four have attended any form of in-service training that is related to the mathematics they teach at the basic level. This also may have contributed to the outcome of having the same algebra knowledge for teaching across the three groups based on years of teaching experience. This is because, in-service training is a key factor to help improve on the knowledge they have as basic school mathematics teachers as they spend more years on the field.

Although the result of the study conflicts the assertion that, teachers' knowledge improves as their years of teaching experience increases, there is a possibility that, basic school mathematics teachers who obtained their teaching certificates from Colleges of Education do improve on their algebra teaching knowledge whiles on the field, but the new crop of basic school mathematics teachers who have spent less than 5 years on the field came out as basic school mathematics teachers having relatively high algebra knowledge which matches the algebra teaching knowledge of those with higher years of teaching experience.

The ANOVA test revealed that there is no statistically significant difference across the three classes of basic school mathematics teachers who obtained their teaching certificates from Colleges of Education. In order words, the test brought to view that, basic school mathematics teachers who obtained their teaching certificates from Colleges of Education and have spent 6 – 10 years and those above 10 years demonstrate the same algebra knowledge as those who have not spent so much time (5 years and below) on the field as in-service mathematics teachers. This may further suggest that, these basic school mathematics teachers do not improve on their algebra teaching knowledge on the field as in-service mathematics teachers as their years of teaching experience increases.

Research Hypothesis Three

The third research hypothesis was, "There is no significant difference between the algebra teaching knowledge of basic school mathematics teachers who obtained their teaching certificates through Distance Education Programmes and their years of teaching Experience".

This research hypothesis was answered using the scores obtained from the teacher-made achievement test on knowledge of algebra for teaching and the information obtained about their years of teaching experience. The statistical tool used was the Analysis of Variance (ANOVA). Teaching experience was categorized into three main groups, namely 5 years and below, 6 – 10 years and above 10 years. The hypothesis was tested at 0.05 level of significance.

Table 10 shows the summary statistics of scores of basic school mathematics teachers who obtained their teaching certificates from Colleges of Education based on their years of teaching experience.

Table 10: Descriptive statistics on test scores of graduates from Distance Education programmes based on years of teaching experience

Teaching Experience	N	Mean	Std. Deviation	Min.	Max.
5 years and below	31	53.50	11.360	29	69
6 - 10 years	36	55.40	9.905	37	71
Above 10 years	30	58.50	6.364	54	63
Total	97				

Source: Field survey (2019)

Table 10 reveals that, all mathematics teachers in the three teaching experience categories had mean scores ranging from 53.50% to 58.50%. This indicates that, collectively they demonstrated fairly average algebra teaching knowledge needed for the teaching of mathematics at the basic school level. The standard deviations of each category of teachers are 11.360 for 5years and below, 9.905 for 6-10 years and 6.364 for above 10 years of teaching experience. The minimum score of 29% was recorded for the 5years and below category while the and maximum score of 71% was recorded for the 6-10 years category.

The mean scores suggest that, their performances improve as their years of teaching experience increases. Figure 8 shows a mean plot of the three categories of basic school mathematics teachers based on years of teaching experience.

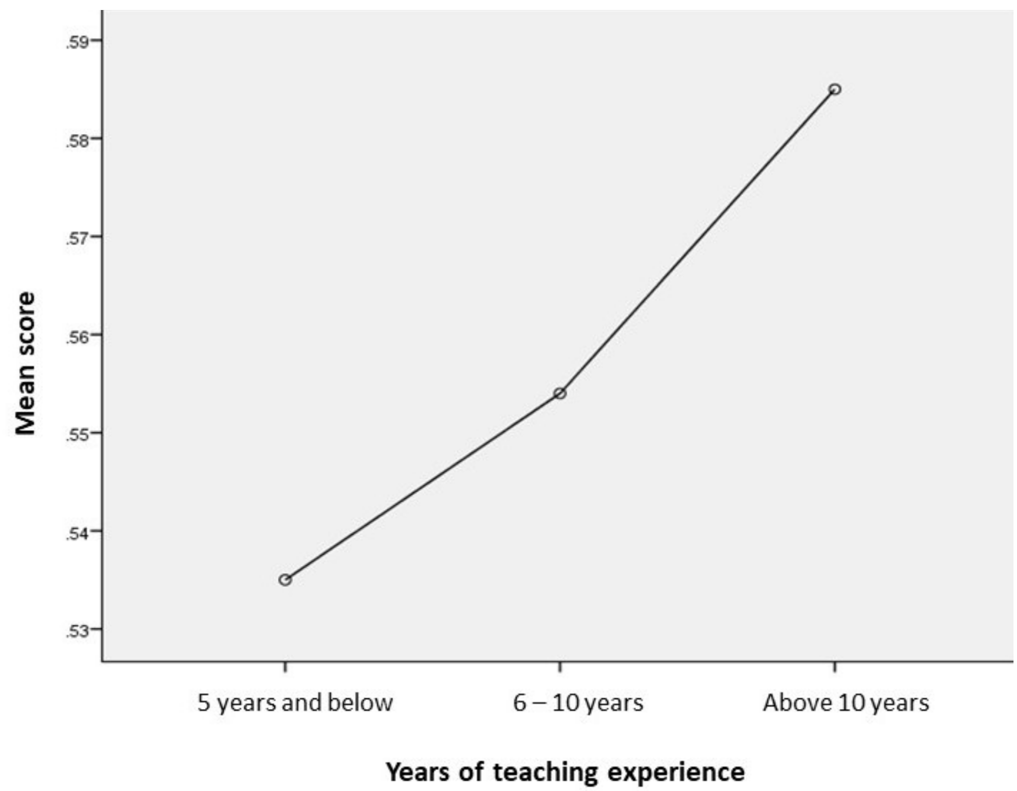

Figure 8: Mean plot of the categories of basic school mathematics teachers who obtained their teaching certificates from Distance Education programmes

Figure 8 shows an increase in mean score as years of teaching experience increase. It also shows that, there is a difference in the mean scores of the three categories of basic school mathematics teachers. The Analysis of Variance brings to view if the differences in means are significant or not.

In order to conduct an ANOVA test, test of homogeneity was necessary to check if the variability of scores for each category is similar. The result of the Levene test for equality variances is indicated in Table 11

Table 11: Test of Homogeneity of Variance of scores of basic school teachers from Distance Education programmes

Levene Statistic	df1	df2	Sig.
.573	2	94	.570

Table 11 shows that, the significant value of .570 is greater than $\alpha = 0.05$. This implies that the test is not significant, which also implies that, the variances of the scores of the three categories of basic school teachers who obtained their teaching certificates from Distance Education programmes are approximately the same or similar. In view of this, the assumption of equality of variances was satisfied hence permitted the ANOVA test.

The output of the Analysis of Variance (ANOVA) test on the scores obtained by basic school mathematics teachers who obtained their teaching certificates from Distance Education Programmes taking into consideration their years of teaching experiences is presented in Table 12.

Table 12: ANOVA test results for the three categories of graduates of Distance Education Programmes based on years of teaching experience

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	.006	2	.003	.273	.777
Within Groups	1.034	94	.011		
Total	1.04	96			

Source: Field survey (2019)

Table 12 shows that, there is no significant difference across the three categories of basic school mathematics teachers trained by the Distance Education Programmes. The sig. value of .777 is greater than $\alpha = 0.05$, which implies that there is no significant difference across the mean scores obtained by basic school mathematics teachers in each category of teaching years of experience. The implication is that, basic school mathematics teachers who obtained their teaching certificates through Distance Education and have spent 6 -10 years and above 10 years on the field as in-service mathematics teachers are not different from those who have spent 5 years and below in terms of their algebra knowledge for teaching. Even though their mean scores differ, the differences are not significant.

This outcome is not in line with the results of Darling-Harmmond, 2000, Klecker, 2002 and Rosenholtz, 1986 which makes it clear that, one of the indicative variables for teacher competence is teachers' years of teaching experience, and that, teachers learn and develop through their teaching experiences. Again, it is also in disagreement with what research has established that, teachers with more years of teaching experience are more effective and have control over the contents they teach than inexperienced teachers; especially those with less than three years of teaching experience (Klecker, 2002; Rosenholtz, 1986). Based on the outcome of Klecker (2002), students of teachers with higher teaching years of experience scored higher marks or performed well as compared to students with teachers of low teaching experiences. This implies that, teachers of the students who performed well demonstrated adequate control over the contents assessed than the teachers of those who performed poorly, which is not the case in the context of this study.

The second phase of the study brought to view that, only 2 out of 15 of the basic school mathematics teachers who obtained their teaching certificates through Distance Education Programmes have attended any form of in-service training that is related to the mathematics they teach at the basic level. This is quite worrying looking at the role of in-service training in the improvement of teacher knowledge. This also may have contributed to the outcome of having the same algebra knowledge for teaching across the three groups based on years of teaching experience.

Although the result of the study conflicts the claim that, teachers' knowledge improves as their years of teaching experience increases. There is a possibility that, basic school mathematics teachers who obtained their teaching certificates through Distance Education Programme do improve on their algebra teaching knowledge whiles on the field, but the new crop of basic school mathematics teachers who have spent less than 5 years on the field came out as basic school mathematics teachers having relatively high algebra knowledge which matches the algebra teaching knowledge of those with higher years of teaching experience.

The ANOVA test revealed that there is no statistically significant difference across the three classes of basic school mathematics teachers (based on years of teaching experience) who obtained their teaching certificates from Distance Education programmes. The implication of this result is that, whether a basic school mathematics teacher trained by Distance Education has spent over 10 years or from 6 - 10years as in-service basic school mathematics teacher, his or her algebra teaching knowledge is not different from those who have spent below 5 years as in-service basic school mathematics teachers.

In order words, the result implies that basic school mathematics teachers who obtained their teaching certificates from Distance Education programmes do not improve on their algebra teaching knowledge on the field as their years of teaching experience increase.

Second Phase of the Study in relation to Research Hypothesis Two and Research Hypothesis Three

Based on the first phase of the study, the findings in relation to Research Hypothesis two and Research Hypothesis three showed that, both categories of basic school mathematics teachers do not improve on their algebra knowledge for teaching as their years of teaching experience increases. There was no statistically significant difference across their test scores based on the three categories of teaching years of experience. The second phase of the study probed some possible causes of this result.

Firstly, it was revealed that, 54.54% of the 33 basic school mathematics teachers who were sampled for the second phase of the study learn or revise what they teach the basic school pupils, (25.16%) also indicated that they sometimes do before lessons. Others (20.30%) also made it clear that, they do not learn or revise what they teach at the basic school level. Those who do not make time to learn or revise, had the feeling that, what they teach at the basic school level are not so challenging that they would need to make time for its study. This implies that, 45.46% of the basic school mathematics teachers do not make time to learn on the job to improve significantly as their years of teaching experience increase.

Secondly, majority (27 representing 81.82%) of the 33 basic school mathematics teachers who participated in the second phase of the study said that, they have not attended any in-service training that is in relation to what they teach. This implies that, algebra contents are no exception with respect to the issue of in-service training. This may have contributed to the results of having the same algebra teaching knowledge possessed by those with high and low years of teaching experience.

Finally, the researcher inquired if the basic school mathematics teachers in the study area assist each other in knowledge acquisition, especially in content areas where a teacher may have difficulty. It was also revealed that, the basic school mathematics teachers in the study area work in isolation. They do not work in teams to enable the weak ones get assistance to improve as in-service basic school mathematics teachers.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Summary

This study was premised on the fact that, basic school mathematics teachers' algebra teaching knowledge affects the algebra knowledge of basic school pupils, hence affecting their general mathematics performance. In view of this, the algebra teaching knowledge levels of basic school mathematics teachers should be monitored and improved to positively affect pupils' algebra knowledge hence their general mathematics performances.

The study focused on two main categories of basic school mathematics teachers, these were: basic school mathematics teachers who obtained their teaching certificates from Distance Education Programmes and those who obtained theirs from Colleges of Education. The re-conceptualized KAT framework was used to bring to light the algebra teaching knowledge levels of basic school mathematics teachers from the aforementioned channels and also checked whether they have the algebra teaching knowledge it takes to effectively teach mathematics at the basic school level to positively affect pupils' general performance in mathematics. It also investigated if there is a significant difference between the algebra teaching knowledge of the two categories of basic school mathematics teachers. The study finally checked if the algebra teaching knowledge of these basic school mathematics teachers improve as their years of teaching experience increase.

The study employed an explanatory sequential mixed method research design. In the first phase of the study, 203 basic school mathematics teachers participated. This number was made up of 106 basic school mathematics teachers who obtained their teaching certificates from Colleges of Education and 97 of those who obtained theirs through Distance Education Programmes. During the second phase of the study, which was meant to probe the outcome of the first phase, 33 of the sampled basic school mathematics teachers were used.

The study revealed that, basic school mathematics teachers from the two groups have fairly satisfactory algebra knowledge for teaching mathematics which needs to be improved because of the effect of teacher knowledge on students' performance. It also revealed that, basic school mathematics teachers who obtained their teaching certificates from Colleges of Education had a relatively higher algebra knowledge for teaching than those who had theirs through Distance Education Programmes. The study also found out that, the algebra teaching knowledge of basic school mathematics teachers from both channels do not improve as their years of teaching experience increase.

Key findings

The findings of the study are as follows:

Basic school mathematics teachers who obtained their teaching certificates from Colleges of Education and those who had theirs through Distance Education programmes have fairly satisfactory knowledge of algebra for teaching.

There is a statistically significant difference between the algebra teaching knowledge of basic school mathematics teachers who obtained their teaching certificates from Colleges of Education and those who had theirs through Distance

Education. The difference is in favour of those who graduated from Colleges of Education.

There is no statistically significant difference in the algebra knowledge for teaching across the three categories (based on years of teaching experience) of basic school mathematics teachers who obtained their teaching certificates from Colleges of Education.

There is no statistically significant difference in algebra knowledge for teaching across the three groups (based on years of teaching experience) of basic school mathematics teachers who obtained their teaching certificates through Distance Education programmes.

Conclusions

The study sought to explore the knowledge of algebra for teaching possessed by basic school mathematics teachers who obtained their teaching certificates from Colleges of Education and those who had theirs through Distance Education programmes. The findings of the study may have some implications for planning concerning improving the algebra teaching knowledge of basic school mathematics teachers. Below are the conclusions that came out of the study.

Basic school mathematics teachers who obtained their teaching certificates from Colleges of Education and those who had theirs through Distance Education programmes have fairly satisfactory algebra knowledge for teaching mathematics and needs to be improved looking at the effect of teacher knowledge on students' performance. Basic school mathematics teachers are expected to demonstrate excellent control over the algebra contents they teach to enable them communicate effectively the knowledge they possess to pupils to influence positively their performance in mathematics.

Basic school mathematics teachers who obtained their teaching certificates from Collages of Education have relatively higher algebra teaching knowledge as compared with their counterparts who obtained their teaching certificates through Distance Education programmes.

The algebra teaching knowledge of basic school mathematics teachers who obtained their teaching certificates from Colleges of Education and their counterparts who obtained theirs through Distance Education programmes do not improve significantly as their years of teaching experiences increase.

Recommendations

Based on the findings of the study, the following recommendations have been made for educational policy and practice in the knowledge of algebra for teaching at the basic school level.

Educational institutions responsible for the training of basic school mathematics teachers should develop and mount more courses that focus on the basics of algebra and also relate more with what basic school mathematics teachers teach at the basic school level. The content of such courses may include; handling of variables, expanding brackets, concept of equality, fractions, exponential equations and expressions etc. This will help the basic school mathematics teachers to get in-depth understanding and will demonstrate adequate control over the algebra contents they teach on the field.

Educational institutions responsible for the training basic school mathematics teachers through Distance Education Programmes should put more emphasis on basic algebra courses. Basic algebra courses should be given more attention to enable prospective basic school mathematics teachers to develop strong foundation in algebra for teaching.

Stakeholders of mathematics education such as Ministry of Education (MoE) and Ghana Education Service (GES) should organize in-service training for basic school mathematics teachers with the aim of helping them to improve on their algebra knowledge for teaching as their years of teaching experience increase to positively affect pupils' performance in mathematics.

Basic school mathematics teachers should form teams to help each other in their respective areas of difficulty so as to improve on their algebra teaching knowledge levels as their years of teaching experience increase.

Suggestions for Further Research

Further research should be conducted at other areas (regions) within the country to check for the algebra teaching knowledge of basic school mathematics teachers.

Further research should be conducted to involve basic school mathematics teachers with varying backgrounds. This will help to ascertain if the issue at hand is peculiar to basic school mathematics teachers who obtained their teaching certificates through Distance Education Programmes and those from Colleges of Education only.

Finally, there should be further research in other domain specific areas in mathematics.

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APPENDICES

APPENDIX A

UNIVERSITY OF CAPE COAST

COLLEGE OF EDUCATION STUDIES

DEPARTMENT OF MATHEMATICS AND ICT EDUCATION

TEACHER-MADE ACHIEVEMENT TEST

INRODUCTION

My name is Williams Osei, an M.Phil. Mathematics Education student of the University of Cape Coast, Ghana. I am researching on the topic: Algebra teaching knowledge of basic school mathematics teachers from Colleges of Education and Distance Education programmes**.** The purpose of the study is to find out the Algebra teaching knowledge levels of the above-mentioned groups. I assure you about the confidentiality of any data collected.

PART 1: DETAILS OF RESPONDENT

Identification Code ☐

Read carefully and select by making a tick (√) in the box beside the appropriate option.

1. Sex

☐ Male

☐ Female

2. How many years have you been teaching mathematics at the Basic School Level?

   ☐ 5 years and below

   ☐ 6 – 10 years

   ☐ Above 10 years

3. How did you obtain your teaching certificate?

   ☐ Distance Education

   ☐ College of Education

4. Indicate the type of teaching certification you have currently.

   ☐ Diploma

   ☐ Degree

   Other (please specify) ……..……………

PART 2: ASSESSMENT QUESTIONS

This instrument contains 35 items on knowledge for teaching Algebra at the Basic School level. Questions 1 to 25 are multiple choice type which require that you circle the correct answer to each question. Questions 26 to 34 require short answers. Question 35 requires that you mark the responses of three Basic School Pupils to a mathematics question as a Basic School Mathematics Teacher.

You have **120 minutes** to answer these questions.

1. Simplify the expression $(9y^2 - 1) - (6y + x^2)$.

   A. $x^2 + 3y - 1$

   B. $x^2 + 3y^3 - 1$

   C. $9y^2 + x^2 - 6y - 1$

   D. $9y^2 - x^2 - 6y - 1$

2. The expression $2a[(a + 3b) + 4(2a - b)]$ can be simplified as

   ……………………

A. $18a^2 + 14ab$

B. $18a^2 - 2ab$

C. $2a^2 + 6ab + 8a - 4b$

D. $18a^2 + 4ab$

3. Simplify the expression $4x^2y + 5xy^2 + 3x^2y - 2xy^2$.

   A. $10x^6y^6$

   B. $-7x^2y + 3xy^2$

   C. $7x^2y + 3xy^2$

   D. $7x^4y^2 + 3x^2y^4$

4. Write the expression $\frac{5}{6r} - \frac{3}{4r}$ as a single fraction.

   A. $\frac{1}{12}$

   B. $\frac{1}{6}$

   C. $\frac{1}{12r}$

   D. $\frac{1}{r}$

5. Find the sum of the expressions $\frac{x}{6}$, $\frac{x-y}{3}$ and $\frac{x+y}{2}$.

A. $\frac{6x}{6} + y$

B. $\frac{6x+y}{6}$

C. $x + y$

D. $6x + \frac{y}{6}$

6. Subtract the product of $3x - 4$ and $2x + 3$ from the product of $6x + 5$ and $x + 3$.

A. $22x - 27$

B. $22x - 3$

C. $24x - 3$

D. $22x + 27$

7. What is the sum of five times of $3x + 5$ and three times of $2x - 7y$?

A. $21x - 7y + 5$

B. $21(x - y) + 25$

C. $21(x + y) + 25$

D. $21(x - y + 25)$

8. Factorize completely the expression $3a^2 + 2ab - 12ac - 8bc$

A. $(a - 4c)(3a - 2b)$

B. $(a - 4c)(3a + 2b)$

C. $(a + 4c)(3a - 2b)$

D. $(a - 4c)(-3a - 2b)$

9. Factorize completely the expression $mp + np - mt - nt$

A. $(m + n) + (p - t)$

B. $(m + n)(p - t)$

C. $(m - n)(p + t)$

D. $(m - n)(p - t)$

10. Factorize the expression $x^2 - 5x + 6$

A. $(x - 2)(x + 3)$

B. $(x - 2)(-x - 3)$

C. $(x + 2)(x - 3)$

D. $(x - 2)(x - 3)$

11. Find the value of x in the equation $\frac{2}{3}(x+2) = \frac{1}{4}x + 3$

A. - 4

B. $-\frac{1}{4}$

C. $\frac{1}{4}$

D. 4

12. Make *d* the subject of the equation $\frac{2}{a} = \frac{1}{b} - \frac{1}{d}$.

A. $d = \frac{2ab}{a-2b}$

B. $d = \frac{ab}{a-2b}$

C. $d = \frac{ab}{a-b}$

D. $d = \frac{ab}{2b-a}$

13. Find an expression for *r* in terms of *V*, *h* and *π* from the equation $V = \frac{4}{3}\pi r^2 h$

A. $r = \left(\frac{3V}{4\pi h}\right)^2$

B. $r = \frac{3V/4\pi h}{r}$

C. $r = \sqrt{(\frac{3V}{4\pi h})}$

D. $r = (\frac{\sqrt{2V}}{4\pi h})$

14. Write the expression $\frac{3}{m+n} - \frac{2}{m-3n}$ as a single fraction

A. $\frac{2m-11n}{(m+n)(m-3n)}$

B. $\frac{m-11}{(m+n)(m-3n)}$

C. $\frac{-11n}{(m+n)(m-3n)}$

D. $\frac{m-11n}{(m+n)(m-3n)}$

15. Simplify the expression $x^2 + 4y^2 - 4xy$

A. $(x + 2y)^2$

B. $(x - 2y)^4$

C. $(x - 2y)^2$

D. $(x + y)^2$

16. Expand the expression $(3x + 4y)^3$.

A. $27x^3 + 64y^3 + 108x^2y + 144x^2y^2$

B. $27x^3 + 64y^2 + 108x^2y + 144xy^2$

C. $27x^3 + 64y^3 + 108x^2y + 144xy^2$

D. $27x^3 - 64y^3 + 108x^2y + 144xy^2$

17. Find the value of *x* in the equation $\frac{4x-3}{2} = \frac{8x-10}{8} + 2\frac{3}{4}$

A. -3

B. 1

C. $\frac{15}{8}$

D. 3

18. Find the value of *x* in the equation $2 = \frac{6\left(\frac{3}{4}\right)+x}{x-2}$

A. -1

B. 1

C. 7

D. 8.5

19. Find the value of *x* if $\frac{3x-2}{5}$ is greater than $\frac{6-4x}{10}$ by 5

A. - 6

B. - 4

C. 4

D. 6

20. Students in Mr Sokpe's class were learning to verify the equivalence of expressions. He asked his class to explain why the expression **a - (b + c)** and **a – b – c** are equivalent. Some of the answers given by the students are listed as options A – D.

Which one of the answers comes closest to explaining why **a - (b + c)** and **a – b – c** are equivalent ?

A. They are equal because of the associative property. We know that a – (b + c) equals (a – b) – c which equals a –b – c.

B. They are the same because, we know that, ***a – (b +c)*** does not equal **a – *b* + *c*,** so must be equal to ***a – b – c.***

C. They are equal because if you substitute numbers, like a =10, b = 2, and c = 5, you will obtain 3 for both expressions.

D. They are equivalent because, what you do to the left hand side, you must do to the right hand side.

21. Which one of the following conclusions is correct about the equation $x^2 = 4$?

A. $x^2 = 2 \text{ } or \text{ } x^2 = -2$

B. $x^2 = 2 \text{ } and \text{ } x^2 = -2$

C. $x = -2 \text{ } or \text{ } x = 2$

D. $x = 2$

22. Gertrude's solution to an equation is shown below

Given: $n + 8(n + 20) = 200$

Step 1: $n + 8n + 20 = 200$

Step 2: $9n + 20 = 200$

Step 3: $9n = 200 - 20$

Step 4: $9n = 180$

Step 5: $\frac{9n}{9} = \frac{180}{9}$

Step 6: $n = 20$

Which of the statements about Gertrude's solution is true?

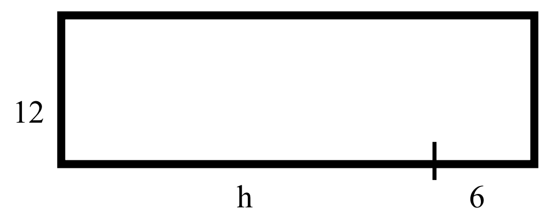
A. Gertrude's solution is correct

B. Gertrude's mistake started from step 1

C. Gertrude's mistake started from step 2

D. Gertrude's mistake started from step 3

23. Write down the perimeter of the figure indicated below



A. $h + 18$

B. $h + 36$

C. $2h + 36$

D. $(h + 18)^2$

24. Find the value of *x* in the equation $4^{2x-1} = \frac{1}{16}$.

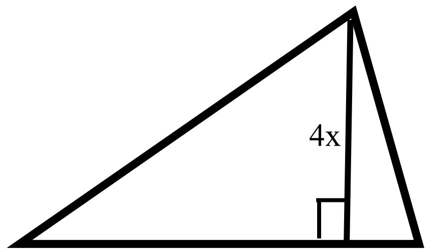
A. $-\frac{1}{2}$

B. $\frac{5}{8}$

C. $\frac{1}{2}$

D. $3\frac{1}{2}$

25. Find the area of the diagram below.



A. $10x$

B. $18x^2 - 16x$

C. $18x^2 - 8x$

D. $36x^2 - 16x$

Part II

For questions 26 – 34, provide your answers in the spaces provided by way of showing working.

26. Find the value of a + *b* + *3*, if *a* +*b* = 92

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27. If *2(e + f) = 8*, then e + f + g =

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28. If $m - w = 10$, then $(m - w) - 3(m - w) =$

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29. What are the values of c, if $c + d = 10$ and c is less than d ? (Note: $d \leq 10$)

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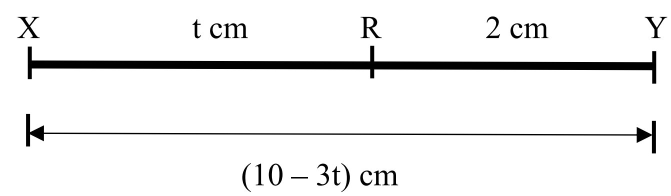
30. Write down an expression for the length XY and proceed to find the value of t.



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31. AM = x cm; MB is 8cm more than AM, write down the length of AB in terms of x and simplify.

AB =

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32. A mathematical club has 25 members. Write a procedure for finding the number of girls, "g", if you know the number of boys, "b"

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33. Four more than three times a certain number is 31. Write down an equation that can help find the number.

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34. Gertrude is *y* years old. Priscilla is 5 years older than Gertrude. Write down the sum of their ages in terms of y and simplify.

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Part III

35. Maud, Priscilla and Gertrude were asked by their mathematics teacher to solve for $\boldsymbol{x}$ in the equation $\boldsymbol{2x^2 = 6x}$ **.**

Below are their respective solutions. Carefully examine their responses by marking and writing your observations. State clearly if any of the solutions are correct.

Maud's Solution

$2x^2 = 6x$

=›› $x^2 = 3x$ Step 1 ------ She divided both sides by 2

=›› $x = 3$ Step 2 ------ She divided both sides by x

Therefore, the value of x is 3

Comment on the correctness of Maud's solution

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Priscilla's Solution

$2x^2 = 6x$

=›› $2x = 6$ Step 1 ------ She divided both sides by x

=›› $x = 3$ Step 2 ------ She divided both sides by 2

Therefore, the value of x is 3

Comment on the correctness of Priscilla's solution

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Gertrude's Solution

$2x^2 = 6x$

=›› $x^2 = 3x$ Step 1 ------ She divided both sides by 2

=›› $x^2 - 3x = 0$ Step 2 ------ She grouped like terms

=›› $x(x-3) = 0$ Step 3 ------ She factorized x out

=›› $x = 0$

and

=›› $x - 3 = 0$ =›› $x = 3$

Therefore, the values of x are 0 and 3

Comment on the correctness of Gertrude's solution

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APPENDIX B

PARAMETRIC ASSUMPTIONS

The assumption of normality was met for this study. This is because the study used 203 basic school mathematics teachers which is greater than 30. Again, a histogram was used to check for the shape of the data set. The bell-shaped indicates that the data is approximately normal.