UNIVERSITY OF CAPE COAST

INTERROGATING CONSTRUCTIVIST APPROACH TO MATHEMATICS TEACHING AND LEARNING IN GHANA: THE CASE OF A JHS ONE TEACHER

BY

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DECLARATION

Candidate's Declaration

I hereby declare that this thesis is the result of my own original research and that no part of it has been presented for another degree in this university or elsewhere.

Candidate's Signature: .... Date: 14/05/18

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Supervisors' Declaration

We hereby declare that the preparation and presentation of the thesis were supervised in accordance with the guidelines on supervision of thesis laid down by the University of Cape Coast.

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ABSTRACT

The purpose of this study was to investigate what happens when a basic school mathematics teacher attempts to interpret and implement his own understanding of constructivist teaching and learning in a Junior High School (JHS) classroom. A total of 25 JHS one mathematics teachers were initially selected using purposive sampling. This number was finally reduced to one research participant using the Mathematics Belief Scale (MBS) and a series of actual class observations. In studying this single "constructivist" teacher, the researcher used ethnographic research methods to collect and analyse data. The results of this study showed that the teacher had a constructivist view of mathematics teaching and learning. The plans the teacher made prior to his teaching sometimes varied from how he actually taught depending on his classroom situations. A connection was also found to exist between the teacher's ideas about constructivist teaching, instructional decisions and his classroom practice. The study also showed that a teacher's experience positively influences his ideas about constructivist teaching and learning, and when constructivist teaching and learning were employed in a JHS classroom, students benefited from sharing knowledge with their friends and became happy and excited about mathematics teaching and learning. It was recommended among others that training institutions paid particular attention to the views pre-service teacher's hold about mathematics before they leave to practice since the view they hold about the subject affects the way they teach.

KEY WORDS

Constructivism

Conception

Formalism

Mathematical objects

Mathematical power

Platonism

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DEDICATION

To my lovely wife Patience and the children Kafui, Elorm, Mawuli and Eyram

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CHAPTER ONE

INTRODUCTION

There is general agreement that every school going child should study mathematics at school; indeed, the study of mathematics is regarded by many people as being very essential (Cockcroft, 1982; NCTM, 2000). In Ghana, mathematics is not only compulsory at the Basic school level but also mandatory at the secondary school level. This decision is to ensure that all students are mathematically empowered to further their studies without any impediment and pursue programmes of their choice without mathematics being a hindrance to their higher pursuits.

However, according to Agudelo-Valderrama (1996), a large number of students still find it difficult entering institutions of higher learning mainly due to poor grades in mathematics. The low pass rates of Junior High School candidates in their Basic Education Certificate Examination, (BECE), as well as the West African Senior High School Examination (WASSCE) over the years, attest to this fact (Gavor, 2015; Amponsah, 2010). Many stakeholders have attributed this low performance to poor teacher methodology and pedagogy and recommended the constructivist strategy to the teaching and learning of the subject (Amponsah, 2010; Mereku, 2003; Fredua-Kwarteng, 2005). Finding a lasting solution to this abysmal performance in mathematics would not only reduce the frustration of students and their parents but also go a long way to help in the development of this nation.

Background to the Study

Ghanaian students over the years have not been performing well in mathematics whether at the BECE or WASSCE level (Gavor, 2015; Amponsah, 2010). Many stakeholders in education have blamed this poor performance over the decades on the two philosophical views which have dominated the teaching industry in Ghana- namely Platonism and Formalism. They have therefore recommended the constructivist approach to mathematics teaching and learning as the solution to this problem (Mereku, 2003; Fletcher, 2005; Amponsah, 2010). It is true that others factors such as teaching and learning resources, socio-economic background of parents etc. also contribute to students' poor performance. However, teacher methodology and pedagogy have been identified as the major contributing factors. I would briefly discuss these three philosophical views and how each of them imparts teaching and learning of mathematics.

Platonism

Three significant philosophies which underpin mathematics are Platonism, Formalism and Constructivism (Davis and Hersh, 1981). Briefly, Mathematical Realism or Platonism holds the view that mathematical entities exist independently of the human mind. Thus, humans do not invent mathematics, but rather discover it, and any other intelligent beings in the universe would presumably do the same. In other words, Platonists believe that mathematical objects and structures exist and that their existence is an objective fact entirely independent of our knowledge of them and they are not dependent on our language or any other human activity (Fletcher, 2005). These objects, they claim, are real finite objects with specific properties some known some unknown and doing mathematics is the process of discovering this pre-existence relationship and the structures connecting them. Any mathematician, in this school of thought, cannot invent anything because it is already there. All that he/she can do is to discover them. As a result, Thompson (1975), a Platonist, called on mathematicians to affirm that mathematical forms indeed have an existence which is entirely independent of the mind considering them. However, the significant problems that other schools of thought have with mathematical Platonism are as follows: (1) precisely where and how do the mathematical entities exist, and how do people who do mathematics get to know of them? Platonists have not adequately addressed these questions. (2) Is there a world utterly different from our physical world that is occupied by these mathematical entities? (3) How do those who do mathematics gain access to this independent world and discover truths about the objects? These are questions pleading for answers so far as Platonism is concern. Even though the indispensability argument for mathematical realism formulated by Quine and Putnam attempted to offer solutions to support the above questions, it does so by stripping mathematics of some of its epistemic status (Busch, 2012). The indispensability argument is as follows: mathematics is indispensable to all empirical sciences, and if we want to believe in the reality of the phenomena described by the sciences, then we ought to also believe in the reality of those entities required for this description.

On the other hand, one of the most persuasive arguments against Platonism is the epistemological argument by Paul Benacerraf in 1973 quoted in

Balaguer (2001). The epistemological argument can be put in the following way: Human beings exist entirely within space-time. If there exist any abstract mathematical objects, then they do not live in space-time. Therefore, it seems very likely that: If there exist any abstract mathematical objects, then human beings could not attain knowledge of them. Thus, if mathematical Platonism is correct, then human beings could not obtain mathematical knowledge. Human beings have mathematical knowledge. Therefore, mathematical Platonism is not correct.

Formalism

The formalist school of thought introduced by the 20th-Century German mathematician David Hilbert holds that all mathematics can be reduced to rules for manipulating formulas without any reference to the meanings of the formulas. The formalists contend that it is the mathematical symbols themselves that are the primary objects of mathematical thought and not any meaning that might be ascribed to them (Fletcher, 2005). For this reason, formalists do not believe in the existence of mathematical objects. They believe that mathematics consists of axioms, definitions, and theorems. They see mathematics as a science of rigorous proofs, and any logical truth must have a starting point- the axiom upon which the theorem is built. The axiom may be true or false, but to the formalist, that is not important. What is essential is the valid logical deductions that can be made from the axiom (Fletcher, 2005). The criticism against formalism is its detachment from context, its foundation on axioms and principles rather than practical understanding, and its formal, syntactic reconstruction of competence.

Also, a major shortcoming of Platonism and formalism is the fact that teaching is teacher-centred rather than student-centred. For those who hold the above philosophical views, "Knowledge is fixed, lying out there for the student to find, like a pebble at the beach" (Clements and Battista, 1990, p.34). Learning is essentially remembering facts and figures and understanding is a secondary matter. The proponents of these philosophies believe that facts are facts and there is only one true reality, and learning occurs as students take in mathematical knowledge in ready-made pieces provided by the teacher. As a result, children are expected to learn, for example, multiplication tables by memorization. Children learn efficient algorithms (methods) to achieve solutions. Platonism and formalism hold the view that understanding is good but not necessary. Learning is by what is popularly referred to as "chalk and talk" where the teacher is the expert and sole authority in the classroom filling empty heads with knowledge.

Teachers, who hold Platonist and formalist philosophical views, do not see learners as people who can construct their own knowledge. To them, mathematics instruction and curricula are based on the transmission or absorption view of teaching and learning. In this view, students passively "absorb" mathematical structures invented by others and recorded in texts or known by authoritative adults. Teaching consists mainly of transmitting these sets of facts, skills, and concepts to students. In these philosophical schools of thought, strict adherence to the fixed curriculum is highly valued by its practitioners and materials are primarily textbooks and workbooks. However, to the constructivist, the real function of the teacher is that of a coach and a facilitator.

Constructivism

Constructivism is fundamentally a theory about how people learn. It says that people construct their knowledge and understanding of the world, through experiencing things and reflecting on those experiences. The basic tenets of constructivism which are embraced to a greater extent by different proponents are as follows:

1. The child actively creates knowledge instead of merely receiving that knowledge from the environment. For example, the idea of "four" cannot be directly detected by a child's senses. It is a relation that the child superimposes on a set of objects.

2. Children create new mathematical knowledge by reflecting on their physical and mental actions and ideas are constructed or made meaningful when children integrate them into their existing structures of knowledge.

3. No single true reality exists, instead, it is the individual interpretations of the world, and experience and social interactions modify these interpretations. As a result, learning mathematics should be thought of as a process of adapting to and organizing one's quantitative world, not discovering pre-existing ideas imposed on us by others.

4. Learning is a social process whereby children grow into the intellectual life of those around them (Burner, 1986). Mathematical ideas and truths, both in use and in meaning, are collectively established by the members of a community. In such a situation, the constructivist classroom is viewed as a culture in which students are involved not only in discovery and invention but also in a social discourse

involving explanation, negotiation, sharing, and evaluation (Amponsah, 2010). According to Crawford and Witte (1999), the best vocabulary to describe a constructivist classroom is energy. The active involvement of students in the teaching and learning process is essential. Obtaining this type of commitment requires a much different classroom from the authoritative and teacher-centred traditional classrooms in which the teacher stands in front of the class directing the content that is presented to the students (Polya, 2002). In a constructivist classroom, knowledge moves in more than one direction. Knowledge shifts from teacher to student, student to student and even from student to teacher. A constructivist teacher, according to Brooks and Brooks (1999), would focus on how students learn and what they must learn together as one.

5. Teachers do not expect students to use the same method to solve questions given to them. When a teacher demands that students use a particular set of mathematical methods in solving a problem, the sense-making activity of students is severely curtailed. Students tend to mimic the processes by rote so that they can appear to achieve the teacher's goals. Their beliefs about the nature of mathematics change from viewing mathematics as sense-making to seeing it as learning a set of procedures that make little sense.

In short, constructivists believe that mathematics does not grow through some established theorems, and axioms neither do it increase by discovering some mathematical objects and structures existing in a world of their own, but through the continuous improvement of guesses by speculation and criticism. Constructivists are of the view that we cannot be sure of any absolute truth but that people construct, or create, "knowledge" based on their experiential reality. To the constructivist, "truth" or "knowledge" is a social construct attained when people agree on particular mental models that appear to be consistent with their collective experiential reality. However, this social agreement does not necessarily imply "universal truth."

For this reason, in the constructivist classroom, the focus tends to shift from the teacher to the students. A class is no longer a place where the teacher ("or expert") is expected to dish out knowledge to students, who wait like empty containers to be filled. Rather in the constructivist paradigm, the students are encouraged to be actively involved in their learning process. The role of the teacher is to function as a facilitator who mediates, prompts, and assists students to develop and assess their understanding, and for that matter, their knowledge. One of the teacher's most significant responsibilities in a constructivist classroom is asking the right questions and pursuit of student questions and interests. The teacher's role is interactive, deeply rooted in negotiation and building on what students already know. Students are supported to construct their own knowledge. In a constructivist environment, the curriculum emphasises big concepts, beginning with the whole and expanding to include the parts instead of the curriculum starting with the parts of the whole. Materials include primary sources and manipulative materials. Both teachers and students think of knowledge not as inert factoids to be memorised, but as a dynamic, ever-changing view of the world we live in and the ability to successfully stretch and explore that view.

In conclusion, even though for several decades, the Platonists and formalists views have dominated the teaching industry in Ghana, they have not made any significant impact on the mathematics landscape (Fletcher, 2005; Eshun and Abledu, 2001; Amponsah, 2010). This study is posited on the assumption that to improve Ghanaian students' mathematical ability there is the need to make a U-turn from the influences of Platonism and formalism regarding the teaching of mathematics in Ghana. The need to make this paradigm shift is further supported by the National Research Council when it said that:

> In reality, no one can teach mathematics. Effective teachers are those who can stimulate students to learn mathematics. Educational research offers compelling evidence that students learn mathematics well only when they construct their own mathematical understanding. There is no way to build this [understanding] except through the process of creating, constructing, and discovering mathematics. (National Research Council, 1989, pp. 59-60)

Since Constructivism offers a sharp contrast to the two earlier views, it is proposed not only to provide a radically different alternative to Platonism and formalism but also to help students better understand, do and apply the mathematics they learn in novel situations. This study is an attempt to find out what happens when a "constructivist" teacher makes frantic efforts to implement his own understanding of constructivism in the classroom. How he tries to demonstrate that the learning and teaching dynamic is a process of negotiation in which the learners come together, try to make sense of the world.

Statement of the Problem

Despite the huge investment the Ghanaian government has been making in both human resources training and infrastructural development in the educational sector, there has not been any corresponding academic improvement to justify those investments (Gavor, 2015). This is particularly true when one considers the fact that it is on record that Ghana has never been represented since 1953 in any International Mathematics Olympiad (IMO). The IMO is an annual mathematics competition that is organised for pre-university students from all regions of the world. It is also sad and disappointing to note that even in sub-Saharan Africa; representation at this prestigious international competition is woefully inadequate (Amponsah, 2010). In the 2003 TIMSS (Trends in International Mathematics And Science Study) mathematics test for grade eight students, Ghana placed 44th out of the 45 countries that participated in the competition. Our Ghanaian students scored a total of 276 compared to the global average which was 466 (Anamuah Mensah and Mereku, 2005; Fredua-kwarteng, 2005). This poor performance in TIMSS again occurred in 2007 and 2011 where Ghana placed 58th out of 59 countries and 63rd out of 63 nations respectively (MoE, 2008; Mullis, Martin, Foy, and Arora, 2012). Many studies have shown that the reasons for this poor performance in mathematics are as a result of teaching methodology and pedagogy and recommend the constructivist approach to the teaching of the subject at least at the Basic and Secondary School levels (Amponsah, 2010; Mereku, 2003; Fredua-Kwarteng, 2005).

Although references to constructivist approaches to mathematics teaching and learning are pervasive in literature, there is not presently any specific description demonstrating this theory in practice (Clement and Battista, 2009; Draper 2002; Medinipur, 2015). For this reason, any research which attempts to address a practical approach to constructivist teaching and learning will serve as a guide to other teachers who are desirous of improving their teaching and learning of mathematics. It would also help as an eye-opener to those who do not even know that certain limitations exist in their teaching of the subject. Practical descriptions of such approaches such as a detailed description of the constructivist teacher in the classroom and taking of video coverage of him as he makes serious efforts in his own small way to implement constructivism in his classroom will serve as a guide to many teachers. The following elements will be expected in a constructivist classroom:

a) The teacher relates lesson to prior knowledge and life experiences.

b) The teacher uses effective questioning techniques at the level of his students.

c) The teacher encourages students to communicate their thinking verbally.

d) The teacher values students thinking

e) The teacher assesses students' continually

f) The teacher motivates his students

g) The teacher engages students in critical thinking and problem solving.

h) The teacher encourages reflection and helps students to extend their thinking beyond the classroom.

i) The teacher builds his instructions on students' articulation of their thought?

j) The teacher teaches students to take responsibility for their learning.

Purpose of the Study

The overall purpose of this qualitative, descriptive study was to determine how students experience mathematics when constructivist teaching and learning is implemented in a JHS classroom. Data were obtained through interviews, class observations, and video recordings of mathematics lessons, examination of lesson notes of the teacher, student assignments and class work. According to Clements and Battista (2009) even though constructivist learning theory is believed to have the potential to be the basis for mathematics teaching mainly because of its emphasis on knowledge construction by the learner instead of knowledge reproduction, functional descriptions of this approach is not readily available. Draper (2002) also explains that one of the major challenges of the constructivist approach is translating this learning theory into a theory of teaching. Draper further argues that constructivism is a descriptive theory of learning and not a theory of teaching and the implementation of this theory requires a change in existing classroom practices. Although some researches have been done to study what constructivist teaching involves (Clements and Battista, 2009; Carpenter and Fennema, 1991, 1992; Fennema et al., 1989) few practical descriptions of this type of teaching exist. In Ghana, there is no such concrete description demonstrating this theory in practice. This investigation would be one such practical description for consideration by mathematics teachers who are looking for ways to enhance teaching and learning of the subject in their classrooms.

Research Questions

The central research question for the study was when constructivist teaching and learning of mathematics is implemented at the Junior High School classroom, how do students experience mathematics? More specifically, the following sub-research questions were of concern to the researcher:

1. How does the teacher view constructivist teaching and learning of mathematics?
2. Do the plans the teacher make before his teaching differ from how he actually teaches?
3. Are there any connections between the teacher's ideas about constructivist teaching and instructional decisions on one hand, and his classroom practice on the other?
4. To what extent does the teachers experience influence his ideas about constructivist teaching?
5. How do students experience mathematics teaching when constructivist methods are employed in their classroom?

Using these questions, I would draw on constructivist learning theory to examine the opportunities for students to learn and understand mathematics.

Significance of the Study

A first step toward changing mathematics teaching and learning is for teachers to accept that problems exist within their teaching (Kraft, 1994: TIMSS, 2007 and 2011). By seeing alternative approaches to teaching mathematics, teachers would become aware of problems that exist within their own teaching.

This research would increase teachers' understanding about an alternative method of teaching mathematics by describing and presenting information about decisions and instructional strategies that were used by a teacher who was constructing his own meanings of constructivist teaching. Teachers who read the study would then become more aware of the strategies that they can use to help students gain mathematical power. Through this awareness, teachers may improve their ability to make effective instructional decisions that would improve their teaching practice and subsequently improve mathematics learning.

Also, it has been argued that constructivist learning theory has the potential to be a basis for mathematics teaching (Von Glasersfield, 1984). However, there is not presently any specific functional description demonstrating this theory in practice (Clements and Battista, 2009). This investigation would be one such practical description for consideration.

Finally, the research will be of great benefit to educational planners like the Ministry of Education, Ghana Education Service, beneficiaries of education as well as organisations that have roles to play with the promotion and development of mathematics education in Ghana. This is because the constructivist approach to mathematics teaching and learning is strongly supported by powerful bodies like NCTM and NRC among others as holding the key to the future of mathematics teaching and learning (National Research Council, 1989; NCTM, 2010).It would also serve as resource material for all stakeholders and others who would like to research further into this area of national concern.

Delimitation

This research was confined to only one JHS 1 teacher at X Community School, Accra. The researcher could not extend his observation to more than one JHS class partly because of the following reasons:

(i) Ethnographic research takes time to build trust with participants who will then be free enough to act or behave in their natural way so that results can reflect reality, hence the concentration on one class.

(ii) The analysis period had to be sufficiently long enough to be thorough; therefore, extending it beyond one teacher would not give the researcher enough time to do an excellent observation.

(iii) The observation and video coverage of the teacher's lessons which was to be done three times a week could not allow for two teachers in two separate classrooms to be studied at the same time because of timetable clashes and cost. This research study was not open to all mathematics teachers in the basic schools. It was limited to only mathematics teachers teaching at JHS one. The reason for the choice of JHS one class was that this is the particular form from which students all over the world are selected to participate in the TIMSS competition of which Ghana was part in 2003, 2007 and 2011. It is also the transition class between upper primary and secondary education.

Through the administration of the mathematics beliefs scale questionnaire in 12 JHS one schools in the X and Y municipalities, 25 mathematics teachers completed the MBS questionnaire and returned them. The second stage of the selection was the class observation by the researcher. The class observation was done to find out whether there was consistency between what the teachers said they do in class and what they indeed practised in the classroom. At the end of the observation which took one whole term, two teachers were finally selected for the study. The single teacher who was chosen finally had two advantages over the other one. Besides his Diploma in Basic Education, he holds a BSc. in statistics from the University of Ghana, Legon and he was very enthusiastic about teaching mathematics. He also showed more interest and zeal in my research than the other teacher.

Limitations

Qualitative research studies are designed to provide a deep understanding of a particular phenomenon, making the results difficult to generalize (Yin, 2003), but all studies have their limits. The limitation of this study is that because of the video coverage of the lessons coupled with classroom observation, I could not cover more than one teacher and for that matter will not be in a position to generalize the results of this study. Secondly, the constructivist teacher who was finally chosen from X Community Basic School, Accra happened to be a mathematics teacher for a class of 58 JHS one students. The classroom was the normal classroom size that should have ideally accommodated only 30 students instead of the 58. As a result, there was overcrowding of students. The dual desks in the classrooms combined with the congestion of students made it difficult to rearrange the desks for small group discussions and cooperative learning whenever the teacher attempted to use any of these strategies to teach. This in my view limited the frequency of the use of the above constructivist methods in the classroom discourse. However, it is worthy to note that 10 out of the 12 JHS schools initially selected for the MBS all used dual desks. The other teacher who was also chosen in addition to Mr Yamoah also had double desks in his class.

Definition of Terms

Conceptions: Thompson (1984) defined conceptions of mathematics as beliefs and views about the nature of mathematical knowledge. The researcher extends this definition to encompass conceptions as the knowledge of and beliefs about mathematics.

Constructivism: Is a theory of learning that provides teachers and educators with an understanding of how students learn. It is underscored by two main principles: (1) that learners are active in constructing their own knowledge, and (2) that social interactions are vital to knowledge construction (Bruning et al. 2004, p.195). The individual learner constructs and gains new knowledge through experience, interaction and active involvement with the learning environment.

Experience As used in this research refers to the academic qualification of the teacher.

Mathematical Objects A mathematical object is an abstract object arising in mathematics. In mathematics, an object is anything that has been (or could be) formally defined, and with which one may do deductive reasoning and mathematical proofs.

Thick description It means to provide a report that moves the reader to the context with all the "insight, understanding, and illumination not only of the facts or the events in the case but also of the texture, the quality, and the power of the context as the participants in the situation experienced it" (Owens, 1982, p.8).

Organization of the Study

The thesis was organised as follows: There was an abstract which did not exceed 250 words summarising the entire study and its findings. Chapter one focused on the background to the study, a definition of the problem to be studied and the rationale for the study. Chapter two was mainly on literature review which highlighted important views and ideas on the topic from other authors and a critique of the literature. Chapter Three was a description of the population and sample, research design and data collection methods. Chapter Four examined the analysis of data. Chapter Five was mainly concerned with the summary, conclusion and recommendations. This was followed by References and Appendices.

Chapter Summary

There is no doubt that every school going child in any part of the world should study mathematics at school. Indeed, the study of the subject is regarded by many people as essential (Cockcroft, 1982; NCTM, 2000). For this reason, mathematics holds a critical position in the school curriculum in virtually every country (Okuma-Nystrom, 2003). In Ghana, despite the enormous investment the government has been making in both workforce training and infrastructural development in the educational sector, there has not been any significant academic improvement to justify those investments (Gavor, 2015; Agudelo-Valderrama, 1996; Cofie, 2000; Eshun and Abledu, 2001). Many stakeholders have attributed this low performance to the traditional method of teaching the subject and recommended the constructivist approach to the teaching and learning of the subject (Amponsah, 2010; Mereku, 2003; Fredua-Kwarteng, 2005;

Fletcher, 2005). In the traditional teaching of mathematics in school, the truth is determined when the teacher says it is correct and knowing mathematics means remembering and applying the proper rules (Lampert, 1990; 1991). For students to know mathematics in ways of the discipline, teachers would have to alter their traditional role of being the authority for correct answers. They should rather become the experts in the group who would support the students' construction of mathematical procedures and principles by choosing and posing the problems and developing the tools for the classroom (Lampert 1989,). This study aimed at interrogating constructivist approach to mathematics teaching and learning in a JHS classroom with the view to finding out how do students experience mathematics when constructivist strategies are employed to teach the students.

CHAPTER TWO

LITERATURE REVIEW

Overview

Although there are various approaches to mathematics teaching and learning, there is enough literature to support the fact that constructivist learning theory has the potential to be a basis for mathematics teaching and learning (Von Glasersfeld, 1984; Larochelle, 2000 and Medinipur, 2015). This study therefore aimed at interrogating constructivist approach to mathematics teaching and learning at a Junior High School level and to find out how this approach influences the students' mathematical experience.

This chapter has been divided into five main sections as follows: The first section is an overview of the chapter. Section two examines the theoretical framework upon which the current study is based. The theoretical framework focuses on constructivism as a theory of learning, Vygotsky's Zone of Proximal Development (ZPD), Bruner's (1985) Theory of Instruction, and Skemp's Mathematical Theory of Relational Understanding. The third section is a review of what researchers have documented as practical ways of how children learn mathematics.

Four groups of influences have also been identified as having an impact on how children learn mathematics. The first one is the demands, constraints and influences from the society in which the mathematics learning takes place. The second set of controls has to do with the knowledge, skills and understanding which the learners develop outside the school setting and which have significance for their learning inside the school. The third set of influences on children's learning of mathematics come from the teaching and learning resources (TLR) used by the teacher and the fourth influence, yet not the least is the teacher.

The fourth section provides the basis why research on mathematics teaching and learning has changed over the years. Our democratic society's needs are continually changing, and consequently, the mathematics that children need to know is also changing. A significant goal of elementary mathematics education is to help children have the power to think about mathematics and do mathematics. This power allows students to have control over their success and helps them to develop autonomy in learning. Section five provides a summary which concludes the chapter. The summary contains the practical significance of the information that was presented in the literature review and justification for improving the instructional practices of teachers in the field of mathematics education.

Theoretical Framework

Four theories provided support and informed this research. They were constructivist learning theory, Vygotsky's (1930) Zone of Proximal Development (ZPD), Bruner's (1985) Theory of Instruction, and Skemp's Mathematical Theory of Relational Understanding. These theories are essential for this study because they share the following standard practices:

a) Learners questions and opinions are valued,

b) Manipulative materials are common components of instruction,

c) Interactive learning approach is a common feature and learning is built on students 'prior knowledge,

d) There are collaborative work and students construct their own knowledge with guidance from the teacher, and

e) Assessment is usually by observation, peer evaluation and testing.

Vygotsky and Bruner both believed in scaffolding, social interaction and learning from adults and other more knowledgeable others. According to Bruner, students are active learners as they construct their knowledge for themselves through their own experiences. The teacher's role is a facilitator to help students discover relationships between information but not to organise it for them. Bruner's emphasis is on discovering learning or student-centred learning. Skemp's relational understanding also means that the child knows what to do and can explain why he has to do it. The four theories are briefly discussed below.

Constructivist Theory

The constructivist theory is grounded in the works of Piaget, Vygotsky, Bruner and the philosophy of John Dewey. A variety of differing views are found within constructivism, as the concept has been defined differently by various researchers and authors (Woolfolk, Hughes, and Walkup, 2008). For example, Confrey (1990) defines constructivism as "a belief that all knowledge is necessarily a product of our own cognitive acts" (p. 107). Similarly, Lowery (1997) sees constructivism as a philosophy that states that students construct new knowledge and understanding for themselves. Lambert et al. (1995) describe constructivism as a theory of learning which contends that:

"Individuals bring past experiences and beliefs, as well as their cultural histories and worldviews, into the process of learning; all of these influences how we interact with and interpret our encounters with new ideas and events" (p. xii). Within this paradigm, the individual learner constructs his/her own knowledge from experiences and interaction with the physical world (Doolittle and Camp 1999). Fosnot (1996) holds that constructivism is a psychological theory which construes learning as an interpretive building process whereby the individual learner actively interacts with the physical and social world. According to Jonassen (1997) and Orton (2004), within this paradigm, students are given the opportunity to utilise their prior knowledge, experience, observation and understanding to formulate new concepts and the emphasis is on concept formation rather than teaching for concept acquisition, as experienced in behaviourist classrooms.

In general, constructivist theories are theories of learning that provide teachers and educators with an understanding of how students learn; two main principles underpin them: "learners are active in constructing their knowledge and that social interactions are essential to knowledge construction (Bruning et al. 2004, p.195). The individual learner gains new expertise through experience, communication and active involvement with the learning environment. Learners create knowledge by building on previously built knowledge, and students can better grasp the concepts and move from merely knowing the material to understanding it (Ward 2001). Within constructivist theory, it is believed that students move from the known to the unknown and possession of a solid foundation of a particular concept is considered to be paramount, as the development of new knowledge is dependent upon what is already known.

Clements and Battista (2009) also define constructivism as an epistemology (knowledge, understanding) which follows certain basic tenets:

a) The student or learner actively creates knowledge.

b) The learner produces new mathematical consciousness through reflections on physical and mental actions.

c) There is no one true reality. Each person has his or her reality based on his/her interpretation.

d) Learning is a social process where meaning is negotiated.

e) Students learn when they are allowed to explore. They tend to memorise when knowledge is "dished out" to them.

The constructivist view of learning is reflected in the developmental theories of Piaget, 1972; Dewey, 1997; Bruner, 1961 and Vygotsky, 1978 among others. In cognitive constructivism, which originated primarily in the work of Piaget, it simply states that an individual's reactions to experiences lead to (or fail to lead to) learning. However, in social constructivism, whose principal proponent was Vygotsky, language and interactions with others such as family, peers and teachers play a primary role in the construction of meaning from experience. Meaning is not simply constructed; it is co-constructed. Proponents of constructivism (Biggs, 2002) offered variations of the following principles for effective instruction:

(i) Instruction should require learners to fill in gaps and extend material presented by the instructor. The goal should be to gradually wean the students away from dependence on instructors as primary sources of required information, helping them to become self- learners.

(ii) Instruction should involve students coming together to work in small groups. This attribute which is considered desirable in all forms of constructivism and essential in social constructivism supports the use of collaborative learning as well as and cooperative education.

Vygotsky's Social Constructivism

Lev Vygotsky was a seminal Russian psychologist who was best known for his sociocultural theory. He believed that social interaction plays a crucial role in children's learning. He put social interaction at the centre of education. Vygotsky, like Piaget, argued that construction of knowledge relied on individuals' own efforts based on their previous knowledge, but he regarded social interaction as a central tenet of the individuals' intellectual development. The social constructivist model accepts that socialisation plays an important role in individuals' learning. That is, social interaction and culture provide the basis for learners' thinking and activities in a given environment. Social interaction influences the individual's cognitive improvement by "explaining reality, transmitting cultural messages and mediating the learning of environmental rules" (Kouzulin and Presseisen, 1995, p.69). From Vygotsky's perspective, culture also catalyses cognitive development, that is, human actions occur in cultural environments and could be incomprehensible outside of these environments

(Woolfolk et al., 2008). Thus, culture offers individuals the tools to think and also directs them how to think. The most important theory, which was introduced by Vygotsky into the world of educational theory, was the idea of "The Zone of Proximal Development (ZPD)" to cope with problems of the assessment of pupils' intellectual skills and the evaluation of instructional behaviour. Vygotsky (1978) defined this theory as the "the distance between the actual developmental level as determined through independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (p. 86). According to his theory, engagement with more capable peers is an effective way of developing abilities and approaches. In this regard, the term „scaffolding" is regularly used in the literature instead of ZPD. This term refers to the context that provides appropriate support (questioning and positive interactions) by teachers, or more able peers or the use of technological tools for pupils to achieve their potential (Bruner, 1985; Stuyf, 2002). ZPD is about learning and would be controlled by the individual's stage of development (Palincsar, 1998). With the purpose of understanding this relationship, it is essential to determine the individual's actual and potential development. Actual development indicated individual's attainment by him/herself without help. Potential development is the maximum level that individuals can accomplish with support (ibid). Hence, communication is an important part of this process (Bruner, 1985). At this juncture, teachers and more skilful peers provide the scaffolding (ibid). Therefore, collaboration becomes an important aspect of learning between pupils and teachers to construct knowledge and skill. Furthermore, this scaffolding (assistance) should come through guidance instead of knowledge transmission (Daniels, 2001).

Zone of Proximal Development

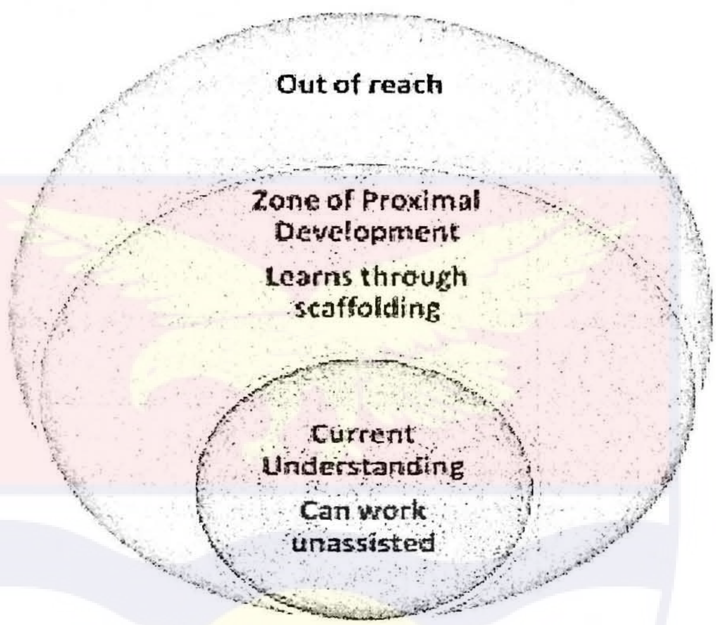

Figure 1 : Zone of Proximal Development

It can be seen from the diagram above that the scaffolding would occur in the ZPD. According to Vygotsky, it was accepted that the individual had to perform in the ZPD, which is regarded as a social period. However, there are some points that need to be monitored during this period. Some of the most important points are as follows:

a. Students' ZPD should be carefully tested.

b. Questions, which will be directed to students, should not be above the level of students" understanding.

c. Students should solve the problems by external support and use their own existing knowledge.

d. Students should develop some new concepts concerning the subject, and they should review the topic.

It is argued that if the problem is complex and beyond the understanding of the student s/he will become more reluctant to solve it, so problems should be set at the right level. Moreover, the constructivist approach aims to form a bridge that focuses on the teacher's pedagogy and mathematics teaching, which relies heavily on the conceptual dimension of mathematics. This conceptual approach is reliant on students using conceptual thinking to make a mathematical definition transform from abstract mathematical thinking to concrete mathematical thinking. To develop their students' ZPD process, teachers need to consistently design innovative tasks; the ability to achieve this is based in part on the teacher's knowledge and beliefs, and also on the beliefs prevalent outside the teacher, e.g. the school, society, etc. (Nielsen, Barry and Staab, 2007).

Applications of the Vygotsky's Social Development Theory

Many schools have traditionally held a transmissionist or instructional model in which a teacher or a lecturer 'transmits' information to students and the students sit passively to receive it. In contrast, Vygotsky's theory promotes learning situations in which students play an active role in the learning process. The roles of the teacher and student are therefore changed. The teacher now collaborates with his or her students to help facilitate meaningful construction by the students. Learning, therefore, becomes a reciprocal experience for the students and teacher.

Many researchers hold different and often conflicting views on whether stages of development guide learning (Piaget, 1970) or whether learning stimulates potential development (Anderson and D'Ambrosio, 2008). Even more points of view exist about the means by which learning occurs in children. While some schools of thought believe learning occurs in isolated instances supported by conditioning (Pavlov, 1927), others feel that natural internal development guides the learning (Bruner, 1985; Piaget and Inhelder (1969). Dewey (1938) suggested that students learn by applying past experiences to new experiences in order to formulate new knowledge. In contrast, Gagne (1985) suggested that students have various levels of learning that require hierarchical prerequisites of instructional strategies for the student to process new intellectual skills. Between these two ideas exist a range of theories about whether learning is enhanced merely by the amount of time spent on learning (Carroll, 1989; Slavin, 1980) or whether it is affected by external influences such as environmental surroundings or socio-cultural interactions (Skinner, 1953). The more integrative, socio-cultural approach described in Vygotsky's (1930). Zone of Proximal Development (ZPD) is the preferred theoretical premise for this study. In his view of cognitive development, Vygotsky (1930) provided evidence that he accepted the idea that inherent developmental stages, similar to those detailed by Piaget (1970), are at the basis of learning. Vygotsky also combined those developmental beliefs with the Marxist view that humans ultimately develop their intellectual potential through the driving forces of their environmental exposure (Vygotsky, 1931). Piaget (1970) and Bruner (1985) shared similar constructivist perspectives with Vygotsky and agreed with his thoughts on the processes of child development. Bandura's (1977) Social Learning Theory also complimented Vygotsky's Social Development Theory (1930). Vygotsky would have supported Bandura's statement that, "learning would be exceedingly laborious, not to mention hazardous, if people had to rely solely on the effects of their own actions to inform them on what to do" (Bandura, 1977, p. 22). Vygotsky (1930) specifically focused on the social-historical environment while developing his integrative theory. He recognised that traditional internal developmental cues, such as imitation and natural curiosity, lead to maturational growth, but theorised those internal forces alone could not direct an individual through more advanced thought processes.

Vygotsky (1935) therefore began to study child behaviour in more depth and to look specifically at students' performance along with school instruction. He realised that conventional tests only evaluated the level of mastery of students performing in isolation. After observing students under directed instruction, Vygotsky discovered that students often performed at higher cognitive levels with modest assistance. In a particular reported instance, Vygotsky described a situation in which two students on the same individually academic performance level achieved at markedly higher levels when assisted with a more advanced problem. This incident revealed to Vygotsky that future cognitive development is determined by not only the level of individual performance but also by the performance the student is capable of with assistance. In 1930, he named this measured ability between a student's actual performance level and the student's potential level the Zone of Proximal Development, ZPD. With the ZPD, Vygotsky proposed children were capable of combining their innate abilities and knowledge with social cues and interactions to further extend their cognitive growth. Vygotsky believed adult instruction assisted and often prompted student development further than what was obtained through self-discovery alone. For this reason, Vygotsky encouraged educators to develop further studies for determining the extent of interaction between intrinsic development and socio-cultural forces. Even though modern educators accept the view that an interaction between natural development and external factors affects the student learning process, uncertainty remains as to the specific implications of certain types of interactions.

Implications for the Classroom

Vygotsky's concept of the zone of proximal development is based on the idea that development is defined both by what a child can do without any support and by what the child can do when assisted by an adult or more knowledgeable other. Knowing both levels of Vygotsky's zone is useful for teachers. The reason is that these levels indicate where the child is at a given moment as well as where the child is expected to be with assistance from more knowledgeable others. The zone of proximal development (ZPD) has several implications for teaching in the classroom. According to Vygotsky, for the curriculum to be developmentally appropriate, the teacher should plan activities that will encompass not only what children are capable of doing on their own but what they can learn with the help of others like the teacher.

Vygotsky's theory does not mean that any topic can be taught to any child at any stage rather only instruction and activities that fall within the zone promote development. As a result, teachers can use information about both levels of Vygotsky's zone of proximal development in organising classroom activities in the following ways:

(a) Instruction can be planned to provide practice in the zone of proximal development for individual children or for groups of children. For example, hints and prompts that helped children during the assessment could form the basis of instructional activities.

(b) Cooperative learning activities can be planned with groups of children at different levels who can help each other learn.

(c) Also, scaffolding is a way of helping the child in his or her zone of proximal development by letting the adult provides hints and prompts at different levels. In scaffolding, the adult does not simplify the task, but the role of the learner is simplified through a well-calculated intervention by the teacher.

Bruner's Theory of Instruction

In his research on the cognitive development of children, Bruner (1966) proposed three modes of representation: enactive representation (action-based), iconic representation (image-based), and symbolic representation (language-based). Bruner theorised that learning occurs by going through these three stages of representation. Where each stage is a "way in which information or knowledge is stored and encoded in memory" (Mcleod, 2008). The stages are more-or-less sequential, although they are not necessarily age-related like Piaget-based theories. According to Bruner, going through the stages is essential for the learner to truly understand the concept, as it helps the learner understand why. The three stages of representation that Bruner wrote about are discussed briefly as follows:

Enactive (Action-based) Stage

The enactive stage is sometimes called the concrete stage; this first stage involves a tangible, hands-on method of learning. Bruner believed that "learning begins with an action - touching, feeling, and manipulating" (Brahier, 2009, p. 52). In mathematics education, manipulatives are the concrete objects with which the actions are performed. Common examples of manipulatives used in this stage in math education are algebra tiles, paper, coins, etc. - anything tangible.

Iconic (Image-based) Stage

Thai is also called the pictorial stage. This second stage involves images or other visuals to represent the concrete situation enacted in the first stage. One method of doing this is to draw pictures of the objects on paper or to picture them in one's mind. Other ways include the use of shapes, diagrams, and graphs.

Symbolic (Language-based) Stage

The symbolic stage is also referred to as the abstract stage. It is the last stage that takes the images and represents them using words and symbols. The use of words and symbols "allows a student to organise information in his or her mind by relating the concepts together" (Brahier, 2009, p. 53). The words and symbols are abstractions; they do not necessarily have a direct connection to the information. Bruner's work also suggests that a learner even of a very young age is capable of learning any material so long as the instruction is organized appropriately. This assertion is in sharp contrast to the view of Piaget and other stage theorists.

Application for Mathematics Teaching

While Bruner has influenced education greatly, it has been most noticeable in mathematical education (Brahier, 2009). This theory is useful in teaching mathematics which primarily is full of concepts. The teaching begins with a concrete representation and then progresses to the symbolic stage and the abstract stage. Initially, the use of manipulatives in the enactive stage is a significant way to sustain the interest of students, who may not be particularly interested in the topic. Eventually, Bruner's stages of representation came to play a role in the development of the constructivist theory of learning as well (Culatta, 2012). Furthermore, Bruner's theory also allows teachers to be able to engage all students in the teaching and learning process irrespective of their cognitive levels of the concept at stake. While more advanced students may have a more well-developed symbolic system and for that matter can successfully be taught at the symbolic level, other students may need other representations of problems to grasp the material (Brahier, 2009)

In addition, by having all students go through each of the stages, it builds a foundation for which the student can fall back on if they forget or as they encounter increasingly difficult problems. For these reasons, it is essential that the teacher goes through each of the stages with the whole class; however, the time spent on each stage can and will vary depending on the student, topic, etc.

Skemp's Mathematical Theory of Relational Understanding

In order to formulate a complete theoretical framework for this study, a need exists for a review of literature related to mathematical learning. Skemp (1987, 1989, 2006), a mathematician and psychologist, devoted much time to researching and analyzing the thought processes which learners implement when they learn mathematics. After much research, Skemp (1987) became increasingly concerned with the discovery that many intelligent students couldn't "do" mathematics. Skemp sought to find the answer as to why otherwise intelligent students had difficulties learning mathematics. Through his research and personal investigations, Skemp (2006) developed and proposed the Mathematical Theory of Relational and Instrumental Understanding.

To best explain the Mathematical Theory of Relational and Instrumental Understanding, Skemp (2006) used the analogy of a person visiting a town for the first time. When unfamiliar with a town, an individual would attempt to learn how to get from point A to point B. The first successful route, regardless of time or distance, would become the fixed plan this individual continues to use to travel between points A and B. The noted references along the way direct the individual between these two points. With these references, the individual may turn right out of the front door, go straight past the church, and so on. After some time, the individual would begin to explore the town, with little or no intention of getting to or from any given point, but rather to create a mental map of the town. During this exploration, however, the individual makes connections to other routes that lead successfully to points A and B. In contrast to the first scenario, if a person with a mental map of the town gets off track, the individual would be able to produce a variety of plans in order to reach points A and point B. Additionally, if this individual made a wrong turn in the derived path plan, the individual, still aware of locations on the mental map, could successfully adjust path routes without getting lost. In the first example, the fixed form of knowledge required to get from one point to another based on memorised references was what Skemp (2006) described as instrumental understanding. In the second example, the formation of overall awareness of point A and point B as they appear on a mental map of the town was what Skemp (2006) referred to as relational understanding.

Skemp's analogy of the person visiting a new town and the learning of mathematics is quite similar. A situation in which students learn what actions to perform using a step-by-step method to reach an answer to a problem is an example of instrumental learning (Skemp, 2006). Instrumental learning occurs many times using direct teaching pedagogy. A situation in which students can produce an unlimited set of plans to solve a problem successfully is an example of relational learning (Skemp, 2006). Relational understanding is more likely to occur with inquiry or discovery pedagogy.

Instrumental learning is what many educators identify as rote memorisation. With helpful understanding, students simply memorize procedures and steps to reach an outcome. This process of instrumental learning utilizes less brain function than relational learning and provides little depth into the reasoning or true understanding of a mathematical problem (Skemp, 1987). Many species other than humans learn using this instrumental or memorization method of learning (Skemp, 1987). Skemp (1987) acknowledged that even the simplest of animal species could learn ordered steps to complete complicated tasks, but for the learner, instrumental learning is absent of any underlying principles or meaning as to the purpose for performing the task, or to what benefit there is in finding the result.

Relational learning is one of the things that separate humans from all other species. With relational learning, individuals can manipulate abstract structures and formulate new conceptual ideas. The new individual structures of knowledge are referred to as schema (Skemp, 2006). Individual schemas are then stored as knowledge and utilised to construct further knowledge when introduced to other structures of information (Skemp, 1987). When schemas are joined to form a new structure of knowledge, they become a concept. According to Skemp (1987), the ability for learners to put new mathematical experiences together with previously learned mathematical schemas and concepts to make a mathematical application is the highest level of learning strategy that learners possess. Skemp (1987) identified this highly developed mathematical learning strategy of utilizing schema and concepts as relational learning. When considering whether one form of learning mathematics was more ideal than the other, Skemp (2006) once again compared these two forms of mathematical learning to the two processes described in learning how to manoeuvre around a new town. Could the individual in both scenarios get from point A to point B? The answer is "yes," but an educator must ask a more probing question: Which is better, instrumental or relational learning, for students to master mathematics and be able to build future concepts of knowledge? Skemp (1987) believed that learners utilizing relational learning rather than instrumental learning would progress more successfully in their mathematical development (Skemp, 1987).

Skemp issued four statements of learning virtues supporting the usage of relational learning versus instrumental learning. First, Skemp (2006) acknowledged that learners who used relational learning strategies became quickly independent in reaching their schema of knowledge. Second, the ability for learners to build their own schema of knowledge was found to be intrinsically satisfying to the learners (Skemp, 2006). Skemp's third virtue of relational learning described that the first two virtues enhanced a learner's level of confidence and therefore, encouraged the learner to pursue additional schemas of knowledge. The final virtue for developing relational learning over instrumental learning was that learners would come to realize that their schemas are never complete (Skemp, 2006). As relational learners developed schemas and concepts of knowledge, Skemp discovered that they became ever more aware of endless learning possibilities. With relational learners' gaining confidence in the third virtue and awareness of endless learning potential in the fourth, Skemp (2006) concluded that relational learning leads learners to self-reward; therefore, the learning process becomes meaningful and self-perpetuating. These four learning virtues of relational learning were not evidenced by the learners of instrumental learning.

On the contrary, Skemp (1987) found students who utilized instrumental learning while developing mathematical skills often reported boredom, disconnect from the content material, and a lack of purpose for performing the mathematical tasks (Skemp 1987). Skemp's (2006) mathematical theory of relational understanding enhances the theoretical framework of this study by suggesting the need for instructional practices that foster specific learning strategies so students can best develop mathematically.

The Implication for the Classroom

The main point of Skemp's argument is that learners construct schemata to link what extensive hierarchy of concepts. We cannot successfully form any particular concept until we have formed the entire subsidiary ones upon which it depends. I would, therefore, argue as a mathematics teacher that, although instrumental teaching has its place, we should always be thinking more about maximizing relational learning opportunities for our students as we engage them in our classrooms. The theoretical framework in this study provides three distinct inferences regarding student learning and instructional practices. First, teachers should not limit the mental growth of students by teaching to their current level of knowledge, but rather challenge and push each learner to new levels of learning (Vygotsky, 1931). Second, teachers should recognize the impact and importance of instructional practices on the student's learning environment (Bruner, 1966). Finally, teachers should foster a style of teaching that specifically promotes relational understanding in mathematical learners (Skemp, 2006). The foundation of this study is the premise that students' mathematical achievement is contingent upon the style of teaching and teaching techniques utilized by the teacher during the learning process.

Constructivism in Mathematics Education

Designing a constructivist context for learners to re-evaluate their mathematical ideas Lerman (1993) argues that constructivism has been overwhelmingly approved within mathematics education research since it seems to blend with existing teaching strategies. At the root of mathematics teaching in a constructivist context is the teacher's perspective on the role of the learner. From this perspective, the teacher should facilitate the learner to take on a central role in constructing his/her own experience and knowledge rather than in imposing knowledge on him/her. Therefore, the acquisition of mathematical knowledge becomes a learner-based activity rather than a passive activity involving the memorization and acceptance of an independent body of truths. Thompson (1992) stated: "Students engage in purposeful activities that grow out of problem situations, requiring reasoning and creative thinking, gathering and applying information, discovering, inventing, and communicating ideas, and testing those ideas through critical reflection and argumentation" (p. 128).

It has been suggested that when teaching, learners should be directed towards "mindful activities" which enable students to articulate their ideas (Adams, 2006) and allow them to construct their knowledge. In this sense, the mathematics teacher's role can be regarded as that of facilitator and coach for students learning rather than expert. From the constructivist perspective, teaching involves designing new environments in which learner's intellectual structures can appear and change (Joyce et al., 2004). Technology allows constructivist teachers of mathematics to design learning environments that foster interest and promote experientially based understanding acquired using collaboration and possibly also through quality social interaction (Pateman and Johnson, 1990). An ideal constructivist context is a "place where learners may work together and support each other as they use a variety of tools and information resources in their guided pursuit of learning goals and problem solving activities" (Wilson, 1996, p. 5). Jaworski's (1991, 1994) work implies that good mathematics teaching takes place only by questioning, experimenting, discovering, constructing, conjecturing, reflecting, and discussing. To suit the rationales of the present study, technology-based professional learning was created in a context wherein participants could engage with investigational mathematical tasks using an interactive process of conjecture, criticism, critical reflection, investigation and collaboration. A constructivist environment is a place where the teacher considers students" activities, thus facilitates students" activities to help them make mathematical relationships and patterns, and discuss mathematical meanings, instead of acting as an expert imposing fixed information. In response to von Glasersfeld's assumptions, Jaworski (1996) implied that "the power of constructivism for mathematics education is encapsulated in this second principle" (p. 2). In this principle, viable mathematical knowledge which fits with experience and evolves through modification and social interaction is based on a learner's experience of the world. Learners can only know what they have built through new experiences and can modify concerning their further experience. This argument can be applied to mathematics teaching: the teacher's role should be to guide and support students' creations of viable mathematical notions instead of conveying correct ways of doing mathematics.

The teacher may simplify this process by designing a constructivist environment where the learner works with conceptual problems and involves a conversation with both peers and teacher. In this situation, rather than just content and prescribed tasks, thinking of students' activities becomes important in establishing effective mathematical learning environments. The evidence from research on psychology illustrated that learning could not take place by passive absorption of information only, but instead, in many situations, learners approach each innovative task with previous knowledge, fit in the new knowledge and create their meaning (Reynolds, 1999). This approach allows learners to establish a link with what they already know with new ideas. The research that will be conducted and described in the following chapters suggests that technology can be used to create an active experientially based learning environment.

Role of the Teacher in a Learner's Cognitive Development

Vygotsky's ZPD concept highlighted the role of the teacher in increasing a child's learning potential. His observational study (1930) provided the evidence necessary for future researchers to explore instructional techniques. Vygotsky's (1935) example of an infant was initially walking by holding on to an adult's hand, even though the infant could not yet walk alone, supported the theory of zone of proximal development (ZPD). The analogy of an adult guiding an infant's steps before the infant was developmentally ready allows educators to make similar assumptions about other cognitive and internal development connections. Therefore, the ZPD directly pertains to teacher-student interaction and provides suitable justification for teachers initiating knowledge beyond students' current performance levels.

One can also apply the ZPD theory to today's educational settings, revealing that conceptual skills and abstract thought can be stimulated in students through teachers' instructional techniques. Once aware of the ZPD and how this takes place in the learning process, a teacher can choose to utilize an instructional practice that guides the learner into greater depths of knowledge rather than merely delegating or dispensing informational titbits to the learner. Teachers can prompt students to question new learning strategies that are just beginning to develop. A teacher's prompt to delve further into the learning will challenge student thinking and possibly extend the learning. Likewise, Bruner (1966) compared mental growth to the rises and rested of a staircase. As the learner's knowledge is nurtured, the concepts eventually mature and advance to the next level of learning. Rather than focusing on a student's readiness to learn, Bruner (1966) believed that the staircase of learning potential was present for every learner, but environmental influences, including instructional tactics, could halt, slow, or advance the learning.

Bruner believed that learning is indeed sensitive to nature, but the learner is rather adaptive to a teacher's range of instructional approaches (Bruner, 1985). In his book Theory of Instruction, Bruner (1966) confirmed Vygotsky's finding that a learner's growth and development were assisted by various external means. Bruner (1985) stated, "Any learner has a host of learning strategies at his or her command," (p.8), but it is the teacher who equips students with the "procedures and sensibilities" (p. 8) to lead them to their potential for obtaining knowledge. Conversely, if a teacher can lead students to their potential, a teacher may also fail to provide students with opportunities that build complex thought patterns. Bruner (1985) recognized that certain educational environments could potentially be responsible for irreversible learning deficits in students. Bruner (1966) warned that these potential learning deficits could be established before entering a formal educational system. Whether the instructor is a parent or a formal instructor, the relationship between instructor and student is significant and has long-lasting effects. Bruner (1966) further stated that, since this is a relation between one who possesses something and one who does not, there is always a special problem of authority involved in the instructional situation. The regulation of this authority relationship affects the nature of the learning that occurs, the degree to which a learner develops an independent skill, and the degree to which he is confident of his ability to perform on his own, and so on. The relation between one who instructs and one who is being instructed is never indifferent in its effect upon learning. (p. 42)

With the current educational dilemma, one may find Bruner's wisdom even more relevant today. Bruner (1966) stated, "We are entering a period of technological maturity in which education will require constant redefinition" (p. 32). He also explained, "The period ahead may involve such a rapid rate of change in specific technology that narrow skills will become obsolete" (Bruner, 1966, p. 32). Indeed, education has evolved greatly over the past few decades, and Bruner correctly acknowledged that without the mastery of simpler skills, more elaborated ones would become increasingly out of reach for students. Therefore, Bruner suggested that educators provide students with opportunities to share in dialogue, paraphrase their thoughts, and internalize their learning to reach higher skills. He also charged educators to facilitate the exploration of learning to encourage learning and problem solving (Bruner, 1966).

To address the accelerated change within the educational system, Bruner proposed a theory of instruction: A theory of instruction is prescriptive in the sense that it sets forth rules concerning the most effective way of achieving knowledge or skill. It provides a yardstick for criticising or evaluating any particular way of teaching or learning. (Bruner, 1966, p. 40). Bruner's (1966) Theory of instruction is prescriptive, rather than descriptive, in the sense that instructional methods are prepared by educators based on how the material can best be learned by the student. Bruner's Theory of Instruction consists of four major principles that are practical for not only analyzing instruction but for determining the best method to lead a child toward learning. First, Bruner declared that instruction should specify which educational experiences most effectively lead a learner toward learning. With various instructional methods producing a range of educational experiences for learners, teachers should be mindful of the means by which material is delivered to students to reap the most benefit from the experience. Second, instruction must identify how a body of knowledge should be organized so that learners can readily grasp the concept. When teaching a complex body of knowledge, teachers should examine the structure of the knowledge to provide students with smaller components of information at a given time. By breaking the body of knowledge down into more manageable bits of information, students build mental frameworks that enable them to obtain the whole body of knowledge. Third, instruction must detail the manner and order in which the material should be disseminated for learning. For example, if a teacher is to prepare lessons about the Laws of Motion, the teacher would need to decide how to best introduce the material, so students gain the most understanding. The teacher may choose to provide students with experimental motion experiences before mentioning the laws. The teacher may probe students' inquiry by questioning the relationship between forces and motion. In contrast, the teacher might simply state the laws of motion, display the mathematical formulas for each, and follow up with the discussion. Bruner (1966) suggested that the sequences teachers choose to deliver material have an impact on how well students attain the overall body of knowledge. Bruner provided educators with the flexibility to determine the pace and choice between extrinsic and intrinsic reward, once the material was released to students. Fourth, Bruner believed that instruction must utilize both types of reward, but that student learning progressed further with intrinsic rewards. Bruner's theory of instruction reminds educators that the purpose of teaching is not only to supply students with a prescribed body of knowledge but also to supply them with the know-how to process knowledge.

A statement of caution about the role of the teacher in Bruner's Theory of instruction prompted, in part, the rationale for this study. Bruner warned educators to avoid the problem of interfering with students' ability to take over their role in the learning process (Bruner, 1966). Grasha (1996) also recognized the importance of teacher-student interactions in the learning process. Student learning remains contingent upon teacher-student encounters and the level of success of those two-way encounters. He said that students "develop through changes in teacher and student perceptions of each other, their actions toward each other, and the 'give and take' inherent in their encounters" (Grasha, 1996, p. 41). Bruner explained that students can develop a dependence on teacher assistance in their learning and that teacher-student communication cycles can sometimes stall or even block the learning process (Bruner, 1966). Although Bruner acknowledged that learners were capable of coping and adjusting to various instructional techniques, certain teaching practices could hinder student development if the teacher's style is not preferred by the student.

The theoretical framework in this study provides three distinct inferences regarding student learning and instructional practices. First, teachers should not limit the mental growth of students by teaching to their current level of knowledge, but rather challenge and push each learner to new levels of learning (Vygotsky, 1931). Second, teachers should recognize the impact and importance of instructional practices on the student's learning environment (Bruner, 1966). Finally, teachers should foster a style of teaching that promotes explicitly relational understanding in learners of mathematics (Skemp, 2006). The foundation of this study is the premise that students' mathematical achievement is contingent upon the style of teaching and teaching techniques utilized by the teacher during the learning process.

Teacher-Centred Approach to Learning

Beginning in the late 1970s researchers in mathematics education focused on what teachers could do to be more effective in helping their students learn to compute. Researchers who studied effective teaching used the teacher-centred model. They focused on the actions of teachers. Two bodies of research within this approach has been reviewed. The first is the research conducted by Good and Grouws that focuses on the actions of teachers. The second review is Leinhardt's body of studies that focused on teacher thinking.

Good and Grouws used the research paradigm called "process-product." These studies focused on teacher behaviour (process) that affects student behaviour which in turn affects student performance or achievement (product). Mathematics education researchers built upon the experimental and correlational research of others in answering questions about what effects teachers' instruction had on student achievement. These researchers identified the behaviour of effective teachers associated with student achievement. By observing classroom teachers, Grouws, a mathematics educator became interested in the effects of teacher behaviour on student mathematics learning (Good and Grouws, 1979). Good and Grouws studied more than 100 third- and fourth- grade teachers who taught mathematics in the same school district. They used student performance on standardized achievement tests in mathematics to compare their teachers' effectiveness. They observed these teachers for more than three years and compared their test scores. The researchers found that some of the teachers produced more mathematics learning than others even though they were using the same textbook and were teaching students with comparable abilities. To follow up this study, Good and Grouws began to observe the teachers to find out the behaviours and instructional strategies that were associated with high and low achievement of students. They found that teachers differed significantly in their classroom behaviour. More effective and less effective teachers taught very differently. The teachers who obtained higher levels of student achievement provided students with a more precise focus on what they wanted their students to learn. These more effective teachers provided more time for students to practice new procedures with immediate teacher feedback. This time was called the developmental portion of the lesson. Good and Grouws further claimed that the developmental time that teachers gave to students affected their achievement. From these studies, Good and Grouws hypothesised that certain teacher behaviours affected higher or lower achievement of students. The teacher behaviours were congruent with the direct instruction model advocated by other researchers in education (Rosenshine, 1978). The model included daily review, development of the lesson by lecture and teacher demonstrations of procedures, supervised practice, independent practice, and conclusion of the lesson. Using this model, Good, Grouws, and Ebmeier (1983) taught teachers the instructional behaviours associated with high student achievement. In an experimental study called the Missouri Mathematics Program, Good et al. studied the effectiveness of their teacher training program. They assigned half of 40 classrooms to the experimental group and the other half to the control group. The experimental teachers were taught the teacher behaviours that the researchers believed increased student achievement. Good et al. gave standardized achievement tests as pre- and posttests. After two and half months, students in the experimental classrooms performed significantly better than those students in the control classrooms. By collecting data from the regular end-of-the-year achievement tests, the researchers found that students who were taught in the experimental classrooms continued to achieve higher test scores than students in the control classrooms.

Good et al. observed both experimental and control classrooms to record the frequency of particular teacher behaviours that influenced student achievement. They studied how teachers sequenced lesson segments: the introductory segment, the developmental segment, and the conclusion. From these observations, Good et al. developed a model of teaching called active teaching. Active teachers increase the amount of time that students practice and experience success on skills before being tested. They evaluated whether the students were performing accurately. These teachers re-taught the content if the need arises. The teachers in the experimental classrooms used whole group instruction to introduce concepts, demonstrate procedures, and provided drill and practice for students with immediate teacher feedback. One of the findings from the Good et al. study was that the teacher should present the lesson in small steps with students practising after each step. The teachers guided the students through frequent successful practice. The researchers concluded that teacher actions did make a difference in student achievement. Their findings further demonstrated that teachers could be trained to provide instruction that would increase student achievement. The findings from the Missouri Mathematics Program that teachers do make a difference in student learning spurred educators to investigate how these active teachers planned and organised their lessons. They identified a model of teaching that could be used to improve performance regardless of content. Even though this type of research was teacher-centred, it had not addressed all of the issues concerning the complexities of teaching. The researchers did not ask questions such as the following: How does the teacher plan what to teach? What is the teacher's thinking during instruction?

The teacher is thinking about teaching-before and during lessons— became an important question and began to play a role in research on effective teaching (Clark and Peterson, 1986). They suggested in their research on teacher thinking that teacher planning and interactive teaching are influenced by the teacher's reflection. They concluded from classroom observations and interviews with teachers, both before and after their lessons, that teacher thinking is influenced greatly by the teacher's beliefs and knowledge about teaching. Therefore, teacher thinking had a powerful effect on what and how children learn.

Leinhardt (1986, 1988, and 1989) compared expert and novice teachers' performance to study how teacher thinking affects classroom teaching. She examined the teachers' planning and decision making before lessons and during teaching. Leinhardt's goal was to develop teacher education programs that made novice teaching more like that of the experts. Leinhardt and Smith (1985) explained that the cognitive skill of teaching draws on two types of knowledge: lesson structure knowledge and subject matter knowledge. Lesson structure knowledge includes "the skills needed to plan and run a lesson smoothly, to pass easily from one segment to another, and to explain material clearly" (p. 247). Subject matter knowledge includes knowledge of "concepts, algorithmic operations, the connections among different algorithm(s) . . . Understanding of . . . Student errors, and curriculum knowledge" (p. 247).

Leinhardt and Greeno (1986) observed eight experts and four novice fourth-grade teachers while teaching lessons on fractions for three and a half months a year for three years. They videotaped the teachers' lessons and interviewed the teachers about planning and evaluation of their lessons and fraction knowledge. The expert teachers had more subject matter knowledge of fractions. They connected the concept of fraction-a part of a whole— with the procedures. Expert teachers understood how the operations on fractions— addition, subtraction, multiplication, and division— were connected to each other. Because novice teachers' knowledge of fractions consisted of isolated concepts rather than connections among concepts, they separated addition and subtraction of fractions into two different types of problems. Novices did not see the connection that the concept of fraction brought to the two types of problems. Novice teachers lacked conceptual knowledge of fractions.

In a similar project, Leinhardt (1986) studied elementary teachers to find out what made an expert teacher. She studied seven experienced teachers. Again, the researchers observed and videotaped teachers during mathematics lessons. They interviewed teachers about their views of pedagogy and math knowledge. Leinhardt found that expert teachers' lessons had clarity and a coherent focus on content. Because experts possessed more lesson structure knowledge, they introduced and represented procedures differently from novice teachers. The expert teacher presented clear explanations of each procedure and demonstrated several methods of approaching a problem. Leinhardt (1988) concluded that "experts are unusually good at constructing series of lessons that successfully transmit the content that needs to be learned" (p. 29). Leinhardt (1989) explained that teaching a lesson is a complicated process that consists of three basic components: a lesson segment, an explanation, and an agenda. A lesson segment is an overarching framework that guides each lesson. Each segment has a unique goal, actions, and task environments. The second lesson component, the explanation, is the "actual transmission of the subject matter content" (p. 56). Leinhardt stated that the explanation of the lesson should be the most carefully constructed segment because it presents the new mathematical material. The explanation segment is equivalent to the developmental portion of the lesson discussed by Good et al. (1983). The last component of a lesson is the agenda. Leinhardt (1989) defined an agenda as "a unique operational plan that a teacher uses to teach a mathematics lesson. It includes both the objective or goals for lesson segments and the action that can be used to achieve them" (p.55). Leinhardt noted that teachers' agendas are not formal written lesson plans but mental notes about what they expect to happen during the lesson.

In her research, Leinhardt (1989) found contrasts in the lesson planning of expert and novice teachers. Novice teachers' agenda followed a structured script that was often incomplete and did not include actions of the students. Experts' lesson segments had patterns. Expert teachers planned agenda that were rich in content. Leinhardt concluded that expert teachers weave lessons together. Their lessons begin with a good presentation of content. This presentation affects student learning and performance. Findings from Leinhardt's research (1986, 1988, and 1989) are that expert teachers' lessons are explicit and direct much like the active teaching model of Good et al. (1983). Expert mathematics teachers design lessons that are well sequenced and connected to previous lessons. Their lessons have the closure that binds the concepts and patterns together.

Experts spend less time in transition between lessons. Leinhardt (1986, 1988, and 1989) extended process/product research in mathematics beyond its focus on specific instructional behaviours that affected students' outcomes on achievement tests to a focus on teacher thinking. She furthered the research of Good et al. (1983) and laid a foundation for further inquiry into cognition. This foundation opened the door for others to investigate student thinking. For example, what are the effects of student thinking on student achievement?

A similar study to that of Good et al. and Leinhardt by Livingston and Borko (1989) investigates the nature of pedagogical expertise by comparing the planning, teaching, and post-lesson reflections of three student teachers. (two secondary and one elementary) With those of the cooperating teachers with whom they were placed. Participants were observed teaching mathematics for one week of instruction and were interviewed before and following each lesson. Differences in the thinking and actions of these experts and novices were analyzed by perceiving teaching both as a complex cognitive skill and as improvisational performance. Novices showed more time to consume, less efficient planning, encountered problems when attempts to be responsive to students led them away from scripted lesson plans and reported more varied, less selective post-lesson reflections than experts. These differences were accounted for by the assumptions that novices" cognitive schemata were less elaborate, interconnected, and accessible than experts 'and that their pedagogical reasoning skills were less well developed.

Constructivist Learner-Centred Paradigm

The constructivist learner-centred paradigm represents a definite shift from teacher-directed instruction to learner-oriented thinking. The role of the teacher changes from one of the transmitter to one of a facilitator. Children help make the decisions about topics to be learned and about how learning would take place. This body of research is similar to cognitively guided instruction because its focus is on children constructing knowledge. However, the learner plays a more active part in deciding what happens in the classroom. Constructivist learning is a process by which an individual actively creates and invents new knowledge. Educators who advocate constructivist learning in mathematics say that children create new mathematical knowledge by reflecting on their thinking and actions while they solve problems. Most constructivists trace their origins about learning back to Piaget (1970, 1980). They believe that people learn through a process of individually acting on the world. Piaget said that during the process of disequilibrium, individuals internally experience cognitive conflict when confronted with new information. During disequilibrium, prior knowledge cannot explain new experiences. Therefore, through accommodating new knowledge and assimilating it with the prior knowledge, individuals form internal structures of knowledge unique to them (Fosnot, 1989; Piaget, 1970, 1980; Schoenfeld, 1992). Piaget (1970) claimed that interactions in the classroom could facilitate knowledge development because interaction can create cognitive conflict that can change thinking. Piaget claimed that peer interactions stimulate student reflection about ideas that other students present.

Also contributing to constructivist learning theory in mathematics are the ideas of Vygotsky (1962, 1978). Vygotsky suggested that individuals construct knowledge in the zone of proximal development through social interaction with more knowledgeable others. During interactions with others, individuals learn as they communicate their thinking. Learning happens when individuals construct their own interpretations through language. Vygotsky (1978) noted that social interaction not only initiates changes in thinking but also alters current thinking. Individuals gradually internalize the talk that occurs during interactions. "Any function in the child's cultural development appears twice or on two planes. First, it appears on the social plane, and then on the psychological plane. . .. Social relations . . . Underlie all higher [cognitive] functions and their relationships" (Vygotsky, 1978, p. 57). With this statement, Vygotsky explained the internalization of knowledge as both a social and psychological activity. Cobb, Yackel, and Wood (1990) and Von Glasersfeld (1989) have developed a way of thinking about constructivist learning that combines the ideas of Piaget and Vygotsky. Learning is a social and cognitive process in which children share their thinking. Both perspectives, the cognitive and the sociocultural, are essential in the learning process and they complement one another (Cobb, 1994). This line of thinking is called social constructivism (Cobb et al., 1990). It puts great emphasis on communicating and negotiating as a process for constructing knowledge.

In the mathematics classroom, teacher and students continually use each other's' contributions to resolve disequilibrium and develop individual knowledge. However, during the process of negotiating and sharing with a knowledgeable teacher, students understand the mathematical meanings of the wider society, taken-as-shared-meanings (Cobb et al., 1991). Students adapt to the actions of others in the course of ongoing interactions. During group discussions, individuals construct their knowledge as they make sense of others' ideas and resolve disequilibrium caused by differences between their ideas and those of others. Teachers who strive to help children construct knowledge of mathematics often change their roles as teachers. A teacher becomes a mediator between students' understandings of knowledge and the culturally established mathematical meanings (Cobb, 1994). Teachers guide and support students' thinking rather than act as the authority for correct answers. The students must decide on the correctness of their knowledge constructions. When children are constructing their knowledge, teachers must be able to pose tasks that help children construct knowledge upon their prior understandings.

Examples of Constructivist Activities

In the constructivist classroom, students work primarily in groups and learning and knowledge are interactive and dynamic. There are a great focus and emphasis on social and communication skills, as well as collaboration and exchange of ideas. This is contrary to the traditional classroom in which students work primarily alone, learning is achieved through repetition, and the subjects are strictly adhered to and are guided by a textbook. Some activities encouraged in constructivist classrooms are:

- Experimentation: students individually experiment and then come together as a class to discuss the results.
- Research projects: students research on a topic and then present their findings to the class.
- Field trips. This allows students to put the concepts and ideas discussed in class in a real-world context. Field trips would often be followed by class discussions.
- Films. These provide visual context and thus bring another sense into the learning experience.
- Class discussions. This technique is used in all of the methods described above. It is one of the most important distinctions of constructivist teaching methods.

The Traditional Classroom

In the traditional classroom, learning begins with parts of the whole expanding to the whole. And the emphasis of the teacher is on basic skills, and there is strict adherence to fixed curriculum, textbooks and workbooks. The instructor "dishes out" knowledge and students are at the receiving end of it. The instructor assumes directive, and authoritative role and assessment are mainly through testing to find out the correct answers; knowledge is inert, and students work individually. Assessment of student learning is viewed as separate from teaching and occurs almost entirely through testing. Every lesson relies heavily on textbooks.

How Constructivist-Based Mathematics Lessons Look Like

Constructivist based mathematics lessons are characterized by student-centred instruction; students are introduced to varying methods of solving problems with different opportunities for students to create their knowledge. That is, learning activities in constructivist classrooms or lessons are characterized by active engagement, reflective thinking and problem solving (Abrams and Lockard 2004). The acquisition of knowledge is affected by the external world within which the individual learner finds him/herself, and it is based on the individual's ability to use his/her cognitive structures to construct knowledge for him or herself (Glasersfeld, 1989). According to Ward (2001), the teacher plays an important role in assisting and guiding students in constructing accurate knowledge as they experience the environment and come into contact with different forms of ideas. In such classrooms, the teacher acts as a facilitator in the teaching-learning process by providing opportunities for students to learn and construct knowledge. The role of the teacher is to act as a knowledgeable adult who supports the learner to achieve ends that would be unattainable if the student worked on his/her own (Goodchild 2002b). One of the main features of a constructivist classroom is social interaction. According to Blanck (1990), individual mental activities are uniquely human, and the individual's creation of knowledge is to a large extent influenced by his/her innate capabilities and characteristics. Blanck, however, adds that, despite the importance of the learner's innate characteristics, the environment influences the creation of new knowledge. The learner is more likely to retain this new knowledge for a longer period if it is constructed through active interaction with the learning environment; giving the learner the opportunity to explore his/her environment helps them to retain newly constructed knowledge and take responsibility for their learning (Goodchild 2002b).

Research on What Constitutes Good Mathematics Teaching

Wilson, Cooney and Stinson (2005) in their study on what constitutes good mathematics teaching and how it develops, examined nine experienced and professionally active teachers' views of good mathematics teaching and how it develops through a series of three interviews. The interviews were couched in the context of the teachers mentoring student teachers. The article compared and contrasts various scholars' and researchers views about what constitutes good mathematics teaching and how it develops with the perspectives held by nine experienced and professionally active high school mathematics teachers of what they also consider good mathematics teaching to be and how it develops. Though there is a large body of research on the notion of becoming a good teacher, what is often lacking in this literature are what teachers themselves think constitutes good teaching and how it develops. The article sought to address this gap by hearing from the experienced practitioners themselves.

Secondly, the article compared and contrasted the nine teachers' notions of what constitutes good mathematics teaching with those of various scholars like Dewey (1916), Polya (1965), Davis and Hersh (1981), and positions stated in various reform documents by other researchers. The study revealed that teachers thought good teaching required: prerequisite knowledge, promoting mathematical understanding, connecting mathematics, visualizing mathematics and, assessing students understanding among others.

Recommendations from the Teachers

(i) The teachers recommended engaging the students by meeting them at their mathematical level. They praised student teachers who could "come down to their students' level."

(ii) They also recommended that teachers adopt a variety of approaches to teaching with the assumption that there would be at least one approach that would reach each learner.

(iii) The teachers highly recommended the experience for their student teachers as a means to becoming good teachers.

(iv) The nine teachers also recommended technology usage, and mathematical activities to help their students to understand the mathematics they learn.

Prerequisite Knowledge and Promotion of Mathematical Understanding

The teachers reported that mathematics teachers needed to have extensive knowledge of both mathematics and their students to teach for understanding, sequence lessons, make transitions between topics, understand student questions, provide good examples and maintain the necessary confidence in front of a class of students. The implication of this finding is that teaching and learning of mathematics in our schools will improve significantly if serious attention should be paid to the prerequisite knowledge of teachers before they are allowed to teach. The implication for mathematical understanding is that, if teachers teach well for students to have good mathematical understanding, they will be able to use their mathematics outside the classroom and in the study of other mathematics courses. Conceptual understanding of the subject will promote the love for the subject and its usefulness and application in the real world.

Connecting and Visualizing Mathematics

The implications of connecting mathematics during teaching and learning of the subject are captured in the following quotation by the National Council of Teachers of Mathematics:

> When students can connect mathematical ideas, their understanding is deeper and more lasting. They can see mathematical connections in the rich interplay among mathematical topics, in contexts that relate mathematics to other subjects, and in their own interests and experience. Through instruction that emphasizes the interrelatedness of mathematical ideas, students not only learn mathematics, they also learn about the utility of mathematics. (NCTM, 2000, p. 64)

The teachers in the study of Wilson et al. (2005) also made specific reference to helping the student visualize mathematics by using computers or calculators or concrete materials since this helps them to retain better the subject taught. This implies that if appropriate teaching and learning materials are used by classroom teachers, students understanding of the subject could be improved significantly. An example to buttress this point is the statement made by a student in Elizabeth's class when she taught the unit circle and trigonometric functions using visual representations. "I have never understood that until this moment," said a student.

Assessing Students Understanding and Refraining from Telling

A statement about assessing students understanding is well captured in a standard document as follows: "The teaching principle is stated as follows: Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well" (NCTM, 2000, p.16). The implication of this context for teaching and learning is that there is the need for frequent, quick evaluation to inform the teacher whether or not to proceed with the lesson or to review what has been taught. This is to avoid a situation where the teacher talked for too long before pausing to find out if students have understood what has been said so far. It also requires sometimes reading students' expressions to assess that understanding if the need be. The teachers in the study agreed that a good lesson is where the teacher moves away from providing information that the students have to memorize or mimic. The reason being that, students just knowing what the teacher tells them do not prepare them to solve real problems or use mathematics outside the classroom. The implication is that students should be guided to discover mathematics for themselves.

Engaging and Motivating Students

Having these types of lessons is an effective way of engaging students. These include using technology, doing group work, hands-on activities such as measuring, having students write, applying mathematics and laboratory activities. Truly, the number one job of teaching is to motivate the learner to learn. Therefore, teaching strategies that help students to understand better would, in turn, motivate the students. The interest of the teacher in the subject he teaches also serves as motivation for the learner.

Effective Management Skills and Reflection

The teachers in the study agreed that classroom management is necessary to achieve other goals. However, the teachers commented that classroom management is learned through experience in the classroom. The implication of this for teaching and learning is that the more experience teachers we have in our schools, the better. Care should be taken to avoid experience teachers leaving for greener pastures.

Reflections serve as a tool to systematically observe, analyse, critique, and reflect on one's classroom practices. Teachers who reflect on their teaching will certainly avoid certain pitfalls thereby improving their teaching.

Summary of Wilson et al. (2005) Study

Some studies have shown the effectiveness of the constructivist approach to teaching and learning in contrast to the traditional drill and learn by rote approach (Hmelo-Silver, Duncan and Chinn, 2007; Steele, 1995). A study by Steele, (1995) on "A constructivist approach to mathematics teaching and learning…." revealed that using constructivist learning strategies has positive gains. For example, such strategies tend to create an exciting environment for students to learn mathematics and enhance their self-esteem. According to this study, when students learn to construct their knowledge, they tend to have control of mathematical concepts and think mathematically. It is very clear that children are not wired to sit and listen obediently to monologues. Hence radical changes have been advocated in many research reports on mathematics education. Unfortunately, many educators are rather focusing on alterations in content rather than the recommendations for fundamental changes in instructional practices. Many of these instructional changes can best be understood from a constructivist perspective. Although references to constructivist approaches are pervasive, practical descriptions of such approaches have not been readily accessible. This research is aimed at producing practical descriptions of a constructivist approach to mathematics teaching and learning.

Chapter Summary

Although there are various approaches to mathematics teaching and learning that research has identified, there is enough literature to support the fact that constructivist learning theory has the potential to be a basis for mathematics teaching and learning (Von Glasersfield, 1984).For this reason, the theoretical framework upon which this study was based focused on constructivism as a theory of learning, Vygotsky's Zone of Proximal Development (ZPD), Bruner's (1985) Theory of Instruction, and Skemp's Mathematical Theory of Relational Understanding. This chapter also identified four groups of influences as having an impact on how children learn mathematics. They are (i) the demands, constraints and influences of society (ii) the knowledge, skills and understanding which the learners developed outside the school setting, (iii) the teaching and learning resources used by the teachers and (iv) the teacher. The chapter ended by providing a justification for improving the instructional practices of teachers in the field of mathematics education.

CHAPTER THREE

RESEARCH METHODS

Overview

This study was basically about how students experience mathematics when constructivist teaching and learning of mathematics are implemented at Junior High School One classroom. This chapter describes explicitly the research procedures and techniques employed in the study. It has also addressed the ethical issues connected with the study and access to the school as well as confidentiality issues. The issues considered include the research design, population, sample and sampling procedure, instruments for data collection, ethical and confidential issues and how the data were analysed. Finally, the issues of validity and reliability were also fully addressed. The study was well suited for qualitative research methods. Specifically, it was an ethnographic research study. In a qualitative ethnographic research study such as the one undertaken by the researcher, description and interpretation of constructivist approach to mathematics teaching and learning is only possible in context, and any effort to share what is learned from the teacher and the learners requires an awareness of the context (Merriam, 2009). Hence, a qualitative ethnographic research design was used to address the research questions of this study. Data for this study were collected using the Mathematics Belief Scale (MBS), lesson observations, interviews, and document analysis to answer the research questions adequately (Ary, Jacobs and Razavieh, 2002). Merriam (2001) suggests that observation becomes a research tool if it serves a formulated research purpose if it is planned deliberately, if it is recorded systematically, and if it is subjected to checks and controls on validity and reliability. Ary, Jacobs and Razavieh (2002) contend that one way to enhance validity is to carefully define the behaviour to be observed and to train the people who will be making the observations. However, Ary, Jacobs and Razavieh identify two sources of bias that affect validity: observer bias and observer effect. According to Ary, et al. (2002) observer bias occurs when the observer's perceptions and beliefs influence observations and interpretations, while observer effects happen when the people being observed behave differently just because they are being observed. These biases were carefully addressed by studying the Teacher Mr Yamoah and his class for a very long time so as not to affect validity in this study.

Apart from observation, interviews were used to collect information that could not be observed directly. Interviewing is necessary when the required information cannot be observed such as feelings, beliefs, perceptions and opinions (Merriam, 2001). About document analysis, Creswell (1998) and Merriam (2001) contend that document analysis, as a data source, is as good as observation and interview. However, it could be argued that document analysis has the potential to reveal information that the interviewee was not ready to share and also information that could not be available during observation. The multiple data sources allow for triangulation of data to reduce bias and at the same time to develop a better understanding of the issues under study.

Research Design

Qualitative research is the method of research in which most of the data is presented in a descriptive form and is conducted in natural settings such as schools, classrooms, families, neighbourhood and other places (Bogdan and Biklen, 1992). Qualitative research has five types of design namely: a case study, phenomenology, ethnography, grounded theory, and applied research. In this particular research, the ethnographic research design was used to investigate this basic school teacher while he taught mathematics at Junior High School One. This design was found to be the most suitable for the five qualitative research types because it has the following characteristics: (a) investigation of a small number of cases, perhaps just one case, in detail (b) The research is conducted within the particular setting under study; (c) the researcher is the main research instrument; (d) the data are descriptive; (e) the researcher obtains meaning from the context and from the participants' perspectives; (f) the data are analysed inductively; (g) the research process is more important than obtained products; and (h) a holistic picture is created (Bogdan and Biklen, 1982; Reichardt and Cook, 1979; Cohen and Crabtree, 2006).

Characteristics of Ethnographic Research

The main characteristics of ethnography are: Contextual, unobtrusive, collaborative, and interpretative. A brief description of each of the above characteristics is as follows:

Contextual

The research is carried out in the context in which the subjects normally live and work. The researcher conducting this type of study is considered a participant-observer, meaning that he lives among the people he is studying and participates in their way of life as much as possible. Under this characteristic, the researcher became an insider, whose aim was to look at what was happening in the classroom through the lens of the teacher. The researcher used the natural setting of the classroom to conduct his observation of the teacher's classroom instruction and the classroom interaction. After school, the researcher interviewed the teacher in the classroom.

Unobtrusive

The researcher avoids manipulating the phenomena under investigation. Throughout the study, the researcher ensured that no student asked him for help in the classroom for any assignment, or group work. During all the interviews conducted, whether audio or video recordings, the researcher tried as much as possible not to provide any leading question.

Longitudinal The research is relatively long. This particular research was conducted for almost one academic year.

Interpretative The researcher carries out interpretative analysis of the data

. **Organic** There is an interaction between questions and data collection/interpretation.

Bogdan and Biklen (2007) offer this description of the features of qualitative research:

> the data collected have been termed soft, that is, rich in descriptions of people, places and conversations, and not easily handled by statistical procedures. Research questions are formulated to investigate topics in all of their complexity, in context. While people conducting qualitative research may develop a focus as they collect data, they do not approach the research with hypotheses to test. They are also concern as well about understanding behaviour from the subject's frame of reference. (p. 43)

In addition, there are five key features of qualitative research which are incorporated into the present study. First, the study is naturalistic (Patton, 2001). In qualitative research, the setting and the people in the setting are the data, and the researcher is the "instrument" which obtains the data. Data are collected primarily through in-depth interviews and participant-observation. Second, the study utilizes descriptive data (Creswell, 2007). The data take the form of words and observations rather than numbers. This form of data is used because the researcher is looking for knowledge and understanding rather than a definitive answer. Third, the study is concerned with a complex social process (Taylor and Bogdan, 1998). By looking at one constructivist teacher and his students over time, one is better able to understand the complex social process in which behaviours occur. Fourth, data are analysed inductively (Bogdan and Biklen, 2007). Data analysis uses a bottom-up rather than top-down approach. As when one puts together the pieces of a puzzle without first seeing the picture, the results of the data analysis take shape as the investigator examines the parts and then assembles them into a theme or series of themes. Fifth, the search for meaning, as the participants understand it, is the primary goal of the study (Merriam, 2009; Wolcott, 2009). Unlike other research approaches, qualitative researchers attempt to answer their research questions holistically (i.e., contextually). The setting, its people, their activities, their interactions, and their points of view are all taken into account. Qualitative researchers, interview participants, spend time in the setting in order to understand the context in which behaviours occur, and review documents related to the focus of the research in order to construct the meaning a particular situation has for the people who are a part of it (Bogdan and Biklen, 2007; Wolcott, 2009).

The Researcher is the Main Research Instrument

During the mathematics lessons, the researcher observed and took notes. He videotaped what was happening in the classroom. While students worked during class, the researcher occasionally observed what the students were doing. During interviews with the teacher, the researcher took notes and audiotaped while the teacher answered the questions. Structured interview was held to help the researcher to understand the teacher's thoughts and why he made certain decisions he made. Questions were also prepared to help evaluate the teacher's knowledge and to understand his beliefs about mathematics and mathematics teaching and learning. Questions were also planned to encourage the teacher to reflect on why he chose particular teaching methods or assessments. The researcher asked the teacher to evaluate specific examples of students' work. The teacher was asked how and why he might teach differently if he had to teach the same lesson again. The researcher asked the teacher what he thought about the students' learning experience.

Being a participant observer in the mathematics classroom helped the researcher to understand how the teacher taught for an understanding of mathematics and how the students reacted to his teaching. Rather than using a quantitative instrument that might only reveal numbers that fit into statistical models, the researcher was the instrument of research, which evaluated what was going on in the classroom. The researcher analysed the data regarding answering his research questions.

Study Area

The X Community Basic School where the researcher carried out his study is located at the heart of the X SSNIT Flats, Accra. It is one of the Basic Schools that serve the needs of those in X SSNIT Flats and its environs. It is about two (2) kilometres from the X main lorry station. Even though the Basic School is located at the heart of the X SSNIT Flats, about 80% of the children from the flats who attend this School are either house helps, maidservants or distant relatives of workers who occupy the flats. Most of the biological children of the occupants of the flats attend basic private schools in other parts of Accra. One interesting thing also discovered by the researcher was the fact that when the biological children of the parents in the flats were on mid-terms, some of these maids, house helps, etc. (who were not on mid-terms) at X Community JHS in particular, were asked to stay home to look after their "younger siblings" while their parents went to work.

Some of these factors coupled with the fact that the JHS students had limited study time at home for reading through their notes and to do their assignments have gone a long way to affect the quality of academic work that these students exhibit in class.

The total population of the Junior High School students was three hundred and fifty (350) with 18 teachers including the headmistress. There were two JHS mathematics teachers of which Mr Yamoah was one. Mr Yamoah has taught JHS mathematics for the past 12 years. According to him, he had attended some workshops and seminars organized for Basic School Mathematics teachers by the Ghāna National Association of Teachers (GNAT), X Municipal Assembly and the Mathematical Association of Ghana (MAG) among others. The number of students in Mr Yamoah's JHS 1 class was 58. The male students were 30, and the female students were 28.

On the average, about 35% to 40% of the students' parents attended PTA meetings whenever it was called. Many of the parents would prefer to send other relations living with them to such meetings. This was partly because a large proportion of the children at the X Community School were not living with their own biological parents which go a long way to affect their commitment to the children's education. The teachers on many occasions complained that the same parents were seen on a regular basis at other PTA meetings where their own biological children attend school. Some of the students the researcher interacted with after class hours said they come from Oyibi, which is about 15km from the school. Their parents prefer to send them to X Community School because they find conditions there more conducive to academic work than the Oyibi Basic School in their own communities.

Population

The population for this study was all JHS one mathematics teachers in two municipalities in Accra. Majority of these teachers were professional teachers with a minimum qualification of Diploma in Basic Education, DBE while the rest were those who had completed Senior High School waiting to better their results or could not continue immediately due to financial constraints. The highest professional qualification among the teachers was Bachelor in Basic Education, BBE.

Sampling Procedure

Purposive sampling was used to select 12 Basic Schools for this study. The researcher started his observation in the 12 Junior High Schools after securing permission from the various headmasters/headmistresses through writing. A minimum of two JHS one Classroom was identified in each school for the observation. Purposive sampling technique was found to be the most suitable because it gave the researcher the opportunity to select the Basic Schools that satisfied his criteria for the study such as mathematics teachers with minimum professional qualification of DBE in addition to having taught the subject for at least three years at the JHS level. Seven Basic Private Schools and Five Basic Public Schools were selected for the study. In all, 25 JHS mathematics teachers were selected from the 12 Schools. A Mathematics Belief Scale, MBS was administered to all the 25 JHS mathematics teachers on the same day. The teachers with the best scores were identified as those who had the strongest beliefs about constructivist teaching and learning. After scoring the MBS, 12 teachers were found to exhibit strong beliefs about constructivism. Most of the questions on the MBS related to whether the teacher believed that children construct their mathematical knowledge or if children need first to be shown how to solve problems and tasks before they are allowed to do it. This number was further reduced to four based on how the teachers performed in the classroom compared to their responses on the MBS questionnaire. The researcher met the remaining four teachers briefly, and the study was discussed with them in general terms. The meeting helped the researcher to identify the four teachers' levels of interest in participating in the study. For two weeks, the researcher observed closely two of the teachers every school day. The following two weeks of the same month, the researcher observed the remaining two teachers every school day for another two weeks. By the end of the first month, the researcher eliminated two more teachers purely because of their classroom practices which were inconsistent with their responses to the MBS questionnaire. The researcher chose two of the teachers for two reasons: (a) They seemed to believe in the constructivist theory of learning and teaching of mathematics as demonstrated in their classroom practice, and (b) they had begun implementing a constructivist approach to mathematics teaching in their way either consciously or unconsciously. The two remaining teachers were regularly observed but due to the use of video coverage, audio recording and documentations involved, the researcher concentrated on only one of the remaining two teachers by name Mr

Yamoah. He was selected mainly because of his zeal and enthusiasm for mathematics. He had a good rapport with his students and students felt more comfortable when they were with Mr Yamoah than the other teacher. He was very popular among his colleague teachers as well as students.

Description of the Sample

Mr Yamoah is a thirty-six-year-old mathematics teacher at X Community Basic School. He holds a Bachelor of Arts degree in Statistics from the University of Ghana, Legon in addition to his Diploma in Basic Education (DBE). Mr Yamoah has been teaching mathematics for the past twelve (12) years. Out of the twelve years, he has spent four (4) at the X Community Junior High School at the time of the study. Mr Yamoah has constantly been striving to improve his knowledge and upgrade his skills in mathematics teaching by attending some workshops and seminars organized at the district and national levels by Ghana Education Service. Mr Yamoah belongs to the Mathematical Association of Ghana (MAG). He enjoys teaching mathematics, especially at the Junior High School level. He readily admits that it is his favourite subject and yearned for students to become involved in the problems or tasks he presents to them in class or asks them to do as project work. His love for teaching mathematics was evident whenever he was teaching the subject. Mr Yamoah was always enthusiastic and ready to go all length to make the class a lively community of learners. For nearly three terms that I was there in the classroom with him, Mr Yamoah never sat down during any mathematics lesson. Not even when he called upon students to show their workings on the marker board. He bends over each desk to explain a point to a student or group of students and mark their work with his red pen which he never forgets to bring to class. With the problems and activities that Mr Yamoah chose, his interaction and questioning of the students, he had established a community in which the children and him (Mr Yamoah) could build a classroom of mathematics thinkers not only about routine problems but also non-routine ones. The students and the teacher Mr Yamoah had constructed, over time, a friendly learning environment through everyday interactions. He encouraged the students to reason about mathematics, and not to learn it instrumentally.

The headmistress of the X Community Basic School also spoke so well about Mr Yamoah's commitment and dedication to duty. She was pleased that among the teachers in 12 Basic Schools that formed my initial sample, their school was the place I found the most suitable teacher for my research.

Data Collection Instruments

Besides the researcher being the main instrument for data collection, three research instruments were used for this particular study (i) mathematics beliefs scale, (ii) pre-observational structured questionnaire, and (iii) post-observational structured questionnaire. All the three can be found in Appendix A, B and C. The Mathematics Beliefs Scales (MBS) was developed by Fennema, Carpenter and Peterson (1987). Since efforts to contact the developers of the MBS to seek permission to use the instrument proved futile, I used the adapted form of the instrument for the research. This instrument has 40 items on it, and the responses have five options A to E where A-strongly agree, B-agree, C-undecided, D-disagree and E-strongly disagree. It was coded in such a way that the teacher with the lowest score had the strongest beliefs about constructivist teaching and learning. The letter A was equivalent to 1, B was equivalent to 2 and so on. Most of the questions related to whether the teacher believed that children construct their mathematical knowledge or if children need first to be shown how to solve problems and tasks before allowing them to do so.

Goetz and LeCompte (1984) stated, "Credibility mandates that standards of reliability and validity be addressed wherever ethnographic techniques are used" (p. 210). They added that ignoring threats to credibility weakens research and that ethnographers must consider lack of reliability and validity to be serious threats to the value of their results. Validity has to do with "how one's finding match reality" (Merriam, 1988, p. 166). Merriam said that reliability deals with "the extent to which one's finding can be replicated" (p. 170). To ensure that this study met the validity and reliability criteria, I spent ten months collecting data so that I could become familiar with the classroom, students, and the teacher. Also, I used several methods of data collection so that I could compare and contrast the patterns of meanings. The combination of several data collection strategies adopted is what is referred to as triangulation (Creswell, 1998). Several scholars have aimed to define triangulation throughout the years. Cohen and Manion (2000) define triangulation as an "attempt to map out, or explain more fully, the richness and complexity of human behaviour by studying it from more than one standpoint." Altrichter et al. (2008) contend that triangulation "gives a more detailed and balanced picture of the situation." According to O'Donoghue and

Punch (2003), triangulation is a method of crosschecking data from multiple sources to search for regularities in the research data. And according to Audrey (2013) triangulation also crosschecks information to produce accurate results for certainty in data collection. It involves documenting evidence from different sources to shed light on a particular theme or issue. Triangulation in qualitative research is important to validity issues such as checking the truthfulness of the information collected. However, the purpose of triangulation is not only to cross-validate data but also to capture different dimensions of the same phenomenon.

I sought to keep my biases in check. I crosschecked data from interviews with data from observations to try to identify any inconsistencies. I reviewed my findings with the participant to see if they agreed. It is also important that my study would be of use to other researchers. Accordingly, I tried to achieve some external validity or generalizability of the findings. Eisner (1981) said that generalizability is possible in qualitative research because of the issue of the belief that the "general resides in particular" (p. 7). Yin (1989) suggested that analytic generalisation does not rely on samples and populations. He also stated that qualitative research might generalize a particular set of results to a broader theory. Based on the above observation, I extended my findings to the constructivist learning theory.

Validity

Validity in qualitative research means "appropriateness" of the tools, processes, and data. Whether the research question is valid for the desired outcome, the choice of methodology is appropriate for answering the research question, the design is valid for the methodology, the sampling and data analysis are appropriate, and finally, the results and conclusions are valid for the sample and context (Leung, 2015).

LeCompte and Preissle (1993) also defined validity as the "measure of the extent to which research conclusions effectively represent empirical reality or whether constructs devised by researchers accurately represent or measure categories of human experience" (p.323). It is a demonstration that a particular research instrument, in fact, measures what it purports to measure (Durrheim, 1999). In simple terms, Validity has to do with "how one's findings match reality." All validity measures that were taken in this study were based on the above conceptions and notions of validity. Also, triangulation ensures that information from one source is not used without cross-checking or validating with information from other sources.

Reliability

Reliability on the other hand, deals with the extent to which one's finding can be replicated. It is important that this particular study be of use to other researchers. Consequently, the researcher tried to achieve some external validity or generalizability of the findings. Eisner (1981) said that generalizability is possible in qualitative research because of the belief that the "general resides in particular" (p. 7). Yin (1989) suggested that analytic generalization does not rely on samples and populations. He also stated that qualitative research might generalize a particular set of results to a broader theory. I extend my findings to the constructivist learning theory. Some of the steps that the researcher took to question, the design is valid for the methodology, the sampling and data analysis are appropriate, and finally, the results and conclusions are valid for the sample and context (Leung, 2015).

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Qualifications and Work Experience of the Researcher

Since in this qualitative research, the researcher is the principal instrument, his qualifications are not only considered relevant but also important in the whole research process. His qualifications are as follows:

a) The researcher had a B.Sc. in Mathematics from the University of Cape Coast in 1989.

b) He taught mathematics at Senior Secondary School level for ten years in Ghana.

c) He also taught mathematics at the University level for another 18 years

d) He had his M.Phil. in Mathematics Education in 1996.

e) He had participated in several mathematics workshops and attended Mathematical Association of Ghana professional workshops many times.

f) He had been a mathematics examiner for West African Examination Council (WAEC) for ten (10) years.

g) He is currently pursuing his PhD in Mathematics Education at the University of Cape Coast and has published some articles in peer-reviewed journals in the field of Mathematics Education.

The Researcher's Beliefs

Nazroo (2006) recommended that researchers reflect on and examine their values when conducting ethnographic research. According to Montoya (2014), it is important to determine the types of biases that could compromise one's research and also take into consideration one's personal beliefs. While there are various types of biases to watch out for, understanding any influences one's research is susceptible to, helps fend off a particularly egregious case of bias. Ross (1998) also suggested that by listing relevant beliefs, the researcher presents awareness of them and, thus, provides the reader with a foundation for evaluating the study. The researcher therefore, identified the following as his beliefs as far as the teaching and learning of mathematics are concern.

1. The researcher believes that children construct their knowledge of mathematics rather than acting as passive "receivers" of knowledge. He believes that children know mathematics by doing mathematics, not computing mathematics.

2. The researcher believes that the teacher's role in teaching mathematics is to guide the student in constructing a relational understanding of mathematics, where the relational understanding of mathematics means knowing "what to do and why to do it" (Skemp, 1976, p.20). This type of understanding enables students to solve mathematical tasks by constructing several plans.
3. The researcher believes that teachers should teach mathematics to children by focusing on how children learn mathematical concepts.
4. He believes that mathematics should be taught within a heterogeneous-grouped class of students.
5. He believes that classrooms are complex environments where teachers and students interact and influence each other's thinking and behaviour. He believes the classroom is a community of thinkers and meaning should be negotiated.

Data Collection Procedure

First day of recording Mr Yamoah's lesson

On the first day of the class observation, the class teacher, Mr Yamoah, introduced the researcher to the students as Mr Robert Akpalu, someone who had come to gather some information about the teaching and learning of mathematics. The teacher further explained to the students that the researcher was not there to help them with their assignments or classwork, but only to observe how teaching and learning were taking place at X Community Junior High School. To the best of the researcher's knowledge, no one told the students anything further than what the class teacher did in his presence on the first official recording day. During class, occasionally, the researcher walked around the room to observe the students during individual and group activities. The researcher directed his observations toward understanding how the teacher constructed his meaning of constructivist teaching. The researcher focused on the interactions between the teacher and his students and observed the discourse during lessons. The researcher was also particular about the kind of questions that the teacher asked during the lesson and the responses students gave; especially what the researcher called the trading of ideas between the teacher and the students.

During the investigation, the researcher collected four types of data-observation, interview, videotape and documents. These four types of data helped the researcher to learn about the culture of the classroom by making inferences. In this study, the researcher observed the teacher's behaviour while teaching mathematics at JHS1, the teacher's use of instructional materials, and the lesson notes prepared for the teaching of the class. The researcher from time to time collected some students' work after the class for examination. Finally, the teacher's and students' discourse about mathematics was also observed. The researcher's level of actual participation in the classroom could be described as moderate by De Walt and De Walt, (2011). According to them, participant observation is not simply showing up at a site and writing things down. On the contrary, participant observation is a complex method that has many components. One of the first things that a researcher or individual should do after deciding to conduct participant observation is to gather data and decide what kind of participant observer he or she would like to be. Spradley (1980) provides five different types of participant observations summarized in the table 1.

Table 1: *Type of Participant Observation*

Type of Participant Observation	**Level of Involvement**	**Limitations**
Non-Participatory	No contact with population or field of study	Unable to build rapport or ask questions as new information comes up.
Passive Participation	Researcher is only in the bystander role	Limits ability to establish rapport and immersing oneself in the field.
Moderate Participation	Researcher maintains a balance between "insider" and "outsider" roles	This allows a good combination of involvement and necessary detachment to remain objective.
Active Participation	Researcher becomes a member of the group by fully embracing skills and customs for the sake of complete comprehension	This method permits the researcher to become more involved in the population. There is a risk of "going native" as the researcher strives for an in-depth understanding of the population studied
Complete Participation	Researcher is completely integrated in population of study beforehand (i.e. he or she is already a member of particular population studied).	There is the risk of losing all levels of objectivity, thus risking what is analyzed and presented to the public.

Participant Observation

According to Kawulich (2005) almost any setting in which people have complex interactions with each other, or with objects, their physical environment can be usefully examined through participant observation. Since doing participant observation means being embedded in the action and context of a social setting, we consider three key elements of participant observation study:

(i) Getting to the location of whatever aspect of the human experience you wish to study. This means going to where the action is—people's communities, homes, workplaces, recreational sites, places of commercial interaction, sacred sites, and the like. Participant observation is almost always conducted in situ. This condition was satisfied in the sense that the researcher was always at the X Community JHS three times a week for almost three terms.

(ii). Building rapport with the participants. The point of participant observation is that one wishes to observe and learn about the things people do in the normal course of their lives. That means the people would have to accept the researcher, to some extent, as someone they can "be themselves" in front of. While the researcher need not be necessarily viewed as a complete insider, a successful participant observer has to inspire enough trust and acceptance to enable his research participants to act much as they would if the researcher were not present.

(iii) Spending enough time is interacting to get the needed data. The informal, embedded nature of participant observation means that one cannot always just delve straight into all the topics that address one's research issues and then leave. The researcher must spend time both building rapport and observe or participating for a long enough period to have a sufficient range of experiences, conversations, and relatively unstructured interviews for his analysis. Depending on the scope of the project and one's research questions, this may take anywhere from days to weeks, months, or even years, and it may involve multiple visits to the research site(s). The researcher used triangulation to increase the credibility of his findings (i.e., researcher rely on multiple data collection methods to check the authenticity of his results.

Ethical issues

Gall et al. (2007) state that when conducting research, it is necessary to follow certain ethical procedures. This is to help obtain permission to conduct the study, and also gain the needed cooperation from individuals who will be affected by the findings of the research. For research of this nature which took nearly three terms to collect data, it was necessary for the researcher to consider ethical issues to make the research stand the test of time. Identifying appropriate sites and working with gatekeepers to obtain permission are critical steps in a case study (Gall et al., 1996). Because the research was conducted in X Community Basic School, I followed ethical procedures (Gall et al., 2003 and 2007). Ethics covers the whole process of research, and it is vital to recognize various sensitive aspects involved in a certain field. As Fluehr-Lobban (1979) says:

> This involves considering ethics in every phase of research, from the conception of the research project to the design of the research methodology. This includes how best to obtain informed consent; beginning and sustaining a dialogue about the intent, funding sources, and

likely outcome(s); ensuring the voluntary participation of persons involved in the study and asking whether they desire anonymity or recognition; thinking about the impact the study may have on those studied through dissemination of results and publication; empowering those who are studied to ask questions, contribute to the research design, or improve methods; and considering reciprocal acts that might benefit the people or community studied. (p.401)

The name of the research participant Mr Yamoah is not the real name of the teacher in this research but rather a pseudonym. The same can be said of all the names of the students used in this research.

Data Processing and Analysis

At the data analysis stage, the researcher tried to get answers from the video recordings, classroom observation notes and interview schedules for the following questions to enable him to answer the research question "What are the teacher's instructional decisions in lesson planning, classroom discourse, and assessment of teaching and learning" These questions were asked during a pre-teaching interview the researcher had with the class teacher.

1. What are you planning to teach today?

Before you enter the class to teach a topic like **shape and space**, tell me the preparations that you made regarding: lesson planning some critical questions you will raise in class for discussion how you plan to handle students' questions and assessment of the students' learning

2. To answer the second research question "what the classroom looks like regarding lesson planning, classroom dialogue, assessment of teaching and learning?" the researcher used the classroom observation notes, the video recordings, class exercises and other materials to address the above question.

3. The interview sessions with the JHS teacher helped the researcher to address the question "How does the teacher define constructivist teaching and mathematics learning about the examination of his beliefs and his knowledge of the subject?"

4. The fourth sub-question "what the connections between the teacher's definitions of constructivist are learning and his instructional decisions and classroom practice?" was also addressed principally through the recorded structured interview

Chapter Summary

This study was basically about how students experience mathematics when constructivist teaching and learning of mathematics strategies are implemented at Junior High School One classroom.

The research design chosen for this qualitative study was ethnographic research design. The data was collected at X Community Basic School where one JHS teacher was the subject of the study for nearly three terms. The sampling procedure for this study was purposive sampling. This is because the researcher purposely chooses the JHS teacher based on the criteria he was looking for. The researcher was the main instrument for the ethnographic study.

The first limitation of this study is that because of the video coverage of the lessons; I could not cover more than one teacher and for that matter will not be in a position to generalise the results of this study. Secondly, the dual desks in the classroom coupled with the overcrowding of students made it difficult to form small group discussions and cooperative learning as frequently as the teacher would have loved to do it.

CHAPTER FOUR

RESULTS AND DISCUSSION

Overview

The purpose of this study was to determine how students experience mathematics when constructivist teaching and learning are implemented at the Junior High School level. The study is guided by the main research question: When constructivist teaching and learning of mathematics is implemented at Junior High School (JHS) One classroom, how do students experience mathematics? An ethnographic research design was used to address the research questions in this study. Data for the study were collected using video coverage, lesson observations, interviews, and document analysis to answer the research questions adequately (Ary, Jacobs and Razavieh, 2002). This chapter specifically deals with the results and its discussions.

Presentation of Results

The main research question was when "constructivist teaching" and learning of mathematics is implemented at Junior High School Classroom, how do students experience mathematics? To adequately address the above research question one-by-one, the results of the five sub-research questions have been presented as follows:

Question 1

How does the teacher view constructivist teaching and mathematics learning?

An interview session, which was pre-arranged with Mr. Yamoah one morning before the class observation began, produced the following results.

Mr. Akpalu: Good morning Mr Yamoah

Mr Yamoah: Good morning Sir.

Mr. Akpalu: For how long have you been teaching mathematics at the basic school?

Mr Yamoah: 12 good years.

Mr. Akpalu: Good! How many years have you been teaching mathematics at the JHS level?

Mr Yamoah: For the past 5 years.

Mr. Akpalu: If somebody asked you to define mathematics, what would be your response?

Mr Yamoah: Ha-ha-ha-ha-ha. This is a big question but I would try to attempt an answer. Mathematics is a way of thinking about problems in order to find solutions. Others may view it differently but to me, it is about solving problems. The problems may be everyday problems like buying from a shop and receiving your change to complex issues like calculating the compound interest on an amount deposited at the bank. Mathematics is part of everything we do as human beings. In cooking, we estimate the quantity of water that would be needed for the rice, or the banku. These are all part of mathematics. I hope I have answered you.

Mr. Akpalu: Considering the answer you have given about what mathematics is, how do you think the subject should be taught in our basic schools?

Mr Yamoah: I think mathematics should be taught in a very practical manner so that students can enjoy the subject and would be able to apply it to their everyday lives-such as the home, the field, office and personal lives. Do you remember when I asked them to bring empty cans of various sizes when I was going to teach them circumference of circles? If you could recall, the students enjoyed the class very well that day and I don't have to tell them that no matter the size of the can or circular object, the formula will always be $C = 2\pi r$. The students discovered this formula on their own. This is how I think teachers of mathematics should teach the subject. I learnt some of these strategies at the training college at Akatsi, from MAG conferences, textbooks and the internet.

Mr. Akpalu: Do you mean you use the internet to do research on how to teach a particular topic?

Mr Yamoah: Oh yes, I do by watching YouTube videos on the particular topic. Though not at all times.

Mr. Akpalu: Can you also share your view on how the students should learn the subject in order to achieve more?

Mr Yamoah: Understanding is very important when it comes to learning of mathematics. Apart from the teacher teaching for understanding, the students should also do well by first understanding the concepts through active involvement in class, asking questions and then practice more examples to deepen their understanding. I also believe in group work because it gives the opportunity to the weaker ones to learn from the good students in the group.

Summary of findings for sub-research question one

According to Mr. Yamoah, mathematics is a way of thinking about problems. The problems may be everyday problems like buying from a shop and receiving your change to complex issues like calculating the compound interest on an amount deposited at the bank. Mr. Yamoah believes that thinking mathematically helps students think better in all areas of life. Mr. Yamoah's knowledge of mathematics is that it is a human activity and part and parcel of our lives. His view also is that children should be encouraged to create or construct their own knowledge. These views are clearly integrated within his conceptions of mathematics.

Research Question 2

Do the plans the teacher make before his teaching differ from how he teaches?

Mr. Akpalu: Mr Yamoah, I would like to know whether you go through any kind of preparation before you teach any particular topic.

Mr Yamoah: Yes, I do. I don't take things for granted. I have learnt my lesson long ago that, you can embarrass yourself in front of your students if you take things for granted.

Mr. Akpalu: What do you mean when you said if you take things for granted?

Mr Yamoah: I mean when you think you know the topic or subject already and you just walk to class feeling big in your shoes that you are a mathematics teacher, you can embarrass yourself before your students.

Mr. Akpalu: Okay, I now understand what you mean.

Mr. Akpalu: What exactly do you do when you are preparing for a class?

Mr Yamoah: I prepare my lesson notes as the first thing. During that preparation, I list the things that I would need to teach the topic and whether I should group the students or allow them to work on their own. I also plan for questions that I would give them at the end of the class as homework and classwork. Many times, I anticipate the type of questions my students are likely to ask concerning the topic and I prepare my responses.

Mr. Akpalu: During your actual classroom teaching, are you able to follow all that you planned to teach for that period?

Mr Yamoah: Yes, I do. But like I told you earlier, sometimes I am not able to teach about 80% of what I plan to teach. I am not able to complete my lessons, especially when there are many questions from students or there are misconceptions to deal with. I had to spend time clearing those misconceptions before teaching the new topic.

Mr. Akpalu: Can you explain further what you mean by misconceptions or any questions from students?

Mr Yamoah: I don't know whether you remember the week I taught the topic length and area. After I taught the topic, I was not expecting students to confuse Perimeter with Area, but unfortunately, some of the students were calculating area when they were asked to find the perimeter. That is what I mean by misconception. I had to leave the topic I planned for that day to go over the difference between area and perimeter and how to calculate each of them again.

Mr. Akpalu: What was the lesson you were unable to teach that day due to students' misconception of area and perimeter?

Mr Yamoah: That was powers of natural numbers.

Mr. Akpalu: How often are you confronted with this kind of situation- I mean not being able to teach what you planned for the day because of shallow understanding of the previous concepts?

Mr Yamoah: Like I said earlier, about 20% of the time I have to go back to teach something that I did not plan for the day.

Mr. Akpalu: Does that frustrate you as a teacher if you had to go back to teach something you had already taught instead of moving forward?

Mr Yamoah: Initially, I was not patient with students who tried to draw the class back. But when I later realized that without helping them, their understanding of the new concepts would be a problem, I finally had, to live with it. Now I am no longer disturbed like the first time. I am used to it now.

Mr. Akpalu: Mr. Yamoah, under the topic area and length you also taught the circle why?

Mr Yamoah: Yes, the circle falls under the main heading Length and Area. You will see this if you check the syllabus. There, you teach the circumference of the circle, the area of the circle, and the relationship between the diameter and the radius of a circle and those kinds of stuff.

Mr. Akpalu: Mr. Yamoah, even though, you provided some of the answers to my questions already, I am sorry to ask you some of them again.

Mr, Yamoah: No problem.

Mr. Akpalu: Mr. Yamoah, before you taught the circumference of the circle you referred to earlier, how did you prepare for that particular class or any other class?

Mr Yamoah: First of all, I prepared my lesson notes. In that lesson note, I listed all the teaching and learning materials I would need for the class. The students were made to look for those materials ahead of time if possible. I also made plans as to how many students to put in a group for effective group work. Some key questions that I would ask in class were also included in the lesson note.

Mr. Akpalu: Are you able to carry out your plans in the classroom during your lesson delivery?

Mr Yamoah: Most of the time, depending on the kind of questions the students raise in class and their level of understanding of the concept, or skill I want to teach, I end up not accomplishing even 20% of what I had planned to do for that lesson. The reason is that when students raise issues that showed that they did not understand something you had taught earlier, you are forced to go back to review that topic before you can proceed to your new lesson to enhance continuity and flow of the lesson.

Mr. Akpalu: When it happens that way, are you frustrated that you are not able to achieve your aim for that lesson?

Mr. Yamoah: Sometimes, it is a mixed feeling. If you aim to complete certain topics before the end of the term, you will be disturbed when you are not able to accomplish what you have planned to do. But if your focus is on helping the students to understand the topic, you will have the patience for them to overcome their challenges before you move on. My main focus is to teach for understanding, so I am not bothered so much when it happens. I can't say whether it is the same for the other teachers.

Summary of findings for sub-research question two

The researcher found out that in lesson planning, Mr. Yamoah's instructional decisions were mostly influenced by what students know already which usually serve as the basis for what they need to learn. These came out during the researcher's interview with Mr. Yamoah as well as from observations of his classroom teaching. The plans Mr. Yamoah made before his teaching sometimes varied from what he taught depending on the classroom situation. If students seemed to have a wrong understanding of the premise on which his lesson was going to be based, then Mr. Yamoah preferred to review that concept before proceeding to teach his main lesson. This was in line with what experts in the field of education said about teaching and learning. According to Fennema and Carpenter (1992), a guiding principle of cognitive guided instruction is that instructional decisions should be based on careful analyses of students' knowledge. If a teacher structures his lessons based on his assessment of students, the learning situation will more appropriately conform to the learner's development. It seems clear that through the decades, scholars envision teaching as an activity that promotes thinking and problem solving rather than the accumulation of information. This perspective is consistent with standards developed by the National Council of Teachers of Mathematics (NCTM). In the set of Standards, it is stated that "Finally, our visions see teachers encouraging students, probing for ideas, and carefully judging the maturity of a student's thoughts and expressions" (NCTM, 1989, p. 10). In the more recent set of standards (NCTM, 2000) the Teaching Principle is stated as follows: Effective mathematics teaching requires an understanding of what students know and need to learn and then challenging and supporting them to learn it well.

Question 3

Are there any connections between the teacher's ideas about constructivist teaching and instructional decisions on one hand and his classroom practice on the other hand?

A connection was found between how Mr. Yamoah planned his lessons and how he taught those lessons. He focused on process, practice and why questions rather than correctness of the answer. Mr. Yamoah's classroom was an open and tolerant learning environment. Below are the data excerpts:

Mr. Akpalu: Mr. Yamoah, I have noticed throughout my class observation that anytime a student answers in class, you just don't agree or disagree with the student's answer, but you always go beyond to ask the student to explain his/her point. Why?

Mr. Yamoah: For me, the why questions are very important because that is how I would know whether students have the correct understanding of the issue at stake or not. I can know whether the student has the right concept or not by probing further through the why and how questions.

Mr. Akpalu: What informs your instructional decisions?

Mr. Yamoah: Please what do you mean by instructional decisions?

Mr. Akpalu: I mean what to teach and how to teach it for better understanding.

Mr. Yamoah: When I am planning to teach, some factors come to mind. These factors were shared with us in a workshop not long ago. Such as: Do my students have the background knowledge to enable them to understand this topic? In other words, what is my knowledge of my students' prior knowledge? What are the potential difficulties students might encounter with these concepts I want to teach? What are the learning needs and interests of students in my class?

What types of problems or learning situations will help me teach the concepts effectively? What teaching strategies should I use to present the problems for better understanding? What materials will students need to solve the problems? What are the main concepts or ideas I want the students to have?

Mr. Akpalu: By your own assessment, who is a good mathematics teacher?

Mr. Yamoah: In my opinion, it is the teacher who does the following things: encourages the students to think and speak their mind freely in class whether they are right or wrong, makes students busy through useful engagement in activities during class, allows the students to communicate verbally what they think, engages the students in various mathematical activities to arrive at solutions and, put students at the centre of learning and allow them to discover mathematics for themselves through guidance.

Summary of findings for sub-research question three.

A connection was found between how Mr. Yamoah planned his lessons and how he actually taught those lessons. He focused on process, practice and why questions rather than correctness of the answer. His classroom was an open and tolerant learning environment. Mr. Yamoah's instructional decisions involved what contents to teach, and how to organize that content for the most effective classroom delivery. When he was asked to state in his own words who a good mathematics teacher was, the answers he provided above showed that he had a constructivist orientation. Mr. Yamoah's definition of constructivist learning was that knowledge is constructed in the mind of the learner and not something transmitted to the learner. In other words, knowledge is constructed by learners through an active, mental process of development; learners themselves are the builders and creators of meaning and knowledge. However, monitoring that construction by the students helps the teacher to guide them properly. According to Mr. Yamoah's definition, a "constructivist teacher" is the one who provides active experiences for students which encourage them to critically reason, analyze and verbally communicate their thinking. This definition of constructivist learning which was reflected in Mr. Yamoah's instructional decisions was based on careful analyses of students' knowledge. This finding is in agreement with the views expressed by Fennema and Carpenter (1992) when they said that "If a teacher structures the lessons based upon his assessment of students' knowledge, the learning situation will more appropriately conform to the learner's development." As Mr. Yamoah assesses his students through listening to their explanations, he said he could understand the students' depth of knowledge and use these explanations as beginning points to lead them to construct new understandings.

Research Question 4

To what extent does the teacher's experience influence his ideas about constructivist teaching?

To a very large extent, Mr. Yamoah's 12 years of teaching mathematics coupled with his academic and professional qualifications, put him in a position to maintain a high level of confidence when he stands before his students. Mr. Yamoah was not "afraid" to leave the textbook and his lesson notes to explore in the classroom with his students. However, the researcher noticed one shortcoming with Mr. Yamoah's teaching: In trying to explore with his students in the class, he would explain almost every question about 2 or 3 times to his students before allowing them to solve it instead of letting the students figure out the demands of the questions themselves. Below are the excerpts from the interview:

Mr. Akpalu: What is your highest academic qualification?

Mr. Yamoah: I hold a B.A. in Statistics from the University of Ghana. That is my highest academic qualification at the moment.

Mr. Akpalu: Why do you say at the moment?

Mr. Yamoah: I am still young and I have plans to further my studies as soon as practicable. I don't intend stopping at the first-degree level.

Mr. Akpalu: I like your spirit of forging ahead.

Mr. Akpalu: Are you a professional teacher?

Mr. Yamoah: Yes. I hold a Diploma in Basic Education from Akatsi College of Education

Mr. Akpalu: For how long have you been teaching mathematics?

Mr. Yamoah: I have been teaching mathematics for the past 12 years.

Mr. Akpalu: Do you enjoy teaching mathematics or you do it because you have to do it?

Mr. Yamoah: I really enjoy teaching mathematics. I don't think I would have enjoyed any other subject more than mathematics.

Mr. Akpalu: In your view, what makes the teaching of mathematics quite easy and enjoyable for you? You teach without sticking to the textbook and your lesson note. Why?

Mr. Yamoah: Having taught mathematics for the past twelve years, I believe I have developed the needed competencies over the years. I also think that my first degree in Statistics has also prepared me to have considerable control over the topics in the syllabus. My educational background also makes it possible for me to move freely from one topic to another and also explore with confidence into unknown areas in the classroom with students.

Mr. Akpalu: Can you describe how you think your own classroom looks like during mathematics lesson?

Mr. Yamoah: Students in my class know that I like grouping them to work together and share ideas. Group work is one common feature in my class. I also encourage the clever ones to mix with the average and below average students during group work. Calling students to come to the marker board to share their results is also a common thing in my class. My classroom is always activity oriented. You can't just sit doing nothing during my mathematics class.

Summary of findings for sub-research question four

It has been found out that to a very large extent the confidence Mr. Yamoah exudes when standing in the presence of his students stems from two things: (i) his experience as a mathematics teacher for 12 years and (ii) his academic qualification which makes him have control over the topics he teaches. Mr. Yamoah has extensive knowledge of both mathematics and his students thereby enabling him to teach for understanding, sequence lessons, make transitions between topics, understand student questions, provide good examples and maintain the necessary confidence in front of his students. In the words of Mr. Yamoah, conceptual understanding of the subject will promote the love for the subject in addition to its usefulness and application in the real world.

Research Question 5

How do students experience mathematics teaching and learning when constructivist methods are employed in their classrooms?

To answer this research question, three boys and two girls were picked randomly to represent the whole class for an interview. The five students were interviewed outside the classroom one after the other. This was done to reduce the noise level from classmates and also to have independent views of the students.

Responses from Student 1

Mr. Akpalu: For how long have you been in Mr. Yamoah's class?

Caleb: For about two years.

Mr. Akpalu: What can you say about Mr. Yamoah's style of teaching mathematics?

Caleb: He challenges us a lot, and he tries to make us better. He gives us work that will make us think. This really helps us to do well in class and in particular mathematics.

Mr. Akpalu: How does Mr. Yamoah's style of teaching affect your mathematics learning?

Caleb: As I said earlier, he makes us think, and he makes us use our minds a lot, and that makes you develop mentally, and that helps us not only to do wonders in mathematics but in other subjects as well. It makes learning mathematics interesting.

Mr. Akpalu: Do you like the way Mr. Yamoah teaches mathematics? Tell me why.

Caleb: As I said before, he challenges us with challenging questions, and that is why I like him. I like his mathematics teaching.

Mr. Akpalu: Thank you for your time. You may go.

Responses from Student 2

Mr. Akpalu: For how long have you been in Mr. Yamoah's class?

Eugenia: For about two years

Mr. Akpalu: What can you say about Mr. Yamoah's style of teaching mathematics?

Eugenia: He helps us understand the topics. There are times we don't understand certain topics. When that happens, the class becomes tense. Mr Yamoah would then bring in some funny jokes to ease the tension, and that makes us relax, and finally, he helps us to understand the topic.

Mr. Akpalu: How does Mr. Yamoah's teaching affect your mathematics learning?

Eugenia: The way he teaches makes me like mathematics.

Mr. Akpalu: If we want to change Mr. Yamoah and bring in another good mathematics teacher what would you say?

Eugenia: I would be very very sad.

Mr. Akpalu: Do you like the way Mr. Yamoah teaches mathematics? Tell me why

Eugenia: Very well. Because teacher Yamoah teaches well and he is good.

Mr. Akpalu: Thank you for your time. You may go.

Responses from Student 3

Mr. Akpalu: For how long have you been in Mr. Yamoah's class?

Doris: For nearly two years

Mr. Akpalu: What can you say about Mr. Yamoah's style of teaching mathematics?

Doris: Very very good. When he teaches, he adds discipline to it. His discipline is very good because you don't get the chance to fool around and he makes us work a lot.

Mr. Akpalu: How does Mr. Yamoah's teaching affect your mathematics learning?

Doris: He helps us a lot. He makes us understand the subject, and this makes learning easy

Mr. Akpalu: Do you like the way Mr. Yamoah teaches mathematics? Tell me why

Doris: Very well. He adds a little bit of entertainment to his teaching, and that makes it very interesting. I like Mr. Yamoah not only as a mathematics teacher, but he is also good in other areas.

Mr. Akpalu: Thank you for your time.

Responses from Student 4

Mr. Akpalu: For how long have you been in Mr. Yamoah's class?

Peggy: For nearly two years

Mr. Akpalu: What can you say about Mr. Yamoah's style of teaching mathematics?

Peggy: He is good. When he teaches mathematics, I understand. Sometimes he starts teaching by giving us scenarios about the topic. This helps us to understand the topic better.

Mr. Akpalu: How does Mr. Yamoah's teaching affect your mathematics learning?

Peggy: He sometimes put us in groups and this helps us to learn from each other especially if you don't understand the topic.

Mr. Akpalu: Do you like the way Mr. Yamoah teaches mathematics? Tell me why

Peggy: Yes. I like the way Mr. Yamoah teaches mathematics. Learning mathematics becomes fun.

Mr. Akpalu: If we decide to send Mr. Yamoah to another school and bring to your class another good mathematics teacher, what would you say?

Peggy: I would be very very sad.

Mr. Akpalu: Thank you for your time and effort.

Responses from Student 5

Mr. Akpalu: For how long have you been in Mr. Yamoah's class?

Kelvin: For nearly two years.

Mr. Akpalu: What can you say about Mr. Yamoah's style of teaching mathematics?

Kelvin: It's fine. The way he approaches the subject is fine.

Mr. Akpalu: How does he approach the subject?

Kelvin: He gives us questions to solve, and if we tried and we can't solve it, then he comes in and solves it for us.

Mr. Akpalu: How does Mr. Yamoah's teaching affect your mathematics learning?

Kelvin: He makes me enjoy mathematics a lot. He puts us in groups and those who have problems understanding the lesson, I sometimes helped them and when they also understand, they helped me.

Mr. Akpalu: How would you feel if we change Mr. Yamoah with an equally good mathematics teacher?

Kelvin: I would feel very very bad.

Mr. Akpalu: Thank you for your time and effort.

Summary of findings for sub-research question five

Students in Mr. Yamoah's class said they like his method of teaching the subject. The relationship between Mr. Yamoah and his students was so strong that they would be very sad if one day, they come to school to find out that their mathematics teacher has been moved to another class or transferred to another school. To these students, even an equally good mathematics teacher would not make the subject interesting (with all the jokes) like Mr. Yamoah. The students during their interview with the researcher were full of praise for their mathematics teacher Mr. Yamoah for his style of teaching which helps them to learn and like mathematics.

The Details of the Classroom Experience for Research Question Five

The summary above for research question five was based on a typical week's lesson of Mr. Yamoah so that readers can have a feel of what the researcher observed in the classroom on a typical day and thereby appreciate the findings presented in summary above. Mr. Yamoah teaches mathematics three (3) times a week at X Community Junior High School 1A on Mondays, Wednesdays and Thursdays. Each lesson was a double period of eighty (80) minutes duration. There were some things that many students in Mr. Yamoah's class knew about him right from the beginning even though they are not written down. Some of them were:

(1) No student can raise up his/her hand and give any answer without verbally explaining how he /she arrived at it to the class.

(2) The class cannot end without students solving some two or three problems on the topic for the day.

(3) Whether a student raised up his/her hand or not, Mr. Yamoah knows all his fifty-two students by their names and could call any student at any time to answer a question or explain a point to the class. This strategy always kept all students

alert in class because no one knows the next person Mr. Yamoah would call to answer a question. Mr. Yamoah has exhibited the above three characteristics in his class to the extent that there was no fear or panic in any student who was called to the board to solve a problem or explain an issue to the class. Mr. Yamoah had managed to build a cordial relationship with his students to the point where the classroom climate had become very friendly and welcoming for students to express their views and opinions and sometimes laughed loud over wrong answers given by their peers.

Pre-activities of the typical day's lesson

A few days before this lesson observation, Mr. Yamoah asked the students to help in the collection of various circular objects for the lesson. He showed a few appropriate examples of the kinds of circular objects to be collected. He emphasized that a variety of various sizes will be needed and the students were going to be placed in groups of five or less, and enough circular objects will be needed for each group.

Through an appeal to the students, the following items were to be secured for the lesson.

Rulers

String or thread for measuring

Collection of circular objects of various sizes (such as plastic lids)

Chart for data collection.

A Typical Day's Lesson by Mr. Yamoah

Topic: Circles, Radius, Diameter and Circumference

Mr. Yamoah: Mr. Yamoah entered the class.

Class: Good morning Mr. Yamoah

Mr. Yamoah: Good morning class. How are you today?

Class: We are fine. Thank you. And you?

Mr. Yamoah after responding to the usual greetings of his class on this particular day, started with a review of the previous lesson. In our previous lesson, we learnt how to find the area of a rectangle, a triangle, a square and other similar figures. You did well on your class assignments which showed that you had understood the lesson. Is there any student who cannot find the area of a triangle, a rectangle or a square?

Mr Akpalu: No hands were raised.

Mr. Yamoah: Good. The topic for today's lesson is: *Area of a Plane Figure*

Mr. Yamoah: Let us begin today's lesson by identifying shapes that look like circles. Name any shape or object that looks like a circle.

Mary: A Wedding ring.

George: Bicycle tyre, orbits,

David: The letter 'o'

Mercy: The big circular objects that we put around our waist at children's park. A student who knew the name mentioned it as Hula hoop. Jane The lid of cylindrical container and bottle tops.

Mr. Yamoah: Good. You have all done well. Even though they all look like circles, they are not circles in the actual sense of the word, why?

Class: No response from the class.

Mr. Yamoah: Who knows why they are not circles in the actual sense of the word?

Class: No response.

Mr. Yamoah Let us check the meaning of a circle from the dictionary and see whether we can get some help.

Class: Students brought out their dictionaries out from their school bags and started looking for the meaning of a circle.

Mr. Yamoah: All these while, Mr. Yamoah was not quick to tell the students the definition to save time but rather allowed the students to come out with the definition themselves. From the dictionary, this answer was provided.

Mavis: A completely round shape like the letter O.

Mr. Yamoah: The reason why they are not circles in the real sense of the word is that most of the things you mentioned are solids, we can hold them and, you can feel them. So, they are three-dimensional objects. But a circle is a two-dimensional shape with its set of points arranged in such a way that they are of equal distance from a given centre. It's a two-dimensional shape made by drawing a curve that is always the same distance from a centre.

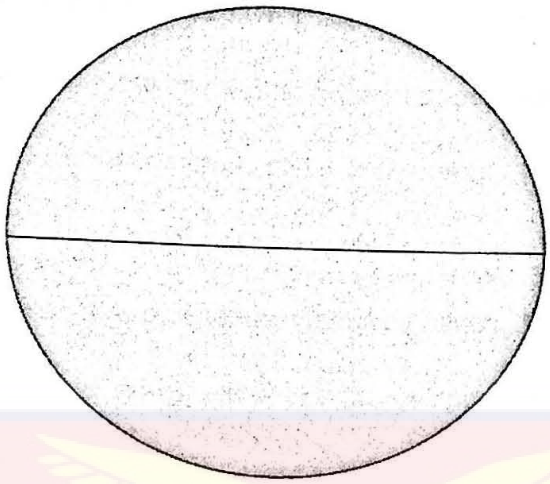


Figure 2: The distance round a circle is the circumference.

Mr. Akpalu: A day before this lesson, Mr. Yamoah had asked all the students to bring thread and circular empty cans to school for a special experiment. Some of the students brought circular lids. The students were excited that they were going to experiment the next day. Students came with milo, tomato, milk, and baked beans tins to school.

Mr. Yamoah: Students, this morning we are going to measure the distance round the circular ends of your tins or lids with the thread.

Mr Akpalu Mr. Yamoah demonstrated what he meant by that with his tomato tin using the thread. He moved the thread round the open end of his tomato tin and marked the thread at where it meets the starting point and then measured the length he got on tape. He encouraged all the students to carefully measure the circumference of their tins and record their readings on a table similar to the one shown below. While the students were doing the measuring, the teacher

Mr. Yamoah was going around guiding those who were facing challenges to understanding the process better. Mr. Yamoah encouraged the students to exchange their tins with other students and carry out the same measurement for at least four different sizes of tins or lids. The activities that took place in the class could be described by the following steps:

Activities

1. The class was divided into small groups of 5 or 6.
2. Each group was given a collection of the circular objects brought by the students.
3. Each group also had rulers, string for measuring, and a chart for collecting their data.
4. It was explained to the students that they were going to measure the circumferences and diameters of the circular objects of different sizes and record those measurements on the chart they were given.
5. The need for the string given to each group was also explained to the students.
6. Students collected and recorded their data on the chart provided.
7. They were required to discuss the relationships they observed and answer the questions on the blackboard later.
8. Small groups shared their findings with the entire class.

9. Students were guided to discover that the circumference is a little more than three times the diameter.
10. Gradually, the concept of Pi was introduced, by letting the students divide the length of each circumference by its diameter.

When Mr. Yamoah was satisfied that they had measured enough circumferences, he also asked them to measure their respective diameters and complete a table as shown below.

Table 2: *Circular Objects and their Ratios*

Circular Object (or Tin)	**Circumference (C)**	**Diameter (D)**	**Ratio of C to D (i.e. $\frac{C}{D}$)**
Gino tomato tin			
Milo tin			
Milk tin			
Cowbell tin			
Baked beans tin			

Mr Yamoah: Mr Yamoah explains diameter and radius to the class and the relationship between the two.

Mr Yamoah: A diameter is a line segment that has both endpoints on the circle and passes through the centre of the circle. A radius is a line segment from the centre of the circle to any point on the circumference.

Mr Akpalu: The students were asked to measure at least four different tins and record their circumferences, diameters and find the ratio of **C** to **D**. Most of the students completed the table and filled the ratio column as well. They observed that irrespective of the size of the can or tin, the ratio of its circumference to its diameter remains approximately the same. This discovery brought a lot of excitement to the students and the class came alive with students interested in trying more circular objects as though they were looking for one situation to disprove this interesting discovery.

Table 3*: Circular Objects and Calculated Ratios*

Circular Object (or Tin)	**Circumference (C)**	**Diameter (D)**	**Ratio of C to D (i.e. $\frac{C}{D}$)**
Gino tomato tin	49.6	16	3.1
Milo tin	25.6	8	3.2
Milk tin	32	10	3.2
Cowbell tin	52.7	17	3.1
Baked beans tin	68.2	22	3.1

Mr Yamoah used the chart above to record their readings and the calculated ratios. He explained that this constant is what is referred to by mathematicians as pi (π). Pi (π) is the ratio of the circumference of a circle to its own diameter. It is approximately $\frac{22}{7}$ or 3.142. Hence C = πd or C = 2πr

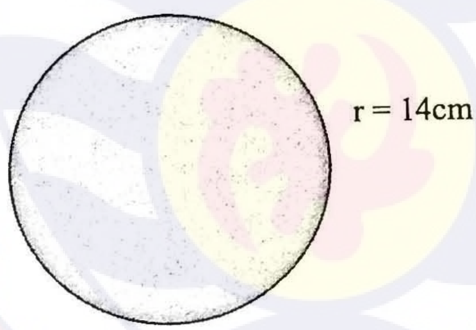
What was particularly interesting about this lesson was the fact that the students were not just given a formula to work with or memorize and use it when it was necessary without knowing how it came about, but they were part of its derivation. By being part of its derivation, they had a sense of ownership and were very excited to use what they had "derived" themselves. As a result, there was active involvement of the students in the learning process.

As usual of Mr Yamoah, he will not end any of his classes without giving the students class assignments. The assignments based on the lesson for the day were as follows:

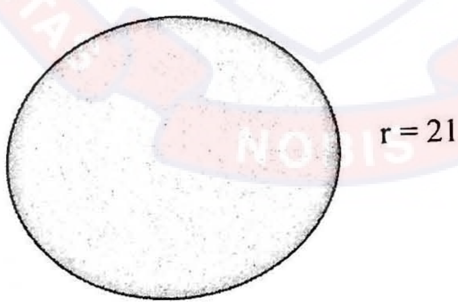
Exercises:

Find the Circumference, C of each circle

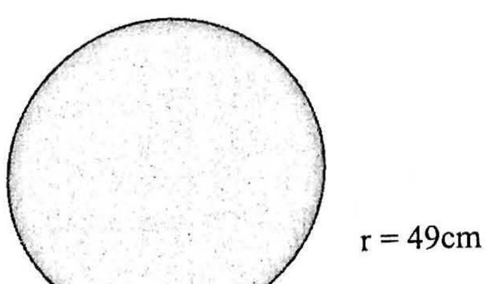
1.



2.



3.



4. The circumference of a circle is 25π meters. What is the radius of the circle?

5. John used a trundle wheel that had a diameter of 1m. How many turns of the wheel would it take to measure the length of a 220 metre dash?

Question five was the most difficult for all the students in the class. First of all, they did not understand the meaning of the object described as trundle wheel. When a picture of the wheel was shown to them, many of them realized that they knew it and related to it very well. Some of the boys said they made it as their "toy cars" when they were younger children. Others said it was bought for them by their parents to play with when they were younger.

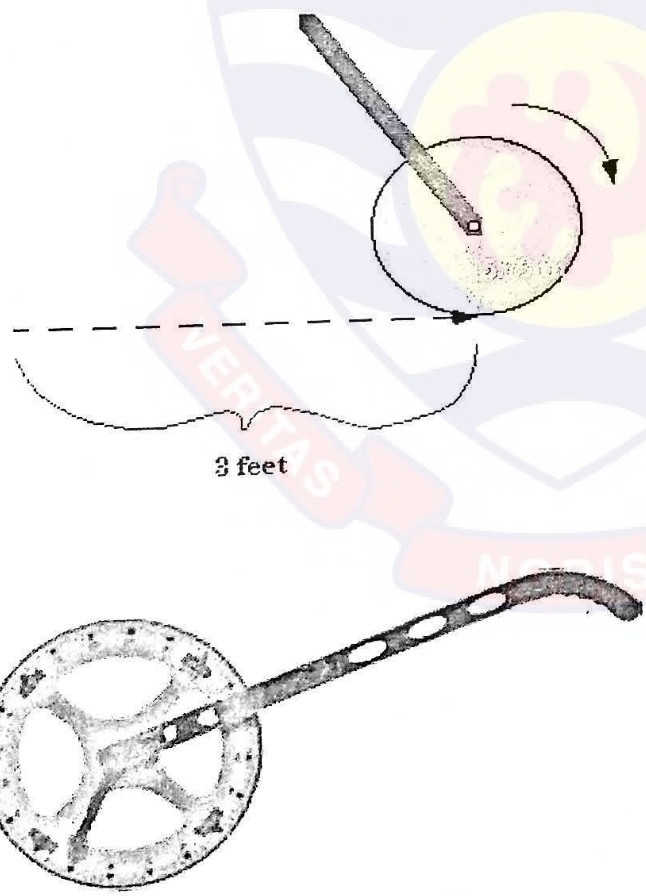


Figure 3: Trundle Wheel for measuring length

Now the issue of understanding the question was cleared and it was the turn of the students to solve the question. Tried as they could, they were not able to solve it before it was time for change over lesson. Mr Yamoah gave all the five questions to the class as homework to be inspected during the next meeting. The solution of the above questions from the students in the class the next time they had mathematics are as shown below.

Figure 4: Figure 5: Photograph of a student's classwork on Circles-1

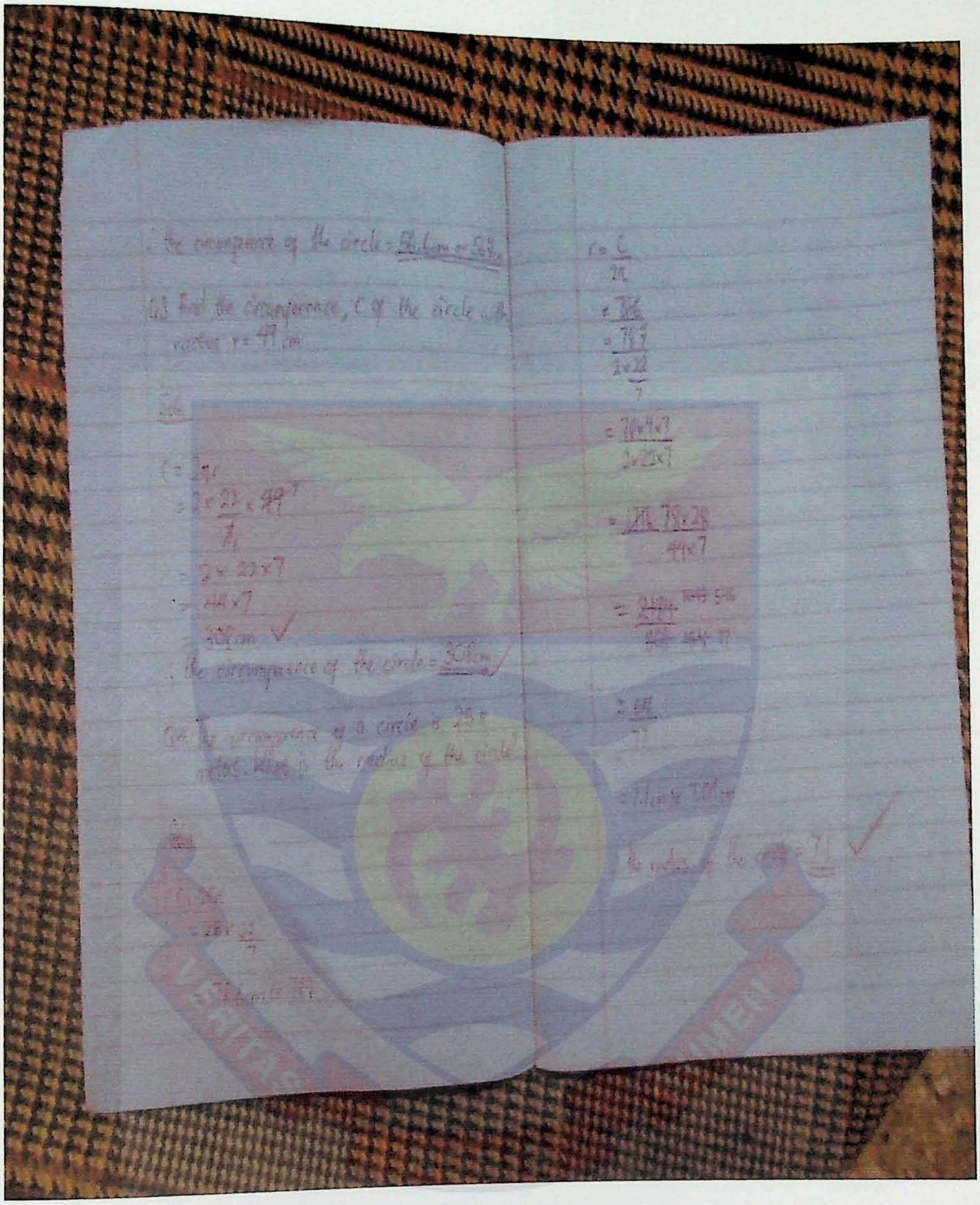


Figure 5: Photograph of a student's classwork on Circles-2

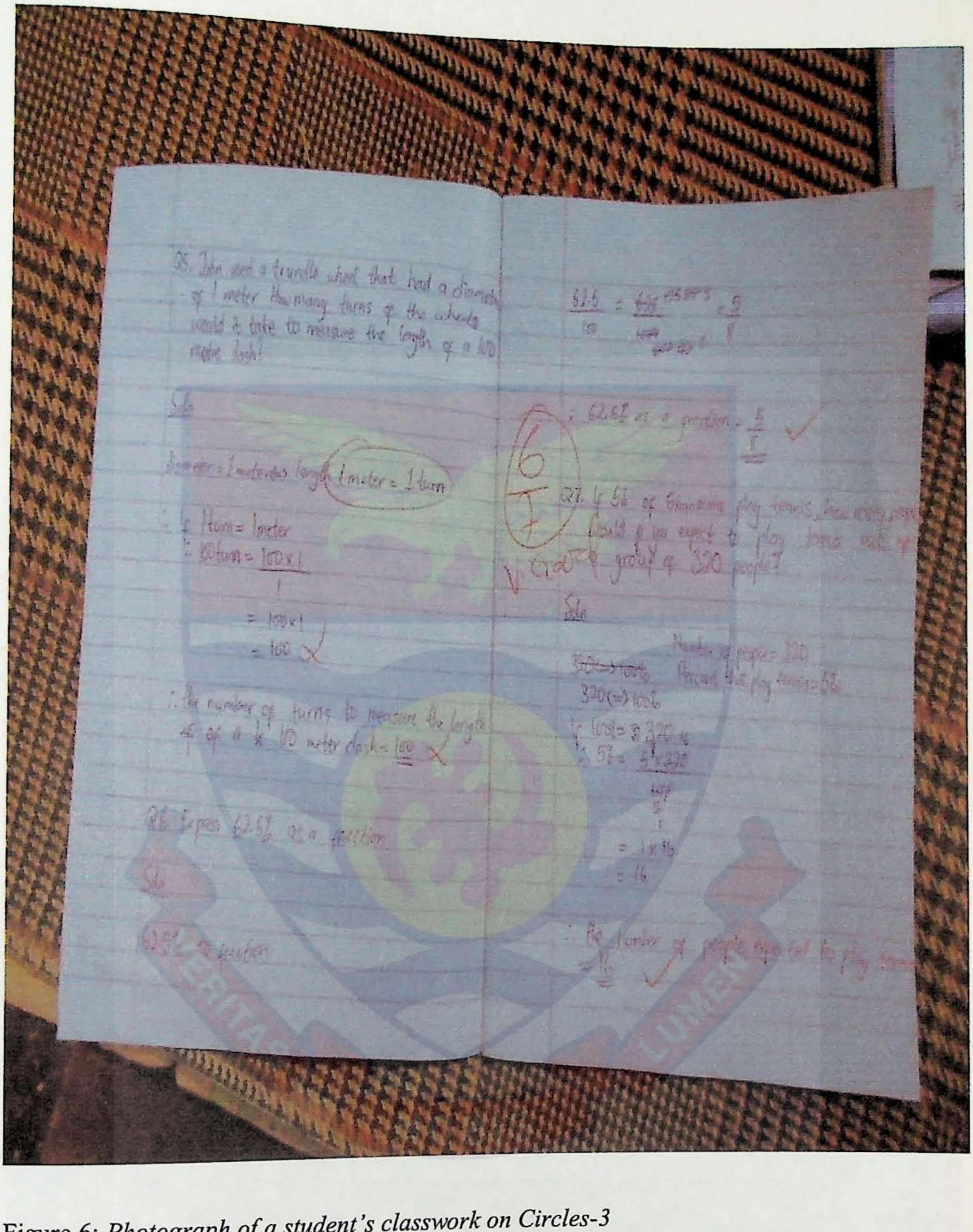


Figure 6: *Photograph of a student's classwork on Circles-3*

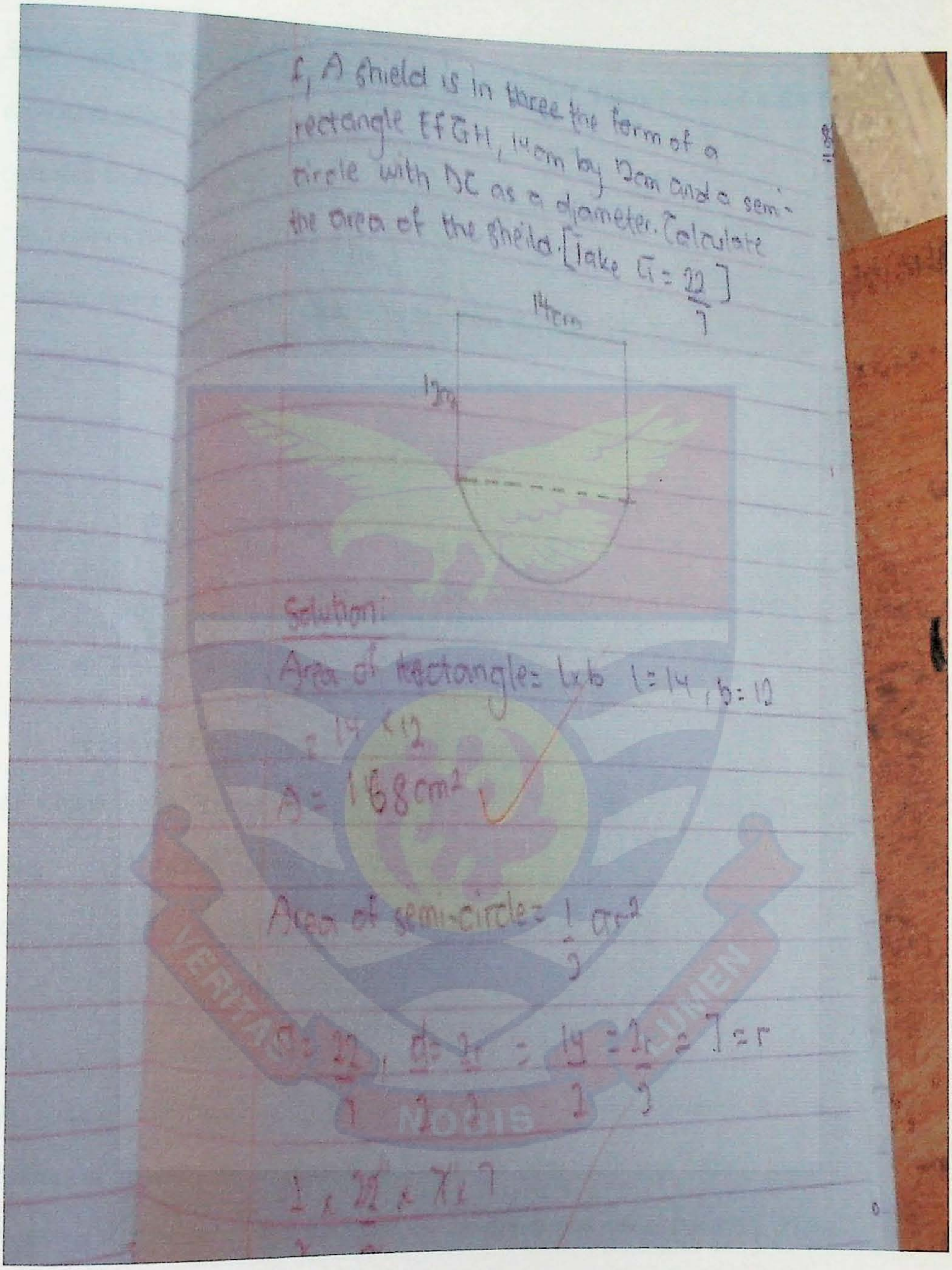


Figure 7: Photograph of a student's classwork on Circles-

Since nobody could solve question five correctly, Mr Yamoah solved it for the class as shown below:

Solution to Question 5

Diameter of the trundle wheel = 1m

Circumference of trundle wheel, $C = 2\pi r = \pi d$

$$= \frac{22}{7} \times 1m$$

$$C = \frac{22}{7} m$$

For a 220m dash it means that

$220 = \frac{22}{7} \times N$ Where N = the number of turns of the wheel

$N = \frac{220 \times 7}{22} = 70$ turns

A Second typical Day's Lesson by Yamoah

Mr Yamoah: Mr Yamoah entered the class.

Class: Good morning Mr Yamoah

Mr Yamoah: Good morning class. How are you doing today?

Class: We are fine and you?

Mr Yamoah: I am also fine.

Mr Yamoah after responding to the greetings in a cheerful manner, told the students to be ready for an interesting lesson. He went further to explain himself by saying that "it is because the topic for the day is percentages and we all use percentages in one way or the other in our everyday lives". Mr Yamoah then wrote the topic on the board.

Topic- Percentages

Mr Yamoah: Most examination scores of students are in percentages, e.g. 40%, 95%, 10% etc. When goods and services are going to be reduced, the new prices are normally advertised by showing how much discounts in percentages the customer will enjoy. For example, an advert may read "Purchase this new bag at a discount of 40%." This is to entice customers to purchase those items.

Mr Yamoah: Write down any two ways that we use percentages in real life.

Class: Some students were seen busily writing down examples in their jotters.

Mr Yamoah: Gladys, tell us your answer

Gladys: Paying of school fees. Parents can pay 50% and later pay the other 50%.

Mr Yamoah: Clap for Gladys.

Class: They clapped for Gladys.

Dan Salaries of workers can increase by 5% or 10%.

Mr Yamoah: Excellent answers from both of you!

Mr Yamoah: **Percent** means **per cent** and the word **cent is** derived from the Latin word "centum" meaning hundred. Therefore, **per cent** means per 100. Therefore, 20% means 20 per 100. Which is the same as $\frac{20}{100} = \frac{1}{5}$

Mr Yamoah went further to give some more examples:

Example 1:

Express 62.5% as a fraction

Solution

62.5 % means $\frac{62.5}{100} = \frac{625}{1000} = \frac{25}{40} = = \frac{5}{8}$

Answer = $^5/_8$

Mr Yamoah explained to the class why in the last example it was necessary to multiply both numerator and denominator by 10 to get $\frac{625}{1000}$. This was done simply to remove the decimal point in the numerator to facilitate simplification.

Below are some of the examples that **Mr Yamoah** worked in class with his students playing active roles.

Finding a Percentage of a Quantity

To find a certain percentage of a given quantity, we multiply the quantity by the corresponding decimal fraction.

Example1

Find:

a. 20% of 65

b. $4\frac{1}{2}$% of ¢40

Solution

a. 20% of 65 = 20% ×65

i. $= \frac{\overset{1}{\cancel{20}}}{\underset{\underset{1}{\cancel{5}}}{\cancel{100}}} \times \frac{\overset{13}{\cancel{65}}}{1}$

ii. $= \frac{13}{1}$

iii. = 13

b. $4\frac{1}{2}\%$ of ¢40 = $\frac{9}{2}\%$ × ¢40 (change the "of" to ×)

$= \text{Gh¢}\frac{9}{200} \times \frac{40}{1}$ $\left[\because \frac{\frac{9}{2}}{100} = \frac{\frac{9}{2}}{\frac{100}{1}} = \frac{9}{2} \times \frac{1}{100} = \frac{9}{200}\right]$

$= ¢\frac{9}{5}$

= ¢1.80

Example 2

If 5% of Ghanaians play tennis, how many people would you expect to play tennis out of a group of 320 people?

Solution:

Number of tennis players = 5% of 320

= 5% × 320

$$= \frac{\overset{1}{\cancel{5}}}{\cancel{100}} \times \frac{\overset{16}{\cancel{320}}}{1}$$

$$= \frac{16}{1}$$

$$= 16$$

16 people would be expected to play tennis.

Example 3

Find 20% of 45.

Solution:

$20\% \text{ of } 45 = 20\% \times 45$

$= \frac{20}{100} \times \frac{45}{1}$ $\quad \{\frac{20}{100} \times \frac{45}{1} = \frac{1}{5} \times \frac{45}{1} = 9\}$

$= 9$

Example 4

Find $3\frac{1}{2}\%$ of ¢80.

Solution:

$3\frac{1}{2}\%$ of Gh¢80 = Gh¢3.5% × 80

= Gh¢$\frac{3.5}{100} \times 80$

= Gh¢0.035 × 80

= Gh¢2.80

Percentage Change (Increase or Decrease)

Subtract the old from the new, then divide by the old value.

Example: You had 5 books, but now you have 7. The change is: 7-5 = 2.

Percentage Change: show that as a percent of the old value ... so divide by the old value and make it a percentage:

So, the percentage change from 5 to 7 is: 2/5 = 0.4 = 40%

Percent Increase

Example 1: Ama works in a supermarket for Gh¢10.00 per hour. If her pay is increased to Gh¢ 12.00 per hour, then what is her percent increase in pay?

Analysis: When finding the percent increase, we take the difference in the pay, and divide it by the original value. The resulting decimal is then converted to a percent.

Solution:

The percentage Increase $=\frac{12-10}{10} \times 100 = \frac{2}{10} \times 100 = \frac{1}{5} \times 100 = 20\%$

Answer: The percent increase in Ama's pay is 20%.

Percent Decrease.

Example 2: The staff at a company was reduced from 40 to 29 employees. What is the percent decrease in staff?

Analysis: When finding the percent decrease, we take the difference without attaching the sign (absolute difference) and divide it by the original value. The resulting decimal is then converted to a percent.

Solution: $\frac{40-29}{40} = \frac{11}{40} = 0.275 = 27.5\%$

Answer: There was a 27.5% decrease in staff.

After solving these questions with the students, Mr Yamoah gave them seven questions on area and percentages as class work. The questions were as follows.

Question 1.

Elorm earns Gh¢ 140 per week for her part-time job. She is to be given a 5% pay rise. How much will she earn per week after the pay rise?

Question 2.

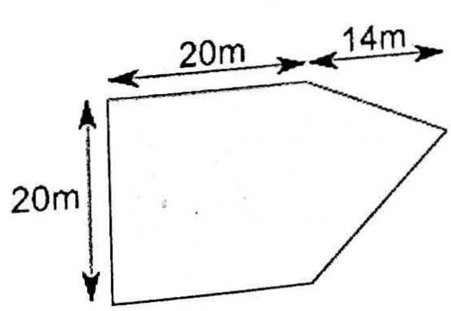
The prices of all the televisions in a shop are to be increased by 8%. Calculate the new price of a television that originally cost Gh¢1,750.

Question 3.

During a reduction sale, the cost of a computer is reduced by 30%. The normal price of the computer was Gh¢ 900. Calculate the selling price of the computer.

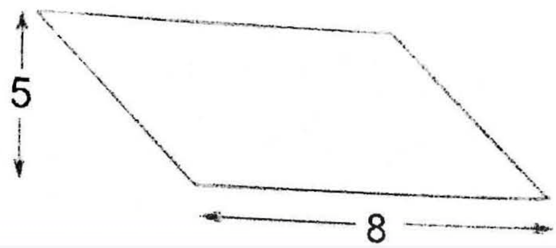
Question 4.

Jonah cuts grass at Gh¢ 0.10 per square meter. How much does Jonah earn cutting this area?



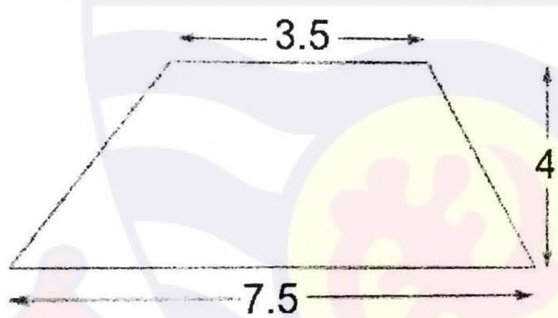
Question 5

What is the area of the parallelogram?

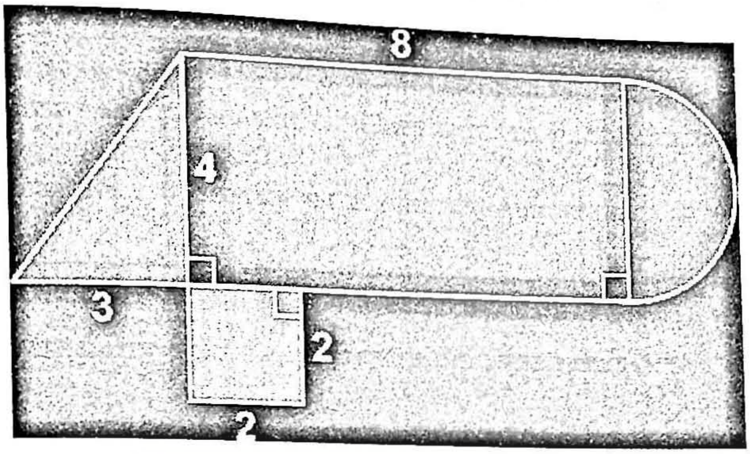


Question 6

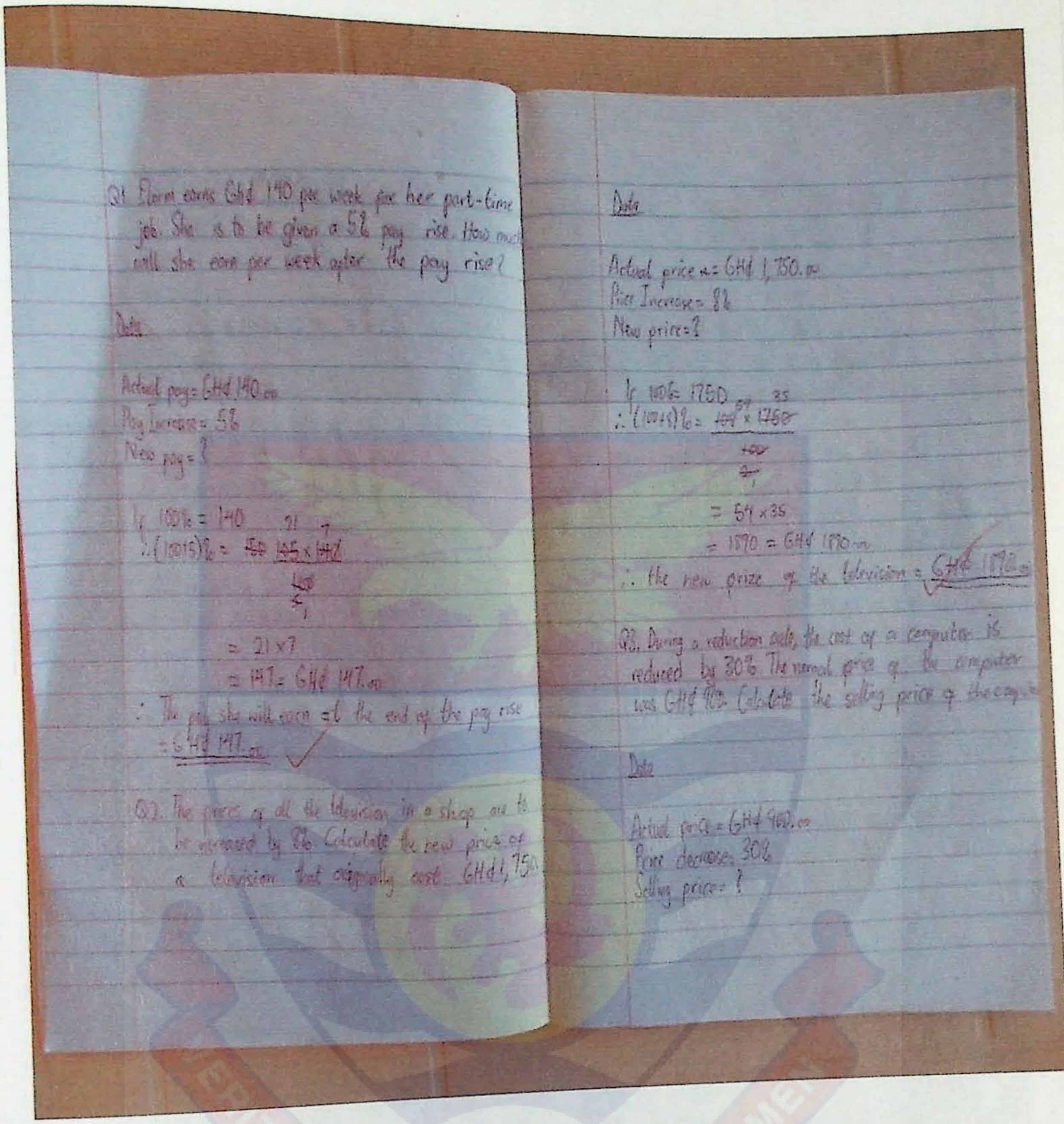
What is the area of the trapezoid?



Question 7: **Find** the area of this strangely shaped object below



When the researcher went around the class, these were some of the works captured by the camera before the class ended.


Q1 Flora earns GH¢ 140 per week for her part-time job. She is to be given a 5% pay rise. How much will she earn per week after the pay rise?

Data

Actual pay = GH¢ 140.00
Pay Increase = 5%
New pay = ?

If 100% = 140
∴ (100+5)% = 105 × 140 / 100
= 21 × 7
= 147 = GH¢ 147.00
∴ The pay she will earn at the end of the pay rise = GH¢ 147.00

Q2. The prices of all the television in a shop are to be increased by 8%. Calculate the new price of a television that originally cost GH¢1,750.

Data

Actual price = GH¢ 1,750.00
Price Increase = 8%
New price = ?

If 100% = 1750
∴ (100+8)% = 108 × 1750 / 100
= 54 × 35
= 1890 = GH¢ 1890.00
∴ the new prize of the television = GH¢ 1890.00

Q3. During a reduction sale, the cost of a computer is reduced by 30%. The normal price of the computer was GH¢ 900. Calculate the selling price of the computer.

Data

Actual price = GH¢ 900.00
Price decrease = 30%
Selling price = ?

Figure 8: *Photograph of a student's classwork on percentages*

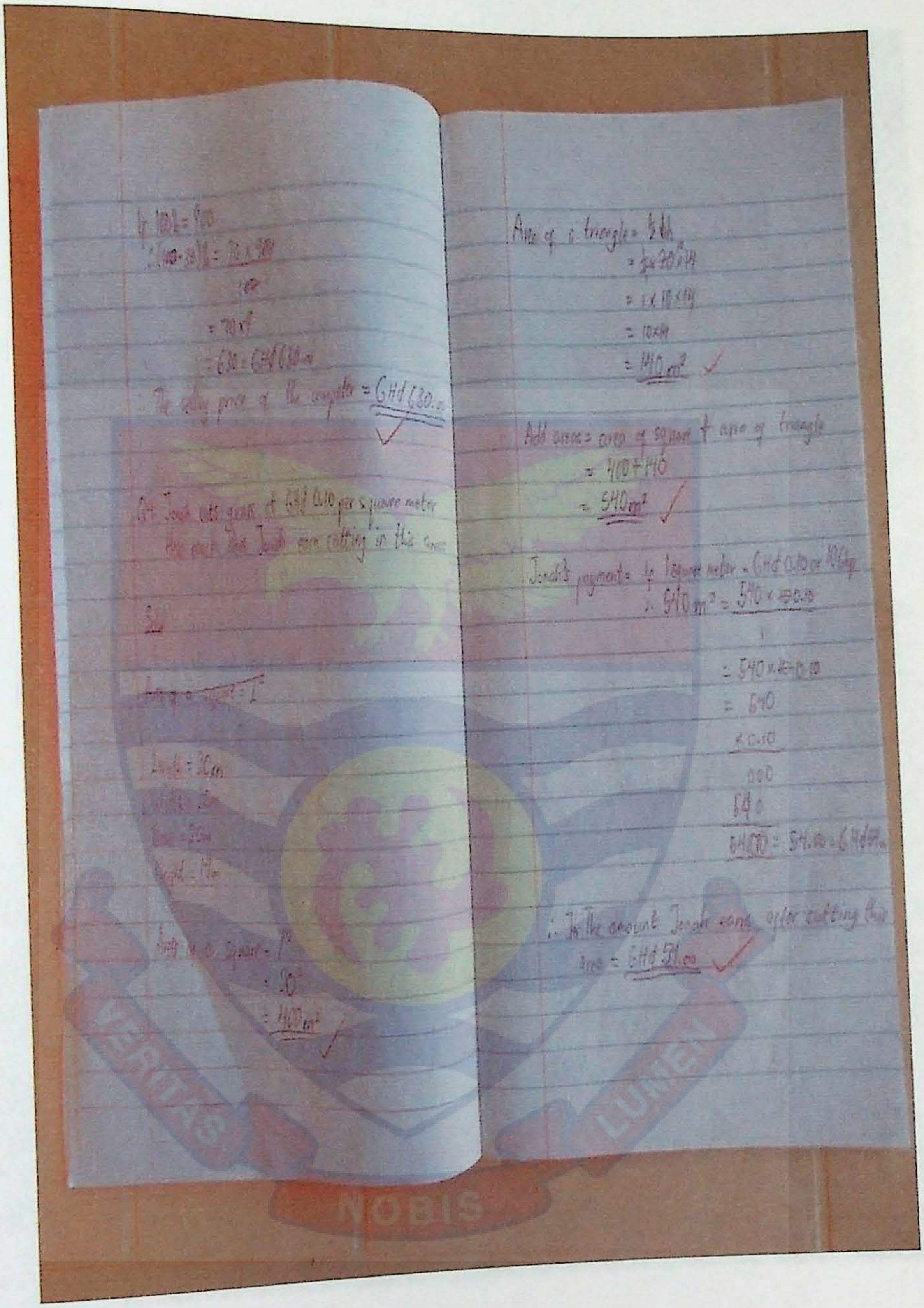


Figure 9: Photograph of a student's classwork on percentages

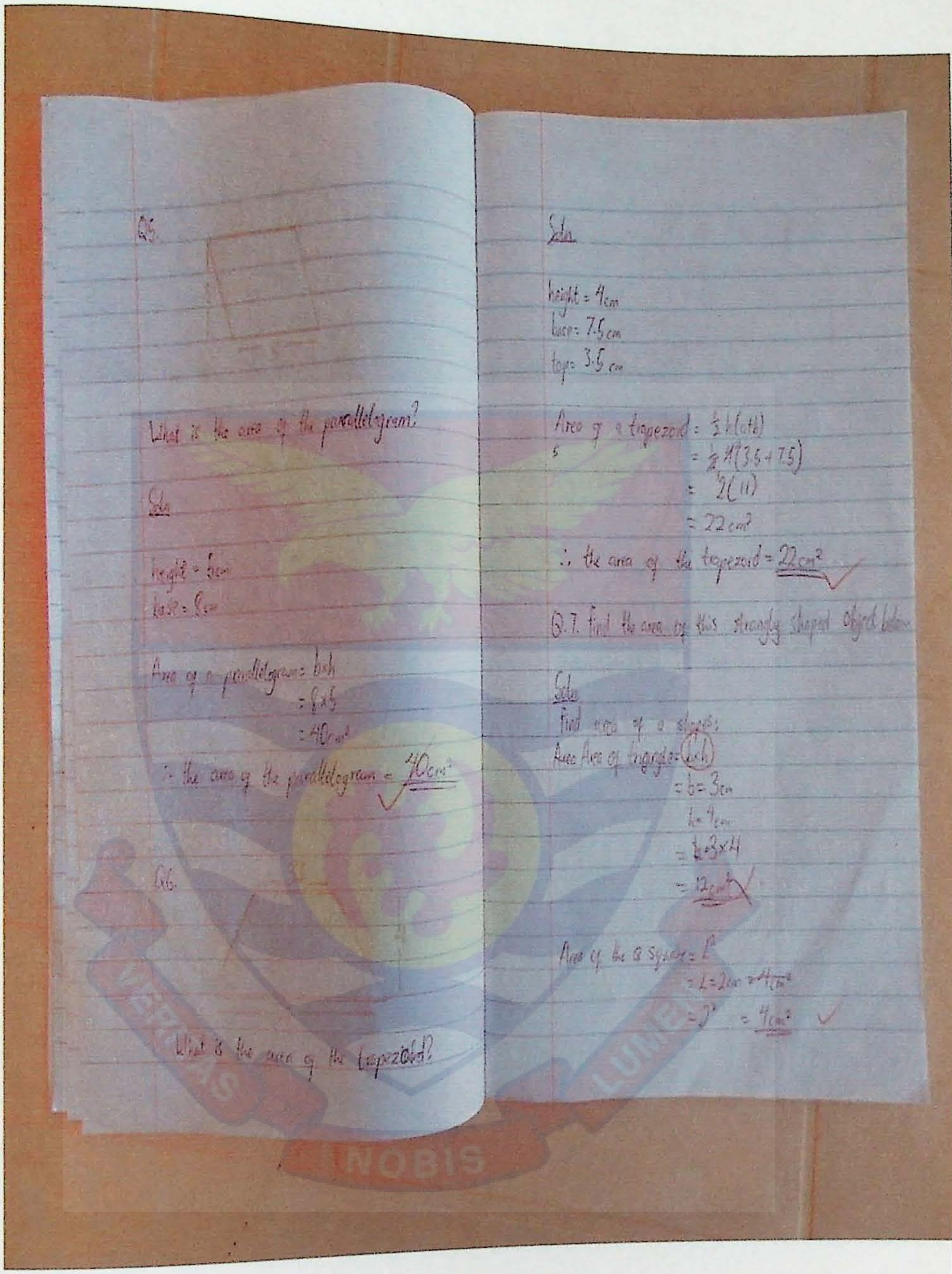


Figure 10: Photograph of a student's classwork on Area of Geometrical *Figures-1*

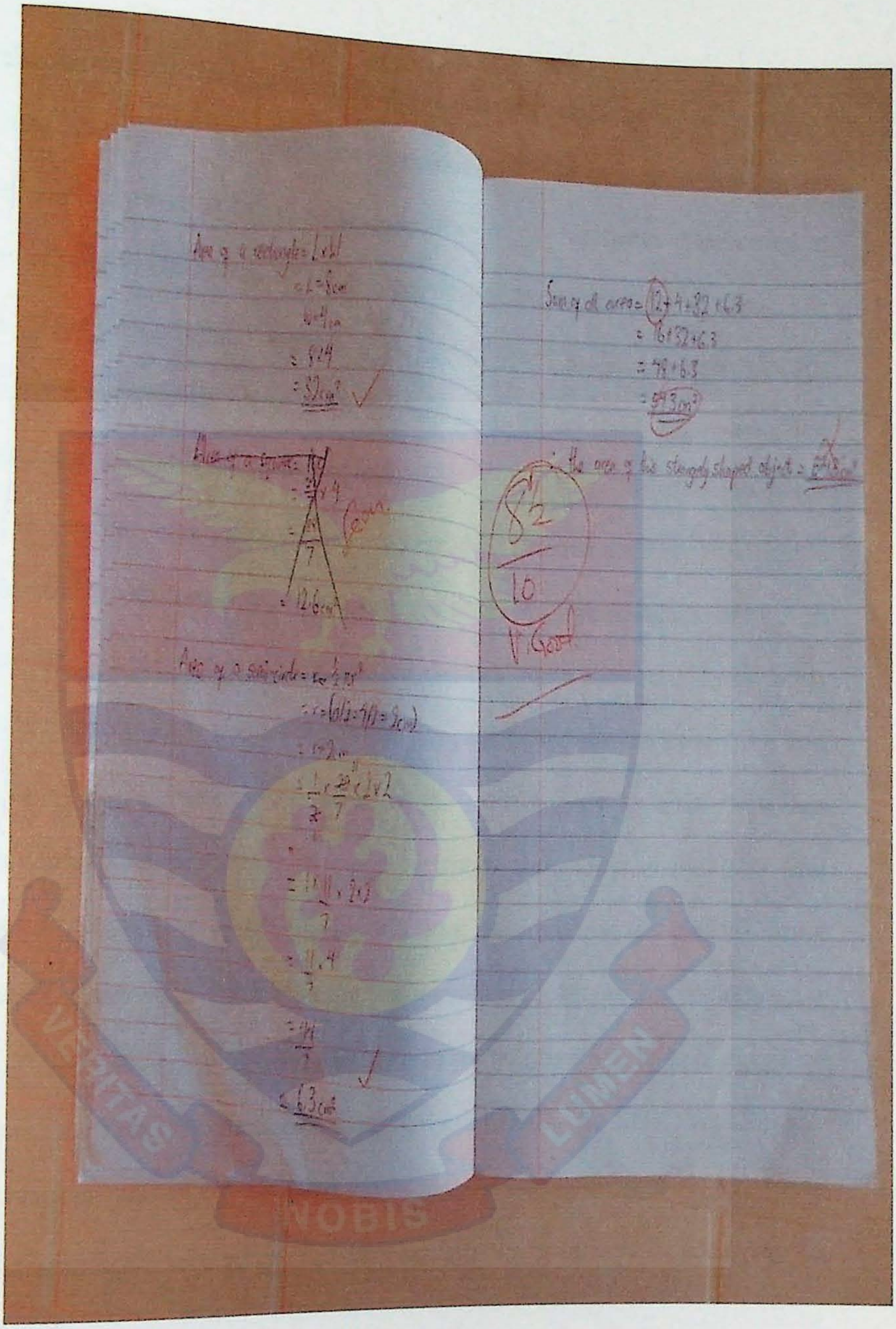


Figure 11: Photograph of a student's classwork on Area of Geometrical Figures

The Teacher Should Use Concrete Problem Solving Contexts

The key to Mr Yamoah's mathematics teaching can be found in the problem solving experiences that he provides for the students. Mathematical problem solving is the centre of all the learning of the content in his class. The problem solving experiences include a variety of word problems, strategies for solving problems, problems using manipulatives to model thinking, open-ended problems, and everyday real-world problems. In this way, Mr Yamoah helps students to see that mathematics is everywhere. Mr Yamoah does not teach skills to be learned for problem solving but teaches skills through problem solving. In his teaching, problem solving is not a distinct topic separate from other mathematics learning. A good example to support this claim was the Cow and chicken in a barnyard question he gave to the students. Mr Yamoah organizes most classes around problem solving experiences that allow students to find solutions to non-routine problems. In this way, he involves students in doing mathematics. Kantowski (1981) defined non-routine problems as "problems for which a problem solver knows no clear path to the solution and has no algorithm which can be directly applied to guarantee solutions]" (p. 114). Mr Yamoah involves students actively in the learning process. Von Glasersfeld (1981) wrote that active learning is taking new information and acting upon it in some way to create meaning.

Students construct knowledge if they are actively involved in solving problems that they understand and want to solve. In this class, students do not sit passively and watch the teacher performs actions to solve the problems but solve the problems themselves. In Mr Yamoah's class, students become involved in experiencing mathematics. Even though Mr Yamoah's mathematics class usually lasts from seventy to eighty minutes, several times during my observations students became so involved that they asked to skip break time so that they could continue with their mathematics. Activities grow out of everyday situations. Mr Yamoah asks open-ended problems that have several answers. Students explore relationships or create solutions with their own methods in open-ended situations where there are many ways to find answers. Their experiences of learning the content help them understand that mathematics is a process of learning to think, not a product. An example of such open-ended questions is shown below:

Write a problem about a pattern that has the ordered pair (2, 20) as the second values to x and y. Write out the ordered pairs for all of your x and y values. Explain the rule you used.

x	2
↓	↓
y	20

Mr Yamoah presents problem solving situations that arouse students' interest and become problematic for them. Mr Yamoah's problems cause them to examine their current knowledge. Kantowski (1980) noted that a problem is a situation for which the individual has no available algorithm to use but must put together their knowledge in a new way to solve the problem. According to Yackel, Cobb, Wood, Wheatley, and Merkel (1990), teachers should provide children with activities that are likely to give rise to genuine mathematical problems.

Mr Yamoah never hurries through problems but concentrates each day on a few significant problems. The problems demand conceptual knowledge and higher-order thinking strategies from his students. He emphasizes primary concepts that are central principles that define mathematics; for example, addition and subtraction are inverse operations; a fraction is a part of a whole; a fraction's value depends on the whole to which it refers, and a circle has only one centre. According to Brooks and Brooks (1993), problems that are structured around primary concepts or conceptual clusters provide a context in which students learn to gather and analyze information that relates to the whole concept rather than to learn the steps linearly. Mr Yamoah continually says, "Tell me what you are thinking." He helps students build on their informal knowledge and guides them to develop a meaningful relational understanding of mathematics that connects the concepts and the procedure

Prerequisite Knowledge and Promotion of Mathematical Understanding

The teachers reported that mathematics teachers needed to have extensive knowledge of both mathematics and their students to teach for understanding, sequence lessons, make transitions between topics, understand student questions, provide excellent examples and maintain the necessary confidence in front of a class of students. This finding implies that teaching and learning of mathematics in our schools will improve significantly if serious attention is paid to the prerequisite knowledge of teachers before they are allowed to teach. This could be in the form of writing certification examinations as in the nursing profession.

The implication for mathematical understanding is that, if teachers teach well for students to have good mathematical understanding, they will be able to use their mathematics outside the classroom and in the study of other mathematics courses. Conceptual understanding of the subject will promote the love for the subject and its usefulness and application in the real world.

Connecting and Visualizing Mathematics

The implications of connecting mathematics during teaching and learning of the subject are captured in the following quotation by the National Council of Teachers of Mathematics. When students can relate mathematical ideas, their understanding is deeper and more lasting than when they do not. They can see mathematical connections in the rich interplay among mathematical topics, in contexts that relate mathematics to other subjects, and in their interests and experience. Through instruction that emphasises the interrelatedness of mathematical ideas, students not only learn mathematics, they also learn about the utility of mathematics. (NCTM, 2000, p. 64). A good example was the connection Mr Yamoah made his students see the simultaneous equation they learnt and the problem below:

There are 18 animals in a barnyard. Some are chickens, and some are cows. Altogether there are 50 legs. How many of the animals are chickens and how many are cows?

Students were excited to discover this beautiful connection between what they learnt in class and its relevance to real-life situations.

Teacher Refraining from Telling

A great teacher is one who creates a classroom environment that makes his students (1) 'curious', 2) want to 'explore' ('investigate') and 3) allows them to 'discover'. Instilling and encouraging these three elements in students makes a great teacher. It is a classroom where curiosity is valued, and students want to explore and investigate the topics being taught. A great teacher creates a classroom environment that makes the students 'wonder' about the things they are learning. You can tell from a classroom situation if the students are asking questions or they are just 'consuming' information given to them by the teacher. A great teacher creates an atmosphere and motivates their students to want to explore and investigate, for example, through experimentation. A great teacher encourages and guides their students to 'discover' answers, information, and solutions and not tell students answers. 'Discovery' makes students happy (Gannon, 2014).

Also, a good lesson is where the teacher moves away from providing information that the students have to memorise or mimic. The reason is that students know what the teacher tells them. It does not prepare them to solve real problems or use mathematics outside the classroom. The implication is that students should be guided to discover mathematics for themselves.

Mr Yamoah in most cases refrained from telling his students answers to questions he posed in class but rather lay a lot of emphasis on processes that lead to one's conclusions. Students most often than not were not told whether their responses were right or wrong but rather asked to explain how they arrived at their answers.

A good example of how a teacher can refrain from telling is this beautiful article published in the Daily Graphic of March 2, 2015. It was a conversation between a teacher and his students.

Teacher: How are trains and cars the same?

Alison: Well, they both have steering wheels.

Teacher: What do you mean by that?

Anna: You know. It's the round thing in the car that you turn to make it go.

Teacher: Well, I'll tell you the truth, I've never been in a train engine before, but someone told me that there are no steering wheels on trains.

Mark: My grandfather took me into the engine of a train once, and I didn't see a steering wheel.

Brian: But I was on a train.

Teacher: Tell us what you saw.

Brian: Well, I was only in the sitting part. But I thought I saw the driver part, and I saw a steering wheel.

Teacher: Now, I'm going to ask you to predict. Do you think the driver could steer the train to our school and pick us up?

Angela: No, there are no tracks here.

Teacher: Who agrees with Angela?

In the discussions, the teacher elicited diverse views leading on to the function of tracks. He then conducted an experiment in front of the children, whereby he took a toy train and rolled it down an incline: first without tracks, and then with curved tracks. To demonstrate the effect, the teacher released the train which followed a random path without tracks and fixed curved path with the tracks. When the teacher asked: "When did the train turn?" Most of the children responded: "When the train was on the tracks". The teacher then referred to the initial enquiry: "Do you think trains need steering wheels to turn?" Mark kept to his original opinion; two other children in the class who originally said yes changed their minds: the remaining children still said "Yes". The teacher, pretending to be still perplexed himself, suggested a field trip to the local train station so that the children could finally look into the engine cabin and discover for themselves whether trains need steering wheels. With probing questions, the teacher sought to know the points of view of the children and found out that they were not quite sure about steering wheels in trains that discovery provided him with the need to set up an appropriate experiment for the children to critically consider two opposing views, and decide for themselves.

Through the experiment, two kids reasoned that the tracks determined the course of the train without the need for a steering wheel. The other students were still uncertain despite the experiment. Finally, the excursion to a train proper settled their curiosity. In the discussions, not once did the teacher ask a question for a mere yes or no answer or offer a right or wrong response. The teacher served as a facilitator or a catalyst by allowing the children to share their point of view. Through an experiment and a supportive trip, the children- in the end – own their answers with certainty. Though small, children can be taught the larger lesson of verifying their beliefs through experiments and checking through sufficient evidence in order not to jump to hasty conclusions. Such basic lessons – once acquired benefit everybody including mature adults.

Using the above illustration as a yardstick to measure how a teacher should refrain from telling his students, Mr Yamoah fell short of being a good example. Whenever students are confused or divided on an issue, he does not allow the students time to search for a long time to find the answers for themselves. He sees it as his responsibility to help students out of their confusion. This is not a good example of a constructivist teacher, and I hope that readers will learn from the correct example from the Daily Graphic of March 2, 2015, illustrated above.

Mr Yamoah's View of Mathematics Influences His Goals

Mr Yamoah wants to provide opportunities for his students to learn to "think mathematically." His goals for his students are influenced by his view of mathematics. According to Mr Yamoah, mathematics is a way of thinking about problems. He views mathematics as a way of solving practical problems relating to everyday life, and he supports his lessons with relevant practical examples. Students were always involved in class discussions. Mr Yamoah gradually and consciously developed a culture of thinking in his class. He says, mathematics "is a way of thinking about the problem of numbers and shapes and concepts and their relationships" He believes that thinking mathematically helps students think better in all areas of life including other subjects that they study in school. Therefore, he extends his main goal of thinking mathematically by saying, "I want my students to learn to reason out things for themselves and value their thinking." Mr Yamoah believes that mathematics is an essential vehicle by which students learn to think and value their thinking.

Doing and teaching mathematics gives Mr Yamoah a great deal of joy. Nevertheless, he does not think that pursuing mathematical knowledge is an end in itself. He wants his students not only to learn to think and reason but also to be able to relate and apply their learning to everyday life. He says the researcher think they learn mathematics by developing their reasoning skills. They need to be encouraged to think and explain and build confidence in their ability to think through situations. They need opportunities to have experiences with mathematics. The experiences the researcher give can be two kinds of things. They can be the real-life situations- like. The researcher needs mathematics to figure my average mathematics score for the year. The researcher wants them to connect mathematics to their real life. But they also need experiences with mathematics that improves their thinking.

Mr Yamoah thinks that learning about mathematics will help his students "have an open mind to analyze problems and not accept anything that comes their way just as it is." Finally, Mr Yamoah says, the researcher wants them to look at the fun side of mathematics, the neat side of mathematics, the challenging side of mathematics and understand that it does connect to a lot of things. Mathematics is valuable in their whole learning experience. It helps them just to be a complete person. Because Mr Yamoah's conception of mathematics is that mathematics is a way of thinking about relationships between concepts and ideas, he designs his instruction to help his students achieve this understanding. Teachers whose conceptions about mathematics lead them to believe in the constructivist approach to learning emphasize that students must internally construct new knowledge through accommodating old knowledge rather than absorb the knowledge that a teacher transmits to them. Changes in teaching that are advocated by the Mathematical Association of Ghana (MAG) and many mathematics educators make it extremely important that we understand the conceptions of teachers who are teaching in our classrooms.

Mr Yamoah's Goals for His Students in Learning Mathematics

A goal is defined as the desired result that a person or a system envisions, plans and commits to achieve usually in the long term. Mr Yamoah tries to help his students come to understand that mathematics is not a collection of rules and procedures but is about making sense of relationships and connections of concepts to one another. He transmits a sense of what mathematics is and what knowing mathematics should be by the activities and problems that he chooses for the involvement of the students. Through his thinking and decision making, he helps students learn the procedures while they develop and refine their conceptual knowledge. Mr Yamoah's teaching exemplifies Skemp's idea of "relational understanding." When Mr Yamoah teaches, he helps students focus on the process of thinking rather than the final correct answer. This focus demonstrates to his students his belief that mathematics is about thinking through the problem and not about only getting the right answer.

Mr Yamoah's Notion about Mathematics Teaching and Learning

Ernest (1989) and Thompson (1984, 1992) have identified three conceptions that teachers usually hold about mathematics. Ernest summarized these three views as follows:

(a) the problem solving view defines mathematics as continually expanding and created by people as they see patterns and synthesize knowledge. Mathematics is a process of inquiry, not a finished product.

(b) The Platonist view defines mathematics as a body of knowledge with established truths that should be discovered not created. Mathematics is a product.

(c) the instrumentalist view defines mathematics as a collection of skills, rules, and facts that people need to memorize. Again, mathematics is a product. The conceptions that Ernest defined involve both an element of beliefs and knowledge. Even though views are usually defined as beliefs (Nespor, 1987; Thompson 1984, 1992), Ernest's definitions combined both beliefs and knowledge. Thompson (1992) noted that researchers should not look at teachers' mathematical beliefs in isolation from teachers' mathematics knowledge.

I came to understand Mr Yamoah's knowledge and beliefs about mathematics by observing his teaching practice and listening to his answers to my questions during the interview sessions. On the Mathematics Beliefs Scales questionnaire developed by Fennema, Carpenter, and Peterson (1987), he strongly agreed that teachers should encourage children to find their solutions to mathematics problems even if they are inefficient. He strongly believed that mathematics should be presented to children in such a way that they can discover relationships for themselves. He strongly agreed that allowing students to discuss their thinking helps them to make sense of mathematics. He did not believe that there was one right way to solve a particular problem. Mr Yamoah's beliefs are in line with the problem solving view of mathematics that Ernest (1989) defined. The problem solving conception is compatible with the constructivist learning principle that learning is a process whereby individuals internally construct new knowledge from prior knowledge. Even though Mr Yamoah never mentioned the words "constructivist learning," by observing him teach, the researcher saw that constructivist principles of learning mathematics were present in his teaching. Mr Yamoah believes that students personally construct mathematics during classroom interactions when they explain and justify their thinking. In his dialogue, he talks about how each student needs to build his or her knowledge in individual ways based upon prior knowledge. He gives students opportunities to construct powerful and correct mathematical ideas and concepts.

Mr Yamoah shows his conception of mathematics as growing and dynamic through both his innovative methods of teaching and the questioning of his students. His questions to students are often "What do you think might happen if we do it this way?" "Why do you think this works?" His conception of mathematics mirrors that of constructivist learning theorists. Piaget (1973) and Kamii (1985, 1989) proposed that students need to construct actively new knowledge internally as they accommodate and assimilate their new knowledge. Vygotsky (1978) proposed that people construct knowledge as they interact and discuss their thinking with more knowledgeable others. However, Von

Glasersfeld (1990) and Cobb (1994) advocated integration of these two processes by suggesting that students' construction of new knowledge is facilitated by interaction with others in a like community.

Mr Yamoah tries to help the students come to understand that mathematics is not a collection of rules and procedures to be memorized but is about making sense of relationships and connections of concepts to one another. He transmits a sense of what mathematics is and what knowing mathematics should be by the activities and problems that he chooses for the involvement of the students. Through his thinking and decision making, he helps students learn the procedures while they develop and refine their conceptual knowledge. Mr Yamoah's teaching exemplifies Skemp's (1978) idea of "relational understanding." When he teaches, he helps students focus on the process of thinking about connections between concepts and procedures. This focus demonstrates to his students his belief that mathematics is about thinking not about only getting the right answer. The process of constructing mathematics through the interactions among the students and with him became the content of his lessons Lakatos (1976) noted that mathematics is created through a process of contradiction while people argue, refine, and improve conjectures. Lakatos suggests that disagreement during the process of proving arguments is central to knowing mathematics.

Mr Yamoah's Depth of Knowledge of Subject Matter

Mr Yamoah also has the depth of knowledge of subject matter. He demonstrates in his teaching practice and decision making a sophisticated knowledge of mathematics as a subject matter. He shows an awareness of important concepts within mathematics and how these concepts are related. He demonstrates within his teaching how mathematical principles are established. He is aware of important questions in the field and the ways that reasoning becomes valid. Through his teaching, Mr Yamoah demonstrates that mathematics content is continually created. He uses the language of mathematics and leads the students to do same. His subject matter knowledge is integrated and well organized. He carefully sequences lessons because he knows which concepts students need to understand before proceeding to others. He knows what to teach and how to teach it. The depth of his knowledge of the subject matter of mathematics was evident within his assessment. His knowledge of mathematics helped him create better questions and problems for the students to reflect on and answer. Mr Yamoah often says, "It is helpful ... to understand the concepts that you are teaching. This [conceptual knowledge] affects the students' learning." Shulman (1986) stated that subject matter knowledge, pedagogical content knowledge, and curricular knowledge are necessary for teaching. Mr Yamoah demonstrates knowledge of mathematics pedagogy. His teaching practice illustrates his knowledge of the conceptual and procedural knowledge that students might bring to a particular learning situation. He shows an awareness of the stages of understanding through which the students might proceed to develop their conceptual thinking. He selects useful and powerful problems, illustrations, and representations to help students learn new concepts. Shulman (1986) believed that teachers need to know "ways of representing and formulating the subject that make it comprehensible to others. . .

. [They need to know] alternate forms of representation. . . [and] an understanding of what makes the learning of specific topics easier or difficult" (p. 9).

Mr Yamoah's knowledge of the subject matter and how children learn the content of mathematics is central to his teaching. His subject matter knowledge is strong enough that he is not threatened by the students' questions. Mr Yamoah uses the knowledge of his children's mathematical thinking to help them learn about their thinking. With this knowledge, he helps them learn to clarify their thinking. In his instruction, he pays attention to individual students' thinking while helping all the students to increase their mathematical power. Mr Yamoah says, "I think a good teacher needs to understand how children learn— that they are not going to be the same. You have to vary the way that you teach so that you include everyone. You have to provide experiences that meet all the needs of your children." Even though Mr Yamoah did not use any of the terms auditory, visual or kinesthetic learners, he demonstrated that he knows the different learning styles of children.

Fennema and Carpenter (1992) emphasized that teachers must know how children learn mathematics. They write that teachers "must know problem difficulty as well as knowledge of distinctions between problems that result in different processes of solutions" (p. 2). These researchers noted that teachers must help students relate new knowledge to the informal knowledge that they already possess. Mr Yamoah places a high priority on choosing the appropriate instruction to help individual students gain their potential level of mathematical power. However, despite his understanding of each student's differences in thinking, Mr

Yamoah was surprised and often astonished by his students' capacity to think at high levels. He said that the most important thing that teachers should remember is "Don't underestimate students' ability to think." He recognized that his students could learn and do many things that he had never expected of them.

Mr Yamoah has fair knowledge about curricula mathematics materials-the range of instructional materials that are available to teach mathematics. This knowledge is evident in the classroom. He makes use of many books, manipulatives, and alternative materials. For instance, he used geoboard and rubber band to help improve students' concepts about perimeter and area. He chooses manipulatives such as solid figures and pattern blocks to increase spatial thinking abilities of his students.

Beliefs might be considered as the lenses through which one looks when interpreting the world, and affect might be thought of as a disposition or tendency one has toward some aspect of his or her world; as such, the beliefs and affect one holds surely affect the way one interacts with his or her world (Philipp, 2007).

Mr Yamoah's Teaching Challenges

According to Clark (1988), teaching is "complex, uncertain, and peppered with challenges" (p. 9). He writes that much of teachers' thinking "energy goes into trying to predict and anticipate potential problems, to guess and estimate what students already know and how they might respond, and to forming plans and routines" (p. 9). Lampert (1985) defined a dilemma simply "as an argument with oneself" (p. 182). However, together Clark and Lampert (1986) gave a more comprehensive definition of challenges of teaching, "the teacher encounters a host of interconnected and competing decision scenarios while planning to teach and during teaching. There are no ideal or optimal solutions to these decisions" (p. 28).

Mr Yamoah openly discussed the challenges he has faced while seeking to obtain a balance between depth and breadth of the mathematics content. He recognises that he is sometimes limited in the quality of experiences he can provide for his students. Most of his concerns are about the students themselves. Are they learning correct knowledge? Am I meeting each student's needs? He said that each class is different and that he must change his approach from one class to another and somewhat every year to adapt to the class.

One of Mr Yamoah's primary challenges is how to get the students to move from their old ways of thinking about mathematics (drill and practice of computations, memorisation of rules and formula etc.) to new ways (such as relational understanding and communicating their thinking among others). How can he help them to begin to articulate their thinking? He talked about how the students were slow to communicate their thinking at the beginning of the year. You have to respond to each class. At the beginning of the year, the students were not used to interacting during the mathematics class. They wanted to give cut and dry answers. Mr Yamoah thinks that after several years in classrooms in which teachers taught mathematics through drill and practice of computations, students are very slow to take risks to explain their thinking about solutions. He struggles with assuring students that he will accept their answers and their ideas without making judgments.

than having to deal with some off-task behaviour, like doing things with the manipulatives that you do not want them to do.

Perhaps, the most serious dilemma that Mr Yamoah reports concerns assessment-how to understand and interpret his students' thinking. In some cases, the students are not clear when they explain their thinking. He thinks that it has been difficult to learn how to understand what they say. He says, "It's hard to respond to the students' responses- understanding and trying to work with [what they say].The researcher had a hard time with understanding in the beginning." Since sometimes it is difficult to understand what they say, Mr Yamoah often needs to help students find other ways to explain their thinking (e.g. drawing diagrams). Because he wants to build his instruction on students' prior knowledge, resolving this dilemma is extremely important. Mr Yamoah also says he sometimes has difficulty in knowing how far he can help students to extend their thinking. Mr Yamoah then discusses how important it is for a teacher to know his students.

Teaching children not only to construct mathematics knowledge but to construct correct mathematical knowledge intensifies the challenges that must be resolved. Lampert maintained that accepting that these will be challenges and cope with them is better than trying to resolve every one of them. Learning to manage gives teachers the power to shape the direction and the outcomes of their work. "Facing a dilemma need not result in a forced choice" (Lampert, 1985, p. 182). Mr Yamoah manages his challenges concerning teaching mathematics by remembering what he thinks his role as a teacher should be—helping his students learn to think mathematically and value their thinking. Mr Yamoah says, 'The key to getting children to improve their problem solving ability is first to make sure teachers are committed to the role of helping students understand."

Planning for Instruction

Clark (1988) pointed out that to understand teachers planning is to understand how teachers transform and interpret their knowledge as they prepare for instruction. To study teacher planning can help explain why and how the teachers make curriculum and instructional choices for classroom instruction. Researchers on teacher thinking and decision making have attempted to differentiate and separate teachers' proactive, interactive, and post active phases of teacher's planning (Brown and Borko, 1992; Clark, 1988; Jackson, 1968). However, Shavelson and Stern (1981) suggested that there is no differentiation in a teachers' thinking before, during, and after instruction. These researchers wrote that the pre-active, interactive, and post-active decision-making processes of teachers are related components in which teachers develop and enact agendas. The researcher chose to agree with Shavelson and Stern; therefore, the researcher discussed Mr Yamoah's thinking by not separating his thinking processes into categories or phases of planning. Mr Yamoah's knowledge and beliefs about mathematics play out in his thinking about instruction and classroom practice. He constantly analyzes and examines his teaching. He makes planning decisions before class based on the information that he has collected about what each child knows and how he can structure his teaching to provide the means for each child to learn.

When Mr Yamoah makes planning decisions, he considers several factors. Some factors that he considers are (a) the potential difficulties students might encounter with specific concepts, (b) the learning needs and interests of students, (c) the concepts that he wants students to learn, (d) the types of problems or the learning situations that will help teach the concepts, (e) the difficulty level of problems, (f) his knowledge of students' prior knowledge, (g) the strategies he will use to present problems, (h) the assessment of students, (i) the materials that students will use to solve problems, (j) the time allowances for concepts or topics, and (k) the organization of students. In the following sections, the researcher will discuss several of the previous factors that Mr Yamoah considers when formulating his plans.

Choices of Mathematical Tasks

The National Council of Teachers of Mathematics (NCTM) first identified the mathematical task in its (1991, 2008) Professional Teaching Standards as "worthwhile mathematical tasks" (p. 24). Boston and Smith (2009) later provided this succinct definition: "A mathematical task is a single complex problem or a set of problems that focus students' attention on a specific mathematical idea" (p. 136).

Mathematical tasks include activities, examples, or problems to complete as a whole class, in small groups, or individually. The tasks provide the rigour (levels of complex reasoning from the conceptual understanding, procedural fluency, and application of the tasks) that students require and thus become an essential aspect of the teacher's collaboration and discussion with his students. In short, the tasks are the problems teachers choose to determine the pathway of student learning and to assess student success along that pathway.

The decisions one makes about what and how a student learns each day is based on the mathematical tasks choices one makes for every lesson, every day. Those choices are a lot of power! As Lappan and Briars (1995) state. "There is no decision that a teacher makes that has a greater impact on students' opportunities to learn and on their perceptions about what mathematics is, than the selection or creation of the tasks with which the teacher engages students in studying mathematics" (p. 139).

The selection of useful mathematical tasks is so important that it is one of the eight research-informed instructional strategies listed in Principles to Actions (NCTM, 2014). And, it is two of the ten high leverage team actions (HLTA's) in Beyond the Common Core Handbooks (Solution Tree, 2015). A key collaborative team decision, then, is to decide which tasks to use in a particular lesson or unit to help students attain the essential learning standards. A growing body of research links students' engagement in higher-level-cognitive-demand tasks to overall increases in mathematics learning, not just in the ability to solve problems (Hattie, 2012; Resnick, 2006).

Mr Yamoah's choice of mathematical tasks seems to follow the above description. He would come to class most of the time with questions that one would not normally find in textbooks, and they were most of the time engaging and challenging. On one typical day Mr Yamoah decided to pose this question to his students almost at the end of the class for discussion during the next meeting:

You are given a bottle of water and offered 1/3 off the price for the same size bottle, or you are offered 1/3 more water at the same price. Which of them is a better offer for you as the consumer?

Mr Yamoah asked the students to try and bring their solutions to class for discussion during the next meeting. Unfortunately, when the students came for the next mathematics class, nobody did it correctly, but they were praised by Mr Yamoah for the serious attempts they made. He told the class to continue trying till they get it right.

Choice of Mathematical Tools

The mathematical tools in Mr Yamoah's classroom are the concrete materials or manipulatives that support the students' efforts to link the concepts with the procedures. Mr Yamoah plans activities that provide an opportunity for students to perform meaningful actions with the concrete materials. Mr Yamoah's choices of mathematical tools depend upon the concepts that he wants his students to think about. Lappan (1993) wrote that manipulative materials should offer the "potential for students to engage in sound and significant mathematics as part of accomplishing the task" (p. 525). Manipulatives are tools to be used thoughtfully by students to help them make sense of mathematics. Mr Yamoah values manipulative materials for students' learning. He said that to teach mathematics so that children would learn to think critically, you "can't just open the book and let them do page such and such. Manipulatives are wonderful aids that help me generate ideas for the students to think about." Mr Yamoah uses manipulatives such as fraction circles and fraction strips to help the students understand fractional concepts. He used base-10 blocks and grid paper to help children learn about perimeter and area. He used the plane and solid shapes, pattern blocks, and geoboards to help children learn to think geometrically. Mr Yamoah explained his considerations when choosing mathematical tools, "I know how my JHS 1 students think…. and the kind of materials I need to bring to class to help them understand the concept I want to teach". Mr Yamoah says one factor in deciding which manipulatives to use is, "I think of what learning materials I can choose that would provide me with the most effective and easiest way for the students to understand the concept I am about to teach."

Lampert (1989) suggested that manipulatives are essential because "representations of a concept contribute to long-term understanding and knowledge use" (p. 256). Students should use manipulative materials that will help them model their mathematical thinking. Using manipulatives can help stimulate thinking and help children build their knowledge. Manipulatives can also help to promote reflective thought (Baroody, 1989).

Sources of Mr Yamoah's Problems for Classroom Discussion

Mr Yamoah sometimes chooses the content of his lessons through reactions to a student's statement or question. For example, one student had stated that he did not find what they were doing under measurement to be very useful in any way— changing inches to feet, to yards and metres. Mr Yamoah used this doubt to plan the next day's lesson. During the next day's activity, Mr Yamoah asked eight students to come up front and hold out their rulers until they reached 96 inches. He then laid three-yard sticks on top of the rulers and asked, "What do you think about that?" They decided during this discussion that there were 2 feet left over without a yardstick laying on top of them. That meant there were 2 2/3 yards. From this demonstration, they decided that 96 inches = 8 feet = 2 2⁄3 yards. Mr Yamoah went further to lay metre sticks on top of the yard and rulers and asked, "What do you think about this also?" The class finally concluded that 96 inches = 8 feet = 2 2⁄3 yards ≅2(2)⁄5 metres.

Mr Yamoah ended that lively discussion by asking the students to brainstorm and find ways that knowing how to change inches to feet to yards and metres could be helpful. He talked about this in the following statement.

Yesterday, when Moses indicated that he didn't think that having to change from inches to feet or yards and metres would be useful to know, I decided to have them look at that in today's measurement activity. I knew that we would be looking at 96 inches. I planned this problem because it was divisible by 12 and also divisible by 3 (12 inches in a foot, 3 feet in a yard). When I saw that I was dividing by 3, it immediately clicked because they know the divisibility rule for 3. So we could look at yards. When Mr Yamoah planned his lessons, he did not use just the teacher's guide. Even though he would look at the chapter to see what the book was emphasising, he used a variety of sources to develop his plans. He would not proceed directly through the book. He would have a scope of concepts and skills that he would want the students to learn, but he did not have a planned sequence. He ordered the concepts that he would teach based on the concepts the children already understand. Duckworth (1987) suggested that teachers who teach with the constructivist approach to learning do not often prescribe a scope, sequence, or a timeline for learning topics. All these can interfere with a student's ability to understand complicated concepts

According to experts, the top eight components of a well written lesson plan are: Objectives (or Goals), Anticipatory Set (approximate time), Direct Instruction (approximate time), Guided Practice (approximate time), Closure (approximate time), Independent Practice, Required Materials and Equipment and Assessment and Follow-Up.

Anticipatory Set: In the Anticipatory Set portion, you outline what you will say and present to your students before the direct instruction of the lesson begins. The purpose of the anticipatory set is to provide continuity from previous lessons, allude to common concepts and vocabulary as a reminder and refresher. Tell the students briefly what the lesson will be all about. Assess the students' level of collective background knowledge of the subject. This will help inform your instruction. Activate the students' existing knowledge base. Whet the class's appetite for the subject at hand. Briefly expose the students to the lesson's objectives and how you will get them to the result

Organization of Students

Mr Yamoah uses the nature of the activity to be undertaken to help him decide how he will organize the students. Mr Yamoah believes that children of all ability levels should work together. He believes that low-achieving and high-achieving students should work with all the others students. He believes that his students learn more if they stay with their classmates. All students in his class learn mathematics at the same time working on the same tasks or activities. The

NCTM (1989, 1991) recommended that students in the same grade learn mathematics altogether rather than in ability groups and that the same topics and activities be used. Different types of classroom organisation that Mr Yamoah uses are cooperative groups, individuals working independently, and the whole group.

Cooperative Groups: Sometimes the cooperative group is four or five students or a pair of students. Sometimes Mr Yamoah puts weaker students together so that they are "forced to communicate to solve their problems." He says, "They have to say the words to explain it." He sometimes puts more vocal students together because they are 'forced to listen to other people's ideas." Yackel, Cobb, and Wood (1991) stated that during whole-group instruction a teacher could encourage in-depth discussions among children as they explain their mathematical thinking. These discussions are important for students' mathematical development as they socially interact. This researcher added that whole-group discussion after students have worked in cooperative groups is valuable for students to explain their solution attempts and get other students' interpretations of those attempts.

Assessment Choices

Mr Yamoah believes that the purpose of assessment is to improve his instruction to improve students' learning. Mr Yamoah understands that to develop instruction that the students need and are ready for he must listen carefully and assess the development of their thinking. While assessing his students' thinking, he looks for what to emphasise, what the next steps should be, and how to challenge their thinking. A guiding principle of cognitive guided instruction is that instructional decisions should be based on careful analyses of students' knowledge. If a teacher structures the lessons based on his assessment of students, the learning situation will more appropriately conform to the learner's development (Fennema and Carpenter, 1992).

As Mr Yamoah assesses his students through listening to their explanations, he says he can understand the students' depth of knowledge and use these explanations as beginning points to lead them to construct new understandings. Mr Yamoah believes that he can learn more about what his students understand by assessing them while he is teaching. He observes their interactions with him and others students and their use of manipulative materials. He says these assessments tell him more about what his students know than written assessments. He says that the complex part of understanding what students are thinking must be done by listening to the students' reasoning. He can assess whether this knowledge is memorized procedures or constructed knowledge that has been internalized. Lambdin (1993) made this point by noting that written tests often tell little about children's strategies. Because of this lack of information, teachers may jump to inaccurate conclusions about the children's performance. Often written tests make it difficult to diagnose children's mathematical difficulties. Researchers, such as Huinker (1993), suggested that listening to student's explanations allows teachers to gain insight into students' conceptual knowledge and reasoning during problem solving. Listening to students allows teachers to "determine the level of understanding, to diagnose misconceptions and missing connections, and to assess verbal ability to communicate mathematics knowledge" (Huinker, 1993, p. 80). With paper and pencil tasks, students' understanding is more likely to be hidden. With these tasks, it is difficult to determine whether an incorrect answer is given because a student lacked the necessary knowledge or simply made an error. Mr Yamoah thinks that this type of assessment is difficult because He is looking for conceptual knowledge which cannot be easily accessed through traditional tests. Nevertheless, he does use written assessment at times

Chapter Summary

In this chapter, the researcher has presented Mr Yamoah's goals for students in learning mathematics. The researcher has described his thinking about instruction and many of his instructional decisions. A common thread is the conception of mathematics as a way of thinking which weaves through Mr Yamoah's thinking, his instructional decisions, his teaching practice, and ultimately to what the students learn about mathematics. His knowledge of mathematics and his belief that children must construct their knowledge is integrated within his conception of mathematics. In his mind, concepts are never isolated topics. Mr Yamoah demonstrates with his thinking and planning how mathematics should be taught is as important as what is taught. With his planning and decision making, he seeks to assure that every child has success at some level in learning a relational understanding of mathematics. To show how his thinking about instruction is focused and coherent, the researcher has detailed his reasons for choosing problems or tasks, his choices of mathematics tools, his decisions about time, his decisions about how to organize students, and his assessment decisions

Mr Yamoah has been able to create a classroom atmosphere that encourages his students to share their thinking about mathematics with their peers freely. The researcher observed a class which was devoid of fear and intimidation largely because of Mr Yamoah's respect for learners' views, the absence of abusive language, derogatory remarks when wrong answers were given, and lack of corporal punishment –such as canning in class. Many of Mr Yamoah's students were free and bold to speak their mind in class. Students even went beyond just sharing their thoughts on a particular mathematics topic to justifying their reasoning and reviewing the solutions of their peers on the marker board on many occasions. Most often, both boys and girls walked confidently to the marker board and wrote their solutions to questions posed by the teacher. Students were not afraid of criticisms or corrections from their peers, their teacher- Mr Yamoah or in the presence of the researcher.

In some instances, Mr Yamoah had to "negotiate" the final answers with his students. A particular case in point was when students worked cooperatively to discover pi (π) by measuring the circumference of circular discs and lids they had brought to class. After measuring several circumferences and their corresponding diameters, the students found their "standard value for pi" by finding the average of the π^' s. Unfortunately, the groups' averages did not tally with what Mr Yamoah wanted the class to accept as the standard value for pi. The 3.142 was eventually accepted by the students after some debate and negotiation with his students. This negotiation was necessary to let the students appreciate the fact that their measurements were not very accurate enough to produce exact value for pi hence the need to accept the standard one.

Furthermore, the researcher observed that whenever students were asked to work in groups, they were often seen actively sharing their thoughts and the group leaders were always excited when they had to represent their groups to share their findings with the whole class.

Some of Mr Yamoah's common questions during his mathematics lessons were:

Who disagreed with this method or the final answer itself?

Who can explain why this answer is wrong or right?

Who has an alternative method of solving this question?

Who solved this question differently from what is on the board?

Where exactly did he/she go wrong and why?

What can we add to this solution to make it complete?

What is missing in this solution on the board?

How do we verify the correctness of this answer?

Who doesn't understand the solution?

CHAPTER FIVE

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Overview

The purpose of this qualitative, descriptive study was to determine how students experience mathematics when constructivist teaching and learning is implemented in a JHS classroom. The research questions which guided this study were:

(i) How does the teacher view constructivist teaching and mathematics learning?

(ii) Do the plans the teacher make before his teaching differ from how he teaches?

(iii) Are there any connections between the teacher's ideas about constructivist teaching and instructional decisions on one hand and his classroom practice on the other hand?

(iv) To what extent does the teacher's experience influence his ideas about constructivist teaching and learning?

(v) How do students experience mathematics teaching when "constructivist methods" are used in their classrooms?

This qualitative research (ethnographic research) study started with the identification of 25 JHS one mathematics teachers in X and Y municipal assemblies in Greater Accra Region. A minimum of two JHS1 classrooms were identified in each school for the observation. After administering an adapted Mathematics Beliefs Scales (MBS) to the teachers to assess their level of appreciation of constructivist approach to mathematics teaching and learning, the number was cut down to 12 and later to four. The researcher finally chose two of the teachers for two reasons:

(a) They seemed to believe in the constructivist theory of learning and teaching of mathematics as demonstrated in their classroom practice during their class observations.

(b) They had begun implementing a constructivist approach to mathematics teaching in their own small way either consciously or unconsciously with enthusiasm. The two remaining teachers were regularly observed but due to the use of video coverage, audio recording and documentation involved, the researcher concentrated on only one of the remaining two teachers by name Mr Yamoah who was so enthused about the study. To study the teaching and learning of this Junior High school one teacher, I used descriptive ethnographic research methods to collect and analyze data. I took field notes and videotaped this class for nearly three terms. I conducted some informal interviews and three formal interviews with the teacher and some students. Also, I carefully examined the teacher's lesson notes, resource materials, and children's classwork and took photographs of students' work. A summary of the findings are as follows:

On the first question of how the teacher views constructivist teaching and learning, it was found that the teacher sees mathematics as a way of thinking about problems. The problems may be everyday problems like buying from a shop or market and receiving a change to complex issues like calculating the compound interest on an amount deposited at the bank. The teacher's view on mathematics teaching was that children should be encouraged to construct their own knowledge to have a relational understanding of the subject. His focus was to make mathematics interesting and easy for his students as well as put into practice teaching strategies that he had learnt at the training college.

The second research question was to find out whether the plans the teacher makes before his teaching differ from how he teaches? The findings revealed that about 20% of the time, Mr Yamoah had to put aside what he planned to teach for that period and rather take his students through what they did not understand fully from previous lessons. Whenever students have a shallow understanding of a concept, he would review that concept before he moves on to a new lesson.

The third research question was "are there any connections between the teacher's ideas about constructivist teaching, instructional decisions and his classroom practice?" The study revealed that Mr Yamoah's view of constructivist teaching; his instructional decisions and classroom practice were connected. Mr Yamoah's notion about constructivist teaching which is basically about providing experiences for his students to think and construct their knowledge influences his lesson preparation as well as his classroom practice. One of the key concerns of Mr Yamoah was his knowledge of his students' prior knowledge which will form the basis of what he intends to teach. His lesson planning mostly involved the teaching and learning materials that would be needed to discover the main idea to be learnt. During his teaching, he would put the students in groups of five or six and tasked them to produce the answers to the questions he would pose in class. A critical study of the JHS teaching syllabus by the researcher showed that Mr

Yamoah was virtually picking most of his ideas from the teaching and learning activity column of the syllabus and adding his twist to it to suit his lesson.

The fourth research question was "to what extent does the teacher's experience influence his ideas about constructivist teaching? Having taught for 12 years at the Basic School as a mathematics teacher, coupled with the fact that he had a professional training as a teacher (DBE) and also a B.A. degree in statistics, Mr Yamoah has extensive knowledge of both mathematics and his students thereby enabling him to teach for understanding, sequence his lessons properly, make transitions between topics, understand student questions, provide good examples and also maintain the necessary confidence in front of his students. Mr Yamoah is of the view that conceptual understanding of the subject will promote the love for the subject and its usefulness and application in the real world. The teaching syllabus also provides useful insight to Mr Yamoah in his lesson planning and delivery.

The fifth and last research question was "how students experience mathematics teaching when constructivist methods are employed in their class?" It was found out through this study that there was a high level of social interaction among the students. Students were free to consult one another for further clarifications and explanation of questions they did not understand from their friends. The students during their interview with the researcher praised their mathematics teacher Mr Yamoah for his style of teaching which helped them to learn and like mathematics. They talked about group discussions which helped them to understand concepts better by listening to their friends' explanations.

Student-centred instruction characterized most of the mathematics lessons of Mr Yamoah. Thus the role of the teacher was that of a knowledgeable adult who supported the learners to achieve ends that would have been unattainable if the students had worked on their own (Goodchild, 2002b).

Conclusions

Examining Mr Yamoah's teaching through the lens of what research has shown as key elements of effective instruction, the following conclusions have been reached based on the findings:

1. A teacher's view of mathematics influences every aspect of the teacher's teaching such as decisions about instruction, planning for instruction, assessment of students, and interaction with his students in the classroom.
2. Effective teachers do not stick to their original plans before teaching. Classroom situations largely determine what they teach and how they should teach for understanding.
3. A constructivist teacher can create a learning environment which is rich for students to become excited about mathematics.
4. A teacher's experience imparts positively on the way he teaches the subject for understanding.
5. By using constructivist teaching methods, a teacher can provide the enabling environment for students to like mathematics and extend their knowledge beyond the classroom.

Recommendations

1. Since a teacher's view about mathematics largely influences every aspect of his teaching, it is recommended that training institutions pay particular attention to the views pre-service teachers hold about mathematics before they go to the classroom to practice.

2. It is also recommended that mathematics teachers who are making serious efforts to use constructivist approaches in teaching and learning in their schools should be identified and recognised by stakeholders. This will energise them to do more to change the way the subject is currently being taught.

3. To change a teacher's methodology and pedagogy for the better, it is recommended that his academic and professional qualifications are enhanced at least to a first-degree level.

4. To make students love mathematics and put the phobia behind them, it is recommended that teachers adopt constructivist approaches to their teaching and learning.

Suggestions for Future Research

To gain a better understanding of how teachers use constructivist approaches in their classrooms, much more remains to be learned about what this teaching "looks like." From the present study, I made four recommendations for future research.

First, it is crucial to have more practical descriptions of teachers using constructivist approaches to mathematics teaching and learning. I would encourage more research in this area at different class levels to see how it influences students learning. Second, the current study suggests that there is a need for more narratives about teachers who are changing their mathematics teaching. Third, this study illustrates the need for long-term qualitative studies of teachers. Fourth, it is important to learn more about how universities and colleges of education programmes are helping pre-service and in-service teachers in learning to teach using constructivist learning principles. This is very crucial because we are far behind in both national and international competitions in mathematics.

REFERENCES

Adams, P. (2006). Exploring social constructivism: Theories and practicalities. *Education, 34*(3), 243-257.

Agudela-Valderrama, A. C. (1996). Improving mathematics education in Columbian schools: Mathematics for all. *International Journal of Educational Development, 16*(1), 15-26.

Ameron H., L. B. (2010). Manipulatives: When they are useful? *The Journal of Mathematical Behaviour, 20*(1), 21-31.

Amponsah, J. (2010). *When will Ghana get represented at the International Mathematical Olympiad? Ghanaweb.* Retrieved April 28, 2010, from https://www.ghanaweb.com/GhanaHomePage/features/When-will-Ghana-get-represented-at-the-International-Mathematical-Olympiad-18083610.

Anamoah Mensah, J. & Mereku,D.K. (2005). *Ghanaian JSS2 Students' abysmal mathematics achievement in TIMMS-2003: A consequence of the basic school mathematics curriculum*. Mathematics Connection. *5*(1), 1-13.

Anamuah-Mansah, J. (2008). *TIMSS 2007 Ghana Report.* Accra: MOES.

Anderson, N. &. D'Ambrosio, B. S. (2008). ZPC and ZPD: Zones of teaching and learning. *Journal for Research in Mathematics Education, 39*(3), 220-246.

Austin, A. (2008). *Educating integrated professionals: Theory and practice on preparation for the professoriate.* San Francisco: Jossey- Bass.

Balaguer, M. (2001). A theory of mathematical correctness and mathematical truth. *Pacific Philosophiccal Quarterly, 82*(2), 87-114.

Ball, D. (1991). *Research on teaching mathematics: Making subject matter knowledge part of the equation*. Greenwich: CT: JAI Press.

Bandura., A. (1977). Self-efficacy. *Encyclopedia of human behavior*, *4*, 71–81.

Baroody, A. (1989). *A guide to teaching mathematics in the primary grades*. Boston,: MA: Allyn and Bacon.

Battista, M. (1994). Teacher beliefs and the reform movement in mathematics education. *Phi Delta Kappan*, 462-463.

Becker, H. S., & Geer, B. (1969). Participants observation and interviewing. In G. J. McCall & J. L. Simmons (Eds.), *Issues in participant observation: A text and reader* (pp. 321-331). Reading, MA: Addison-Wesley.

Biggs, A. (2002). Enactivism and some implications for Education: A Personal Perspective. *Vinculum, 39*(2), 4-12.

Blakey, E. (1990). *Developing Metacognition*. Retrieved January 2002, from http://www.ed.gov/database/ERIC_Digests/ed327218.html.

Blanck, G. (1990). Vygostky: The Man and his Cause. In L. C. Moll (Ed.), *Vygotsky and Education: Instructional Implications of Sociohistorical Psychology* (pp. 31-58). London: Cambridge University Press.

Blum, M. (2002). *Enhancement of students learning and attitudes towards mathematics through authentic learning experiences.* Sydney: Curtin University of Technology,Australia.

Blumer, H. (1967). Society as symbolic interaction. In J. Manis & B. Meltzer (Eds.), *Symbolic interaction* (pp. 139-148). Boston: Allyn and Bacon.

Blumer, H. (1969). *Symbolic interactionism.* Eaglewoods Cliffs, NJ: Prentice Hall.

Bogdan, R., & Biklen, S. K. (1982). *Qualitative research for education: An introduction theory and methods.* Boston: Allyn and Bacon.

Bogdan, R.C., & Biklen, S. K. (2007). *Qualitative research for education: An introduction to theories and methods* (5th ed.). Boston: Pearson Education.

Boston, M. D., & Smith, S. S. (2009). Research in mathetics education. *Journal for Research in Mathematics Education, 40*(2), 119-156.

Brahier, D. J. (2009). *Teaching secondary and middle school mathematics.*(3rd ed.). Bowling: Pearson Education, Inc.

Brooks, J. G., & Brooks. M. G. (1999). *In search of understanding: The case for constructivist classrooms.* Alexandria: Association for Supervision and Curriculum Development.

Brooks, J. G. (1993). *The case for constructivist classrooms.* Alexandria, VA: Association for Supervision and Curriculum Development.

Brophy, J. E. (1979). Teacher behaviour and its effects. *Journal of Educational Psychology,71*(6), 733-750.

Brown, C.A., & Borko, H. (1992). Handbook of research on mathematics teaching and learning. *A Project of National Council of Teachers of Mathematics*, 209-239.

Brown, J. G., & Brooks, M. G. (1992). The case for constructivist classrooms. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 209-239). New York: Macmillan.

Brown, S. I., & Cooney, T. J. (1986). Stalking the dualism between theory and practice. In P. F. L. Verstappen (Ed.), *Second conference on systematic co-operation between theory and practice in mathematics education* (pp. 21-40). Lochem, The Netherlands: National Institute for Curriculum Development.

Brown, S. I., Cooney, T. J., & Jones, D. (1990). Mathematics teacher education. In W. R. Houston (Ed.), *Handbook of research on teacher education* (pp. 639-656). New York: Macmillan.

Bruner, J. (1961). *The act of discovery.* Cambridge: Harvard Educational Review.

Bruner, J. (1966). *Toward a theory of instruction.* Cambridge, Mass: Belkapp Press.

Bruner, J. (1985). Vygotsky: A historical and conceptual perspective. In J. V. Wertsch, *Culture, communication and cognition* (pp. 21-34). Cambridge: Cambridge University Press.

Bruner, J. (1986). *Actual minds, possible worlds.* Cambridge, MA: Harvard University Press.

Bruner, J. (1996). *The culture of education.* Cambridge, MA: Harvard University Press.

Bruner, J. (2009). *The process of Education* . London: Harvard University Press.

Bruning, R. H., Schraw, G., Norby, M. M., & Ronning, R. R. (2004). *Cognitive psychology and instruction* (4th ed.). Columbus, OH: Merrill.

Busch, J. (2012). Is the indispensability argument dispensable? *Theoria*, *77*, 139-158.

Bush, W. (1982). *Preservice secondary mathematics teachers' knowledge about teaching mathematics and decision-making process during teacher training(Doctoral Dissertation,University of Goergia,1982).* Dissertation Abstracts International, 43, 226A.

Capps, L.R., & Pickreign. (1993). Language connections in mathematics: A critical part of mathematics instruction. *Arithmetic Teacher*, *4*(1), 8-12.

Carpenter, T. (1984). The acquisition of addition and subtraction . *Journal for Research in Mathematics Education*, *15*(3), 179-201.

Carpenter, T. A., Fennema, E., & Franke, M. L. (1992a). *Cognitively guided instruction: Fractions*. Madison: University of Wincousin, Wincousin Center for Education Research.

Carpenter, T. A., Fennema, E., & Franke, M. L. (1992b). *Cognitively guided instruction: Multiplication and division.* Madison: University of Wincousin, Wincousin Center for Education Research.

Carpenter, T. A., Fennema, E., & Franke, M. L. (1992c). *Cognitively guided instruction: Building the primary mathematics curriculm on children's informal mathematical knowledge.* Madison: University of Wiscousin, Wincousin Center for Education Research.

Carpenter, T. A., Hiebert, J., & Moser, J. M. (1981). Problem structure and first grade children's initial solution processes for simple addition and subtraction problems. *Journal for Research in Mathematics Education, 12*, 27-39.

Carpenter, T. P., & Fennema, E. (1991). Research and cognitively guided instruction. In T. P. Carpenter and E. Fennema, *Integrating research on teaching and learning mathematics* (pp. 2-19). Albany, NY: State University of New York Press.

Carpenter, T. P., & Fennema, E. (1992). Cognitively guided instruction: Building on the knowledge of students and teachers. *International Journal of Educational Research,, 17*(5), 457-470.

Carpenter, T. P., Fennema, E, Peterson, P. L., Chiang, C. P., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experiment. *American Educational Research Journal,*, *26*(4), 499-531.

Carroll, J. (1989). The Carroll model: A 25-year Retrospective and prospective view. *Educational researcher, 18*(1), 26-31.

Chambers, D. L. (1993 Yearbook,). Integrating assessment and instruction. In N. L. Webb & A. F. Coxford (Eds.), *Assessment in the mathematics classroom* (pp. 17-25). Reston, VA: National Council of Teachers of Mathematics.

Clark, C. (1988). Asking the right questions about teacher preparations: Contributions of research on teacher thinking. *Educational Researcher, 17*, 5-12.

Clark, C. M., & Peterson, P. L. (1986). Teachers' thought process. In M. Wittrock (Ed.), *Handbook of research on teaching* (pp. (3rd ed., 255-296)). New York: Macmillan.

Clarke, D., Stephens, M., &Waywood, A. (1992). Communication and the learning of mathematics. In T. A. Romberg (Ed.), *Mathematics assessment and evaluation* (pp. 184-212). Albany, NY: University of New York Press.

Clement, D. (1999). *Concrete manipulative, concrete ideas, contemporary: Issues in Early Childhood.* Canada: Wiley & Sons Inc.

Clement, D. H. (1990). Constructivist learning and teaching. *Arithmetic Teacher, 38*(1), 34-35.

Cobb, P. & Steffe, L. P. (1983). The constructivist researcher as teacher and model builder. *Journal for Research in Mathematics Education, 14*(3), 83-94.

Cobb, P. (1988). The tension between theories of learning and theories of instruction in mathematics education. *Educational Psychologist,, 23*, 87-104.

Cobb, P. (1993). *Cultural tools and mathmatics learning: A case study.* Atlanta, GA.

Cobb, P. (1994). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher, 23*, 13-19.

Cobb, P. Y. (1989). Young children's emotional acts while engaged in mathematical problem solving. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving* (pp. 1, 17-148). New York: Springer Verlag.

Cobb, P., Yackel, E., & Wood, T. (1991). Curriculum and teacher development: Psychological and anthropological perspectives. In T. P. E. Fennema, *Integrating research on teaching and learning mathematics* (pp. 92-131). Albany, NY: State University of New York Press.

Cobb, P., Yackel, E., & Wood, T. (1990). Classrooms as learning environments for teachers and researchers. In C. A. R. B. Davis, *Constructivist views on the teaching and learning of mathematics* (pp. 125-146). Reston, VA: National Council of Teachers of Mathematics.

Cobb, P., Yackel, E.,& Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics Education,*, *23*, 2-233.

Cockcroft, W. (1982). *Mathematics counts.* London: Her Majesty's Stationary Office.

Cohen, D. C. (2006). *Qualitative Research guideline reports*. Retrieved from http://www.qualves.org/HomesEval-3664.html.

Cohen, L. M. (2000). *Research Methods in Education.* London: Routledge Flamer.

Confrey, J. (1990). What constructivism implies for teaching. *Journal for Research in Mathematics Education Monographs, 4*, 107-122.

Corwin, R. B., & Storeygard, J. (1992). Talking mathematics. *Hands On,*, 15, 6-8.

Crawford, M. A. (1999). Strategies for Mathematics : Teaching in context. *Educational Leadership, 57*(3), 34-38.

Creswell, J. (1998). *Qualitative enquiry and research design choosing among five traditions.* Thousand Oaks,CA: Sage Publication.

Creswell, J. W. (2007). *Qualitative inquiry and research design: Choosing among five approaches*. Thousand Oaks, CA: Sage.

Culatta, R. (2012). *Constructivist Theory (Jerome Bruner).Instructional Design*. Retrieved January 20, 2012, from iconic representation: http://www.youtube.com/watch?v=gIW0mjMo9IE

Custodio, H. (2016). Hospital aquired infections clinical presentation.

Daniels, H. (2001). *Vygotsky and Pedagogy*. London: Routledge Falmer.

Davis, P. J., & Hersh, R. (1981). *The mathematical experience*. Boston: Birkhauser.

Davis, R. B., Maher, C. A., & Noddings, N. (Eds). (1990). *Contructivist views of the teaching and learning of mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Denzin, N. K. (1978). *The reasearch act: A theoretical introduction to sociological methods (*2nd ed.). New York: McGraw-Hill.

Dewey, J. (1938). *Logic: The theory of inquiry*. New York: Holt.

Dewey, J. (1997). *Experience and Education*. New York: Kappa Delta.

Doolittle P. E., & Camp G.W. (1999). Constructivism: The Career and Technical Education Perspective. *Journal of Vocational and Technical Education, 16*(1), 23 – 46 .

Draper, R. J. (2002). School mathematics reform, constructivism, and literacy: a case for literacy Instruction in the reform-oriented math classroom. *Journal of Adolescent and Adult Literacy, 45*(6), 520-529.

Duckworth, E. (1987). *The having of Wonderful ideas and other essays on teachhing and learning.* New York: Teachers College Press.

Hmelo-Silver, C. E., Duncan, R. G., & Chinn, C. A. (2007). Scaffollding and achievement in problem-based and enquiry learning: A response to Kirschner, Sweller and Clark. *Educational Pyschologist*, *42*(2), 99-107.

Eisner, E. W. (1981). On the difference between scientific and artistic approaches to qualitative research. *Educational Researcher, 10*(4) , 5-9.

Erickson, F. (1986). Tasks in time: Objects of study in a natual history of teaching. In K. Zumwalt (Ed.), *Improving teaching* (pp. 131-147). Alexandria, VA: Association for Supervision and Curriculum Development.

Ernest, P. (1991). *The philosophy of mathematics teacher: A model.* London: Falmer.

Eshun, B., & Abledu, G. K. (2001). The effect of alternative assessment on the attitudes and achievement in mathematics of female pre-service teachers in Ghana. *African Journal of Educational in Mathematics and Science, 1*, 21-30.

Eshun, B. A. (2004). Sex Differences in Attitude of Students Towards Mathematics in Secondary Schools. *Mathematics Connection, 4*, 1-13.

Eshun, B. A., & Famiyeh, J. (2005). Early Number Competencies of Children at the Start of Formal Education. *African Journal of Educational Studies in Mathematics and Sciences 3*(1), 21- 31.

Fennema, E, Carpenter, T. A., & Peterson, P. L. (1991). *Learning mathematics with understanding: COgnitively guided instruction.* Madison: University of Wiscousin, Wiscousin Center for Educationa Research.

Fennema, E, Franke, M. L., Carpenter, T. P., & Carey, D. A. (1993). Using children's mathematical knowledge in instruction. . *American Educational Research Journal, , 30*, 555-584.

Fennema, E. C. (1987). *Mathematics beliefs scales Studies of the application of cognitive and instructional science to mathematics instruction. (National Science Foundation Grant No. MDR-8550236).* Madison: University of Wisconsin-Madison.

Fennema, E., & Carpenter, T. P. (1992). *Cognitively guided instruction readings (Report No. MDR-8550236).* Madison: University of Wincousin-Madison.

Fennema, E., Carpenter, T. P., & Peterson, P. L. (1987). *Mathematics beliefs scales. Studies of the application of cognitive and instructional science to mathematics instruction. (National Science Foundation Grant No. MDR-8550236).* . Madison: University of Wincousin-Madison.

Fennema, E., Carpenter, T. P., & Peterson, P. L. (1989). Learning mathematics with understanding: Cognitively guided instruction. In J. Brophy (Ed.), *Advances in research on teaching* (pp. 195-221). Greenwich, CT: JAI Press.

Fletcher, J. (2005). Constructivism and mathematics education in Ghana. *mathematics connection*, *3*, 29-36.

Fluehr-Lobban, C. (1979). *Ethics and the profession of Anthropology : Dialogue for Ethically conscious practice.* (2nd ed.). Walnut Creek, CA: AltaMira Press.

Fosnot, C. T. (1989). *Enquiring teachers, enquiring learners.* New York: Teachers College Press.

Fosnot, C. T. (1996). *Constructivism: Theory, perspective, and practice.* New York: Teachers College Press.

Franser, D. W. (2005). Five tips for creating independent activities aligned with the common core state standards. *Teaching Exceptional Children, 45*(6), 6-15.

Fredua-Kwarteng, Y. (2005, January 8). *Ghana Flunks at Math and Science: Analysis (1)*. Retrieved from Ghanaweb: www.ghanaweb.com

Gage, N. L. (1978). *The scientific basis of the art of teaching* . New York: Teachers & Bacon.

Gagne, R. M. (1985). *The Conditions of Learning and Theory of Instruction.* New York: CBS College Publishing .

Gall, M. D. (2007). *Educational Research.* Boston, MA: Allyn and Bacon.

Gall, M. D., Gall, J. P., & Borg, W. R. (2003). *Educational Research: An introduction* (7th ed.). Boston, MA: A & B Publication.

Gavor, M. (2015, August 20). *GhanaWeb/Too many students are failing the WASSCE.* Retrieved from Ghanaweb.com: https://www.ghanaweb.com/GhanaHomePage/features/Too-many-students-are-failing-the-WASSCE-376575

Geertz, C. (1973). *The interpretation of cultures.* New York: Basic Books.

Goetz, J., & LeCompte, M. (1981). Ethanographic research and the problem of data reduction. *Anthropology and Education Quarterly, 12*(1), 51-70.

Goetz, J., & LeCompte, M. (1984). *Ethnography and qualitative design in educational research.* New York: Academic Press.

Good, T. L., & Grouws, D. (1979). The Missouri mathematics project: An experimental study of fourth-grade classrooms. *Journal of Educational Psychology, 71*, 355-362.

Good, T. L., Grouws, D., & Ebmeier, H. (1983). *Active mathematics teaching.* New York: Longman.

Goodchild, S. (2002b). Exploring students goals in classroom acitivity. In Goodchild S. and English L. (Eds.), *Researching Mathematics Classrooms: A Critical Examination of Methodology.* Greenwich Information Age Publication.

Green, J. L., Kantor, R., & Rogers, T. (1991). Exploring the complexity of language and learning in the classroom. In Idol & B. F. Jones (Eds.), *Educational values and cognitive instruction: Implications for reform* (pp. Vol. 2, 333-364). Hillsdale, NJ: Lawrence Erlbaum Associates.

Heddens, J. (1997). *Improving mathematics teaching by using manipulatives James W. Heddens Kent State University.* Retrieved from http://www.fed.cuhk.edu.hk./-filee/mathfor/edumath/9706/13hedden.html.

Hiebert, J. (1992). Mathhematical, cognitive, and instruction analyses of decimal fractions. In R. P. G. Leinhardt, *analysis of arithmetic for mathematics teaching* (pp. 283-322). Hillsdale, NJ: Lawrence Erlbaum Associates.

Hiebert, J., & Lefevre, P. (1986). Conceptual and knowledge in mathematics: An introductoryanalysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1-27). Hillsdale, NJ: Lawrence Erlbaum Associates.

Hiebert, J., & Wearne, D. (1986). Procedures over concept: The acquisition of decimal number knowledge. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 199-223). Hillsdale, NJ: Lawrence Erlbaum Associates.

Hiebert, J., & Wearne, D. (1991). Methodologies for studying learning to inform teacher. In T. P. Carpenter and E. Fennema, *Integrating research on teachig and learning mathematics* (pp. 153-176). Albany: State University of New York Press.

Hiebert, J.,& Carpenter, T. A. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65-100). New York: Macmillan.

Huinker, D. M. (1993). Interviews: A window to students' conceptual knowledge of the operations. In N. L. Webb & A. F. Coxford (Eds.), *Assessment in the mathematics classroom* (pp. 80-86). Reston, VA: National Council of Teachers of Mathematics.

Jackson, P. W. (1968). *Life in classrooms.* New York: Holt, Rinehart & Winston.

Jaworski, B. (1991). *Interpretations of a constructivist philosophy in mathematics teaching.* Milton Keynes: Open University.

Jaworski, B. (1994). *Investigating mathematics teaching: A constructivist enquiry.* London: Falmer Press.

Jaworski, B. (1996, June 7). *Constructivism and teaching: The social-cultural context.* Retrieved from http://www.grout.demon.co.uk/Barbara/chreods.htm:

Jaworski, B. (2003). Research practice into/influencing mathematics teaching and learning development: towards a theoretical framework based on co-learning and partnerships. *Educational Studies in Mathematics, 54*(23), 249-282.

Johnson, D. W., & Johnson, R. T. (1989). Cooperative learning in mathematics education. In P. R. Trafton & A. P. Shulte (Eds.), *New directions for elementary school mathematics* (pp. 234-245). Reston, VA: National Council of Teachers of Mathematics.

Jonassen, D. (1997). Instructional design models for well-structured and ill-structured problemsolving learning outcomes. *Educational Technology Research and Development, 45*, 65–94.

Joyce, B. W. (2004). *Models of teaching* (7th ed.). Boston: Allyn.

Kamii, C, & Lewis, B. A. (1990). Constructivism and first-grade arithmetic. *Arithmetic Teacher, 35*, 36-37.

Kamii, C. (1985). *Young children arithmetic.* New York: Teachers College Press.

Kamii, C. (1989). *Young children continue to reinvent arithmetic, 2nd grade: Implications of Piaget's theory.* New York: Teachers College Press.

Kamii, C. (1990). Constructivism and beginning arithmetic (K-2). In T. J. Cooney & C. R. Hirsch (Eds.), *Teaching and learning mathematics in the 1990s.* (pp. 22-30). Reston, VA: National Council of Teachers of Mathematics.

Kamii, C. (1991). Toward autonomy: The importance of critical thinking and choice making. *School Psychology Review, 20*, 382-388.

Kantowski, M. G. (1980). Some thoughts on teaching for problem solving. In S. Krulik & R. E. Reys (Eds.), *Problem solving in school mathematics* (pp. 195-203). Reston, VA: National Council of Teachers of Mathematics.

Kantowski, M. G. (1981). Problem solving. In E. Fennema (Ed.), *Mathematics education research: Implications for the 80s* (pp. 111-130). Reston, VA: Association for Supervision and Curriculum Development.

KATH. (2009). Infection prevention and control policy for Komfo Anokye Teaching Hospital, Kumasi.

Kawulich, B. (2005). Participant observation as a data collection method. *Forum: Qualitative Social Reseacrh*, 43.

Kirk, J., & Miller, M. L. (1986). *Reliability and validitty in qualitative research.* Beverly Hills, CA: Sage Publications Inc.

Koehler, M. S., & Grouws, D. A. (1992). Mathematics teaching practices and their effects. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 115-126). New York: Macmillan.

Kosko K.W., & Wilkins, J. L. (2010). Mathematical communication and its relation to the frequency of manipulative use. *International Electronic Journal of Mathematics Education,*, *5*(2), 79-90.

Kraft, R. J. (1994). *Teaching and Learning in Ghana: A curriculum, textbooks, syllabus and handbook analysis.* Colarado: University of Colarado.

Lampert, M. (1991). Connecting mathematical teaching and learning. In T. P. Carpenter & E. Fennema, *Integrating research on teacing and learning mathematics* (pp. 132-167). Albany: State University of New York Press.

Lampert, M. (1992). Teaching and learning long division for understanding in school. In R. P. G. Leinhardt, *Analysis of arithmetic for mathematics teaching* (pp. 221-282). Hillsdale: Lawrence Erlbaum Associates.

Larochelle, L. (2000) Radical Constructivism: Notes of viability, ethics. and other issues. In L. P. Steffe & P. W. Thompson (Eds.). *Radical constructivism in action building on the pioneering work of Ernest Von Glasersfeld*, (pp.55-68). New York: Routledge Falmer.

LeCompte, M. D., & Preissle, J. (1993). Qualitative Research: What it is, what isn't, and how it is done. In B. Thompson (Ed.), *Advances in Social Science methodology* (Vol. 3, pp. 141-163). Indiana: JAI Press.

Lee, S. J. (2014). Early childhood teachers' misconceptions about mathematics education for young children in the United States. *Early Education and Development, 8*(1), 111- 143.

Leinhardt, G., & Smith, D. A. (1985). Expertise in mathematics instruction: Subject Matter Knowledge. *Journal of Educational Psychology, 77*, 247-271.

Leinhardt, G. (1988). Expertise in instructional lessons: An example from fractions. In T. J. Cooney, D. Jones & D. A. Grouws (Ed.), *Perspectives* *on research on effective mathematics teaching* (pp. 47-66). Reston, VA: National Council of Teachers of Mathematics.

Leinhardt, G. (1989). Math lessons: A contrast of novice and expert competence. *Journal for Research in Mathematics Education, 20*, 52-75.

Leinhardt, G., & Greeno, J. C. (1986). The cognitive skill of teaching. *Journal of Educational Psychology, 78*(2), 75-95.

Lerman, S. (1993). Can we talk about Constructivism? *British Society for Research into Learning Mathematics, 13*(3), 20-23.

Lira, J., & Ezeife, A. N. (2008). *Strengthening intermediate level mathematics teaching using manipulatives: A theory-backed discourse.* Ontario: Academic Exchange-Extra.

LIston, D. P., & Zeichner, K. (1991). *Teacher education and the social conditions of schooling.* New York: Routledge.

Lowery, L. (1997). The Nature of Teaching. *FOSS Newletter, 10*, 6-8.

Martinez, J. G. (1987). Preventing math anxiety: A prescription. *Academic Therapy Publications, 23*, 117-125.

Maslen, G. D. (2014). The regulation of cognitive enhancement devices. Extending the medical model. *Journal of Law and the Biosciences, 1*(1), 68 – 93.

Mathematics, N. C. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

McLeod, D. B. (1989). The role of affect in mathematical problem solving. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving* (pp. 20- 36). New York: Springer- Verlag.

McLeod, D. (1992). Research on affect in mathematics education: A reconceptualization. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National council of teachers of mathematics*. (pp. 575-595). New York: Macmillan Publising Company.

McLeod, D. B. (1989). The role of affect in mathematical problem solving. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving* (pp. 20-36). New York: Springer-Verlag.

McLeod, D. B. (1991). Research on learning and instruction in mathematics: The role of affect. In T. P. E. Fennema, *Integrating research on teaching and learning mathematics* (pp. 55-82). Albany: State University of New York.

Mcleod, S. A. (2008). *Bruner. Simply Psychology*. Retrieved January 20, 2012, from http://www.simplypsychology.org/bruner.html:

Medinipur, K.P. (2015). Constructivist approach in mathematics teaching and assessment of mathematical understanding. *Basic Research Journals of Educational Research and Review*, *4*(1), 8-12

Mereku, D. K. (2003). *Methods in Ghanaian primary mathematics textbooks and teachers' classroom practices, 23* (2). In J. Williams, (Ed.) Proceeding of the British society for research into learning mathematics. (p.61-66)

Meriam, S. B. (1988). *Case study research in education: A qualitative approach.* San Francisco: Jossey-Bass.

Merriam. (2009). *Qualitative Research: A guideline to design and implementation.* San Francisco: John Willey and Sons.

Merriam, S. (2001). *Qualitative research and case study application in education.* San Francisco: Jossey-Boss Publishers.

Merriam, S. (2009). *Qualitative research:A guide to design and implementatioon.* San Francisco, CA: John Wiley & Sons.

Milgram, R. J. (2014, May 14th). *The key topic in a successful math curriculum.* Retrieved from math.berkeley.edu: http: // math. berkeley. Edu

Ministry of Education. (2002). *Meeting the Challenges of Education in the twenty first century: Report of the president's committee on review of education reforms in Ghana.* Accra: Ministry of Education.

Ministry of Education. (2005). *The Ontario Curriculum: Grades 9 and 10 mathematics Queen's Printer for Ontario.* Ontario: Ministry of Education.

Ministry of Education and Sports . (2007). *Teaching Syllabus for Mathematics (Senior High School).* Accra: Curriculum Research and Development

Mereku, D. K. (2003). *Methods in Ghanaian primary mathematics textbooks and teachers' classroom practices*, *23* (2). In J. Williams, (Ed.) Proceeding of the British society for research into learning mathematics. (p.61-66)

Meriam, S. B. (1988). *Case study research in education: A qualitative approach.* San Francisco: Jossey-Bass.

Merriam. (2009). *Qualitative Research: A guideline to design and implementation.* San Francisco: John Willey and Sons.

Merriam, S. (2001). *Qualitative research and case study application in education.* San Francisco: Jossey-Boss Publishers.

Merriam, S. (2009). *Qualitative research:A guide to design and implementatioon.* San Francisco, CA: John Wiley & Sons.

Milgram, R. J. (2014, May 14th). *The key topic in a successful math curriculum.* Retrieved from math.berkeley.edu: http: // math. berkeley. Edu

Ministry of Education. (2002). *Meeting the Challenges of Education in the twenty first century: Report of the president's committee on review of education reforms in Ghana.* Accra: Ministry of Education.

Ministry of Education. (2005). *The Ontario Curriculum: Grades 9 and 10 mathematics Queen's Printer for Ontario.* Ontario: Ministry of Education.

Ministry of Education and Sports . (2007). *Teaching Syllabus for Mathematics (Senior High School).* Accra: Curriculum Research and Development

Division. Research and Development Division (CRDD) of the Ministry of Education, Science and Sports (MOES). .

National Coiuncil of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

National Research Council. (2002). *Helping children learn mathematics.* Washington, DC: National Academy Press.

National Research Council. (1989). *Everybody count: A report to the nation on the future of mathematics education.* Washington, DC: National Academy Press.

NCTM. (2000). *Principles and Standards for School Mathematics.* Reston, VA: National Council of Teachers of Mathematics.

NCTM. (2003). *Focus in High School mathematics : Research and sense making.* Reston Va: Author.

Nespor, J. (1987). The role of beliefs in the practices of teaching. *Journal of Curriculum Studies.*, *19*, 317-328.

Newman, D., Griffin, P., & Cole, M. (1989). *The construction zone: Working for cognitive change in school.* Cambridge, MA: Cambridge University Press.

Nickson, M. (1992). The culture of the mathematics classroom: An unknown quantity? In D. Grouws (Ed.), *Handbook research on mathematics teaching and learning* (pp. 101-114). New York: Macmillan.

Nielsen, D. C. (2007). Teachers' reflections of professional change during a literacy-reform initiative. *Teaching and Teacher Education, 24*(5), 1288–1303.

O'Donoghue, T. &. (2003). *Qualitative educational research in action: doing and reflecting.* London: Falmer Press.

Okuma-Nyström, M. K. (2003). *God turns the chapter and everything changes: children's socialization in two gambian villages.* Stockholm: Institute of International Education,Stockholm University.

Orton, A. (2004). *Learning Mathematics: Issues, Theory and Classroom Practice.* (3rd ed.) London: Cassell Education.

Owens, R. G. (1982). Methodological rigor in naturalistic inquiry: Some issues and answers. *Educational Administration Quaterly, 18*, 1-21.

Palincsar, A. S. (1998). Social Constructivist Perspectives on Teaching and Learning. *Annual Review of Psychology, 49*, 345-375.

Pateman, N. A., & Johnson, D. C. (1990). Curriculum and constructivism in early childhood mathematics: Sources of tension and possible resolutions. In L. P. Steffe., and T. Wood (Eds.), *Transforming children's mathematics* *education: International perspectives* (pp. 346-356). Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Patton, M. Q. (1980). *Qualitaive evaluation methods.* Beverly Hills, CA: Sage Publication. Inc.

Patton, M. Q. (2002). *Qualitative research and evaluation methods.* New Yorke: Sage Publications, Inc.

Piaget, J., & Inhelder, B. (1969). *The Psychology of the child.* New York: Basic Book Publishers.

Piaget, J. (1972). *The principles of genetic epistemology.* London: Routledge and Kegan Paul.

Piaget, J. (1973). *To understand is to invent.* New York: Grossman.

Piaget, J. (1977). *The development of thought: Equilibration of cognitive structures.* New York: Viking Press.

Piaget, J. (1980). *Adaptation and intelligence: Organic selection and phenocopy.* Chicago: University of Chicago Press.

Polya, G. (1965). *Mathematical Discovery* (Vol. *2*). New York: Willey and Sons.

Polya, G. (1981). *Mathematical Discovery: On understanding, Learning, and Teaching Problem Solving.* Vol. *2*, New York: John Wiley & Sons, Inc.,.

Polya, G. (2002). The goals of mathematics education. *Journal of califonia mathematics council, 26*(1), 6-8.

Reichardt, C. C., & Cook, T. A. (1979). *Qualitative and quantitative methods in evaluative research.* Beverly Hills, CA: Sage.

Reynolds, D. H. (1999). What Does the Teacher Do? Constructivist Pedagogies and Prospective Teachers' Beliefs about the Role of a Teacher. *Teaching and Teacher Education, 16*, 21-32.

Riedesel, C. A. (1990). *Teaching elementary school mathetics.* Englewood Cliffs, NJ: Prentice Hall.

Rosenshine, B. (1976). Recent research on teaching behaviors and students' achievement. *Journal of Teacher Education,, 27*, 61-64.

Rosenshine, B. (1978). Review of teaching styles and pupil progress. *American Educational Research Journal, 15*, 163-169.

Ross, D. D. (1978). Teaching beliefs and practices in three kindergatens (Doctoral dissertation, University of Virginia, 1978). *Abstracts International*, 40, 661A.

Ross, D. D. (1987). Action research for preservice teachers: A description of why and how. *Peabody Journal of Education*, *64*, 131-150.

Schifter, D., & Fosnot, C. T. (1993). *Reconstructing mathematics education.* New York: Teachers College Press.

Schoenfeld, A. H. (1992). Radical constructivism and the pragmatics of instruction. *Journal for Research in Mathematics Education*, *23*, 290-295.

Shavelson, R. J., & Stern, P. (1981). Research on teachers' pedagogical thoughts, judgments, decision, and behavior. *Review of Educational Research, 51*, 455-498.

Sherrod. (2009). Developing science and mathematics integrated activities for middle school students. *International Journal of Mathematical Education in Science and Technology*, *40*(2), 247-257.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*, 4-14.

Simon, M. A. (1995). Reconstructing mathematics pedegogy from a constructivist perspective. *Journal for Research in Mathematics Education*, *26*(2), 114-145.

Simon, M. A., & Schifter, D. E. (1991). Towards a constructivist perspective: An intervention study of mathematics teacher development. . *Educational Studies in Mathematics*, *22*, 309-331.

Skemp, R. (1976/2006). Relational understanding and instrumental understanding. *Mathematics Teaching in the Middle School*, 88-95.

Skemp, R. K. (1978). Rational understanding and intrumental understanding. *Arithmetic Teacher*, *26*, 9-15.

Skemp, R. R. (1987). *The psychology of learning* . Hillside, NJ: Lawrence Erlbaum Associate Inc. Publishers.

Slavin, R. E. (1980). Cooperative learning. *Review of Educational Research, 50*(2), 315-342.

Smith, J. K., & Heshusius, L. (1986). Closing down the conversation: The end of the quantitative-qualitative debatte. . *Educational Researcher, 15*, 4-13.

Smith, M. (2009). *Jerome S. Bruner and the process of education. the encyclopedia of informal education*. Retrieved January 20, 2012, from http://www.infed.org/thinkers/bruner.htm:

Sovchik, R. (1989). *Teaching mathematics to children.* New York: Harper & Row.

Spradley, J. P. (1979). *The ethnographic interview.* Chicago: Holt, Rinehart, and Winston.

Spradley, J. P. (1980). *Participant observation.* Chicago: Holt, Rinehart, and Winston.

Stake, R. E. (1978). The case study method in social inquiry. *Educational Researcher,* 7, 5-8.

Steedly, K. D. (2008). Effective Mathematics instruction. Evidence for Education. *Evidence for Education, 3*(1), 1 – 12.

Steedly, K., Dragso, K., Arafeh, S., & Luke, S. D. (2008). Effective mathematics instruction: Evidence for education. *Institutes of Education Sciences, 3*(1),1-12.

Steele, D. F. (1995). A constructivist approach to mathematics teaching and learning by a fourth grade teacher. PhD Desertation, Florida, USA.

Steen, L. A. (1990). Mathematics for all Americans. In T. J. Cooney & C. R. Hirsch (Eds.), *Teaching and learning mathematics in the 1990s* (pp. 130-134). Reston, VA: National Council of Teachers of Mathematics.

Steffe, L. P., & Cobb, P. (1988). *Construction of arithmetical meanings and strategies*. New York: Springer- Verlag.

Steffe, L. P., & Killion, K. (1986). Mathematics teaching: A specification in a constructionist frame of reference. In L. Burton & C. Hoyles (Eds.), *Proceedings of the Tenth International Conference, Psychology of Mathematics Education* (pp. 207-216). London: University of London Institute of Education.

Steffe, L. P., Von Glasersfeld, E., Richards, J., & Cobb, P. (1983). *Children's counting types: Philosophy, theory, and applications*. New York: Praeger Scientific.

Stuyf, R. (2002). *Scaffolding as a Teaching Strategy. Adolescent Learning and Development*. Retrieved from http://condor.admin.ccny.cuny.edu/~group4/.

Swenne, C. (2006). Wound infection following coronary artery bypass graft surgery. Risk factors and the experiences of patients. *ACTA Universities Upsaliensis UPPSALA*.

The West African Examination Council. (2006). *Chief Examiners' Report On the "What is the importance of algebra in today's world."*. Accra: The West African Examination Council.

Thompson, A. G. (1984). The relationship of teachers' conceptions of mathematics teaching to instructional practice. *Educational Studies in Mathematics*, *15*, 105-127.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York: Macmillan.

Thompson, D., & Rubenstain, R. (2000). Learning mathematics vocabulary: Potential pitfalls and instructional strategies. *The Mathematics Teachers, 93*(7), 568-574.

TIMSS. (2007). *International Mathematics report:Findings from IEA's trends in International Mathematics and Science Study at the Fourth and Eighth Grades Students in an International Context, NCES.*

Törner, G. & Sriraman B. (2006). *A brief historical comparison of tendencies in 300 Mathematics Didactics/Education in Germany and the United States* ZDM *1*(38), 14-21

Uma, S. (2003). *Research method for business: A skill building approach,* (4th ed), John Wiley & Sons.

Van de Walle, J. A. (1993). *Elementary school mathematics: Teaching developmentally.* White Plains, NY: Longman.

Von Glasersfeld, E. (1984). An introduction to radical constructivism. In P. Watzlawick (Ed.), *The InPented Reality.* London: W.W. Naughton and Co.

Von Glasersfeld, E. (1989). Cognition, construction, of knowledge, and teaching. *Synthese, 18*(1),121-140.

Von Glasersfeld, E. (1990). An exposition of constructivism: Why some like it radical. In C. A. R. B. Davis, *Constructivist views on the teaching and learning of mathematics* (pp. 19-300). Reston, VA: National Council of Teachers of Mathematics.

Von Glasersfeld, E. (1993). An attentional model for the conceptual construction of units and number. *Journal for Research in Mathematics Education,, 12*, 83-94.

Vygotsky, L. (1930). *The instrumental method in psychology.* Retrieved from https://www.marxists.org.

Vygotsky, L. (1978). *Interaction between learning and development, Mind in society.* Cambridge: Harvard University Press.

Vygotsky, L. S. (1962). *Thought and language.* Cambridge, MA: M.I.T Press.

Vygotsky, L. S. (1978). *Mind in society: The developmental of higher psychological processes.* Cambridge, MA: Harvard University Press.

Ward, C. (2001). Under Construction: On Becoming a Constructivist in View of the Standards. *Mathematics Teacher, 94*(2), 94-96.

Wearne, D., & Hiebert, J. (1989). Cognitive changes during conceptually based instruction on decimal fractions. *Journal of Educational Psychology, 81*, 507-513.

Webb, N., & Romberg, T. A. (1992). Implications of the NCTM standards for mathematics assessment. In T. A. Romberg (Ed.), *Mathematics assessment and evaluation* (pp. 37-60). Albany, NY: State University of New York Press.

Weiss, I. B. (2001). *Report of the 2000 national survey of science and mathematics education.* Chapel Hill, NC: Horizon Research, Inc.

Weiss, I. B., Pasley, J. D., Smith, P.S.,Banilower, E. R., & Heck, D. J. (2003). *Looking inside the classroom: A study of K-12 mathematics and science education in the United States.* Chapel Hill, NC: Horizon Research Inc.

Wheatley, G. H. (1990). Spatial sense and mathematics learning. *Arithmetic Teacher, 37*, 10-11.

Wigfield, A., & Meece, J. L. (1988). Math anxiety in elementary and secondary school students. *Journal of Educational Psychology, 80*, 210-216.

Wilmot, E. (2010). An Investigation of primary and J.H.S teachers' attitudes towards mathematics in some selected schools in the central region of Ghana. *Mathematics Connection, 9*, 1-10.

Wilson, P. S., Cooney T. J., & Stinson, D. W. (2005). What constitutes good mathematics teaching and how it develops: Nine high school teachers perspectives. *Jornal of mathematics teacher education, 8*(2), 83-111.

Wolcott, H. (1976). Criteria for an ethnographic approach to research in schools. In J. Roberts & S. Akinsanya (Eds.), *Schooling in the cultural context* (pp. 23-44). New York: David McKay.

Wolcott, H. F. (2001). *The art field work.* Walnut Creek, C.A: AltaMira Press.

Woods, A. (2005). Keypoints of the CDC's surgical site infection guideline. *Skin Wound Care. 18*(4), 215-220.

Woolfolk A.V., Hughes M., & Walkup, V. (2008). *Psychology in Education.* Harlow: Pearson Longman Ltd.

Yackel, E, Cobb, P., Wood, T., Wheatley, G., & Merkel, G. (1990). The inportance of social intraction in children's construction of mathematical knowledge. In T. J. Cooney & C. R. Hirsch (Eds.), *Teaching and learning mathematics in the 1990s* (pp. 12-21). Reston, VA: National Council of Teachers of Mathematics.

Yackel, E. C. (1990). The importance of social interaction in children's construction of mathematical knowledge. In T. J. Hirsch (Ed.), *Teaching and learning mathematics in the 1990s* (pp. 12-21). Reston, VA: National Council of Teachers of Mathematics.

Yackel, E., Cobb, P., & Wood, T. (1991). Small-group interactions as a source of learning opportunities in second-grade mathematics. *Journal for Research in Mathematics Education,*, *22*, 390-408.

Yin, R. (1989). *Case study research: Design and methods* (2nd ed.). Newbury Park, CA: Sage.

Yin, R. (2003). *Case study research: Design and methods* (3rd ed.). Thousand Oaks: Sage Publications.

Yin, R. K. (1982). Studying phenomenom and context across sites. *American Behaviorial Scientists,*, *26*, 84-100.

APPENDICES

APPENDIX A

ADAPTED MATHEMATICS BELIEFS SCALE

BACKGROUND INFORMATION ABOUT THE TEACHER

1. Which class (s) do you teach? JHS 1 JHS 2 JHS 3

2. Gender: Male Female

3. How old are you?

Age (Years)	Tick
Below 20	
20 – 25	
26-30	
31-35	
36-40	
41-45	
46-50	

4. What is your academic qualification?

5. Please state.............................. ....

....

6. What is your Professional Qualification?

Professional Qualification	Qualification Tick
Teacher's Certificate 'A'	
Diploma (Education)	
Degree (B. Ed)	
Masters	
Untrained Teacher	

On the following pages is a series of sentences. You are to mark your answer sheets by telling how much you agree that the statements are true.

As you read the sentence, you will know whether you agree or disagree. If you strongly agree, circle A on your answer sheet. If you agree, but not so strongly, or you only "sort of" agree, circle B. If you disagree with the sentence very much, circle E for strongly disagree. If you disagree, but not so strongly, circle D. If you are not sure about a question or you can't answer it, circle C. Do not spend much time with any statement but be sure to answer every statement. Work fast but carefully.

There is no "right" or "wrong" answer, the only correct responses are those that reflect what you believe to be true. Be sure to respond to each item in a way that reflects your beliefs.

THIS INVENTORY IS BEING USED FOR RESEARCH PURPOSES ONLY AND NO ONE WILL KNOW WHAT YOUR RESPONSES ARE.

1. Children should solve word problems before they master computational procedures.

A=Strongly Agree B=Agree C=Undecided D=Disagree E=Strongly Disagree

2. Teachers should encourage children to find their own solutions to math problems even if they are inefficient.

A=Strongly Agree B=Agree C=Undecided D=Disagree E=Strongly Disagree

3. Children should understand computational procedures before they spend much time practicing them.

A=Strongly Agree B=Agree C=Undecided D=Disagree E=Strongly Disagree

4. Time should be spent solving simple word problems before children spend much time practicing computational procedures.

A=Strongly Agree B=Agree C=Undecided D=Disagree E=Strongly Disagree

5. Teachers should NOT teach exact procedures for solving word problems.

A=Strongly Agree B=Agree C=Undecided D=Disagree E=Strongly Disagree

6. Children should understand the meaning of an operation (addition, subtraction, multiplication, or division) before they memorize number facts.

A=Strongly Agree B=Agree C=Undecided D=Disagree

E=Strongly Disagree

7. The teacher should demonstrate how to solve simple word problems before children are allowed to solve word problems.

A=Strongly Agree B=Agree C=Undecided D=Disagree

E=Strongly Disagree

8. The use of key words is an effective way for children to solve word problems.

A=Strongly Agree B=Agree C=Undecided D=Disagree

E=Strongly Disagree

9. Mathematics should be presented to children in such a way that they can discover relationships for themselves.

A=Strongly Agree B=Agree C=Undecided D=Disagree

E=Strongly Disagree

10. Even children who have not learned basic facts can have effective methods for solving problems.

A=Strongly Agree B=Agree C=Undecided D=Disagree

E=Strongly Disagree

11. It is important for a child to be a good listener in order to learn how to do mathematics.

A=Strongly Agree B=Agree C=Undecided D=Disagree

E=Strongly Disagree

1 2. Most young children can figure out a way to solve simple word problems.

A=Strongly Agree **B=Agree** **C=Undecided** **D=Disagree**

E=Strongly Disagree

13. Children should have many informal experiences solving simple word problems before they are expected to memorize number facts.

A=Strongly Agree **B=Agree** **C=Undecided** **D=Disagree**

E=Strongly Disagree

14. An effective teacher should NOT demonstrate the right way to do a word problem.

A=Strongly Agree **B=Agree** **C=Undecided** **D=Disagree**

E=Strongly Disagree

15. Children should NOT be told to solve problems the way the teacher has taught them.

A=Strongly Agree **B=Agree** **C=Undecided** **D=Disagree**

E=Strongly Disagree

16. Most young children need NOT be shown how to solve simple word problems.

A=Strongly Agree **B=Agree** **C=Undecided** **D=Disagree**

E=Strongly Disagree

17. Children's written answers to paper-and- pencil mathematical problems indicate their level of understanding.

A=Strongly Agree **B=Agree** **C=Undecided** **D=Disagree**

E=Strongly Disagree

18. The wrong way to teach problem solving is to show children how to solve one kind of problem at a time.

A=Strongly Agree **B=Agree** **C=Undecided** **D=Disagree**
E=Strongly Disagree

19. It is better to provide a variety of word problems for children to solve.

A=Strongly Agree **B=Agree** **C=Undecided** **D=Disagree**
E=Strongly Disagree

20. Children learn math best by figuring out for themselves the ways to find answers to simple word problems.

A=Strongly Agree **B=Agree** **C=Undecided** **D=Disagree**
E=Strongly Disagree

21. Children usually can figure out for themselves how to solve simple word problems.

A=Strongly Agree **B=Agree** **C=Undecided** **D=Disagree**
E=Strongly Disagree

22. Recall of number facts should NOT precede the development of an understanding of the related operation (addition, subtraction, multiplication, or division).

A=Strongly Agree **B=Agree** **C=Undecided** **D=Disagree**
E=Strongly Disagree

23. Children CAN understand an operation (addition, subtraction, multiplication, or division) before they master some of the relevant number facts.

A=Strongly Agree B=Agree C=Undecided D=Disagree

E=Strongly Disagree

24. Most children cannot figure mathematics NOT out for themselves and must be explicitly taught.

A=Strongly Agree B=Agree C=Undecided D=Disagree

E=Strongly Disagree

25. Children should understand computational procedures before they master them.

A=Strongly Agree B=Agree C=Undecided D=Disagree

E=Strongly Disagree

26. Children learn math best by attending to the teacher's explanations.

A=Strongly Agree B=Agree C=Undecided D=Disagree

E=Strongly Disagree

27. It is important for a child to discover how to solve simple word problems for him/ herself.

A=Strongly Agree B=Agree C=Undecided D=Disagree

E=Strongly Disagree

28. Children should be allowed to invent ways to solve simple word problems before the teacher demonstrates how to solve them.

A=Strongly Agree B=Agree C=Undecided D=Disagree

E=Strongly Disagree

29. Time should be spent practicing computational procedures before children are expected to understand the procedures.

A=Strongly Agree **B=Agree** **C=Undecided** **D=Disagree**

E=Strongly Disagree

30. The goals of instruction in mathematics are best achieved when students find their own methods for solving problems.

A=Strongly Agree **B=Agree** **C=Undecided** **D=Disagree**

E=Strongly Disagree

31. Allowing children to discuss their thinking helps them to make sense of mathematics.

A=Strongly Agree **B=Agree** **C=Undecided** **D=Disagree**

E=Strongly Disagree

32. Teachers should allow children who are having difficulty solving a word problem to continue to try to find a solution.

A=Strongly Agree **B=Agree** **C=Undecided** **D=Disagree**

E=Strongly Disagree

33. Children can figure out ways to solve many math problems without formal instruction.

A=Strongly Agree **B=Agree** **C=Undecided** **D=Disagree**

E=Strongly Disagree

34. Teachers should tell children who are having difficulty solving a word problem how to solve the problem.

A=Strongly Agree **B=Agree** **C=Undecided** **D=Disagree**

E=Strongly Disagree

35. Frequent drills on the basic facts are essential in order for children to learn them.

36. Most young children can figure out a way to solve many mathematics problems without adult help.

A=Strongly Agree **B=Agree** **C=Undecided** **D=Disagree**

E=Strongly Disagree

37. Teachers should allow children to figure out their own ways to solve simple word problems.

A=Strongly Agree **B=Agree** **C=Undecided** **D=Disagree**

E=Strongly Disagree

38. It is better to teach children how to solve one kind of word problem at a time.

A=Strongly Agree **B=Agree** **C=Undecided** **D=Disagree**

E=Strongly Disagree

39. Children should not solve simple word problems until they have mastered some number facts.

A=Strongly Agree **B=Agree** **C=Undecided** **D=Disagree**

E=Strongly Disagree

40 To be successful in mathematics, a child must be a good participator in the learning process.

A=Strongly Agree **B=Agree** **C=Undecided** **D=Disagree**

E=Strongly Disagree

A=Strongly Agree **B=Agree** **C=Undecided** **D=Disagree**

E=Strongly Disagree

APPENDIX B

STRUCTURED INTERVIEW QUESTIONS

Post-observation Questions

These questions are those that I pre-planned for interviews. However, these questions often lead to many others that I have not written here.

1. How do you deduce what students are saying in class?

2. When you are interpreting and rephrasing students' answers are you trying to get their ideas across or are you trying to help them make arguments?

3. Do you have a purpose for letting students act as "teachers?"

4. Do you think students like this interaction in mathematics?

5. How long does it take to really get them talking and justifying their answers?

6. Have you found that every student become accustomed to this interaction?

7. Have you changed the way you approach this type of teaching at all?

8. On many occasions, you said, 'The answer is not important. What is important is how you are thinking about it." Would you elaborate on that?

9. Do you think students would have handled a test in a different way if they are not taught how to justify their answers?

10. Why are you so strong about students estimating their answers before they solve them?

11. What are the reasons for your choices of things to measure?

12. Why do you emphasize prediction?

13. Why is it important for students to use correct vocabulary?

14. What mathematics do you see in the perimeter activities?

15. How do you distinguish between a students' valid or invalid answer?

16. Generally, how do you choose what manipulatives or materials that you are going to use?

17. What is it you are trying to accomplish with the connection between standard and metric measure?

18. What advantages do you see with cooperative groups? What disadvantages?

19. When students are in cooperative groups, how do you evaluate individuals? How do provide for individual accountability?

20. Does everyone in one group get the same grade if they turn in one thing?

21. How do you keep some students from doing more of the work and more of the thinking?

22. How do you choose the problem of the day?

23. Why is it important for a student to rephrase another student's answer?

24. What is your role during your teaching?

25. How would you tell a new teacher about this kind of teaching? How to plan for it?

How to do it during class? How to evaluate it?

26. What is mathematics to you?

27. What are your goals for your students in learning mathematics?

28. Do you ever have to explain your mathematics teaching to parents?

29. How do you decide which student's response to build the lesson upon?

30. If students come to a way of thinking about some concept that is mathematically incorrect, do you tell them it is incorrect? Who is the authority?

33. What is the role of being a student in your class? What do you expect from your students?

34. Can you tell me some of the progression in how you got to where you are now in your approach to teaching mathematics?

35. With the type of mathematics teaching that you practice, what kinds of knowledge do you think a teacher needs to know?

APPENDIX C

PRE-OBSERVATIONAL INTERVIEW QUESTIONS

(These questions are the ones for which the researcher asked the teacher to write answers.)

Pre-observational Questions

1. Could you tell me what you are planning to do when I come to observe your lesson?

2. Can you give me some details about what the students will actually be doing?

3. Why did you decide to do this lesson? How does it relate to the rest of your work in mathematics?

4. Is there anything in particular you are hoping to have happen today?

5. How likely is it that this (#4) will happen? What will it depend on? What might upset your plan?

6. Will this be difficult for any of your students? Why?

7. Is there anything I should especially pay attention to while I am observing your lesson?

8. How much preparation time did this lesson take?

9. How will this lesson relate to the outside world? How will it help the students in their real life?

10. Were you prompted to teach this lesson because you found the students lacking some ability or knowledge?

11. When you plan your lessons do you like to think through the entire lesson and plan for the kinds of questions you will ask or do you set aside the children's comments or questions?

12. Suppose that during the lesson you ask a child a question and he/she hesitates. Will you rephrase the question, ask a similar question, explain the problem, or ask another student.