UNIVERSITY OF CAPE COAST

MATHEMATICAL FORMULATION OF NEW IBFS TECHNIQUE IN COMPARISON WITH VAM FOR SOLVING A TRANSPORTATION PROBLEM

BY

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Thesis submitted to the Department of Mathematics of the School of Physical Sciences, College of Agriculture and Natural Sciences, University of Cape Coast, in partial fulfilment of the requirements for the award of Master of Philosophy degree in Mathematics

JUNE 2023

DECLARATION

Candidate's Declaration

I hereby declare that this thesis is the result of my own original research and that no part of it has been presented for another degree in this university or elsewhere.

Candidate's Signature .... Date ....

Name: Albert Dadzie

Supervisor's Declaration

I hereby declare that the preparation and presentation of the thesis were supervised in accordance with the guidelines on supervision of thesis laid down by the University of Cape Coast.

Supervisor's Signature .... Date ....

Name: Dr. Martin Anokye

ABSTRACT

This study is an extension of the research initiated by Somani (2015), to improve the Initial Basic Feasible Solution (IBFS) of a transportation problem (TP). The results of this new IBFS technique were compared to the Somani Approximation Method (SAM) and the well-known Vogel's Approximation Method (VAM), using two literature values adopted in the study.

From the analysis of the basic feasible solutions from the three methods in relation to the first literature data: the IBFS values of the given problem obtained from SAM, VAM and the new IBFS method were $555, 303$ and 267 respectively. The corresponding results from the Modified Distribution (MODI) method on the IBFS of SAM, VAM and the new IBFS method also yielded an optimal solution value of 267. The total iterations involved in computing IBFS through to the optimal solution for SAM, VAM and the new IBFS method were $8, 6$ and 4 respectively.

In the second literature data, the IBFS values of the given transportation problem obtained from SAM, VAM and the new IBFS method were also $640, 625$ and 625 respectively. Applying the MODI method on the IBFS of SAM, VAM and the new IBFS method provided an optimal solution value of 625. The total iterations involved in computing IBFS through to the optimal solution for SAM, VAM and the new IBFS method were $6, 4$ and 4 respectively.

Comparatively, the results from the study, aside from optimal solution obtained from fewer iterations, showed that the new IBFS Technique produces an IBFS value that is better than the one produced by either the SAM or the VAM. It is recommended that the new IBFS technique will be adopted by companies and businesses to solve their transportation problems.

KEY WORDS

Destination

Initial basic feasible solution

Optimal solution

Source

Supply

Transportation problem

ACKNOWLEDGEMENTS

Without the support of a lot of people, this study would not have been possible. First and foremost, I respectfully express my gratitude to God for turning what had seemed unachievable into a reality.

My deepest thanks also goes to Dr. Martin Anokye (my supervisor), who carefully examined the study as a whole and provided many helpful criticisms and recommendations that helped to shape this work.

Last but not least, I thank my parents, siblings, and other loved ones, for their physical and spiritual support. Thank you very much and God bless you all.

Finally, I am grateful to Dr. Benedict Barnes and Dr. Frederik Asenso Wireko for their friendly counsel and contributions and anybody who contributed in one way or the other, who for luck of space was not mentioned. Thank you very much and God bless you all.

DEDICATION

To my Parents

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LIST OF ABBREVIATIONS

TP : Transportation problem

TPs : Transportation problems

LP : Linear programming

IBFS : Initial basic feasible solution

VAM : Vogel's approximation method

SAM : Somani's approximation method

NWCR : North West Conner rule

RMM : Row minima method

CMM : Column minima method

LCCM : Least Cost Cell Method

BCM : Best candidate method

MODI : Modified distribution

CHAPTER ONE

INTRODUCTION

Introduction

One of the first use of linear programming (LP) was the transportation problem (TP). Reducing transportation costs for moving goods from source to destination while meeting supply and demand constraints is the main objective of the TP. The number of sources and the number of destinations are the primary causes of TPs.

When goods are moved from production plants to warehouses, warehouses to wholesalers, wholesalers to retailers, and from retailers to customers, a transportation problem occurs. Inventory, allocation of production plants, staffing decisions, scheduling, management, and many other problems are just a few examples of how the TP can be applied.

Background to the Study

One of the normal operational tasks performed for a company's long-term viability is distribution. Generally, every business and company, involved in production, distributions, supplies, and many others, executes the delivery process. Distribution is also a component of marketing because it deals with how goods are transported from a manufacturer to a customer (Hasibuan, 2017).

These companies and businesses incur distribution charges, such as transportation costs when delivering goods or services. Several factors contribute to transportation costs, such as the distance between a source and the destination, the mode of transportation, and many others. The assumption of the TP is that, the cost of delivery along a specific route is directly proportionate to the quantity of the products being conveyed. In the work of Jude (2016), he stated that, utilizing operations research optimization approaches, transportation expenses can be reduced.

According to Yadav, Boadh, Singh, and Rajoria (2020) the distribution process has become a great concern to organizations, in particular for enterprises that assemble and transport products. This concern is taken care of by utilizing transportation problem-solving strategies that lower the cost of distribution.

The TP is a specific type of LP that deals with the daily activities of businesses or organizations and, for the most part, controls coordination. The goal of the TP is to decrease the cost of the distribution processes and in turn increase revenue or profit (Hasibuan, 2017).

In the work of (Yadav et al., 2020), they stated that a French mathematician known as Gaspard Monge discovered the transportation problem in 1781 and offered fascinating details concerning actual problems. Unfortunately, not much research was done into his discovery until 1941, Frank Lauren Hitchcock made substantial contributions by applying the transportation problem (TP) to model business TP. His study was on “The distribution of a product from several sources to numerous localities”. With his work, he is credited with making the first significant input to the process of solving TPs (Hitchcock, 1941).

In 1947, a Dutch American mathematician and economist known as Tjalling Charles Koopmans came out with a study relating to Hitchcock’s on Optimum utilization of the transportation system (Koopmans, 1949). These two inputs aided in creation of TP strategies that take into account a variety of delivery sources and destinations.

Many researchers since then have made a lot of contributions in resolving transportation problems. There are many algorithms available for solving transportation problems using the linear programming (LP) model (Hlayel, 2012).

A TP can be modeled as balanced or unbalanced problem. In solving TPs, the initial basic feasible solution (IBFS) is computed first and the optimal solution is sought after (Srinivasan, 2010). There are many existing methods for finding IBFS which include; Column Minimum Method (CMM), Row Minimum Method (RMM), Least Cost Cell Method (LCCM), Northwest Conner rule (NWCR), Vogel's approximation method (VAM), Somani's approximation method (SAM), Best Candidate Method (BCM) and many more (Hlayel, 2012; Reeb & Leavengood, 2002). Utilizing the Modified Distribution (MODI) approach, the optimum solution is attained. This method is applied to IBFS which helps optimize the results it yielded. On the other hand, Stepping Stone Method proposed by (Charnes & Cooper, 1954) may also be used instead of the MODI method.

These methods for finding IBFS come with their strengths and weaknesses. Considering the North West Corner Rule (NWCR), it is very easy to apply, it does not take a longer time in its computation. It also involves less iteration when computing IBFS of TPs. It is also effective since it provides step-by-step solution. However, the NWCR does not take into consideration the important factor, that is, the cost which is sought to be minimized. The NWCR produces IBFS that is far from the optimal solution leading to lengthy iterations in finding optimal solution (Sudirga, 2017).

The Least Cost Cell Method (LCCM), CMM, and RMM take into account the important factor, thus, the cost component. In other words, the least cost is considered before allocating resources. These approaches also offer a precise solution as well. Computing IBFS with these methods requires lesser time. However, it is discovered that these methods rely more on observation than a step-by-step procedure for arriving at the optimal solution. When there is tie among candidates for the lowest cost, it does not adhere to any set of procedure. Again, these methods produced IBFS that are far from optimal solution (OS) to any given TP (Hossain & Ahmed, 2020).

Hlayel (2012), in his study, proposed a method known as the Best Candidate Method (BCM). This method produces better IBFS than NWCR, CMM, RMM, and Least Cost Cell Method but its computational time is lengthy. It also involves lengthy iterations when finding optimal solution.

Regarding Vogel's approximation method (VAM), it allocates resources based on the cell's opportunity cost. Also, it produces a better basic feasible solution as compared to other existing methods (SAM, BCM, NWCR, CMM, RMM, and LCCM). Comparatively, applying MODI method on VAM's IBFS value is the best option to get an optimal solution. Nevertheless, it is time consuming when computing IBFS. Also, computing IBFS using VAM is tedious especially when the given matrix is a large one (Srinivasan, 2010).

In the work of Somani (2015), he came out with an algorithm known as Somani's Approximation Method (SAM). His method also tried to resolve some of the challenges faced by some existing methods like the NWCR, BCM, VAM, and others. The SAM is very easy and simple to apply. It also takes into consideration the cost factor when making allocations to a particular cell. Computing IBFS using the SAM is the fastest. Thus, it takes lesser time. Its application is less stressful. Nevertheless, it is discovered that in many TPs it produces IBFS that is far from the optimal solution, leading to time-consuming iterations when computing optimal solution.

Statement of the Problem

The recent increase in local and international fuel prices have had a substantial influence on the cost of transporting goods. Due to this, market share has declined, profits have decreased, and competition among several businesses has intensified. As a result, manufacturers now include transportation expenses in the price of their products, requiring consumers to pay these rates. To maintain market share, expand markets, and advance the level of technology use, manufacturers and businesses are now challenged to develop an effective transportation mechanism that will help them lower their transportation expenses (Asare, 2011).

This study seeks to resolve these problems by reviewing the existing methods to help produce cost-effective results and fewer iterations for getting an optimal solution. This study builds on an IBFS technique which was proposed by Somani (2015) whose IBFS values are far from optimal solution value.

This study will mathematically formulate a new IBFS technique from the one initiated by Somani (2015) and compare the outcome with the well-known Vogel's Approximation Method (VAM) IBFS' value.

Many researchers have come out with different methods to help solve transportation problems. Most of these existing methods are not cost-effective and computation of optimal solution involves lengthy iterations. Therefore, it is important to formulate new IBFS method to help reduce cost associated with transportation issues.

Purpose of the Study

The main aim of the study is to develop a solution to transportation problems by formulating a new technique for finding IBFS from a transportation problem and then find the optimal solution from the existing optimal solution methods.

Research Objectives

The research objectives of the study are as follows:

1. to formulate a new IBFS method for solving TPs by modifying the SAM,
2. to apply the new IBFS method to solve a balanced TP,
3. to conduct a comparative analysis between the new IBFS method, the SAM and VAM,
4. to optimize the results of the new IBFS method, the SAM and VAM algorithm using MODI.

Research Methodology

This paper develops a new IBFS method for solving TPs. TPs are modeled as the LP model of TP type, and represent the TP in tableau and solved using the new IBFS Method, the SAM and VAM. Numerical examples are used for the analysis.

Significance of the Study

The study of transportation problems helps to identify optimal transportation routes along with units of the commodity to be transported to minimize total transportation costs.

It is believed that the study will provide the stakeholders with a new technique for resolving basic feasible solution to their transportation issues.

Delimitation

The study was delimited to literature values and numerical illustration on transportation data. It was also delimited to balanced transportation problem.

Limitations

The study was limited to the application of the new IBFS method on literature values and we could not extend it to real industrial transportation data due inaccessibility. It was also limited only the balanced transportation problem.

Definition of Terms

Source / Origin: It is the point from which finished products are distributed (Azizi, Birafane & Boueddine, 2015).

Destination: It is the point where products are moved to (Hasan, 2012).

Unit Transportation cost: It is the expense incurred when shipping a product from its place of origin to its final destination (Kawser, 2016).

Degeneracy: this occurs when there are less filled cells than there are rows (m) + columns (n) minus one (m + n - 1) (Ma, Lin, & Wen, 2013).

Perturbation Technique: It is a technique applied to resolve transportation problems when it is degenerate (Ma et al., 2013).

Feasible Solution: It is when a given solution satisfies the non-negativity and the row and column constraints (Hlayel, 2012).

Basic Feasible Solution: A feasible solution is said to be basic if and only if is not degenerate and satisfy rows (m) + columns (n) minus one (m + n – 1) (Ma et al., 2013)..

Optimal Solution: this is when the least cost for the total transportation cost is obtained (Supattananon & Akararungruangkul, 2020).

Balanced transportation problem: a TP is balance if the supply is same as the demand (Harrath & Kaabi, 2018).

Unbalanced transportation problem: a TP is unbalanced if the supply is less or greater than the demand (Harrath & Kaabi, 2018).

Organization of the Study

There are five chapters in this study. In particular, Chapter One serves as the study's introduction. A review of the literature on the transportation model or problem is presented in Chapter Two. The third chapter discusses the research methodology for developing the proposed IBFS method. The findings and conclusions of the study are also discussed in Chapter Four. The summary, conclusion, and recommendation of the study are also presented in Chapter Five.

Chapter Summary

This chapter presents the introduction of the study, as well as background to the study. The statement of the problem was also expressed, purpose of the study, research objectives, research methodology, significance of the study, limitations of the study, definition of terms and the lastly, organization of the research.

CHAPTER TWO

LITERATURE REVIEW

Introduction

This chapter provides an overview of previous research on the transportation problem and its various solution techniques.

The transportation model is highly useful for making location decisions. When picking between two or more sites, the model might be used to allocate new facilities, a manufacturing source, or an office. The transportation model can minimize or reduce total transportation, distribution, and production costs while maximizing profit (Chokanat, Pitakaso, & Sethanan 2019).

According to Poler, Mula, and Díaz-Madroñero (2014) practically all operations research textbooks, publications, and mathematical programming include the TP. The LP mathematical model is used to describe the transportation problem, which is commonly shown in a transportation tableau. Linear programming has been successfully applied to challenges relating to personnel assignment, banking, distribution, education, petroleum, engineering, transportation, and many others (Stoilova & Stoilov, 2021).

Linear programming is a major model in mathematical programming that is widely used in operations research. Several optimization models are used in mathematical programming, including stochastic, nonlinear, dynamic, integer, and goal programming. Each of these programming is a useful optimization technique that aids in resolving problems with a specific structure dependent on the rules used in the model's formulation (Murthy, 2007).

The Transportation Problem

Akpan, Ugbe, Usen, and Ajah (2015) presented a study on Modified Vogel's Approximation Method (MVAM) for Solving Transportation Problems. In their research, they modified VAM to find an IBFS for the TP. Comparatively, their outcomes showed that the MVAM gives minimum transportation cost better than the VAM in many cases, and better than NWCM and LCM.

Ezekiel and Edeki (2018) conducted research on transportation issues because they affect the majority of organizational decisions in a decomposed setting. Dangote cement factory (in Ibese, Nigeria) was used as a case study in their work, with three source and four destination centers. They applied MVAM to solve the TP and their results from the MVAM was found to be the best method for calculating IBFS of TP.

Hlayel (2012) also conducted research on solving TPs using the best candidate's method (BCM). His approach was utilized to save shipping costs and time. In his research, he discovered that the BCM provides the best IBFS to a TP and outperforms other approaches in terms of computing time and complexity.

Girmay and Sharmay (2013) presented a study on heuristic approach to balancing an unbalanced transportation problem. The research looked at VAM and its modification in order to find an IBFS to an unbalanced TP. They improved VAM to obtain an IBFS to an unbalanced TP.

To handle a fuzzy transportation problem, Dinagar and Keerthivasan (2018) introduced the Modified Best Candidate Method (MBCM). By choosing the best candidate, they were able to decrease the number of possibilities and find the optimum solution for their TPs. They also used Interval Valued Triangular Fuzzy Numbers (IVTFN) in conjunction with their mathematical operations to arrive at the best results.

Ashraful, Halel, Hasan, and Kanti (2014) published a study on an alternate technique for finding an IBFS to the TP. It was demonstrated in their study that their suggested technique, called the "Implied Cost Method (ICM)," yields IBFS that are lesser than VAM and are extremely near to the optimal solutions.

Somani (2015) published a paper on an innovative method for determining optimal transportation costs. This method provides a fundamentally possible solution to a particular transportation problem. The solution was derived via NWC, Diagonal Minima, Row Minima, and Column Minima and is found to provide the best results to any form of TP.

Karagul and Sahin (2020) published a study on the Novel approximation method (NAM) for solving TPs. In their research, they developed an innovative approach for identifying a working solution to the transportation problem. They used twenty-four test cases to compare their suggested approach, the Karagul-Sahin Approximation Method, and six other methods. In their analysis, it was discovered that their suggested strategy produced the best initial result with impressive computation speeds. They concluded that the results produced by their suggested method were just as accurate as VAM approach and as quick as the NWCR.

Amaliah, Fatichah and Suryani (2022) published a paper on a novel heuristic procedure for deriving an IBFS to a TP. In their work, they presented the Bilqis Chastine Erma (BCE) method for determining the IBFS of a TP. The performance of their suggested strategy was evaluated using numerical examples. When the BCE results were compared to those of other methods such as Total Opportunity Cost Matrix - Minimal total (TOCM-MT), Total Differences Method 1 (TDM1), Juman and Hoque Method (JHM), and VAM, the BCE obtained the lowest total minimal cost, as well as the shortest solving time.

Aini, Shodiqin and Wulandari (2021) solved a fuzzy TP using the Assigning Shortest Minima method and the zero-suffix method. Their method adopted solved TPs without requiring an IBFS in many TPs.

Ahmad (2020), demonstrated how Goyal's modification of VAM for the unbalanced TP may be improved by deleting or adding appropriate constants to the cost matrix's rows and columns. The approach solves an unbalanced TP by providing an IBFS near-optimal solution.

Hossain and Ahmed (2020) did comparison research of an IBFS using the Least Cost Mean Method (LCMM) of TP. In their presentation, they proposed the LCMM, which uses the mean of the least and next-least costs for each column and row of the cost matrix to compute column and row penalties to provide IBFS. The technique was presented using numerical examples, and comparison research was also conducted to validate the performance of the suggested method, which revealed that, it is computationally at ease and yields a relatively better IBFS.

Gani and Abbas (2014) investigated a novel average strategy for tackling intuitionistic fuzzy transportation problems. Their solution is fairly basic in terms of arithmetic computations and eliminates a significant number of iterations. An accuracy function was also employed to defuzzify the Triangular Intuitionistic Fuzzy Number. The optimal solution to the Intuitionistic Fuzzy transportation problem was computed and the method validated through a real transportation data.

Quddoos and Shakeel (2016) present a revised version of Assigning Shortest Minima (ASM) Method. Comparatively, the modified ASM produces much better results than those obtained by VAM and Zero Suffix Method (ZSM).

Chapter Summary

Various studies and research by other researchers were presented in this chapter. The outcomes of their work were discussed in this chapter.

CHAPTER THREE

RESEARCH METHODS

Introduction

This chapter presents the mathematical representation of a TP, solution to a transportation problem, the stepwise algorithm of existing TP methods and the proposed method.

Mathematical Representation of Transportation Problem

All stations are considered as an origin or source (S) where units are transported from. On the other hand, a city is considered as a destination (D) where units or products are demanded. Every destination has a certain demand (D), and every source has a certain supply (S).

Additionally, every system of roads connecting the specified collection of stations or cities has a certain transportation cost (Latunde, Richard, Esan, & Dare, 2019; Mahmoodirad, Dehghan, & Niroomand, 2019). Figure 1 shows the standard setup for cities on the highway in a network form.

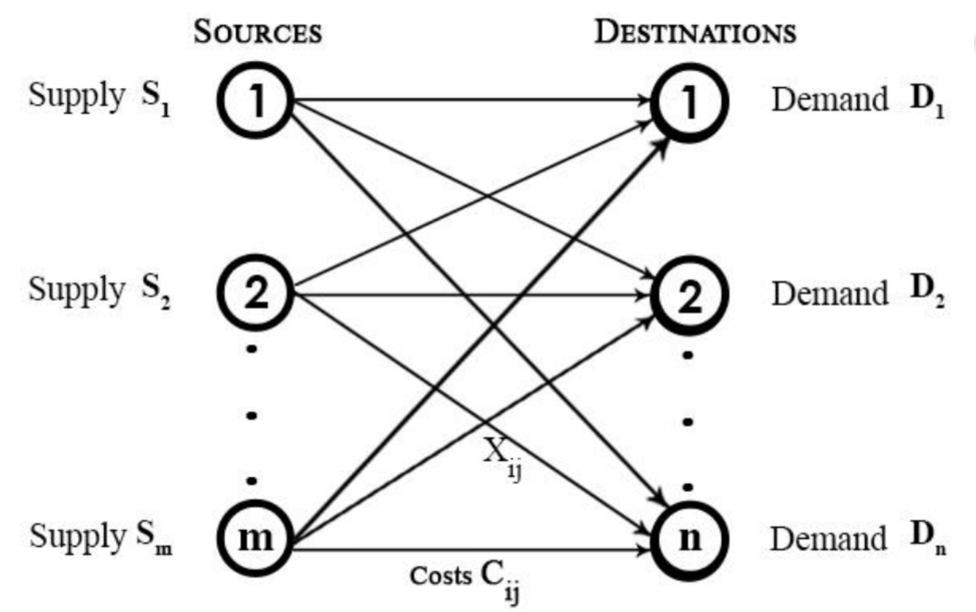

Figure 1: The Transportation Problem's Network Flow Model

Suppose there are m points of origin (supply), S_1, S_2, S_3, $\ldots$ S_m and n destinations D_1, D_2, D_3, $\ldots$ D_n. The point S_i where *i =1, 2, 3 . . ., m* can supply p_i units, and the destination D_j, where $j = 1, 2, 3..., n$ needs d_j units. The quantity of goods conveyed from the supply point i to the demand j is represented by X_{ij} $\forall i = 1, 2, 3..., m$ and $\forall$ $j = 1, 2, 3..., n$. The unit cost of transportation from the supply point i to the demand point j is represented by C_{ij} $\forall$ $i = 1, 2, 3..., m$ and $\forall$ $j = 1, 2, 3..., n$.

By application these quantities are written in specific units, thus, the supplies available, demand in tonnes or hundreds of tonnes, and the transportation cost are written in cost unit per ton.

Mathematical Representation of the Transportation Tableau

A TP can be represented in tabular form with all the necessary parameters contained in it. The transportation tableau, which depicts a typical TP in standard matrix form, shows the destination demand (d_j) in the bottom row and the supply availability (p_i) at each source in the far-right column.

Each route is represented by a cell. Each cell's top right corner displays the amount of transported products (X_{ij}), while the lower left corner displays the unit shipping cost (C_{ij}). The constrains on demand and supply as well as the transportation cost between each demand and supply point are implicitly expressed in the transportation tableau.

Table 1: **The General Transportation Problem Tableau**

		Destination				Supply
		D_1	**D_2**	**$...D_j...$**	**D_n**	
Source	**S_1**	C_{11} X_{11}	C_{12} X_{12}	$...C_{1j}...$ $...X_{1j}...$	C_{1n} X_{1n}	p_1
	S_2	C_{21} X_{21}	C_{22} X_{22}	$...C_{2j}...$ $...X_{2j}...$	C_{2n} X_{2n}	p_2
	$...S_i...$	C_{i1} X_{i1}	C_{i2} X_{i2}	$...C_{ij}...$ $...X_{ij}...$	$...C_{in}...$ $...X_{in}...$	$...p_i...$
	S_m	C_{m1} X_{m1}	C_{m2} X_{m2}	$...C_{mj}...$ $...X_{mj}...$	C_{mn} X_{mn}	p_m
Destination Demand		d_1	d_2	$...d_j...$	d_m	Σp_i Σd_j

Source: Srinivasan (2010).

Formulation of the TP

Minimize:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} \mathrm{C}_{ij}\mathrm{X}_{ij}$$

$$i = 1, 2, 3, \ldots, m \;\&\; j = 1, 2, 3, \ldots, n$$

Subject to:

1. Demand Constraint:

$$\sum_{j=1}^{n} X_{ij} \geq d_i, \forall i = 1, 2, 3, \ldots, m$$

2. Supply Constraint:

$$\sum_{i=1}^{m} X_{ij} \leq p_i, \forall j = 1, 2, 3, \ldots, n$$

The supply constraint ensures that the quantity transported from all sources is less or equal to the quantity available whiles the demand constraint also ensures that the quantities that reach every destination are greater or equal to the demand.

Solution to Transportation Problem (TP)

The following list outlines the steps involved in finding a transportation problem's solution:

Step 1: Balancing the given problem: to balance is to determine whether the total of the availability constraints matches the total of the required constraints. In other words, if $\sum p_i = \sum d_j$ move on to step 2. If not, you can balance the TP by opening a dummy row or column. Dummy cells' cost coefficients are zero. If $\sum p_i > \sum d_j$, open a dummy column whose cost coefficient is zero and whose requirement constraint is equal to $\sum p_i - \sum d_j$. If $\sum p_i < \sum d_j$, open a dummy row whose cost coefficient is zero and whose requirement constraint is equal to $\sum d_j - \sum p_i$. Proceed to the second step after the balancing is complete.

Step 2: Finding the IBFS: three of the available methods for finding IBFS are listed below:

1. The North-West Corner Rule,
2. Least Cost Cell Method,
3. Vogel's Approximation Method (VAM).

Step 3: Finding Optimal Solution: a test for optimality is conducted to determine if the solution is optimal or otherwise after finding the IBFS. Two approaches can be used to compute the optimal solution:

1. Stepping Stone Method,
2. Modified Distribution Technique (MODI) method.

Finding Initial Basic Feasible Solution

In this section, we provide in details the various existing IBFS methods and how each is applied to find the solution.

The North-West Corner Rule (NWCR)

TPs can be solved using the NWCR, which selects the fundamental variable from the top left corner. The NWCR approach steps are summarized below:

Step 1: Considering the supply and demand conditions, assign available X_{ij} to cell in the upper-left corner.

Step 2: Distribute every available possible X_{ij} to the next feasible adjacent cell.

Step 3: Replicate the second step until all rim specifications are met.

The Least Cost Cell Method

An IBFS to a TP can be found by applying Least Cost Cell technique, where the fundamental variables are selected based on the unit cost of transportation. The minimum-cost approach focuses on the least routes to obtain a better IBFS. The steps involve are shown below;

Step 1: Locate the cell with the lowest delivery costs.

Step 2: Give the chosen matrix cell the smallest of either supply or demand.

Step 3: Prepare a new matrix by removing the column or row whose demand or supply has been satisfied.

Step 4: As much as possible, choose the value of the corresponding X_{ij} within the supply and demand constraints.

Step 5: If the demand is met, remove the column.

Step 6: If the supply is depleted, remove the row.

Step 7: Repeat steps 1 to 6 till all constraints are met.

Vogel's Approximation Method (VAM) Algorithm

The VAM makes allocations subject to the opportunity cost of the cell. Among the three existing IBFS models, the VAM is considered to produce the best IBFS value. Below are the steps involved;

Step 1: Find the difference between the smallest and the next smallest cost value in each column and row. The difference is the penalty.

Step 2: Locate and assign the corresponding cell's Min(p_i d_j) to the call with the maximum penalty. Make the allocation at your convenience when there is a tie-in maximum penalty of either row or column.

Step 3: Delete the appropriate row if the assignment in the previous satisfies the supply at the origin. Delete the associated column if it meets the need of the demand.

Step 4: If all available supply and demand have been met, stop the process. Otherwise, repeat the steps.

Somani's Approximation Method (SAM)

The SAM method was an innovation to solve for IBFS of any given TP. The steps involve in how the SAM works are listed below;

Step 1: Create a transportation table for any given TP and, if it is not balanced, transform it into balanced form.

Step 2: Locate the least cost element in each row.

Step 3: Every row's minimal element is compared, and the one with the lowest mathematical value is used to place the demand in relation to the supply.

Step 4: Go to the next step if the supply and demand are exhausted in the column or row. When the demand or supply is depleted, identify the remaining rows' least element and follow step 3 again.

Step 5: If demand and supply are unequal, step 3 is repeated after finding the smallest element in each row.

Step 6: The smallest element whose demand is lowest can be allocated first if there is tie in the least cost value.

Step 7: If the total supply and the total demand are not satisfied, repeat the same process.

Step 8: When the total demand and total supply are exhausted, multiply the cost value and with its assigned value in each cell.

The New IBFS Method

This new IBFS method is a modification of the IBFS technique developed from the previous studies, which is believed to produce a much better IBFS value than that of Somani (2015). The steps of the new IBFS Model are listed below:

Step 1: Create a transportation table for any given TP and, if it is not balanced, transform it into balanced form.

Step 2: Locate the least cost element in each row. In each row, subtract the least cost value from the other values in the row. Do the same to the columns afterwards.

Step 3: Every row's minimal element is compared, and the one with the lowest mathematical value is used to place the demand in relation to the supply.

Step 4: Go to the next step if the supply and demand are exhausted in the column or row. When the demand or supply is depleted, identify the remaining rows' least element and follow step 3 again.

Step 5: If demand and supply are unequal, step 3 is repeated after finding the smallest element in each row.

Step 6: The smallest element whose demand is lowest can be allocated first if there is tie in the least cost value.

Step 7: If the total supply and the total demand are not satisfied, repeat the same process.

Step 8: When the total demand and total supply are exhausted, multiply the cost value and with its assigned value in each cell.

Step 9: Find the sum of the results in step 8 to get the total minimum transportation cost.

Finding the optimal solution

The optimal solution is obtained after finding the IBFS. Two well-known methods are available for finding optimal solution. These available methods with the steps involved are shown below:

The Stepping Stone Method

The stepping stone is the oldest technique for computing optimal solution. The steps involve are listed below:

Step 1: Find each unallocated cell in the tableau, and identify the paths of the stepping-stone and cost changes.

Step 2: As much as possible X_{ij} should be allocated to the unallocated cell with the largest cost reduction.

Step 3: In order to confirm that an optimal solution has been found, repeat steps 1 and 2 until all unallocated cells have positive cost values.

The Modified Distribution (MODI) Method

The MODI technique (or the U-V method) is the modified version of the stepping-stone approach. The MODI uses mathematical equations instead of the stepping-stone routes. In 1955 Ferguson and Dantzig discovered this method (Srinivasan, 2010). The steps for the MODI Method are listed below:

Step 1: Determine the IBFS

Step 2: Using the formula $C_{ij} = u_i + v_j$, calculate the values of the dual variables u_i and v_j.

Step 3: Use the formula $X_{ij} = C_{ij} - (u_i + v_j)$ to determine the opportunity cost.

Step 4: The solution is optimal if all $X_{ij} \geq 0$; if not, it is not optimal and the transportation cost can still be decreased.

Step 5: Pick the unallocated cell that has the lowest opportunity cost as the cell to be included in the next solution.

Step 6: Create a loop for the unallocated cell you chose in step 3.

Step 7: Place a plus sign at the cell that is being evaluated and alternate plus and negative signs at the vacant cells on the closed path's corner points.

Step 8: Decide how many units at most should be delivered to the unallocated cell. The smallest value in a negative position along the closed path represents the maximum quantity of units that can be transported to the entering cell. Add this amount to all of the cells on the closed path's corner points that are marked with plus signs, and subtract it from the cells that are marked with minus signs. An unallocated cell becomes occupied in this manner.

Chapter Summary

This chapter presented the methodology adopted in the study. Specifically, details of existing TP solving algorithms were discussed in this chapter. The steps involved in solving TPs were also listed. The study showed that there are three steps in solving TPs, namely: Balancing the given problem, finding the IBFS (the three existing models for finding IBFS were shown) and lastly how an optimality test is computed. The researcher also proposed a new IBFS method and showed how it works.

CHAPTER FOUR

RESULTS AND DISCUSSION

Introduction

The focus of this study is to formulate a new IBFS method and then use it find a basic feasible solution to a typical transportation problem. The results from the proposed method will be compared with the basic feasible solution from the Vogel's Approximation method and Somani Approximation Method to ascertain the dynamics of the two solution techniques. The data to be used for this comparison test is obtained from the previous work by Reeb and Leavengood (2002) and Somani (2015).

Numerical illustration 1

Table 2: **A Balanced Transportation Problem 1**

	D_1	D_2	D_3	D_4	D_5	SUPPLY
S_1	3	4	6	8	9	20
S_2	2	10	1	5	8	30
S_3	7	11	20	40	3	15
S_4	2	1	9	14	16	13
DEMAND	40	6	8	18	6	78

Source: Somani (2015).

Table 2 is a literature values adopted by the researcher from Somani (2015).

Solution to the Numerical Illustration 1

The total supply is equal to the total demand. That is, 78. Hence the transportation problem is a balanced one.

IBFS by SAM

Iteration 1:

	D_1	D_2	D_3	D_4	D_5	SUPPLY
S_1	**11** 3	4	6	**9** 8	9	**20**
S_2	**22** 2	10	**8** 1	5	8	**30**
S_3	7	11	20	**9** 40	**6** 3	**15**
S_4	**7** 2	**6** 1	9	14	16	**13**
DEMAND	**40**	**6**	**8**	**18**	**6**	**78**

Table 3: **Minimum Cost in the SAM Iteration 1**

Source	Minimum cost on Row							
S_1	3	3	3	3	(3)	8	(8)	X
S_2	1	(1)	2	(2)	X	X	X	X
S_3	3	3	3	3	3	(3)	40	(40)
S_4	(1)	2	(2)	X	X	X	X	X

Table 3 shows the details of the identified and selected minimum cost in each row for allocation in iteration 1

Iteration 2:

	D_1	D_2	D_3	D_4	D_5	SUPPLY
S_1	**11** 3	4	6	**9** 8	9	**20**
S_2	**22** 2	10	**8** 1	5	8	**30**
S_3	7	11	20	**9** 40	**6** 3	**15**
S_4	**7** 2	**6** 1	9	14	16	**13**
DEMAND	**40**	**6**	**8**	**18**	**6**	**78**

Total transportation cost =

$$(3\times 11)+(2\times 22)+(2\times 7)+(1\times 6)+(1\times 8)+(8\times 9)+(40\times 9)+(3\times 6)=555$$

The IBFS obtained by SAM is 555.

Optimal Solution by MODI Method

Iteration 1

	$V_1 =$	$V_2 =$	$V_3 =$	$V_4 =$	$V_5 =$	SUPPLY
$U_1 = 0$	**11** 3	4	6	**9** 8	9	**20**
$U_2 =$	**22** 2	10	**8** 1	5	8	**30**
$U_3 =$	7	11	20	**9** 40	**6** 3	**15**
$U_4 =$	**7** 2	**6** 1	9	14	16	**13**
DEMAND	**40**	**6**	**8**	**18**	**6**	**78**

We now compute the value of the dual variables u_i where $i = 1, 2, 3, 4$ and v_j where $j = 1, 2, 3, 4, 5$ using the equation $C_{ij} = u_i + v_j$ for occupied cells. For this, we arbitrarily assign $u_1 = 0$. Thus, we have,

$C_{11} = u_1 + v_1 \Rightarrow 0 + v_1 = 3 \Rightarrow v_1 = 3,$

$C_{14} = u_1 + v_4 \Rightarrow 0 + v_4 = 8 \Rightarrow v_4 = 8,$

$C_{21} = u_2 + v_1 \Rightarrow u_2 + 3 = 2 \Rightarrow u_2 = -1,$

$C_{23} = u_2 + v_3 \Rightarrow -1 + v_3 = 1 \Rightarrow v_3 = 2,$

$C_{34} = u_3 + v_4 \Rightarrow u_3 + 8 = 40 \Rightarrow u_3 = 32,$

$C_{35} = u_3 + v_5 \Rightarrow 32 + v_5 = 3 \Rightarrow v_5 = -29,$

$C_{41} = u_4 + v_1 \Rightarrow u_4 + 3 = 2 \Rightarrow u_4 = -1,$

$C_{42} = u_4 + v_2 \Rightarrow -1 + v_2 = 1 \Rightarrow v_2 = 2.$

We now compute the values of $X_{ij} = C_{ij} - (u_i + v_j)$ for each unoccupied cell. Thus, we have,

$X_{12} = C_{12} - (u_1 + v_2) \Rightarrow X_{12} = 4 - (0 + 2) \Rightarrow X_{12} = 2,$

$X_{13} = C_{13} - (u_1 + v_3) \Rightarrow X_{13} = 6 - (0 + 2) \Rightarrow X_{13} = 4,$

$X_{15} = C_{15} - (u_1 + v_5) \Rightarrow X_{15} = 9 - (0 - 29) \Rightarrow X_{15} = 38,$

$X_{22} = C_{22} - (u_2 + v_2) \Rightarrow X_{22} = 10 - (-1 + 2) \Rightarrow X_{22} = 9,$

$X_{24} = C_{24} - (u_2 + v_4) \Rightarrow X_{24} = 5 - (-1 - 1) \Rightarrow X_{24} = 7,$

$X_{25} = C_{25} - (u_2 + v_5) \Rightarrow X_{25} = 8 - (-1 - 29) \Rightarrow X_{25} = 38,$

$X_{44} = C_{44} - (u_4 + v_4) \Rightarrow X_{44} = 14 - (-1 - 1) \Rightarrow X_{44} = 16,$

$X_{32} = C_{32} - (u_3 + v_2) \Rightarrow X_{32} = 11 - (32 + 2) \Rightarrow X_{32} = -23,$

$X_{31} = C_{31} - (u_3 + v_1) \Rightarrow X_{31} = 7 - (32 + 3) \Rightarrow X_{31} = -28,$

$X_{33} = C_{33} - (u_3 + v_3) \Rightarrow X_{33} = 20 - (32 + 2) \Rightarrow X_{33} = -15,$

$X_{43} = C_{43} - (u_4 + v_3) \Rightarrow X_{43} = 9 - (-1 + 2) \Rightarrow X_{43} = 8,$

$X_{45} = C_{45} - (u_4 + v_5) \Rightarrow X_{45} = 16 - (-1 - 29) \Rightarrow X_{45} = 46.$

Iteration 2

	$V_1 = 3$	$V_2 = 2$	$V_3 = 2$	$V_4 = -1$	$V_5 = -29$	SUPPLY
$U_1 = 0$	- **11** 3	(2) 4	(4) 6	+ **9** 8	(38) 9	**20**
$U_2 = -1$	**22** 2	(9) 10	**8** 1	(7) 5	(38) 8	**30**
$U_3 = 32$	+ **(-28)** 7	(-23) 11	(-15) 20	- **9** 40	**6** 3	**15**
$U_4 = -1$	**7** 2	**6** 1	(8) 9	(16) 14	(46) 16	**13**
DEMAND	**40**	**6**	**8**	**18**	**6**	**78**

Now we select the most negative value of $X_{ij} = C_{ij} - (u_i + v_j)$, thus, $X_{31} = -28$. We look for a loop when we enter X_{31}. The loop identified is $X_{31} - X_{11} - X_{14} - X_{34}$. Now increasing X_{31} and X_{14} by θ, and decrease X_{11} and X_{34} by θ. The maximum increase in X_{31} is 9 beyond which X_{11} is negated. This will increase X_{14} to 18 and decrease X_{11} to 2 and $X_{31} = 0$

Iteration 3

	$V_1 =$	$V_2 =$	$V_3 =$	$V_4 =$	$V_5 =$	SUPPLY
$U_1 = 0$	**2** 3	4	6	**18** 8	9	**20**
$U_2 =$	**22** 2	10	**8** 1	5	8	**30**
$U_3 =$	**9** 7	11	20	40	**6** 3	**15**
$U_4 =$	**7** 2	**6** 1	9	14	16	**13**
DEMAND	**40**	**6**	**8**	**18**	**6**	**78**

We now compute the value of the dual variables u_i where $i = 1, 2, 3, 4$ and v_j where $j = 1, 2, 3, 4, 5$ using the equation $C_{ij} = u_i + v_j$ for occupied cells. For this, we arbitrarily assign $u_1 = 0$. Thus, we have,

$C_{11} = u_1 + v_1 \Rightarrow 0 + v_1 = 3 \Rightarrow v_1 = 3,$

$C_{21} = u_2 + v_1 \Rightarrow u_2 + 3 = 2 \Rightarrow u_2 = -1,$

$C_{23} = u_2 + v_3 \Rightarrow -1 + v_3 = 1 \Rightarrow v_3 = 2$

$C_{14} = u_1 + v_4 \Rightarrow 0 + v_4 = 8 \Rightarrow v_4 = 8,$

$C_{31} = u_3 + v_1 \Rightarrow u_3 + 3 = 7 \Rightarrow u_3 = 4,$

$C_{41} = u_4 + v_1 \Rightarrow u_4 + 3 = 2 \Rightarrow u_4 = -1,$

$C_{42} = u_4 + v_2 \Rightarrow -1 + v_2 = 1 \Rightarrow v_2 = 2,$

$C_{35} = u_3 + v_5 \Rightarrow 4 + v_5 = 3 \Rightarrow v_5 = -1.$

We now compute the values of $X_{ij} = C_{ij} - (u_i + v_j)$ for each unoccupied cell. Thus, we have,

$X_{12} = C_{12} - (u_1 + v_2) \Rightarrow X_{12} = 4 - (0 + 2) \Rightarrow X_{12} = 2,$

$X_{13} = C_{13} - (u_1 + v_3) \Rightarrow X_{13} = 6 - (0 + 2) \Rightarrow X_{13} = 4,$

$X_{15} = C_{15} - (u_1 + v_5) \Rightarrow X_{15} = 9 - (0 - 1) \Rightarrow X_{15} = 10,$

$X_{22} = C_{22} - (u_2 + v_2) \Rightarrow X_{22} = 10 - (-1 + 2) \Rightarrow X_{22} = 9,$

$X_{24} = C_{24} - (u_2 + v_4) \Rightarrow X_{24} = 5 - (-1 + 8) \Rightarrow X_{24} = -2,$

$X_{25} = C_{25} - (u_2 + v_5) \Rightarrow X_{25} = 8 - (-1 - 1) \Rightarrow X_{25} = 10,$

$X_{44} = C_{44} - (u_4 + v_4) \Rightarrow X_{44} = 14 - (-1 + 8) \Rightarrow X_{44} = 7,$

$X_{32} = C_{32} - (u_3 + v_2) \Rightarrow X_{32} = 11 - (4 + 2) \Rightarrow X_{32} = 5,$

$X_{33} = C_{33} - (u_3 + v_3) \Rightarrow X_{33} = 20 - (4 + 2) \Rightarrow X_{33} = 14,$

$X_{43} = C_{43} - (u_4 + v_3) \Rightarrow X_{43} = 9 - (-1 + 2) \Rightarrow X_{43} = 8,$

$X_{34} = C_{34} - (u_3 + v_4) \Rightarrow X_{34} = 40 - (4 + 8) \Rightarrow X_{34} = 28.$

Iteration 4

	$V_1 = 3$	$V_2 = 2$	$V_3 = 2$	$V_4 = 8$	$V_5 = -1$	SUPPLY
$U_1 = 0$	+ **2** 3	(2) 4	(4) 6	- **18** 8	(10) 9	**20**
$U_2 = -1$	- **22** 2	(9) 10	**8** 1	+ **(-2)** 5	(10) 8	**30**
$U_3 = 4$	**9** 7	(5) 11	14) 20	28) 40	**6** 3	**15**
$U_4 = -1$	**7** 2	**6** 1	(8) 9	(7) 14	(18) 16	**13**
DEMAND	**40**	**6**	**8**	**18**	**6**	**78**

Now we select the most negative value of X_{ij} = C_{ij} - $(u_i + v_j)$, thus, $X_{24} = -2$. we look for a loop when we enter X_{24}. The loop identified is X_{14}

- $X11$ - X_{21} - X_{24}. Now increasing $X24$ by θ, decrease X_{14} by θ, increase X_{11} by θ, and decrease X_{21} by θ. The maximum increase in X_{24} is 18 beyond which X_{14} will be negated. This will increase X_{11} to 20 and decrease $X21$ to 4.

Iteration 5

	$V_1 =$	$V_2 =$	$V_3 =$	$V_4 =$	$V_5 =$	SUPPLY
$U_1 = 0$	**20** 3	4	6	8	9	**20**
$U_2 =$	**4** 2	10	**8** 1	**18** 5	8	**30**
$U_3 =$	**9** 7	11	20	40	**6** 3	**15**
$U_4 =$	**7** 2	**6** 1	9	14	16	**13**
DEMAND	**40**	**6**	**8**	**18**	**6**	**78**

Again, we compute for the value of the dual variables u_i where $i = 1, 2, 3, 4$ and v_j where $j = 1, 2, 3, 4, 5$ using the equation $C_{ij} = u_i + v_j$ for occupied cells and the values of $X_{ij} = C_{ij} - (u_i + v_j)$ for each unoccupied cell. For this, we arbitrarily assign $u_1 = 0$. Thus, we have,

$C_{11} = u_1 + v_1 \Rightarrow 0 + v_1 = 3 \Rightarrow v_1 = 3,$

$C_{21} = u_2 + v_1 \Rightarrow u_2 + 3 = 2 \Rightarrow u_2 = -1,$

$C_{23} = u_2 + v_3 \Rightarrow -1 + v_3 = 1 \Rightarrow v_3 = 2,$

$C_{24} = u_2 + v_4 \Rightarrow -1 + v_4 = 5 \Rightarrow v_4 = 6,$

$C_{31} = u_3 + v_1 \Rightarrow u_3 + 3 = 7 \Rightarrow u_3 = 4,$

$C_{41} = u_4 + v_1 \Rightarrow u_4 + 3 = 2 \Rightarrow u_4 = -1,$

$C_{42} = u_4 + v_2 \Rightarrow -1 + v_2 = 1 \Rightarrow v_2 = 2,$

$C_{35} = u_3 + v_5 \Rightarrow 4 + v_5 = 3 \Rightarrow v_5 = -1.$

We now compute the values of $X_{ij} = C_{ij} - (u_i + v_j)$ for each unoccupied cell. Thus, we have,

$X_{12} = C_{12} - (u_1 + v_2) \Rightarrow X_{12} = 4 - (0 + 2) \Rightarrow X_{12} = 2,$

$X_{13} = C_{13} - (u_1 + v_3) \Rightarrow X_{13} = 6 - (0 + 2) \Rightarrow X_{13} = 4,$

$X_{15} = C_{15} - (u_1 + v_5) \Rightarrow X_{15} = 9 - (0 - 1) \Rightarrow X_{15} = 10,$

$X_{22} = C_{22} - (u_2 + v_2) \Rightarrow X_{22} = 10 - (-1 + 2) \Rightarrow X_{22} = 9,$

$X_{14} = C_{14} - (u_1 + v_4) \Rightarrow X_{14} = 8 - (0 + 6) \Rightarrow X_{14} = 2,$

$X_{25} = C_{25} - (u_2 + v_5) \Rightarrow X_{25} = 8 - (-1 - 1) \Rightarrow X_{25} = 10,$

$X_{44} = C_{44} - (u_4 + v_4) \Rightarrow X_{44} = 14 - (-1 + 6) \Rightarrow X_{44} = 9,$

$X_{32} = C_{32} - (u_3 + v_2) \Rightarrow X_{32} = 11 - (4 + 2) \Rightarrow X_{32} = 5,$

$X_{33} = C_{33} - (u_3 + v_3) \Rightarrow X_{33} = 20 - (4 + 2) \Rightarrow X_{33} = 14,$

$X_{43} = C_{43} - (u_4 + v_3) \Rightarrow X_{43} = 9 - (-1 + 2) \Rightarrow X_{43} = 8,$

$X_{34} = C_{34} - (u_3 + v_4) \Rightarrow X_{34} = 40 - (4 + 6) \Rightarrow X_{34} = 30.$

Iteration 6

	$V_1 = 3$	$V_2 = 2$	$V_3 = 2$	$V_4 = 6$	$V_5 = -1$	SUPPLY
$U_1 = 0$	20 3	(2) 4	(4) 6	(2) 8	(10) 9	20
$U_2 = -1$	4 2	(9) 10	8 1	18 5	(10) 8	30
$U_3 = 4$	9 7	(5) 11	(14) 20	(30) 40	6 3	15
$U_4 = -1$	7 2	6 1	(8) 9	(9) 14	(18) 16	13
DEMAND	40	6	8	18	6	78

Total transportation cost =

$(3 \times 20) + (2 \times 4) + (7 \times 9) + (2 \times 7) + (1 \times 6) + (1 \times 8) + (5 \times 15) + (6 \times 3) = 267.$

Since all $X_{ij} > 0$ the solution is optimal. That is, The total minimum transportation cost needed is 267.

IBFS by VAM

The VAM is an existing method which is known to produce the best IBFS as compared to the other existing methods.

Iteration 1:

	D_1	D_2	D_3	D_4	D_5	Supply	Penalties							
S_1	$\boxed{2}$ 3	4	6	$\boxed{18}$ 8	9	20	1	1	1	1	1	1	(3)	
S_2	$\boxed{22}$ 2	10	$\boxed{8}$ 1	5	8	30	1	3	3	(3)				
S_3	$\boxed{9}$ 7	11	20	40	$\boxed{6}$ 3	15	4	4	(4)					
S_4	$\boxed{7}$ 2	$\boxed{6}$ 1	9	14	16	13	1	1	1	1	1	1	2	(2)
Demand	40	6	8	18	6	78								
Penalties	1	3	(5)	3	5									
	1	3		3	(5)									
	1	3		3										
	1	3		3										
	1	3		(6)										
	1	(3)												
	1													

Iteration 2:

	D_1	D_2	D_3	D_4	D_5	SUPPLY
S_1	3 **2**	4	6	8 **18**	9	**20**
S_2	2 **22**	10	1 **8**	5	8	**30**
S_3	7 **9**	11	20	40	3 **6**	**15**
S_4	2 **7**	1 **6**	9	14	16	**13**
DEMAND	**40**	**6**	**8**	**18**	**6**	**78**

Total transportation cost =

$(1\times6)+(22\times2)+(7\times9)+(2\times7)+(3\times2)+(1\times8)+(8\times18)+(6\times3) = 303.$

The IBFS obtained by VAM is 303.

Optimal Solution by MODI Method

Iteration 1

	V_1=	V_2=	V_3=	V_4=	V_5=	SUPPLY
U_1= 0	2 3	4	6	18 8	9	20
U_2=	22 2	10	8 1	5	8	30
U_3=	9 7	11	20	40	6 3	15
U_4=	7 2	6 1	9	14	16	13
DEMAND	40	6	8	18	6	78

We now compute the value of the dual variables u_i where $i = 1, 2, 3, 4$ and v_j where $j = 1, 2, 3, 4, 5$ using the equation $C_{ij} = u_i + v_j$ for occupied cells. For this, we arbitrarily assign $u_1 = 0$. Thus, we have,

$C_{11} = u_1 + v_1 \Rightarrow 0 + v_1 = 3 \Rightarrow v_1 = 3,$

$C_{21} = u_2 + v_1 \Rightarrow u_2 + 3 = 2 \Rightarrow u_2 = -1,$

$C_{23} = u_2 + v_3 \Rightarrow -1 + v_3 = 1 \Rightarrow v_3 = 2\ ,$

$C_{14} = u_1 + v_4 \Rightarrow 0 + v_4 = 8 \Rightarrow v_4 = 8,$

$C_{31} = u_3 + v_1 \Rightarrow u_3 + 3 = 7 \Rightarrow u_3 = 4,$

$C_{41} = u_4 + v_1 \Rightarrow u_4 + 3 = 2 \Rightarrow u_4 = -1,$

$C_{42} = u_4 + v_2 \Rightarrow -1 + v_2 = 1 \Rightarrow v_2 = 2,$

$C_{35} = u_3 + v_5 \Rightarrow 4 + v_5 = 3 \Rightarrow v_5 = -1.$

We now compute the values of $X_{ij} = C_{ij} - (u_i + v_j)$ for each unoccupied cell. Thus, we have,

$X_{12} = C_{12} - (u_1 + v_2) \Rightarrow X_{12} = 4 - (0 + 2) \Rightarrow X_{12} = 2,$

$X_{13} = C_{13} - (u_1 + v_3) \Rightarrow X_{13} = 6 - (0 + 2) \Rightarrow X_{13} = 4,$

$X_{15} = C_{15} - (u_1 + v_5) \Rightarrow X_{15} = 9 - (0 - 1) \Rightarrow X_{15} = 10,$

$X_{22} = C_{22} - (u_2 + v_2) \Rightarrow X_{22} = 10 - (-1 + 2) \Rightarrow X_{22} = 9,$

$X_{24} = C_{24} - (u_2 + v_4) \Rightarrow X_{24} = 5 - (-1 + 8) \Rightarrow X_{24} = -2,$

$X_{25} = C_{25} - (u_2 + v_5) \Rightarrow X_{25} = 8 - (-1 - 1) \Rightarrow X_{25} = 10,$

$X_{44} = C_{44} - (u_4 + v_4) \Rightarrow X_{44} = 14 - (-1 + 8) \Rightarrow X_{44} = 7,$

$X_{32} = C_{32} - (u_3 + v_2) \Rightarrow X_{32} = 11 - (4 + 2) \Rightarrow X_{32} = 5,$

$X_{33} = C_{33} - (u_3 + v_3) \Rightarrow X_{33} = 20 - (4 + 2) \Rightarrow X_{33} = 14,$

$X_{43} = C_{43} - (u_4 + v_3) \Rightarrow X_{43} = 9 - (-1 + 2) \Rightarrow X_{43} = 8,$

$X_{34} = C_{34} - (u_3 + v_4) \Rightarrow X_{34} = 40 - (4 + 8) \Rightarrow X_{34} = 28.$

Iteration 2

	$V_1 = 3$	$V_2 = 2$	$V_3 = 2$	$V_4 = 8$	$V_5 = -1$	SUPPLY
$U_1 = 0$	+ **2** 3	(2) 4	(4) 6	- **18** 8	(10) 9	**20**
$U_2 = -1$	- **22** 2	(9) 10	**8** 1	+ **(-2)** 5	(10) 8	**30**
$U_3 = 4$	**9** 7	(5) 11	(14) 20	(28) 40	**6** 3	**15**
$U_4 = -1$	**7** 2	**6** 1	(8) 9	(7) 14	(18) 16	**13**
DEMAND	**40**	**6**	**8**	**18**	**6**	**78**

Now we select the most negative value of $X_{ij} = C_{ij}$ - $(u_i + v_j)$, thus, $X_{24} = -2$. we look for a loop when we enter X_{24}. The loop identified is X_{14} - $X11$ - X_{21} - X_{24}. Now increasing X_{24} by θ, decrease X_{14} by θ, increase X_{11} by θ, and decrease X_{21} by θ. The maximum increase in X_{24} is 18 beyond which X_{14} will be negated. This will increase X_{11} to 20 and decrease X_{21} to 4.

Iteration 3

	V_1=	V_2=	V_3=	V_4=	V_5=	SUPPLY
U_1= 0	3 [20]	4	6	8	9	20
U_2=	2 [4]	10	1 [8]	5 [18]	8	30
U_3=	7 [9]	11	20	40	3 [6]	15
U_4=	2 [7]	1 [6]	9	14	16	13
DEMAND	40	6	8	18	6	78

Again, we compute for the value of the dual variables u_i where $i = 1, 2, 3, 4$ and v_j where $j = 1, 2, 3, 4, 5$ using the equation $C_{ij} = u_i + v_j$ for occupied cells and the values of $X_{ij} = C_{ij} - (u_i + v_j)$ for each unoccupied cell. For this, we arbitrarily assign $u_1 = 0$. Thus, we have,

$C_{11} = u_1 + v_1 \Rightarrow 0 + v_1 = 3 \Rightarrow v_1 = 3,$

$C_{21} = u_2 + v_1 \Rightarrow u_2 + 3 = 2 \Rightarrow u_2 = -1,$

$C_{23} = u_2 + v_3 \Rightarrow -1 + v_3 = 1 \Rightarrow v_3 = 2,$

$C_{24} = u_2 + v_4 \Rightarrow -1 + v_4 = 5 \Rightarrow v_4 = 6,$

$C_{31} = u_3 + v_1 \Rightarrow u_3 + 3 = 7 \Rightarrow u_3 = 4,$

$C_{41} = u_4 + v_1 \Rightarrow u_4 + 3 = 2 \Rightarrow u_4 = -1,$

$C_{42} = u_4 + v_2 \Rightarrow -1 + v_2 = 1 \Rightarrow v_2 = 2,$

$C_{35} = u_3 + v_5 \Rightarrow 4 + v_5 = 3 \Rightarrow v_5 = -1.$

We now compute the values of $X_{ij} = C_{ij} - (u_i + v_j)$ for each unoccupied cell. Thus, we have,

$X_{12} = C_{12} - (u_1 + v_2) \Rightarrow X_{12} = 4 - (0 + 2) \Rightarrow X_{12} = 2,$

$X_{13} = C_{13} - (u_1 + v_3) \Rightarrow X_{13} = 6 - (0 + 2) \Rightarrow X_{13} = 4,$

$X_{15} = C_{15} - (u_1 + v_5) \Rightarrow X_{15} = 9 - (0 - 1) \Rightarrow X_{15} = 10,$

$X_{22} = C_{22} - (u_2 + v_2) \Rightarrow X_{22} = 10 - (-1 + 2) \Rightarrow X_{22} = 9,$

$X_{14} = C_{14} - (u_1 + v_4) \Rightarrow X_{14} = 8 - (0 + 6) \Rightarrow X_{14} = 2,$

$X_{25} = C_{25} - (u_2 + v_5) \Rightarrow X_{25} = 8 - (-1 - 1) \Rightarrow X_{25} = 10,$

$X_{44} = C_{44} - (u_4 + v_4) \Rightarrow X_{44} = 14 - (-1 + 6) \Rightarrow X_{44} = 9,$

$X_{32} = C_{32} - (u_3 + v_2) \Rightarrow X_{32} = 11 - (4 + 2) \Rightarrow X_{32} = 5,$

$X_{33} = C_{33} - (u_3 + v_3) \Rightarrow X_{33} = 20 - (4 + 2) \Rightarrow X_{33} = 14,$

$X_{43} = C_{43} - (u_4 + v_3) \Rightarrow X_{43} = 9 - (-1 + 2) \Rightarrow X_{43} = 8,$

$X_{34} = C_{34} - (u_3 + v_4) \Rightarrow X_{34} = 40 - (4 + 6) \Rightarrow X_{34} = 30.$

Iteration 4

	$V_1 = 3$	$V_2 = 2$	$V_3 = 2$	$V_4 = 6$	$V_5 = -1$	SUPPLY
$U_1 = 0$	20 3	(2) 4	(4) 6	(2) 8	(10) 9	20
$U_2 = -1$	4 2	(9) 10	8 1	18 5	(10) 8	30
$U_3 = 4$	9 7	(5) 11	(14) 20	(30) 40	6 3	15
$U_4 = -1$	7 2	6 1	(8) 9	(9) 14	(18) 16	13
DEMAND	40	6	8	18	6	78

Total transportation cost =

$(3 \times 20) + (2 \times 4) + (7 \times 9) + (2 \times 7) + (1 \times 6) + (1 \times 8) + (5 \times 15) + (6 \times 3) = 267.$

Since all $X_{ij} > 0$ the solution is optimal. That is, The total minimum transportation cost needed is 267.

IBFS by the New IBFS Model

From Table 2, we first locate the lowest cost value in each row and subtract each identified least value from its respective row cost values. After doing that identify the lowest cost value in each column and subtract each identified lowest value from its respective column cost values. There will be at least a zero (0) cost value in each row and column. Then start making the allocations by identifying minimum cost value in each row and make allocation to the cell that has the minimal cost value. This complete the first iteration.

Iteration 1

	D_1	D_2	D_3	D_4	D_5	SUPPLY
S_1	0 **20**	1	3	1	6	**20**
S_2	1 **4**	9	0 **8**	0 **18**	7	**30**
S_3	4 **9**	9	17	33	0 **6**	**15**
S_4	1 **7**	0 **6**	8	9	15	**13**
DEMAND	**40**	**6**	**8**	**18**	**6**	**78**

Table 4: **Minimum Cost in the New IBFS Method Iteration 1**

Source	Minimum cost on Row							
S_1	0	0	0	0	(0)	X	X	X
S_2	0	0	(0)	(0)	1	(1)	X	X
S_3	(0)	4	4	4	4	4	4	(4)
S_4	0	(0)	1	1	1	1	(1)	X

Table 4 shows the details of the identified and selected minimum cost in each row for allocation in the iteration 1

Following the steps of the new IBFS Model listed in Chapter Three, the minimum value for each row is 0. Allocation is made at $S_3 = 0$. We allocate 6 to cell $(S_3\ D_5)$. Now the next allocation exclude D_5 because its demand is exhausted.

The next minimum value is identified at S_1, S_2, and S_4 which is 0. We make allocation to $S_4 = 0$ because its demand value is lesser than S_1 and S_2. Therefore we allocate 6 to cell $(S_4\ D_2)$. The demand of D_2 is exhausted.

The next minimum value is identified at S_2 and S_1, which is 0. Allocation is made at $S_2 = 0$ in the cell $(S_2\ D_3)$. D_3 is exhausted. Again, allocation is made at $S_2 = 0$ in the cell $(S_2\ D_4)$. Now D_4 is exhausted.

The next allocation is done at $S_1 = 0$ in the cell $(S_1\ D_1)$. The supply of S_1 is exhausted. We allocate 4 to $(S_1\ D_2)$, 9 to $(S_1\ D_3)$ and lastly 7 to $(S_1\ D_4)$.

This complete the first iteration. Now we proceed to find the total minimum transportation cost in Iteration 2. The original table is redrawn and the allocation in Iteration 1 is maintained in the redrawn table. Thus, allocations in the cells do not change in the redrawn table.

Iteration 2

	D_1	D_2	D_3	D_4	D_5	SUPPLY
S_1	3 **20**	4	6	8	9	**20**
S_2	2 **4**	10	1 **8**	5 **18**	8	**30**
S_3	**7** **9**	11	20	40	3 **6**	**15**
S_4	2 **7**	1 **6**	9	14	16	**13**
DEMAND	**40**	**6**	**8**	**18**	**6**	**78**

Minimum Transportation Cost =

$$(20\times3)+(4\times2)+(9\times7)+(7\times2)+(6\times1)+(8\times1)+(18\times5)+(6\times3)=267$$

The IBFS obtained by the New IBFS Model is 267.

Optimal Solution by MODI Method

Iteration 1

	$V_1 =$	$V_2 =$	$V_3 =$	$V_4 =$	$V_5 =$	SUPPLY
$U_1 = 0$	20 3	4	6	8	9	20
$U_2 =$	4 2	10	8 1	18 5	8	30
$U_3 =$	9 7	11	20	40	6 3	15
$U_4 =$	7 2	6 1	9	14	16	13
DEMAND	40	6	8	18	6	78

We compute for the value of the dual variables u_i where $i = 1, 2, 3, 4$ and v_j where $j = 1, 2, 3, 4, 5$ using the equation $C_{ij} = u_i + v_j$ for occupied cells and the values of $X_{ij} = C_{ij} - (u_i + v_j)$ for each unoccupied cell. For this, we arbitrarily assign $u_1 = 0$. Thus, we have,

$C_{11} = u_1 + v_1 \Rightarrow 0 + v_1 = 3 \Rightarrow v_1 = 3,$

$C_{21} = u_2 + v_1 \Rightarrow u_2 + 3 = 2 \Rightarrow u_2 = -1,$

$C_{23} = u_2 + v_3 \Rightarrow -1 + v_3 = 1 \Rightarrow v_3 = 2,$

$C_{24} = u_2 + v_4 \Rightarrow -1 + v_4 = 5 \Rightarrow v_4 = 6,$

$C_{31} = u_3 + v_1 \Rightarrow u_3 + 3 = 7 \Rightarrow u_3 = 4,$

$C_{41} = u_4 + v_1 \Rightarrow u_4 + 3 = 2 \Rightarrow u_4 = -1,$

$C_{42} = u_4 + v_2 \Rightarrow -1 + v_2 = 1 \Rightarrow v_2 = 2,$

$C_{35} = u_3 + v_5 \Rightarrow 4 + v_5 = 3 \Rightarrow v_5 = -1.$

We now compute the values of $X_{ij} = C_{ij} - (u_i + v_j)$ for each unoccupied cell. Thus, we have,

$X_{12} = C_{12} - (u_1 + v_2) \Rightarrow X_{12} = 4 - (0 + 2) \Rightarrow X_{12} = 2,$

$X_{13} = C_{13} - (u_1 + v_3) \Rightarrow X_{13} = 6 - (0 + 2) \Rightarrow X_{13} = 4,$

$X_{15} = C_{15} - (u_1 + v_5) \Rightarrow X_{15} = 9 - (0 - 1) \Rightarrow X_{15} = 10,$

$X_{22} = C_{22} - (u_2 + v_2) \Rightarrow X_{22} = 10 - (-1 + 2) \Rightarrow X_{22} = 9,$

$X_{14} = C_{14} - (u_1 + v_4) \Rightarrow X_{14} = 8 - (0 + 6) \Rightarrow X_{14} = 2,$

$X_{25} = C_{25} - (u_2 + v_5) \Rightarrow X_{25} = 8 - (-1 - 1) \Rightarrow X_{25} = 10,$

$X_{44} = C_{44} - (u_4 + v_4) \Rightarrow X_{44} = 14 - (-1 + 6) \Rightarrow X_{44} = 9,$

$X_{32} = C_{32} - (u_3 + v_2) \Rightarrow X_{32} = 11 - (4 + 2) \Rightarrow X_{32} = 5,$

$X_{33} = C_{33} - (u_3 + v_3) \Rightarrow X_{33} = 20 - (4 + 2) \Rightarrow X_{33} = 14,$

$X_{43} = C_{43} - (u_4 + v_3) \Rightarrow X_{43} = 9 - (-1 + 2) \Rightarrow X_{43} = 8,$

$X_{34} = C_{34} - (u_3 + v_4) \Rightarrow X_{34} = 40 - (4 + 6) \Rightarrow X_{34} = 30.$

Iteration 2

	$V_1 = 3$	$V_2 = 2$	$V_3 = 2$	$V_4 = 6$	$V_5 = -1$	SUPPLY
$U_1 = 0$	**20** 3	(2) 4	(4) 6	(2) 8	(10) 9	**20**
$U_2 = -1$	**4** 2	(9) 10	**8** 1	**18** 5	(10) 8	**30**
$U_3 = 4$	**9** 7	(5) 11	(14) 20	(30) 40	**6** 3	**15**
$U_4 = -1$	**7** 2	**6** 1	(8) 9	(9) 14	(18) 16	**13**
DEMAND	**40**	**6**	**8**	**18**	**6**	**78**

Total transportation cost =

$(3\times20)+(2\times4)+(7\times9)+(2\times7)+(1\times6)+(1\times8)+(5\times15)+(6\times3) = 267.$

Since $\forall\ X_{ij} > 0$ the solution is optimal.

Therefore the total minimum transportation cost needed is 267.

Table 5: **Resulted values from Numerical Illustration 1**

Method	IBFS	Optimal Solution	Total Iterations
SAM	555	267	8
VAM	303	267	6
New IBFS Method	267	267	4

Results in Table 5 shows the comparison of SAM, VAM and the New IBFS Model on their performance. It is clearly shown that SAM produced an IBFS value of 555 and the total iterations involved in computing IBFS through to the optimal solution is 8.

Again VAM produced an IBFS value of 303 and the total iterations involved in computing IBFS through to the optimal solution is 6.

Also, the new IBFS method produced an IBFS value of 267 and the total iterations involved in computing IBFS through to the optimal solution is 4. By applying the MODI Method, the optimal solution of 267 was obtained for numerical illustration 1.

Numerical Illustration 2

Table 6: **A Balanced Transportation Problem 2**

	D_1	D_2	D_3	D_4	Supply
S_1	19	7	3	21	10
S_2	15	21	18	6	30
S_3	11	14	15	22	20
Demand	15	10	20	15	60

Source: Reeb and Leavengood (2002).

Table 6 literature values adopted by the researcher from Reeb and Leavengood (2002).

Solution to Numerical Illustration 2

IBFS by SAM

Iteration 1

	D_1	**D_2**	**D_3**	**D_4**	**Supply**
S_1	19	7	**10** 3	21	**10**
S_2	15	**5** 21	**10** 18	**15** 6	**30**
S_3	**15** 11	**5** 14	15	22	**20**
Demand	**15**	**10**	**20**	**15**	**60**

Table 7: **Selected Minimum Cost in SAM Iteration 1**

Source	Minimum cost on Row					
S_1	(3)	X	X	X	X	X
S_2	6	(6)	15	18	(18)	(21)
S_3	11	11	(11)	(14)	X	X

Table 7 shows the details of the identified and selected minimum cost in each row for allocation in the iteration 1.

Iteration 2

	D_1	D_2	D_3	D_4	Supply
S_1	19	7	3 **10**	21	10
S_2	15	21 **5**	18 **10**	6 **15**	30
S_3	11 **15**	14 **5**	15	22	20
Demand	15	10	20	15	60

Total transportation cost =

$(11 \times 15) + (21 \times 5) + (14 \times 5) + (3 \times 10) + (18 \times 10) + (6 \times 15) = 640.$

The IBFS obtained by SAM is 640.

Optimal Solution by MODI Method

Iteration 1

	$V_1 =$	$V_2 =$	$V_3 =$	$V_4 =$	Supply
$U_1 = 0$	19	7	3 **10**	21	10
$U_2 =$	15	21 **5**	18 **10**	6 **15**	30
$U_3 =$	11 **15**	14 **5**	15	22	20
Demand	15	10	20	15	60

We compute for the value of the dual variables u_i where $i = 1, 2, 3$ and v_j where $j = 1, 2, 3, 4$ using the equation $C_{ij} = u_i + v_j$ for occupied cells and the values of $X_{ij} = C_{ij} - (u_i + v_j)$ for each unoccupied cell. For this, we arbitrarily assign $u_1 = 0$. Thus, we have,

C_{13}=$u_1 + v_3$⇒$0 + v_3 = 3 \Rightarrow v_3 = 3$,

C_{23}=$u_2 + v_3$⇒$u_2 + 3 = 18 \Rightarrow u_2 = 15$,

C_{22}=$u_2 + v_2$⇒$15+ v_2 = 21 \Rightarrow v_2 = 6$,

C_{24}=$u_2 + v_4$⇒$15 + v_4 = 6 \Rightarrow v_4 = -9$,

C_{32}=$u_3 + v_2$⇒$u_3 + 6 = 14 \Rightarrow u_3 = 8$,

C_{31}=$u_3 + v_1$⇒$8 + v_1 = 11 \Rightarrow v_1 = 3$.

We now compute the values of $X_{ij} = C_{ij} - u_i + v_j)$ for each unoccupied cell. Thus, we have,

$X_{11} = C_{11} - (u_1 + v_1) \Rightarrow X_{11} = 19 - (0 + 3) \Rightarrow X_{11} = 16$,

$X_{12} = C_{12} - (u_1 + v_2) \Rightarrow X_{12} = 7 - (0 + 6) \Rightarrow X_{12} = 1$,

$X_{14} = C_{14} - (u_1 + v_4) \Rightarrow X_{14} = 21 - (0 - 9) \Rightarrow X_{14} = 30$,

$X_{21} = C_{21} - (u_2 + v_1) \Rightarrow X_{21} = 15 - (15 + 3) \Rightarrow X_{21} = -3$,

$X_{33} = C_{33} - (u_3 + v_3) \Rightarrow X_{33} = 15 - (8 + 3) \Rightarrow X_{33} = 4$,

$X_{34} = C_{34} - (u_3 + v_4) \Rightarrow X_{34} = 22 - (8 - 9) \Rightarrow X_{34} = 23$.

Iteration 2

	$V_1 = 3$	$V_2 = 6$	$V_3 = 3$	$V_4 = -9$	Supply
$U_1 = 0$	(16) 19	(1) 7	**10** 3	(30) 21	**10**
$U_2 = 15$	+ (-3) 15	- **5** 21	**10** 18	**15** 6	**30**
$U_3 = 8$	- **15** 11	+ **5** 14	(4) 15	(23) 22	**20**
Demand	**15**	**10**	**20**	**15**	**60**

Now we select the most negative value of $X_{ij} = C_{ij} - (u_i + v_j)$, thus, $X_{21} = -3$. We look for a loop when we enter X_{21}. The loop identified is $X_{21} - X_{22} - X_{31} - X_{32}$. Now increasing X_{21} by θ, decrease X_{22} by θ, increase X_{32} by θ, and decrease X_{31} by θ. The maximum increase in X_{21} is 5 beyond which X_{22} will be negated. This will increase X_{32} to 10 and decrease X_{31} to 10 and X_{22} to 0

Iteration 3

	V_1 =	V_2 =	V_3 =	V_4 =	Supply
$U_1 = 0$	19	7	**10** 3	21	**10**
U_2 =	**5** 15	21	**10** 18	**15** 6	**30**
U_3 =	**10** 11	**10** 14	15	22	**20**
Demand	**15**	**10**	**20**	**15**	**60**

We compute for the value of the dual variables u_i where $i = 1, 2, 3$ and v_j where $j = 1, 2, 3, 4$ using the equation $C_{ij} = u_i + v_j$ for occupied cells and the values of $X_{ij} = C_{ij} - (u_i + v_j)$ for each unoccupied cell. For this, we arbitrarily assign $u_1 = 0$. Thus, we have,

$C_{13}{=}u_1 + v_3{\Rightarrow}0 + v_3 = 3 \Rightarrow v_3 = 3,$

$C_{23}{=}u_2 + v_3{\Rightarrow}u_2 \ + 3 = 18 \Rightarrow u_2 = 15,$

$C_{24}{=}u_2 + v_4{\Rightarrow}15 + v_4 = 6 \Rightarrow v_4 = -9,$

$C_{21}{=}u_2 + v_1{\Rightarrow}15 + v_1 = 15 \Rightarrow v_1 = 0\ ,$

$C_{31}{=}u_3 + v_1{\Rightarrow}u_3 + 0 = 11 \Rightarrow u_3 = 11\ ,$

$C_{32}{=}u_3 + v_2{\Rightarrow}11{+}\ v_2 = 14 \Rightarrow v_2 = 3.$

We now compute the values of $X_{ij} = C_{ij}{-}(u_i + v_j)$ for each unoccupied cell. Thus, we have,

$X_{11} = C_{11}{-}(u_1 + v_1){\Rightarrow}X_{11} = 19{-}(0 + 0)\ {\Rightarrow}X_{11} = 19,$

$X_{12} = C_{12}{-}(u_1 + v_2){\Rightarrow}X_{12} = 7{-}(0 + 3)\ {\Rightarrow}X_{12} = 4,$

$X_{14} = C_{14}{-}\ (u_1 + v_4)\ {\Rightarrow}X_{14} = 21{-}\ (0 - 9)\ {\Rightarrow}X_{14} = 30,$

$X_{22} = C_{22}{-}\ (u_2 + v_2)\ {\Rightarrow}X_{22} = 21{-}\ (15 + 3)\ {\Rightarrow}X_{22} = 3,$

$X_{33} = C_{33} - (u_3 + v_3) \Rightarrow X_{33} = 15 - (11 + 3) \Rightarrow X_{33} = 1,$

$X_{34} = C_{34} - (u_3 + v_4) \Rightarrow X_{34} = 22 - (11 - 9) \Rightarrow X_{34} = 20.$

Iteration 4

	$V_1 = 0$	$V_2 = 3$	$V_3 = 3$	$V_4 = -9$	Supply
$U_1 = 0$	(19) 19	(4) 7	**10** 3	(30) 21	**10**
$U_2 = 15$	**5** 15	(3) 21	**10** 18	**15** 6	**30**
$U_3 = 11$	**10** 11	**10** 14	(1) 15	(20) 22	**20**
Demand	**15**	**10**	**20**	**15**	**60**

Total transportation cost =

$(15 \times 5) + (11 \times 10) + (14 \times 10) + (3 \times 10) + (18 \times 10) + (6 \times 15) = 625.$

Since all $X_{ij} > 0$ the solution 625 is the optimal transportation cost for the TP.

IBFS by VAM

Iteration 1

	D_1	D_2	D_3	D_4	Supply	Penalties				
S_1	19	7	**10** 3	21	**10**	4	4	X	X	X
S_2	**5** 15	21	**10** 18	**15** 6	**30**	9	3	3	3	**(3)**
S_3	**10** 11	**10** 14	15	22	**20**	3	3	3	3	X
Demand	**15**	**10**	**20**	**15**	**60**					
Penalties	4	7	12	**(15)**						
	4	7	(12)	X						
	4	**(7)**	3	X						
	4	X	3	X						
	(4)	X	3	X						

Iteration 2

	D_1	D_2	D_3	D_4	Supply
S_1	19	7	**10** 3	21	**10**
S_2	**5** 15	21	**10** 18	**15** 6	**30**
S_3	**10** 11	**10** 14	15	22	**20**
Demand	**15**	**10**	**20**	**15**	**60**

Total transportation cost =

$(15 \times 5) + (11 \times 10) + (14 \times 10) + (3 \times 10) + (18 \times 10) + (6 \times 15) = 625.$

The IBFS obtained by VAM is 625

Optimal Solution by MODI Method

Iteration 1

	$V_1 =$	$V_2 =$	$V_3 =$	$V_4 =$	Supply
$U_1 = 0$	19	7	3 **10**	21	**10**
$U_2 =$	15 **5**	21	18 **10**	6 **15**	**30**
$U_3 =$	11 **10**	14 **10**	15	22	**20**
Demand	**15**	**10**	**20**	**15**	**60**

We compute for the value of the dual variables u_i where $i = 1, 2, 3$ and v_j where $j = 1, 2, 3, 4$ using the equation $C_{ij} = u_i + v_j$ for occupied cells and the values of $X_{ij} = C_{ij} - (u_i + v_j)$ for each unoccupied cell. For this, we arbitrarily assign $u_1 = 0$. Thus, we have,

$C_{13}{=}u_1 + v_3{\Rightarrow}0 + v_3 = 3 \Rightarrow v_3 = 3,$

$C_{23}{=}u_2 + v_3{\Rightarrow}u_2\ + 3 = 18 \Rightarrow u_2 = 15,$

$C_{24}{=}u_2 + v_4{\Rightarrow}15 + v_4 = 6 \Rightarrow v_4 = -9,$

$C_{21}{=}u_2 + v_1{\Rightarrow}15 + v_1 = 15 \Rightarrow v_1 = 0,$

$C_{31}{=}u_3 + v_1{\Rightarrow}u_3 + 0 = 11 \Rightarrow u_3 = 11\ ,$

$C_{32}{=}u_3 + v_2{\Rightarrow}11{+}\ v_2 = 14 \Rightarrow v_2 = 3.$

We now compute the values of $X_{ij} = C_{ij}{-}(u_i + v_j)$ for each unoccupied cell. Thus, we have,

$X_{11} = C_{11}{-}(u_1 + v_1){\Rightarrow}X_{11} = 19{-}(0 + 0)\ {\Rightarrow}X_{11} = 19,$

$X_{12} = C_{12} - (u_1 + v_2) \Rightarrow X_{12} = 7 - (0 + 3) \Rightarrow X_{12} = 4,$

$X_{14} = C_{14} - (u_1 + v_4) \Rightarrow X_{14} = 21 - (0 - 9) \Rightarrow X_{14} = 30,$

$X_{22} = C_{22} - (u_2 + v_2) \Rightarrow X_{22} = 21 - (15 + 3) \Rightarrow X_{22} = 3,$

$X_{33} = C_{33} - (u_3 + v_3) \Rightarrow X_{33} = 15 - (11 + 3) \Rightarrow X_{33} = 1,$

$X_{34} = C_{34} - (u_3 + v_4) \Rightarrow X_{34} = 22 - (11 - 9) \Rightarrow X_{34} = 20.$

Iteration 2

	$\mathbf{V_1 = 0}$	$\mathbf{V_2 = 3}$	$\mathbf{V_3 = 3}$	$\mathbf{V_4 = -9}$	**Supply**
$\mathbf{U_1 = 0}$	(19) 19	(4) 7	**10** 3	(30) 21	**10**
$\mathbf{U_2 = 15}$	**5** 15	(3) 21	**10** 18	**15** 6	**30**
$\mathbf{U_3 = 11}$	**10** 11	**10** 14	(1) 15	(20) 22	**20**
Demand	**15**	**10**	**20**	**15**	**60**

Total transportation cost =

$(15 \times 5) + (11 \times 10) + (14 \times 10) + (3 \times 10) + (18 \times 10) + (6 \times 15) = 625.$

Since all $X_{ij} > 0$ the solution 625 is the optimal transportation cost for the TP.

IBFS by the New IBFS Method

Iteration 1

	D_1	D_2	D_3	D_4	Supply
S_1	16	1	**10** 0	18	**10**
S_2	**5** 9	12	**10** 12	**15** 0	**30**
S_3	**10** 0	**10** 0	4	11	**20**
Demand	**15**	**10**	**20**	**15**	**60**

Table 8: **Selected Minimum Cost in the New IBFS Method Iteration 1**

Source	Minimum cost on Row					
S_1	0	0	0	(0)	X	X
S_2	0	0	0)	9	(9)	(12)
S_3	(0)	(0)	X	X	X	X

Table 8 shows the details of the identified and selected minimum cost in each row for allocation in iteration 1

Iteration 2

	D_1	D_2	D_3	D_4	Supply
S_1	19	7	10 3	21	10
S_2	5 15	21	10 18	15 6	30
S_3	10 11	10 14	15	22	20
Demand	15	10	20	15	60

Total transportation cost =

$(15 \times 5) + (11 \times 10) + (14 \times 10) + (3 \times 10) + (18 \times 10) + (6 \times 15) = 625.$

The IBFS obtained by the new IBFS method is 625.

Optimal Solution by MODI Method

Iteration 1

	$V_1 =$	$V_2 =$	$V_3 =$	$V_4 =$	Supply
$U_1 = 0$	19	7	10 3	21	10
$U_2 =$	5 15	21	10 18	15 6	30
$U_3 =$	10 11	10 14	15	22	20
Demand	15	10	20	15	60

We compute for the value of the dual variables u_i where $i = 1, 2, 3$ and v_j where $j = 1, 2, 3, 4$ using the equation $C_{ij} = u_i + v_j$ for occupied cells and the values of $X_{ij} = C_{ij} - (u_i + v_j)$ for each unoccupied cell. For this, we arbitrarily assign $u_1 = 0$. Thus, we have,

$C_{13}=u_1+v_3 \Rightarrow 0+v_3=3 \Rightarrow v_3=3,$

$C_{23}=u_2+v_3 \Rightarrow u_2\ +3=18 \Rightarrow u_2=15,$

$C_{24}=u_2+v_4 \Rightarrow 15+v_4=6 \Rightarrow v_4=-9,$

$C_{21}=u_2+v_1 \Rightarrow 15+v_1=15 \Rightarrow v_1=0,$

$C_{31}=u_3+v_1 \Rightarrow u_3+0=11 \Rightarrow u_3=11\ ,$

$C_{32}=u_3+v_2 \Rightarrow 11+\ v_2=14 \Rightarrow v_2=3.$

We now compute the values of $X_{ij}=C_{ij}-(u_i+v_j)$ for each unoccupied cell. Thus, we have,

$X_{11}=C_{11}-(u_1+v_1) \Rightarrow X_{11}=19-(0+0) \Rightarrow X_{11}=19,$

$X_{12}=C_{12}-(u_1+v_2) \Rightarrow X_{12}=7-(0+3) \Rightarrow X_{12}=4,$

$X_{14}=C_{14}-\ (u_1+v_4) \Rightarrow X_{14}=21-\ (0-9) \Rightarrow X_{14}=30,$

$X_{22}=C_{22}-\ (u_2+v_2) \Rightarrow X_{22}=21-\ (15+3) \Rightarrow X_{22}=3,$

$X_{33}=C_{33}-\ (u_3+v_3) \Rightarrow X_{33}=15-(11+3) \Rightarrow X_{33}=1,$

$X_{34}=C_{34}-\ (u_3+v_4) \Rightarrow X_{34}=22-(11-9) \Rightarrow X_{34}=20.$

Iteration 2

	$\mathbf{V_1=0}$	$\mathbf{V_2=3}$	$\mathbf{V_3=3}$	$\mathbf{V_4=-9}$	**Supply**
$\mathbf{U_1=0}$	(19) 19	(4) 7	**10** 3	(30) 21	**10**
$\mathbf{U_2=15}$	**5** 15	(3) 21	**10** 18	**15** 6	**30**
$\mathbf{U_3=11}$	**10** 11	**10** 14	(1) 15	(20) 22	**20**
Demand	**15**	**10**	**20**	**15**	**60**

Total transportation cost =

$(15\times 5)+(11\times 10)+(14\times 10)+(3\times 10)+(18\times 10)+(6\times 15)=625.$

Since all $X_{ij}>0$ the solution 625 is the optimal transportation cost for the TP.

Table 9: **Resulted values from Numerical Illustration 2**

Method	IBFS	Optimal Solution	Total Iterations
SAM	640	625	8
VAM	625	625	4
New IBFS Method	625	625	4

Results in Table 9 also shows the comparison of SAM, VAM and the New IBFS method on their performance. It is clearly shown that SAM produced an IBFS value of 640 and the total iterations involved in computing IBFS through to the optimal solution is 6.

Again VAM produced an IBFS value of 625 and the total iterations involved in computing IBFS through to the optimal solution is 4.

Also, the new IBFS method produced an IBFS value of 625 and the total iterations involved in computing IBFS through to the optimal solution is 4. By applying the MODI method, the optimal solution of 625 was obtained for numerical illustration 2.

Chapter Summary

This Chapter provided a comprehensive analysis of the results and findings from the comparison of the New IBFS technique with SAM, and VAM for solving transportation problems. It validates the effectiveness of the new technique and highlights its advantages over existing methods, contributing to the body of knowledge in transportation problem-solving techniques.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Introduction

This chapter presents the summary and the conclusion based on the findings from the study. It also provides recommendations for stakeholders and suggestions for further studies.

Summary

This study developed a new IBFS method for solving TPs from one of the existing methods called SAM. The new IBFS Method was applied to a balanced TP and a comparative analysis was conducted with its IBFS results with that of the SAM, and the VAM. Finally, the MODI method was used to find the optimal solution using the various results from the three IBFS methods. The study adopted literature values for a balanced transportation problem from studies conducted by Reeb and Leavengood (2002) and Somani (2015).

Conclusions

The researcher solved a balanced TP using three IBFS methods, and then examined how well each method performed in bringing down the overall cost of TP using the same problem. These methods gave basic feasible solution of any given TP. Relating to the numerical illustration 1, the IBFS values of the given problem obtained from SAM, VAM and the new IBFS method were 555, 303 and 267 respectively. Applying the MODI method on the IBFS of SAM, VAM and the new IBFS method yielded an optimal solution value of 267. The total iterations involved in computing IBFS through to the optimal solution for SAM, VAM and the new IBFS method were 8, 6 and 4 respectively. In the second illustration (numerical example 2), the IBFS values of the given problem obtained from SAM, VAM and the new IBFS method are 640, 625 and 625 respectively. The corresponding results from the MODI method on the IBFS of SAM, VAM and the new IBFS method yielded an optimal solution value of 625. The total iterations involved in computing IBFS through to the optimal solution for SAM, VAM and the new IBFS method were 6, 4 and 4 respectively. From the comparison above, it is obvious that, the new IBFS method produces the best IBFS value and in lesser iterations the optimal solutions are obtained. The VAM is known to produce the best IBFS value but this study shows that the new IBFS method proposed in the study produces an IBFS value which is better than the IBFS value obtained by VAM and SAM. The study also revealed that the IBFS value produced by the new method is either the same as the optimal solution value or very close to the optimal solution value. It can be concluded that the new IBFS Method (which was an improvement on SAM) produces better IBFS results than the original SAM and VAM. In some cases, the new IBFS method produces an IBFS value which is the same as VAM.

Recommendations

From the results the researcher recommends that the new IBFS Model should be adopted by companies and businesses for their transportation problem planning.

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