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VALUE ALIGNMENT IN MATHEMATICS LEARNING BETWEEN TRAINEE TEACHERS AND THEIR TUTORS IN THE COLLEGES OF EDUCATION IN GHANA

BY

DANIEL ASHONG

Thesis submitted to the Department of Mathematics and ICT Education, Faculty of Science and Technology Education, College of Education Studies, University of Cape Coast, in partial fulfilment of the requirements for the award of Doctor of Philosophy degree in Mathematics Education

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DECLARATION

Candidate's Declaration

I hereby declare that this thesis is the result of my own original research and that no part of it has been presented for another degree in this university or elsewhere.

E. Date 27-03-2023

Name: Daniel Ashong

Supervisor's Declaration

I hereby declare that the preparation and presentation of the thesis were supervised in accordance with the guidelines on supervision of thesis laid down by the University of Cape Coast.

Supervisor's Signature Date 27 - 03 - 2023 Name: Prof. Ernest Kofi Davis

VOBIS

ABSTRACT

This study explored the value alignment in mathematics learning between trainee teachers and their tutors in the colleges of education in Ghana. A sequential explanatory mixed-methods design was employed to explore the attributes 34 mathematics tutors and their 1,050 students' value in mathematics learning. These participants were sampled using purposive and systematic sampling techniques. The data were collected through the use of questionnaires and semi-structured focus group interview guides. The data collected were analysed using descriptive and inferential statistics. The results showed that the trainee teachers valued understanding, collaboration, strategies, accuracy, fluency, relevance, mystery and learning technologies; while their mathematics tutors valued understanding, strategies, mastery, problem-solving and fun. Two of these attributes valued (understanding and strategies) were observed to be common for the two groups. It was further revealed that the most prioritised attribute valued by both trainee teachers and their tutors was understanding. Moreover, though all the two groups accepted that valuing was an important component of the trainee teachers' mathematics learning, there was generally no relationship between the attributes valued by the two groups. Again, it was found that there was no significant difference in the levels of valuing between trainee teachers and their tutors on the two common attributes. The reasons for the observed attributes valued in mathematics learning were provided by both groups of participants. Finally, the study recommends that though there is an existing course that introduces trainee teachers to the value concept, there is the need to pay deeper attention to values in mathematics education by instituting it as a course on its own at the colleges of education.

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KEY WORDS

Colleges of Education Mathematical Values Valuing in Mathematics Learning Mathematics Tutors

Trainee Teachers



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DEDICATION

To my mother Madam Florence Yaa Boadi, my wife Dr. (Mrs.) Helena Naa Korkor Ashong, and my children Michelle, Kenneth, David, and Justin.



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CHAPTER ONE

INTRODUCTION

Any human society or institution exists based on its values and as such values are considered to be crucial in education, and more importantly in mathematics education. Values are also an important underlying factor when it comes to curriculum development as every curriculum is pivoted on values as Seah, Anderson, Bishop, and Clarkson (2016) noted, and reiterated by Howson, Keitel, and Kilpatrick (2008) that to see contrasts in curriculum development one has to look for the style and underlying values. This means that every educational curriculum is differentiated from another by the underlying values defined in a specific context. Seah (2002) confirmed that values are the most important element in raising the quality of mathematics teaching-learning. Besides, Davis, Seah, Howard, and Wilmot (2021) in their recent survey argued that valuing constitutes an important aspect of mathematics pedagogy and that, it has an association with students' learning outcomes.

However, developed and developing countries are finding new and better ways of improving educational standards. Improved educational standards depend mostly on the quality of interactions that takes place in the classroom environment. One such new way of improving the classroom learning experiences of students is teaching through values. This value approach allows students to learn based on what they consider important in their classroom learning. This approach has received much attention in countries like Australia, China, Singapore and Hong Kong.

Therefore, the need for research study in the area of values in Ghana is long overdue, especially at the tertiary level since the problem of poor performance in mathematics has engaged the attention of stakeholders in education in the country for some time. Besides, research indicates that the performance of students in Sub-Sahara Africa appears to be the least among their counterparts elsewhere (Madosi, Ramdhany & Spangenberg, 2020; Ngware, Ciera, Musyoka & Oketch, 2015) and TIMSS (2011, 2012) results also suggested same. It is important to highlight that these students whose performance is seen to be weak, if not poor are the same students who are admitted into tertiary institutions like the colleges of education in the country and consequently, become professional teachers who teach at the basic level. Hence, there is an urgent need for an empirical research study that will explore values as a means to improve education in Africa, especially in Ghana. It is in this direction that this current study is being conducted to explore values at the colleges of education level in Ghana.

This introductory chapter therefore will set the context for the whole study by providing the background, statement of the problem, the purpose of the study, research objectives, research questions and hypothesis, significance of the study, delimitations and limitations.

Background to the Study

For every educational system to succeed in providing quality education to its learners, teachers and their roles cannot be ignored. Teachers can bring up students to become functional members of society through the teaching of values. Besides, teachers are the ones who have direct contact with the students in the school and classroom. Again, teachers are the pivot of the curriculum implementation in every educational system and is based on the important roles

teachers play in the delivery of quality mathematics education. In Ghana, there was peoples' representative assembly on the best ways of improving teacher education in the country, which led to the formation of a committee to come up with a report that addresses the difficulties we as a nation were encountering in the educational system. Hence, Anamuah-Mensah and Benneh committee was formed.

Based on the Anamuah-Mensah and Benneh report (NTECF, 2017) formulated through a public consultative decision to improve the quality of teaching-learning at the basic level a new curriculum framework was designed, and as a result in 2018, all 48 colleges of education admitted the first batch of students to pursue a 4-year Bachelor of Education (B. Ed) degree programmes. This was intended to phase out the 3-year diploma programmes ran by the colleges of education and to expand the curriculum to equip trainee teachers to acquire the needed knowledge, skills, and competences required to improve performance at the basic school level. Currently, there are 48 public colleges of education in Ghana. The first batch of the current (B. Ed.) programme admitted in the 2018/2019 academic year are currently in their third year in the 2020/2021 academic year. These particular third-years are being mentored, examined, and certificated based on the UCC bachelor of education curriculum. This batch of trainee teachers were selected for this study due to the homogeneity in characteristics that they share across the colleges. It is also important to note that subsequent batches (i.e., 2019/2020 academic year upwards) of trainee teachers admitted were shared through college affiliations among five public universities (University of Cape Coast (UCC), University of Education, Winneba (UEW), University of Ghana (UG), Kwame Nkrumah University of Science and Technology (KNUST) and University of Development Studies

(UDS)) for a curriculum designed, mentorship, examination, and certification. In such a case, there might be some sort of differences based on the practices of the affiliated universities. Given these affiliations, non-teacher training institutions such as UG, UDS, and KNUST have started various teacher training programmes such as Bachelor of Education and Master of Education programmes to minimize such differences.

Given the developmental stages at the colleges of education, it is undeniable fact that the programmes for training teachers in Ghana will continue to be reviewed to meet the needs and aspirations (Ronde, 2010) of society and Ghana as a whole, as society keeps on changing, more especially in technological advancement. The curriculum review is therefore planned to incorporate the societal values expected (Offorma, 2016) in its individuals for them to become functional members of society.

The programmes for the new Bachelor of education degree were designed to train competent teachers who would teach at the basic levels of Ghana's educational system. This new programme trains Early Childhood, Primary and Junior High School (JHS) teachers. In the case of the mathematics curriculum structure, the trainee teachers are put under core mathematics and elective mathematics. The core mathematics trainee teachers are those taking the general programmes (non-mathematics elective). While the mathematics elective trainee teachers are those taking mathematics and ICT programmes.

The mathematics courses designed for core mathematics trainee teachers include; Elementary Algebra (EBS 101) for first-year first-semester; Geometry and Trigonometry (EBS 143) for first-year second-semester; Methods of teaching primary school mathematics (EBS 322) for third-year first-semester and Statistics & Probability I (EBS 350) third-year second-semester. The

courses for the fourth year have not been provided and therefore, the researcher cannot account for such courses at the moment. The content area covered in the three mathematics content courses above includes Elementary Algebra (EBS 101) - Binary operations; Application of Sets; Ratio, Proportions, Percentages and Rates; Number bases and their applications $(+, -, \times, \div)$; Indices and Logarithms; Relations and Functions; Algebraic expressions and Equations (Linear and Quadratic); and Linear Inequalities. Geometry & Trigonometry (EBS 143) – Polygons; Geometrical constructions; Circles theorems; Application of Pythagoras theorem to Measurement of 2-D and 3-D Shapes; Co-ordinate geometry; Trigonometric functions including drawing graphs and solving simple trigonometric equations; and Vectors and Movement geometry. For Statistics & Probability I (EBS 350) - Collection, and organization of data; Representation, analysis and interpretation of data; Measures of central tendency; Measures of dispersion; Experiments and probability of simple events; and Combined events, tree diagrams, and conditional probability (IoE, 2017).

The mathematics courses for the elective mathematics trainee teachers include; Elementary Geometry (EBS 145) and College Algebra (EBS 102) for first-year first-semester; College Geometry (EBS 124) and Trigonometry (EBS 169) for first-year second-semester; Nature of Mathematics (EBS 285) and Psychological Basis of Teaching-learning Mathematics (EBS 277) for secondyear first-semester; Curriculum Studies in Mathematics (EBS 243) and Algebraic Thinking (EBS 201) for second-year second-semester; Methods of Teaching JHS Mathematics (EBS 356J), Pedagogical Content Knowledge in Mathematics (EBS 371) and Calculus (EBS 301J) for third-year first-semester; Statistics & Probability II (EBS 351) for third-year second-semester. The

fourth-year mathematics courses could not be reported for because they have not been provided by the University of Cape Coast (mentoring institution). Hence, the core mathematics trainee teachers take 4 courses at least up to the third-year whiles the elective mathematics trainee teachers take 12 mathematics courses at least up to the third year. As a result, the core mathematics trainee teachers are engaged less in mathematics as compared to their colleagues taken mathematics as their elective subject and might not acquire the needed knowledge, skills, and values required to teach mathematics at the foundation (Primary) level. It is important to note that these core mathematics trainee teachers are expected to teach all subjects at the Primary School level including mathematics. This means for an individual to develop the right values, such a person must be introduced to the right mathematical tools and activities at the foundation stage or primary school level. This makes the role of teachers at this level very important so far as the development of values is concerned.

Values taught in mathematics classrooms have been grouped into three forms. These are general educational values, mathematical values, and mathematics education values (Bishop, 1996). General educational values are not subject-specific concepts but desired attributes that are expected from every individual who has gone through formal education. Such values include; good behavior, honesty, respect, humanity, and humility. Mathematical values are the values that mirror the type of mathematical knowledge and are associated with western mathematics. However, mathematics education values connect the pedagogical approaches and the appropriate teaching practices (Madosi, Spangenberg & Ramdhany, 2020).

The values observed to be constructed through western mathematics were categorized by Bishop (2008, 1988) and Dede (2006) as three

complementary pairs of mathematical values; ideological (rationalismobjectism), sentimental (control-progress), and sociological (opennessmystery). According to them, rationalism value is presented as a deductive logic that concerns the correctness and explanations in mathematics. Objectism shows the nature, objects, and symbols which are used to concretize mathematical ideas considered to be abstract language. Value control is demonstrated through the application of mathematical ideas to social or environmental problems. Whiles, the progress value is invested in the growth of mathematical ideas anytime and used in other fields of study. The value of openness lies in the fact that mathematical knowledge and truth are open to accessibility and verification by everybody concerned with mathematics. Finally, the power of mystery value is provided through the wonders and mystiques found and experienced in mathematics.

These mathematical values have revolutionized value research in mathematics education around the world which has also impacted greatly on theory and practice. Value research has been conducted in the area of students valuing, teachers valuing, relationships between teachers and students valuing and, values conveyed by mathematics curriculum materials such as textbooks.

In the area of student valuing, research studies have been done in countries such as China, Singapore, Australia, Japan, Turkey, Germany, New Zealand, Bruni, South Africa and Hong Kong by researchers such as Abdullah and Leung (2019); Dede (2006); Hill, Hunter, and Hunter (2019); Kalogeropoulos and Clarkson (2019); Madosi, Ramdhany, and Spangenberg (2020); Seah (2011b); etc. In Ghana, the limited mathematics value research conducted were all identified to be on students' valuing at the pre-tertiary level (Primary, Junior High School and Senior High School); Davis, Carr, and

Ampadu (2019); Davis, Howard, Seah and Wilmot (2021); and Seah, Davis and Carr (2017b). This means research on students' values in mathematics is very young and promising in the country and needs to be explored further. Besides, there is no known mathematics value research conducted on students at the tertiary level especially, at the colleges of education.

For mathematics teachers valuing, few research studies have been identified around the world, for instance, under the VAMP Project, Bishop, Clarkson, Fitzsimons, and Seah (2000) investigated and documented mathematics teachers' understanding of their own intended and implemented values in China and Australia. Dede (2012), also investigated values in mathematics teaching by comparing Turkish and German mathematics teachers. In similar studies, Aktas and Argun (2018) revealed secondary school teachers' values as reflected in their classroom practices. Finally, the values of Japanese mathematics teachers were investigated by Corey and Ninomiya (2019). Considering how important the values held by teachers are, it is therefore surprising that no research study was identified on teachers' valuing in Africa. This means a big gap exists in the teaching and learning of mathematics as teachers' understanding of values impacts greatly on the learning experiences offered to their learners in the classroom.

Moreover, few studies have also reported on the relationship between teachers and their students' valuing. Kalogeropoulos and Clarkson (2019) reported how teachers' and students' roles of engagement and values were aligned in the mathematics classroom. Besides, Anderson and Österling (2019) investigated democratic actions in school mathematics and the dilemma of conflicting values between teachers and their students in Swedish Schools. Given these studies, it makes sense to conclude that limited studies have been

conducted that report on the relationship or alignment between students and their teachers valuing across the world and such studies were not identified in Africa, not to mention Ghana.

In countries like China and Australia, a study reported on values conveyed in mathematics curriculum (curriculum projected values) materials such as textbooks. The curriculum projected values are also known to influence values exhibited by both students and their teachers (Seah, Anderson, Bishop & Clarkson, 2016).

The context (Colleges of Education) is very important because these institutions train professional teachers to teach at the basic schools in the country and therefore, the trainee teachers must possess the right values which would be impacted to their prospective pupils after their professional training. This study is specifically focused on trainee teachers taking mathematics as a core subject because they are being trained and are expected to teach all subjects including mathematics at the foundation levels in the country after their professional training. This makes such trainee teachers agents of mathematics education, and as such possessing the right mathematics education values which they would impact on their prospective students is a key to improving the learning experiences of our pupils.

Statement of the Problem

In Ghana, the search for new strategies for improving the teachinglearning of mathematics (Davis *et al.*, 2021) has not been successful. The problem of poor performance in mathematics has persisted for a long time now at the colleges of education. The evidence of this assertion is provided in the trend analysis presented below which gives the situational picture of the problem at the colleges of education in Ghana, particularly among the third-year

trainee teachers. It is important to emphasize that the third-year trainee teachers' performance in all subjects are assessed based on the University of Cape Coast grading system which is provided as a letter grade and its' corresponding Mark (%) range as follows; A (80 - 100); B+ (75 - 79); B (70 - 74); C+ (65 - 69); C (60 - 64); D+ (55 - 59); D (50 - 54); and E (0 - 49). The results for the trainee teachers (IoE, 2019, 2021) from the five selected colleges in Ahafo, Bono and Bono East Regions of Ghana offering Mathematics as a core subject in Elementary Algebra (EBS 101), Geometry and Trigonometry (EBS 143) and Statistics and Probability I (EBS 350), first-year first and second semesters, and third-year second-semester content courses respectively are presented in Table 1 below.

It should be noted that in some cases the percentages in Table 1 do not add up to 100 percent and the difference was caused by the number of candidates who had incomplete (IC) in their results or were "absent" from the examination but had registered for these courses.

College	Year	Total Number of Trainee Teachers for Semester	% that had A to C+	% that had C to D	% that had E	Total Number of Trainee Teachers for	% that had A to C+	% that had C to D	% that had E	Total % that had E for the Academic Year
		One		-		Semester Two				
А	2019	215	12.1	27.0	61.0	212	3.30	48.1	48.1	54.6
	2021	-	-	-	-	134	12.9	35.7	51.4	51.4
В	2019	443	32.3	61.6	5.6	439	21.7	70.6	8.2	6.9
	2021	-	-	-	-	216	23.5	64.5	12.0	12.0
	2019	197	15.0	33.2	51.9	197	8.1	50.8	41.1	45.2
С	2021	-	-	-0	-	85	41.1	46.7	12.2	12.2
D	2019	229	15.7	48.9	35.4	227	16.3	60.4	23.4	29.4
	2021	-	-	-18	-	220	34.3	57.5	8.2	8.2
Е	2019	396	24.8	52.5	22.7	390	32.3	60.8	6.9	14.9
E	2021	-	1-	-	-	380	38.6	54.5	6.90	6.9

Table 1: Mathematics Performance of Trainee Teachers in 2018/2019and 2020/2021 Academic Years (First-year and Third-year)

SOURCE: Institute of Education (2019; 2021).

It can be seen from Table 1, that the performance of third-year Trainee teachers currently and when they were in the first year were very weak as the majority fell within the grade C to E bracket for all the three end-of-semester examinations. The corresponding grades (C to E) percentages were observed as; 87.91 percent (i.e., Sum of percentages of C to D and E), 67.72 percent, 85.03 percent, 84.28 percent, and 75.25 percent respectively for colleges A, B, C, D and E for the first semester. Besides, the second semester was not different as a percentage for grades C to E were recorded as 96.70 percent, 78.35 percent, 91.88 percent, 83.70 percent, and 67.70 percent. Hence, the corresponding

percentages for grades A to C+ are far less than that of grades C to E, suggesting generally very weak mathematics performance. This conclusion was arrived at based on the fact that these three content mathematics core courses (Elementary Algebra, Geometry & Trigonometry and Statistics & Probability I) are the main mathematics content courses taken by these mathematics core trainee teachers at the colleges of education. Furthermore, the percentages for grade E (fail) were significant as compared to the total number of trainee teachers who sat for the three mathematics courses except in college B where the number of failures seems to be less as compared to the other four colleges.

Also, according to Course coordinators' reports (IoE, 2019) for Elementary Algebra (EBS 101) course analyzed above indicated that trainee teachers' performance was below average. The report explained that in a sample of 9.804 trainee teachers who sat for the examination in the first semester 5.643 candidates representing 57.56 percent scored below 50 percent. The report further indicated that trainee teachers showed weak content knowledge in areas such as formulating appropriate algebraic equations from given word statements, simplifying mathematical problems in fractional forms, and definition of functions and using them to distinguish functions from relations. The report further recommended that Tutors should expose trainee teachers to real-life or application problems, encourage trainee teachers to pay attention to weak content areas identified, and should teach trainee teachers how to present their solutions including the use of correct notations. From the report, mathematics tutors attributed the weak performance of trainee teachers to difficulty in determining coverage as far as topics are concerned and the weak mathematics background of trainee teachers.

For the Geometry and Trigonometry (EBS 143), course coordinators' report revealed that in a sample of 6,029 trainee teachers, 4,426 representing 73.4 percent scored less than 50 percent of the marks. Hence, there was a clear indication that performance was below average. Besides, trainee teachers showed weak knowledge in the following content areas; applying Pythagoras' Theorem to determine whether the given dimensions are those of a named triangle, applying constructions ideas in solving everyday problems such as the best location of a project, and slope of a straight line. Again, the report recommended that tutors should expose trainee teachers to real-life/application problems, encourage trainee teachers to pay attention to weak content areas identified, and tutors to teach trainee teachers how to prove mathematical statements and provide justification for their answers (Course Co-coordinator's Report, IoE, 2019). In line with the content areas where trainee teachers showed weakness in solving real-life/application problems and their inability to use the right mathematical notations are areas that need serious attention. Since the essence of education is to prepare individuals to be able to solve their everyday problems using the appropriate mathematical tools (such as symbols and notations) to fit well in their respective societies. If at the tertiary level trainee teachers are not able to perform such mathematical activities, what would be the situation like for their prospective learners?

For Statistics and Probability I (EBS 350), the assessment coordinators' report from the institute of education for all colleges of education across the country suggested that general performance was low. It was additionally stated that candidates who scored grade D+ or D were 879 (26.49 %), with 353 (10.64 %) scoring grade E out of 2,059 total candidates who sat for the course (Assessment Coordinators' Report, IoE, 2021).

The performance provided in Table 1 also appears confirm course and assessment coordinators' reports on both first-year and third-year mathematics content courses for the core mathematics trainee teachers. The trainee teachers' weak performance in mathematics at the colleges of education perhaps could be attributed to their weak foundation carried on from the Senior High Schools (SHS).

Though there has been some improvement in students' performance in mathematics at the Senior High School level in recent times, such improvement observed were not significant as reflected in students' performance in the Senior High School Certificate Examination results (passes from A1 to C6 in percentages) 42.73%, 38.33%, 65.31%, 65.71% (WAEC, 2017; 2018; 2019; 2020) respectively. These students from the Senior High Schools are consequently, the ones admitted into the colleges of education institutions in the country. This means the poor foundation in mathematics is curried to the tertiary (colleges of education) level. This problem of poor performance in mathematics, therefore, becomes a continuum at the colleges of education (tertiary) level.

It should therefore be a concern to every individual involved in mathematics education to find out the possible causes and strategies to improve performance at the colleges of education. It is also a known fact that to improve performance, there must be an improvement in the effectiveness of mathematics learning. This means if learning is not effective it manifests in learners' performance. Hence, providing the learners with appropriate learning experiences improves their performance. Consequently, the weak performance of trainee teachers as illustrated in the trend analysis and coordinator's reports

can largely be associated with the ineffectiveness of the learning processes that exist in mathematics classrooms.

From the perspective of mathematics education research, different studies have been conducted and interventions implemented at all levels of the educational system in Ghana. Such research studies conducted at the colleges of education were geared towards improving the performance of trainee teachers. Some of these studies include Eshun and Sokpe (2009) who evaluated the effect of training on the mathematics knowledge of trainee teachers in Ghana. In another study, Kumah, Akpandja, Djonda, and Kumi (2016) investigated the factors influencing trainee teachers' mathematics achievement at some colleges of education in the Volta Region of Ghana and proposed that achievement in mathematics usually depends on how trainee teachers can construct mathematics concepts and their attitude towards the subject (mathematics). Also, Asiedu (2020) after exploring the causes of pre-service teachers' poor performance in algebra at the colleges of education identified attitudes and motivation, and lesson presentation on the part of the mathematics tutors as the factors that influence performance at the colleges of education. Though these studies have contributed to a better understanding of the problem of poor performance in mathematics, the problem still persists at the colleges of education. Against these that this study is being conducted to improve the quality of mathematics learning at the colleges.

For the problem of poor performance in mathematics, Peng and Nyroos (2012) argued that one factor that perhaps has the potential to impact performance in mathematics is many teachers' unawareness of learners' values in mathematics education. These situations of unsuccessful significant improvement in the implementation of research interventions and proposals in

mathematics education may perhaps be accounted for the lack of attention and seriousness on the part of mathematics educators towards the development of mathematics education values among tutors at the colleges of education. As Seah (2013) proposed, if teachers can incorporate what learners value in the learning of mathematics, it would improve conceptual understanding, hence minimizing learners' learning difficulties. This means a better understanding of learners' values could help teachers incorporate values into their pedagogies to teach mathematics which may enhance learners' performance in mathematics (Madosi *et al.*, 2020).

It is therefore important to note that the few research studies conducted in the area of values in Ghana were all focused on pre-tertiary (Primary to SHS) students. Such studies also identified students' values from only the perspective of students. Hence, those studies did not report from the perspective of teachers, the relationship between students' and their teachers' valuing in mathematics learning. It is also a fact that no known mathematics value research study has been conducted at the tertiary level in Ghana. Besides, such study in Ghana has the potential of starting meaningful educational research debate in the field of mathematics, especially at the tertiary level.

It is against this background that this study is being carried out to explore values in trainee teachers' mathematics learning from the perspective of their tutors. In this sense, the colleges of education are important due to their strategic position in the development of education in Ghana, as these institutions train teachers to teach at the basic school level. By this, the trainee teachers would be able to incorporate the right values in their lessons and improve the performance of their prospective students. Also, a better understanding of trainee teachers' values could assist tutors to incorporate values in their pedagogies to teach

mathematics, which may lead to better performance in mathematics. Nevertheless, there is no known study conducted on the relationship (alignment) between values espoused by both trainee teachers and their tutors in mathematics learning at the colleges of education in Ghana. To fill these gaps that exist in the literature space, this study is being designed to explore the value alignment in mathematics learning between trainee teachers and their tutors at the colleges of education in Ghana.

Purpose of the Study

The purpose of this study was to explore what trainee teachers and their tutors value in mathematics learning and, it also sought to ascertain whether what tutors value in their students' mathematics learning aligns with what trainee teachers value at the colleges of education in the Ahafo, Bono and Bono East Regions of Ghana.

Research questions

The following research questions were formulated to guide the study:

- 1. What do trainee teachers value in their mathematics learning at the colleges of education?
- 2. What do tutors value in their students' mathematics learning at the colleges of education?
- 3. What is the relationship between trainee teachers and their tutors' valuing in mathematics learning?
- 4. What are the differences, if any, in valuing between trainee teachers and their mathematics tutors?

Research hypothesis

The hypothesis below was formulated to address research question three (3).

 H_0 : There is no significant difference between trainee teachers and their tutors' valuing in mathematics learning at the selected colleges of education.

Significance of the Study

This study aims at exploring what trainee teachers and their tutors value in mathematics learning and, it also sought to ascertain whether what tutors value in their students' mathematics learning aligns with what trainee teachers value at the colleges of education in the Ahafo, Bono and Bono East Regions of Ghana.

This study is significant to the researcher in that it will help him improve his skills in educational research and classroom practice as far as teachinglearning of mathematics is concerned. Besides, the study will expose the researcher to understand students' valuing in mathematics learning in the classroom and take action appropriately to improve their learning.

This study also brings to lamplight the importance of values in the teaching-learning process, more specifically in the learning of mathematics. That, based on the findings of this study both tutors and trainee teachers will realize the importance of values to both individuals and society as a whole. This study, therefore, will help mathematics tutors plan and design strategies that put great emphasis on appropriate mathematical pedagogy and practices that promote the development of various attributes of valuing in the classroom.

The results from the study will also bring to light what trainee teachers value in their mathematics learning. This information will enable tutors to be aware of what trainee teachers value in mathematics learning. Tutors' awareness

of trainee teachers valuing will enable them to come out with interventions that will allow trainee teachers to acquire the appropriate values where necessary and to pass on the same to their prospective students. The results of value alignment will also enable both tutors and trainee teachers to be aware of each other's values and engage in value negotiations to ensure effective classroom discourse.

This study is intended to serve as the springboard for more attention to be geared towards the need to pay attention to values in mathematics learning at the colleges of education and the entire educational system in the country. As such, the results of the study will influence policy formulation in mathematics education. Again, the results from this study will help experts in the development of mathematics curricula at all levels of education, especially at the colleges, to focus on the needed attributes of values the curriculum must project to train qualified mathematics education graduates in the country.

Finally, the findings of the study will add to the literature on values in mathematics learning across the globe especially, at the tertiary level. Many value studies that have been carried out have focused on the pre-tertiary level and hence, the results of this study will serve as the source for literature on values at the tertiary level.

Delimitations

This research study was limited to the subject of mathematics specifically and the concept of values concerning mathematics education (Seah, 2011a). The concept of mathematical values was based on the three pairs of complementary values conceptualized by Bishop (1988) and Dede (2006). The definitions of mathematical values and mathematics education values were adapted to reflect the context of colleges of education in Ghana. These sources

were selected for the conceptualization of value based on the fact that other studies in the area of values in mathematics and mathematics education have drawn from the same sources. Also, five identical colleges of education in the formerly Brong-Ahafo Region and currently Ahafo, Bono and Bono East Regions of Ghana were used for the study. The mathematics tutors and trainee teachers taking mathematics as a core subject in the selected colleges of education were purposively selected for the study.

Limitations

The samples of the colleges of education used in this study were all coeducational institutions and therefore, did not include single-sex colleges. However, in the national picture, there are single-sex colleges of education which when included could have affected the results of this study. This assertion was based on the fact that studies have shown that the attitudes of students in co-educational institutions seem to differ from that of students from single-sex institutions.

The results were, therefore, generalized only to the selected colleges of education in which the study was conducted. Besides, the results of the study were restricted to mathematical and mathematics educational values in the context of mathematics learning at the colleges of education in Ghana.

Definition of Terms

Mathematics Tutor: Refers to an instructor/lecturer teaching mathematics at the colleges of education in Ghana (My interpretation for this study).

Trainee Teachers: Refers to prospective teachers who are undergoing teacher preparatory training at the colleges of education to teach at the basic schools in the near future. In this study, these are prospective teachers who are being

trained at the colleges of education to teach mathematics at the basic schools (Akyeampong, Pryor, & Ampiah, 2006).

Mathematical Values: Refers to attributes of importance and worth that are internalized by an individual that provides him or her with the desire and determination to pursue any course of action chosen in mathematics (Bishop, 1988).

Values in Mathematics learning: Any learner activity or practice that leads to the construction of mathematical knowledge in the classroom (My interpretation for this study).

Least Prioritised Values: Refer to the value(s) identified based on the quantitative data analysis of the study to be less demonstrated by the trainee teachers and their tutors (My interpretation for this study).

Core Mathematics: Mathematics courses that may be taken by all trainee teachers who are not doing mathematics as a major or elective subject (My interpretation in this study).

Organization of the Study

This thesis has been organized into five main chapters, namely chapters one to five.

Chapter One was the introduction of this study and consisted of the background to the study, statement of the problem, and objective of the study are provided. Besides, research questions, the hypothesis of the study, delimitations of the study, limitations, the significance of the study, the definition of terms, and the organization of the study are also illustrated.

Chapter Two presented the related literature reviewed concerning values in the teaching-learning of mathematics for this study. The review began by providing a conceptual review of values and conceptual framework for the

study. A review of variables as sentimental, ideological, and sociological values. Other headings that were reviewed included teachers' valuing; students' valuing; high and least prioritised values; value-centered mathematics curriculum, empirical review and finally, the summary of the chapter.

Chapter Three described the processes involved in conducting the study. The chapter specifically looks at the research design, the study area, population, participants, the sampling procedure, data collection instrument, data collection procedures and data processing and analysis. The chapter finally discusses the validity and reliability of the data collection instruments.

Chapter Four presents the results from the analysis and findings, statements of observations, including statistical tables. Discussions on the findings based on the research questions and hypothesis. This chapter ends with a chapter summary.

Chapter five was Summary, Conclusions, and Recommendations. It gives a summary of the key findings and conclusions. Finally, recommendations based on the outcomes of the study and suggestions for further studies are provided.

CHAPTER TWO

LITERATURE REVIEW

Overview

The literature review was organized under two main headings namely a conceptual review of values and a review of empirical studies. The conceptual framework for the study, review of mathematical values (Sentimental, Ideological and Sociological values), mathematics education values, teachers' valuing, students' valuing, curriculum projected values, high and least prioritised values and value-centered mathematics curriculum were presented under conceptual review. Followed by an empirical review and finally, the summary of the chapter.

Conceptual Review of Values

The concept of values has been defined by different authors from various perspectives as presented in this section. Values were seen as the choices about what one regards as important, such as freedom to choose, having alternatives to choose from, as well as choosing after keen thought of the results of each option, prizing and appreciating (Raths, Harmin, & Simon, 1987). Bishop (1988) also argued that value is a belief one holds deeply, even to the point of cherishing, and acts upon and that one's value is hidden and therefore not apparent. In general view, Halstead and Taylor (2000) defined values as the principles and fundamental convictions which act as a general guide to behaviour, the standards by which particular actions are judged as good or desirable. In this sense, the value was pictured as the guide to judge one's action in terms of appropriate or inappropriate and hence, being more judgmental. Values are preferences of individuals related to their standards of thoughts and actions that are important and worthwhile to themselves (Chin & Lin, 2001).

Again, Dede (2006) also described values as personal choices considering the worth or importance of behaviour or idea, or general aims that are adopted or followed by an individual as a member of society. Besides, coverage of values is unlimited as it cuts across different disciplines and fields. Lastly, values were visualized by Cai and Garber (2012) to be continuous principles in similar conditions leading to the preferences and decisions taken into consideration of the significance, worth and demand of an object, idea, notion, or behaviour.

Based on the above definitions presented, the idea synonymous with the concept of values is the provision of convictions that enable individuals to judge the importance and worth of action and that values are projected as the bases upon which decisions are made. This means understanding why individuals make certain decisions is to understand the concept of values. In other words, the choices that we make in our everyday life are influenced by the accumulated values developed based on our past experiences. Hence, values activate an individual's belief and attitudinal domains (cognitive, affective and conation) associated with judging and decision-making to take a course of action. So, the actions of judging the importance or worth and decision making require conscious coordination between cognitive and affective (Davis et al., 2021) domains that generate the "I want to do" mindset. Studies in the area of values in the teaching-learning of mathematics conceptualized values as cognitive and affective processes leaving the conative aspect (Seah, 2019) which gives the course of action to be repeated many times over a period. In the mathematics classroom teachers and students are constantly engaged in judging and decisionmaking (Clarkson, 2019) concerning the selection of appropriate curriculum materials, assessment techniques to employ, pedagogical activities, resources, concepts, procedures, and manipulative materials that suit students best in

achieving the set goals. This means values are an integral part of any successful classroom discourse at all levels of education. It is also a fact that teachers' understanding of the value concept and their students' values would assist them to capitalize on the everyday knowledge of the students and associating such knowledge with the classroom knowledge to ensure effective teaching-learning processes. Thus, the various definitions demonstrate more intersecting ideas than differences when contextualized in the mathematics classroom of which colleges of education are no exception. Given these definitions, values were considered in this study as attributes of importance and worth that are internalized by trainee teachers that provide them with the desire and determination to pursue any course of action chosen in the learning process of mathematics at the colleges of education level in Ghana.

Conceptual Framework of the Study

The variables were contextualized and operationalized in the conceptual framework of the study to reflect colleges of education and the Ghanaian situation. The conceptual framework that directed the focus of the study is illustrated in *Figure 1* below. The conceptual framework in its first part shows the directional relationships between the variables (Sentimental, Ideological and Sociological values) and their sub-constructs (*control-progress, rationalism-objectism and openness-mystery*) as conceptualized by Bishop (1988). The second part of the framework was based on Seah and Bishop (2000)'s and Dede (2006)'s conceptualization of the five-pairs of mathematics educational values (*formalistic view – activist view, instrumental learning – relational learning, relevance – theoretical knowledge, accessibility – special, evaluating – reasoning*). These dimensions are deep-rooted in the mathematics educational values which connect the pedagogical approaches to the teaching-learning

practices that take place in the mathematics classroom. Based on the two concepts, trainee teachers and their mathematics tutors' values were identified. The later part of the framework further evaluates the interaction between values espoused by trainee teachers in their mathematics learning, and values espoused by tutors in their students' mathematics learning as presented in Figure 1 below.

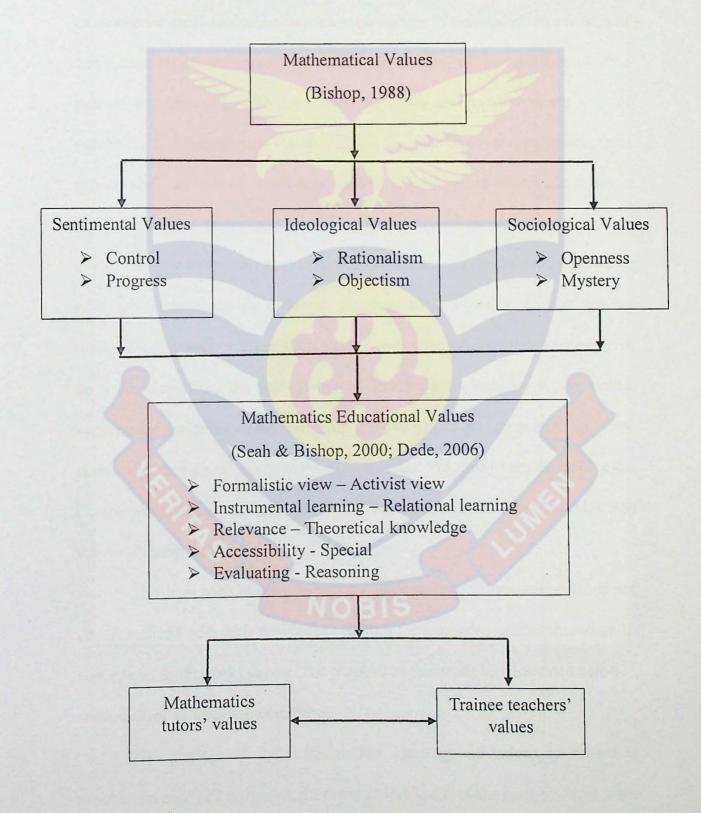


Figure 1: Conceptual Framework of the Study.

Mathematical values

Value in mathematics was presented by Seah and Bishop (2001) as one's internalization, cognitisation, and de-contextualization of affect variables (such as beliefs and attitudes) in one's socio-cultural context. They are inculcated through the nature of mathematics and one's experience in their socio-cultural environment such as the mathematics classroom. These values form part of one's ongoing developmental personal value system, which equips one with a pair of cognitive and affective lenses to shape and modify one's way of perceiving and interpreting the world, and to guide one's choice of course of action. They also influence the development of one's other beliefs and needs in life. Madosi et al. (2020) also referred to mathematical values as the values that mirror the type of mathematical knowledge and are shaped by mathematicians who have grown up in different cultures, such as rules, conjectures, or inferences. Besides, mathematical values were presented as those which have been developed as the knowledge of mathematics has developed within westernized cultures. The mathematical values were presented in three complementary pairs: Sentimental (Control and Progress) values, Ideological (Objectism and Rationalism) values and Sociological (Openness and Mystery) values (Bishop, 2008, 1988).

The three pairs of complementary mathematical values were defined and contextualized with their appropriate themes for measurement (as shown in the conceptual framework) to meet the purpose of this study as illustrated below.

Rationalism as ideological value

The genesis of the mathematical ideology of values is shown in rationalism and for that matter, for every individual to make mathematical sense depends on how logical that person's speech or argument has been presented.

Hence, mathematical logic and reasoning are manifested in the rationality of the use of mathematical objects. Rationalism is the demonstration of authority and power of mathematics and therefore has been considered to be the heart of mathematics (Bishop, 2008, 1988). The rationalism of mathematics controls the systematic, arrangement, patterns, orderliness, and inductive reasoning which shape the human mind and besides allow mathematicians to make constructive arguments (Anderson & Österling, 2019). The mathematical explanation which is characterized by logic, completeness, and consistency is highly associated with the mathematical value of rationalism. This means rationalism provides the trait that allows students to communicate effectively and argue reasonably. It is as well participatory in nature through the sharing of ideas and understanding of the arguments of others. Moreover, the mathematical world has been revolutionized by the introduction of computers and other ICT tools in the teaching-learning of mathematics and has further strengthened the rationality powers of mathematics as mathematical operations have gained more logical sense. This has increased the explanation power of mathematics in terms of consistency in its argumentation.

In recent times, the superiority of mathematicians does not only reside in the ability to use symbols and other mathematical objects, but also in the ability to integrate technology in the teaching-learning of mathematics (Ashong, Agyei, & Tetteh, 2020) which has further widened the scope of mathematical ideology. Rationalism as a mathematical value allows individuals to understand and manipulate their environment to get insight into problems that border human development and to generate logical solutions and conclusions for such problems. Without rationalism, mathematical language and symbols become meaningless and of no relevance to human society. This means that the

mathematical tools that empower mathematicians to make sense of the world lie in the powers of logic which is the spine of the value rationalism. In research, some studies defined rationalism as the promotion of reasoning among students in the mathematics classroom. For instance, Aktas, Yakıcı-Topbas, and Dede (2019) argued that mathematical reasoning plays a key role as a means in individuals' communication and connection processes for mathematical learning and as such, allows them to use what they learn in daily life. Also, the value of rationalism through reasoning aligns with the idea of actively communicating (Anderson & Österling, 2019) mathematical ideas. This means that the value of rationalism aids in improving the communication skills of the learners which allows them to make sense of their presentation and argumentation.

Teachers' lesson designs must aim at developing students' skills in expressing and defending their opinions, interpreting data obtained from experience, and attempting at making predictions, which are all outcomes of logical reasoning. They also emphasized that Bishop's (1991) categorization of values of rationalization has been addressed as mathematical reasoning under their study's findings. The basis of rationalism was described as reason, logical thinking, explanation, abstraction, theories and proofs, (Bishop, 1988; Dede, 2006) and defined as sub-themes that possessed the characteristics of measuring rationalism as a variable. In identifying what constitutes the value of rationalism in mathematics classroom lessons Abdullah and Leung (2019) discussed such activities as; revising previous knowledge, given problems to solve, asking conceptual questions (definitions), procedural questions, encouraging students to explain their answers, class discussions, group discussions, allowing for interactions using strategies such as think-pair-share.

For this study, the mathematical value of rationalism was measured using questionnaire items constructed from sub-themes and classroom activities that are associated with teaching-learning of mathematics in the colleges of education mathematics classroom. The value of rationalism, therefore, was defined in this study as a demonstration of practices that encourage logical explanations and critical thinking such as; (reasoning, logical thinking, explanation, classification, use of theories, procedural questions, revising previous knowledge, encouraging students to explain their answers, class discussions, group discussions) in the mathematics classrooms at the colleges of education.

Objectism as ideological value

The mathematical value of objectism is that, it deals with the reality of mathematics in terms of its nature and character. In simple words, is that which brings symbolizing, concretizing and applying mathematical ideas to light (Anderson & Österling, 2019). Mathematics is seen by many as abstract in nature. The nature that allows for concretization and symbolization of mathematical ideas is the objective nature (Bishop, 2008). All mathematics educators agree that mathematical knowledge is well understood by learners when it is transformed into its practical reality. The teachers, therefore, have to let learners see mathematical concepts or ideas, and these are done through the use of symbols and concrete materials. It is, therefore, imperative that mathematical ideas are best understood through interaction with nature. Hence, human development has been dependent on man's ability to interact with mathematical objects which serve as sources of mathematical ideas. The concretization of mathematical ideas demonstrates hypothetical reality to explore and analyze human advancement. Besides, the concretization of

mathematical ideas and the use of symbols become a mathematical habit and define every mathematician. Objectism is, therefore, a network of logical connections developed with mathematical ideas as a result of proofs, extensions, examples, counter-examples, abstractions and generalizations that contribute to giving the objective meaning to such ideas and dealing with them as if they were real objects. Bishop (1988) and Dede (2006) identified the themes that describe or measure objectism as materials, methods, concretization, and symbolization. To Abdullah and Leung (2019), objectism can also be described or measured using activities that occur during the instruction process (teaching-learning process) in the mathematics classroom. Such activities were mentioned to include; remembering, recalling, solving a problem, reminding students of strategies and concepts used previously, showing correct workings, recall of procedural skills learned, demonstrations of skills, encouraging students, concretizing mathematical concepts, reciting objectives before the start of lessons, use of drawing/illustrations as an aid to explanation and to communicate mathematical ideas.

In this study, the variable objectism refers to classroom activities that promote the concretization of mathematical ideas using appropriate objects and symbols. Objectism was measured by the use of constructed items in the designed questionnaire. The items were constructed to reflect the mathematics classroom practices at the colleges of education context in Ghana. The mathematical value objectism was measured as the ability to demonstrate the following; (recalling, solving problems, relating mathematics ideas to real-life situations, reminding students of strategies and concepts used previously, showing correct workings, recall of procedural skills learned, encouraging students, concretizing mathematical concepts, keeping in mind the objectives

through to the end of lessons, use of drawing/illustrations to aid explanations and to communicate ideas) in the mathematics classroom at the colleges of education.

Control as sentimental value

It is interesting to understand the sort of contribution mathematics provides to human existence and life in general. One of such is the values provided by mathematics as sentimental which is associated with feelings and attitude. One side of the double-faced coin is the control value of mathematics under sentimental values. The control value of mathematics describes activities that reinforce the sense of correctness, mastery, and certainty as presented by Abdullah and Leung (2019). Besides, Anderson and Österling (2019) as well described it as a sense of certainty and power through which mastery of rules are valued.

In that, it is the mathematical spirit that provides feelings of control and security for human understanding of his environment. It also implies the ability to acquire knowledge, explain nature, make projections, and predict natural occurrences within the environment to provide security for such occurrences as the power of mathematical control. With this power of mathematical control, people can study our natural environment such that earthquakes, storms, and sea levels are predicted and measures put in place to protect lives and properties. The control value is developed as mathematicians concretized and manipulate such materialized mathematical ideas to the benefit of humanity. Through these processes, humans attain mastery over the physical environment and increase the power of the human mind. Besides, the respect and general acceptance of mathematics through science has been propagated through description, explanation and prediction. This has been a major component of scientific

research in the modern day's development of scientific knowledge. The control power of mathematics cut across almost if not all fields of human knowledge, in social sciences, economics, medicine, agriculture, and engineering to mention but a few. It is an undeniable fact that our social environment cannot be better understood without description and interpretation in mathematical terms and, that mathematics has been used in gaining control over the environment.

In a mathematics classroom, the power of mathematical knowledge is understood through the use of facts and algorithms which are familiar to the learners, and as such provide a feeling of security and control to them. It also provides self-fulfilling moments to learners when facts, theorems, and algorithms are followed to arrive at the correct answer to a problem. It reveals the intrinsic satisfaction and aesthetic pleasure invoke in the learner as a result of the mathematical power of control. Studies have shown that when ICT is integrated into classroom lessons it tends to influence (Abun, Magallanes, & Incarnacion, 2019; Altun & Cakan, 2006: Ashong et al., 2020; Hwa, 2018) attitudes of learners to a greater extent and hence turn to control their behaviour. When rules and guidelines are attached to mathematical ideas and objects, it provides magic feelings in a game sense for the enjoyment and entertainment of society. The sense of control and security provided by technology (computers) in all fields of human lives is one of the greatest gifts offered by mathematicians to the world, not to mention the introduction of Robotics which offers a clearer idea about how our environment could be controlled and provide security. These technologies are the products of mathematical ideas.

The value control was characterized and described (Bishop, 1988; Dede, 2006) as a demonstration of prediction, knowing, security, mastery over the

environment, rules, and power over mathematical objects and ideas. Abdullah and Leung (2019) further provided mathematics classroom activities that embodied the nature of the value control as; explaining students' presentations, explaining steps, emphasizing the lesson development to the class and verbal closure of lessons. In this study, value control as a construct was defined to constitute the demonstration of mathematical classroom activities that involve the manipulation of mathematical objects and symbols. This was measured using specific practices such as; explaining students' presentations, explaining steps, emphasizing the lesson development to the class, verbal closure of lessons, manipulation of teaching-learning materials, and the use of ICT resources.

Progress as sentimental value

The complementary partner of the control feeling is the progress feeling under the mathematical value sentimental. It is the fundamental reason for life and phenomena to have the ability to make progress, grow, develop, or change from the present state to a new or desired state. Anderson and Österling (2019) contended that progress is the sense of ideas growing through questioning and investigation in that progress relates to activities where students have the sense of having the opportunity to develop their ideas through questioning and exploration. Progress, allowing ideas to grow through questioning rhymes with the description of democratic participation as exploring, collaborating, and communicating in the mathematics classroom.

The valuing of progress offers such feelings to recipients of mathematical knowledge. Amongst the fundamentals of teaching-learning is that in terms of knowledge, growth or progress is achieved when the unknown becomes known to an individual and the feeling associated with this growth or

development in mathematical knowledge is the spirit of progress. It is based on this premise that the review of relevant previous knowledge and the lesson objectives are important in teaching-learning situations in the mathematics classroom. The link between what is already known and what is yet to be known is very important in the learning process of this all-important and unique subject. It is also important to emphasize that through all the mathematical values the evidence of the existence of the other five identified values is rested in the value of progress. This feeling of progress also is well demonstrated within the research community as ideas, methods and statistical tools are being developed to solve more complicated mathematical problems. The results of these are manifested in the discovery of new theories, concepts, and strategies that ensure the continuity of mathematical knowledge. Besides, the progress made in the field of technology has further empowered the valuing of progress as more control and security are envisaged among mathematicians in recent times. Thus, the ability to integrate technology into the teaching-learning of mathematics is considered to be substantial progress in the teaching of the subject (Baba & Shimada, 2019). The resultant feelings as a product of achievement made in mathematics education further provide confidence and trustworthiness to the mathematical knowledge when viewed from the positive perspective of a double-sided coin. The spirit of value progress (Bishop, 1988) is deeply rooted in the definitions, procedures, algorithms, axioms, and proofs which are all sources of needed variations in the teaching-learning of mathematics. It is indeed the exploratory character of mathematics that brings the power of research to bear in the development and construction of human knowledge. Abdullah and Leung (2019) argued that growth, improvement, development of knowledge, generalization, questioning, and use of alternative strategies are

manifestations of valuing progress. Dede (2006) as well presented the characteristics of valuing progress as a review of related literature, working on class projects, and solving real-life problems and that evidence of these activities in the teaching-learning process can be considered as a yardstick for measuring the progress value in the mathematics classroom.

In this study, the mathematical value progress was defined as mathematics classroom activities that enhance growth in trainee teachers' learning including their performance. It was measured as a demonstration of mathematics classroom activities such as; questioning, participation, collaboration (class projects, group presentations, and group discussions), use of alternative strategies, use of ICT resources, and the use of examples and nonexamples as having the ability to unearth the value progress in the mathematics classroom at the colleges of education level in Ghana. These themes were specifically used to construct the questionnaire items that tutors and trainee teachers responded to.

Openness as a sociological value

Sociological values are mostly known to be associated with democratic (Anderson & Österling, 2019) actions and are found in the sociological dimension of mathematical values. The first component of the complementary pairs of sociological values is the value of openness (Bishop, 1988) which is concerned with the fact that mathematical truth, prepositions, and ideas are open to verification and examination by all persons within the society. Mathematical knowledge is therefore accessible to all members of society who seek to find the purity and alternativeness of such truth. The openness in its pure mechanical sense is demonstrated in the proofs of mathematical truth as one goes through the acceptable logical and explanatory procedures. The identification of the

nature of truth makes an individual have control and become more secure in terms of being able to offer stronger opinions and argumentative stands in society. The mathematical truth itself cannot be contaminated by culture as in this sense it becomes independent of context and cannot be changed.

Though generally, the valuing of openness has two sides to a coin, the emphases are on the positive perspectives as they result in a democratic society, where each person's opinion is considered paramount to the developmental process of the society. In mathematics, the value of openness is demonstrated in the classroom through a democratic classroom environment, where mathematical knowledge construction is not the sole responsibility of only the teacher but a shared responsibility of both teachers and learners. Anderson and Österling (2019) argued that mathematics is democratically open for anyone to use and explain. They further elaborated that openness is the idea of mathematics being open for inquiry and explanations, thus a description of activities that invite students to actively participate in mathematical activities. The evidence of openness in the classroom answers the question of 'Who can do mathematics?' as mathematical ideas are freely available for verification by all members of the class, and that such knowledge is not for only a few people who are specially prepared by nature (born with mathematical ideas) to do mathematics. In recent times, the value of openness has gained a broader dimension through connectivism or networking (Siemens, 2005; Downes, 2010) as technology has made it possible to connect and share ideas for the development and transfer of mathematical knowledge. The provision of the internet which has resulted in networking has made transmission of knowledge (mathematical knowledge) readily available, for easy accessibility and verification as applied in mathematics and research through a collaboration of

ideas. Having access to this mathematical truth further enhances the confidence and infallibility of mathematical knowledge which tends to increase the validity of its truth. The value of openness is experienced according to Dede (2006) as facts, articulation, demonstration, verification, universality, individual freedom, sharing, and accessibility. Besides, Abdullah and Leung (2019) also in their mathematics classroom observation reported that the value of openness is demonstrated in a lesson by engaging learners in the following lesson activities; encouraging students' presentations, sharing their work in front of the whole class, presenting ideas to peers, establishing the targeted outcome for students to focus on lesson objectives, and writing closure of lessons on board.

In this study, the mathematical value openness was defined as the demonstration of mathematics activities that ensure open accessibility and usage of mathematical knowledge in the teaching-learning process as exists in the colleges of education mathematics classrooms in Ghana. The variable openness was measured using themes (items) as constructed on both questionnaires for tutors and trainee teachers. Hence, the evidence of openness was specifically measured through the demonstration of; encouraging trainee teachers to present and share their work, group discussions, highlighting the outcomes (objectives) of the lesson, recapping the main ideas in the lesson, provision of both individual and group assignments, the use of ICT-resources in the lessons and accessing mathematical knowledge from the internet.

Mystery as sociological value

The second complementary pair of sociological values of mathematics was identified as mystery (Dede, 2006; Bishop, 1988) and described to include abstraction, unclear origin, wonder, and mystique, above human understanding and magical. According to Anderson and Österling (2019) the value mystery is

a concern with the mystique of mathematical ideas, origin, where it comes from and who possesses the power to explore such ideas? One of the familiar subjects that are studied all over the world is mathematics due to its' numerous utilitarian values but is the subject most people do not want to talk about or encounter in their lives. To be precise, most people are afraid of the subject itself. This has been as a result of society viewing mathematicians as mysterious as the subject itself. Hence, mathematical ideas are seen to have come from outside this world and are for only a few supernatural beings who can make sense of such ideas. Besides, the powers of mathematical ideas and their objects lie in the dehumanized nature of the subject and the people associated with it. When these mathematical ideas are polished and refined they become magical and a source of entertainment to society. The bedrock of such power is rooted in its abstractness or pattern generation (magic square numbers, Pythagorean triple, and choose a number game) of logical ideas found in mathematics. In technology, the value of mystery is becoming increasingly powerful due to the advancement in artificial intelligence (artificial minds) development and the use of computers has further deepened the wonders that mathematics has to offer. It is also revealing that the analysis and interpretation of data and results respectively have made more questions to be asked by non-mathematicians in the area of research.

In the mathematics classroom, the value of mystery is considered (Anderson & Österling, 2019) to be associated with the fact that mathematical activities are usually already outlined and described by somebody. Hence, mathematical algorithms already exist to be employed in solving problems. In their lesson study, Abdullah and Leung (2019) also demonstrated that the value of mystery can be defined in terms of classroom activities to include;

introducing students to mathematical games, storytelling, history about particular concepts, and showing interesting mathematical patterns, shapes and designs. For this study, the value mystery was defined to include mathematics classroom activities that emphasize aesthetics and wonder in mathematical ideas at the colleges of the education mathematics classroom. The evidence of value mystery was measured using specific themes such as; the introduction of trainee teachers to mathematical games (e.g. choose a number game), history about particular concepts (e.g. geometry and algebra), classification of mathematical objects, evidence of abstractions, and generalizations, showing interesting patterns, shapes and design, the use of mathematical ideas in reallife situations (e.g. in construction of houses, farming, fishing, etc.), and ICTintegration in mathematics lessons.

Mathematics educational values

Values in mathematics education were defined as attributes of importance and worth that are internalized by an individual that provides him or her with the will and determination to maintain any course of action chosen in the learning and teaching of mathematics (Seah & Anderson, 2015). In recent times, values in mathematics education have been referred to as an individual's acceptance of convictions which are considered to be of importance and worthwhile that provides the individual with the will and grit to maintain any 'I want to' mindset in the learning and teaching of mathematics. In the process, this conative variable shapes how the individual's reasoning, emotions, and actions relating to mathematics pedagogy are developed and established (Seah, 2019, 2018). It was also revealed that mathematics education values are those which are embedded in curricula, textbooks as well as teachers' professional practices (Bishop, 2008; Davis *et al.*, 2021). These are attributes of mathematics

learning or teaching that students or teachers would emphasize and which they think are important to success in mathematics education, and that mathematics education values connect the pedagogical approaches and norms of teaching (Madosi *et al.*, 2020). Sam and Ernest (1997) contended that mathematics education values are connected to the teaching and learning of mathematics and refer to the degree to which standards and procedures that can be associated with teaching and learning mathematics are viewed as important. Besides, Bishop (2008) demonstrated that mathematics education values are the values embedded in a particular curriculum, textbooks, and classroom practices as a result of the other sets of values.

Furthermore, Worsley (1984) located values as one of three dimensions of culture: cognitive (represented via ideas), conative (manifested via performance), and normative (represented by values). Seah and Bishop (2002) went further to define culture as an organized system of values that are "transmitted to its members both formally and informally" and that mathematics education values are seen as a product of culture within society. In a recent study, Seah (2004) argued that the values of practicing teachers reflect their cultural backgrounds. Hence, mathematics education values are considered to be culturally bound and differ from one sociocultural environment to another. On this basis, values taught and learned in the mathematics education classroom may differ from one country to another, from one region to another, and from one school to another. This means mathematics educational values developed by students are not constant and change depending on the environment (Davis *et al.*, 2019) and the specific mathematics education values exposed to by their mathematics teachers. As teachers would show evidence of mathematical

activities they consider to be important as reflected in their views and classroom practices.

Different studies have been conducted based on the conceptualization of five-value dimensions involved in mathematics education proposed by Seah (2011b) as relevance, accessibility, formalistic view, relational view, and the process (procedures and tools). In this sense, mathematics education values were defined in this study as attributes in mathematics learning that students consider important in their mathematics learning experiences.

Formalistic view - activist view as a mathematics education value

The formalist view is concerned with actions that encourage logical thinking and make mathematics enjoyable. This view of mathematics education values shows the deductive and receptive learning in mathematics, while the activist view value shows its intuition and discovery learning. The development of the mind depends largely on the formalist view in the learning of mathematics and critical thinking is invested in this value which allows individuals to develop new ideas and the ability to create. This means the advancement of human society depends on such aspects of mathematics (Madosi, *et al.*, 2020). In this study, the formalist view was considered to be an attribute of mathematics learning activities that encourage the development of logical thinking and enjoyment in trainee teachers' mathematics learning.

Instrumental learning - relational learning as a mathematics education value

The relational view was defined as mathematics activities that are associated with the learning of rules, procedures, operations and formulas (Madosi, *et al.*, 2020) with their applications to special questions. Relational learning shows the demonstration of the relationships among concepts and

forming appropriate graphics. This aspect of mathematics educational values is manifested in instrumental and relational attributes in mathematics learning. Hence, mathematical concepts are better understood based on relationships and powerful tools in theory generation. The relational view was observed as the pedagogical activities that help in developing trainee teachers' use of rules, procedures, operations and formulas in establishing relationships among mathematics concepts.

Relevance - theoretical knowledge as a mathematics education value

Accordingly, the value of relevance refers to mathematics learning activities that promote the application of mathematical ideas in solving everyday life problems through which society develops. Besides, relevance is used to reflect the theoretical knowledge in mathematics. It is the valuing that shows the importance of mathematical knowledge in solving real-life problems. These real-life problems and their demands differ across cultures. The value relevance manifests the power of mathematics in solving everyday problems in society. Teaching the theoretical aspect of Mathematics provides the backbone for mathematical knowledge (Dede, 2006). The relevance value in this study was referred to as the attributes of mathematics learning that promote everyday problem-solving skills through the application of mathematical ideas of trainee teachers.

Accessibility - special as a mathematics education value

This mathematics education value refers to valuing that focus on doing and preparing mathematics activities for a group or individual people to implement. It suggests the special nature of doing mathematics in the classroom. This includes mathematizing vertically or horizontally and the activities within which these are done. Hence, doing and preparing mathematics activities by

those interested in mathematics and, help erase the misconception that mathematics is meant for special people or super natural beings. For this, the accessibility dimension was measured as the pedagogical activities that encourage the doing, preparation of activities and participation of trainee teachers in the mathematics classroom.

Evaluating - reasoning as a mathematics education value

This dimension of mathematics educational value is very important so far as critical thinking is concerned. It improves the development of the mind towards higher order thinking leading to divergent thinking and allows students to concentrate effectively on different activities at the same time. Students are asked to realise the steps of knowing, applying routine operations, searching solving problem, reasoning and communicating in order to solve a problem. The first three of this five steps demonstrate using mathematical knowledge about evaluating an unknown answer; while the last two demonstrate the capability of using mathematical knowledge, reasoning more and the ability of spreading the knowledge (Seah & Bishop, 2000).

Seah and Bishop's (2000) conceptualization has been widely applied in different studies and countries as in Japan, Seah, Baba and Zhang (2017a) reported in their survey conducted, that Japanese primary and secondary school students demonstrated evidence of creativity, discussion, ICT, know-how, mystery, others' involvement, reality, results and wonder as the attributes of mathematics learning they consider important in their mathematics classroom. Additionally, Seah (2002) also proposed twelve different mathematics education values as; accuracy, clarity, conjecturing, consistency, creativity, effective organization, efficient working, enjoyment, flexibility, openmindedness, persistence, and systematic working. These sources serve as the

basis where most studies in the area of values in mathematics education around the world draw from.

In a cross-cultural study conducted in Turkey between Turkish, Turkish immigrants, and Germen students, Dede (2019) reported that students in grade nine after they were interviewed valued mathematics education attributes like practice, relevance, rationalism, and fun for the Turkish students. Again, the German students also valued attributes such as relevance, fun, rationalism and consolidating values. Attributes such as relevance, rationalism and communication were valued by the immigrants.

In a similar study, Madosi *et al.* (2020) identified hard work and effort when doing mathematics; numerous different methods to obtain the answer to a mathematics problem; authentic examples of shapes to understand their properties; demonstration and explanation of mathematics concepts and proofs; and teaching and explaining mathematical concepts as the attributes valued by grade 9 Gauteng, South African learners in their mathematics learning experiences.

In Ghana, Davis *et al.* (2021) suggested eight attributes senior high school students view as important in their mathematics learning experiences connections, understanding, fluency, learning technologies, feedback, instructional materials, open-endedness and problem-solving. Again, Davis *et al.* (2019) presented in their survey the attributes valued by Ghanaian primary, junior high school, and senior high school students demonstrated in their mathematics learning as; achievement, relevance, fluency, authority, the use of ICT, versatility, and strategies.

In the same way, Zhang (2019) after investigating the aspects of mathematics learning attributes valued by Chinese Mainland primary and

secondary school learners reported evidence of ability, effort, diligence, use of formulas, memory, knowledge, and thinking, as the latter two attributes were espoused by the secondary school learners.

The aim of various studies conducted in the area of mathematics education is to identify what the learners consider to be important in their mathematics learning so that teachers can design appropriate strategies to help such learners in their learning to improve performance. Hence, this current study would help identify attributes of mathematics teaching-learning that tutors and trainee teachers consider important and the relationship between such attributes valued at the selected colleges of education in Ghana.

It's worth noting that the WIFI instrument has been widely adopted or adapted for mathematical and mathematics educational value research. It has been used to conceptualize both mathematical values and mathematics education values. Some studies conceptualized mathematical values (Bishop, 1988), and adopted or adapted the WIFI instrument (questionnaire) items while others also conceptualized Seah's (2011b) five-value dimension of mathematics education values based on which the WIFI instrument was developed. The WIFI instrument has been operationalized for both mathematical values (Anderson & Osterling, 2019; Dede, 2019) and mathematics education values (Davis et al., 2019; Davis et al., 2021; Madosi et al., 2020; Hill et al., 2019; Seah et al., 2017b; Zhang, 2019) depending on the perspective of the researcher. Hence, the WIFI questionnaire was adapted for this study. The questionnaire instrument was used to collect quantitative data that was used to obtain attributes valued in mathematics learning by the trainee teachers and their tutors. The attributes of mathematics learning (mathematics education values) demonstrated by both trainee teachers and their tutors were generated through the principal component

analysis conducted. The focused group interview guides also helped to sought for reasons from both trainee teachers and tutors on why they valued the observed attributes in mathematics learning after the quantitative analysis.

Moreover, the relationship between Bishop's (1988) mathematical values and Seah's (2011a) mathematics education values was provided by Madosi et al. (2020) when they linked the value relevance to the progresscontrol dimension of mathematical values; accessibility was viewed as an openness-mystery dimension of mathematical values; the formalistic view was equated to rationalism-objectism dimension; relational view as rationalismcontrol dimension and process (tool or procedural) was compared to controlopenness-mystery dimensions of mathematical values. Based on the above relationships it is clear that the mathematics education values are generated through the combination of two or more mathematical values or some specific themes under two or more mathematical values when linked to pedagogy. For instance, there is an aspect of ICT usage in mathematical values of control, openness, and mystery, and hence, questionnaire items on the three variables that incorporate ICT can be assembled to formulate the learning technologies (Davis et al., 2021) under the mathematics education values. Such combinations can result in possible mathematics education value attributes (components) formed based on the component analysis conducted. Here, the generation of the mathematics education value attributes are not necessarily based on the complementary pairs of mathematical values but depends on the internal relationships between the individual questionnaire items.

Mathematics tutors' (teachers) values

In the area of teachers valuing mathematics, not much has been done so far as value research is concerned. Teachers' knowledge of values has been

noted to be very important as it contributes to effective teaching-learning of the subject. Teachers' teaching has been identified to be one of the major sources of conveying values either explicitly or implicitly. In some countries, textbooks are only seen to determine the implemented mathematics curriculum because the teachers' ideas are dominant (Cao *et al.*, 2006) and curriculum materials are merely an aid for the teacher. In this section, the researcher reviews the literature on some of the studies conducted on teachers in the area of valuing.

In the (VAMP) project, Bishop, Fitzsimons, Clarkson and Seah (2000) contextualized the project by investigating and documenting mathematics teachers' understanding of their own intended and implemented values; the extent to which these teachers can gain control over their values of teaching; increase the possibilities for more effective mathematics teaching through values of teachers, and of teachers in training. The study collected qualitative data (interview and lesson study) on 30 mathematics practicing teachers in Victoria, in Australian schools and reported that the actions and the choices teachers make in classroom discourse are largely influenced by their values and the choices the teacher makes also help to shape the values of their students. They also highlighted that hypothetical situations that occur in the classroom and their corresponding decisions revealed teachers' values and that they recognized the presence of values but little do they know about values.

Meanwhile, it was reported that the choices teachers make influence or shape the values of their students, but no data was collected and reported that indicated the relationship between attributes valued by mathematics teachers and their students. Though mathematics education research has shown that values are an integral part of any mathematics teaching, little is known or has been written about values in mathematics education. It is also important to

stamp the fact that values do exist throughout all levels of human relationships, at the individual level, learners have their preferences and abilities which allow them to consider some activities more important than others and through classroom activities, teachers would not only be aware of different values that they are teaching but also be more in control of their values when teaching. This means value research offers a greater range of teaching strategies and a more comprehensive mathematics education to all its students.

To assist mathematics teachers to nurture mathematics thinking in their students using findings from research on mathematical values under the (VAMP) project, qualitative data were collected on 6 mathematics teachers from Australian schools using personal interviews (before and after lessons), lesson observations, and video recordings of lessons. After analyzing the data descriptively, Bishop (2008) observed some key findings that needed attention within the mathematics education community as teachers held values about mathematics and mathematics education to a degree; teachers also had many goals in planning for lessons, often revealing their values, which included the development of individual children, helping them acquire Mathematics knowledge and gain confidence with doing mathematics, and finally, showing how mathematics can be relevant to them personally as well as to society.

However, it was reported that the teachers found it difficult to discuss values concerning mathematics, but the introduction of some of the theoretical terminology used in the instrumentation helped the teachers to discuss their teaching. They also chose to make explicit certain mathematics or mathematics education values or they 'showed' them implicitly. Again, it was easier for the teachers to think about and recognize the values they were teaching, rather than to implement new values. The implications of the latter finding were presented

for the implementation of new policies in mathematics education with any educational policy developments, it is essential to have good research as the foundation, as well as good theories to support and structure that research work; mathematical thinking has been studied in many ways, but in relation to values it is useful to consider it as an aspect of meta-cognition and affectivity; the context for the developments should be the classroom, as it is there that the community of practice significantly influences the values of mathematical thinking; equally important to consider in this kind of research is the students' socio-cultural context, as any educational values are embedded in the culture of that particular society; finally, the teachers need special support in this research and development activity, as values teaching involves the teachers' pedagogical identity, which must be respected.

In a similar study, conducted to reveal the mathematics values of secondary school mathematics teachers reflected in their classroom practices based on the conceptualization of Bishop's (1988) and Bishop *et al.* (1999) three complementary pairs of values on five secondary school mathematics teachers sampled purposively and with varying teaching experiences. After using interview forms based on the classroom scenarios, lesson video recording and unstructured observation forms presented some interesting findings as teachers showed evidence of mathematical values, but were demonstrated in varying proportions. Besides, the teachers demonstrated some of the value, but not all dimensions. It was particularly reported that the teachers highly prioritised the value dimensions of objectism, control, and openness over the complementary value duals. Also, it was observed that the application of control-progress (Sentimental) values together in the classroom practices emerged significantly in the results. Notwithstanding, the study reported on only the mathematics

teachers' values, reporting on students' values as well could have given a balanced view of the problem and some interesting theories could have also been formulated. Hence, more studies in the area of mathematical values are required, especially studies that report on both teachers' and students' mathematics values to investigate the relationships between them (Aktas & Argun, 2018).

In order to investigate the values of the Japanese mathematics teacher community, Corey and Ninomiya (2019) analyzed three particular community practices to uncover the potential values of the mathematics teachers. The study used personal observation and video recordings to collect data on 2nd to 9thgrade teachers. Based on the pre and post-observation discussions and critical analysis after observing the teachers on a mathematics education course and professional development, particularly in Japan. The study revealed the importance of how mathematics teachers' values influence their students' values. Again, the study stated that due to variation between individuals, it is valuable to think of the values of communities and cultures. It was strongly established that values that communities hold can influence the values of individuals who are enculturated into that community. The deduction made from the above findings suggests that the community could be a school or a group of people in a defined setting such as churches, society, clubs, etc. which have a direct impact on the values held by an individual.

However, some weaknesses were identified in this study as it was difficult associating different responses given on reasons why mathematic teachers engage in a particular behavior and accepted the fact that, it still might take some methodological analysis to articulate the shared values because the values might still be implied by the responses. It is therefore important to

emphasize that the values of an individual are influenced by external factors, as students' mathematical values are directly influenced by their teachers' values and that teachers' values are in turn influenced by other factors such as the intended curriculum.

Based on the literature reviewed on mathematics teachers' (tutors') valuing, it can be concluded that there is no known study that reports on students' values from the perspective of their mathematics teachers. It is, therefore, imperative to consider mathematics education value research study that would investigate teachers valuing in their students' learning as a component in understanding the dynamics in valuing in the classroom, more importantly at the colleges of education classrooms in Ghana. For this study, the mathematics tutors' values in their students' learning are considered to be the various attributes of mathematics learning that tutors consider important at the colleges of education level. Tutors' values would be obtained by the use of principal component analysis on the responses provided to the items on the questionnaire for mathematics tutors. The individual value constructs would be named based on the characteristics of items that are loaded to a specific component that is generated from the analysis. To balance the research perspective of values, a literature review on students' values in their mathematics learning is presented in the subsequent section below.

Trainee teachers' (students) values

In recent times, values possessed by students have gained prominent attention in the area of mathematics education research across the world due to their importance in developing the students based on what attributes of mathematics learning (Davis *et al.*, 2021; Madosi *et al.*, 2020; Seah, 2011a) they consider important in their learning experiences in the classroom. It has been

suggested by different researchers that knowing specific values possessed by students and designing appropriate lessons based on such values are identified to improve students' academic performance significantly which for many years research interventions have failed to provide. Therefore, this section presents some of the studies conducted in the area of students' valuing as conceptualized in the framework. The focus of this review is based on research conducted in the area of mathematical values and mathematics educational values as operationalized in this study.

To determine the mathematical values of mathematics students towards learning the concept of function on undergraduate students studying at Cumhuriyet University in Siavas, Turkey, Dede (2006) sampled 343 from 1st to 4th-year students of the Primary Mathematics Education Programme. Students were asked to answer 5 out of 10 questions provided in a test and the questions were carefully designed to contain one of the mathematics education values. The students were as well supposed to provide the reasons for selecting a particular question to respond to. Their responses were then analyzed using rubrics which allowed for responses to be transcribed into themes. They reported that all students across the grade levels prefer learning function concepts, those questions that hold the formalistic view, relevance, instrumental understanding/learning, accessibility, and reasoning values respectively. Besides, their findings showed some similarities and differences with Seah and Bishop's (2000) study that examined mathematics educational values projected by Singapore and Victoria (Australian) mathematics textbooks. The students' reasons for selecting a particular question in the function test were determined by familiarity, logic, application to daily life, addresses to the theory, it involves the use of rules, operations and formulas, showing of conceptual relations, it

was so easy, etc. Moreover, the study could have collected data on mathematics lecturers to determine the source of the mathematics educational values exhibited by the students through comparison to make a stronger argument. They as well reported that students often preferred the questions showing the symbolic (objectism and control) aspect of the function concept. They posited that these students are potential teachers and naturally, would transmit their mathematics educational values to their prospective students either explicitly or implicitly and concluded that such students would focus in their future education more on the mathematics educational values such as formalistic view, theoretical, and instructional understanding/learning values.

It is clear from the concluding part of the study that fits the context of the colleges of education in Ghana, as the trainee teachers are also potential teachers and for that matter expected to build the right values in their prospective students explicitly or implicitly in the future. This means if the trainee teachers are not assisted to develop the appropriate values through their tutors and the curriculum materials, the problem becomes cyclical and would be passed on from one generation to another.

A study conducted in Palembang, purposively sampled six students in a class to identify the mathematical values demonstrated by the students in solving non-routine problems. Safura *et al.*, (2018) conceptualized Bishop's (1988) categorization of three complementary pairs of mathematical values and collected data using lesson observation, tests, and interviews which were analyzed qualitatively. Their report suggested that students demonstrated more values of objectism, control, rationalism, and progress whiles dimensions of mystery and openness were least demonstrated in the solving of non-routine problems. However, in the opinion of the researcher, the sample of six students

is too small to suggest any strong relationship that would result in valid findings being formulated, and besides, the mixed methods design approach could have allowed for more participation and triangulation (Chan & Wong, 2019) of data. The study also reported on only students' mathematical values but relationships could have been established between teachers' and students' valuing giving a broader perspective of the situation. Based on their findings, the argument was made that values are not constant and are influenced by context or environment, and differ from a person to person depending on the individuals' past experiences and the environment. Hence, students would exhibit some of the value dimensions more than others (prioritised) in their learning of mathematics which appears to agree with Davis *et al.* (2019).

In another study conducted on Guanteng, South African grade 9 students from the public schools after conceptualizing Bishop's (1996) and Seah's (2011b) five value dimensions, using the WIFI instrument which was analyzed quantitatively, revealed that learners value more hard work and effort when doing mathematics; alternative methods to obtain answers; authentic examples of shapes to understand their properties; demonstration and explanation of mathematics concepts and proofs; teaching and explaining mathematical concepts. Meanwhile, the study made use of only ten semantic questions to determine what values the learners consider important in mathematics, and therefore, strong conclusions could not be made out of such questionnaire items. The study again reported on only what learners consider important in their mathematics learning, but not on teachers' values or the alignment between them. The study further considered only quantitative data, but collecting both quantitative and qualitative data could have given more confirmatory findings through triangulation of results. The study was also limited in relating the

students' values identified to their performance. This means that, in order to ensure better performance in mathematics, teachers should focus on values in mathematics classrooms. In that, values influence the way learners choose to engage with the mathematical tasks, and eventually how they perform in the subject. Finally, the study provided mindfulness by teachers about what values learners consider important in their learning of mathematics and allow teachers to use appropriate pedagogical approaches that would include these values (Madosi *et al.*, 2020).

Recently, a study conducted in Ghana by Davis et al. (2021) to explore the attributes of mathematics learning that Ghanaian Senior High School students from the Cape Coast Municipality, after using a principal component analysis on 416 students' responses to questionnaire items, identified eight components of students' mathematics learning attributes as; connections, understanding, fluency, learning technologies, problem-solving, feedback, instructional aid and open-endedness which were comparable to attributes reported on students by similar studies elsewhere (Hong Kong and Japan) in the world. However, the study concentrated on students' values at the Senior High School level only and in one region of Ghana. They could not report on valuing among teachers at the Senior High School level, which limited the focus of the study and hence, could not explore the relationships between teachers' and students' values (value alignment). The study as well, could not establish whether the identified students' values reflected the values projected by the Senior High School mathematics curriculum or not. These limitations of the study added to the fact that research on values in mathematics education in Ghana is very young and promising. Therefore, more research studies need to be conducted in this new and exciting field of mathematics education.

Furthermore, to examine and explore the mathematics educational values of middle school Pasifika students in New Zealand based on their significance in contributing to more effective and equitable mathematics learning, Hill, Hunter and Hunter (2019) surveyed through questionnaire and interview guide from 131 Pasifika middle school students (year 7 and 8) from four selected schools. The students were made to rank value activities from highest to least valued. Based on the students' responses, it was revealed that the four most valued activities in mathematics education for these students were utility, peer collaborations/group work, effort and practice, and family/familial support respectively. They further argued that having insight into what your students value most helps teachers to ensure the right classroom culture and pedagogy are developed to align with these students' values. Besides, the study expressed the need for educators to examine the mathematics educational values of all students including minority students to provide an equitable mathematics classroom.

From their suggestion above, it expressed the importance of identifying the mathematics educational values of all students in a mathematics classroom but their study failed to collect data on all the students as only the minority cultural group were considered in the study and could have provided more dynamism if the results were compared with the majority cultural groups, and or their mathematics teachers' values. To ensure such dynamism, this current study would collect data on the intact classes of third-year trainee teachers offering mathematics as a core subject in the selected colleges of education together with their mathematics tutors.

In this current study, the trainee teachers' values were operationalized as mathematics activities trainee teachers consider important in their learning

experiences at the colleges of education level in Ghana. The trainee teachers' values would be identified through principal component analysis that would be conducted on the students' questionnaire responses generated using the SPSS software. Based on the identified named component the respective relationships would be formulated.

Value alignment between students (trainee teachers) and their teachers (tutors)

In searching for mathematics values research that reports on both teachers' and students' values resulted in limited studies. This also emphasizes the scarcity of mathematics value research studies in general as a new and promising research area in mathematics education. This was evident when Carr (2019) conducted a search on value research in mathematics across the world through various search engines. The study considered works on teachers, students, or parent/guardians valuing in mathematics at the primary and secondary school levels and, that these articles were published in the English language in a peer-reviewed journal from 2013 to 2017. Carrs' search found 34 studies of mathematics valuing across the world and only one article was found in Africa (South Africa). Besides, most of the studies reported were conducted using qualitative approaches (interviews or self-reported data) and few also adopted the quantitative approach (pre and post-test data) assessment on valuing. Though Carrs' study was limited in reporting on the mathematics value research studies reported in other languages apart from the English language and at the higher level of schooling, the study presents the limited situation of mathematics value research across the world and in Africa to be precise.

Another mathematics value research conducted to investigate both teachers' and students' valuing was presented by Kalogeropoulos and Clarkson

(2019) after investigating how teachers and their students' mathematical values were aligned in the classroom, using Kalogeropoulos (2016)'s four levels of value alignment of engagement in mathematics learning and reported how both teachers and their students negotiated their mathematical values through scaffolding, balancing, intervention and refuge strategies of value alignment. The study collected data on four teachers and their 10 and 11-year students through lesson study and was analyzed descriptively to establish how teachers accommodate the unanticipated critical situation in a typical mathematics classroom. Their findings showed that teachers' and their students' values were negotiated to bring dynamic interaction between the parties and that unanticipated values of the students can be aligned with their teachers' values to maintain activities through interactions in their mathematics learning. The study further deepens the importance of values in ensuring effective and continued interaction in the mathematics classroom and that, any undesirable mathematical values that are identified with the students by the teachers in the classroom are deemphasized or corrected. The study also confirmed the strong relationship between the mathematical values of both teachers and their students.

Though the study has shown a great approach to dealing with undesirable values amongst students in the mathematics classroom, the researcher believes that collecting both quantitative and qualitative data could have provided more in-depth knowledge about the initial values held by the teachers and their students, and a larger sample could have been used as multiple data sources could have allowed for triangulation.

Based on their recommendations, more research is required on values and value alignment between teachers and their students. It is also important to

note that values held by teachers and students are not constant and can be negotiated (teachers' values can influence their students' values and vice-versa) through the classroom culture or environment. Hence, when both teachers and students reflect on and discuss their thoughts, ideas, approaches embraced and acknowledges the mathematical identity of both parties. Then, these interactions reveal values for consideration, and value alignment may be the catalyst for facilitating meaningful mathematics learning. Based on the above review, it has been demonstrated that there is a stronger relationship between teachers and their students if both parties' mathematical values are well negotiated. Therefore, it is the expectation of the researcher that such relationships are established between trainee teachers and their mathematics tutors based on the results of this current study.

The existence of relationships would therefore be established by comparing the named components (value constructs) generated through the principal component analysis for both mathematics tutors and their trainee teachers for similarities and differences by the use of correlational analysis and qualitative means (inspection) by the researcher. To determine whether significant differences exist between trainee teachers' and their tutors' values, an independent samples t-test would be conducted for both parties' responses to the questionnaire items for tutors and trainee teachers. The reasons for demonstrating the identified value constructs would be obtained through semistructured focus group interviews conducted after the quantitative data analysis for all mathematics tutors who participated in the quantitative data collection and 50 (10 each from the participated college) trainee teachers would be systematically sampled for the focused group interview sessions.

Mathematics curriculum projected values (MCPV)

Recognizing the importance of values in mathematics education, a curriculum may be carefully designed to convey or project certain underlying values and which in turn influence the mathematical values exhibited by both teachers and their students (Seah, Anderson, Bishop, & Clarkson, 2016). The curriculum and its' materials such as textbooks have been seen as a major transmitter of values in mathematics education, as in countries like Australia and China, textbooks are almost the major determinant of the implemented curriculum (Cao, Seah, & Bishop, 2006) as well as considered to be part of the intended curriculum.

These curricula projected values may be observed explicitly or implicitly (Dede, 2011; Madosi *et al.*, 2020) by experts in mathematics education as incorporated in the curriculum or other curriculum materials such as textbooks. The specific values incorporated in a particular nation's mathematics curriculum can be classified into the three complementary pairs of mathematical values conceptualized (Bishop, 1988, 2008; Dede, 2006; Haciomeroglu, 2020) as presented in the conceptual framework for this study. The pairs are indicated as; rationalism-objectism, control-progress, and openness-mystery, which are individually defined as demonstrated in the previous sections.

Based on the comparative study conducted by Cao *et al.* (2006) after comparing values conveyed in Chinese and Australian mathematics textbooks using Seah's (1999) coding protocol for identifying mathematical values based on Bishop's (1988) complementary pairs of mathematical values revealed that both countries textbooks emphasized the value of objectism more than its complementary value of rationalism. Again, the value of control was observed

more than the value of progress, and the value of mystery was projected more than the value of openness. The study as well reported that in comparing each category of values projected between China and Australia, objectism appeared to be projected more in Australian textbooks than in China textbooks. However, the values of control, mystery, rationalism and openness are all emphasized more in the Chinese textbooks. This means the curriculum and its materials may project different values in different proportions in different places or countries.

Least and high prioritised values

The least prioritised mathematics values (LPMV) refer to the attributes valued based on the quantitative data analysis of the study to have low mean rankings (less demonstrated) by both mathematics tutors and trainee teachers at the respective selected colleges of education. While the highly prioritised values (HPMV) were identified to have high mean (highly demonstrated) rankings based on the quantitative data analysis. For instance, Davis et al. (2019) reported that the mathematical value of control seems to be demonstrated more by the students at the higher grade levels from JHS to SHS in Ghana. Besides, Ghanaian senior high school students exhibited high evidence of connections, understanding and fluency when eight mathematics educational values were reported (Davis et al., 2021). A similar study also revealed that Palenbang students highly prioritised mathematical values of objectism, control, rationalism and progress respectively. Whiles, they least prioritised the values of mystery and openness. These were reported when the students' ability in solving non-routine problems was explored (Safura, Aisyah, Hiltrimartin, & Indaryanti, 2018). Moreover, after examining the mathematical values of five mathematics teachers in their classroom practices at the secondary school level, Aktas and Argün (2018) reported that teachers highly demonstrated values of

objectism, control and openness respectively. It was as well showed that teachers in their democratic actions in the mathematics classroom practices in Sweden demonstrated highly in the values of objectism and control than rationalism and openness (Anderson & Österling, 2019).

Based on the above literature, it is clear that these highly and least prioritised values may differ (Davis, et al., 2019) from one college to another depending on the initial data analysis. The least prioritised values may be a result of different factors, in this sense the design and planning of lesson activities (Abdullah & Leung, 2019) by tutors can be geared towards specific mathematical or mathematics education values if the tutor is not previewed to the concept and types of values associated with teaching-learning of mathematics. The trainee teachers may also come into the mathematics classroom with some already developed mathematical values from the basic and secondary school levels. This may influence the development of mathematical values at the colleges of education level especially when there are imbalances in the already held values. The nature of classroom discourse can as well aid in some of the mathematical values being more prioritised than others when class activities (Aktas & Argün, 2018; Nakawa, 2019) and communication are not purposively designed to reflect all the mathematical values. More especially, when trainee teachers find it difficult to express their ideas and thinking in mathematics classrooms as it can impede the development of a particular mathematical value. The least prioritised and highly prioritised values would be determined based on descriptive statistics (Mean Rankings) of the various constructs on mathematics educational values. Hence, the least the constructs mean, the least prioritised that construct (mathematics education value) and

vice-versa.

Value-centered mathematics curriculum (VMC)

Education in recent times has become a necessity as it serves as the bedrock of every nation's developmental agenda. This means that depending on the developmental needs of a particular nation, the educational curriculum should, therefore, be structured and designed to reflect or address such developmental needs. In designing the educational curriculum, three components that should be followed are; the intention of the curriculum, the implementation process, and the attained (Smith, 2000). These three levels of curriculum development help inculcate in their generation the societal values of that particular nation. As part of the definition of curriculum, a field of study involved what is taught in the schools, how it is taught, how what is taught is planned for, and how it is evaluated. From this definition, it is clear that all three stages (intended, implemented, and attained) are reflected in the aims of every curriculum and, are to improve teaching-learning in a given environment. In simple terms, the curriculum as a field of study and research is to inform or improve practice. Considering the various definitions and illustrations proposed by different authors in societal and cultural perspectives (sociocultural views) one of the main aims or purposes of every educational curriculum is to produce individuals who will be in the capacity to serve the needs of their community or society of which Ghana is no exception. Thus providing them with competencies that reflect the elements of the learners' culture and as such produce learners who would be functional citizens (Offorma, 2016) of their society.

The researchers' conceptualization of mathematics value-centered curriculum is not different from the general categorization of an educational curriculum which is based on Robitaille and Garden's (1989) three levels of

curriculum (Intended-planned, implemented, and attained curriculum). In linking these levels of curriculum to a value-centered curriculum, the intended is referred to as the explicitly and implicitly planned values in the mathematics curriculum (Seah, Anderson, Bishop, & Clarkson, 2016). The evidence of the planned values is seen in the curriculum materials such as the curriculum itself, textbooks, teachers' handbooks, and other printing documents which are used as teaching-learning resources in mathematics education. In line with this, Cao et al. (2006) applied Seah's (1999) mathematical values coding protocol to compare the values conveyed in mathematics textbooks in China and Australia. Their studies revealed that generally mathematical value of Objectism was projected more than Rationalism and similarly, the value of Control was emphasized more than the value of Progress and the value of Mystery was observed more than the value of Openness. Moreover, in comparing each category of value signals between China and Australia, Objectism seems to be very much more emphasized in Australian textbooks than in the Chinese ones, while the value signals of the other categories such as Control, Mystery, Rationalism and Openness were all emphasized more in the Chinese textbooks.

The implemented mathematics value-centered curriculum is considered as the values held by teachers as demonstrated in their teaching practices and are transferred either implicitly or explicitly (Dede, 2011) onto their students. Such values are best observed in teachers' classroom practices (Enacted teacher and classroom values) through lesson observations. To some extent, the teachers' implemented values can be exhibited through their perceptions and opinions (Teachers espoused classroom values) during mathematics discussions. Lastly, the attained of the mathematics value-centered curriculum is associated with the mathematical values that are developed by the students as

exhibited in their mathematics learning through values stated by learners as acquired and those evidenced in learners' behaviour. However, it is important to state that already values that promote positive cognitive and affective developments are being enacted and instilled in Ghanaian mathematics classrooms, although not all teachers are professionally aware (Madosi *et al.*, 2020) of them. Hence, turning the lenses to a value-centered mathematics curriculum will create the necessary awareness for curriculum planners, teachers and students. That would contribute to the development of individuals who fit well into society.

Review of Empirical Studies on Values

Different research studies have been conducted to investigate the nature and theories surrounding values in mathematics education. These studies were conducted across the world and in some cases, their results were compared (Davis et al., 2021; Dede, 2012; Madosi et al., 2020) to ascertain the global nature of the situation. Most of these studies were conducted on teachers' and students' values and values in the mathematics classroom while few have also looked at values projected by the various curriculum materials such as syllabus and textbooks. For instance, Davis et al. (2019) explored the effect of grade levels (Primary, Junior High School (JHS) and Senior High School (SHS)) on what students consider important in their mathematics learning in Ghana. Using a survey of 1265 students across the grade levels which adopted the WIFI instrument. After employing a one-way multivariate analysis of variance (MANOVA), the study reported some interesting differences in valuing mathematics learning across the grade levels. Their result suggested that significant differences between the grade levels existed for seven components which were identified as; achievement, relevance, fluency, authority, ICT,

versatility and strategies. Besides, the mathematical value of control was more evident at the higher grade levels from JHS to SHS. They also suggested that all the constructs of values used were embraced by the students, except that some of them were more highly demonstrated at different levels of schooling. They further argued that due to the learning of more application problems at the SHS level, such students demonstrated more of a control dimension as compared to students of the other levels. In their conclusion, they proposed that in order to understand students mathematical valuing across grade levels better, it is important to conduct research that would provide evidence on how students' values reflect those of their teachers and curriculum materials such as syllabus and textbooks. Nevertheless, the study provided enough evidence that suggests values held by students keep on changing from one grade level to another and it is, therefore, important to investigate the values possessed by trainee teachers at the colleges of education level in Ghana.

A comparative study conducted by Dede (2012) investigated values in mathematics teaching among nine Turkish and 13 German mathematics teachers using a questionnaire consisting of Likert-type multiple-choice and open-ended questions. After categorizing the teachers' values in isolated thinking and connected thinking reported some similarities and differences among both groups of mathematics teachers. German mathematics teachers' responses related to the category of "*Isolated Thinking*" were grouped under eleven sub-categories; intellectual activity (rationalism, mental development), logical thinking (rationalism, reasoning), generalization (rationalism, progress), regularly (rationalism), systematic (rationalism), cognition (rationalism, process), structural (rationalism), clear scientific (clarifying, openness), abstract (rationalism), accurate (rationalism), and precise (rationalism). Among others,

rationalism as a mathematical value was more dominant. Besides, the values of mental development, reasoning, progress, process, and openness were also demonstrated. Under connected thinking, German mathematics teachers' values were grouped into fourteen sub-categories as; usefulness in the real world (relevance, usefulness) language (communication, tool), flexibility in thinking (progress, flexibility), aesthetics (esthetics), understanding real-world (tool, control), culture (communication, tool), applicability (applicability), combination (tool), problem-solving (process, progress), composition (tool), creativity (creativity, intuitiveness, mystery), game (fun, enjoyment), classification (tool, process), and historical background (process). Looking at these sub-categories, it is clear that German mathematics teachers were more dominant in values of usefulness, communication, tool, flexibility, applicability, esthetics, process, creativity, and fun.

However, Turkish mathematics teachers' responses to the isolated thinking category were grouped into four sub-categories as; intellectual activity (rationalism, mystery, mental development), logical thinking (rationalism, reasoning), calculation (rationalism), and intelligence indicator (rationalism, tool). These sub-categories show that Turkish mathematics teachers generally hold the value of rationalism. Besides, they emphasize the values of mental development and reasoning. Turkish teachers also showed more evidence of mathematics educational values such as usefulness, communication, flexibility, aesthetics, tool, economics, product, consistency, and applicability underconnected thinking. Based on the above, it is clear that both mathematical values and mathematics education values can be conceptualized effectively for a broader understanding of the value concept in mathematics education.

Seah *et al.* (2017a) reported that primary and secondary school students in Japan highlighted nine valuing attributes in mathematics learning to be creativity, discussion, ICT, know-how, mystery, others' involvement, reality, results, and wonder. A similar study involving primary and secondary students in Hong Kong showed that learners valued alternative approaches, applications, effort, exposition, explorations, feedback, ICT, (mathematics) identity, and recall so far as learning mathematics is concerned. In another study, Hill, Hunter, and Hunter (2019) also reported demonstration of utility, peer collaboration/group-work, effort and practice, and family/familial support as attributes of mathematics learning that New Zealand primary school learners consider important.

Summary of Literature Review

This chapter concentrated on the review of the related literature that was in line with the purpose and objectives of this study. The review started with the conceptual review, a conceptual framework that showed all the variables, relationships and processes involved in the study based on which the relationships from the study were generated. The study variables conceptualized were reviewed and their operational definitions were provided for clear and coincided measurement. These included mathematical values, mathematics education values, mathematics tutors' values, trainee teachers' values, value alignment between trainee teachers and their tutors, value-centered mathematics curriculum and ended with an empirical review. The summary of the review which highlights the main gaps identified in the literature which informed the focus of this study was as well elaborated.

Based on the literature reviewed, it is clear how important values are in mathematics education (Hill, Hunter, & Hunter, 2019; Seah, 2002). More

attention must be geared towards its development and awareness in our educational system. It is also evident that generally little has been done in the area of mathematical and mathematics education values research (Carr, 2019; Zhang, 2019), especially in Africa and specifically Ghana. Besides, these few studies conducted largely concentrated on the values exhibited by students from the perspective of only students and at the pre-tertiary level while few also investigated values of mathematics teachers. Moreover, there is no or little evidence that shows that a relationship exists between teachers' and their students' values when it comes to mathematics learning (Bishop, FitzSimons, Clarkson, & Seah, 2000; Davis, Carr, & Ampadu, 2019). It is also important to mention that few studies were conducted to identify the mathematics values that are more dominant (highly prioritised) and the ones that are less dominant (least prioritised), such studies failed to find the reasons for such least demonstration of some values. Additionally, these studies could not suggest strategies that might help improve such least prioritised values exhibited by students. Arguably, there is a scarcity of empirical research studies on values that deal with what tertiary students consider valuable in their learning experiences (Zhang, 2019). The only empirical study that was cited on curriculum analysis was Cao et al. (2006) which investigated the values projected in China and Australian mathematics textbooks. This implies that, if any, limited studies have investigated the values projected by the mathematics curriculum in Ghana. The choice of the topic for this study was largely influenced by the proposal for research that would provide evidence on how students' values reflect those of their teachers (Davis Carr & Ampadu, 2019). It is clear from the literature that no known study has been conducted on value and its' alignment in mathematics between trainee teachers and their tutors at the tertiary level, more especially at

the colleges of education level in Ghana. Therefore, this study is to fill the gaps that exist in the literature on values in mathematics learning at the colleges of education in Ghana.

The next chapter presents the methods that were used to explore the formulated research questions and the hypothesis for the study.



CHAPTER THREE

RESEARCH METHODS

Overview

This study aimed at exploring what trainee teachers and their tutors' value in mathematics learning and, it also sought to ascertain whether what tutors value in their students' mathematics learning aligns with what trainee teachers value at the colleges of education in the Ahafo, Bono and Bono East Regions of Ghana. This chapter, therefore, presents the methodological procedures and tools used in carrying out the study. For this purpose, the processes involved in the conduct of the study were outlined in this section as; research design, study area, population, sampling procedure, data collection instruments, data collection procedure and, data processing and analysis. Besides, the chapter also discusses the validity and reliability of the research instruments as well as how ethical issues were addressed. The chapter ends with a summary.

Research Design

Research paradigm refers to the philosophical worldview (Aliyu, Bello, Kasim, & Martin, 2014; Cohen, Manion, & Morrison, 2018; Creswell & Creswell, 2018) of the existence of knowledge or truth from the perspective of a researcher. It includes an integrated association of concepts, variables, and problems with its respective methodology and appropriate tools. The processes involved in arriving at the findings and conclusions in research are that it should be able to stand in the midst of rival hypotheses. This means the study was not limited to a specific research paradigm but the combination of both positivist and interpretivist research paradigms to obtain more stable findings that can be defended in the midst of a rival hypothesis. The researcher believes that the

study would have a stronger foundation if positivist and interpretivist paradigms are triangulated. The pragmatic paradigm is, therefore, the resultant when both positivist and interpretivist paradigms are triangulated. Pragmatists believe that reality is constantly renegotiated, debated and interpreted, and therefore the best method to use is the one that solves the problem (Creswell & Creswell, 2018).

From the ontological perspective, concepts and variables (mathematical and educational values) exist naturally in the social environment and, there is a causal relationship between mathematics tutors' and trainee teachers' concepts of values. This study aligns itself with this assumption by investigating the relationships between variables (mathematical/mathematics educational values) as identified by Bishop (2008, 1991, 1988); Dede (2006) and Seah (2011b). It is also a known fact that values are socially and culturally constructed through formal or informal interactions and for that matter, can best be understood through the interpretation of the concepts or constructs in a defined social context. The classroom interactions that occur between teachers, students and peers resulting in an effective classroom discourse serve as means of transmitting both mathematics values and knowledge that make sense in terms of a defined socio-cultural context.

The epistemological position of this study exists in two main directions. The first perspective deals with the ideological (Rationalism and Objectism) perspective of mathematical values. This aligns itself with the values people have about mathematics in terms of dependency on theories, logic, and hypothesis (Bishop *et al.*, 2000) and that the mathematical (solutions and explanations) truth is absolute and infallible. This power of mathematics lies in its nature, objects, and symbols that are used to represent abstract mathematical language. This ideological epistemological perspective situates itself with the

positivist epistemological worldview. However, the second perspective emphasized both the sentimental (control and progress) and sociological (openness and mystery) perspectives of mathematical values. The former deals with the applicability of mathematical knowledge to real-life situations and the provision of solutions to social problems. These are achieved through the manipulation and interpretation of mathematical knowledge, objects, and symbols. This makes mathematical truth and knowledge fallible. The latter also opens the mathematical truth and knowledge for verification to society. Hence, mathematical knowledge can be accessed or verified by individuals interested or who find it useful. The surprises (mystery) in its (Bishop, 1999; Seah & Bishop, 2000) own nature makes individuals or society wonder, and when polished (manipulated) it entertains. Also, teaching and learning mathematical truth and knowledge are sociocultural and are interpreted to make more sense to a social group of people through interactions. These satisfy the assumption that knowledge from multiple sources is more reliable than from a single source and this aspect of the epistemological worldview attaches itself to the interpretivist point of view.

Consequently, this study employed the sequential explanatory mixedmethods design (SEMMD) approach. It incorporates both quantitative and qualitative approaches to data collection and data analysis. This is to ensure higher internal validity, external validity, reliability, and objectivity through triangulation, confirmation, and further explanation of data collection procedures and data analysis.

However, more people are becoming aware and cautious of the importance that research play in the socio-cultural development of every nation and the world at large. Therefore, every researcher must be mindful of any

information that is put out there as it has a great influence on people. Hence, to present a relatively impartial situation about a problem, multiple sources (Fraenkel & Wallen, 2009) of data are required to cross-check, confirm or support the results from each data source (triangulation). For this reason, mixed methods are becoming increasingly popular (Chan & Wong, 2019) among the research community. Mixed methods involve combining or integrating qualitative and quantitative data in a single research study. A sequential explanatory mixed methods design (SEMMD) was selected for this study where quantitative data were collected first and analysed to obtain the results based on which a follow up qualitative data were collected to provide further explanations for the attributes valued in the quantitative data (Creswell & Creswell, 2018; Cohen et al., 2018). The researcher must identify which part of the quantitative data needs to be explained or confirmed and design qualitative instrument(s) that can solicit the needed data for the explanations. This mixed-methods design allows the researcher to seek further and better explanations to support the attributes that would be valued by both trainee teachers and their tutors in their students' mathematics learning through quantitative analyses. The operational structure and processes in the sequential explanatory mixed-methods design are shown in Figure 2 below.

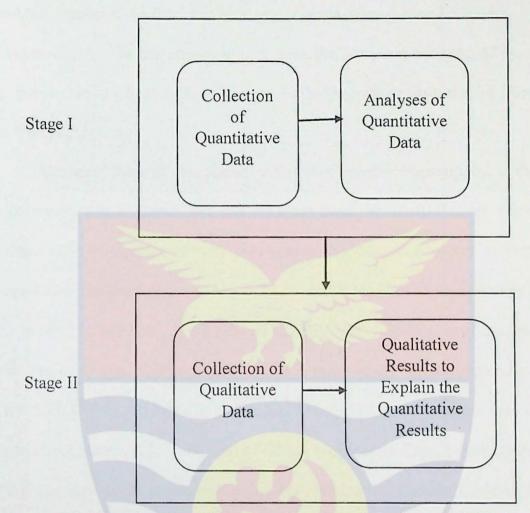


Figure 2: The Operational Structure of Sequential Explanatory Mixed-Methods Design (SEMMD)

SOURCE: Adapted from Cohen et al. (2018).

Study Area

The study was conducted in three (3) regions of Ghana (Ahafo, Bono and Bono East Regions) formerly the Brong-Ahafo Region of Ghana. Ghana is situated on the coast of the Gulf of Guinea in western Africa, sharing borders to the northwest and north with Burkina Faso, to the east with Togo, to the south with the Atlantic Ocean, and the west with Côte d'Ivoire. Ghana lies only a few north of the Equator, degrees therefore giving it a warm climate. Ghana occupies an area of 238,535 km². Ghana can boast of more than

seventy ethnic groups. The major ethnic groups in Ghana include the Akan, the Mole-Dagbane, the Ewe, Ga-Adangme, Gurma, Guang, Grusi, Kusaasi, Konkomba, etc. The largest ethnic groups are the Akans representing 47 % of the population in Ghana followed by the Mole-Dagbane constituting 16 % of the total population.

However, three of the selected colleges of education are situated in the Bono region, one in Bono East, and the other from the Ahafo Region. These colleges offer a Bachelor of Education general (Mathematics as core subject) programme. The three regions (Ahafo, Bono, and Bono East regions) occupy a land area of 11,481 Km²; 5,193 Km² and 22,952 Km² respectively of the total land area of Ghana. The population of these three newly formed regions are 1,168,807 (3.93 %), 613,049 (0.21 %), and 1,034,421 (3.47 %) respectively of the total population (29.77 million) of Ghana (Population Census Projections, 2020) and most of its populace are farmers (Ministry of Gender, Children & Social Protection - MGCSP, 2014) growing both cash crops (Cocoa, Cashew, Coffee, Rubber, and Tobacco) and food staffs (maize, yam, rice, etc.). They can also boast of civil servants like teachers, nurses, doctors, security officers, etc. The people in these three regions are mostly from the Akan ethnic group who speak Twi as the local language. This means trainee teachers admitted into colleges in these regions are made to study the Twi language as their second language.

For educational institutions, the three regions currently house five universities (two public and three private universities), five colleges of education (all public), and quite many Senior High Schools (both public and private). The location of the three regions (Ahafo, Bono, and Bono East Regions of Ghana) involved in this study are shown on the map below.

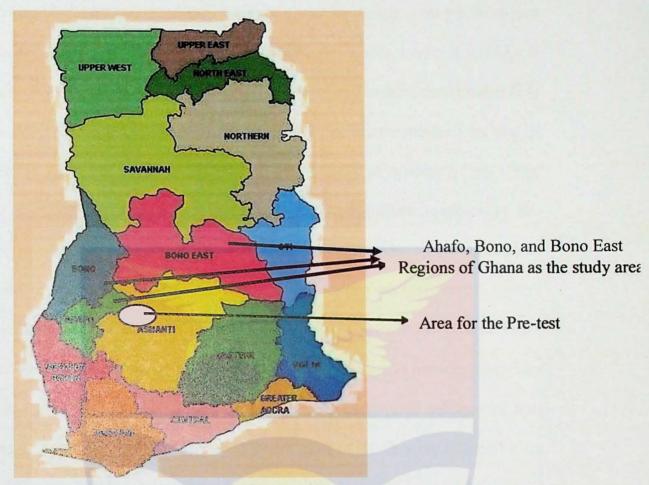


Figure 3: Map Showing the Study Area in Ghana. SOURCE: Adopted from Historical Dictionary of Ghana (Owusu-Ansah, 2018)

Population

The general population for the study was made up of all trainee teachers (5,077) and their mathematics tutors (34) in the five selected colleges of education in the Ahafo, Bono, and Bono East Regions (formerly Brong-Ahafo Region) of Ghana. For anonymity, the colleges have been coded as colleges A, B, C, D, and E. Hence, the population distribution of all the trainee teachers across the five selected colleges is; 813, 1693, 600, 681, and 1290 for colleges A, B, C, D, and E respectively. The target population for the study was all 2020/2021 academic year third-year trainee teachers (1, 487) in the five selected colleges of education and all their (34) mathematics tutors. The distribution of the target population of trainee teachers in the selected colleges is as follows; College A (172); College B (525); College C (190), College D (220) and

College E (380). The accessible population and the sample were the third-year trainee teachers (1, 050) offering Mathematics as a core subject (A (134); B (231); C (85); D (220) and E (380)) and their respective mathematics tutors (34) in the five selected colleges of education in three aforementioned regions of Ghana. All the selected colleges started the Bachelor of education programme in the 2017/2018 academic year and admitted trainee teachers to the mathematics core programmes in the same academic year. The programmes, curriculum and courses for all these third-year trainee teachers were designed, monitored, and supervised by the University of Cape Coast, Ghana. These colleges have equivalent teaching-learning resources such as libraries, computer laboratories and lecture rooms. The trainee teachers in these colleges are also taught by qualified and experienced tutors most of which were trained by the same educational universities in Ghana. Their tutors sometimes attend workshops, conferences, and seminars organized by both the University of Cape Coast and Transforming Teacher Education and Learning (T-tel). These make the subjects or participants possess common characteristics (homogeneity) and, as such all other factors are marginalized to reduce their tendency of influencing the outcome of the study.

Participants

Mathematics tutors

A total of thirty-four (34) mathematics tutors consisting of 31(91.18%) males and 3 (8.82%) females were purposively sampled from the five (5) selected colleges of education that were used in this study. The distribution of the tutors from colleges A, B, C, D and E were 5, 10, 8, 5 and 6 respectively.

Third-year trainee teachers

The third-year Primary education and JHS education trainee teachers (taking mathematics as a core subject) from the five selected colleges of education in the Ahafo, Bono and Bono East Regions took part in this study. A total of one thousand and fifty (1,050) third-year trainee teachers who participated in this study were made up of 575 (54.80 %) males and 475 (45.20 %) females, of which 134 of them were from College A, 231 were from College B, 85 were from College C, 220 of them from College D and 380 from college E.

Sampling Procedure

This study aimed at exploring what trainee teachers and their tutors' value in mathematics learning and, it also sought to ascertain whether what tutors value in their students' mathematics learning aligns with what trainee teachers value at the colleges of education in the Ahafo, Bono and Bono East Regions of Ghana. To achieve this aim, a multi-stage sampling technique was used. This allowed for different sample techniques to be employed at different stages of the data collection.

In the first stage, based on the National Conference of Principals of Colleges of Education (PRINCOF) zoning of the colleges of education into five (5) zones namely Ashanti-Brong Ahafo (ASHBA), Eastern-Greater Accra (EAGA), Central-Western (CENTWEST), Northern and Volta. One zone (Ashanti-Brong Ahafo, ASHBA) was selected randomly. The ASHBA zone was then split into two, Ashanti and Brong-Ahafo, from which the researcher randomly selected Brong-Ahafo (now Ahafo, Bono and Bono East Regions of Ghana).

In the second stage, a purposive sampling technique was used to select the five colleges of education, all the mathematics tutors (34) and the third-year trainee teachers (1,487) because as compared to the first and second years trainee teachers, they have had more exposure to mathematics at the colleges of education. The third-year trainee teachers taking mathematics as a core subject (1,050) were used because they are more likely to teach mathematics at the primary school level.

In the third stage, a systematic sampling technique was used to select 50 third-year trainee teachers (10 from each college) out of the main sample for focus group interview sessions. The systematic sampling technique was applied as follows:

A frequency interval (*f*) was calculated first as the total number of group population being represented divided by the sample size required.

That is, $f = \frac{N}{Sn}$, where;

f = frequency interval

N = the total number of the group population

sn = the required number in the sample.

Note that after the starting point of the selection has been made randomly, every f^{th} person is selected until the required sample is achieved.

Hence, for college A, N = 134 and sn = 10; this means f = 13.4 (which was rounded to 14.0).

Hence, every 14th trainee teacher was selected after selecting the starting point. The selection continued until all 10 trainee teachers were obtained from college A to respond to the focused group interview guide items. Note that, the first person from the ten trainee teachers required from college A was randomly

selected and that the particular sample of the ten trainee teachers could have changed if another trainee teacher had randomly been selected first. That means settling on a particular trainee teacher as a first person to be selected was purely based on randomization. Repeating the above process for colleges B, C, D, and E yielded 23^{rd} , 9^{th} , 22^{nd} and 38^{th} frequency intervals from group populations of 231, 85, 220, and 380 respectively, based on which 50 trainee teachers were obtained for the focus group interview sessions.

Data Collection Instruments

Two main data collection instruments were designed for this study namely questionnaires and interview guides. Both the questionnaire and the interview guide were of two kinds with parallel items. One for the mathematics tutors and the other for the trainee teachers. The questionnaires were adapted from the WIFI instrument questionnaire and were designed based on the constructs operationalized in the conceptual framework. The questionnaires were designed to four-point Likert-scale type using Absolutely Important (AI = 4) as the highest scale, Important (I = 3), Unimportant (U = 2) to Absolutely Unimportant (AU = 1) as the lowest scale. Each questionnaire was divided into two sections of which the first part sought biographical information about the respondents, and included age, gender, programme, and the college of the respondent. The other section was made up of 64 items (values in mathematics learning) that elicit the values of trainee teachers and that of their tutors in their students' mathematics learning. The questionnaire instrument items were constructed in the English language since that is Ghana's official mode of instruction at all levels of the country's educational system.

The most widely used value research instrument in recent years has been the "What I Find Important (WIFI)" instrument which was developed by Seah

(2011b) to measure students' values in their mathematics learning experiences. The WIFI instrument has been used to conceptualize both mathematical values (Anderson & Osterling, 2019; Dede, 2019) and mathematics educational values (Davis *et al.*, 2019; Davis *et al.*, 2021; Hill *et al.*, 2019; Madosi *et al.*, 2020; Seah *et al.*, 2017a; Zhang, 2019) which the instrument was originally made to measure. In Ghana, the WIFI instrument was first used by Seah *et al.* (2017b) and Davis *et al.* (2019) when they explored 'valuing in mathematics leaning across grade levels' and was adapted into a five-point Likert- scale type questionnaire by Davis *et al.* (2021) when they explored 'the attributes of mathematics learning which Ghanaian senior high school students value'. Hence, the WIFI instrument has been proven to work in the context of Ghana, especially at the pre-tertiary levels of the educational system.

However, the WIFI instrument has been critiqued for its' limitation in covering all the components of mathematical values (Anderson & Österling, 2019) and that its items are mostly classified under values of openness, progress, and rationalism. Hence, the original WIFI instrument has weaknesses in reporting on mathematical values of objectism, control, and mystery. Therefore, the WIFI questionnaire was modified to cover all the components of mathematical values based on Bishop's (1988) categorization. Again, the WIFI instrument is known to provide only quantitative data and does not provide qualitative information on the respondents. Again, given this weakness of the WIFI, the current study employed a sequential explanatory mixed-methods design to provide reasons for exhibiting the identified values in mathematics learning observed as a result of the quantitative analysis. However, currently, there is no known value instrument in mathematics education that measures values from both students' and their teachers' perspectives. Therefore, the

instrument was adapted in two folds, one to measure trainee teachers' valuing and the other to measure mathematics tutors' valuing in their students' learning (see Appendices A and B respectively). The questionnaires were contextualized to suit Ghanaian mathematics classrooms and, more especially in the colleges of education.

The interview guide was made up of opened-ended semi-structured items that sought to find out the trainee teachers' and tutors' reasons for demonstrating the observed attributes valued in mathematics learning. The focused group interview guide items were made up of two main opened-ended questions and their corresponding follow-up questions depending on the views expressed by the respondents. The first interview guide item was constructed to find out why the observed values from the quantitative analysis for each of the two groups were important to the trainee teachers' mathematics learning. The second interview item was also intended to seek views on attributes that were not loaded in other group values (see Appendices C and D respectively).

Dealing with threats to validity of instruments

In dealing with threats to validity in research, the researcher must indicate the grounds and the evidence that were used to connect their data with the claims made from or conclusions drawn from the data collected (Cohen *et al.*, 2018; Creswell & Creswell, 2018; Creswell, 2012). It is therefore important for the researcher to demonstrate the logical link made between data collected and propositions, data and conclusions that support the sense of explanations given in the face of alternative or rival hypotheses. In this section, the researcher looked at different kinds of validity (Content-Related, Construct-Related, Internal-Related, External-Related and Cultural-Related) and how their threats were neutralized in the study.

Content-related validity of instruments

The content-related evidence of validity refers to the content and format of the instrument in the sense that it covers fairly and comprehensively the domain or items that it intends to cover (Cohen et al., 2018; Creswell & Creswell, 2018; Creswell, 2009; Fraenkel et al., 2012). In this direction, all the domains of the mathematical values identified in the literature were considered in this study as reflected in the operationalization of the conceptual framework and the construction of both questionnaires and the interview guide items. The components of the concept of mathematical values were operationally defined in terms of mathematics classroom practices as existing at the colleges of education in Ghana based on Bishop (1988) and Dede (2006) three complementary pairs of mathematical values as Sentimental, Ideological and Sociological values which were further categorized into their sub-constructs respectively as; control-progress, rationalism-objectism, and openness-mystery. These are the known and exhaustive variables for measuring an individual's mathematical and mathematics education values, including technology which was all considered in the operationalization and defined in the sense of this study. Besides, both questionnaires and interview guide items were carefully sampled to obtain a fair representation of all the variables. Also, the interview guide items were carefully crafted to solicit all possible reasons for demonstrating the identified mathematics educational values that aroused from the quantitative data analysis. In conclusion, the content of values was exhaustively covered and fairly sampled for the construction of both questionnaire and interview guide items used for the data collection in this study. Hence, content-related validity was appropriately dealt with in this study.

Construct-related validity of instruments

The construct-related evidence of validity involves the definition, operationalization, and exhaustion of elements of the studying constructs in a research project (Cohen et al., 2018; Creswell & Creswell, 2018; Creswell, 2009; Fraenkel et al., 2012). Given this, the constructs involved in this study were generated from the extensive literature review on the concept of mathematical/mathematics educational values. The constructs were defined following the definitions provided by different researchers (Bishop, 1988; Dede, 2019, 2006; Seah, 2011a) in the field of mathematics education. The elements for each construct were demonstrated and represented in the instrumentation of both questionnaires and the interview guide items used in measuring the respective constructs that were involved in the study. For instance, the construct objectism was operationally defined as mathematics classroom activities that encourage symbolization and concretization of mathematical ideas and its elements listed to be the use of teaching-learning materials, use of mathematical symbols, formulas, looking for mathematics in real-life situations, showing of correct working, etc. which were reflected in the instrumentation accordingly.

This means findings and inferences made from this research study reflected the views and opinions of trainee teachers and their tutors on the concept of mathematical/mathematics educational values. As such, specific behaviour obtained as a result of this study was also defined in terms of highly prioritised (more dominant) and least prioritised (less dominant) values depending on the level of demonstration or exhibition (using mean ratings as the yardstick for measuring) of each construct by trainee teachers and their mathematics tutors at the selected colleges of education in Ahafo, Bono and

Bono East Regions of Ghana. Hence, the above demonstrations indicate how threats to construct validity were neutralized in this study.

Internal-related validity

Internal validity in educational research refers to the truthfulness and correctness of the inferences drawn from the findings and conclusions within the defined context or group of people (Cohen *et al.*, 2018; Creswell, 2009). Hence, the reason for ensuring that internal validity issues are dealt with in a research study is to make sure that no alternative hypotheses exist to explain the outcomes of the study. For that matter, any relationship observed between the variables themselves, and the groups are solely based on the data collected. Besides, when threats of internal validity are addressed in a study it increases the applicability, consistency, neutrality, dependency, and credibility in its interpretations and conclusions within a particular setting or group of people (Cohen *et al.*, 2018). This section, therefore, presents evidence of how the threats to internal validity were controlled in this study.

To begin with, the threats to internal validity can be minimized when convenient or purposive sampling is used in research by providing adequate information on the subjects (sample) that suggest homogeneity (Fraenkel *et al.*, 2012) among the sample used for the study. The common characteristics shared by the sample have been demonstrated in this chapter under the description of population and sample selection. The homogeneity was explained in terms of common characteristics shared between the selected colleges, between the thirdyear mathematics core trainee teachers and their tutors used in this study. Besides, due to the use of the mathematics core intact classes, the trainee teachers did not show many disparities in their ages and were drawn from almost the same catchment areas for admission into the selected colleges of education.

The administration and collection of both quantitative and qualitative data were planned to take place within 4 weeks to reduce the tendency of mortality and maturation among respondents. A high return rate of the questionnaire instruments was achieved due to the effective organization of the administration of the questionnaires.

On the part of instrumentation, all the questionnaire items were constructed using a uniformed four-point Likert scale from Absolutely Important (AU = 4) as the highest to Absolutely Unimportant (AU = 1) being the lowest scale. The questionnaire items were checked for simple errors to be corrected before processing them for analysis. The appropriate statistical tools (Principal component analysis, correlation and independent samples T-test) in the SPSS software were used for the quantitative analysis. For the qualitative data, the transcriptions of the interview responses were done by two independent experts in mathematics education after which the results were compared for the sake of data triangulation. Also, the transcriptions were sent to the respondents just to be sure that the transcriptions reflected the opinions expressed by the respondents before using them in the write-up.

The use of the sequential explanatory mixed methods design (SEMMD) enabled the researcher to look at the issues reported from multiple perspectives (Creswell, 2009) in order not to report a narrow view of the research problem and to present the findings as objectively as possible. The sequential explanatory mixed methods design also allowed for further explanations and reasons to be sought for a better understanding of the issues being studied. Finally, the findings from the study were sent to the selected colleges of education for mathematics tutors and administrators to study and for suggestions before making the findings public.

© University of Cape Coast https://ir.ucc.edu.gh/xmlui External-related validity

To ensure high external validity of a study is to provide enough evidence that suggests the findings of the study can be generalized to a wider population, cases, settings, times, or situations (Cohen *et al.*, 2018; Creswell & Creswell, 2018; Fraenkel *et al.*, 2012). This means the inferences drawn from the findings of the study must be transferable based on the characteristics highlighted above. Hence, the researcher demonstrates how external validity issues were dealt with in the study in this section.

To start with, the purpose of the study was to explore what trainee teachers and their tutors value in mathematics learning and, it also sought to ascertain whether what tutors value in their students' mathematics learning aligns with what trainee teachers value in the colleges of education in the Ahafo, Bono and Bono East Regions of Ghana. The study was delimited to the trainee teachers and their mathematics tutors in the five selected colleges of education. The study was further designed to focus on mathematics tutors and their trainee teachers taking mathematics as a core subject per the new four-year Bachelor of Education programme introduced in the colleges of education. The sample was 1,050 third-year (core mathematics) trainee teachers out of 1,487 total trainee teachers admitted to the selected colleges of education in the 2017/2018 academic year. For the trainee teachers interviewed, 50 (10 from each college) participants were systematically sampled from the five selected colleges. The sample was considered to be sufficiently large per Creswell and Creswell (2018) and Fraenkel et al. (2012) who suggested a sample of 100 to be sufficiently large out of 1000 participants for mixed-methods designs. All the mathematics tutors in the selected colleges (34) were used for the study. Due to the limited number of mathematics tutors in the selected colleges used in this study, both

quantitative and qualitative data were collected to obtain fair, multiple, and explanatory views from the respondents. This was supported by the assertion that views from multiple sources are more likely to be valid and reliable than views from a single source (Seah, 2019).

Based on these interventions used in the study, it was, therefore, possible to generalize the findings to cover the accessible population of mathematics tutors and their trainee teachers taking mathematics as a core subject at the selected colleges of education in the Ahafo, Bono and Bono East Regions of Ghana, but not to generalized to cover all trainee teachers in the Ahafo, Bono and Bono East Regions or all trainee teachers in Ghana. It is also important to emphasize that due to the design and the structure of the study variables, it is likely to produce similar findings when replicated over a similar environment (context) or over a period of time. These strengths of the study lie in the fact that the study's variables were well defined and operationalized for clear understanding and implementation in the field of mathematics education. In conclusion, the findings of this research study would be generalized within the confines (accessible population) of the selected colleges of education.

Cultural-related validity

The concept of values is known to be socio-culturally constructed (Seah & Bishop, 2000) through the interactions that take place in and out of the mathematics classroom. For this reason, the instruments used in collecting data on views or opinions on individuals valuing in mathematics learning must be able to provide culturally valid information (Cohen *et al.*, 2018; Creswell & Creswell, 2018; Creswell, 2009) based on which relevant and meaningful inferences can be made from its findings. In line with this, the questionnaire instruments were constructed to reflect the college of education mathematics

classroom environment bearing in mind the culture and the belief systems of the people of these areas where the selected colleges are situated including the college used for the pre-test of the instruments. Since the intact classes of core mathematics trainee teachers together with their regular mathematics tutors were used for the administration of the questionnaire, the normal classroom atmosphere (culture) was not interrupted. This made trainee teachers behave normally which ensured good classroom discourse through the interactions.

However, the items on the instrument were constructed in the form of a four-point Likert scale type from Absolutely Important (AI = 4) to Absolutely Unimportant (AU = 1). Some of the items were modified or replaced to get a fair representation of all the variables in the study. The questionnaire items were constructed to reflect the practices that occur in the colleges of education situation and context of Ahafo, Bono and Bono East Regions of Ghana. The interview guide items were self-constructed by the researcher to reflect the college of education situation and the culture within the communities where the study was conducted. The English language was used in the construction of the instruments which were employed for the data collection. The English language has been the mode of instruction in Ghanaian schools due to the colonization by the British and for that matter, reading and understanding the instrument items was not a problem for both mathematics tutors and trainee teachers. There were no communication barriers observed because the researcher originates from the area (locality) where the study was conducted. Notwithstanding, the instruments were further explained by the researcher to all the respondents for more clarification and understanding. Hence, the inferences drawn from the findings of this study would be more meaningful and relevant to societies where the selected colleges are situated and Ghana as a whole.

Dealing with threats to the reliability of instruments

Reliability is concerned with precision and accuracy in measuring a given variable. From the perspective of research, reliability is the ability to demonstrate that if the study is replicated on a similar group of respondents in a similar defined context it would yield similar results. This calls for internal consistency in the research instruments used for the data collection. To achieve high reliability in this study, the researcher validated the questionnaire instruments to fit well with the colleges of education mathematics classroom situation and the context of the Ghanaian mathematics classroom. The items on the questionnaires (for both tutors and trainee teachers) covered all the variables involved in the study. The questionnaires were given to colleagues who are experts in mathematics education to check for mistakes and effect necessary corrections. Besides, the questionnaires and interview guides were pre-tested in a nearby college that shared similar characteristics as the colleges selected for the actual research study. Based on the results obtained from the pre-test as the result of the Principal Component Analysis, the scale constructions were made to determine the reliability coefficients of the constructs using Deville's (2003) reliability benchmarks which indicated acceptable Cronbach alpha values of minimum (0.791) and maximum (0.812) on the reliability scale. In the process, the weaker questionnaire items were deleted or reconstructed to ensure a highreliability coefficient for each construct observed in the study.

Besides, the items on the interview guides were also constructed to sought for tutors' and trainee teachers' reasons for demonstrating the identified mathematics learning attributes valued at the colleges of education classrooms. These items were carefully crafted to bring out trainee teachers and their tutors' reasons for the purpose and objectives of the study. The hypothetical nature of

the items had the tendency of bringing out respondents' reasons unconsciously to reflect all the concepts and themes aroused from the quantitative data analysis. The interview sessions were all (both pre-test and the main study) done by the researcher himself to ensure consistency in the interview process and the questioning. The data collected from the pre-test interview sessions were transcribed to see whether the responses obtained were in the position to provide answers and explanations to the research questions that guided the study. The results and findings from the main study were objectively reported without imposing the researchers' perspective or influencing the outcomes. The sequential explanatory mixed-methods design approach used allowed the results and findings from the quantitative approach to be explained by the qualitative data for better understanding.

Data Collection Procedure

Following the institutional ethical clearance approval, a formal introduction letter from the Department of Mathematics and ICT education, the University of Cape Coast, and an official letter from the researcher were sent to the respective colleges to seek permission and agreed on the proposed date for the data collection. Further discussions were made with the heads of departments who helped in the data collection process. On the day of the administration of questionnaires, a formal introduction was made to the principals of the respective colleges of education, mathematics tutors, and their trainee teachers. The purpose of the study was then explained to them and the questionnaires were administered to the trainee teachers by the researcher personally with the assistance of the various respective mathematics heads of department (HOD).

All trainee teachers in the core mathematics intact classes were used for the administration of the questionnaire. They were made to sit in their lecture rooms and were given copies of the questionnaire to respond to. The items on the questionnaire were then explained to them. Trainee teachers were given adequate time to read and respond to the items. Close supervision was made so that the participants did not have the chance to confer or discuss with each other.

For the tutors, the questionnaires were administered individually depending on their free periods on that particular day at the mathematics department offices and in one college in the staff common room due to the absence of a departmental office. The allocated time for responding to the questionnaire items was 30 minutes for both trainee teachers and their mathematics tutors.

Two weeks after the analysis of the questionnaire responses (quantitative data) to obtain the attributes valued by the trainee teachers and their mathematics tutors, the researcher went back to the respective colleges to conduct the focus group interview sessions for all the mathematics tutors and the systematically sampled trainee teachers (10 from each college). A process that had already been followed for the pre-test of the instruments. The interview sessions were conducted for each focus group (trainee teachers and mathematics tutors in each college) at the mathematics department offices and one college in a classroom after permission and assistance from the mathematics HOD in the respective colleges were sought. The interview sessions were conducted by the researcher himself to ensure the confidentiality of responses and views. The sessions were conducted in turns during their break time on that day. Each interview session took a maximum of 45 minutes depending on the follow-up questions that aroused during the interview. The responses resulting from the

interviews were recorded using the coding book and, two tested and verified audio recorders to cater for any unforeseen circumstances during the interview sessions.

Data Processing

For the quantitative data from the questionnaires, the responded questionnaires were checked for facial errors such as double ticking and unticked items. After all the questionnaire items had been entered into the SPSS application, the responses were coded using the four Likert scale given on the questionnaires. Coding was as follows: Absolutely Important = 4, Important = 3, Unimportant = 2 and Absolutely Unimportant = 1. This was to convert the ordinal data to numerical data in order to carry out the various quantitative analyses. The data was then cleaned for obvious entry and typographical errors in the SPSS application. The questionnaire items that showed negative correlations during the test run were recoded for them to correlate positively before the actual run of the analysis.

For the qualitative analysis, the statements from the interview session recordings were listened to carefully by two independent mathematics education experts (the researcher himself and one other mathematics tutor) and wrote exact statements played from the recorded audio tapes in a transcription book after listening to the same statement several times. The written statements were then rewritten to read well and compared to the mathematics education value that was under discussion during the interview session. This is then assigned to the appropriate attribute valued as a reason. However, if the statement is not clear in the first instance, then the statement is reframed making sure that the keywords or the themes are maintained in order not to alter the main idea of that statement or sentence. This process was repeated for all the

five colleges selected for the study and all the reasons gathered for each mathematics education attribute valued by both trainee teachers and their tutors were generated. These included attributes valued by trainee teachers and not their tutors and vice versa. The statements were transcribed using themes and sentence meaning developed (Hill, Hunter & Hunter, 2019) from the concepts operationalized in the study. To ensure the reliability of the qualitative data, a coding book (Yin, 2009) was used to record the definitions and themes of concepts used in the study. The transcriptions were checked to avoid obvious mistakes made during the transcription process (Gibbs, 2007). The transcription was independently done by the researcher and another independent expert in mathematics education after which the resulting responses were compared for triangulation purposes to increase the validity and reliability of the semistructured interview guides.

Data Analysis

To answer the research questions raised, both quantitative and qualitative data were collected and analyzed based on the sequential explanatory mixed-methods design (SEMMD) that was used for the study.

Quantitative analysis

For the quantitative analysis, both descriptive (means and standard deviations) and inferential statistics (principal component analysis, correlational, and independent samples t-test analysis) in the SPSS software application were used to analyze the quantitative data collected for the study. The attributes valued by trainee teachers and their mathematics tutors were obtained through the principal component analysis. The internal characteristics of the named components were reported using the means and the standard deviations. The high and least prioritised values were identified using the means

of the various named component variables (mathematics education values). The relationships between the attributes valued by the trainee teachers and their mathematics tutors were also established by the use of correlational analysis. Lastly, the independent samples t-test was applied to determine if significant differences exist between the common attributes valued by both trainee teachers and their mathematics tutors.

The statistical tools used for the data analyses based on which the results were obtained are presented as follows:

Principal component analysis

The Principal Component Analysis (PCA) is the default analytical tool in the SPSS software for Exploratory Factor Analysis (EFA) in the Factor Analysis (FA). The principal component analysis is used to reduce the data set into a smaller number of linear combinations (Fletcher & Alhassan, 2016) of the initial variables in a way that captures most of the variability in the pattern of correlations. In principal components analysis, the original variables are transformed into a smaller set of linear combinations, with all of the variances in the variables being used (Pallant, 2005).

PCA for trainee teachers' questionnaire

A Principal Component Analysis (PCA) was used to analyze the 1050 trainee teachers' questionnaire responses, the original 64 items were loaded onto 12 components with eigenvalues of more than one, with a cut-off point of 0.4. Out of these 64 items, 15 items (i.e. items; 11, 12, 14, 17, 18, 21, 22, 24, 31, 34, 37, 38, 50, 61 and 63) were deleted since items 17, 61 and 63 loaded onto two different components and the rest showed loadings below the cut-off point of 0.4. The remaining 49 items were subsequently subjected to another PCA for the second time and 9 components were obtained. As a result, 5 items

(i.e. items; 5, 29, 30, 44 and 58) were further eliminated because items 5 and 58 loaded significantly at two different components. However, items 29, 30 and 44 showed factor loadings below the cut-off point.

The remaining 44 items were used to run PCA again for the third time which resulted in 8 components. Whiles item 2 showed double loadings, items 20 and 27 were loaded below the cut-off point and hence, they were deleted.

The 41 items were used for the fourth and the final PCA run which produced 8 components again with no double loadings or loadings below the cut-off point. These 8 components with 41 items retained from the original questionnaire for the trainee teachers demonstrate the existence of values in trainee teachers' mathematics learning at the selected colleges of education in Ghana. The qualitative naming process for these eight value components with their corresponding items that loaded onto them are presented in chapter four.

PCA for mathematics tutors' questionnaire

In conducting the PCA for the tutors' questionnaire responses, the initial 64 items were loaded onto 18 components with eigenvalues of more than one, with a cut-off point of 0.4. However, out of the 18 components, 13 questionnaire items (i.e. items; 11, 14, 18, 25, 31, 32, 34, 36, 38, 45, 49, 50, and 53) were deleted since their factor loadings were below the cut-off point. Items 30, 55, 58 and 62 were also deleted because they were the only items that loaded onto their respective components whiles item 13 was as well deleted since it loaded significantly onto two different components. All 18 questionnaire items were deleted and hence, 46 items were left to conduct the second principal component analysis.

The remaining 46 items were subjected to a second principal component analysis with varimax rotation and again 13 components having eigenvalues

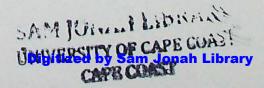
greater than one were obtained. However, out of the 13 components, 5 items (i.e. items; 6, 7, 8, 40 and 56) were deleted for displaying factor loadings below the cut-off point. A third principal component analysis with varimax rotation was conducted on the remaining 41 items and resulted in 12 components having eigenvalues of more than one. However, out of the 12 components, 3 of the items (i.e. items; 3, 21 and 37) were deleted because they loaded below the cutoff point whiles item 27 was deleted for loading onto more than one component. The 38 items retained were subjected to a fourth principal component analysis which resulted in 12 components being generated. Items 2, 27 and 60 were deleted based on the fact that they showed double loadings. This means 35 items were retained after the fourth analysis and therefore, were subjected to a fifth principal component analysis which yielded 11 components. In this case, only item 48 was deleted for loading onto two different components. Hence, the 34 items that were retained after the fifth analysis was used in the sixth principal component analysis and obtained 11 components again, with items 4, 5, 9, 19, 41 and 51 deleted for double loadings and one item loading onto a component alone. In the seventh analysis, the remaining 28 items were used to conduct another principal component analysis which resulted in 9 components. For this stage. 8 items (i.e. 1, 12, 16, 26, 28, 29, 33 and 64) were further deleted for double loadings and the rest loaded onto their components alone. This resulted in 20 items being retained for the eighth analysis and 5 components were observed.

The five (5) attribute components obtained from the remaining 20 questionnaire items showed the existence of values in trainee teachers' mathematics learning at the colleges of education in Ghana from the perspective of their mathematics tutors. The 20 items are presented under the various value

components in chapter four. The five attributes valued were named qualitatively based on what the items under each attribute valued collectively or mostly represent. The naming process and individual value names are presented in chapter four.

Pearson product-moment correlation

Pearson product-moment correlation is a measure of the parallel or complementary relationship that exists between two interval or ratio variables (Chee, 2013). It is denoted by r, and which takes on the range of values from -1 to +1 which is expressed in quantitative form. A value of positive one (+1) means variables are positively related linearly, zero (0) means there is no linear relationship, and a negative one (-1) implies the linear relationship is completely negative. Pearson product-moment correlation is frequently used as a measure of association (Chee, 2013) because it is a simple method of determining the association (relationship) that exists between two variables. Pearson productmoment correlation cannot identify relationships that are not linear. The coefficient of correlation informs the researcher about the association between two variables. However, when more than two variables are included, other measures such as multiple correlations and partial correlations (Fletcher & Alhassan, 2016) are employed. For instance, in the case of multiple correlations, the extent of association between three or more variables is measured at the same time. To this end, a multiple correlational analysis was conducted between individual attributes valued obtained from both trainee teachers' and their tutors' perspectives. Whiles, the bivariate correlational analysis was conducted to determine the overall relationship between trainee teachers and their tutors' valuing in mathematics learning. Both multiple and bivariate correlational analyses were performed to provide bases for answering research question three



which seeks to determine the relationship between trainee teachers and their tutors' valuing in mathematics learning.

Independent samples t-test

The independent samples t-test analytical tool in the SPSS application software is used to compare the mean scores on some continuous variable (dependent variable) for two different groups of subjects (Pallant, 2005). In this study, the continuous variables (dependent variables) were the attributes valued in mathematics learning obtained from the principal component analysis. These attributes in mathematics learning valued by both trainee teachers and their mathematics tutors were computed into continuous (dependent) variable based on the items retained from the questionnaires for the two respective groups. This was done by the use of 'transform and compute variable' in the SPSS software. However, the trainee teachers and their mathematics tutors constituted the categorical (independent) variable. The independent samples t-test analytical tool aided in testing the research hypothesis which sought to test whether significant differences exist between trainee teachers and their tutors' attributes in mathematics learning. The results from the independent samples t-test are presented in chapter four.

Qualitative analysis

The qualitative data collected through the interview guides were analyzed by the use of thematic and sentence-meaning transcriptions. In all, five focus groups were obtained from the five colleges of education for the trainee teachers which were represented as FGSA, FGSB, FGSC, FGSD and FGSE for trainee teachers focus group from colleges A, B, C, D and E respectively. The five focus groups for the mathematics tutors were also coded as FGTA, FGTB, FGTC, FGTD and FGTE for mathematics tutors focus groups from colleges A,

B, C, D and E respectively. These codes were signed to the responses to credit each response to a specific focus group. Different codes may be assigned to one statement if multiple focus groups gave the same or similar reason for the same attribute valued. For analysis, themes were formulated from the grouped statements for each attribute valued by either the trainee teachers or the mathematics tutors. Characteristics expressing common attributes were further grouped and the appropriate name was assigned to that set of attributes that served as a reason for that valued attribute.

The new named themes as a result of the transcriptions were then used as main reasons for demonstrating the observed attribute valued. For instance, some of the responses provided by the trainee teachers to explain why valued attribute 'understanding' was important in their mathematics learning were given as follows:

- 1. "When you form the correct concepts it allows you to apply the right concept in solving mathematics problems".
- 2. "Developing the right concepts allow you to transfer the mathematical knowledge to other academic disciplines such as Physics, Chemistry, Engineering and Economics which are very important in learning other subjects."
- 3. "The essence of learning is to understand and therefore, education is built on sound understanding".
- 4. "Progress can be achieved only through the understanding of concepts".
- 5. "Understanding the concepts allows us to develop our critical thinking abilities".

6. "In Bloom's taxonomy, we need an understanding of concepts to be able to apply, analyze, synthesize and evaluate. This means understanding forms the basis of learning".

In analyzing these statements thematically, all the key characteristics relating to valuing were identified in each statement by highlighting them. The identified key characteristics were then put under one group called 'Subthemes' as; "correct concept, apply, right concept, solving problems, right concepts, transfer, knowledge, discipline, other subjects, understand, built on sound, progress, achieve, concepts, develop, critical thinking, apply, analyze, synthesis, evaluate and basis of learning".

The sub-themes were again grouped based on the common attribute they expressed and these new sets of attributes were named 'Themes'. Each set of themes were then named based on the common attribute that group represented. For example, the theme consisting of value characteristics; "correct concepts, right concept, right concept, knowledge, concepts and basis of learning" was given the name '*knowledge acquisition*'. Also, the theme comprising "apply, solving problems, transfer, other subjects, critical thinking, apply, analyze, synthesis and evaluation was named '*control*'. The last theme for attribute understanding were made up of "discipline, build on sound, basis of learning, progress and achieve" was named '*progress*'. Hence, knowledge acquisition, control and progress were assigned as the reasons why understanding was valued by the trainee teachers. Control was considered to be the dominant among the three reasons, since it had more sub-themes (characteristics) than the other two reasons.

The various reasons provided by both trainee teachers and their mathematics tutor in support of their respective valued attributes are presented under research questions one and two respectively in chapter four.

Ethical Issues

It is expected that some ethical issues are likely to arise during this study, hence it is necessary to demonstrate how these issues were addressed in this study. Some of the ethical issues that were dealt with in this study are:

Ethical issues in the research problem

This is to enable the researcher to build trust and respect with the respondents by making sure that the questionnaires and the interview guide items are well structured to meet the demands of the problem being studied and that the study has no tendency of marginalizing any individual involved in the research study. This was duly checked during the pre-test of the data collection instruments. Hence, all participants were treated equally as co-researchers to avoid any biases.

Ethical issues in the purpose and questions

In this case, the researcher conveyed and explained the purpose of the research to the respondents, so that both participants and researcher had the same purpose in mind. This was achieved through the use of the common language (English Language) which is the official language and mode of instruction in all levels of education in Ghana. All the participants could read and write fluently in the English language and for that matter, understanding and interpreting the instructions for the questionnaires and interview guide items did not pose any problem to the respondents.

Ethical issues in data collection

To protect and value participants, research plans were sent to the IRB for review. Also, an Informed Consent Form was designed for respondents to sign before taking part in the research. This was to acknowledge the right and protection of the respondents during data collection and assured them that information that may infringe on their rights would remain concealed. Hence, the participants had the right to opt-out of participating in this research study. The questionnaire for trainee teachers was administered in well-spaced and ventilated classrooms to avoid stressed conditions in the classrooms.

Ethical issues in data analysis and interpretation

This was to ensure that the anonymity of the respondents was protected during and after the data collection process. Hence, the names and index numbers of the participants were disassociated from the responses at all stages of data processing and analysis, especially in both the questionnaire and the interview administration. The data once collected and analyzed were owned by only the researcher and would not be accessible to anybody or association. Again, once the data has been analyzed, it would be kept under lock and key for one year after which it will be discarded by the researcher so that it does not fall into second hands.

Ethical issues in psychological effects on pedagogy

The study did not attach any particular pedagogical approach and only identified the values held by trainee teachers, mathematics tutors, and explored their alignment. For that matter, mathematics tutors would design lessons based on the resulting findings that were observed from the study using the pedagogical approaches familiar to the trainee teachers. However, it was explained to the participants that there were no psychological effects on

pedagogical approaches that are expected since no additional or new pedagogical approach was introduced. Though there are no known direct psychological effects from learning outcomes, the study would allow the mathematics tutors in the selected colleges to design appropriate lessons to improve trainee teachers' values that would be identified as least prioritised or demonstrated values.

Ethical issues in writing and disseminating the research

Language or words that are biased against persons because of gender, sexual orientation, racial or ethnic group, disability, or age was not entertained in this research. Also, this study did not involve the falsification or invention of findings that resulted from the results of the study. In other words, the researcher reported objectively on the findings generated to reflect the views of trainee teachers and their mathematics tutors valuing mathematics learning in the Ahafo, Bono and Bono East Regions of Ghana.

Chapter Summary

The main purpose of this chapter was to elaborate and present the main research methods and tools employed in conducting this research study. The chapter first discussed the research design that situates the study. The design was identified to be the sequential explanatory mixed-methods design. The study area for the research was also described to give a fair idea about the characteristic nature of the area. The population, participants, the sampling procedure, the research instruments, the data collection procedure and, data processing and analysis were as well presented in this chapter. Also, this chapter discussed how the pre-test was conducted to validate the instruments used for the data collection. Moreover, illustrations of how the validity and reliability of the research instruments were ensured. The chapter ends with the chapter

summary. Notwithstanding, the necessary conditions set to achieve the general validity and reliability of findings for this study, some factors could have provided strong findings but were not included in the methods of this study. For instance, widening the scope of the study to include more trainee teachers and the number of colleges in other regions could have provided stronger findings.

Chapter Four that follows presents the results obtained from the data analysis based on the data collected and appropriate discussions on the research questions and hypothesis that guided the study.



CHAPTER FOUR

RESULTS AND DISCUSSION

Overview

The purpose of the study was to explore what trainee teachers and their tutors' value in mathematics learning and, it also sought to ascertain whether what tutors value in their students' mathematics learning aligns with what trainee teachers value at the colleges of education in the Ahafo, Bono and Bono East Regions of Ghana. In chapter three, the methods employed to achieve the purpose were presented. In this chapter, the results obtained from the study are presented based on the research questions and the hypothesis that were formulated to guide the study in chapter one. The presentation of the results based on the research questions for the statistical tools that were used for the study. This was followed by a discussion of the results and ended with a chapter summary.

Results

The next section that follows illustrates the results from the data analysis and how the research questions and hypothesis for the study were addressed.

Biographic data of respondents

Mathematics tutors

A total of thirty-four (34) mathematics tutors consisting of 31(91.18%) males and 3 (8.82%) females were purposively sampled from the five (5) selected colleges of education that were used in this study. The distribution of the tutors from colleges A, B, C, D and E were 5, 10, 8, 5 and 6 respectively. 15 (44.10%) of the mathematics tutors were within the age range of 30 to 39 years, while 16 (47.10%) of them were within the age range of 40 and 49 years.

Again, 3 (8.82 %) of them were 50 years and above. However, 14 (41.20 %) of the mathematics tutors have taught below 5 years in their respective colleges of education and 12 (35.30 %) of them have a teaching experience of between 6 to 10 years. The remaining 8 (23.50 %) have a teaching experience of 11 years and above at their present colleges.

Third-year trainee teachers

The third-year Primary education and JHS education trainee teachers (taking mathematics as a core subject) from the five selected colleges of education in the Ahafo, Bono and Bono East Regions took part in this study. Out of the total of one thousand and fifty (1,050) third-year trainee teachers who participated in this study 575 (54.80 %) were males and 475 (45.20 %) females. The mean age of the third-year trainee teachers taking mathematics as a core subject was approximately 23.96 years implying that majority of the third-year trainee teachers were around the age of 24 years.

Test of assumptions on statistical tools used in this study

Assumptions for conducting principal component analysis

For principal component analysis to be conducted three assumptions have to be satisfied. These assumptions include the Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy should be acceptable. Hence, KMO greater or equal to 0.5 is considered acceptable or better (Kaiser, 1970).

Therefore, KMO was found for the trainee teachers and their mathematics tutors' attributes valued to be 0.950 and 0.642 respectively which were better than the minimum acceptable value of 0.5.

The second assumption is Bartlett's test of sphericity and should be (Davis *et al.*, 2021) significant (i.e., p < 0.05). Bartlett's test of sphericity was

significant at 0.000 for the two respective data collected and hence, the second assumption was not violated for this study.

If these conditions are satisfied then it means principal component analysis can be applied to the correlation matrix generated. It also suggest that the reliability of the resultant identity matrix of the questionnaire was strong enough and therefore, the principal component analysis is applicable. It must be noted that the third assumption, sample size depends on the first two assumptions in the sense that if those assumptions are met then, it automatically means the sample size was adequate. This suggests that the condition of sample size adequacy has not been violated. Since the first two assumptions were met, it means the sample size was adequate for PCA analysis in the case of both trainee teachers and their tutors.

Assumptions for conducting correlational analysis

For correlational analysis to be conducted there are four assumptions that should be considered namely independence of observations, normality, linearity and homoscedasticity. These four assumptions were checked before the actual analysis. Firstly, the independence of observations was not violated since the two sets of respondents (trainee teachers and mathematics tutors) used in this study were independent groups. The administration of the questionnaires and data analysis were done independently. Secondly, the normal curve imposed on the histogram for the trainee teachers' data set seems normally (bell-like shape) distributed. For the mathematics tutors' data set, the distribution seems slightly skewed to the right (positively skewed), but the bell-like shape can be observed. The normality graphs for both trainee teachers and their mathematics tutors are presented in Figures 4 and 5 respectively. **Trainee Teachers**

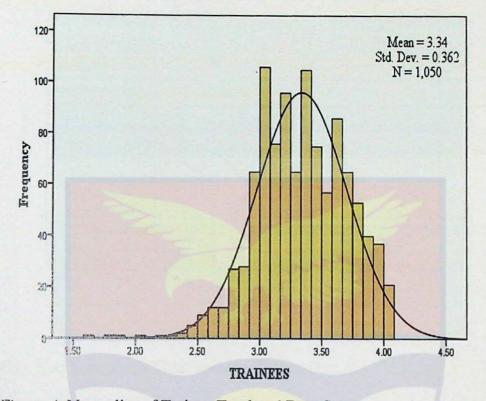
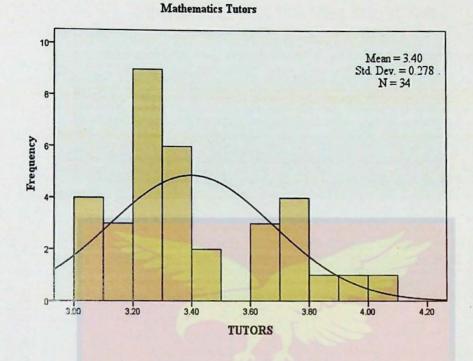
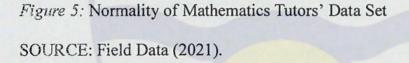


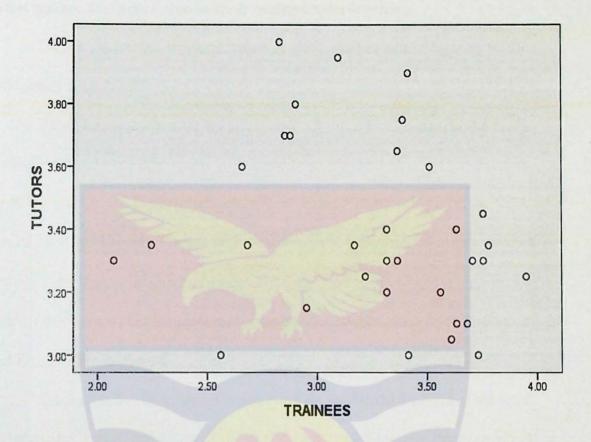
Figure 4: Normality of Trainee Teachers' Data Set SOURCE: Field Data (2021).

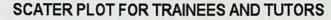
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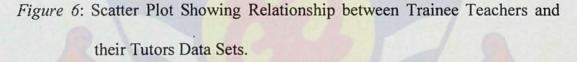




Thirdly, the linearity between the two data sets was observed to be a roughly negative straight line, suggesting there exists a sort of weak negative relationship between them. Hence, the linearity assumption was not violated in this study. For the fourth assumption, homoscedasticity was also checked by observing the resultant scatter plot for the two data sets. It was observed that there appear to be a cigar shape in the minor diagonal direction. The linearity and homoscedasticity assumptions are illustrated in the scatter plot presented in Figure 6 below.







SOURCE: Field Data (2021)

Assumptions for conducting independent samples t-test

These sample sizes were considered to be generally large enough to produce valid results as they were more than 30 (1050 and 34) in each case. Also, the independent samples t-test was employed since the two groups (trainee teachers and mathematics tutors) were independent of each other. To conduct independent samples t-test, there must be one continuous dependent variable and one categorical independent variable (independent groups). In this study, the dependent continuous variable was the attributes valued from the questionnaire items retained after the PCA. The independent categorical

variable (groups) were the trainee teachers and their tutors. Therefore, the observed data satisfies the conditions for conducting independent samples t-test.

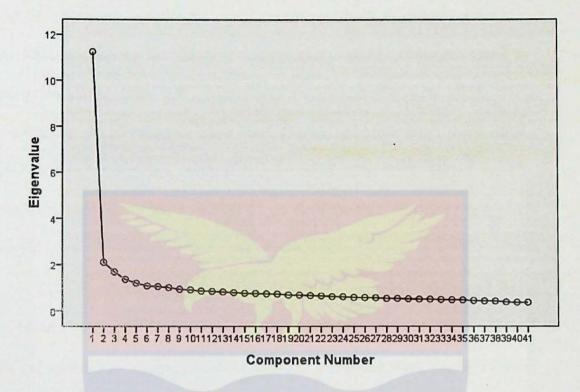
What trainee teachers value in their mathematics learning

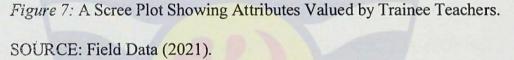
"What do trainee teachers value in their mathematics learning at the colleges of education?"

In addressing research question one, a Principal Component Analysis (PCA) was conducted on responses obtained from the third-year trainee teachers from the colleges in the selected regions of Ghana. The final run of the PCA resulted in eight (8) components with 41 items from the initial 64 questionnaire items for the trainee teachers retained. The cutoff point of .40 and a significant level of 0.05 was set for the principal component analysis with varimax rotation and Kaiser Normalization.

Consequently, these components together explained 50.60% of the total variance and overall reliability of 0.930 for the 41 questionnaire items retained from the trainee teachers. The distribution of the 41 items retained for the eight components was as follows: Trainee Teachers' Component one (TTC1) 13 items, Trainee Teachers' Component two (TTC2) 7 items, Trainee Teachers' Component three (TTC3) 4 items, Trainee Teachers' Component four (TTC4) 4 items, Trainee Teachers' Component five (TTC5) 4 items, Trainee Teachers' Component seven (TTC7) 3 items, Trainee Teachers' Component seven (TTC7) 3 items and Trainee Teachers' Component eight (TTC8) 3 items. (See Table 2)

The scree plot below shows the attributes valued components that were obtained in the final principal component analysis for the trainee teachers in their mathematics learning. It is clear from the scree plot that, though several attributes were valued, only those with eigenvalues greater than one were retained as shown in Figure 7 below. Scree Plot





The naming of the eight attribute components valued by the trainee teachers in their mathematics learning is demonstrated below. The naming of each attribute component was agreed upon based on the qualitative approach and the items that loaded onto a particular attribute valued component collectively or mostly represented. Besides, the names were influenced by the relationship that existed between their interrelated attributes and the fusibility of the items with the higher factor loadings. This means that all the items that loaded onto an attribute component were considered in naming that particular component and, named based on what the majority of these items collectively represented.

TTC1 (fluency). The 13 items that loaded onto this component were: "Understanding my solutions (item 60). Revising previous knowledge (item 55).

Explaining my solution to the class (item 52). Finding different ways of solving problems (item 56). Learning through mistakes (item 62). Recalling formulas (item 54). Knowing reasons for using specific methods in solving questions (item 48). Appreciating the beauty of mathematics (item 53). Using diagrams to explain solutions (item 59). Classifying mathematics objects (item 57). Remembering the correct procedure (item 45). Using mathematics terms (item 46). Practicing how to use mathematics formulas (item 49)". With these 13 items, attribute fluency accounted for 12.25 % of the total variance explained. (See Table 2).

TTC2 (understanding). Seven items loaded onto this attribute and they are: "Asking questions (item 1). Involvement in-class activities (item 3). Using different methods in solving problems (item 9). Feedback from teachers (item 16). Showing correct workings (item 23). Explaining mathematical concepts (item 19). Identifying mathematics ideas (item 4)". Besides, this attribute understanding contributed 9.00 % of the total variance explained.

TTC3 (mystery). The four items that loaded onto this attribute mystery were: "Wonder in mathematics (item 64). History of mathematics (item 42). Mathematics puzzles (item 36), Using mathematics symbols (item 40)". These four items together accounted for 5.51 % of the total variance explained.

TTC4 (strategies). Its four loaded items were: "Accepting different views from colleagues (item 43). Using teaching-learning materials (item 28). Explaining the origin of mathematics rules and formulae (item 26). Using concrete materials to understand mathematics (item 39)". This accounted for 5.31 % of the total variance explained.

TTC5 (learning technologies). Its items included: "Learning mathematics with the internet (item 10). Learning mathematics with computers

(item 13). Accessing information from the internet (item 41). Using ICT tools in mathematics (item 51)". This attribute contributed to 5.10 % of the total variance explained.

TTC6 (relevance). Its items loaded were: "Outdoor mathematics activities (item 33). Whole-class discussions (item 47). Mathematics debates (item 25)". The three items for this valued attribute contributed 4.85 % of the total variance explained.

TTC7 (collaboration). With loaded items "Answering group assignments (item 8). Doing homework (item 6). Group discussions (item 7)". Hence, attribute collaboration contributed 4.32 % of the total variance explained.

TTC8 (accuracy). This attribute valued by trainee teachers accounted for 4.27 % of the total variance explained. The three items that loaded to the attribute accuracy included: "Using the calculator to verify the answer (item 15). Using the calculator in mathematics learning (item 32). Getting assistance from colleagues during presentations (item 35)". (See Table 2)

The named attributes valued by the trainee teachers in their mathematics learning as a result of the Principal Component Analysis (PCA) with their respective item (factor) loadings are presented in Table 2 below.

Table 2: Attributes Valued by Trainee Teachers in their Mathematics

Learning

Values in Mathematics learning	Items (N = 1050)	Factor loadings		
Fluency	Understanding my solutions.	.616		
, achey		.595		
	Revising previous knowledge			
	Explaining my solution to the class.	.581		
	Finding different ways of solving problems.	.565 .561		
	Learning through mistakes.			
	Recalling formulas.			
	Knowing reasons for using specific methods in solving questions.	.531		
	Appreciating the beauty of mathematics.	.522		
	Using diagrams to explain solutions.	.515		
	Classifying mathematics objects.	.497		
	Remembering the correct procedure.	.485		
	Using mathematics terms.	.477		
	Practicing how to use mathematics formulas.	.431		
Understanding	Asking questions.	.658		
o, i i i i i i i i i i i i i i i i i i i	Involvement in class. activities	.625		
	Using different methods in solving problems.	.582		
	Feedback from teachers.	.548		
	Showing correct workings.	.534		
	Explaining mathematical concepts.	.530		
	Identifying mathematics ideas.	.476		
Mystery	Wonder in mathematics.	.667		
iviystory	History of mathematics.	.603		
	Mathematics puzzles.	.570		
	Using mathematics symbols.	.465		
Strategies	Accepting different views from colleagues.	.559		
Strategies	Using teaching-learning materials.	.559		
	Explaining the origin of mathematics rules and formulae.	.520		
	Using concrete materials to understand mathematics.	.516		
Learning Technologies	Learning mathematics with the internet.	.812		
	Learning mathematics with computers.	.732		
	Accessing information from the internet.	.493		
	Using ICT tools in mathematics.	.464		
Relevance	Outdoor mathematics activities.	.602		
	Whole-class discussions.	.501		
	Mathematics debates.	.445		
Collaboration	Answering group assignments.	.668		
	Doing homework.	.609		
	Group discussions.	.569		
Accuracy	Using the calculator to verify the answer.	.738		
	Using the calculator in mathematics learning.	.630		
	Getting assistance from colleagues during presentations.	.484		

SOURCE: Field Data (2021)

The descriptive statistics of attributes valued by the Trainee Teachers in their mathematics learning are presented in Table 3. From the four-point Likert

scale used in the questionnaire for trainee teachers, a score of 2.50 is considered a borderline and thus, any mean score below the borderline is considered low or least ranked whiles a mean score above the borderline means that the attribute valued is considered highly ranked. These criteria were used in interpreting the means of the named attributes valued in trainee teachers' mathematics learning. For the standard deviation, the least the deviation the better the spread within that attribute and the higher the deviation the poorer or wider the spread within that attribute valued. The above conditions aided the interpretation of the descriptive statistics for the eight attributes valued by the trainee teachers in their mathematics learning as presented in Table 3.

Table 3: Descriptive Statistics, Reliability (Cronbach's Alpha) andVariance Explained on Attributes Valued by Trainee Teachers

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Valuing in Trainee	Number	Mean	Std.	Cronbach's	Variance
Teachers' Mathematics	of		Deviation	Alpha	Explained
Learning	items				(%)
Fluency	13	3.34	0.441	0.878	12.25
Understanding	7	3.48	0.429	0.787	9.00
Mystery	4	3.14	0.579	0.687	5.51
Strategies	4	3.46	0.452	0.619	5.31
Learning Technologies	4	3.10	0.580	0.669	5.10
Relevance	3	3.23	0.541	0.562	4.85
Collaboration	3	3.47	0.497	0.627	4.32
Accuracy	3	3.44	0.483	0.590	4.27
OVERALL	41	3.34	0.362	0.930	50.60

SOURCE: Field Data (2021)

As observed in Table 3, eight (8) attributes valued components were obtained from the trainee teachers as a result of the PCA. These attributes include; Fluency (M = 3.34, SD = 0.441), Understanding (M = 3.48, SD = 0.429), Mystery (M = 3.14, SD = 0.579), Strategies (M = 3.46, SD = 0.452), Learning Technologies (M = 3.10, SD = 0.580), Relevance (M = 3.23, SD = 0.541), Collaboration (M = 3.47, SD = 0.497) and Accuracy (M = 3.44, SD = 0.483) respectively. Besides, all eight attributes valued were highly ranked by the trainee teachers in their mathematics learning. However, the best-ranked attribute valued by trainee teachers was understanding, followed by collaboration, strategies, accuracy, fluency, relevance, mystery and lastly, learning technologies. Moreover, the best spread was also observed within the valued attribute of understanding, followed by fluency, strategies, accuracy, collaboration, relevance, mystery and the poorest spread was seen within the attribute of learning technologies. It was also observed that the attribute understanding was the highest ranked and as well had the best spread around its mean. However, the least ranked and poorest spread were observed for the attribute learning technologies. It was as well interesting to note that the least three ranked attributes valued were also having the poorest spread respectively. The attributes valued by the trainee teachers in their mathematics learning showed an overall mean of 3.34 and a standard deviation of 0.362 based on the 41 questionnaire items retained after the PCA.

In response to research question one, the trainee teachers seem to value attributes of understanding, collaboration, strategies, accuracy, fluency, relevance, mystery and learning technologies respectively in their mathematics learning.

However, to provide reasons for the attributes valued by the trainee teachers, the observed attributes trainee teachers valued in their mathematics learning were shared with them. This was to enable them to assign reasons why they valued those attributes in their mathematics learning.

The open-ended semi-structured interview guide was made up of two items for the trainee teachers to respond to. The first item was to find out why they consider these observed attributes (fluency, understanding, mystery, strategies, learning technologies, relevance, collaboration and accuracy) important in their mathematics learning. With some follow-up questions depending on the need.

The most prevailing responses for each value are summarized below after the thematic transcription of the data (views) collected from the trainee teachers through focused group interviews across all the five colleges involved in this study.

Trainee teachers' responses to interview guide item one

Fluency

The analysis of the responses for attribute fluency showed that reasons provided by the trainee teachers can be grouped into four namely control, problem-solving, respect, and social status. The majority of the trainee teachers gave reasons relating to control. Typical examples of the reason relating to control were: "Being fluent in mathematics gives you an upper hand in all aspects of life because you can use concepts and symbols with ease (FGSC); Fluency in mathematics allows you to study mathematics to a higher level of education (FGSA); Fluency allows you to apply mathematics in your everyday life (FGSB, FGSC)".

Understanding

For understanding, the reasons were categorized into three main themes. That is knowledge acquisition, control and progress. The most prevailing reason was identified to be relating to control and specific examples were: "Understanding the concepts allows us to develop our critical thinking abilities (FGSE); In Bloom's taxonomy, we need an understanding of concepts to be able to apply, analyze, synthesize and evaluate. This means understanding forms the basis of learning (FGSD); When you form the correct concepts it allows you to apply the right concept in solving mathematics problems (FGSB, FGSC)".

Mystery

The reasons cited for attribute mystery were analysed under three (3) main themes namely aesthetic value, intrinsic motivation and usefulness. The dominant among these three attributes were aesthetic value and intrinsic motivation. Typical examples were given as Aesthetic value ("*Mystery allows for beauty through the arrangement of solutions or order of presentations in class (FGSE); Sometimes the beauty in the handwriting can influence examiners and can let them fall in love with such a student without knowing the student personally (FGSB, FGSC))*" and Intrinsic motivation ("It arouses your interest and influences you to participate fully in the lesson (FGSE, FGSA); Mystery in mathematics can help reduce some misconceptions in mathematics learning. For instance, seeing mathematics as a male domain or mathematics is for few special people in society (FGSD, FGSA, FGSE))".

Strategies

The reasons the trainee teachers gave were organized into three main themes that were named as creativity, control and examinations. Most of them gave reasons relating to creativity. Examples included for creativity were:

"Using different strategies in solving problem allow for alternative solutions and it brings dynamism to learning mathematics (FGSC); Knowing different ways of doing or solving problems makes that student more fluent and flexible (FGSE)".

Learning technologies

For learning technologies, the main themes generated from trainee teachers' reasons were grouped into three themes namely future demand, innovations and affordability (which was a hindrance in the use of technology). Major reasons given by the trainee teachers were related to innovations and typical examples were: "Using technology in our learning allows us to manipulate such tools during our prospective lessons (FGSD, FGSE); There is the need to adapt to new ways of learning mathematics and technology is one of such ways (FGSC,); We are in an advanced world so in two to four years you would get to the classroom and without technology, teaching will be impossible. Hence, we need it in our learning today so we can use it in our future lessons (FGSA, FGSE)".

Relevance

The main reasons cited for relevance were themed as application and communications. Besides, most cited examples were related to application. Some of which were presented as: "*Relating mathematics ideas to real-life allow for the application of mathematical ideas in our everyday life such as buying and selling (FGSB, FGSC); (Relevance in mathematics learning is important since it allows for economic projections to be made; Even our steps when walking are guided by mathematics as steps need to be estimated well to cross gutters (FGSC)".*

Collaboration

Two main themes were organized for the attribute collaboration namely socialization and leadership. The majority of the reasons were relating socialization. Some of the examples given for the theme of socialization were "It leads to effective social interactions (socialization) and to know others better (FGSA, FGSD); Collaboration ensures sharing of ideas that are of mutual benefit to all the students involved (FGSE, FGSC); It allows the weak students to be helped in terms of learning by the students considered to be good (FGSE)".

Accuracy

The trainee teachers' reasons assigned to the valued attribute accuracy, were put into four groups which were understanding, control, progress and intrinsic motivation. The analysis of the results showed that their reasons relating to accuracy mostly had to do with understanding. Typical examples relating to understanding were "Accuracy also can be considered as the product of your understanding. Hence, accuracy is evidence of understanding concepts or problems (FGSC); Getting the right answer shows that you understand the concept (FGSB, FGSD)".

Again, the researcher wanted to find out the views of trainee teachers on the attributes valued by their mathematics tutors which were not loaded as attributes on their part. To determine whether these attributes were important in their mathematics learning as expressed by their tutors. The interview guide item two was formulated to provide answers to these questions. Generally, the trainee teachers accepted that the attributes valued by their tutors which did not load on their part were equally important. The reasons given by the trainee

teachers for their tutors' espoused attributes (fun, mastery and problem-solving) are presented below.

Trainee teachers' responses for interview guide item two

The views expressed by the trainee teachers on some of their tutors' valued attributes that were not observed in their case were presented as:

Fun

The reasons provided by the trainee teachers for fun were grouped under three main themes enjoyment, confidence and limited time (which is cited as a challenge to the use of fun in the classroom). The majority of the reasons were related to enjoyment. Examples of such reasons were: "*Fun is indeed important in our mathematics learning because it makes us active and the classroom lively (FGSC); Fun makes us learn difficult concepts without realizing that it is difficult (FGSD); Fun reduces anxiety among students as they become happy in class (FGSC)"*.

Mastery

The reasons for mastery attributes were put under four main themes namely fluency, accuracy, control and applications. Some of the reasons included: "*Mastery is achieved through fluency, strategies and accuracy*. *Hence, it takes a lot to master mathematics content or concept (FGSA, FGSC); Mastery can only be achieved if the student has control over basic concepts and facts in mathematics (FGSE)*".

Problem-solving

The main themes that resulted from the reasons given for problemsolving were application, understanding and control. Some of the typical examples were cited as: *The problem-solving skills allow us to apply in solving problems we encounter outside the classroom (FGSC); Our development as*

people depends on our ability to solve problems. This means being able to solve problems is an indication of our understanding and control over our environment (FGSA)".

The above reasons provided by the trainee teachers demonstrated why the mathematics learning attributes espoused by them and their tutors were important in their learning. In some cases, they even provided how these valued attributes are linked with each other. Besides, they also suggested in certain situations some strategies that can aid in developing attributes valued by them by their mathematics tutors.

In the next section, research question two provides the results obtained from the PCA based on the responses to questionnaire items from the mathematics tutors. It provides what attributes tutors valued in their students' mathematics learning at the five selected colleges of education.

What tutors value in their students' mathematics learning

"What do tutors value in their students' mathematics learning at the colleges of education?"

To answer this research question, a principal component analysis was conducted on the questionnaire responses obtained from the mathematics tutors from the five colleges purposively selected for this study. The final run of the principal component analysis resulted in a Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy of 0.642 and was significant at the .000 level, using Bartlett's test of sphericity. The rotated component matrix was reached indicating that the identity matrix of the questionnaire instrument for mathematics tutors was reliable and, that the assumptions for conducting the principal component analysis were satisfied. The cutoff point of .40 and a significant level of 0.05 was set for the principal component analysis with

varimax rotation and Kaiser Normalization. The resultant rotated matrix retained 20 items from the original 64 questionnaire items for the mathematics tutors and was loaded onto components as indicated in the scree plot presented in *figure 8* below. The distribution of 20 retained items was as follows: Mathematics Tutors' Component (MTC1) 4 items, Mathematics Tutors' Component (MTC2) 5 items, Mathematics Tutors' Component (MTC3) 4 items, Mathematics Tutors' Component (MTC4) 4 items and Mathematics Tutors' Component (MTC5) 3 items. The five attribute components together explained 70.35 % of the total variance, with an overall Cronbach's alpha (reliability) of 0.821. (See Table 4).

The scree plot below illustrates the attributes valued components that were obtained in the final principal component analysis from the perspective of the mathematics tutors. It is evident from the scree plot that, though several attributes were valued, only those with eigenvalues greater than one were retained as shown in Figure 8 below.

Scree Plot

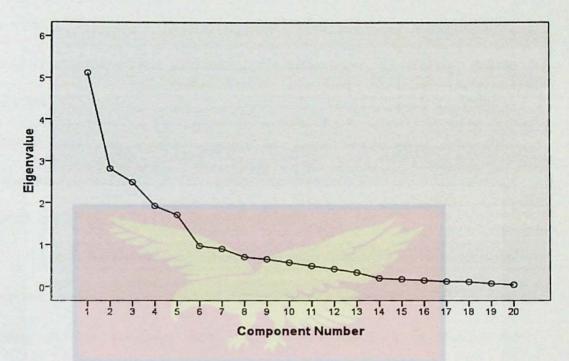


Figure 8: A Scree plot Showing Attributes Valued by Mathematics Tutors in their Students' Learning.SOURCE: Field Data (2021)

The naming of the five attribute components valued by the tutors in their students' mathematics learning is presented below. The naming of each valued attribute component was reached based on a qualitative approach and what items loaded onto a particular component collectively or mostly represent. Besides, the names were influenced by the relationship that existed between their interrelated attributes and the fusibility of the items with the higher factor loadings. Thus, all the items loaded onto a particular attribute component were considered in the naming as indicated under research question one.

MTC1 (fun). The 4 items that loaded onto this attribute were: "Students accepting different views from colleagues (item 43), Students knowing the history of mathematics (item 42), Students using the calculator to verify the

answer (item 15) and Students engaging in mathematics games (item 44)". The attribute fun accounted for 14.72 % of the total variance explained.

MTC2 (mastery). The five items that loaded onto this attribute included: "Students explaining their solutions to the class (item 52), Students getting assistance from colleagues during presentations (item 35), Students recalling formulas (item 54), Students identifying mathematical ideas from their environment (item 17) and Students verifying theorems (item 63)". The attribute mastery explained 14.70 % of the total variance.

MTC3 (understanding). The items that loaded onto this attribute component were: "*Teacher step-by-step delivery of lessons (item 24), Students showing correct workings (item 23), Students engaging in small-group discussions (item 22) and Students engaging in whole-class discussions (item 47)*". The four items that loaded onto this attribute together explained 14.18 % of the total variance.

MTC4 (problem-solving). The four items that loaded unto this attribute component were: "Students learning by proofs (item 61), Students making up their mathematics questions (item 20), Students using mathematics terms (item 46) and Students learning mathematics with the internet (item 10)". The attribute problem-solving accounted for 14.16% of the total variance explained.

MTC5 (strategies). With its three items contributed 12.59 % of the total variance explained. The items were: "Students classifying mathematics objects (item 57), Students using diagrams to explain solutions (item 59), Students using concrete materials to understand mathematics (item 39)". (See Table 4)

The attributes valued by the tutors in their students' mathematics learning as a result of the Principal Component Analysis (PCA) with their respective item (factor) loadings are presented in Table 4 which follows.

Valuing in	Items $(N = 34)$	Factor loadings
Mathematics learning		
Fun	Students accepting different views from colleagues	.824
	Students knowing the history of mathematics	.773
	Students using the calculator to verify the answer.	.694
	Students engaging in mathematics games	.690
Mastery	Students explaining their solutions to the	.863
	class. Students getting assistance from colleagues during presentations.	.681
	Students recalling formulas	.643
	Students identifying mathematical ideas from their environment	.608
	Students verifying theorems	.506
Understanding	Teacher step-by-step delivery of lessons	.835
E	Students showing correct workings	.799
	Students engaging in small-group discussions	.748
	Students engaging in whole-class discussions	.694
Problem-solving	Students learning by proofs	.838
	Students making up their mathematics questions	.766
	Students using mathematics terms	.691
	Students learning mathematics with the internet	.686
Strategies	Students classifying mathematics objects	.833
	Students using diagrams to explain solutions	.809
	Students using concrete materials to understand mathematics	.519

Learning

SOURCE: Field Data (2021)

The descriptive statistics of attributes valued by the Mathematics Tutors in their students' mathematics learning are presented in Table 5. It is also important to note that in interpreting the standard deviations (spread) of the named attribute valued by tutors in their trainee teachers' mathematics learning, the least the deviation the better the spread within that value and the higher the

deviation the poorer or wider the spread within that attribute. The above criteria aided the interpretation of the descriptive statistics presented in Table 5.

Table 5: Descriptive Statistics, Reliability (Cronbach's alpha) and

Valuing in Mathematics	Number		AL PROPERTY	Cronbach's	Variance
Learning from Tutors'	of		Std.	Alpha	Explained
Perspective	Items	Mean	Deviation		(%)
Fun	4	3.18	0.558	0.802	14.72
Mastery	5	3.34	0.357	0.754	14.70
Understanding	4	3.62	0.405	0.811	14.18
Problem-solving	4	3.30	0.518	0.801	14.16
Strategies	3	3.53	0.395	0.670	12.59
OVERALL	20	3.40	0.278	0.821	70.35

Variance Explained by Attributes Valued by Tutors (N = 34)

SOURCE: Field Data (2021)

From Table 5, it can be observed that all the five attributes valued by the tutors in their trainee teachers mathematics learning were highly ranked and, with the best ranked attribute being understanding (M = 3.62, SD = 0.405), this was followed by strategies (M = 3.53, SD = 0.395), mastery (M = 3.34, SD = 0.357), problem-solving (M = 3.30, SD = 0.518) and lastly, fun (M = 3.18, SD = 0.558). In addition, the best data spread was seen in mastery, which was followed by strategies, understanding, problem-solving and lastly fun. It was as well observed that fun that was ranked least also had the poorest data spread among the five attributes valued by the mathematics tutors in their students' learning. Besides, each of these five attributes contributed significantly to the total variance explained (70.35 %) towards what tutors value in their students' mathematics learning from the selected colleges of education in Ghana. Finally, tutors in the selected colleges of education appear to value attributes in their

order of priority; understanding, strategies, mastery, problem-solving and fun respectively in their students' mathematics learning.

Moreover, to understand why these attributes were valued by the mathematics tutors, a semi-structured interview was conducted on the part of the tutors as in the case of the trainee teachers. The purpose of this focus group interview was to solicit tutors' views on why the attributes espoused by them and their students were important.

The first interview item for the tutors was to find out why attributes (fun, understanding, mastery, strategies and problem-solving) valued by them were important in their students' mathematics learning. With some follow-up questions depending on the need. Their reasons are summerised below after the thematic transcription and sentence meaning of the data collected from the mathematics tutors through the focus group interviews.

Mathematics tutors' responses to interview guide item one

The responses to the focus group interview guide item one from the tutors for each of their attributes valued in their students' mathematics learning are presented as:

Fun

Analysis of the reasons given by the mathematics tutors for fun, three main themes were obtained namely socialization, concentration and motivation. Among these the dominant reason was socialization. Some of its examples were: "If students learn through fun or games, it allows students to associate themselves well with the teacher. The reason is that most students see mathematics to be too difficult. So if the classroom is very lively it helps them to understand the topic the teacher is going to teach and can serve as the introductory aspect of the lesson. Through this, the students will be eager to

take part in what the teacher is doing (FGTD); Once the lesson is fun, it reinforces learning because it will boost their interest. So intrinsically, it allows learners to enjoy mathematics. Any time there is mathematics class, students feel like they are going to play and mathematics will not be difficult as people think. So the kind of misconception that mathematics is difficult will be erased. Hence, the fun aspect of mathematics motivates the students to do better in mathematics; Fun makes the lessons more interactive (FGTC, FGTA)".

Mastery

The three main reasons generated for mastery after the analysis were application, control and precision. The majority of the tutors gave reasons relating to application, some of which were: "Mastery is very important in mathematics learning because mathematics is intertwined. Thus, the topics are related which means our students need to master the first topic learned which serves as the basis for the second topic. The students need to master the first topic to apply it to the second topic (FGTE); Mathematics is more of problemsolving and if you do not have the mastery of the subject area you cannot apply it to solving real-life problems or daily activities. So students need to master it to be able to solve real-life problems or activities (FGTB, FGTD)".

Understanding

The four main reasons relating to understanding were namely application, progress, relevance and mastery of concepts. Most of the reasons tutors gave were related to application and specific instances were: "Understanding is very important and the key in the sense that if the activities do not bring the proper understanding of the concept being taught the students will not be able to apply. For proper application of the concept learned there must be a good understanding of that concept (FGTC); Understanding is very

important when it comes to solving problems. For instance, Polya's indicated that understanding a problem is the initial step in solving that particular problem (FGTA)".

Problem-solving

Reasons relating to skills acquisition, strategies and relevance were given by the mathematics tutors on problem-solving. The majority gave reasons relating to skills acquisition. Typical examples relating to skills acquisition were: "Mathematics is all about problem-solving and therefore, if students do not have the skills of solving problems, then basically nothing can be achieved. For this, students need to be problem solvers and these activities provide the prerequisite skills needed for the actual problem-solving process (FGTE); The problem-solving skills are very important as it is being encouraged to be used as an approach or strategy in learning concepts as indicated in the new national curriculum for mathematics (National Council for Curriculum and Assessment – NaCCA, 2019) for basic schools. Hence, problem-solving can be employed as a strategy in teaching other concepts (FGTC, FGTD)".

Strategies

The strategies which were the last attribute valued by the mathematics tutors resulted in four main reasons after the analysis of the results which were: application, control, relevance and hindrances. The hindrances are things that impede the use of alternative approaches in teaching and learning mathematics, such as teaching many contents in a short period of time. The application was the most dominant reason among the four reasons. Specific examples of reasons relating to application were: "*Learning using different strategies also help students in their future learning as they have different mathematical tools to apply (FGTE); Using different strategies ensures that students are aware that* there is no one unique way of solving a problem and therefore, mathematical investigations must be encouraged by tutors in the classroom (FGTD)".

The interview guide item two for the mathematics tutors was to determine their views on some of the attributes valued by their students that did not load from their part, with some follow-up questions for emphasis and clarification as the need arises. The responses from the tutors for interview guide item one were on only three sampled students' valued attributes (relevance, learning technologies and mystery). Since the time allocated for the interview could not allow us to go through all the six attributes valued by only the trainee teachers. The reasons given by the tutors for the above-listed trainee teachers' attributes valued are presented below.

Mathematics tutors' responses to interview item two

Relevance

From the tutors, three main reasons were generated for attribute relevance namely cultural identity, usefulness and understanding. Most of the reasons cited for relevance are related to usefulness. Examples of reasons relating to usefulness were: "Most times teachers resort to pamphlets and textbooks so our teaching does not make use of everyday situations and how they relate to mathematics (FGTA); Students normally asked questions like "What am I going to use this topic for? (FGTC)" this means tutors do not emphasize the relevance of the topics or concepts to the students (FGTB); If students are made to know the usefulness of a concept or topic they are likely to participate fully in the lessons and would concentrate (FGTA)".

Learning Technologies

For learning technologies, two main groups were obtained as a result of the analysis namely relevance and hindrances that have to do with the difficulty

in usage, know-how, affordability and not allowed to be used in examinations. The majority of them gave reasons relating to relevance. Specific excerpts were: "Tutors emphasize that Smart-boards are important in teaching-learning of mathematics but most colleges do not have them (FGTB); The use of technology is very important and wondered why it was not loaded as a value under the tutors (FGTC); Tutors accepted the fact that students nowadays are more interested and use technology more than their tutors. Therefore, the reason for students espousing it as a valued attribute (FGTE)".

Mystery

Analysis of results shows that reasons tutors gave relating to the mystery were mainly grouped into three namely openness, wonder and control. The majority of the reasons tutors gave were in relation to wonder. Typical examples of the reasons relating to wonder were: "Sometimes when students are kept in suspense it allows them to pay attention and concentrate (FGTE); It also arouses their interest when the lesson is fascinating (FGTC); Mystery help boost the confidence of the learners when they realized that mathematicians are ordinary people like them (FGTB, FGTE)".

Indeed, the above reasons presented by the mathematics tutors explain why valuing is important in their students' mathematics learning at the five colleges used for this study. Tutors, in addition, highlighted some wonderful approaches that can assist to develop the right mathematics attributes valued by both groups in this study. Through the interview, most tutors did not only accept attributes valued by themselves to be important but also attributes valued by their trainee teachers as well. Again, tutors expressed concern about the need to pay more attention to what attributes their students' value as perhaps that is the key to unlocking students' difficulties in learning mathematics. Besides, they

gave reasons why some of the important pedagogical activities are not being practiced in mathematics classrooms despite their significance in developing the attributes valued by their students.

The next section that follows (research question three) presents the relationship between the attributes valued by the trainee teachers and that of their mathematics tutors.

Relationship between trainee teachers and their tutors valuing in mathematics learning

"What is the relationship between trainee teachers and their tutors valuing in mathematics learning?"

This question was aimed at finding out if any relationship existed between attributes valued in mathematics learning from the perspective of trainee teachers and their tutors. For the relationship between trainee teachers and their tutors' valuing in mathematics learning to be established, a Pearson product-moment correlation was conducted. The Pearson-moment correlation was used because the various attributes in mathematics learning identified from both perspectives were scaled into continuous independent (Pallant, 2005) variables based on which the correlation was conducted.

In exploring the relationships, the researcher first looked at the facial (by inspection) relationships that existed between the individual computed attributes from the trainee teachers (fluency, understanding, mystery, strategies, learning technologies, relevance, collaboration and accuracy) and their mathematics tutors' (fun, mastery, understanding, problem-solving and strategies) perspectives qualitatively. It was observed that two of these mathematics education attributes valued were common between the two groups. These two common attributes were strategies and understanding. Consequently,

combining the attributes valued in mathematics learning by both trainee teachers and their tutors resulted in eleven attributes. These attributes of mathematics learning included; fun, mastery, understanding, fluency, strategies, mystery, learning technologies, relevance, collaboration, problem-solving and accuracy.

For the second part, the researcher looked at the relationship that exists between the computed mathematics learning attributes valued by the trainee teachers (eight attributes) and that of their tutors (five attributes) by the use of multiple correlational analysis.

Lastly, the computed overall attributes valued by both trainee teachers and their tutors. The responses obtained from the 1050 trainee teachers and their 34 mathematics tutors were used for the bivariate correlational analysis in this study. The correlation matrix indicating the correlation coefficients and their corresponding significant values among the valued attributes from both perspectives was generated. The summary correlation from the analysis is presented in Table 6 below (See Appendix "E" for the correlation matrix table).

Table 6: Pearson Correlation for Attributes Valued between Traince

Values in Mathematics Learning	Pearson Correlation Coefficient (r)	Sig. value	
Learning Technologies (TTC5) and Mastery (MTC2)	-0.417**	0.007	
Relevance (TTC6) and Mastery (MTC2)	-0.335*	0.027	
Relevance (TTC6) and Strategies (MTC5)	-0.317*	0.034	
Accuracy (TTC8) and Problem-solving (MTC4)	-0.293*	0.046	
Overall Trainee Teachers (TTC) and Tutors (MTC) (no correlation)	-0.224	0.101	

Teachers and their Tutors $(N_1 = 1050, N_2 = 34)$

*. Correlation is significant at the 0.05 level.

**. Correlation is significant at the 0.01 level.

SOURCE: Field Data (2021)

Based on the results from Table 6, there were four significant relationships observed from the attribute variables from the trainee teachers' and their mathematics tutors' perspectives. The first relationship was observed between learning technologies (from trainee teachers) and mastery (from mathematics tutors). These two attributes are related inversely with a Pearson Correlation Coefficient (r) of -0.417 and sig. value of 0.007. Again, inverse relationships were recorded for the following pairs of attributes; relevance (from trainee teachers) and mastery (from mathematics tutors) with Pearson Correlation Coefficient (r) of -0.335 and sig. value of 0.027; relevance (from trainee teachers) and strategies (from mathematics tutors) with Pearson Correlation Coefficient (r) of -0.317 and sig. value of 0.034. The last inverse relationship was seen between accuracy (from trainee teachers) and problem-solving (from mathematics tutors) with Pearson Correlation Coefficient (r) of -0.317 and sig. value of 0.034. The last inverse relationship was seen between accuracy (from trainee teachers) and problem-solving (from mathematics tutors) with Pearson Correlation Coefficient (r) of -0.317 and sig. value of 0.034. The last inverse relationship was seen between accuracy (from trainee teachers) and problem-solving (from mathematics tutors) with Pearson Correlation Coefficient (r) of -0.317 and sig. value of 0.034.

From the first part of the correlational analysis, four different kinds of inverse relationships were observed between trainee teachers' attributes valued and that of their mathematics tutors. These relationships were also observed to be weak and negative but, these relationships cannot be ignored due to their significant nature. However, it is important to mention that no positive or direct relationships were seen between the attributes from trainee teachers and their mathematics tutors so far as mathematics learning at the selected colleges of education was concerned.

Consequently, there was no relationship observed between the overall attributes computed from both trainee teachers and that of their mathematics tutors (i.e., sig. value of 0.101 > 0.05 alpha-level). This means that what attributes trainee teachers valued in their mathematics learning appear to be different from what their mathematics tutors also consider important in their students' learning. Though both trainee teachers and their mathematics tutors valued some attributes in the trainee teachers' learning. These attributes generally do not relate statistically as seen in the correlation of the overall computed attributes. This implies that the mathematics tutors seem not to be aware of what attributes their students consider important in their mathematics learning.

In concluding this section, it was observed from the correlational results that some weak inverse relationships were reported between individual attributes valued by the trainee teachers and that of their mathematics tutors. But, generally, there was no relationship existing between the overall attributes valued by the two groups.

Furthermore, the next section that follows explores the differences in valuing between trainee teachers and their mathematics tutors.

Differences in valuing between trainee teachers and their mathematics tutors

"What are the differences, if any, in valuing between trainee teachers and their mathematics tutors?"

This research question was addressed through qualitative comparison between the individual items that loaded onto the common pairs of attributes valued by trainee teachers and their mathematics tutors.

To explore the differences between trainee teachers and their mathematics tutors' valuing, a qualitative comparison was made on the two pairs of common attributes (understanding and strategies) valued by both groups based on the items that loaded onto them after the PCA. The common pairs of attributes valued and their respective items are presented in Table 7 below.

Table 7	: Differences	between	Trainee	Teachers an	nd Mathematics	Tutors

Common Attributes	Trainee Teachers	Mathematics Tutors
Valued		
Understanding	> Asking questions (item 1).	> Teacher step-by-step
(TTC2/MTC3)	> Involvement in-class activities	delivery of lessons (item
	(item 3).	24).
	> Using different methods in	> Students showing correct
	solving problems (item 9).	workings (item 23).
	> Feedback from teachers (item	Students engaging in
	16).	small-group discussions
	Showing correct workings	(item 22).
	(item 23).	> Students engaging in
	> Explaining mathematica	whole-class discussions
	concepts (item 19).	(item 47).
	> Identifying mathematics idea	
	(item 4)	
Strategies	Accepting different views from	Students classifying
(TTC4/MTC5)	colleagues (item 43).	mathematics objects (item
	> Using teaching-learning	g 57).
	materials (item 28).	> Students using diagrams
	Explaining the origin of	f to explain solutions (item
	mathematics rules and	d 59).
	formulae (item 26).	> Students using concrete
	> Using concrete materials to	o materials to understand
	understand mathematics (iten	n mathematics (item 39).
	<i>39).</i>	

Valuing	
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SOURCE: Field Data (2021)

From Table 7, comparing the attribute understanding qualitatively between the trainee teachers and their tutors, it was obvious that only one item

(Showing correct workings, item 23) was common between them. The other items do not even share any characteristics that can be compared. By inspection, the trainee teachers and their mathematics tutors seem to value differently on the attribute of understanding in mathematics learning. Therefore, their differences in understanding far outweigh their similarities.

On the other hand, it appears for the attribute strategies the similarities outweigh the differences. In comparing item to item for strategies valued by both trainee teachers and their mathematics tutors, the item "Using concrete materials to understand mathematics (item 39)" was loaded to both groups and it focused on the use of concrete materials in the learning of mathematics. Also, comparing items "Explaining the origin of mathematics rules and formulae (item 26)" and "Students using diagrams to explain solutions (item 59)" that loaded for trainee teachers and their tutors respectively, it can be observed that both items were concerned with the common characteristic of students "explaining". Again, the items "Using teaching-learning materials (item 28)" and "Students classifying mathematics objects (item 57)" loaded for trainee teachers and their tutors respectively, had to do with using and classifying mathematical objects. Hence, the main attribute for both items relates to the manipulation of materials including symbols. This appears to suggest that the two items seem to relate more than their differences. The only difference observed for the attribute strategies was from the item "Accepting different views from colleagues (item 43)" which was loaded to only the trainee teachers.

The next section that follows explores whether significant differences exist between the mean score rankings from both trainee teachers and their mathematics tutors' common attributes valued.

Difference between trainee teachers and their tutors' valuing in mathematics learning

The research hypothesis formulated for this study was " H_0 : there is no significant difference between trainee teachers and their tutors' valuing in mathematics learning at the selected colleges of education"

To explore the differences that exist statistically between trainee teachers (1050) and their tutors (34), an independent samples t-test was conducted on the common attributes valued. This statistical tool was used because the trainee teachers and their mathematics tutors were considered independent samples in this study. The result for the independent samples t-test is presented in Table 8 below.

Table 8: Differences in Valuing on Common Attributes between Trainee Teachers and their Mathematics Tutors (M, SD, P-Value, and Effect Size)

Common Attributes Valued	Trainee Teachers (N = 1050)		Mathematics Tutors (N = 34)		p-value	t-value	Effect
	Mean (M)	SD	Mean (M)	SD			
Understanding	3.48	0.43	3.62	0.40	0.06	-1.910	0.003
Strategies	3.46	0.45	3.53	0.39	0.37	-0.903	0.001

SOURCE: Field Data (2021)

From Table 8, the results was not significant for the attribute understanding (trainee teachers M = 3.48, SD = 0.43; mathematics tutors M = 3.62, SD = 0.40) because the t-test (sig = 0.06) results indicated no significant difference between the two groups with a very small effect size difference (effect size d = 0.003). This seems to suggest that trainee teachers and their

mathematics tutors' levels of valuing for attribute understanding were the same. Hence, the two groups valued understanding equally.

Again, the results indicated that there was no significant difference in valuing for the attribute strategies (trainee teachers M = 3.46, SD = 0.45; mathematics tutors M = 3.53, SD = 0.39) since the t-test (sig = 0.37) results seem to suggest that both trainee teachers and their mathematics tutors valued strategies equally with very small effect size difference (effect size d = 0.001). Hence, the results from Table 8 seem to suggest that there is an equal level of valuing between the trainee teachers and their mathematics tutors on the common attributes valued in mathematics learning at the selected colleges of education.

The section that follows presents the discussion on the findings from the results illustrated in the previous presentations in the order of how the research questions and the hypothesis were answered with their appropriate headings.

Discussion of Results

The purpose of the study was to explore what trainee teachers and their tutors value in mathematics learning and, it also sought to ascertain whether what tutors value in their students' mathematics learning aligns with what trainee teachers value at the colleges of education in the Ahafo, Bono and Bono East Regions of Ghana. This was achieved through the use of adapted questionnaires and semi-structured interview guides.

This section provides a discussion on the results of the findings from the study. This discussion was important as the findings of this study would be evaluated with other previous studies conducted in the area of valuing in mathematics learning.

Attributes Valued by trainee teachers in their mathematics learning

The above theme was generated from research question one which stated "What do trainee teachers value in their mathematics learning at the colleges of education?" This research question was addressed through the use of principal component analysis (PCA). Consequently, the results from the analysis appeared to suggest that the third-year trainee teachers (core mathematics trainee teachers) valued eight (8) attributes in their mathematics learning. These attributes were fluency, understanding, mystery, strategies, learning technologies, relevance, collaboration and accuracy. Though this study was conducted at the tertiary level, some of its attributes valued reflected the findings of some existing studies conducted at the pre-tertiary level around the world. For instance, out of the eight attributes valued in mathematics learning by the trainee teachers in this study, five of them (fluency, relevance, strategies, ICT and accuracy) were valued by Ghanaian Primary, Junior High School and Senior High School students as part of the WIFI study (Seah et al., 2017a). This is an indication that some of these attributes might have been carried through from the pre-tertiary level to the tertiary level. Again, the trainee teachers are being purposely trained to teach at the basic school level of our educational system and therefore, there is a possibility that some of these attributes valued might be transferred (Bishop et al., 2000) to their future students at the basic school level.

Also in Japan, primary and secondary school students reported attributes such as creativity, discussion, ICT, know-how, mystery, others' involvement, reality, results and wonder (Seah *et al.*, 2017b). Besides, in New Zealand students valued attributes such as utility, peer collaboration, effort and practice and family support (Hill *et al.*, 2019).

In another study, attributes of relevance, accessibility, formalistic view, relational view and process (tool or procedure) were valued by Australian primary school pupils (Seah, 2011a). South African students valued hard work and effort, numerous different methods, and authentic examples of shapes to understand their properties, demonstration and explanation of concepts and proofs, and teaching and explaining mathematical concepts (Madosi *et al.*, 2020). These findings from the literature appear to have some strong relationships with that of this current study. Hence, the theoretical basis of this study was grounded in literature and practice.

Though the eight attributes of mathematics (fluency, understanding, mystery, strategies, learning technologies, relevance, collaboration and accuracy) valued by the third-year trainee teachers in their mathematics learning were all highly prioritised, some were more prioritised than others. This was observed from their mean rankings as all the eight attributes valued had a mean value greater than the baseline value of 2.5. It can, therefore, be concluded that all the attributes valued by the trainee teachers were highly prioritised in their mathematics learning. However, some of these attributes were seen to be more prioritised than others. The best-prioritised mathematics attribute valued was observed to be understanding, followed by collaboration, strategies, accuracy, fluency, relevance, mystery and learning technologies respectively. Besides, the best spread of data was reported in understanding, followed by fluency, strategies, accuracy, collaboration, relevance, and mystery. The poorest data spread was observed for learning technologies.

So to the trainee teachers, understanding is the most important attribute in their mathematics learning and for that matter, teachers can capitalize on it to design lessons that would include pedagogical activities that promote

understanding of mathematics concepts. After all, the essence of teaching and learning mathematics is for students to understand. It is, therefore, not surprising that understanding was the most prioritised attribute valued, and had the best data spread among all the other seven observed attributes. The mathematics learning attribute of understanding demonstrated by the trainee teachers in this study was also reported in Davis *et al.* (2021) and Madosi *et al.* (2020). In support of this valued attribute, the trainee teachers highlighted the reasons why they consider understanding as most valuable in their mathematics learning. Three reasons assigned to understanding by the trainee teachers related to knowledge acquisition, control and progress. Among these reasons the most dominant was control. Besides, as prospective teachers, the trainee teachers require a good understanding of concepts as part of their training which would enable them to teach mathematics effectively at the basic school level as prospective professional teachers.

The second most prioritised attribute valued was the collaboration that was grounded in teamwork. Therefore, trainee teachers see collaboration as the second most important attribute of their mathematics learning. This valued attribute was also the fifth in terms of its data spread among the other attributes espoused by the trainee teachers in this study. This valued attribute also draws its importance from the fact that there is strength in teamwork and therefore, trainee teachers believe that they can make much progress in their mathematics learning through collaboration or working together as a team in the classroom. The educational implication of this valued attribute is that teachers can use learning activities that allow for cooperation and teamwork in their lessons as trainee teachers see these activities as important in their mathematics learning. This would ensure that each trainee teacher has a shared responsibility

(Anderson & Österling, 2019) in the mathematics classroom and would urge them to participate fully in the lessons. Also, the attribute of collaboration due to its impact was reported in other studies like Madosi *et al.* (2020); Seah *et al.* (2017b) and Hill *et al.* (2019). For their views on why they consider collaboration important in their mathematics learning, the trainee teachers cited socialization and leadership as the main reasons for valuing collaboration. The majority of them gave reasons relating to socialization based on the interaction that goes on in the mathematics classroom.

The third most prioritised value espoused by the trainee teachers was strategies that highlight the different ways of doing mathematics and solving problems. Interestingly, it was also the third attribute with good data spread among the attributes valued by the trainee teachers. This attribute was important since no concept can be taught or learned without a strategy or an algorithm. Hence, the intended strategy to be used would suggest its appropriate learning activity and teaching-learning materials that should be employed in that lesson. The attribute strategies valued by the trainee teachers in this study were in line with studies conducted by Davis et al. (2019); Hill (2019); Madosi et al. (2020) and Seah et al. (2017b). Based on the value studies conducted at the different stages of the Ghanaian educational system, it can be concluded that strategies was more dominant among students at all levels so far as mathematics learning was concerned. The three main reasons given by the trainee teachers as to why they considered strategies important in their mathematics learning included creativity, control and examinations. Most of the reasons cited were related to creativity which is one of the important attributes of mathematics learning. Again the trainee teachers accepted that strategies help reduce cheating in the examination due to alternatives available to the student. From the reasons given

by the trainee teachers above, it is clear that the attribute strategies was very powerful in the trainee teachers' mathematics learning as it contributes to the development of other valuing attributes such as fluency and flexibility. It also leads to the development of other learning abilities and brings dynamism to mathematics learning.

Accuracy, the fourth most prioritised attribute, as well as its data spread as valued by the trainee teachers. It is known that achieving high accuracy in mathematics, requires a lot of practice and effort. Hence, accuracy cannot be attained overnight and therefore, teachers need to include appropriate techniques in their mathematics lessons in order to ensure that the attribute accuracy is well developed in their students. This accuracy was also reported in Seah *et al.* (2017b) and Hill *et al.* (2019). The reasons provided by the trainee teachers in support of this attribute were categorized into four main groups namely understanding, control, progress and intrinsic motivation. The most dominant among the four main reasons was understanding. For the above reasons, the importance of accuracy cannot be ignored in the teaching and learning of mathematics as it has links with other attributes such as progress, understanding and control. It also tends to motivate intrinsically and boost trainee teachers' confidence in learning the subject.

The fifth attribute prioritised by the trainee teachers was fluency which deals with the pedagogical activities that promote effective use and manipulation of mathematical objects such as symbols and concrete materials in the learning of mathematics. Besides, it was the second-best data spread among the eight mathematics education attributes valued by the trainee teachers. The power of fluency is vested in the effective manipulation and use of mathematical symbols and materials. Therefore, understanding and

application cannot be achieved without evidence of fluency. This attribute was also confirmed in Davis *et al.* (2021); Davis *et al.* (2019) and Seah *et al.* (2017b). For fluency, the reasons offered by the trainee teachers were categorized into four main themes namely control, problem-solving, respect and social status. The majority of the trainee teachers gave reasons relating to control. Hence, learning activities that promote the four main attributes are encouraged to be employed by mathematics tutors at the colleges of education.

Relevance was the sixth attribute valued by the trainee teachers in terms of both prioritization and the spread of its data. The attribute relevance is very important due to its ability to relate mathematics ideas to real-life situations. This means it provides the link between mathematics and society. This valuing provides the basis for which society develops through the use of mathematical ideas and it makes people experience the power of mathematics in their lives. This mathematics learning attribute valued by the trainee teachers in this study was in line with the findings of most studies that report on values in literature. These studies included; Davis et al. (2019); Dede (2019, 2006); Seah et al. (2017b) and Seah (2011a). The reasons suggested by trainee teachers were grouped into two main themes which were application and communication. The most cited examples had to do with application. The reasons above suggest that trainee teachers through relevance can apply mathematical ideas in their everyday life. Besides, mathematics tutors in their response to interview guide item two suggested cultural identity, usefulness and understanding as the main reasons why they think attribute relevance was important in the trainee teachers' mathematics learning.

The seventh valued attribute prioritised by the trainee teachers was the mystery which was also the seventh so far as the spread of data was concerned.

This attribute underlines the surprises in learning mathematics vested in its wonders and mystique. From the trainee teachers, activities that promote such mathematics learning attributes are very important to them. The mystery is one of the widely reported attributes in the area of mathematics. This attribute observed in this study was also reported in other studies such as; Bishop (2000); Carr (2019); Corey and Ninomiya (2019; Dede (2006); Seah (2008) and Safura *et al.* (2018). However, the main reasons given by the trainee teachers were aesthetic value, intrinsic motivation and usefulness. Besides, the majority of them gave reasons relating to aesthetic value and intrinsic motivation. While their tutors in responding to interview guide item two agreed that mystery was important in their students' mathematics learning and suggested reasons such as openness, wonder and control.

Lastly, one important valuing attribute that was prioritised by the trainee teachers was learning technologies (Davis *et al.*, 2021; Davis *et al.*, 2019; Seah *et al.*, 2017b; Seah, 2011a) which coincidentally appeared to have the poorest data spread among the eight attributes valued. The importance of this attribute lies in the fact that the world is being moved by the advancement of technology and the powers of nations are measured based on their advancement and development in technology. For this, education has got its fair share such that education is being catalyzed by the use of technological tools in the classroom. In mathematics teaching and learning, technology integration has become the order of the day and teachers in mathematics education are no exception. For that matter, the effectiveness of a lesson is now being measured using technology integration as a yardstick.

The main reasons given to support the importance of learning technologies in trainee teachers' mathematics learning were illustrated as

demand and innovations. Additionally, they suggested the hindrances that impede the use of learning technologies as affordability and know-how. Thus, technology comes with its own challenges. A typical example was during the Covid'19 era when the survival of our educational system depended on such technological tools. But accepted that this technology could not be sustained due to poor internet connectivity problems at the colleges of education. They were quick to add that such technology comes with a cost and most colleges do not have the financial means to provide such technologies. Hence, these challenges associated with the use of technology might have affected its prioritization.

On the other hand, though learning technologies did not load for them, the mathematics tutors accepted that the use of technology in mathematics lessons was very important and wondered why such attributes did not load in their case. Instead, the tutors provided reasons why they were not using the technological tools in their classrooms as their response to the interview guide item two.

This was seen as a way to defend themselves for not using such tools in their lessons, a clear indication of their acceptance that they do not employ such technologies in their students' mathematics learning. The reasons given by them related to attributes of relevance and some hindrances such as affordability, know-how and students not being allowed to use such tools in examinations.

Also, they agreed that most of their colleague tutors were "born before computers" and hence, it is difficult for them to use such technological tools in their teaching and would not encourage such tools in their students' mathematics learning. Tutors again, accepted that students nowadays are more interested and use such technology more than their tutors. Tutors were also

aware that students would not use these technological tools in examinations and therefore, do not pay attention to them during the teaching-learning process. Besides, tutors stated that they were forced to use technological tools such as Zoom, telegram, WhatsApp and so on due to the covid'19, but could not be sustained because the college authorities were reluctant to buy these technology applications for integration in the classroom and tutors could not afford them on their own. Moreover, most of these tools need special know-how to be able to use them and therefore, special training was required by tutors. For instance, Telegraph, Geogebra and Smart Notebook need training before they can be used effectively. Based on the above responses from both trainee teachers and their mathematics tutors, it emphasizes why the learning technologies was least prioritised by the trainee teachers and did not load at all as an attribute for the mathematics tutors.

Attributes Valued by tutors in their students' mathematics learning

The results from the analysis suggest that the mathematics tutors seem to value (in order of priority) attributes of understanding, strategies, mastery, problem-solving and fun respectively in their students' mathematics learning. The best data spread among these five attributes valued by the mathematics tutors were presented as follows; mastery, strategies, understanding, problemsolving and fun respectively. These attributes valued together explained 70. 35% of the total variance in tutors' valuing in their students' mathematics learning at the five selected colleges of education used in this study. It is also important to note that two of these attributes valued by the tutors were also valued by the trainee teachers themselves in their mathematics learning. These two common attributes were understanding and strategies.

Moreover, out of the five attributes valued (understanding, strategies, mastery, problem-solving and fun) by the mathematics tutors, four were also confirmed in other important studies conducted in the area of values in mathematics education. In that line, understanding and problem-solving were reported by Davis *et al.* (2021). Again, fun and strategies were also observed by Seah *et al.* (2017b). Besides, the attribute of fun was reported in Dede (2019) while the strategies was observed in Ghanaian pre-tertiary students as presented in Davis *et al.* (2019).

Mastery was the third-best prioritised among the other attributes valued by the tutors in their students' mathematics learning. The main reasons tutors gave relating to mastery were observed to be attributes of applications, control and precision. Besides, the majority of the mathematics tutors' reasons were related to the applications. Mastery was important to the tutors because it has links with other attributes such as control, applications and precision. In responding to interview guide item two, the trainee teachers also agreed that mastery of concepts was important in their mathematics learning, though it was not prioritised by them. The main reasons given by the trainees were fluency, accuracy, control, problem-solving and applications. Based on the reasons provided by both tutors and their students suggest that they all agreed that mastery was very important in mathematics learning.

Understanding was the most prioritised attribute valued by tutors in their students' mathematics learning. Besides, it was the third attribute with a good spread within its data set. Understanding also happens to be one of the two attributes valued by both trainee teachers and their mathematics tutors. As already discussed for trainee teachers, it is one of the important attributes reported in the literature. The reasons given by the mathematics tutors on why

this attribute was important were applications, progress, relevance, and mastery of concepts. The most dominant reason given by the tutors related to application. This makes understanding a very important attribute because it appears to be linked to other attributes such as application, progress, relevance and mastery since these attributes require understanding. The mathematics tutors indicated through the interviews that to achieve a better understanding, learning activities should be relevant to the concept that is to be developed. They said it was also important to use materials that are familiar and not above the level of the students for better understanding.

The second most prioritised valued and data spread respectively attribute valued by the mathematics tutors was strategies. Three main reasons provided by the tutors in support of this attribute were applications, control and relevance. Moreover, the majority of the tutors gave reasons relating to applications. However, they mentioned hindrances in the implementation of alternative strategies in the mathematics classroom to be teaching several topics within a short period. Strategies was one of the common attributes valued by both groups of participants. This affirms how important strategies was in the trainee teachers' mathematics learning. It is, therefore, obvious that without strategies mathematics would be boring as there would be only one way of doing mathematics. Thus, a sense of variety and alternatives would not be experienced by learners.

The fourth attribute valued by the mathematics tutors in their trainee teachers' learning was problem-solving. Problem-solving is one of the important attributes in the teaching and learning of mathematics, as mathematics is known to be made up of problems that require solving. Again, our everyday life is engraved with problems that need to be solved for our

communities to develop. The three main reasons offered by the mathematics tutors in support of why problem-solving activities are important in their students' mathematics learning were skills acquisition, strategies and relevance. However, most of the reasons suggested by the tutors were related to skills acquisition. Therefore, problem-solving skills have to be acquired for effective application. Besides, in responding to the interview guide item two, the trainee teachers also agreed that problem-solving was important for the following three reasons i.e. applications, understanding and control. Therefore, both groups agreed that it was an important attribute in the students' mathematics learning.

The fifth and last attribute valued by the mathematics tutors was fun and was as well had the poorest spread in its data structure. Fun refers to the attributes that promote enjoyment in mathematics learning. This attribute was valued by only the tutors in their students' mathematics learning. The attribute fun was in line with Dede (2006). In highlighting the importance of fun, the mathematics tutors' reasons were analyzed into three main themes namely concentration, motivation and socialization. The majority of the reasons given by the tutors were related to socialization. This reinforces the importance of social interactions that ensures effective class discourse in mathematics learning. Though fun was valued by only mathematics tutors, responding to trainee teachers' interview guide item two, the trainee teachers gave two main reasons why fun was important in their mathematics learning namely enjoyment and confidence. They were also quick to add that the time allocation for a course does not allow tutors to engage in the fun aspect of mathematics learning.

Relationships between attributes valued by trainee teachers and their mathematics tutors

In order to ascertain whether or not relationship exist between what tutors and their trainces valued in students' mathematics learning, a Pearson product-moment correlation was conducted between the eight attributes valued by the trainee teachers themselves and the five valued by their tutors. Based on the analysis, it was revealed that four significant weak inverse relationships existed between attributes valued by the trainee teachers themselves and that of their mathematics tutors (Anderson & Österling, 2019). The inverse relations were observed between learning technologies (from trainee teachers) and mastery (from mathematics tutors); relevance (from trainee teachers) and strategies (from mathematics tutors) and lastly, accuracy (from trainee teachers) and problem-solving (from mathematics tutors). The significance of these relationships suggests that they are very important and therefore, cannot be ignored so far as valuing in mathematics learning is concern at the five selected colleges of education where this study was conducted.

Also, the inverse relationships between the respective pairs of valued attributes were an indication that when there is a change in one of these attributes it would affect the other pair to change in the opposite direction. For instance, if the attribute of learning technologies (from trainee teachers) is emphasized in trainee teachers' learning it would cause mastery (from mathematics) to be de-emphasized and verse versa. The same analogy can be drawn for the other three inverse relationships reported above. Suggesting conflicting valuing between the trainee teachers and their mathematics tutors in the students' learning experiences (Clarkson *et al.*, 2019). This means teaching

attributes valued in mathematics at the five selected colleges would not be a clear-cut process as experience is required on the part of the mathematics tutors to teach it. Perhaps, the learning activities employed in the trainee teachers' mathematics learning by the tutors have no value intentions and are not intended to expose them to any values. This assertion was confirmed in the correlation conducted on the overall attributes (using items retained for each group) valued by both groups. The result appeared to indicate that there was no significant relationship between the overall attributes valued by both groups of participants. This finding was seen to contradict Kalogeropoulos (2016)'s assertion that a strong relationship existed between students and their teachers' valuing. The implications of this finding show that what trainee teachers value in their mathematics learning has no relationship with what their tutors think the student's value. The possible reason that could be proposed here was that the mathematics tutors were not aware (Peng & Nyroos, 2012) of what their students value in their mathematics learning. From this, it appears that there is value misalignment between trainee teachers and their mathematics tutors. The misalignment in valuing may have serious implications on the trainee teachers' mathematics learning. In that, decisions taken by the tutors in the students' mathematics learning might not be in the interest of the students due to the differences in valuing. It was also said that the decisions teachers make in their classrooms are influenced (Bishop et al., 2000) by the values held and therefore, the learning activities implemented by the mathematics tutors in the college classroom might have been influenced by their values but not that of their students. Deducing from the above discussions, it would be in the right direction that mathematics tutors from the five selected colleges be exposed more to value

teaching to help their students develop appropriate values that would be useful in their professional journey.

Differences in valuing between trainee teachers and their mathematics tutors

In comparing the attributes valued (fluency, understanding, mystery, strategies, learning technologies, relevance, collaboration and accuracy) by trainee teachers and their tutors (fun, mastery, understanding, problem-solving and strategies) qualitatively, the results revealed that six attributes were valued by only the trainee teachers. These six attributes were fluency, mystery, learning technologies, relevance, collaboration and accuracy. However, attributes of fun, mastery and problem-solving were valued by only the mathematics tutors. This means the attributes valued differently by the two groups supercedes those valued by both groups of respondents.

Additionally, the items that loaded onto each common attribute valued were compared respectively for their differences. After the comparison of the respective items, it was observed that attribute understanding shared only one item "Using concrete materials to understand mathematics (item 39)" between the two groups. All the remaining items (4 items for trainee teachers and 3 items for tutors) were different between the two groups. Hence, understanding the differences outweigh the similarities based on the individual items that were loaded onto them. Though for strategies, one item "Using concrete materials to understand mathematics (item 39)" loaded to both trainee teachers and their mathematics tutors directly, the other remaining items (3 items for trainee teachers and 2 items for tutors) shared some similar characteristics. For example, items 26 and 59 for the trainee teachers and their tutors respectively had "explaining" as a common characteristic, though those items were different.

Again, items 28 and 57 that loaded for trainee teachers and their tutors respectively had related characteristics as "teaching-learning materials" and "mathematics objects" respectively. The two characteristics relate in that teaching-learning materials are specific examples of mathematical objects. The only direct observable difference was on the item "*Accepting different views from colleagues (item 43)*" which loaded for only the trainee teachers on strategies. Hence, for strategies, it appears the similarities outweigh the differences.

Significant differences in valuing between traince teachers and their mathematics tutors

The discussion in this section was to demonstrate how the research hypothesis "there is no significant difference between trainee teachers and their tutors valuing in mathematics learning in the selected colleges of education" was addressed. The differences were determined by the use of an independent samples t-test on the two common attributes valued by both trainee teachers and their mathematics tutors. In achieving this, the two common attributes (understanding and strategies) valued were computed into continuous dependent variables using the items that loaded onto them as a result of the PCA conducted for the two groups.

The results showed that both trainee teachers and their mathematics tutors ranked values in mathematics learning highly as observed in the overall means for the two groups. In both cases, the overall means were above the baseline of 2.50 on the four-point Likert scale used in this study. This appears to suggest that both trainee teachers and their tutors consider valuing as an important component in mathematics learning at the selected colleges of education in Ghana. It is also important to mention that not much has been done

in the area of research that investigated the differences in valuing at the tertiary level. The existing studies that investigated differences were conducted at the pre-tertiary level. Such studies also concentrated on only attributes valued by students (Davis *et al.*, 2019; Seah *et al.*, 2017b). This current study has, therefore, presented the nature of differences between tutors and their students' valuing in the context of mathematics learning at the colleges of education in Ghana. Thus, this study would serve as a springboard for future studies aimed at exploring the differences between teachers' and their students' valuing, especially at the tertiary level.

Chapter Summary

This chapter aimed at presenting the results and discussions on attributes valued by trainee teachers and their tutors in mathematics learning with their differences and relationships at the five selected colleges of education in Ghana. From research questions one and two, eight attributes (understanding, collaboration, strategies, accuracy, fluency, relevance, mystery and learning technologies) of mathematics learning were valued by the trainee teachers, while five attributes (understanding, strategies, mastery, problem-solving and fun) were also valued by their tutors respectively. It was also observed that two of these attributes (understanding and strategies) were common to both groups. Based on the correlational analysis conducted, it was revealed that four inverse relationships existed between the attributes valued by trainee teachers and that of their tutors in mathematics learning which cannot be ignored. However, generally, there was no relationship between the overall computed attributes valued by both groups. Again, it was revealed that there were no significant differences in the common attribute valued by both trainee teachers and their is trained by the groups.

tutors. Finally, the reasons for the attributes valued by the trainee teachers and that of their tutors were presented.

The next section which is chapter five presents a summary, conclusions and recommendations for this study.



CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Overview

This chapter presents the summary of the study, conclusions drawn from its findings and recommendations based on the findings generated from this study.

Summary

The purpose of the study was to explore what trainee teachers and their tutors value in mathematics learning and, it also sought to ascertain whether what tutors value in their students' mathematics learning aligns with what trainee teachers value at the colleges of education in the Ahafo, Bono and Bono East Regions of Ghana.

The research design used in the conduct of this study was a sequential explanatory mixed-methods design (SEMMD). The quantitative data through the four-point Likert scale questionnaires for both trainee teachers and their mathematics tutors were collected and analyzed first. Based on the findings obtained, the qualitative data were then collected through the semi-structured interview guides administered to both groups of respondents to explain the results generated from the quantitative analysis. This enabled the researcher to ascertain from the respondents why they valued attributes observed from the quantitative analysis were important in trainee teachers' mathematics learning.

A purposive sampling technique was used to select five colleges of education, core mathematics third-year trainee teachers and their mathematics tutors. Again, a systematic sampling technique was used to select some trainee teachers out of the main sample for the focused group interviews in each

selected college of education. All the mathematics tutors in each selected college were used for the interview session.

A sample of 1050 third-year core mathematics trainee teachers from the five colleges of education, and all their mathematics tutors (34) were selected for this study. Besides, fifty (50) of these trainee teachers (10 from each college) were as well selected systematically for the focused group interview sessions in each college of education. Four research questions and one research hypothesis guided the conduct of this study. These research questions were: "What do trainee teachers value in their mathematics learning in the colleges of education?; what do tutors value in their students' mathematics learning in the colleges of education?; what is the relationship between trainee teachers and their tutors' valuing in mathematics learning?; what are the differences, if any, in valuing between training teachers and their mathematics tutors?; The hypothesis, however, was "there is no significant difference between trainee teachers and their tutors valuing in mathematics learning at the selected colleges of education".

Furthermore, descriptive statistics were employed to describe the data collected. Again, principal component analysis was used to determine the attributes valued by the third-year trainee teachers and their mathematics tutors. The focused group interviews were analysed qualitatively and the responses from both groups were used to provide the main reasons why the observed attributes were important in the trainee teachers' mathematics learning. Both qualitative comparison and independent samples t-test were as well used to explore the differences between the common attributes valued by the trainee teachers and their mathematics tutors.

the attributes valued by the two groups, a Pearson product-moment correlation was used.

The summary of the main findings generated from the results is presented in the section below.

Summary of key findings

The first research question sought to identify the attributes valued by the third-year core mathematics trainee teachers in their mathematics learning at the colleges of education in Ghana. The following findings were obtained:

- The trainee teachers valued eight (8) attributes in their mathematics learning. These attributes were understanding, collaboration, strategies, accuracy, fluency, relevance, mystery and learning technologies.
- All the eight attributes valued were highly ranked as their computed means were greater than the baseline of 2.50 suggesting that all were highly prioritised.
- The best three highly prioritised attributes were understanding, collaboration and strategies respectively. While the least prioritised attributes were relevance, mystery and strategies respectively.
- 4. The eight attributes valued together explained 50.60% of the total variance towards the trainee teachers' valuing in their mathematics learning at the five colleges of education used in this study.
- 5. Trainee teachers gave various reasons why the attributes espoused by them were important in their mathematics learning. For instance, understanding was valued because it has links with

knowledge acquisition, control and progress. Of these reasons cited, the most dominant reason was related to control. Again, the reasons given in support of attribute collaboration were related to socialization and leadership. The results showed that the majority of them gave reasons relating to socialization.

The second research question elicited attributes valued by the tutors in their students' mathematics learning. The key findings observed after the PCA are presented as follows:

- 1. Five attributes were valued in trainee teachers' mathematics learning from the perspective of their tutors and these were understanding, strategies, mastery, problem-solving and fun.
- 2. All the five attributes valued by the mathematics tutors were highly ranked as their computed means were all greater than the baseline of 2.50 and they were all highly prioritised as reported for the trainee teachers as well.
- 3. The best-prioritised attributes valued by the tutors' were understanding, strategies, mastery, problem-solving and fun respectively.
- The attributes valued by the tutors in their students' mathematics learning together accounted for 70.35 % of the total variance explained.
- 5. Some of the reasons given by the tutors in support of the attributed valued by them in their students' mathematics learning were; mastery of concepts was important because it relates to application, control and precision. Besides, the majority of the reasons cited for this attribute were related to application.

Problem-solving was valued since it promotes application, control and relevance. Also, the majority of the reasons given by the tutors for problem-solving had to do with application.

The third research question sought to determine if there were any relationships between the attributes valued by the trainee teachers and their mathematics tutors. A qualitative comparison was first made, followed by multiple and bivariate correlational analysis to determine the Pearson productmoment correlation coefficient among the attributes valued by both groups. The findings from the results showed that:

- The combined attributes valued by trainee teachers and their mathematics tutors were eleven and two of these attributes were common to the groups. The eleven attributes espoused in this study were strategies, mastery, understanding, fluency, relevance, mystery, learning technologies, fun, problem-solving, collaboration and accuracy.
- 2. Four weak negative significant relationships were recorded between the attributes valued by the two groups; learning technologies (from trainee teachers) and mastery (from mathematics tutors); relevance (from trainee teachers) and mastery (from mathematics tutors); relevance (from trainee teachers) and strategies (from mathematics tutors); and accuracy (from trainee teachers) and problem-solving (from mathematics tutors).
- Generally, there was no significant relationship between the attributes valued by the trainee teachers and their mathematics tutors.

The fourth question found out if there were differences between the attributes valued by trainee teachers and their tutors. The findings obtained after the qualitative comparison were:

- Six attributes (fluency, mystery, learning technologies, relevance, collaboration and accuracy) were valued by only the trainee teachers. However, attributes of fun, mastery and problem-solving were valued by only the mathematics tutors.
- For the common attribute understanding, the differences outweigh the similarities based on the individual items that loaded for the trainee teachers and their tutors respectively.
- Strategies appears to share similar characteristics between their corresponding items than differences.

The only research hypothesis of the study aimed at finding out whether there were significant differences between the common attributes valued by the trainee teachers and their mathematics tutors. After addressing this research hypothesis based on the independent samples t-test that was conducted on the responses collected from the questionnaire items. The common pairs valued (with their respective questionnaire items retained) were used for this t-test analysis. The result indicated that there was no significant differences between the common attributes valued by the trainee teachers and their mathematics tutors.

In the next section that follows the researcher presents the conclusions drawn from the findings generated from this study.

Conclusions

This study has revealed that the third-year core mathematics trainee teachers valued eight attributes (understanding, collaboration, strategies,

accuracy, fluency, relevance, mystery, and learning technologies) in their mathematics learning. While their mathematics tutors valued five attributes (understanding, strategies, mastery, problem-solving and fun) in their students' learning. Two of these attributes (understanding and strategies) were common in both traince teachers and their mathematics tutors valuing. Besides, six attributes (collaboration, accuracy, fluency, relevance, mystery, and learning technologies) valued by the trainee teachers' were not observed in their tutors' valuing. Similarly, three attributes (mastery, problem-solving and fun) were valued by only the mathematics tutors. Hence, by inspection, the attributes valued by trainee teachers and their mathematics tutors showed more differences than similarities between the two sets of valued attributes. Moreover, for the two common attributes, understanding showed more differences in terms of the individual items that loaded onto their respective attributes. Though strategies showed some differences in the items that loaded, they shared some similar characteristics within the corresponding items respectively.

From the computed means for the reported attributes for the trainee teachers, it was observed that the most prioritised was understanding, followed by collaboration, strategies, accuracy, fluency, mystery and learning technologies. While the most prioritised attribute from their mathematics tutors' perspective was understanding, followed by strategies, mastery, problemsolving and fun. However, it was observed that the attribute of understanding and strategies were ranked as the best three most prioritised valued attributes by both groups. This means that both trainee teachers and their mathematics tutors consider these two attributes (understanding and strategies) as important

components of students' mathematics learning experiences at the colleges of education in Ghana.

On the other hand, the two least prioritised attributes recorded for the trainee teachers were mystery and learning technologies, while that of the mathematics tutors were problem-solving and fun. These least valued attributes were observed as such based on the fact that for mystery, perhaps mathematics tutors were not aware of its presence in their students' mathematics learning and therefore, do not include pedagogical activities (mathematics puzzles, history of mathematics and some great mathematicians) that promote mystery in their mathematics lessons. For learning technologies, though it was not loaded for the mathematics tutors, in responding to interview guide item two, tutors accepted that it was very important in their students' learning. According to the tutors, the value attribute learning technologies was not prioritised on their part possibly because most of the tutors were considered "born before computers" and therefore, find it difficult to use such technological tools in their teaching. As a result, they would not encourage the use of such tools in their students' learning. Again, since most of these technological tools are not allowed to be used during examinations by trainee teachers, mathematics tutors do not see the need to include such tools in their students' learning experiences.

In the case of the mathematics tutors, problem-solving was least prioritised. Perhaps, they see teaching through problem-solving as a difficult task and as such would not encourage such attributes that they are not familiar with in their students' learning. Another reason was that the mathematics tutors are not able to model such problems with familiar materials and context to make the problems relevant and meaningful to their students. Fun was observed as the least prioritised attribute by the mathematics tutors since the time allocated

for the teaching of mathematics does not allow the fun aspect of mathematics learning to be fully realized in students' learning. Though the fun was least prioritised, both mathematics tutors and their students agreed that it was a very important attribute in the teaching-learning of mathematics. Most of these positive reasons elaborated have to do with effective discourse, motivation and reduction in anxiety among trainee teachers.

The study also revealed four weak inverse relationships that could not be ignored between the attributes valued by the trainee teachers and their mathematics tutors. These relationships were; learning technologies (from trainee teachers) and mastery (from mathematics tutors); relevance (from trainee teachers) and mastery (from mathematics tutors); relevance (from trainee teachers) and strategies (from mathematics tutors) and accuracy (from trainee teachers) and problem-solving (from mathematics tutors). The inverse relationships between these attributes means that there are cases of value misalignment among such pairs of attributes in the trainee teachers' mathematics learning. As emphasizing one of these pair of valued attributes would de-emphasize the other. However, it was observed that there was generally no relationship between the overall identified attributes valued by the trainee teachers and their mathematics tutors. It, therefore, appears to suggest that both groups valued different sets of attributes in mathematics learning. Thus, although value education is not part of the trainee teachers' curriculum, they value certain attributes in their mathematics learning which appears to diverge from what their tutors valued. Hence, there was value misalignment in mathematics learning between trainee teachers and their tutors at the five selected colleges of education.

Finally, this current study has contributed adequately to the understanding valuing at the colleges of education and also contributed to the academic debate on values in mathematics learning at the five selected colleges of education in Ghana. The study revealed that both trainee teachers and their mathematics tutors accept and consider the identified attributes as very important in the students' mathematics learning.

Recommendations

Based on the findings generated from this study, the following recommendations were made for tutors, trainee teachers, college authorities, curriculum planners and policymakers.

- 1. The existence of attributes valued in the learning of mathematics has some implications on curriculum development, pedagogies used by mathematics tutors, teacher preparation and in-service training programmes for the colleges of education. This study, therefore, recommends the following;
 - i) curriculum of teacher education programmes need to include courses on mathematics education values to give trainee teachers exposure to values that would enable them to help learners to learn mathematics meaningfully.
 - ii) college authorities should organise professional development programmes for practicing tutors to enhance their competencies and experience to enable them integrate some of these learning attributes valued in their lessons.
 - iii) curriculum developers when developing or revising the curriculum for mathematics teacher education should put priority on such valued attributes.

- iv) though there is an existing course that introduces trainee teachers to the value concept, there is the need for the mentoring universities to pay deeper attention to values in mathematics education by instituting it as a course on its own at the colleges of education.
- The study further found that generally there was no relationship between the attributes valued by the trainee teachers and their mathematics tutors. This study, again, recommends that;
 - i) colleges of education in collaboration with the mentoring universities should organize in-service training for mathematics tutors to expose them more to the valuing in mathematics learning.
 - ii) mathematics tutors should abreast themselves with the pedagogical activities that their trainee teachers consider important in their learning to support trainee teachers' cognitive, affective and conative development in their mathematics learning.
- The outcome of this study revealed that some of these attributes valued had a critical influence on mathematics learning at the colleges of education. The study, therefore, recommends that;
 - the attributes of understanding and strategies, which were the most prioritised by both trainee teachers and their mathematics tutors should be given the needed attention as both groups consider them very important in mathematics learning.
 - ii) trainee teachers should be made aware of the presence of these valued attributes in their mathematics learning to identify the associated pedagogical activities which are of interest to them.

- iii) special in-service training should be organized for mathematics tutors for effective use and integration of some of these technological tools in their lessons.
- iv) mathematics tutors should be encouraged more by the college authorities to employ problem-solving attributes in their teaching to expose their trainee teachers to such an approach for them to also apply in their future lessons.

Suggestions for Further Research

This study was conducted on third-year trainee teachers taking mathematics as a core subject. A further study is, therefore, recommended to explore elective mathematics trainee teachers' valuing in their mathematics learning at the colleges of education.

Again, this study was conducted in only one PRINCOF zone Ash-BA. Hence, it is recommended that this study be replicated in other zones to present the national picture on valuing in trainee teachers' mathematics learning at the colleges of education in Ghana.

Finally, this study recommends that further studies should be conducted to explore how attributes valued in mathematics learning might reflect trainee teachers' mathematics performance at the colleges of education in Ghana.

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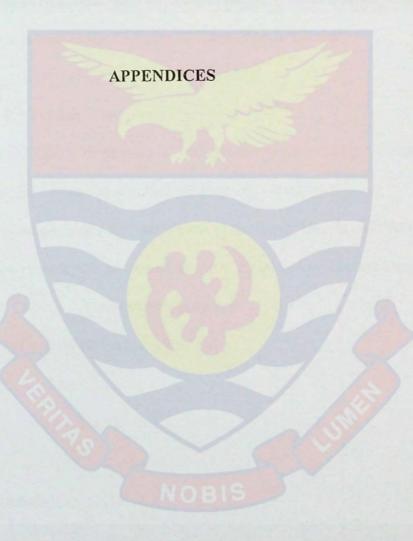
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APPENDIX "A"

UNIVERSITY OF CAPE COAST

FACULTY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS AND ICT EDUCATION

QUESTIONNAIRE FOR Ph.D. (MATHEMATICS EDUCATION)

THESIS

Value Alignment in Mathematics Learning between Trainee Teachers and

their tutors in the Colleges of Education in Ghana

(UCCIRB/CES/2021/100)

Questionnaire for Trainee Teachers

This questionnaire elicits information on what trainee teachers at the Colleges

of Education value in their mathematics learning.

Please, provide your genuine response to each of the items. Be assured that the

information you provide will be treated as confidential and will be used only

for academic purposes.

SECTION A: BIOGRAPHICAL DATA

College.....

Programme

Age (In Years)

Sex (Male or Female)

SECTION B

This section elicits information on what students' value in mathematics learning.

For each of the following statements provided, tick ($\sqrt{}$) the appropriate box which applies to you indicating the extent to which you value the statement. **Key**: The options are defined and weighted for this questionnaire as follows: Absolutely Important (AI) = 4; Important (I) = 3; Unimportant (U) = 2;

Absolutely Unimportant (AU) = 1

Item	Items on Valuing in Mathematics Learning	4	3	2	1
No.					
1	Asking questions				
2	Keeping objective(s) of a lesson in mind				
3	Involvement in class activities				
4	Identifying mathematics ideas				
5	Group presentations.				
6	Doing homework				
7	Group discussions				
8	Answering group assignments.				
9	Using different methods in solving problems				
10	Learning mathematics with the internet	-			
11	Discussing answers	1.	5		
12	Giving enough time to think about the problem before			1	
	solving it.		E		
13	Learning mathematics with computers	5			
14	Using mathematical ideas in solving real-life				
	problems.				
15	Using the calculator to verify the answer.				
16	Feedback from teachers				
17	Identifying mathematical ideas from the environment				
18	Using both examples and non-examples				
19	Explaining mathematical concepts				
		1	_	1	

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20	Making up my mathematics questions	1.	T	1	
21					
	Knowing the keywords on a topic				
22	Small-group discussions				
23	Showing correct workings				
24	Step-by-step delivery of lessons.				
25	Mathematics debates.				
26	Explaining the origin of mathematics rules and				
	formulae				
27	Individual presentation of solutions				
28	Using teaching-learning materials				
2,9	Students getting feedback from their colleagues				
30	Relationship between mathematics concepts				
31	Knowing the theoretical aspect of mathematics		1		
32	Using calculator in mathematics learning	17	1		
33	Outdoor mathematics activities	1	1	3	
34	Remembering strategies used previously		5		
35	Getting assistance from colleagues during	X		1	
	presentations.				
36	Mathematics puzzles				
37	Explaining mathematics procedures				
38	Using real objects in mathematics learning				
39	Using concrete materials to understand mathematics				
40	Using mathematics symbols				
41	Accessing information from the internet				
42	History of mathematics				
		Am	1		

43	Accepting different views from colleagues				
44					
44	Mathematics games				
45	Remembering the correct procedure				
46	Using mathematics terms				
47	Whole-class discussions				
48	Knowing reasons for using specific methods in				
	solving questions.				
49	Practicing how to use mathematics formula	72			
50	Memorising facts	7			
51	Using ICT tools in mathematics				
52	Explaining my solution to the class.				
53	Appreciating the beauty of mathematics				
54	Recalling a formulas	-	1		
55	Revising previous knowledge	17			
56	Finding different ways of solving problems.	1	5	1	
57	Classifying mathematics objects	7	5		
58	Stories of mathematicians		3	7	
59	Using diagrams to explain solutions	NO)	-		
60	Understanding my solutions				
61	Learning by proofs				
62	Learning through mistakes				
63	Verifying theorems				
64	Wonder in mathematics				

Thank you for participating!!!

APPENDIX "B"

UNIVERSITY OF CAPE COAST

FACULTY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS AND ICT EDUCATION

QUESTIONNAIRE FOR Ph.D. (MATHEMATICS EDUCATION)

THESIS

Value Alignment in Mathematics Learning between Trainee Teachers and

their tutors in the Colleges of Education in Ghana

(UCCIRB/CES/2021/100)

Questionnaire for Mathematics Tutors

This questionnaire elicits information on what tutors at the Colleges of

Education value in their students' mathematics learning.

Please, provide your genuine response to each of the items. Be assured that the

information you provide will be treated as confidential and will be used only

for academic purposes.

SECTION A: BIOGRAPHICAL DATA

College.....

Age (In Years)

Sex (Male or Female)

Teaching experience at the college level (In Years)

SECTION B

This section elicits information on what tutors value in their **students**' mathematics learning.

For each of the following statements provided, tick ($\sqrt{}$) the appropriate box which applies to you indicating the extent to which you value the statement. **Key**: The options are defined and weighted for this questionnaire as follows:

Absolutely Important (AI) = 4; Important (I) = 3; Unimportant (U) = 2;

Absolutely Unimportant (AU) = 1

Item	Items on Valuing in Mathematics Learning	4	3	2	1
No.					
1	Students asking questions				
2	Students keeping objective(s) of a lesson in mind				
3	Students involvement in class activities				
4	Students identifying mathematics ideas	-/			
5	Students group presentations	2			
6	Students doing homework				-
7	Students engaging in group discussions				-
8	Students taking part in group assignments				
9	Students using different methods in solving				
	problems		_/		
10	Students learning mathematics with the internet	1	1	0	
11	Students discussing their answers				
12	Teacher giving enough time to students to think		1		
	about the problem before solving it	1			
13	Students learning mathematics with computers				
14	Using mathematical ideas in solving real-life				
	problems.				
15	Students using the calculator to verify the				
	answer.				
16	Teacher giving feedback				

17	Students identifying mathematical ideas from				
	their environment		•		
18	Teacher using both examples and non-examples				
19	Teacher explaining mathematical concepts				
20	Students making up their mathematics questions				
21	Students knowing the keywords on a topic				
22	Students engaging in small-group discussions				
23	Students showing correct workings				
24	Teacher step-by-step delivery of lessons				
25	Students engaging in Mathematics debates				
26	Teacher explaining the origin of mathematics				
	rules and formulae				
27	Individual student presentation of solution				
28	Teacher using teaching-learning materials		1		
29	Students getting feedback from their colleagues			A	
30	Students establishing relationships between		5	K	
	mathematics concepts		10	12	
31	Students knowing the theoretical aspect of	5	Y		
	mathematics	1			
32	Students using calculators in their mathematics				
	learning				
33	Students engaging in outdoor mathematics				
	activities				
34	Students remembering strategies used previously				
		1		_	1

.

35	Students getting assistance from colleagues		-		
	during presentations.				
36	Students unravelling Mathematics puzzles				_
37	Students explaining mathematics procedures				
38	Students using real objects in mathematics				
	learning				
39	Students using concrete materials to understand		2		
	mathematics				
40	Students using mathematics symbols	2			
41	Students accessing information from the internet				
42	Students knowing history of mathematics				
43	Students accepting different views from				
	colleagues				
44	Students engaging in mathematics games		7		
45	Students remembering the correct procedure			X	
46	Students using mathematics terms	/	5	K	
47	Students engaging in whole-class discussions			7	
48	Students knowing reasons for using a specific	6	SV.		
	method	1			
49	Students practicing how to use mathematics				
	formula				
50	Students memorising facts				
51	Students using ICT tools in mathematics				
52	Students explaining their solutions to the class.				
53	Students appreciating the beauty of mathematics		-		

54	Students recalling formulas		
55	Teacher revising previous knowledge		
56	Students finding different ways of solving		
	problems.		
57	Students classifying mathematics objects		
58	Stories of mathematicians		
59	Students using diagrams to explain solutions		
60	Students understanding their solutions	70	
61	Students learning by proofs		
62	Students learning through mistakes		
63	Students verifying theorems		
64	Students identifying wonders in mathematics		

Thank you for participating!!!

APPENDIX "C"

UNIVERSITY OF CAPE COAST

FACULTY OF SCIENCE AND TECHNOLOGY DEPARTMENT OF MATHEMATICS AND ICT EDUCATION INTERVIEW GUIDE FOR Ph.D. (MATHEMATICS EDUCATION)

THESIS

Value Alignment in Mathematics Learning between Trainee Teachers and

their tutors in the Colleges of Education in Ghana

ETHICAL CLEARANCE-ID (UCCIRB / CES / 2021 / 100)

Interview Guide for Trainee Teachers

This interview session is a follow up to the value questionnaire you responded to earlier and it is meant to find out reasons or explanations to your espoused values in mathematics learning.

Please, provide your genuine response to each of the items that will be asked.

Be assured that the information you provide will be treated confidential and

will be used only for academic purposes.

Hence, your permission to audio record views is sorted by your agreement to participate in this interview session. Thank you for your participation.

College.....

Gender (Male or Female)

Interview Items:

NOTE: These interview guide items are follow-up to the questionnaire items that you responded to earlier on values in mathematics learning. The analysis of the questionnaire items resulted in eight (8) values espoused by the trainee teachers. These values included; **fluency, understanding, mystery, strategies, learning technologies, relevance, collaboration and accuracy.**

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1. What makes each of these above-listed values important in your

mathematics learning?

A. Fluency;

- B. Understanding;
- C. Mystery;
- D. Strategies;
- E. Learning technologies;
- F. Relevance;
- G. Collaboration;
- H. Accuracy.

......Follow-up question(s) may apply).

- NB: Operational definitions for the values will be provided to participants.

Thank you!!

APPENDIX "D" UNIVERSITY OF CAPE COAST FACULTY OF SCIENCE AND TECHNOLOGY DEPARTMENT OF MATHEMATICS AND ICT EDUCATION INTERVIEW GUIDE FOR Ph.D. (MATHEMATICS EDUCATION) THESIS

Value Alignment in Mathematics Learning between Trainee Teachers and their tutors in the Colleges of Education in Ghana ETHICAL CLEARANCE-ID (UCCIRB / CES / 2021 / 100) Interview Guide for Mathematics Tutors

This interview session is a follow up to the value questionnaire you responded to earlier and it is meant to find out reasons or explanations to your espoused

values in your students' mathematics learning.

Please, provide your genuine response to each of the items that will be asked.

Be assured that the information you provide will be treated confidential and

will be used only for academic purposes.

Hence, your permission to audio record views is sorted by your agreement to

participate in this interview session. Thank you for your participation.

College.....

Gender (Male or Female)

Interview Items

NOTE: These interview guide items are follow up to the questionnaire items that you responded to earlier on your students' values in mathematics learning. The analysis of the questionnaire items resulted in five (5) values espoused by you in relation to your students' mathematics learning. These values include; fun, mastery, understanding, problem-solving and strategies.

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- What makes each of the above-listed values important in your students' mathematics learning?
 - A. Fun;
 - B. Mastery;
 - C. Understanding;
 - D. Problem-solving;
 - E. Strategies.

..... (Follow up question(s) may apply).

 Do you consider values such as learning technologies, fluency, mystery, relevance, collaboration and accuracy important in your

students' mathematics learning?Why?

NB: Operational definitions for the values will be provided to participants.

Thank you!!!

APPENDIX "E"

Pearson Correlation Matrix Table on Values espoused by Trainee Teachers and their Mathematics Tutors ($N_1 = 1050$, $N_2 = 34$)

	MTCI	MTC2	MTC3	MTC 4	MTC 5	TTCI	TTC2	TTC3	TTC4	TTC5	TTC6	TTC7	TTC8	TTC OVERA LL
MTC2	Pearson Correlation	.274	1											
	Sig. value	.058												
MTC3	Pearson Correlation	.304*	.243	1										
	Sig. value	.040	.083											
MTC4	Pearson Correlation	.425**	.313*	.196	1	15	0							
	Sig. value	.006	.036	.133		T A			1					
MTC5	Pearson Correlation	.026	.365*	.152	088	1	6	þ ľ	1	A				
	Sig. value	.441	.017	.196	.310		0100	//		1 2				
TTC1	Pearson Correlation	071	185	173	274	.072	1	·	/	25				
	Sig. value	.345	.147	.164	.058	.343				2/1				
TTC2	Pearson Correlation	.081	184	.089	271	009	.646**	1		1				
	Sig. value	.325	.148	.307	.060	.480	.000		14/					
TTC3	Pearson Correlation	069	203	049	225	081	.620**	.439**	1					
	Sig. value	.348	.125	.392	.101	.324	.000	.000						

there is the more than

TTC4	Pearson Correlation	.142	111	178	206	.122	.609**	.535**	.445**	1				
	Sig. value	.211	.266	.157	.122	.247	.000	.000	.000					1.081-52
TTC5	Pearson Correlation	015	417**	.003	142	122	.447**	.350**	.456**	.331**	1			
	Sig. value	.467	.007	.494	.212	.246	.000	.000	.000	.000				
TTC6	Pearson Correlation	125	335*	122	138	317*	.588**	.471**	.482**	.457**	.395**	1		
	Sig. value	.241	.027	.246	.217	.034	.000	.000	.000	.000	.000			
TTC7	Pearson Correlation	.111	023	.098	216	175	.419**	.532**	.319**	.428**	.257**	.368**	1	
	Sig. value	.266	.449	.291	.109	.161	.000	.000	.000	.000	.000	.000		
TTC8	Pearson Correlation	155	196	.169	293*	130	.470**	.418**	.384**	.464**	.303**	.367**	.316**	1
	Sig. value	.190	.133	.169	.046	.231	.000	.000	.000	.000	.000	.000	.000	
MTC OVERALL	Pearson Correlation			151		100		b K	1	9				224
	Sig. value				N.S.	2	14/15	15		0				.101

*. Correlation is significant at the 0.05 level.

**. Correlation is significant at the 0.01 level.

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APPENDIX "F"

Sample Letter for Data Collection from Head of Department

UNIVERSITY OF CAPE COAST COLLEGE OF EDUCATION STUDIES FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION DEPARTMENT OF MATHEMATICS AND I.C.T EDUCATION

Telephone: 0332096951 2552, UCC, GH Telegrams & Cables: University, Cape Coast Email: dmiete@ucc.edu.gb

Your Ref:

Our Ref: DMICTE/P.3/V.3/022

University Post Office Telex: Cape Coast, Ghana

Date: 30th July, 2021

The Principal

Dear Sir/Madam,

RESEARCH VISIT

The bearer of this letter, Daniel Ashong, with registration number BT/DME/19/0005 is a PhD, (Mathematics Education) student of the Department of Mathematics and ICT Education, College of Education Studies, University of Cape Coast.

As part of the requirements for the award of a doctor's degree, he is required to undertake a research visit at your outfit with the purpose of collecting data on the topic "VALUE ALIGNMENT IN MATHEMATICS BETWEEN TRAINEE TEACHERS AND THEIR TUTORS IN THE COLLEGES OF EDUCATION IN GHANA".

I would be grateful if you could give his the necessary assistance he may need.

Thank you for your usual support.

Yours faithfully,

Dr (Mrs) Christina Boateng HEAD



APPENDIX "G"

Ethical Clearance for the Study

UNIVERSITY OF CAPE COAST INSTITUTIONAL REVIEW BOARD SECRETARIAT

TEL: 0558093143 / 0508875309 E-MAIL: irb @ucc.edu.gh OUR REF: UCC/IRE/A/2016/1118 VOUR REF: OMB NO: 0990-0279 IORG #: IORG0009096



7TH OCTOBER 2021

Mr. Daniel Ashong Department of Mathematics and ICT Education University of Cape Coast

Dear Mr. Ashong,

ETHICAL CLEARANCE - ID (UCCIRB/CES/2021/100)

The University of Cape Coast Institutional Review Board (UCCIRB) has granted Provisional Approval for the implementation of your research titled *Value Alignment in Mathematics between Traince Teachers and their Tutors in the Colleges of Education in Ghana.* This approval is valid from 7th October 2021 to 6th October, 2022. You may apply for a renewal subject to submission of all the required documents that will be prescribed by the UCCIRB.

Please note that any modification to the project must be submitted to the UCCIRB for review and approval before its implementation. You are required to submit periodic review of the protocol to the Board and a final full review to the UCCIRB on completion of the research. The UCCIRB may observe or cause to be observed procedures and records of the research during and after implementation.

You are also required to report all serious adverse events related to this study to the UCCIRB within seven days verbally and fourteen days in writing.

Always quote the protocol identification number in all future correspondence with us in relation to this protocol.

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Yours faithfully.

Dr. Samuel Ascidu Owusu, UCCIRB Administrator

ADMINISTRATOR INSTITUTIONAL REVIEW BOARD UNIVERSITY OF CAPE CORST

APPENDIX "H"

VITA

Name: Daniel Ashong

Address: Department of Mathematics and ICT Education						
Faculty of Science and Technology Education						
University of Cape Coast. Ghana						
Educational Background (Institution, Qualification, and	Date)					
University of Cape Coast, Doctor of Philosophy (Mathem Education)		Submitted				
Kwame Nkrumah University of Science and Technology	(Kumasi),	June, 2011				
Master of Science (Industrial Mathematics)						
University of Cape Coast, Bachelor of Education (Mathe	matics	June, 2005				
Education)						
Sunyani Senior Secondary School, S.S.S.C.E. Certificate	Dec., 1997					
Kaneshie North '1' Junior Secondary School, B.E.C.E. C	Certificate	July, 1993				
Employment Records						
Teaching Positions	Date					
Senior Tutor at Berekum College of Education	2011 – to dat	e,				
Teacher at Notre Dame Girls Senior High School	Oct., 2007 -	Aug., 2011				
(Fiapre-Sunyani)						
Sunyani Secondary School (National Service)	Sept., 2005 –	July, 2006				
Administrative Positions						
College Governing Council Member Sept., 2017 –						
(Academic Staff Rep.)	Feb., 2018 –	Nov., 2020				
Vice Chairman (Colleges of Education Teachers						
Association of Ghana, CETAG; BECOLED Local)						
Head of Mathematics/ICT Department	Sept., 2014 –	Aug., 2017				
(Berekum College of Edu.)						
College Academic Board Member	Sept., 2014 –	Aug., 2017				

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College Council Finance Committee Member	
Departmental Rep. to Call	Nov., 2015 – Mar., 2017
Departmental Rep. to College Academic Board	Sept., 2012 – Aug., 2014
Academic Counselor (Berekum College of Edu.)	
Form Tutor (Notre Dame Girls SHS)	Sept., 2012 – Aug., 2014
(riche Dame Onis SHS)	Sept., 2007 – Aug., 2011

Recent Publications

Ashong, D., Agyei, D. D. & Tetteh, H. N. K. (2020). Examining Pre-Service Teachers' Attitude towards ICT-Integration in Teaching and Learning of Geometrical Constructions. *International Journal of Scientific and Research Publications 10 (3)*, 731-739.

Tetteh, H. N. K., Wilmot, E. M., Alhassan, M. N., & Ashong, D. (2018). Gender Differences in attitudinal variables towards Mathematics among Pre-service Teachers in the Brong-Ahafo Region of Ghana. *The International Journal of Humanities and Social Studies 6 (4)*, 186-193.

Tetteh, H. N. K., Agyei, D. D., & Ashong, D. (2018). Correlates of Senior High School Students' Attitude and Internet Usage. *International Journal of Innovative Research and Development* 7 (4), 64-72.

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Ashong, D., Darkwah, K. F., & Tetteh, H. N. K. (2018). Comparison of Floyd-Warshall and Mills algorithms for solving all-pairs shortest path problem. Case study: Sunyani Municipality. *International Journal of Physical Sciences Research* 2(1), 1-17.

Ashong, D. & Tetteh, H. N. K. (2018). Effect of Mathematics Colouring Worksheets on the Performance of Basic School Pupils. *Researchjournali's Journal of Education 6(5)*, 1-10.

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