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SENIOR HIGH SCHOOL MATHEMATICS TEACHERS AND STUDENTS' CONTENT KNOWLEDGE IN CIRCLE THEOREMS AND THEIR VAN HIELE'S LEVELS OF GEOMETRIC THINKING

FREDERICK QUARSHIE

2023

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SENIOR HIGH SCHOOL MATHEMATICS TEACHERS AND STUDENTS' CONTENT KNOWLEDGE IN CIRCLE THEOREMS AND THEIR VAN HIELE'S LEVELS OF GEOMETRIC THINKING

BY

FREDERICK QUARSHIE

Thesis submitted to the Department of Mathematics and Information Communication Technology Education of the Faculty of Science and Technology Education, College of Education Studies, University of Cape Coast, in partial fulfilment of the requirements for the award of Master of Philosophy Degree in Mathematics Education

NOVEMBER 2023

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DECLARATION

Candidate's Declaration

I hereby declare that this thesis is the result of my own original research and that no part of it has been presented for another degree in this university or elsewhere.

Candidate's Signature: .... Date: ....

Name: Frederick Quarshie

Supervisors' Declaration

I hereby declare that the preparation and presentation of the thesis were supervised in accordance with the guidelines on supervision of thesis laid down by the University of Cape Coast.

Supervisor's Signature: .... Date: ....

Name: Dr. Forster D. Ntow

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ABSTRACT

This study specifically sought to assess the content knowledge in Circle Theorems and the geometric thinking levels of in-service mathematics teachers and their students in the Senior high schools in Ghana using van Hiele's levels. This assessment was conducted in the Tarkwa – Nsuaem and Prestea – Huni Valley Municipalities in the western region with teacher population of 104 and a student sample of 280. A survey research design was used through an adopted geometry achievement test (GAT). A 100% expected return rate was achieved since it was administered directly and was collected on the same day of administration. It was realised from the study that 89.42% of the teacher population passed with an average mean of 27.50 out of 40 marks. It was also identified that although the percentage pass was high, their van Hiele geometric thinking level was up to the level 3 out of the five levels. Again, it was also identified that the most of the student respondents had below average content knowledge in Circle Theorems with an average mean of 22.90 and a percentage pass of 48.21%. Out of the five levels of geometric thinking, the students could demonstrate their understanding of the Circle Theorems up to van Hiele level 3. The means of the test scores for the teachers and the students differed statistically significantly, and the effect size was big, according to the results of an independent-samples t-test. It was recommended that regular in-service training and workshops must be organised by educational stakeholders for mathematics teachers in Circle Theorems to help enhance teachers' van Hiele levels of geometric thinking. Assessing students in Circle Theorems must be organised using the van Hiele levels. This will highlight their strengths and weaknesses at each level for remediation.

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May God aboundingly bless you all.

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DEDICATION

To my sons Riley, Johan and Jaden without them this work would not have been completed with joy. I appreciate their unwavering love and well-wishes.

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LIST OF ACRONYMS

CRDD	Curriculum Research and Development Division
GAT	Geometric Achievement Test
NCTM	National Council of Teachers of Mathematics
PCK	Pedagogical Content Knowledge
PST	Pre-Service Teachers
SMK	Subject Matter Knowledge
TK	Teacher Knowledge

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CHAPTER ONE

INTRODUCTION

Chapter one explains the background as well as the problem statement that underpin this study. The purpose of assessing the in-service SHS mathematics teachers and their students' content knowledge is spelt out. Research questions and the hypothesis are clearly stated. The significance of this study, limitations and delimitations are emphasized.

Background of the Study

In Ghana, mathematics is a crucial subject at all levels of school. It is therefore required that all Senior high school students study mathematics at the second cycle level. In the nation, a passing grade is necessary for entry into post-secondary education. A passing grade in Mathematics is a prerequisite for enrolment in institutions such as the Nursing Schools, Training Colleges, Universities, Police Force, Armed Forces, Immigration Services and Universities.

The Senior high school Mathematics Curriculum highlights seven paramount areas of study (Senior high school Mathematics Syllabus, 2010). The topics addressed in the curriculum include Numeration and Numbers, Trigonometry, Algebra, Vectors and Transformation in a Plane, Plane Geometry, Statistics and Probability and Mensuration. Therefore, Geometry is a subfield of Mathematics. A subfield that studies the characteristics of objects' surrounding and the shapes of individual objects and the spatial relationships between them. Mensah-Wonkyi and Adu (2016) cited Drickey (2001) that geometry is an area of mathematics that provides a wealth of visualizations for comprehending mathematical, algebraic, and statistical

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concepts. Trigonometry and vectors, mensuration, Plane geometry and transformation in a plane are the four key topics covered by geometry. A crucial component of the study of mathematics is geometry.

The connections and properties of points, lines, planes, and solid objects are studied in this area of mathematics, giving the world a meaningful understanding through geometry (Schopenhauer, 2016). In terms of architecture, machinery, and pretty much everything else that is built with mathematics, it is a crucial component of the modern world. It supports to decide on what materials to use and what designs to make and it plays an important function in building and construction process. In the classroom, geometry helps the students to understand spatial relationships and this helps them to understand their place in the world.

It is important to help Ghanaian elementary and secondary school students to learn and understand geometry. When teaching geometry, it is important to assist students see objects they might not otherwise be able to see or understand (Noss, Healy & Hoyles, 1997). However, it is because of this that geometry is a challenging subject to study. To assist the geometry teaching and learning, a variety of resources have been created (Schopenhauer, 2016). Compass, dividers, protractors, and set squares are a few examples of building tools in this category. Teachers also employ geometry charts to aid in enhancing conceptual knowledge of the subject (Hooper & Rieber, 1995). The explanation makes it abundantly obvious that the syllabus is created to aid in the student's development of classification and generalization skills (Senior high school Mathematics syllabus, 2010). This implies that the student must

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be exposed to a learning environment that will enable them to understand this justification.

According to the curriculum, the teacher must lead the class in teacher-student activities such as "identifying the link between angles subtended at the centre and that at the perimeter by an arc." Again, exercises like assisting students in figuring out how vertical angles of a cyclic quadrilateral relate. Additionally, when teaching plane geometry II, it is crucial to demonstrate to students how to determine whether the tangent at the circumference of a circle is verrtical to the radius of the circle and equal when drawn from exterior points to the same circle (p. ii).

According to O'Connor, Kanja, and Baba (2000), both the teacher's material expertise and the instructional method they use have a significant impact in the acquisition of instructional content for meaningful learning and the development of necessary skills. Stigler and Hiebert (1999) came to the conclusion that the teaching methods used in Japanese schools provide greater opportunities for learning mathematics because Japanese school students excelled than their peers from other countries in the Trends in International Mathematics and Science Study (TIMSS). According to Atebe (2008), instructors need knowledge, abilities, and judgment in order to effectively teach geometric ideas. According to Alex and Mammen (2016), instruction based on van Hiele's approach fosters the acquisition of crucial geometrical information as well as lifetime learning abilities. In conclusion, it is most likely that students will demonstrate a proper knowledge of geometric concepts when instructional materials are combined with experience in van Hiele's frame and learning phases. As a result, mathematics teachers can

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utilize van Hiele's theory as a scheme for teaching geometry as well as for evaluating their students' level of knowledge during geometry sessions. Once more, it can assist the teacher in coming up with appropriate lessons and activities so that comprehension advances from one level to the next.

Statement of the Problem

All areas of science such as Mathematics, Physics, Chemistry and other educational stakeholders have all expressed special interest in the teaching and study of geometry (Mesa, Gómez, & Cheah, 2012). This is due to the fact that it fosters the growth of rational reasoning, deductive reasoning, inquiring, analytical reasoning, and problem-solving capabilities in the individual. Thus, geometry supports the holistic development of the learner. According to Clement (2004), geometry instruction fosters students' scope for intellectual perception and problem-solving. The value of studying geometry is that it aids in practical problem-solving tasks like architecture and construction.

Students can relate mapping concepts learned in the classroom to location and orientation in the outside world using geometry. Students can better understand spatial relationships thanks to it. Geometry is taught in Ghana's Senior high schools to ensure students have the skills, understanding, and mindset required to solve mathematical problems. In order to apply his or her knowledge to addressing problems in real life, the student must also acquire the necessary mathematical proficiency. Secondly, be prepared for additional study and related careers in science, business, industry, and a number of other professions (SHS Core Mathematics Syllabus, 2010).

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However, majority of the students perform below expectations in Circle Theorems, despite the stakeholders' best attempts to raise their performance. Some of these students' poor geometry results can be associated with teacher's lack of expertise in the subject (Unal, 2005). He continued, saying that failing geometry classes could be due in part to mathematics teachers not providing their students with the necessary learning opportunities. Tahir (2006) ascribed the students' persistently subpar geometry performance to a variety of factors, as stated in Hassan, Kajuru, and Abari (2019), to the mathematics teachers' lack of important abilities and competency in both content knowledge and pedagogy.

In their Chief Examiners' Report, the West African Examinations Council (WAEC), the examining organization in charge of grading Senior high school students in Ghana, also mentioned a couple of the students' struggles with Circle Theorems. The Chief Examiner for Mathematics noted that candidates who solved question 3(a) of May/June 2011 Core Mathematics examination faced a significant challenge. They were unable to reproduce the relevant Circle Theorems relations to provide an answer (WAEC, 2011). According to the 2012 reports, students who write the West African Senior School Certificate Examination (WASSCE) perform poorly on Circle Theorems questions (WAEC, 2012). The majority of the time, they avoid these questions, and the few who attempt them show only a limited understanding of the subject. WAEC (2014) report cited that ‘students lack understanding of basic concepts of plane geometry’. The report of 2015 also indicated that ‘students lack the ability to recall and apply knowledge in Circle Theorems to solve related problems.’ The 2017 report was identical to

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previous ones. "Difficulty in solving geometry problems, such as cyclic quadrilaterals, tangent and chord theorems" was highlighted (WAEC, 2017). These same problems were reported again in 2018 and 2019. It was also observed that all the WASSCE examinations from 2014 to date, had questions involving Circle Theorems but students struggled to answer them satisfactorily.

The reports did not only emphasize students' weaknesses in Circle Theorems but also suggested that teachers should desist from specializing in topics they are familiar with and give equal attention to all the topics in the syllabus. Again, teaching relating to Circle Theorems should be thorough (WAEC, 2015; 2016). Furthermore, Mifetu, Kpotosu, Raymond and Amegbor's (2019) research on geometry indicated that one of the reasons students struggled to understand Circle Theorems was because their teachers did not adequately explain the concept to them. It is evident that most students find geometry to be difficult (Brannon, Liengme, &Liengme, 2018). Due to the substantial quantity of geometrical knowledge needed to understand the subject, Luneta (2015) stated that both teachers and the students exhibit some level of difficulties when Circle Theorems are taught and learned.

To overcome this issue, lot of research has been done on Circle Theorems in geometry, including how GeoGebra can be used to teach and learn geometry. Geometry teaching and learning has made substantial use of the computer programme "GeoGebra" (Hohenwarter, & Fuchs, 2004). In order to determine the impact of utilizing GeoGebra on Senior high school students' performance in Circle Theorems, Tay and Mensah-Wonkyi (2018) conducted a study and came to the conclusion that utilizing GeoGebra to understand

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Circle Theorems significantly improved student learning, and as a result, students taught using the GeoGebra method outperformed those who were taught conventionally. Moreover, the GeoGebra approach made the teachings more captivating, practical, and simple to comprehend. They advised teachers to use GeoGebra while instructing students on Circle Theorems. Mwingirwa and Miheso-Connor (2016) did another study on the "state of teachers' technology uptake applications of GeoGebra in teaching secondary school Mathematics in Kenya." It was found that the trained teachers appeared excited about utilizing GeoGebra. The teachers highlighted the difficulties they faced because of inadequate teaching resources, the distinctive attributes of geometry, and the students' inability to visualize geometrical objects.

Other researchers have also looked into teaching methods as a way of solving Circle Theorems problems. Hissan and Ntow (2021) investigated the “Effect of Concept-Based Instruction on Senior high school Students’ Geometric Thinking and Achievement in Circle Theorems”. They came to the conclusion that concept-based instruction, which incorporates conversation, hands-on activities, and guided discovery, can help students attain better outcomes and higher geometric thinking. Also, a case study was conducted by Susuoroka, Baah, Assan-Donkoh, Baah-Duodu, and Puotier (2019) on “Cooperative Learning Strategy in Teaching and Learning of Circle Theorems in Mathematics”. The authors deduced that this approach brings on board frequent interactions among students, increases students’ participation in class and enables the teacher to get through to students with different learning strengths.

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According to Marchis (2012), the geometry content knowledge of teachers should be evaluated within the content knowledge that the teacher should have in order to teach it and studies have shown that the content knowledge of teachers and/or prospective teachers is lower in geometry content than in other subjects. Research has proven that Van Hiele's theory is a crucial resource for comprehending teachers' pedagogical topic knowledge about the teaching of geometry (Erdogan & Durmus, 2009). Additionally, it is said to be the theory that describes geometry students' levels of thinking the best (Alex & Mammen, 2016). The van Hiele theory describes a methodology of instruction that teachers might follow to improve their students' levels of geometry comprehension. It categorizes students' geometric reasoning abilities into five distinct hierarchical levels.

Numerous researchers in numerous nations have been inspired by this theory (Howse & Howse, 2015), and as a result, their geometry curriculum and instructors' teaching strategies have changed. But according to the literature, there haven't been many studies on the van Hiele theory in Ghana. Only 33 of the 351 Level 200 Pre-Service Teachers (PSTs)—representing 2.2 percent of the sample—of Salifu, Fuseini, and Yakubu's (2018) study on van Hiele's Geometric Thinking Levels of Pre-Service Teachers (PSTs) of the E.P. College of Education in Bimbilla, Ghana, are qualified to teach geometry when assigned to their various schools. The outcome confirms the Institute of Education's professional board report (University of Cape Coast) that 28.8% of Pre-Service Teachers (PSTs) failed on the geometry examination while 42.3% who received weak passes in Ghana's Colleges of Education's 2015 academic year. Also, 23.2 percent of PSTs failed the end-of-semester

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examination for the 2017 academic year, or received a mark of D+ or D that was less favourable (Institute of Education, 2015 and 2017).

In Ghana, various researches have been conducted on Circle Theorems and geometry as a whole, but none has been conducted to assess the geometric content knowledge regarding Circle Theorems of the in-service mathematics teachers and that of their students at the SHS level and their level of van Hiele geometric thinking. Therefore, this research intends to assess the content knowledge and the geometric thinking levels of SHS mathematics teachers and students and their van Hiele's levels of geometric thinking.

Purpose of the Study

This study utilized the van Hiele's levels to evaluate the geometric thinking levels of in-service core mathematics teachers and their students in Senior high schools in Ghana. Specifically, the study sought to:

1. explore the geometric content knowledge of the senior high school core mathematics teachers?
2. explore the geometric content knowledge of the senior high school students?
3. identify the operating levels of the senior high school core mathematics teacher in circle theorems using the van Hiele's levels of geometric thinking.
4. identify the operating levels of the senior high school students in circle theorems using the van Hiele's levels of geometric thinking.
5. determine whether there is a statistically significant difference between the mean scores of the SHS core mathematics teachers and the mean scores of the SHS students.

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Research question/hypothesis

The following research questions and hypothesis guided the study's design.

Research Question

1. What is the geometric content knowledge of Senior high school Mathematics Teachers?
2. What is the geometric content knowledge of Senior high school students?
3. At what levels of van Hiele's geometric thinking are Senior high school mathematics teachers operating?
4. At what levels of van Hiele's geometric thinking are Senior high school students operating?

Hypothesis

H_0: There is no statistically significant difference between the mathematics teachers' geometric content knowledge and their students' geometric content knowledge in Circle Theorems.

Significance of the Study

By drawing teachers' attention to the shortcomings in the teaching and learning processes and advising teachers to update their subject-specific knowledge and teaching skills, the study's findings will disclose some of the issues that arise in the teaching and learning of Circle Theorems.It will answer some of the reasons why most students perform poorly when answering questions on Circle Theorems or tend not to answer them in examinations. The findings of this study, however, will help in identifying a solution to certain fundamental issues in mathematics, particularly Circle Theorems. When planning teacher professional development programmes about Circle

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Theorems, it will once more inform stakeholders. Finally, the study will add to the body of writings surrounding geometry education and instruction.

Limitation

The reluctance of some of the mathematics teachers to respond to the Geometry Achievement Test (GAT) was a significant limitation of this study. The geometry achievement test scores for teachers might not be adequately represented as a result.

Delimitation

The study's scope was on mathematics teachers at the Senior high school level that were sampled from Ghana's Tarkwa-Nsuaem and Prestea-Huni Valley municipalities situated in the Western Region of Ghana. The study's only focus was on assessing the content knowledge and the van Hiele levels of geometric thinking of Senior high school Core Mathematics teachers and learners.

Organisation of the Study

This thesis is structured into five chapters. The background to the study, problem statement, purpose, research questions, and hypothesis are all presented in chapter one along with the study's importance, delimitation, limitations, and organizational structure. The associated literature is reviewed in Chapter two. The methodology is covered in Chapter three while data analysis is covered in Chapter four. Chapter five, which includes the summary, findings, and suggestions, ends the thesis's sequence.

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CHAPTER TWO

LITERATURE REVIEW

The chapter examines variety of literature works related to assessing senior high school teachers and students' content knowledge in geometry (Circle Theorems) as well as the van Hiele levels of geometric thinking. The following content: history and sources of geometry, background history of Euclidean geometry, importance of Euclidean geometry, problems with teaching and learning of geometry were discussed under the concept of geometry. There are additional explanations provided for the causes of students' conceptual and learning challenges in classroom geometry. A thorough review is done on teacher content knowledge, the effects of that knowledge on student achievement, and how students perceive and perform in school geometry. Theories underpinning this research are spelt out and a summary of the literature concludes the chapter.

History and Sources of Geometry

"Geometron" is an ancient Greek word for geometry. "Geo" means 'Earth' and "metron" means 'measurement'. The study of geometry is very ancient and has been there across all civilizations such as the Egypt, India, Babylonia, China, Greece, Incas and others (Origin of geometry, n.d.). This is the case because geometrical issues are so pervasive in daily life that it makes sense that they would predate civilisation. Because it works with planes, points, lines, spatial figures and space, geometry aids people in solving challenges they encounter in their daily lives (Oflaz, Bulut, &Akcakin, 2016), such as surveying and land construction. Egyptians were the first surveyors, using geometry to create large constructions and property boundaries in shapes

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like rectangles, circles, squares and triangles because doing it by eye was impossible. (Van Manen, 2016, p. 11). Because geometry is a branch of mathematics that focuses on the measurement and relationships of lines, angles, surfaces, solids, and points, it aids students in understanding the world around them and fosters their capacity for critical thought, argumentation based on logic and deduction (Jupri, 2017). Geometry teachers' knowledge and comprehension are therefore essential (Kovács, Recio, & Vélez, 2018).

By analysing, characterizing, and comprehending their environment, students who study geometry gain certain fundamental abilities that they can use in other contexts as well as other mathematical disciplines (Kovács et al., 2018). It is a prerequisite skill for careers in mechanical drawing, astronomy, art, physics, and architecture. Despite how crucial it may appear, learning it can be challenging for most students (WAEC, 2014).

Background History of Euclidean Geometry

The Greeks made geometry a more rigorous subject that placed more emphasis on logic than on outcomes. They first brought up the idea of "proofs." One of the noteworthy figures is Euclid, an ancient Greek mathematician who is thought to have been the first to formulate the axioms, postulates, and definitions of geometry built on lines and points, laying the groundwork for what is now known as Euclidean geometry (King, 2018). One of their achievements in the field of geometry is the "Euclid Element" by Euclid around 300 BC which includes a set of 13 books covering theorems, constructions, and geometric proofs. Fitzpatrick (2008) highlighted some of Euclid's elements: Five significant postulates are present. These are;

- Any two points can be connected to form a line segment.

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- There is no end to the length of a line.
- A circle can be created by using a point as the centre and the length of a line segment as the radius.
- Right angles are all equivalent to one another.
- “If a straight line falling across two (other) straight lines makes internal angles on the same side (of itself whose sum is) less than two right-angles, then the two (other) straight lines, being produced to infinity, meet on that side (of the original straight line) that the (sum of the internal angles) is less than two right-angles (and do not meet on the other side)” (Fitzpatrick, 2008, p.6).

These components are crucial to geometry education and are still applicable to today's teaching and learning of geometry. The creators of non-Euclidean geometry, Carl Friedrich Gauss, Janos Bolyai, and Nikolai Lobachevsky, support the first four postulates described in Euclidean geometry; but, the fifth postulate, also known as the parallel postulate, was not universally accepted. The development of "hyperbolic geometry" and "elliptical (spherical) geometry" was made possible by this. Algebraic geometry, analytic geometry, differential geometry, affine geometry, topology, conformal geometry, and projective geometry are other subfields of geometry.

Importance of Euclidean Geometry

"Let no one destitute of geometry enter my doors." Plato (427–348 B.C.). This was an inscription that was found at the entrance to Plato’s room. Again, a beginning student of Euclid confronted him and asked, “What shall I get by learning these things?” It was told that Euclid corresponded with his bondservant, saying, “Give him a coin [Boyer, three pence], and since he must

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make gain out of what he learns" (Greenberg, 1974, p. 7). These expressions emphasize how geometry is significant and should be advantageous to anyone who studies it. Greenburg (1974) asserts that geometry was primarily studied for its aesthetic qualities; despite evidence in the literature to the contrary, many current mathematicians continue to hold this opinion that geometry was invented with applications to measure the earth. (Clements & Battista, 1992).

The beauty of geometry cannot be underrated. It can be found everywhere on the planet, from a country's symbols of identity, such as flags, to its currency. Some cities in the world are known for their aesthetic beauty, such as their architecture. The Eiffel Tower in Paris (France) is an edifice that attracts lots of tourists around the world. The independent square is also used to represent Ghana. The identity of most cities is defined based on their geometrical designs. This demonstrates how important geometry is worldwide.

The Ghanaian SHS Core Mathematics curriculum seeks to teach students how to select and apply generalization and classification criteria, communicate effectively with mathematical terms, symbols, and explanations through logical reasoning, and use mathematics in everyday life by recognizing and applying the appropriate mathematical problem-solving techniques. When the student has finished studying geometry, they should be able to:

- Specifically solve two- or three-dimensional problems by the use of spatial relationships;
- Remember, use and interpret mathematical knowledge in the context of commonplace circumstances;

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- Accurately organize, understand and present information in written, graphical and diagrammatic forms;
- Analyse a situation, choose an appropriate course of action, and use the proper methodology to find a solution (Curriculum Research and Development Division – CRDD, 2010).

The main goal of teaching geometry in schools is to "improve [the students'] logical reasoning ability," according to expectations (French, 2004, p. 2). This goal, which is also the main goal of mathematics instruction in schools, is what spurs geometry's inclusion in the mathematics curriculum. Thus, as mentioned in Nojiyeza (2019), geometry is an essential area of mathematics taught in Ghanaian classrooms and other countries. It is impossible to separate geometry instruction from the overall mathematics curriculum (van Hiele, 1986; French, 2004).

Geometry "assists students [to] depict and make sense of their reality," according to the National Council of Teachers of Mathematics, NCTM (1989, p. 112), in the United States, for instance. The Council went on to say that students can use geometry to strengthen their spatial awareness and logical reasoning skills (ibid.). Since the world in which we find ourselves is "inherently geometric" (Clements & Battista, 1992, p.420), understanding geometry is a crucial mathematical skill. One of the fundamental objectives of mathematics education is to raise students' levels of geometric thinking because it is essential in many specific, technical, and occupational fields (Olkun, Sinoplu&Deryakulu, 2005).

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Geometry should be taught in secondary schools for seven reasons, according to Sherard (1981), who outlined why it is a fundamental mathematical skill. The seven points made by Sherard (1981) are as follows:

- Due to its significance as a tool for communication, geometry is a fundamental ability. We use several geometric terminologies in our everyday speech and writing, including point, line, angle, parallel, perpendicular, plane, circle, square, triangle, and rectangle. This use of geometric language enables us to express our ideas to others in a clear manner.
- Numerous situations in everyday life can benefit from the use of geometry. Geometrical applications are necessary for many elements of our daily activities, including measurements around our homes.
- Numerous topics in elementary mathematics need the use of geometry. Many mathematical, algebraic, and statistical ideas make more sense when they are explained in terms of geometry.
- A strong mathematical foundation is provided by geometry for future study. Euclidean geometry, for instance, was a requirement for admission to universities in the United Kingdom (French, 2004).
- Because it is a part of humanity's cultural heritage, geometry is a fundamental skill. It appeals visually right away on an instinctive level. Studying it has educational, aesthetic, and cultural benefits.
- Geometry offers an environment for improving students' logical reasoning capacity (French, 2004).
- The "development of students' spatial sense and understanding" is improved by geometry (NCTM, 1989, p.49).

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The aforementioned seven factors lead to the logical conclusion that geometry is a necessary mathematical knowledge that must be mastered because it has applications in all areas of daily life.

Problems with the teaching and learning of Geometry

When urged to create a simpler way to teach his elements, Euclid replied to the king of Egypt, "There is no royal route to geometry" (Dimakos, Nikoloudakis, Ferentnos&Choustoulakis, 2007, p. 90) cited in Longwi (2012). His response highlights the fact that teaching and understanding geometry will be challenging for both teachers and students. Burger and Shaughnessy (1985) conducted an experimental study, and their interviews with secondary school students revealed that many of them had vague conceptions about fundamental shapes and their characteristics. Students typically struggle to define and recognize geometric shapes and are unable to think deductively in terms of geometry, according to research on the "understanding of geometric concepts by students" (Nikoloudakis, 2009).

There are a ton of reports on how geometry is tough in many academic subjects. Fuys, Geddes and Tischler (1988) acknowledged that the geometry curriculum for elementary schools placed an excessive emphasis on formal symbolism and identification. Contrarily, according to Senk (1989), many high school students in the United States of America are unprepared for geometry lessons. Additionally, Weber (2003), cited in Nikoloudakis (2009), observed that it was extremely challenging for students to construct simple geometric proofs. In their study, Atebe and Schafer (2011) found that the secondary school student participants “had a limited and possibly inadequate grasp of basic geometric terminology” (p. 63). Also, this can be attributed to

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the curriculum gap between the geometry objectives in the Senior high schools and that of the Junior High Schools or that of the learners (van Hiele, 1986). De Villiers (1996) attributed the students' failure in geometry in most high schools to the communication gap between the teacher and the learner in a study of Grade 12 students in KwaZulu Natal, based on the fact that about 45% of the learners had only mastered the Level 2 or lower of the van Hiele levels whereas the examination assumed mastery level 3 and above. Unal (2005) concluded that subject matter incompetence on the part of the teacher contributes to the low performance of learners in geometry. He added that the refusal to equip the learner with the learning opportunity in geometry is one of the possible reasons why students are failing in geometry. Luneta, (2015) also observed that geometry as a topic is difficult for teachers to teach and equally difficult for learners to learn because of the large amount of geometrical knowledge required to deal with the topic.

According to research by Salifu, Fuseini, and Yakubu (2018) on the geometric thinking levels of pre-service teachers in Ghana, just 2.2 percent of the sample size of 351 were qualified to teach geometry when posted. Again, the Institute of Education Professional Board (University of Cape Coast) announced that in the 2015 academic year's end-of-semester examination for the colleges of education, 28.8% of the pre-service teachers failed while 42.3 percent received weak passes on the geometry course.

In my view and experience as a mathematics teacher, the problems identified above are not far from the classroom experiences I have encountered. The graduates from the basic school either lack or exhibit a very low understanding of basic geometric concepts when they enter Senior high

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school. Again, not enough time is allowed for students to study and understand the idea of geometry. Additionally, the idea of teaching geometry causes some teachers problems. The lack of geometric knowledge and experience that many junior high school students bring to the Senior high schools frequently frustrating. The apparent inability of students to reason geometrically at a higher level is due to a variety of factors, not merely their own motivation or learning preferences. The teacher's guidance and selection of tasks are equally crucial to the students' learning. The majority of these influencing elements are encountered in the geometry classroom by every mathematics teacher. These issues led the van Hieles to develop these levels of geometric thinking theory in order to help teachers determine their students' levels and build their knowledge accordingly. Literature has revealed that various factors contribute to the difficulties with geometric conceptualization. This study is designed specifically to investigate the teacher knowledge factor using the van Hiele's levels.

As a result, when teaching geometry, as with any other topic in mathematics, teachers should carefully plan sequence of teaching events that will help develop the student's geometric conceptualization using van Hiele's theory. The van Hiele theory (van Hiele, 1959) sparked my interest and provided the basis for my work. If learning geometry requires developing critical thinking skills and conceptualization at the "highest possible level", then all mathematics teachers should be familiar with the features of effective geometry training (Van Hiele, 1986).

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Causes of learning difficulty in school Geometry

There is a worldwide problem with geometry education, according to literature. Textual, academic, and instructional/pedagogical elements have all been cited as the main causes of this (Clements & Battista, 1992). Although there are other factors, these stated factors play a major role in the teaching and learning of geometry.

Textual factor

Textbooks, in the traditional sense, should reflect the curricular description of what the student must acquire and master in terms of body of knowledge and skill. Varying geometry textbooks' organizational methods frequently lead students to develop different levels of geometry problem-solving proficiency (Fujita & Jones, 2002). The United Kingdom (U.K.) textbooks were "designed around a set of exercises with mathematical theorems merely stated rather than developed or proved," while the Japanese "textbooks attempt to develop students' deductive reasoning through 'proof' using various approaches," according to Fujita and Jones (2002, p. 82). The consequences realised were that, in the U.K., there was a consistently poor performance in constructing proofs by older students such as 14- to 15-year-olds, but “most 14- to 15-year-old students in Japan can write down a geometric proof” even though “around 70% of the students cannot understand why proofs are needed” (p. 81).

In Ghana, the mathematics curriculum is spiral which sees to it that topics learned in one class are revised and built upon as students’ progress from one class to the next. By observation, teachers mostly depend on the textbook for teaching. However, the chronological arrangement of the content

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of the textbooks must be rearranged closely for content and sequencing (Suydam, 1985, p. 482) when teaching geometry.

Curricular factor

The school curriculum outlines which topics are to be taught and how they must be taught. This has significant effects on how well students succeed in geometry (Clements & Battista, 1992). Every nation has its own curriculum, which may vary slightly in terms of the geometric material, time, and importance placed on practical techniques, proofs, and applications (French, 2004, p. 7).

Lack of a comprehensive, cogent, and well-organized junior high school geometry curriculum is to blame for students' inability to perform well or show much interest in geometry in Senior high school (Siyepu, 2005). In South Africa, the geometry curriculum is "heavily loaded in the senior secondary school with formal geometry, and with relatively little content done informally in the elementary school," according to De Villiers (1997, p. 42). According to De Villiers (2010), the primary reason for geometry failure is "high expectations levels in the curriculum that are higher than the learning abilities of the student. For instance, the curriculum can call for students to demonstrate geometric reasoning at Van Hiele's level three even while they can only do so up to the second van Hiele level.

On the other hand, the Ghanaian Senior high school core mathematics curriculum (CRDD, 2010) has a true reflection on the junior high school mathematics curriculum, but students still have lots of issues in geometry learning. For example, the SHS syllabus outlines angles, properties of parallel lines, polygons and their properties under Plane geometry 1 (p. 12-14) which

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is the same as that of the Junior High School but has been named shape and space.

Pedagogical factor

One of the roles of the teacher is to translate the curricular intentions into the classroom through potential learning experiences. The teacher does this by selecting, preparing and presenting a series of learning activities for the students. Proper learning cannot take place in the classroom without good organisation and effective teaching. Also agreeing that "learning is significantly and necessarily tied to instruction," Stoker (2003, p. 11). This suggests that "teachers in the classroom are the most crucial individuals in any educational institution" (Evans, 1959). The amount of learning that occurs in the classroom is largely influenced by the teachers' familiarity with the subject matter (Circle Theorems). Thus, most teachers' classroom behaviour is "affected by their understanding" of the subject and pedagogy unique to that subject (Nieuwoudt & van der Sandt, 2003).

Students' struggles in class are closely tied to teachers' inadequate pedagogical topic knowledge in the subject area, such as Circle Theorems (van Hiele, 1986; Shulman, 1987; Mji&Makgato, 2006). According to Van Hiele (1986), many teachers fail to foster their students' conceptual grasp of the subject because they are unable to fit their instruction to their students' level of thinking.

In Ghana, the examining body for Senior high schools, WAEC, has reported in their chief examiner's report that students refuse to answer questions relating to Circle Theorems, and those who do tend to answer them either have little knowledge of or show their lack of knowledge of the basic

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concepts in the subject matter. Some also lack the ability to recall and apply knowledge in the form of Circle Theorems to solve related problems (WAEC, 2011; 2012; 2014; 2015). They went on to say that teachers should stop refusing to teach Circle Theorems and start teaching them thoroughly (2016). Mifetu et. al. (2019) discovered that students struggle to answer questions about Circle Theorems, and Luneta (2015) discovered that some teachers found geometry difficult to teach. This confirms that the teachers either have little knowledge of Circle Theorems or are unable to use the appropriate process or methodology to teach it to the students' understanding.

Students' Conceptual Difficulties with School Geometry

According to literature, some of the difficulties students face when learning geometry include concepts of angles, angle sums of triangles, parallel and perpendicular lines, properties of shapes, groupings of fundamental shapes, misconceptions, imprecise terminology, class inclusion of shapes, and proof writing (Usiskin, 1982; Mayberry, 1983; Clements & Battista, 1992; French, 2004; &Siyepu, 2005). Some of the problems pertaining to this work is discussed.

Properties of Shapes

Most high school students cannot give an unambiguous description of shapes based on their characteristics. As a result, they frequently fail to recognize shape's characteristics (Mayberry, 1983). For instance, Clements and Battista (1992, p. 422) reported that "less than 25% of 11th-grade [U.S.] students correctly identified the lines of symmetry of given shapes" in their study. It was also realised that only a few students were able to identify that a

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rectangle is a parallelogram when the 11th-grade students were sorting activities that involved different triangles and quadrilaterals.

Misconceptions

Students' misconceptions about geometric concepts are many and varied yet interesting (French, 2004). According to the literature, there are some common misconceptions among secondary school students regarding geometric shapes and the connections between their properties. Many high school students, according to Clements and Battista (1992), believe that "a square is not a square if its base is not horizontal." This means that they cannot identify some basic shapes when their standard orientation is changed. There is widespread concern about students' misunderstanding of diagonals, with some students unable to identify the number of diagonals in a given shape and even identifying edges as diagonals (French, 2004). Oberdorf and Taylor-Cox (1999) opined that "lack of exposure to proper terminology and too few authentic experiences in the primary school, together with misinformation by adults, have been identified as some of the possible reasons for students' misconceptions in geometry." Students come to a geometry class with various ideas and perceptions.

Imprecise Terminology

Language is an essential tool in all forms of communication. As every subject area has its own unique language, so does geometry. Bloom (1956) cited in Atebe (2008) asserts that "the most basic type of knowledge in any particular field is its terminology". Lack of language proficiency, according to Feza and Webb (2005), prevents geometric comprehension from progressing. However, too frequently, Students lack the language skills needed to

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systematically compare shapes or describe a figure's special characteristics (Feza & Webb, 2005). According to Oberdorf and Taylor-Cox (1999, p.340), one of the causes of students' errors about geometry is "lack of exposure to suitable language." Precise terminologies should be utilized in the geometry classroom to address students' imprecise use of geometric terms (Hoffer, 1981).

Teacher content knowledge

Teacher content knowledge refers to the information or a body of knowledge i.e., facts, theories, principles and concepts that students are expected to learn and to be taught by a teacher in a given subject area such as mathematics.

Teachers' content knowledge (CK) for teaching mathematics

In his seminal presentation on the results of the research programme aimed at finding knowledge issues relating to teacher development and teacher education, Shulman (1986) identified teacher content knowledge as one of the original three categories of teacher subject matter knowledge. He intended for his first category, subject knowledge, to stand for "the amount and organization of knowledge in teachers' minds" (p.9). According to Shulman (1986), content knowledge is the general conceptual grasp of a subject area that a teacher possesses and is acquired through completing the necessary coursework (Shulman, 1986). The theories and principles that are taught and learned in certain academic courses are also included in Shulman's definition of content knowledge, in addition to the facts and concepts in a subject. It serves as the foundation for PCK development.

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The level of content knowledge of the mathematics teacher has a significant influence on the instructional practices (Hughes, Swars, Auslander, Stinson, & Fortner, 2019). Content knowledge is basically what is to be taught. It is also known as “subject matter knowledge (SMK)”. Take Euclidean geometry as an illustration. All mathematicians should have a certain amount of this information, but it does not necessarily have to be for teaching purposes. For teachers, this knowledge must be pertinent to the mathematics taught in schools.

Mathematics teachers’ pedagogical content knowledge for teaching

The second category by (Shulman, 1986) was pedagogical content knowledge. With this category, he went “beyond knowledge of subject matter *per se* to the dimension of subject matter knowledge *for teaching*” (p. 9, italics in original). Shulman (1986) identified representations of certain subject concepts as well as a grasp of what makes students learn a particular topic easily or difficult as components of pedagogical content knowledge. Shulman (1986) specified that the representation of ideas exhibited in a taught topic in one’s subject area, illustrations, examples, explanations, demonstrations as well as powerful analogies aims at formulating subject matter that makes it comprehensible to others. According to Shulman, whether or not students of all ages and backgrounds can understand the teachings and topics that are most frequently taught depends on their concepts and preconceptions (p. 9). The Pedagogical Content Knowledge (PCK) derives two dimensions from Shulman. These include "knowledge of subject matter representations" and "knowledge of particular learning difficulties and students' conceptions."

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When considering PCK, these two dimensions were frequently used as reference points.

According to Doody and Noonan (2013), pedagogical knowledge refers to a teacher's in-depth understanding of the procedures and techniques used in teaching and learning, i.e., what to teach and how to teach. There is no assurance that teachers with strong subject-matter expertise would also have strong pedagogical understanding, according to Novak & Tassell's (2017) opinion. According to Hörsch, Schuler, Rosenkränzer, Kramer, and Rieß (2016), pedagogical knowledge includes not just subject-specific material but also mastery of non-content-related techniques and procedures for running lessons and classrooms productively and effectively.

The body of knowledge that enables the instructor to translate their own expertise into the understanding of the students is referred to as pedagogical content knowledge, on the other hand (Steffe et. al., 2016). PCK connects classroom education to familiarity with the pertinent subject (Vermeulen & Meyer, 2017). PCK is the result of a teacher's subject-matter competence and teaching methods, an understanding of the various levels of student subject-matter comprehension, and the various ways that content knowledge is applied in teaching and learning in the classroom (Wei et al., 2017).

As noted in Nojiyeza (2019), the teachers' PCK of geometry is a crucial piece of knowledge needed to make geometry engaging. Due to the complexity of teaching, teachers need to be knowledgeable in a variety of areas (Masduki, Suwarsono, & Budiarto, 2017). Subject knowledge and pedagogical content knowledge are the two most important facets of teachers'

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knowledge that affect students' performance. Teachers must have in-depth knowledge and grasp of topic, curriculum, student characteristics, teaching and learning methods, and effective classroom management because the appropriate combination of CK, PK, and PCK may make them successful and competent (Masduki et al., 2017).

Curriculum Knowledge

Curricular knowledge, according to Shulman, is the third category. It includes knowledge of how subjects are organized both during the course of a school year and across time, as well as strategies for using curriculum resources like textbooks to set up a study schedule for students. In order to assist students in developing helpful cognitive maps, connecting one idea to another, and addressing misconceptions, teachers in today's classrooms must possess a strong and flexible understanding of their subject matter. Teachers need to be aware of the connections between their subjects and the real world. The foundation of teachers' pedagogical content knowledge is this kind of comprehension, which helps them explain concepts to others (Shulman, 1987). Teachers cannot help children learn what they themselves do not comprehend, as Ball (1990a) put it concisely (p. 5).

Effects of teacher knowledge on students' achievement

Teaching in a specific field of study necessitates a specific knowledge base. It is the knowledge of the subject matter they teach that attracts employers to hire teachers. Education stakeholders strive to provide students with highly qualified teachers who are competent enough to demonstrate good subject matter knowledge through certification and experience. Does the level

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of certification guarantee the teacher's content knowledge? Does it really have a direct relationship with the students' performance?

Despite the interest and concern, only a few studies have been conducted on how students' achievement relates to the subject matter knowledge of their teachers. For example, only three studies were conducted on both teachers' and students' mathematical knowledge and students' mathematical achievement as of 1997 (Rowan, Chiang & Miller, 1997). Ball (1990) made the claim that American teachers lack the required knowledge to teach mathematics, and also that teachers' intellectual resources have a substantial impact on students' learning, which sparked a rise in interest in subject matter knowledge.

Reviews on the initial approaches to measuring students' achievement based on teacher experience, teacher preparation, education level, courses taken, and others were disputed (Hanushek, 1996), as cited in Hill, Ball, & Rowan (2015). Other studies were conducted to measure teachers' knowledge using certification, examination, or tests of subject matter knowledge. These studies proved a positive correlation between teacher knowledge and students' test achievement (Hanushek, 1986 cited in Ball, 1990). Shulman's (1986) work elaborated more on how knowledge matters in teaching. Emphasizing that knowledge of subject content combined with knowledge of how to teach such content is what determines teachers' effectiveness. According to studies, what other adults would know about fractions, place value, or slope, for example, would be quite different from what teachers should know (Ball, 1988, 1990).

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Despite the fact that these studies have been significant in helping to define the mathematical content knowledge that teachers must possess, they were not designed to evaluate theories on the ways in which particular aspects of this knowledge benefit in student learning. As a result, although many believe that teachers' knowledge is important in raising student accomplishment, its impact on student learning has not yet been proven via empirical research.

Hill, Rowan, and Ball (2015) therefore explored the "effect of teachers’ mathematical knowledge for teaching on students’ mathematics achievement." A sample of 1190 and 1773 grade one and three students, and 334 and 365 grade one and three teachers respectively were used. Using linear mixed model methodology, they found out that the teachers’ mathematical knowledge had a direct relationship with grade one and three students’ results. They emphasized that their findings supported those found in the literature on educational production. They concluded that the positive correlation of teacher content knowledge with students’ achievement implies that teacher content knowledge plays a major role even in the teaching and learning of every elementary mathematics content. They also reported that teachers’ content knowledge must be measured based on the content on which the student is to be assessed. This study took a similar approach, assessing the teacher's subject matter knowledge of Circle Theorems through test items, which were parallel to the students' achievement test. The outcome of the test will be analysed using mean scores of both tests and t-test.

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Theoretical Framework

In order to guide the researcher in determining the nature and extent of the study in connection to the research questions, aims, and purposes of the study, a theoretical framework presents a road map of the research process. The van Hiele theory of geometric cognition serves as the theoretical foundation for this investigation.

The van Hiele theory

The van Hiele theory of geometric thought, was developed in 1957 by Dina van Hiele-Geldof and her husband Pierre Marie van Hiele (van Hiele, 1986). It emerged from their doctoral dissertations, which were completed simultaneously at the University of Utrecht. The theory was later clarified, amended, and advanced in the 1960s, when the geometry curriculum was revised by Pierre after Diana died shortly after her dissertation.

When Wirszup (1976) authored and gave talks on the theory in North America, it became well-known in the 1970s. In his monumental book, Mathematics as an Educational Task, the van Hieles' professor Hans Freudenthal from the University of Utrecht also drew attention to their works (1973). The English translations of 1984 have done a lot to improve this (Fuys, Geddes, & Tischler, 1984).

Recognition, analysis, order (informal deduction), deduction (formal deduction), and rigor are the five sequential and hierarchical discrete levels of geometric thought that make up the van Hiele theory, which are arranged from level 0 to level 4 (Usiskin, 1982; Burger & Shaughnessy, 1985, p. 420). According to the idea, which explains thought processes, students advance successively from the lowest level (visualization) to the highest level (rigor).

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Later, the labels of the levels were adjusted to 1-5 to accommodate a new level known as "pre-recognition level," sometimes known as "level 0." (Stols, Long & Dunne, 2015).

The van Hiele's levels of geometric thought

According to Pierre and Diana, when learning geometry, students pass through various stages of reasoning (van Hiele, 1986). The van Hiele hypothesis sought to enhance geometry instruction and assist students in expanding their understanding of geometry by designing activities that would take their capacity for thought into account as new skills were presented (Alex & Mammen, 2016). The van Hiele theory cannot be isolated from the teaching and learning of geometry, according to Piaget (2000).

Although the theory is intended to evaluate students' geometric knowledge, it has also positively impacted how geometry is taught to students (Al-Ebous, 2016) by providing teachers with a model to use and put into practice in order to raise students' levels of geometric thinking. It can also be used to explain why many students struggle with geometry (Seah & Horne, 2019). Using this framework, which is applicable to all disciplines of geometry, it is feasible to evaluate students' and/or teachers' geometrical thinking in geometry classes (McIntyre, 2017). Teachers of geometry who have a solid grasp of this theory will be able to determine the pace and degree of their students' learning (Luneta, 2015), and they may use this framework to forecast their performance both now and in the future.

Level 1: Visualisation (recognition)

Students use nonverbal reasoning and visual perception at this level. Geometric forms are recognized by their "total" shape, and they are compared

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to their prototypes or common objects. A cube is comparable to a box or a die, for instance. According to McAndrew, Morris & Fennell (2017), students can cite the name of an object based on its outward appearance (shape or form); nevertheless, they are only able to recognize the shape and not the qualities. Shape is determined by how it looks.

Level 2: Analysis

At this level, students start to describe and analyse the features of geometric forms. They do not comprehend the relationships between properties and think that each property is important (there is no difference between necessary and sufficient properties). They do not think empirically derived facts needs to be supported by evidence.

Level 3: Abstraction (Informal deduction)

At this stage, students might make links between characteristics and numbers. They produce definitions that have a purpose. They are able to support their judgments with convincing arguments. They are adept at creating logical diagrams and maps.

Level 4: Deduction (Formal deduction)

Students are able to provide logical geometric proofs at this level. They are able to distinguish between conditions that are required and those that are sufficient. They list the attributes that already exist in others. They are aware of the function that definitions, theorems, axioms, and proofs serve.

Level 5: Rigor

Students are able to describe the creation of mathematical systems at this level. They have access to all possible types of proof. Both Euclidean and

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non-Euclidean geometry are understood by them. They can describe the outcomes of adding or removing an axiom from a certain geometric system.

Table 1 is a summary of the geometric thought levels identified by Van Hiele.

Table 1: Summary of the van Hiele's levels

Level	Description	
Level 1	*Basic level of visualisation or recognition*	Students can group like shapes or things together and recognize, name, and compare geometric figures like rectangles, squares, and triangles based on their forms, or on how they appear physically. They might not understand concepts like the parallelism of opposite sides.
Level 2	*Analysis*	The properties of geometric forms can be used by students to analyse them. For instance, a parallelogram has equal opposed angles, as do all the angles in a square. Students are aware of the qualities of different geometric shapes, but they are unable to compare them. For instance, a square and a rectangle both have four right angles.
Level 3	*Informal deduction*	Students can show how particular properties are logically related to one another by using informal arguments. For example, they can show how, if two sides of a quadrilateral are parallel, then the two opposite angles must also be equal, or how a rectangle is made up of two squares. The students can now understand definitions and follow informal arguments. Students start to recognize arguments and can follow them without having to write and organize them.
Level 4	*Formal deduction*	Deductively proving theorems is a skill that students can develop. Students now comprehend the significance of deductions and the functions of theorems, postulates, and proofs, i.e., they can build and write proofs with comprehension. This is the initial stage of formal deductions.
Level 5	*Rigor*	At this point, pupils have seen geometry in its most abstract form. They can now compare and evaluate theorems. Van Hiele acknowledged his interest in the first three levels because level 3 is where the majority of high school geometry is taught and because it is rather uncommon for this level to exist in secondary schools.

Adapted from Subbotin and Voskoglou (2017, pp. 1-2)

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The theory has the following characteristics, as described by Crowley (1987): These qualities are particularly important to educators since they offer direction for choosing instructional strategies.

Sequential (order)

Hierarchical and sequential layers are used. Students must have mastered a sizable amount of the lower levels in order to perform satisfactorily at one of the advanced levels in the van Hiele hierarchy (Hoffer, 1981). Without passing level (N - 1), a student cannot be at level N. The student must therefore proceed through the levels in order. Instruction is more important for level progression than age or biological maturation. Van Hieles' asserts that a significant portion of the geometry students' difficulties stem from the fact that they are being taught at the deduction level when they have not yet attained the abstraction level.

Intrinsic and Extrinsic (Adjacency)

At each level, what was intrinsic in the preceding level becomes extrinsic in the current level. "At each level, there appears in an extrinsic way that which was intrinsic at the preceding level." "At the base level, figures were in fact also determined by their properties, but someone thinking at this level is not aware of these properties" (van Hiele, 1984, p. 246).

Linguistic (Distinction)

According to the following explanation, each level has its own language. "Each level has its own linguistic symbols and its own system of relations connecting these symbols." An association that appears to be "right" on one level may turn out to be erroneous on another. Consider the connection between a square and a rectangle, for instance. Different degrees of reasoning

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prevent two people from comprehending one another. "Neither is able to follow the other's cognitive processes (van Hiele, 1984, p. 246).

Mismatch (Separation)

A key element in progressing through the levels is language structure. Different levels of understanding are incomprehensible to two people. At a lower level, the teacher uses a different "language" to communicate with the student. Although teachers think they are expressing themselves logically and clearly, their level 3 or level 4 reasoning is not accessible to students at lower levels, and the teachers do not comprehend the mental processes of their students. The van Hieles believed that this characteristic was one of the primary causes of geometry failure.

Attainment/Advancement

Age is less of a factor in level progression (or lack thereof) than the instruction's content and delivery methods. A student cannot skip a level during learning; certain methods speed up learning while others slow down or even block students from moving between levels. In van Hiele's words, “a skilled student can be taught abilities above his actual level, just as one can teach young children fractional arithmetic without explaining what fractions mean, or older children differentiation and integration even though they do not understand what differential quotients and integrals are” (Freudenthal, 1973, p. 25).

The features of this theory give teachers precise instructions on how to help students advance from one level to the next by developing geometric activities that are appropriate for the students' level (Armah et al., 2018). Prior knowledge of all geometry ideas is required before introducing any new

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concepts; nevertheless, because students and teachers work at different levels in geometry classes, there is a chance that they won't comprehend one another (Subbotin &Voskoglou, 2017). For instance, a student using level N reasoning will not comprehend an instructor using level N+1 reasoning (Al-Ebous, 2016). Levels 1 through 3 see the development of procedural geometrical fluency, whereas levels 4 through 5 see the development of conceptual comprehension (Luneta, 2015). Before going on to the next level, the learner must complete each one, according to Luneta.

Chapter Summary

The study evaluated the literature to illustrate the numerous studies that have shed light on the geometry teaching and learning process in schools. Although Euclidean geometry has received a lot of attention, Circle Theorems have received very little attention in comparison. Studies involving the teacher's content knowledge, the problems and the causes of learning difficulties in the teaching and learning of geometry, and assessing students' van Hiele's levels of geometric thinking have been explored, but little focus has been seen on the measurement of the teacher's content knowledge of Circle Theorems using the van Hiele's levels of geometric thinking using achievement tests. This makes this study unique because of its attention to geometric content knowledge at the Senior high school level. The outcome will contribute significantly to the field of geometry as well as inform policymakers in their decision-making.

In this study, the teacher content knowledge was further discussed and presented in this chapter. It was shown that the teacher's content knowledge is made up of curriculum knowledge, student knowledge and characteristics,

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knowledge of educational contexts, knowledge of educational outcomes and goals, general pedagogical knowledge, classroom management knowledge, and pedagogical content knowledge (PCK). Again, it emphasizes how important these are for teaching and for improving students' comprehension of any given idea. To enhance the teaching and learning of geometry, it is crucial to strengthen instructors' pedagogical content knowledge (PCK).

Additionally, it was mentioned that teachers should make connections between the van Hiele theory and the geometric thought framework when educating students in geometry. This method is particularly important for secondary schools. The van Hiele framework's suggested geometry teaching sequence has a good effect on both geometry teaching and learning and may help students comprehend and master higher-order geometrical skills. Instructors must be conscious of the levels at which their students are thinking when they enter the classroom and employ a variety of dynamic teaching strategies to help them master geometry and overcome the many challenges they face while learning geometry.

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CHAPTER THREE

RESEARCH METHODS

This chapter outlines the research methodologies and procedures carried out to assess the "senior high school teachers and their students' content knowledge in Circle Theorems and their van Hiele's levels of geometric thinking". Information on the research design, the population from which the sample was drawn, the sampling technique, the instrument used to collect the data, its validity and reliability, the methods used to collect the data, and the methods used to process and analyse the data are all included in the description.

Research Study Design

According to Leavy (2017), this is a tool to be used when attempting to solve a research topic. It is also the methods for carrying out a study, which may include details like how, when, and from whom the data would be obtained, according to Creswell and Poth (2017). According to Walliman (2017), research designs come in different types and forms and are all suitable for various types of research. The type of the research problem determines the design of study. So, for this study, a cross-sectional survey research approach was used. Cross-sectional surveys are mostly used for collection of diverse quantitative data. Such data could be on the prevalence of disease, behaviours, knowledge, attitudes and respondent opinions (Polit & Beck, 2014). Connelly (2016) added that researchers most of the time, in a cross-sectional survey, explore the relationship between variables such as knowledge of assessment skills and work environment on the performance of physical assessment skills.

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A cross-sectional survey has the benefit of helping to gather data from a sample that has been selected from a pre-set population. Although it may take more than a day to acquire or retrieve the data, it is only collected once (Fraenkel & Wallen, 2000). It could be viewed as a snapshot that illustrates the subject the researcher wants to investigate. Results, however, may be limited or skewed if a variable changes over time (Hofer, Silwinski, & Flaherty, 2002). Cross-sectional studies can be applied with a variety of groups and can cover a wide range of human behaviour, situations, and activities (Polit & Beck, 2014). Furthermore, it may be completed rather quickly if the necessary data is already at hand.

The researcher chose cross-sectional design, which helped the researcher collect data from different mathematics teachers and year two Senior high school students in a relatively short period and also be able to use it to compare many different variables at the same time. This offered the opportunity to measure the content knowledge and the geometric thinking levels of the SHS mathematics teachers in the western region and their students in the teaching and learning of Circle Theorems and compare their results using the van Hiele's levels. It also helped the researcher assess the content knowledge of the SHS mathematics teachers.

The researcher in this study was aware of any potential drawbacks of utilizing a cross-sectional survey. It is challenging to determine time, which is a drawback. That is, if you collect data from research participants at a single point only, you cannot directly measure changes that occur over time (Johnson & Christensen, 2012).

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Study Area

The study area selected for the study was the Tarkwa Nsuaem Municipality and the Prestea-Huni Valley Municipality, which are located in the western part of Ghana, called the Western Region. Each of these two municipalities has three public high schools. These districts were chosen at random using the lottery technique from among the thirteen districts in the area.

Population of the Study

Population was defined by Polit and Hungler (1996) as the whole collection of cases that satisfy a certain set of requirements. But regardless of the fundamental unit, the population is always made up of the full collection of factors that the researcher is particularly interested in. The study was carried out in the Tarkwa-Nsuaem and Prestea-Huni Valley Municipalities in the Western region of Ghana. There were six (6) Senior high schools in these Municipalities as the time of this research. All the six schools were selected for the study. All SHS students and Core Mathematics teachers in the chosen municipalities made up the target population. However, the accessible population included all year two students with the total population of 3528 students and the 104 in-service mathematics teachers in the six (6) schools in the said municipalities in the 2021/2022 academic year.

Sample

According to Krejcie and Morgan (1970), a sample size of 346 is appropriate for a total population of 3500. In all, 384 respondents were used as the sample for this study. This consists of 280 SHS 2 students and 104 mathematics teachers. The teachers' sample comprised 77 males (74%) and 27

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females (26%). The SHS 2 students were chosen for the study because, in accordance with the SHS mathematics curriculum, the mathematical content "Circle Theorems" is taught in form two. Again, the SHS 2 students had been taught Circle Theorems. Additionally, the form 3 students were already out of school when the study's data were collected.

Sampling Technique

The two municipalities (Tarkwa-Nsuaem and the Prestea-Huni Valley Municipality) for the study were selected purposively. This was based on the students' performance in core mathematics in the 2019 and 2020 WAEC examination statistics in the selected Municipal Education Offices (MEOs). Presented in Table 2 is the percentage performance of the selected schools.

Table 2: Percentage Pass Performance of Selected Schools

Schools	2019	2020
SHS A	24	35
SHS B	41	50
SHS C	17	14
SHS D	43	36
SHS E	33	32
SHS F	20	22
Mean	29.67	31.50

Source: Municipal Education Offices (MEOs), 2022.

The performance in Table 2 indicates that, only SHS B had 50% pass in mathematics in 2021. No school in 2020 had 50% or more. They all had less than 50% pass. It implies that, majority of the students who wrote WASSSCE in these schools obtain Grade F. The means obtained for 2020 and

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2021 was 29.67 and 31.50 respectively. Although, there was an improvement in the performance in 2021, the result was not encouraging. This is an indication that, the performance of core mathematics in the selected municipalities is not encouraging and needs to be investigated. Moreover, the core mathematics questions for 2019 and 2020 had questions on circle theorems as in previous years such as 2011 to 2018.

The two selected districts had three senior high schools each totalling six (6) in all. The researcher purposively chose all the six schools in the municipalities because the number of schools were not many. Again, taking sample from all the schools will show a fair representation of the participant schools in the municipalities and for the study.

Mathematics teachers in each of the six (6) schools were chosen purposively in that they possessed the characteristics of interest to the researcher. To take part in the study, all teachers were invited to participate. They were given the consent form for teachers (see Appendix C) to complete if they wanted to take part in the study. The researcher used census technique in obtaining the 104 mathematics teachers since the purpose was to include all the mathematics teachers in each of selected schools.

On the part of the students' sample, a multi-stage sampling technique was used. First of all, the SHS 2 students were chosen purposively from each of the six schools selected for the study in that they had been taught the mathematics content 'Circle Theorems'. The form 3 students were not included since they were out of school. Secondly, a simple random technique was used to select an intact class from each of the six schools to represent each school. This was done to ensure that every SHS 2 student in each of the six

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schools had equal chance of participation. The students in the selected classes were given student consent form (See Appendix D) to fill if they wished to participate in the study. The actual student sample size used for the study was obtained by adding all the students in the randomly selected intact classes in all the selected schools for the study. A total student sample size of 280 was realized from the six (6) randomly selected classes. Presented in Table 3 are the details of the sample size.

Table 3: Description of the Sample Size

Schools	Number of Teachers	Number of Students
SHS A	14	42
SHS B	21	50
SHS C	17	46
SHS D	17	45
SHS E	15	48
SHS F	20	49
Total	104	280

Source: Field Data, 2022

Data Collection Instrument

Data collection instruments are tools for data collection. Some of these instruments include questionnaires, tests, inventories, rating scales, observation and interviews. For both the participating teachers and the SHS 2 students, a geometry achievement test (GAT) item served as the primary data gathering tool for this study.

Teachers' Geometric Achievement Test

The teachers' geometric achievement test items had two sections labelled A and B (See Appendix A). Section A requested data on the

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participant's demographics and Section B requested data on the participant's content knowledge. The section B part of the teachers' GAT and the students' GAT items were adopted from Hissan and Ntow (2021) because the questions and the arrangement of the questions in the adopted GAT items matched the purpose of this research study. The specific goals of Plane Geometry II (Circle Theorems) in the SHS Core Mathematics syllabus served as the basis for the GAT items and they were constructed to fit into the Ghanaian context. The five levels of van Hiele's theory of geometric teaching and learning were used to produce the fifteen achievement test questions, with each question reflecting a different level of geometric thought and comprehension. When assessing the prospective van Hiele level for each question, Mayberry's (1983) level descriptors (Level 1 to 5) of van Hiele's geometric reasoning were used to confirm the acceptability of the questions for a particular level.

The questions 1 to 3 assessed the visualisation or recognition level. The respondent was required to use visual perception to answer the questions. The question one at the visualization level:

In the Figure 1, line AT is a tangent to the circle at A. What is the name given to;

a) triangle AOT?

b) angle TAO?

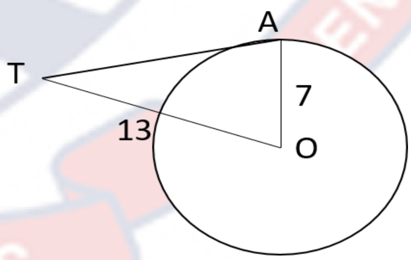

Figure 1: A sample of visualization question

The questions 4 to 6 were used to assess the analysis level of the respondent. The questions required the respondent to analyse and name properties of geometric figures. A sample is shown in Figure 2.

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Given that O is the centre of the circle, determine the value of angle y.

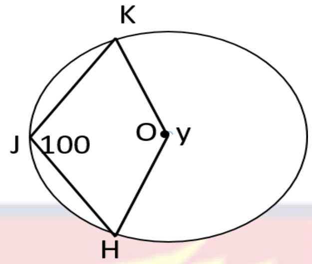

Figure 2: A sample of analysis question

The questions 7 to 10 were used to assess the abstraction (informal deduction) level of the respondent where the respondent was expected to perceive relationships between properties and figures. Figure 3 is an example of an abstraction question.

In Figure 3, O is the centre of the circle with TP as a tangent at T and <AOB = 60^o. Find the size of angles x and y.

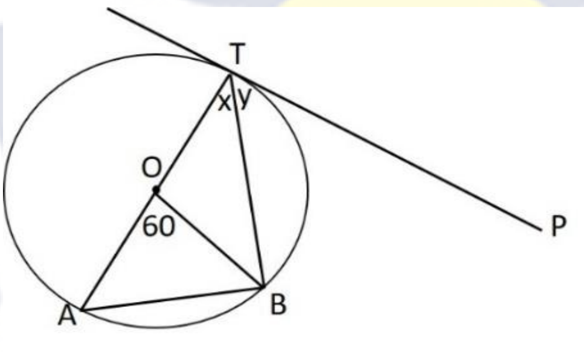

Figure 3: A sample of an abstraction question

The questions 11 to 14 assessed the deduction (formal deduction) level. This level requires the respondent to be able to give deductive geometric proofs.

An example of a question measuring the deduction level of the respondent is the question 11 on the teacher questionnaire indicated in Figure 4.

In the diagram below O is the centre of the circle, show that p + q = 90^o.

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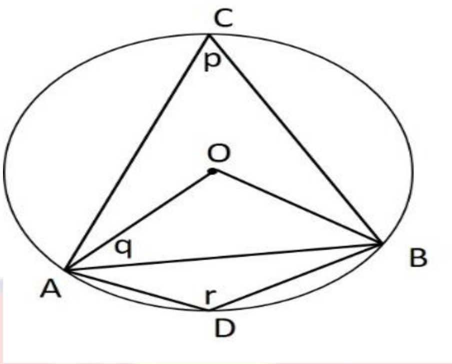

Figure 4: A sample of a deductive question

The question 15 assessed the rigor level of the participants. Here, the respondent was required to use all types of proofs and be able to describe the effect of adding or removing an axiom from a given geometric system.

Question 15: Prove the theorem: a line drawn from the centre of a circle perpendicular to a chord, bisects the chord.

Open-ended questions, short responses, and the mathematical proof that passed professional review made up the test items. Additionally examined were item difficulty and item discrimination analyses. The allocation of the GAT scores is shown in Table 4.

Table 4: Van Hiele's levels question distribution and mark allocation

Van Hiele's levels of geometric understanding	Question number	Total marks
Visualization (Recognition)	1 –3	6
Analysis	4 – 6	6
Abstraction (Informal Deduction)	7 – 10	12
Deduction (Formal Deduction)	11 – 14	12
Rigor	15	4

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The geometry achievement test was chosen for this study because it is characterized as a collection of knowledge-acquisition questions and is frequently used in research to get data on opinion, interest, and experiences that cannot be obtained through observation (Archibald, 2016). According to Walliman (2017), tests are an effective study technique for obtaining first-hand information about people's activities, experiences, social interactions, opinions, and awareness of events. It also provides perceptiveness into the enumeration you want to appeal.

Students' Geometric Achievement Test

The students' GAT is a parallel assessment test to the teachers' GAT (See Appendix B). A parallel assessment test is a different version of a test that measures the same content areas as a given test and has the same item difficulty level but contains different sets of items. It was also made up of 15 circle theorem questions that reflected the five levels of Van Hiele's theory of geometric thinking in teaching and learning, with each question assigned to reflect each level of geometric thought and understanding. It had no demographic characteristics. The question distribution and mark allocation are the same as those of the teacher's GAT.

Validity of test instrument

The degree to which a test accurately measures what it claims to measure determines the validity of the test instrument. The content validity, concurrent validity, and predictive validity are three methods for estimating a test's validity. Only the GATs' content validity was assessed in this study. Lawshe (1975) defined "content validity" as "means for measuring expert or qualified judges' agreement over the significance of a given test item." The

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GATs for this study were adopted from Hissan and Ntow (2021). The adopted test items were given to the researcher’s supervisor for inspection and scrutinize the content validity of the instruments and its appropriateness to this research. He agreed that the adopted GAT's content was good to help in achieving the study's goals.

Pre-testing

A pre-test was conducted using 10 SHS mathematics teachers and 20 SHS 2 students in the Sekondi-Takoradi Metropolitan Assembly in the Western region of Ghana. The chosen school for the pilot testing had similar characteristics as the selected schools for the study in the Tarkwa-Nsuaem and Prestea-Huni Valley Municipalities of Western region. The responses from the administered test instruments collected were scored and recorded. Two weeks later, the same test instruments were retested with the same 20 SHS 2 students and 10 SHS Mathematics teachers. Test instruments collected were scored and recorded too. The reliability of the test instruments was tested. The viability of the main investigation was determined using the pre-test data.

Reliability of test instruments

“Reliability is the extent to which an evaluation instrument generates steady and consistent outcomes,” claim Phelan and Wren (2005). For both teacher and student GAT results obtained from the pre-tests, the Pearson's Product-moment Correlation was performed to ensure test-retest reliability. The Pearson’s moment correlation coefficient obtained for the pilot testing was 0.84 for the student GAT items and 0.86 for the teacher GAT items. However, for the student GAT and teacher GAT, respectively, the reliability coefficients obtained during the actual data gathering exercise were 0.82 and

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0.85. Pallant (2005), indicated that the ideal coefficient for reliability should be 7.0 and above. This indicates that the instrument chosen was appropriate and suitable for use for this study since the reliability coefficient was more than 7.

Data Collection Procedures

An introduction letter was obtained from the Mathematics and ICT Education Department, University of Cape Coast for data collection after the researcher have been cleared by the Institutional Review Board (IRB) of the same University on April 1, 2022. With the help of the introductory letter that was acquired from the University, permission was requested from the heads of the chosen schools. After permission was sought, the researcher was introduced to the heads of mathematics departments who in turn introduced the researcher to the members of the mathematics department and the SHS 2 students to explain the purpose and the intent of the research. The participants were assured of confidentiality of any information they will provide. Consent forms were distributed to the participants.

With the assistance of two field research assistants, direct administration of the GAT was used under the researcher's supervision. They were briefed on their duties and told not to interfere in any of the students' work but rather assist the researcher in coordinating and collecting data from the student sample selected. They also supervised the work of the student respondents and guided the learners where necessary. Their support in the administration of the GAT items helped to ensure a 100% return rate. The presence of the researcher together with that of the assistants helped clarify any doubts and misconceptions some of the teachers and students had.

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The items for the geometry achievement test (GAT) were given to the students, and they were given one hour and thirty minutes to complete them in one sitting after being notified two weeks prior. The teachers' GAT was administered to all the mathematics teachers through the heads of the mathematics department in all the selected SHS schools to complete and was collected on the same day. Teachers who were absent during the administration of the test were later visited and they answered the GAT with the help of the various heads of the mathematics department. The direct involvement of the various mathematics heads of department in conducting of the test aided in the achievement of a 100% return rate.

Data Processing and Data Analysis

After marking, the completed test items were checked for errors, and the collected data was appropriately coded before being loaded into the statistical package for social sciences (SPSS) for data analysis. For the score of the GAT, a scoring scheme was developed with allocated marks according to the van Hiele's levels. The created scheme was applied to analyse both the hypothesis and each research question.

Table 5 is the presentation of the scoring scheme and its corresponding remarks used for the analyses.

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Table 5: Scoring scheme based on van Hiele's levels

Level	Question numbers	Question type	Total marks	Mark allocation
Visualization	1 – 3	Name, discriminate	6	0 Incorrect visualization 1 Partly correct 2 correct visualizations
Analysis	4 – 6	Properties	6	0 Incorrect analysis 1 Partly correct 2 Correct analyses
Abstraction	7 – 10	Definition, relationships	12	0 Incorrect abstraction 1 Low abstraction 2 Partly correct 3 Correct abstractions
Deduction	11 – 14	Formal deduction	12	0 Incorrect deduction 1 Low deduction 2 Partly correct 3 Correct logical deduction
Rigor	15	Proof	4	0 Incorrect 1 Analysis level 2 Abstraction level 3 Deduction level 4 Rigor level

At the visualisation and analysis levels, a correct score earned the respondent two marks. However, a zero (0) mark was awarded for a wrong solution, whereas one mark was awarded for a partly correct solution. At the abstraction and deduction level, a total score of three is earned for answering a question correctly. Wrongly solved questions attract zero marks, whereas one mark was awarded for a low deduction response, two marks for a partly correct response, and three marks for a fully correct response. At the rigor level, a correct solution was scored four marks, whereas three marks are awarded for a solution up to the deduction level, two marks was awarded for an abstraction level solution, a mark for an analysis level solution, and zero for a total incorrect response.

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The geometrical content knowledge of SHS mathematics teachers and students for research questions one and two respectively were analysed using the total raw score obtained from the tests.

Table 6 is the presentation of the range of scores and its corresponding remarks used for analysing research questions one and two.

Table 6: Range of scores and remarks for GAT

Range of scores	Remarks
10 and below	Very Low Content Knowledge
11 – 20	Below Average
21 – 25	Average Content Knowledge
26 – 35	Above Average
36 – 40	High Content Knowledge

The range of scores in Table 6 was used to determine the level of geometric content knowledge of the respondents as described by Usiskin (1982). He recommended "3 out of 5" right success criterion for level assignment to determine the van Hiele's levels of the respondents as identified in research questions three and four. If the respondent successfully answers at least three out of the five questions in any one of the five subcategories, the respondent is deemed to have mastered the particular van Hiele Geometric Thinking (VHGT) level. That is, more than half of the answers are correct in any of the 5 subgroups.

In this research, any respondent who scored more than half the total score passes the test but the pass mark of above 20 was categorised into average, above average and high content knowledge. Those who had 20 and below were classified as below average or have very low content knowledge based on the mark obtained by the respondent. Mean scores, percentages,

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standard deviation, contingency tables and frequency distribution tables were also employed in analysing the content knowledge and the van Hiele's levels and the geometric thinking of both the teachers and the students.

The analysis performed for the hypothesis: "There is no statistically significant difference between the mathematics teachers' geometric content knowledge and students' geometric content knowledge in Circle Theorems" was a t-test with independent samples. Independent samples t - test was used because the purpose of the test was to compare the mean scores of two continuous variables for two different groups of people.

The data analysis procedures employed in the analysis of the research questions and the hypothesis have been summarised in Table 7.

Table 7: Summary of data analysis techniques

Number	Research questions/Hypothesis	Analysis
1	What is the geometric content knowledge of senior high school mathematics teachers?	Frequency tables, percentages.
2	What is the geometric content knowledge of senior high school students?	Frequency tables, percentages.
3	At what levels of van Hiele's geometric thinking are senior high school mathematics teachers operating?	Percentages, frequency tables.
4	At what levels of van Hiele's geometric thinking are senior high school students operating?	Percentages, frequency tables.
5	There is no statistically significant difference between the mathematics teachers' geometric content knowledge and their students' geometric content knowledge using van Hiele's levels.	Contingency table, mean, histogram, standard deviation, box plot, t – test.

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Chapter Summary

The focus of this thesis was to assess the content knowledge of the circle theorem and the van Hiele levels of geometric thinking of Senior high school mathematics teachers and students. A cross-sectional descriptive survey design was used to gather and analyse the quantitative data needed to address this problem and answer the research questions. Both the teacher and the student participants in the study were chosen using purposive and simple random sampling techniques. The primary tool utilized to gather data from both participants was the Geometry Achievement Test (GAT). Before being used on the two groups, both instruments underwent pilot testing, reliability and validity tests.

The collected data was quantitatively analysed using descriptive and inferential statistical techniques to see if there was a statistically significant difference between the two groups. Percentages, mean scores, standard deviation, contingency table, histogram and frequency distribution tables were used under descriptive analysis and an independent samples t-test was conducted under inferential statistics. The acquired teacher data underwent item analysis as well. The study also complied with ethical standards such informed permission, reducing the risk of damage, anonymity, and secrecy, as well as ethical clearance by the university. The evaluation and interpretation of the study's data are discussed in the next chapter.

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CHAPTER FOUR

RESULTS AND DISCUSSION

The main research objective was to assess the in-service Senior high school mathematics teachers' and second-year SHS students' content knowledge in Circle Theorems and their van Hiele's levels of geometric thinking. Grand total of 104 teachers and 280 students responded to the GAT. The rate of return was 100%. The research questions are discussed based on the quantitative data collected and the outcome of the study are presented based on the following research questions and hypothesis.

Research Questions:

1. What is the geometric content knowledge of senior high school mathematics teachers?
2. What is the geometric content knowledge of senior high school students?
3. At what levels of van Hiele's geometric thinking are senior high school mathematics teachers operating?
4. At what levels of van Hiele's geometric thinking are senior high school students operating?

Research Hypothesis

There is no statistically significant difference between the mathematics teachers' geometric content knowledge and their students' geometric content knowledge in Circle Theorems.

Geometric content knowledge of the SHS mathematics teachers

Research question one sought to assess the Senior high school mathematics teachers' content knowledge of Circle Theorems. A Geometry

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Achievement Test (GAT) was conducted to examine carefully the content knowledge of SHS mathematics teachers on Plane Geometry II (Circle Theorems). The test outputs were scored out of 40. The test items were scored based on van Hiele's level of geometric thinking. Table 8 shows the descriptive information of the test results obtained from the teachers' GAT.

Table 8: Descriptive statistics for teachers' GAT results

Statistics	Outcome
N	104
Mean	27.50
Standard deviation	5.422
Skewness	-.173
Range	25
Minimum	15
Maximum	40
Mode	20 and 33

From Table 8, the minimum and maximum scores obtained from the test were 15 and 40 respectively. The mean score of 27.50 was recorded for the total score of the GAT and the standard deviation recorded was 5.422. This stipulates that the data are not so widely spread from the mean since the deviation is 5.422. The modal marks for the study were 20 and 33. The recorded skewness was -.173 as presented in Table 8. This indicates that the mark distribution for the collected data is moderately negatively skewed, or skewed left. This indicates that majority of the marks obtained in the test were above the mean (M = 27.50). The results pointed out that the majority of teacher respondents scored above average. The range of marks obtained by the teacher respondent is tabulated in Table 9. It shows the range of scores

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obtained by the teachers, their frequencies, corresponding percentages and remarks.

Table 9: Performance of Teachers

Score	Frequency	Percentage	Remarks
0 – 10	0	0.00	Very Low Content Knowledge
11 – 20	11	10.58	Below Average
21 – 25	18	17.31	Average Content Knowledge
26 – 35	61	58.65	Above Average
36 – 40	14	13.46	High Content Knowledge
Total	104	100.00	

It is observed from Table 9 that out of the 104 teacher respondents, none of them scored below 11 which represents "very low content knowledge" in the Circle Theorems concept. The number of teacher respondents who scored "below average," thus 11–20, was 11 (10.58%). Moreover, 18 (17.31%) scored marks from 21 to 25, representing "average content knowledge" in Circle Theorems. Out of the 104 respondents, 61 (58.65%) scored from 26 to 35 marks, representing above-average content knowledge, and 14 (13.46%) scored from 36 to 40 marks, indicating high content knowledge.

The mean mark for the teacher distribution was 27.50, which is within above-average level of content knowledge. A standard deviation of 5.422 indicates that, majority (79 out of 104) of the teacher respondents had ‘Average’ and ‘Above average’ content knowledge in Circle Theorems. That is, their GAT scores range from 22.078 to 32.92. Again, out of the 104 teachers, 56 (53.85%) scored above the mean and 48 (46.15%) scored below the mean. Moreover, 93 (89.42%) out of the 104 teachers scored more than

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50% of the total. Only 10.58% had below-average content knowledge marks, representing 11 teacher respondents. Although only 14 (13.46%) achieved a “high content knowledge level” in the test conducted, the quality of performance was impressive since the pass rate was high.

Discussion on Research Question One

From research question 1, “What is the geometric content knowledge of SHS Mathematics Teachers?” it was deduced that 93 (89.42%) of the respondents passed the test based on Usiskin’s (1982) more than half correct success criteria with a pass mark of “3 out of 5”. In this research, the more than half was from a score of 21. This indicates that the majority of the teacher respondents exhibited geometric content knowledge, which is above average to high content knowledge level, except for 11 (10.58%) teachers who had below average content knowledge in the GAT.

The report by the Institute of Education (University of Cape Coast) in 2015 based on the academic end of semester examination showed that 42.3% of the Pre-Service teachers have low content knowledge in geometry. In 2017, they also reported that 23.2% have low content knowledge in geometry. Only 33 out of the 351 pre-service teachers (PSTs), representing 2.2% of the sample, were found to be qualified to teach geometry when they were posted to their respective schools, according to another study by Salifu, Fuseini, and Yakubu (2018). This study has shown that the in-service mathematics teachers have content knowledge in Circle Theorems ranging from ‘average content knowledge’ to ‘above average content knowledge’ to as only 11 (10.58%) failed. Although some failed, it is not as alarming as reported by the Institute of Education Professional Board (University of Cape Coast) and that of Salifu,

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Fuseini and Yakubu. It was expected that as the performance was not that encouraging in the pre-service teachers, it should be replicated in the in-service teachers' performance. But the outcome showed that, most of the teachers had either average or above average content knowledge in Circle Theorems.

However, it was realized that, only 14 (13.46%) of the in-service teachers have 'high content knowledge' pertaining to teaching and learning of Circle Theorems. This can be deduced that some core mathematics teachers have limited and possibly inadequate grasp of basic geometric terminology

Geometric content knowledge of the SHS 2 students

The geometric achievement test (GAT) conducted for the SHS students was used to assess their content knowledge in Circle Theorems. The assessment was graded using the same grading system as the teachers. Table 10 is the presentation of the performance of the students with their corresponding remarks.

Table 10: Performance of students

Score	Frequency	Percentage	Remarks
0 – 10	46	16.43	Very Low Content Knowledge
11 – 20	99	35.36	Below Average
21 – 25	61	21.78	Average Content Knowledge
26 – 35	64	22.86	Above Average
36 – 40	10	3.57	High Content Knowledge
Total	280	100	

Table 10 is the tabulation of the performance of the students after the test and their corresponding remarks. From Table 10, only 3.57%, or 10 out of 280 student respondents, demonstrated high content knowledge in the

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administered test, scoring above 35. The percentage of students who had "below average content knowledge" had the highest frequency with 99 (35.36%) out of 280 respondents. This was followed by "above average" knowledge with 22.86%, and 61 (21.78%) out of the 280 respondents had "average content knowledge" in the Circle Theorems.

Again, the mean of 22.9 indicates that the average performance of the student respondents was at “average content knowledge” and a standard deviation of 4.637 of indicates that the general performance of the respondents was from “below average” to “above average” content knowledge. Therefore, the analysis of research question 2, “What is the geometric content knowledge of SHS 2 students?” has revealed that only 48.21% of the student respondents passed the test. This connotes that the majority (51.79%) of the examinee had below-average content knowledge in geometry (Circle Theorems). However, their knowledge levels were from “below average” to “above average” content knowledge.

Discussion on Research Question Two

The output of the test is a reflection of Atebe and Schafer’s (2011) work as majority (51.79%) of the student respondents had some issues with their responses in the conducted test. In their carried-out research, they ascertained that the secondary school students “had a limited and possibly inadequate grasp of basic geometric terminology” (p. 63). The van Hieles again agrees to it that there are limitations and possible inadequate grasp of geometry content on the part of the students. This inadequate grasp of these geometry content can be attributed to the curriculum gap between the

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geometry objectives in the Senior high schools and that of the junior high schools or that of the learners (van Hiele, 1986).

De Villiers (1996) attributed students' failure in the learning of geometry in most high schools to the communication gap between the teacher and the learner in his study. In this study, the researcher identified that most of the students lacked the ability to recognise geometric shapes and could not think deductively. Moreover, some student respondents could not transfer their knowledge in basic geometry when making geometric deductions and proofs.

SHS Teachers Van Hiele's levels in Circle Theorems

This research question is to help the researcher identify the levels of geometric thinking of the teacher participants in the teaching and learning of the Circle Theorems. Van Hiele divided the levels into five categories. These categories are; visualization, analysis, abstraction, deduction and rigor. The goal is to categorize the teacher participants according to their geometric thinking levels.

Teacher performance in the VHGT level 1 (Visualization)

At level 1, the aim is to determine whether the respondent can recognize the given shapes by looking at the shape of the figure given. Three questions carrying two marks each were asked. Table 11 shows the marks allotted and their percentages for each question at VHGT level 1.

Table 11: Teacher performance of each test item in the VHGT level 1

Question	Incorrect (0) N (%)	Partly correct (1) N (%)	Correct (2) N (%)
1	5 (4.81%)	31 (29.81%)	68 (65.38%)
2	12 (11.54%)	7 (6.73%)	85 (81.73%)
3	3 (2.88%)	1 (0.96%)	100 (96.15%)

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For question 1, the respondents were to give the specific names for (a) the triangle AOT and (b) the angle TAO as shown in Figure 5, and were essential to use their understanding of angles and apply the 'tangent to a radius of a Circle Theorem' to answer. The number of fully correct answers given by the respondents was 68 (65.38%).

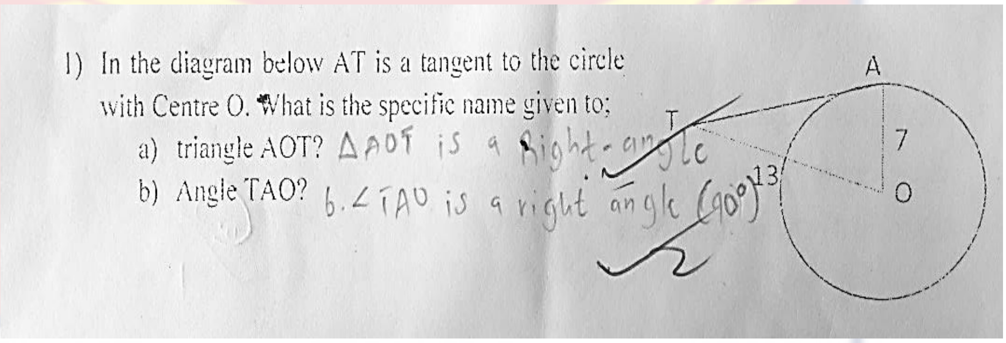

Figure 5: A snapshot of a sampled marked script for question 1

However, 31 (29.81%) scored one mark out of two and 5 (4.81%) were incorrect. The percentage pass was good, as more than half of the respondents had scored all the 2 marks. Figure 6 identifies an example of an incorrect response by a respondent.

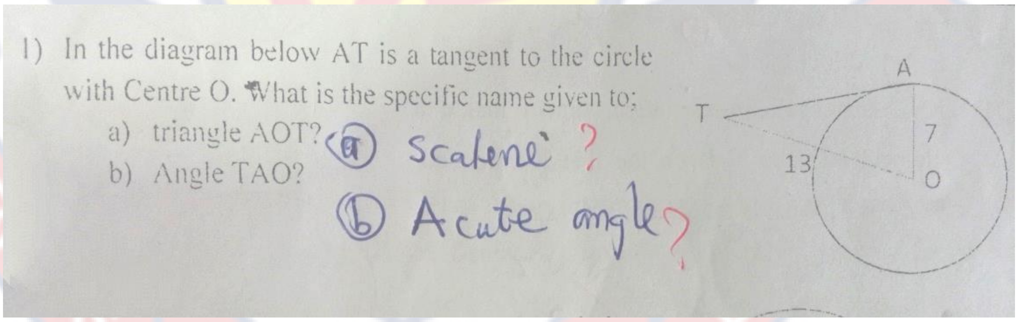

Figure 6: A snapshot of an incorrect response for question 1

The tangent to a radius of a Circle Theorem was required of the respondent to answer the question one. That is, a tangent to the circle makes an angle of 90° to the radius at the point of contact. This property makes triangle AOT a right-angled triangle. It was identified that the respondent

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could not recall or had no knowledge about the property. Therefore, he could not identify the type of angle and triangle AOT forms.

The responses from respondents for question 2 was better than question one as 85 (81.73%) had scored all correct. In question 2, respondents were to answer: Why line AC is equivalent to line BC? The specific name given to triangle ABC? as shown on Figure 7.

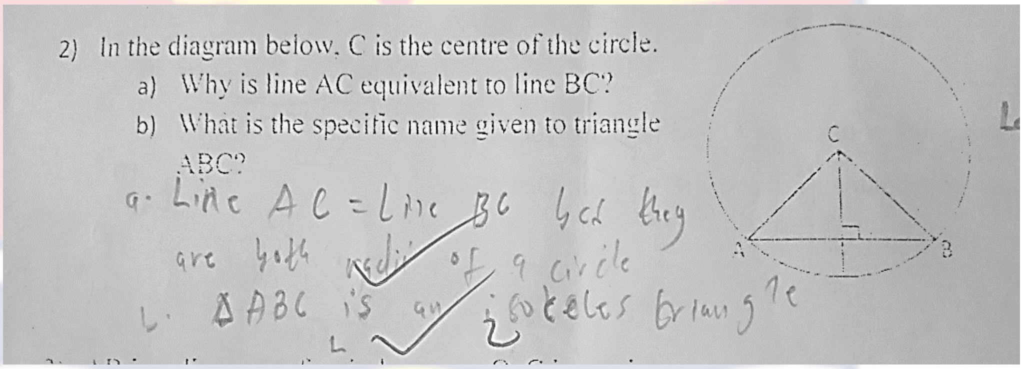
2) In the diagram below, C is the centre of the circle.
a) Why is line AC equivalent to line BC?
b) What is the specific name given to triangle ABC?
a. Line AC = Line BC bcs they are both radii of a circle
b. ΔABC is an isoceles triangle

Figure 7: A marked sample of correct responses of teacher GAT question 2

The incorrect responses were only 12 (11.54%).

Figure 8 shows an example of an incorrect response.

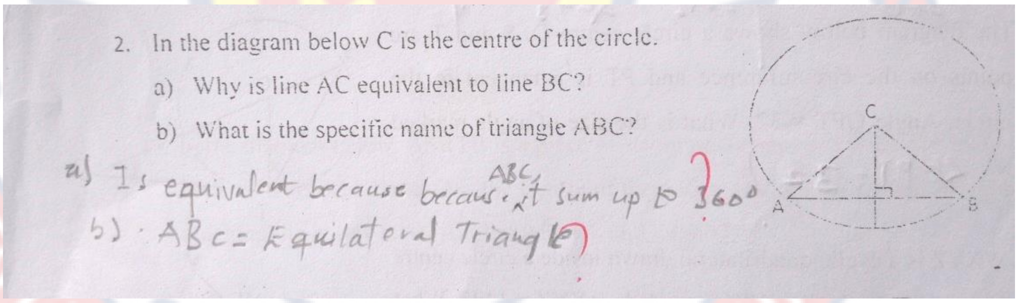
2. In the diagram below C is the centre of the circle.
a) Why is line AC equivalent to line BC?
b) What is the specific name of triangle ABC?
a) Is equivalent because because ABC it sum up to 360°
b) ABC = Equilateral Triangle

Figure 8: A marked sample of incorrect response for teacher GAT question 2

The respondent could not identify that line AC and line BC are all representing the radius of the circle. Therefore, they are of equal lengths. Without this guide, the respondent could not identify the type of triangle formed in the diagram.

Question 3 required the respondent to calculate the size of angle CBA as shown in Figure 9.

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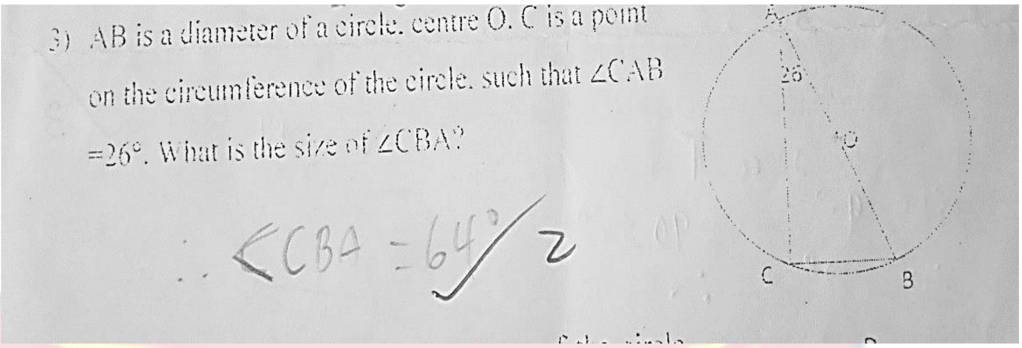

Figure 9: A snapshot of teachers sampled marked script for question 3

Out of the 104 respondents, 100 (representing 96.15%) scored all correct and only 3 (2.88%) had no score for question 3. The percentage of respondents who passed question 3 was higher compared to questions 1 and 2. It demonstrated that the respondents were well-versed in the circle theorem property “angle subtended from a diameter is 90°”.

The average percentage score at VHL 1 was 81%, representing 84 out of 104 respondents. This indicates that the performance of the respondents was outstanding at level 1. Therefore, the respondents at this stage can perceive geometric diagrams by their shapes. They can also identify shapes defined by their appearance, as described by the van Hiele theory.

Teacher performance in the VHGT level 2 (Analysis)

Three (3) questions were used to evaluate the respondents' performance at VHGT level 2, and the results are shown in Table 12, which summarizes their performance in terms of percentages.

Table 12: Teacher performance of each test item in the VHGT level 2

Question	Incorrect (0) N (%)	Partly correct (1) N (%)	Correct (2) N (%)
4	13 (12.50%)	1 (0.96%)	90 (86.54%)
5	13 (12.50%)	1 (0.96%)	90 (86.54%)
6	20 (19.23%)	0 (0%)	84 (80.77%)

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As indicated in Table 13, it was deduced that out of the 104 respondents, 90 (86.54%) had question 4 correct, only 1 (0.96%) scored 1 mark out of 2 and the remaining 13 (12.50%) scored nothing. The question requested the respondents to measure the angle PTR on the diagram shown on the marked script in Figure 6. The respondents were required to apply the angles subtended on the circumference of a circle by a chord theorem to help obtain the value of angle PTR. The percentage pass was impressive as almost 86.54% of the teachers scored the question correctly. A sample of a marked script is shown in figure 10.

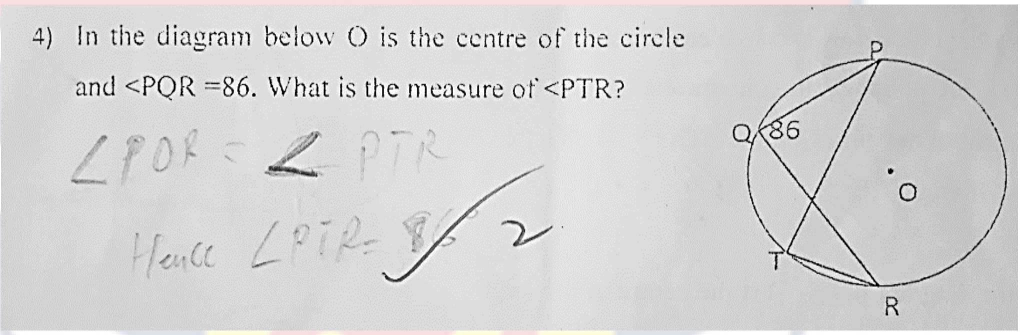

Figure 10: A marked sample of teacher GAT question 4

A sample of a marked script from the 13 respondents who had it wrong is indicated in Figure 11.

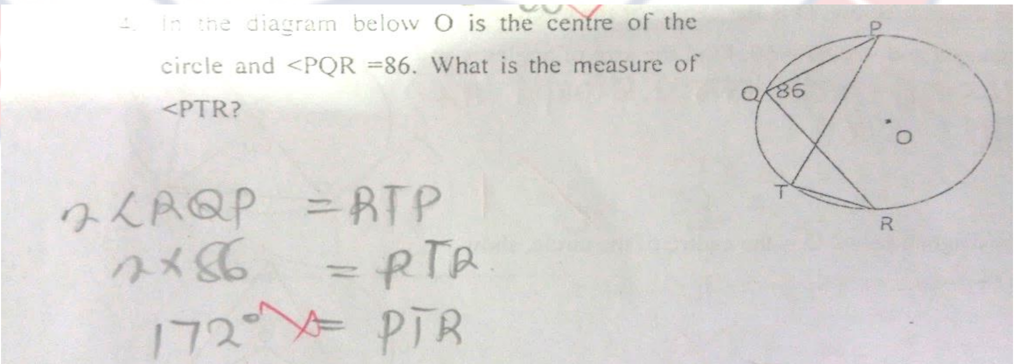

Figure 11: A sample of an incorrect response to question 4

Here, the respondent used ‘an angle subtended at the centre by the circumference of the circle theorem’ instead of ‘angles subtended on the

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circumference of a circle by a chord theorem'. That made the calculations wrong.

The percentage score for question 5 was the same as question 4 as 90 respondents, representing 86.54%, also scored all correct and 13 (12.50%) gave an incorrect response. However, the percentage pass of question 6 declined to 84 (80.77%) compared to the percentage pass of questions 4 and 5. Question 6 requested the respondents "determine the value of the angle y" as indicated in Figure 12.

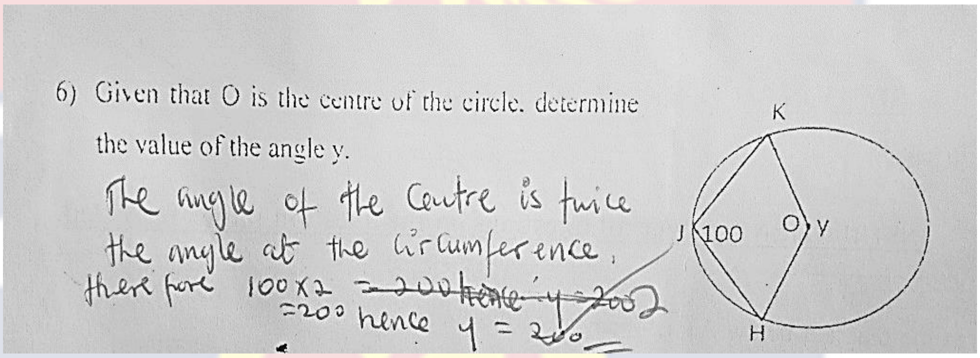

Figure 12: A sample of a marked question 6 script

Twenty (19.23%) of the teachers could not analyse the angles of incidence at the centre of a circle theorem correctly. A sample of an incorrect response for question 6 is also indicated in Figure 13.

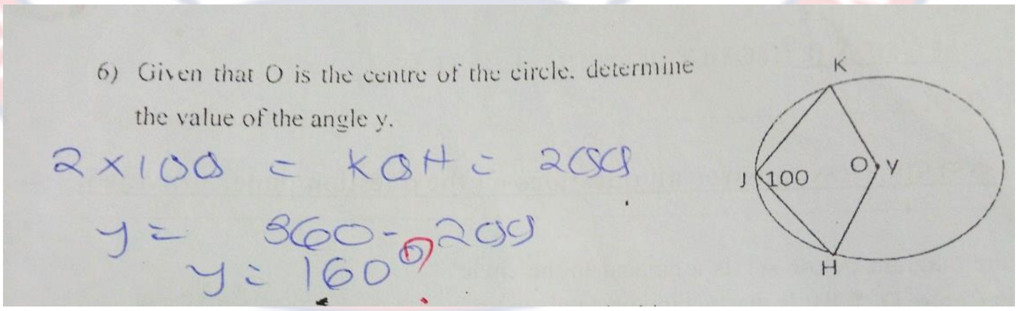

Figure 13: A sample of an incorrect response of question 6

It was identified that the respondent had knowledge about the angle subtended at the centre by the circumference of the circle theorem but could

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not identify that it was the angle KOH in the major sector that was being determined.

At van Hiele level 2, an average of 88 respondents responded correctly and 16 responded incorrectly, indicating 84.62% correct and 14.74% incorrect. These findings at this level showed that an average of 88 (84.62%) of the respondents could identify given shapes as well as analyse the properties of given geometric figures fully.

Teacher performance in the VHGT level 3 (Abstraction)

The results of the van Hiele Level 3 are presented in Table 13. This level included four questions, each worth three points, for a total of twelve points. The scores obtained by the respondents at this level is discussed. At this level, it is expected of the respondent to relate the properties of shapes by giving formal arguments. Here, definitions are now meaningful to learners who can see proofs and make simple deductions.

Table 13 is the display of the frequency distribution of the results of the teacher respondents at the VHGT level 3.

Table 13: Teacher performance of each item in the VHGT level 3

Question	Incorrect (0) N (%)	Low understanding (1) N (%)	Partly meaningful (2) N (%)	Correct logical deduction (3) N (%)
7	19 (18.27%)	3 (2.88%)	14 (13.46%)	86 (82.69%)
8	30 (28.85%)	1 (0.96%)	10 (9.62%)	63 (60.58%)
9	22 (21.15%)	9 (8.65%)	9 (8.65%)	64 (61.54%)
10	37 (35.58%)	10 (9.62%)	10 (9.62%)	47 (45.19%)

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From Table 14, question 7, which required the respondent to calculate the size of the angle VWZ of a cyclic quadrilateral, showed that 86 out of the 104 respondents scored all correct, representing 82.69%, and 19 representing 18.27% could not score any mark. The pass rate was high and encouraging. A sample of the marked script is shown in Figure 14.

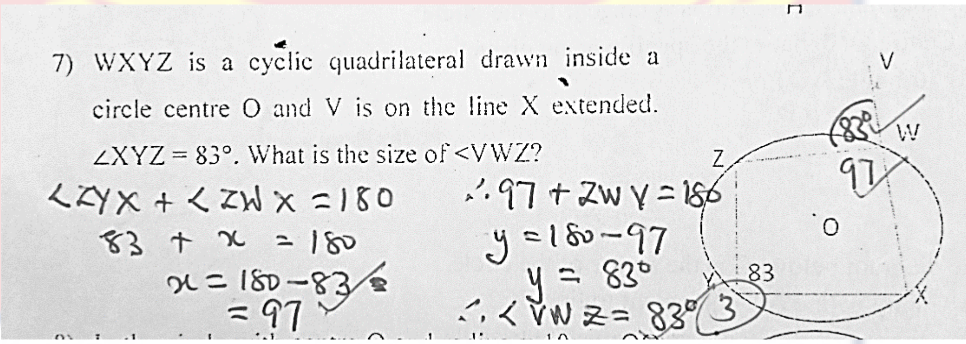

Figure 14: A sample of a marked script of question 7

However, a sample of the respondents who could not answer the question 7 correctly is analysed in Figure 15.

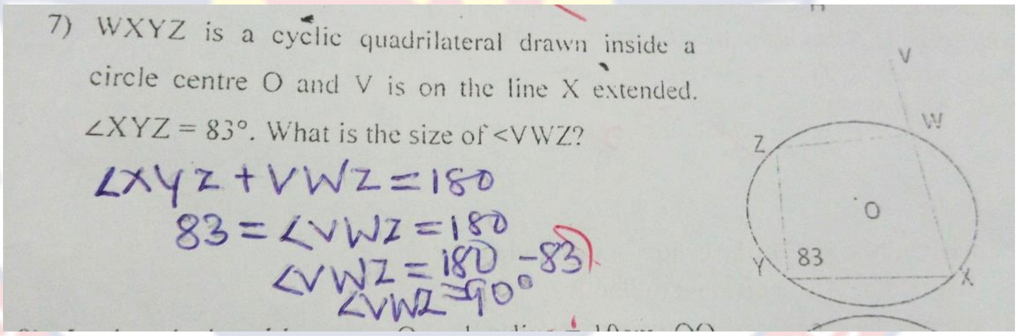

Figure 15: A sample of an incorrect marked script of question 7

The respondent was required to add angle XWZ to angle XYZ to sum up to 180^{o} but it was angle VWZ that was added to XYZ. That was incorrect because in cyclic quadrilateral, the sum of opposite interior angles adds up to 180^{o}.

Question 8, which required the respondents to deduce and determine the value of x using the Pythagoras' theorem, had 63 (60.58%) of its

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respondents scoring all correct with 30 (28.85%) scoring zero. Only one (0.96%) had low understanding of the question and 10 (9.62%) gave a partly meaningful response to the question.

The outcome of the correct logical deduction response for question 9 was almost the same as that of question 8, as 64 (61.54%) scored all correct and the incorrect response was 22 (21.15%). The respondents were to employ the "tangent to diameter" theorem, "angles on a diameter" theorem, or "angles subtended at the centre" theorem, as well as previous knowledge of triangle properties, to guide them in solving question 9. A sample of an answered question is presented in Figure 16.

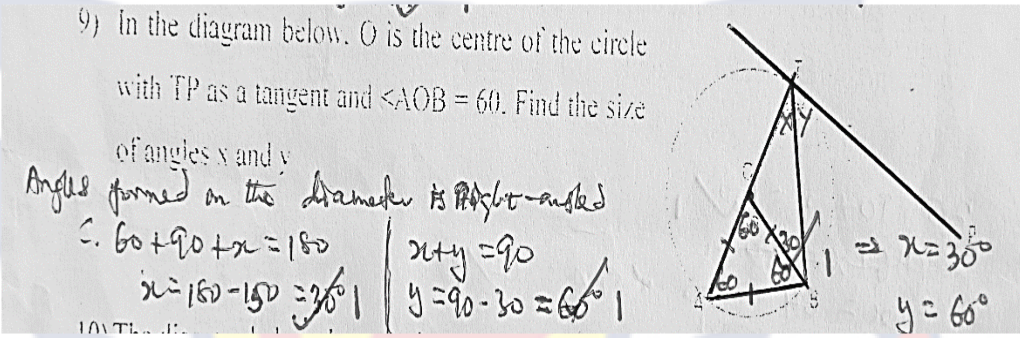

Figure 16: A sample of an answered teacher GAT question 9

It was however realised that few of the respondents could not answer this question fully based on simple arithmetic error. An example is discussed as follows. In finding the value of y, 90° – 30° should give a result of 60° but the respondent gave an input of 50°. The output of the respondent is shown in Figure 17.

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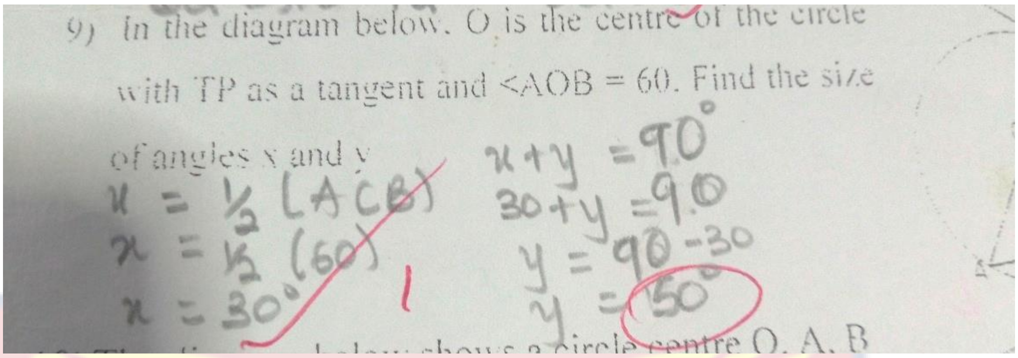

Figure 17: A sample of an incorrect response of question 9

The performance of the respondents for questions 7, 8 and 9 was good as 86, 63 and 64 teachers out of 104 passed respectively for giving a correct logical deduction response. The outcome of question 10, where the respondents were to find the size of angle ABC using the "tangent to a radius" and "angles subtended at the centre of a circle" theorem properties was below average. Only 47 students (45.19%) gave a correct logical response to the question.

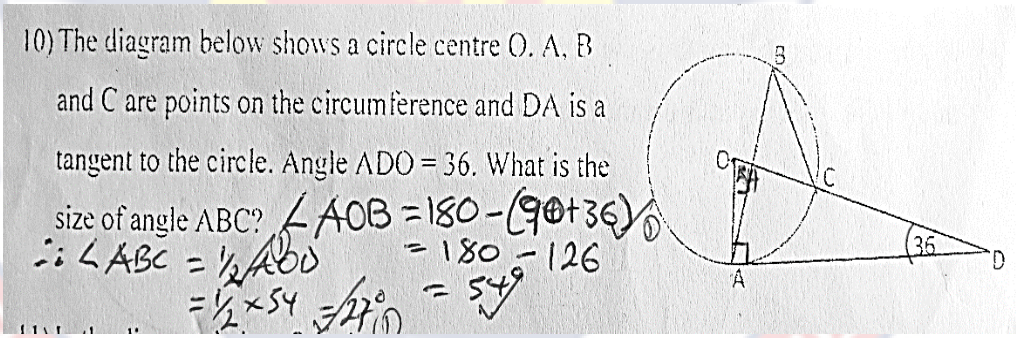

Figure 18: A correct sample solution to question 10

The incorrect responses given were 37 (35.58%) and the partly meaningful response was 10 (9.62%). A sample of an incomplete response is indicated in Figure 19.

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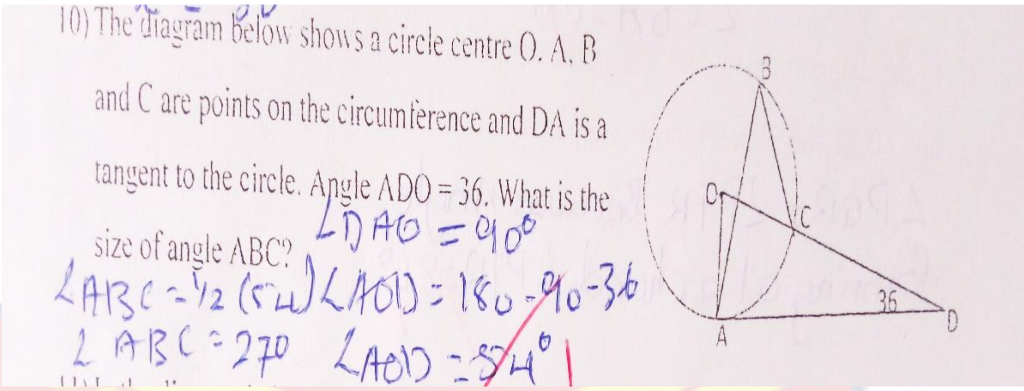

10) The diagram below shows a circle centre O. A, B and C are points on the circumference and DA is a tangent to the circle. Angle ADO = 36. What is the size of angle ABC?

∠DAO = 90°

∠ABC = ½ (54) ∠AOD = 180 - 90 - 36

∠ABC = 270 ∠AOD = 54°

Figure 19: A sample of a partly meaningful response to question 10

The respondent was able to determine the angle at the centre (<AOD) using the “tangent to a radius” theorem but was unable to determine the size of <ABC using the “angles subtended at the centre of a circle” theorem.

In all, it was realised from Table 13 that more than 50% of the respondents had good content knowledge in the “abstraction” level (VHGT level 3). An average of 65 respondents representing 62.5% scored questions 7 to 10 correct. Also, the percentage average of those who scored zero from questions 7 to 10 is 25.96%. Therefore, the average percentage of those who scored above 50% in VHL 3 was recorded to be 72.84%. This implies that about a quarter of the 104 respondents could not pass at level. Conclusively, 72.84% (76 out of 104) of the teacher respondents perceived relationships between properties of shape and could also give simple arguments to justify their reasoning on geometric shapes.

Teacher performance in the VHGT level 4 (Deduction)

This level had four questions, with each question carrying three marks, totalling 12 marks. These questions were numbered 11 to 14. Table 14 below shows the frequency distribution of the results obtained by the respondents with their corresponding percentages.

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Table 14: Teacher performance of each item in the VHGT level 4

Question	Incorrect (0) N (%)	Low understanding (1) N (%)	Partly correct (2) N (%)	Correct logical deduction (3) N (%)
11	35 (33.65%)	1 (0.96%)	4 (3.85%)	64 (61.54%)
12	42 (40.38%)	4 (3.85%)	8 (7.69%)	50 (48.08%)
13	81 (77.88%)	11 (10.58%)	0 (0.00%)	12 (11.54%)
14	46 (44.23%)	2 (1.92%)	4 (3.85%)	52 (50%)

From Table 14, 35 (33.65%) gave incorrect responses, while 64 (61.54%) gave fully correct responses to question 11. Those who scored two marks out of three were 4 and only one respondent scored one out of the three marks. At 65%, the passing rate was higher than average. This is an indication that more than half of the respondents had the prerequisite knowledge in Circle Theorems to answer the question. Respondents were asked to demonstrate that p + q = 90, as shown in Figure 20, by using the subtended angle at the centre and isosceles angles properties as guides. A sample of the teacher's GAT response is shown in Figure 20.

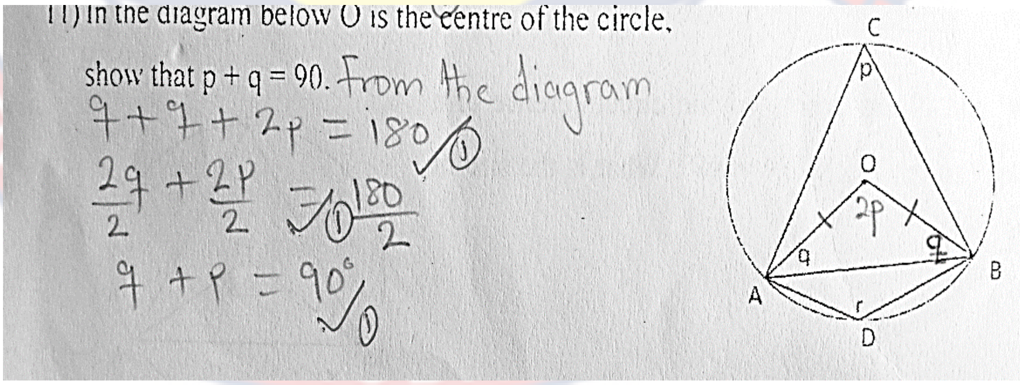

Figure 20: A screenshot of correct sample solution to question 11

Figure 21 is also a response from a teacher respondent who gave a partly correct response to question 11.

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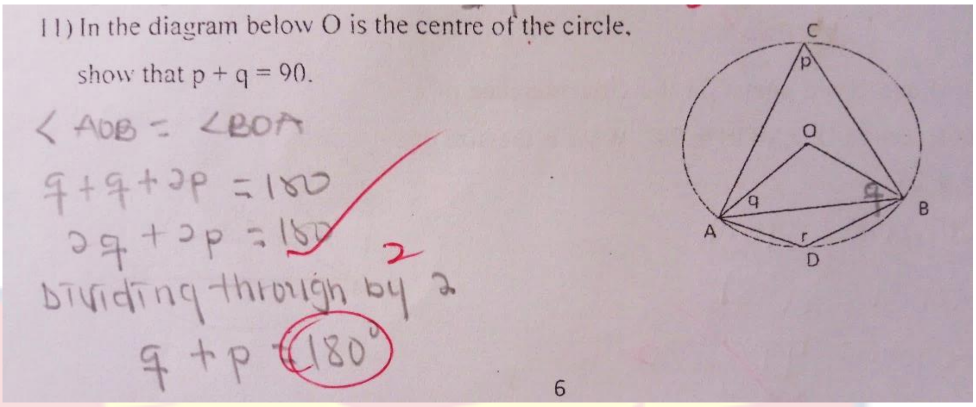

Figure 21: A sample of a partly correct response to question 11.

The respondent was able to deduce that |OA| and |OB| are both radius and equal making triangle OAB an isosceles triangle. It was also deduced that <AOB = 2<ACB. Therefore, q + q +2p = 180°. However, the only problem identified was the inability to divide both sides of the equation by 2 to get q + p = 90°.

Question 12 revealed the following outcomes: 42 (40.38%) of respondents gave incorrect responses, 50 (48.08%) gave correct responses and eight (7.69%) had partly correct answers. The passing percentage was 55%. In this case, respondents were asked to calculate the values, $x°$ *and* $y°$, as shown in Figure 22. It was essential to use the "angle subtended at the centre of a circle theorem" to find the value of y° and combine it with angle properties to calculate $x°$.

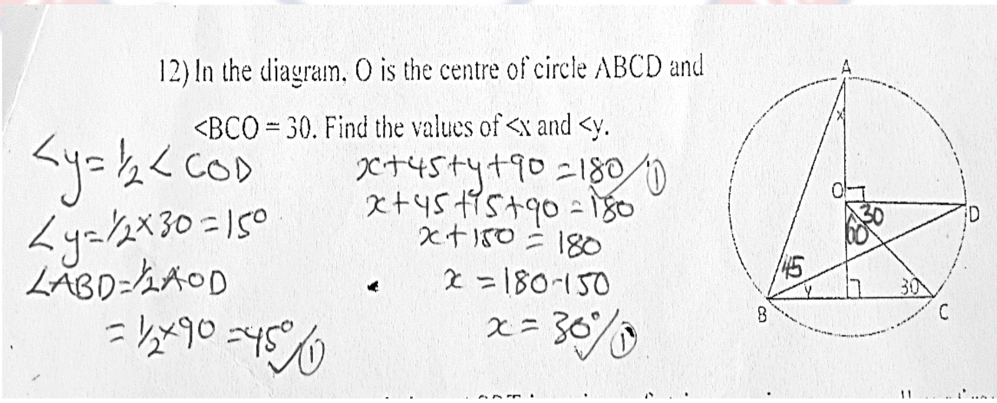

Figure 22: A snapshot of correct solution of teacher GAT question 12

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A sample of an incorrect response of question 12 is presented in Figure 23.

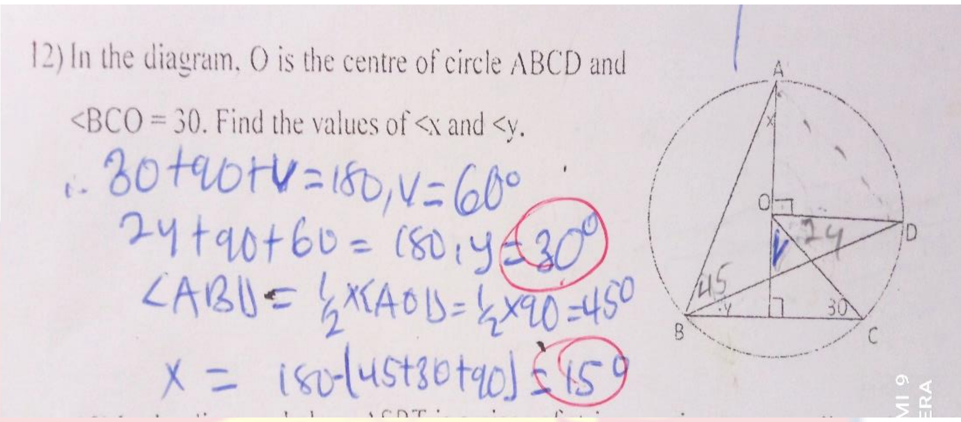

Figure 23: A snapshot of an incorrect response of question 12

The respondent was able to apply the right – angled triangle property to identify the value of angle 'V' in Figure 23 but could not deduce the values of $x°$ *and* $y°$ using any of the circle theorem properties making the solution incorrect.

The percentage pass in question 13 was very abysmal, as only 12 (11.54%) scored correctly and 81 (77.88%) scored incorrectly. The percentage pass was 11.54% and the percentage fail was 88.46% as 11 (10.58%) had low understanding at the van Hiele level 4. The question required respondents to use a combination of their knowledge of Circle Theorems and previous knowledge of angles to make logical deductions in order to calculate the obtuse angle SOT, as shown in Figure 24. Their response was evident: only a few had the conceptual knowledge of the theorems involved and therefore could not provide meaningful arguments to answer this question.

"In the diagram below. ASRT is a piece of string passing over a pulley of a radius 10cm in a vertical plane. O is the Centre of the pulley and is a horizontal straight line touching the pulley at M. Angle SAB=90, and TBA=60. Calculate the obtuse angle SOT."

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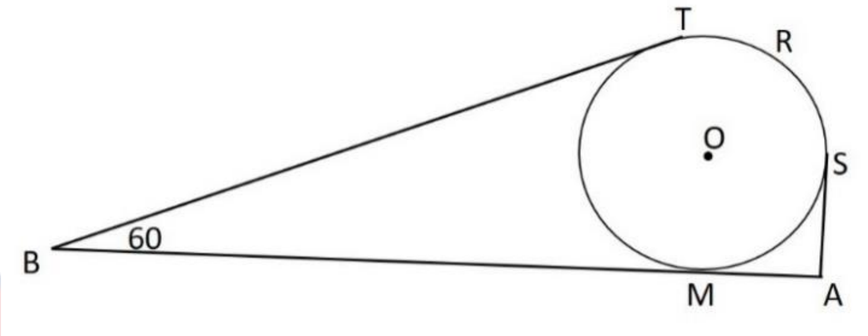

Figure 24: A sample of question 13 of teacher GAT

Figure 25 is a sample of the 11 (10.58%) respondents who gave a partially correct response.

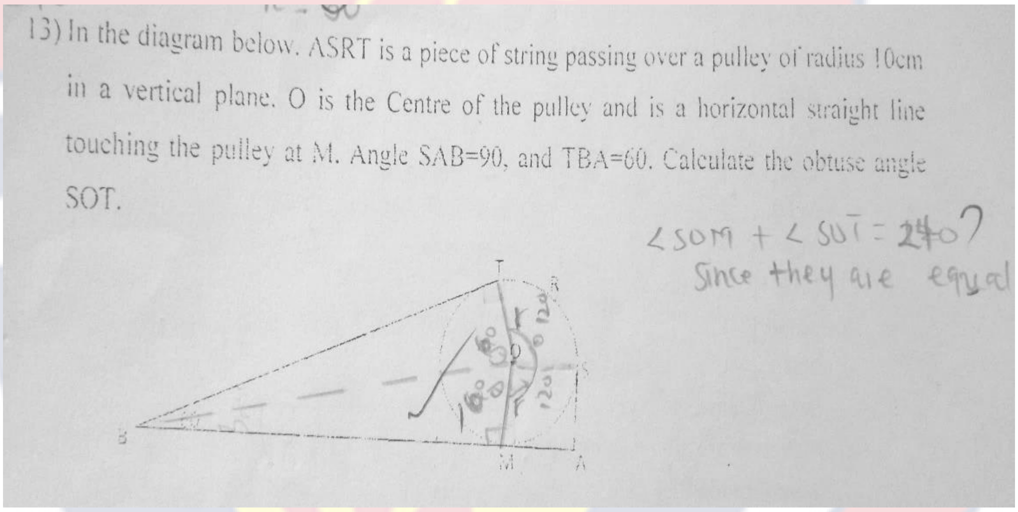

Figure 25: A sample of a partially correct response to question 13.

The respondent was able to identify that “two tangents from an external point to a circle are equal” property was one of the appropriate rules to use to answer question 13. However, the “tangent to a circle” property was not identified. Therefore, the respondent could not continue with the solution to the question.

For question 14, the respondents were required to prove that $<ROS=2x$ using the concepts ‘alternate segment theorem’ and ‘angles subtended at the centre of a circle theorem’ properties to find < ROS as indicated on the diagram in Figure 26.

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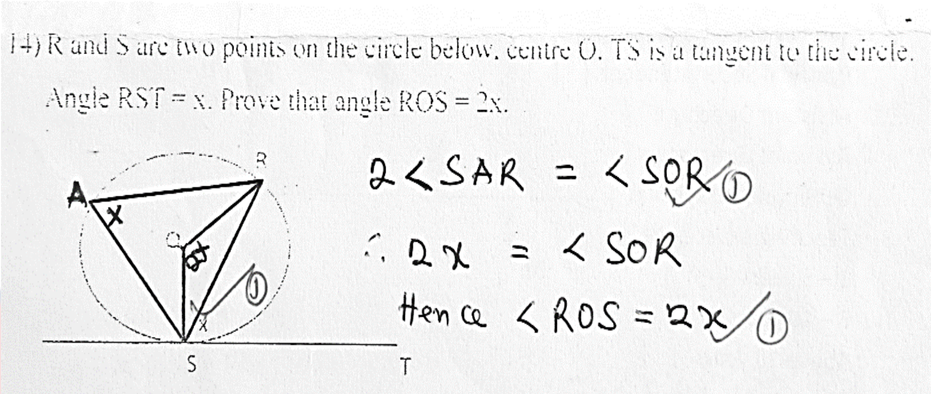

Figure 26: A sample of correctly answered question 14.

Those who responded correctly were 52 (50%) and 46 (44.23%) responded wrongly. Partly correct score respondents were 4 (3.85%), while low understanding score respondents were 2 (1.92%). However, the pass percentage was 53.8%. Partly correct response of question 14 is presented in Figure 27.

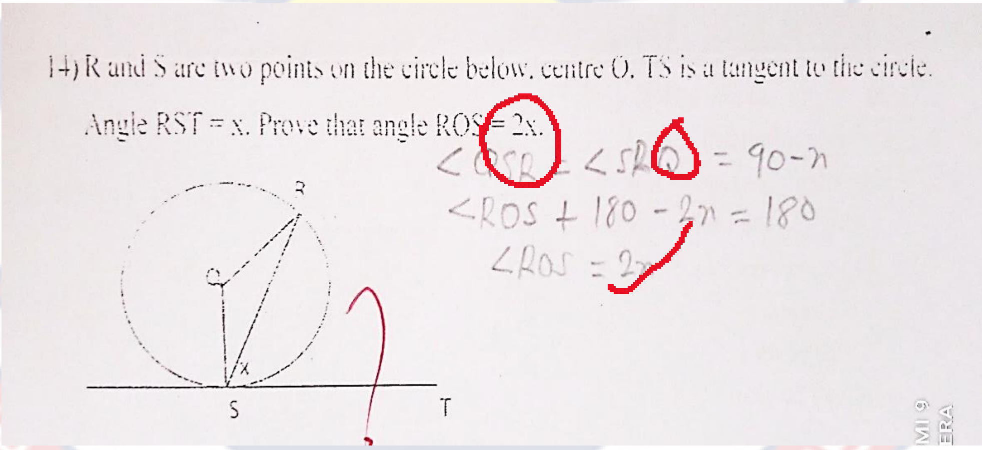

Figure 27: A sample of a partly correct response of question 14

Some of the letters used on the diagram in question 14 was misquoted in the solution procedure which gives a different picture to the diagram. The centre, O was misquoted as ‘Q’. However, the respondent used “tangent to a circle” theorem and isosceles triangle property to prove that <ROS = 2X.

Level 4 had a different turn where the percentage of the incorrect increased drastically as compared to that of the preceding levels. It was

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deduced that an average of 49.04 percent of the respondents gave incorrect answers, compared to an average percentage of 42.79% correct responses at this level. However, the percentage of passes at level 4 was 46.63% and 53.37% failed at this level. This is a realisation that more than half of the respondents could not construct or deductively prove the theorems. It implies that 53.37% of the respondents cannot structure or write geometric proofs with understanding. Again, they do not understand the importance of deduction and the role of theorems as described by van Hiele level 4 (Subbotin &Voskoglou, 2017).

Teacher performance in the VHGT level 5 (Rigor)

The question at level 5 was a proof of a theorem: a line drawn from the centre of a circle perpendicular to a chord bisects the chord. The respondents were expected to prove the theorem using a diagram. The proof will determine whether the respondent has reached Level 5 of the van Hiele theory of geometric thinking or not. Correct answers receive four points. The scores obtained by respondents are presented in Table 15, along with their corresponding percentages.

Table 15: Teacher performance of each item in the VHGT level 5

Score (mark)	Number of respondents, N	Percentage (%)
Incorrect (0)	73	70.19%
Clear visualization and analysis (1)	1	0.01%
Clear visualization, analysis and abstraction (2)	1	0.01%
Clear visualization, analysis, abstraction and deduction (3)	12	11.54%
Correct proof (4)	17	16.35%

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From Table 15, the percentage of respondents who had incorrect scores outweighed that of the correct scores. The percentage of 70.19% was realised from the incorrect score as compared to 16.35% in the correct proof. Only 11.54% had 3 out of 4 marks, representing 12 respondents. These findings imply that most of the respondents could not prove the question given. Thus, out of the 104 teacher respondents, only 17 could answer question 15. However, only 29 (27%) scored more than half.

The observations made were that most of the respondents were unable to visualise or properly recognise the theorem they were to prove, hence their inability to solve it. The respondents were expected to draw a circle and make deductions as shown in Figure 28. Respondents were required to demonstrate that chord OX bisects chord AB such that AX = BX.

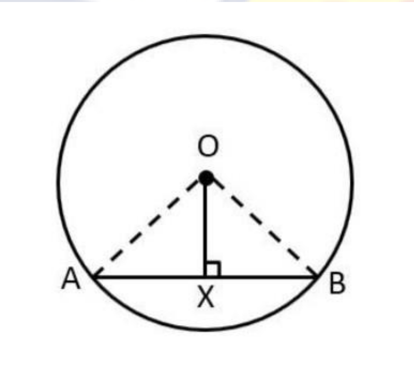

Figure 28: A sketch to prove question 15.

To prove that OX bisects chord AB. Thus, AX = BX

Proof:

In ΔOAX & ΔOBX, from Figure 28.

$\angle OXA = \angle OXB$ (They are both 90°) [VHL 1]

OA = OB (Both radius) [VHL 1 and 2]

OX is the common height for ΔOXA and ΔOXB [VHL 3 and 4]

$\therefore \Delta OAX \equiv \Delta OBX$ (RHS rule) [VHL 3 and 4]

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AX = BX [VHL 4 and 5]

Hence, the theorem is proved.

The 12 (11.54%) respondents who attempted to demonstrate their understanding proved it up to the abstraction level. The observation was that they could not explain the relationship between the radius and the right triangles they drew. Some just drew the diagrams without identifying the congruent parts. Therefore, they were not able to describe or deduce the relationship between them to justify their proof. Figure 29 is a sample of one of the answers.

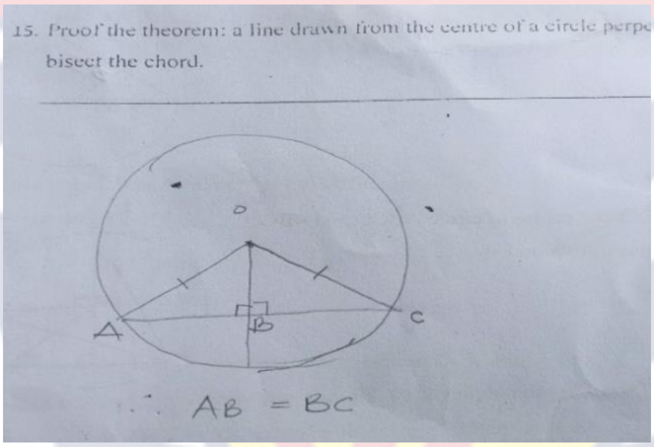

Figure 29: A sample of an answered question 15 of the teacher's GAT

Overview of the results at each level

Table 16 summarizes the percentage pass mark to the nearest whole number for each level as well as the total number of respondents who passed.

Table 16: Summary of percentage pass marks at each level

Level	Name	Percentage pass (%)	Number passed
1	Visualization	81	84
2	Analysis	85	88
3	Abstraction	73	76
4	Deduction	47	49
5	Rigor	27	29

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Given that 84, 88, and 76 teachers, respectively, had pass rates of 81 percent, 85 percent, and 73 percent, the data in Table 16 shows that majority of the teachers had control over levels 1-3. As a result, they can analyse 'Circle Theorem' concepts up to the abstractive level of van Hiele's levels with ease. This indicates that an average of 83 (79.67%) of the teacher participants had acquired the levels of visualization, analysis, and abstraction. Deduction and the rigor stage performance at levels 4 and 5 fell short of the pass mark. Only 49 and 29 respondents representing 47 and 27 percent respectively passed. This demonstrates that majority of participants have not reached van Hiele's geometric level of rigor and reasoning. In other words, the larger percentage of these respondents were unable to provide deductive geometric proofs and were unable to comprehend the significance of definitions, theorems, axioms, and proofs. Furthermore, they were unable to explain how changing an axiom in a particular geometric system would affect it as indicated in the van Hiele theory's stages.

Discussion on Research Question Three

For research question 3, "At what levels of van Hiele's geometric thinking are SHS mathematics teachers operating?". The findings showed that the teacher respondents have acquired much content knowledge up to level 3 of van Hiele's theory. The percentage pass rate was 81, 85 and 73 for levels 1 to 3 respectively. The percentage pass rate at levels 4 and 5 were 47 and 27 respectively showing that the teachers' performance was below average. From research question one, it was realized that the general teacher performance was good (89.42%), however categorizing the teacher content knowledge using the van Hiele's levels has revealed that the mathematics teachers exhibited some

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lapses when dealing with questions involving proofs and deductions. This confirms Luneta (2015) assertion that geometry as a topic is difficult for teachers to teach because of the large amount of geometrical knowledge required to deal with the topic. It can be deduced that lacking knowledge and control over deduction and proofs in Circle Theorems has adverse implications in the teaching and learning processes. Just as Ball (1990) suggested that teachers lack of essential knowledge and intellectual resources for teaching mathematics affect student learning significantly. Shulman (1986) was also of the view that the level of this knowledge obtained by the teacher has a relative influence on the teaching practises in the classroom.

SHS 2 students' Van Hiele's levels in Circle Theorems

This research question sought to determine each of the van Hiele's levels attained by the SHS 2 students.

Student performance in the VHGT level 1 (Visualization)

This level requires the respondent to recognise geometric figures by their appearance alone but not their properties. The GAT questions were three, each carrying two marks, for a total of six marks. The total score for getting all the marks correct at level 1 was six. The performance of the students after answering the questions at VHL 1 is presented in Table 17 with their corresponding percentages.

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Table 17: Frequency distribution for student performance at VHL 1

Score	Frequency	Percent	Cumulative Percent
0	19	6.8	6.8
1	5	1.8	8.6
2	11	3.9	12.5
3	30	10.7	23.2
4	24	8.6	31.8
5	34	12.1	43.9
6	157	56.1	100.0
Total	280	100.0	

Table 17 confirms that most respondents passed and had achieved level 1 of the van Hiele level of geometric thinking, with 157 (56.1%) students scoring all the questions under level 1 correct, 34 (12.1%) scoring 5 and 24 (8.6%) scoring 4 correct. Only 19 (6.8%) scored zero, 5 (1.8%) scored one, 11 (3.9%) scored 2 and those who scored three were 30 (10.7%). It was realized that out of the 280 students, 215 passed with a corresponding percentage of 76.79, using Usiskin's (1982) pass mark criteria. The number of respondents who could not pass was (65) 23.21%.

Analysis of the student GAT is as follows:

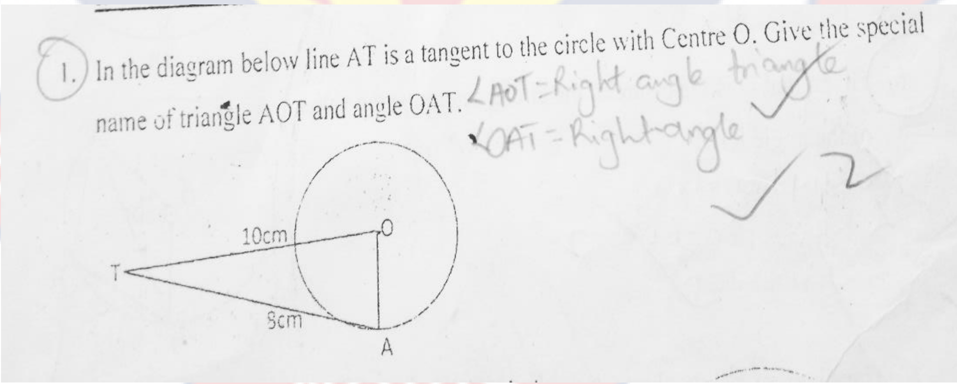

Figure 30: A snapshot of a sampled fully correct marked script for question 1

The question requested the respondent to give the special name of triangle AOT and name angle OAT. Out of 280 students, 182 (65%) passed by scoring

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all the two marks, 70 (25%) scored one and 28 (10%) scored zero. Sample of partly answered question is presented in Figure 31.

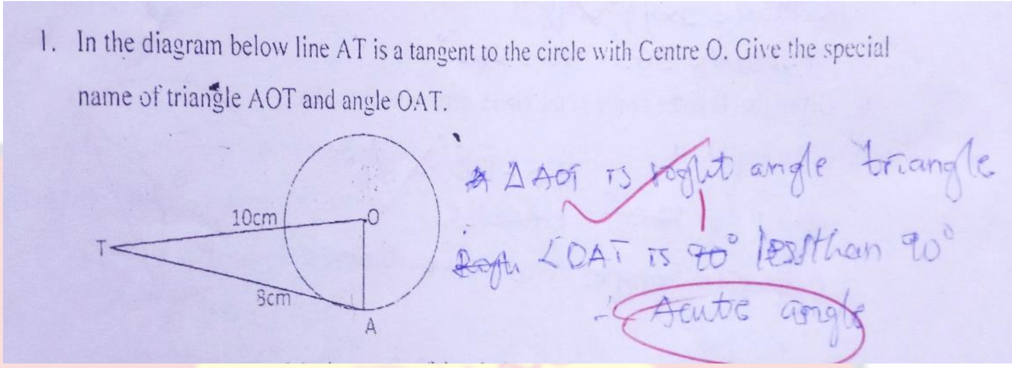

Figure 31: A marked sample of incorrect response of student GAT question 1

The respondent was able to name the type of triangle given but could not identify angle OAT. This implies that, the respondent does know that an angle formed by the intersection of a tangent to the circumference of a circle and the radius makes an angle of 90°.

Question 2 required the respondent to explain why |AC| is equivalent |BC| and give the specific name of triangle ABC as drawn in Figure 32.

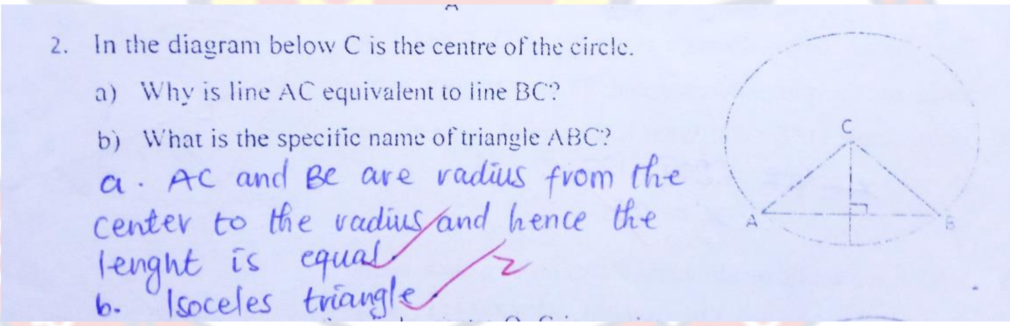

Figure 32: A marked sample of correctly answered student GAT question 2

Out of 280 respondents, 194 (69.29%) scored all correct, 49 (17.5%) scored one mark out of two and 37 (13.21%) did not score any. The pass rate for question 2 was good. However, some of the respondents could not answer the question fully. Sample of a partly correct answered question is shown in Figure 33.

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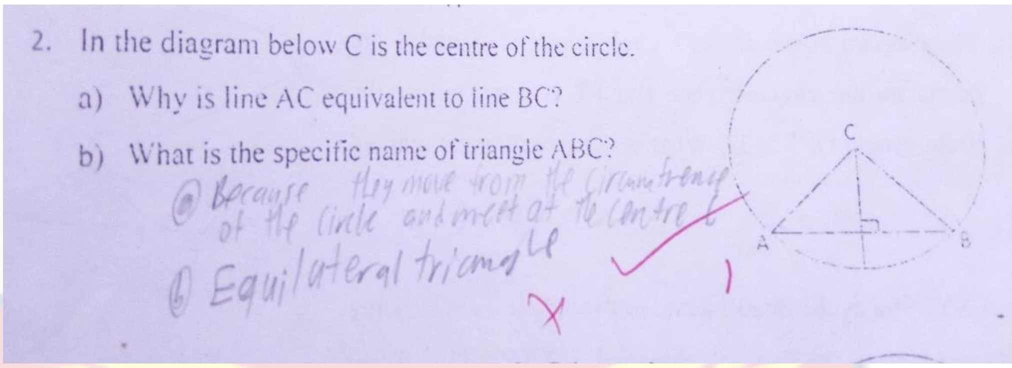

Figure 33: A marked sample of partly correct question 2

It was identified that, the respondent was unable to deduce that a triangle formed on a chord that subtend to the centre of the circle is isosceles. The triangle was described as an equilateral instead of isosceles.

For question 3 of the student GAT questions, the respondents were to find a missing angle in a triangle an a diameter that subtend to the circumference of a given circle. They were to calculate the size of angle BCA as shown in Figure 34.

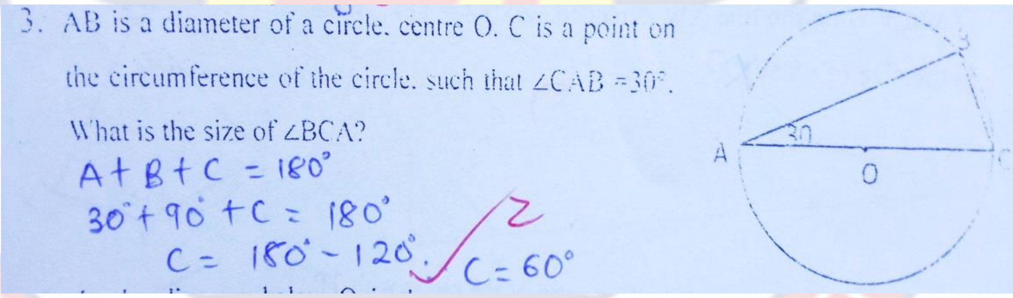

Figure 34: A sample of correctly answered GAT question 3

A triangle formed on a diameter of a circle that subtend to the circumference of the circle makes an angle of 90^{o} at the circumference. Therefore, the sum of the three interior angles of the triangle add up 180^{o} as shown in Figure 34. The number of respondents who answered it correctly were 215 (76.79%) while 24 (8.57%) scored one mark out of 2. The remaining 41 (14.64%) scored zero. Sample of an incorrect solution is presented in Figure 35.

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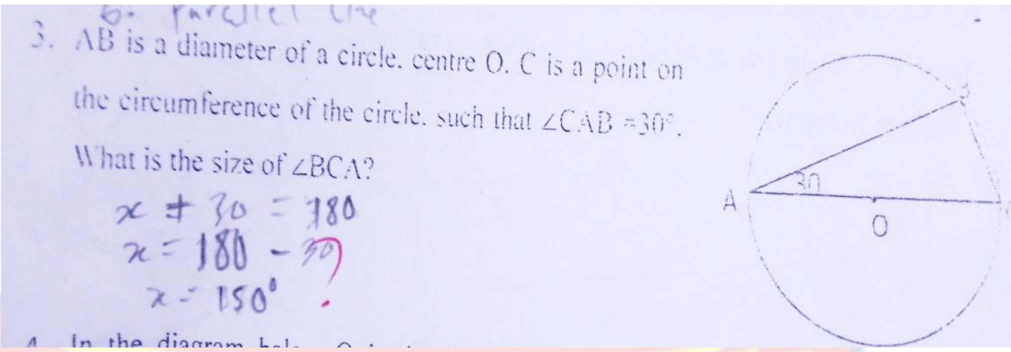

Figure 35: A sample of an incorrect GAT question 3

Here, the respondent summed only two angles out three. This implies that the respondent did not know that the sum of the interior angles adds up to 180°. There was lack of knowledge about which Circle Theorem property to use to answer this question.

Student performance in the VHGT level 2 (Analysis)

At this level, the questions required the respondent to analyse figures using their geometric properties, such as observation or measurement. It is also expected that students at this stage be able to illustrate, model and relate the properties and attributes of geometric concepts. The questions answered were three, with two marks each, totalling six marks.

Table 18 presents the frequency of the marks obtained by the respondents at VHL 2 and their corresponding percentages.

Table 18: Frequency distribution for student performance at VHL 2

Score	Frequency	Percent	Cumulative Percent
0	23	8.2	8.2
1	2	0.7	8.9
2	29	10.4	19.3
3	9	3.2	22.5
4	56	20.0	42.5
5	12	4.3	46.8
6	149	53.2	100.0
Total	280	100.0	

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The respondents who passed at this level were 217 (77.50%) and the number who could not pass were 63 (22.50%). Those who scored all the marks correct were 149 (53.2%). Out of the 280 students, the total percentage pass rate was great, as more than 50% of the respondents scored all three questions correct. This implies that the students have achieved a high level of geometric thinking at the level 2.

For GAT question 4, the respondents were to find angle PTR as shown in Figure 36.

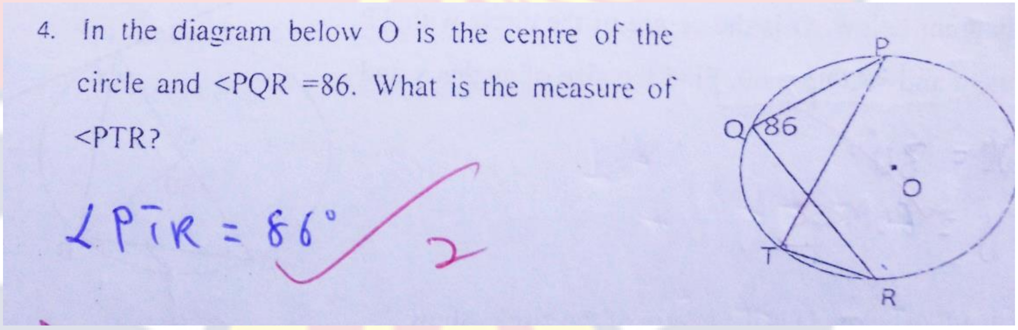

Figure 36: A sample of a correct GAT question 4.

The respondent was able to analyse that two or more angles formed on the circumference of a circle subtended from the same chord are the same. The respondents who answered it correctly were 210 (75%) and the remaining 70 (25%) of the respondents could not answer it correctly. Sample of an incorrect response is shown in Figure 37.

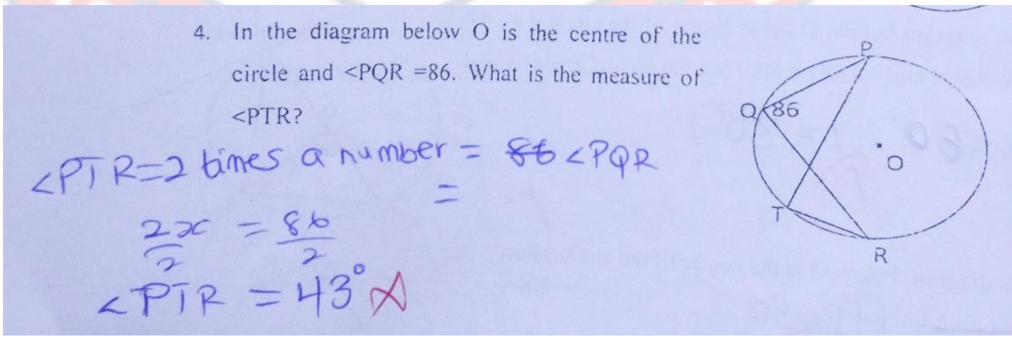

Figure 37: A sample of an incorrect GAT question 4

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The respondent used 'angle at the centre of a circle is twice any angle at the circumference standing on the same arc theorem' to solve for angle PTR instead of using 'angles at the circumference standing on equal arc theorem'.

For question 5, the respondents were to analyse what the size of an acute angle MON is. They were required to use 'angle at the centre of a circle is twice any angle at the circumference standing on the same arc theorem' to solve for angle MON. A sample of a correctly answered question 5 is shown in Figure 38.

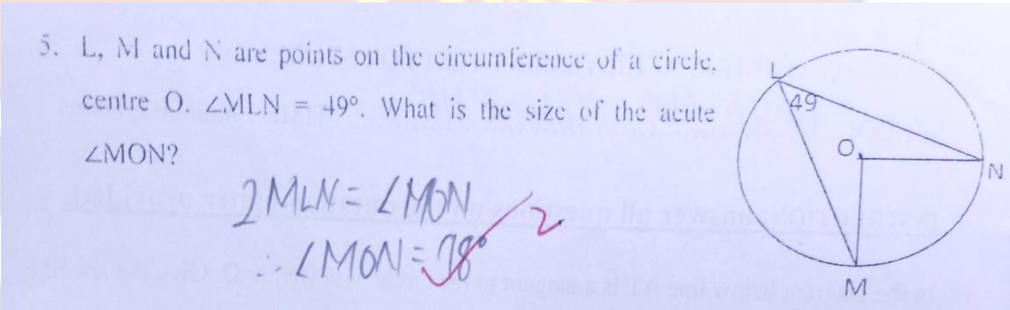

Figure 38: A correctly answered student GAT question 5

Out of 280 respondents, 220 (78.57%) answered it correctly and the remaining 60 (21.43%) could not answer the question correctly. A sample of the incorrect response provided by the respondent is shown in Figure 39.

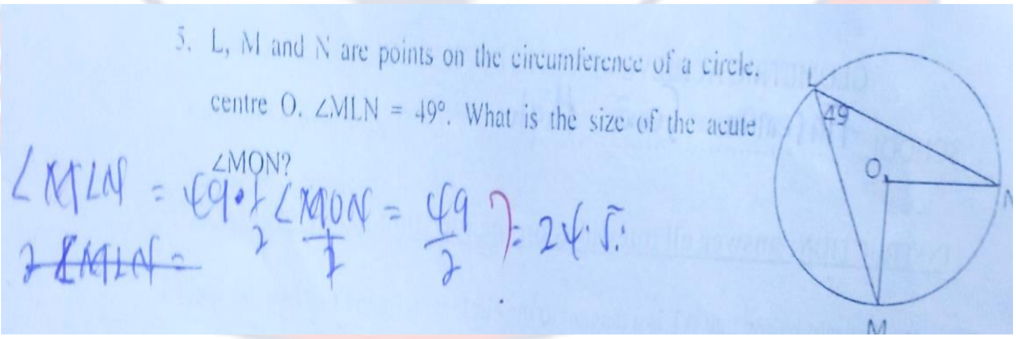

Figure 39: An incorrect response to student GAT 5 question

Here, the respondent quoted the rule for obtaining angle MON correctly but could not deduce further to obtain the value of angle MON using the 'angle at the centre is twice the angle subtended at the circumference theorem'.

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For student GAT question 6, the respondents were required to determine the value of the angle KHJ of a circle with centre O as shown in Figure 40.

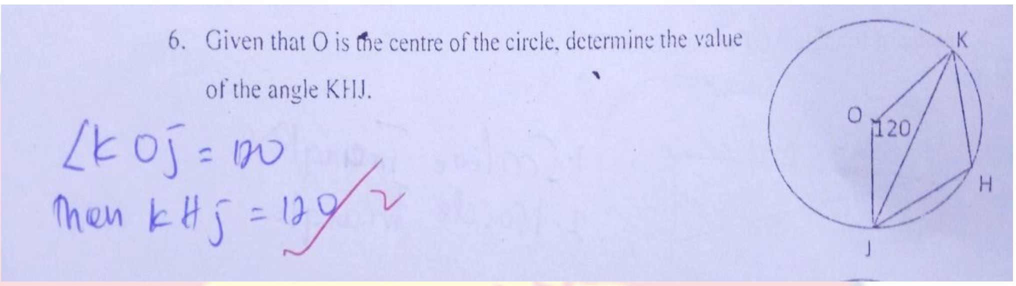

Figure 40: A marked sample of a correctly answered GAT question 6

The respondents were to use the ‘angle at the centre is twice the angle subtended at the circumference theorem’ to answer. The number of respondents who got it correct were 171 (61.07%) and 20 (7.14%) scored one mark. The remaining 89 (31.79%) scored zero.

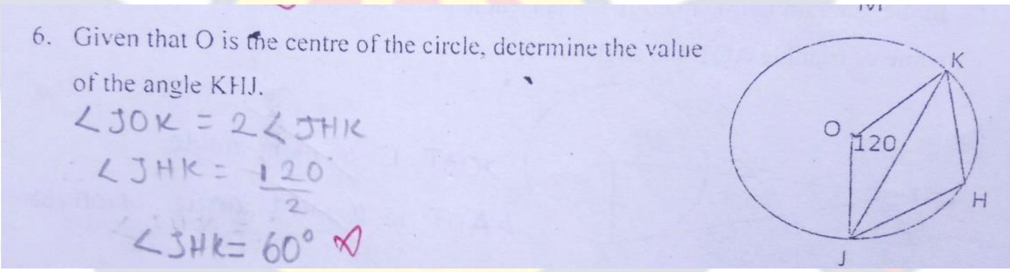

Figure 41: An incorrect response to student GAT 6 question

A sampled incorrect marked script as shown in Figure 41 indicates that, the respondent had knowledge about the ‘angle at the centre is twice the angle subtended at the circumference theorem’ but could not apply it properly to obtain the right value of angle KHJ.

Student performance in the VHGT level 3 (Abstraction)

At level 3, the respondents were required to exhibit the ability to order figures logically and understand the relationships among the properties of figures. They are required to have the ability to follow an informal argument

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and see proofs without writing or structuring them. There were four questions answered, each carrying three points, for a total of 12 points. Table 19 exhibits the performance of the students by score and their corresponding percentages.

Table 19: Frequency distribution for student performance at VHL 3

Score	Frequency	Percent	Cumulative Percent
0	59	21.1	21.1
1	18	6.4	27.5
2	7	2.5	30.0
3	14	5.0	35.0
4	13	4.6	39.6
5	4	1.4	41.1
6	21	7.5	48.6
7	12	4.3	52.9
8	5	1.8	54.6
9	41	14.6	69.3
10	15	5.4	74.6
11	14	5.0	79.6
12	57	20.4	100
Total	280	100.0	

From Table 19, 59 (21.1%) scored zero, 18 (6.4%) scored 1 and 7 (2.5%) scored 2. Only 57 students (20.4%) scored all correct and 59 (21.1%) scored nothing. The percentage pass for those who scored all correct has dropped drastically at level 3 to 20.4% as compared to levels 1 and 2, which had corresponding percentages of 56.1% and 53.2% respectively. This implies that, 79.6% of the students had some difficulty answering the question.

However, the pass rate achieved at this level was 51.43% (144) as compared to that of 48.57% (136) who failed. The pass rate was impressive on paper, but the quality of the results was very poor, as only 57 out of 280 students (20.4%) could answer all the questions correctly. Conclusively, about 20% of the respondents had the ability to follow an informal argument and see proofs without writing or structuring them.

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At this level, the questions were to assess the deductive thinking level of the student respondents. Question 7 requested the respondent to identify the size of an angle marked x as shown in Figure 42

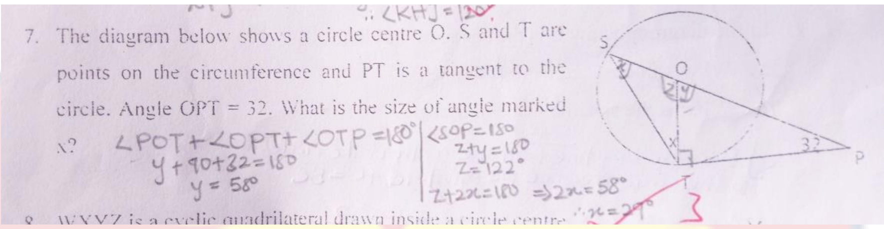

Figure 42: A marked sample of a correctly answered GAT question 7

The question required the use of 'tangent to a circle theorem', the right-angled triangle and the isosceles triangle properties to find the angle x. The number of respondents who scored all correct were 148 (52.86%) and 17 (6.07%) scored 2 marks. Therefore, 165 (58.93%) passed and the remaining 115 (41.07%) could not pass. A sample of an incorrect response to question is shown in Figure 43.

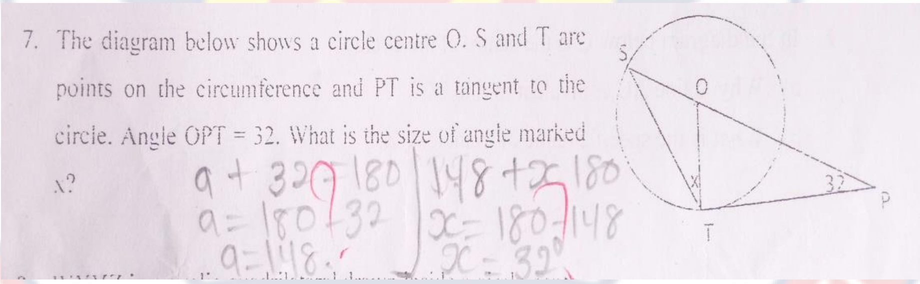

Figure 43: A sample of an incorrect response to GAT question 7

It can be concluded that the respondent had little or no clue on how to answer the question.

For question 8, the respondents were to find the size of angle VWX of a cyclic quadrilateral as shown in Figure 44.

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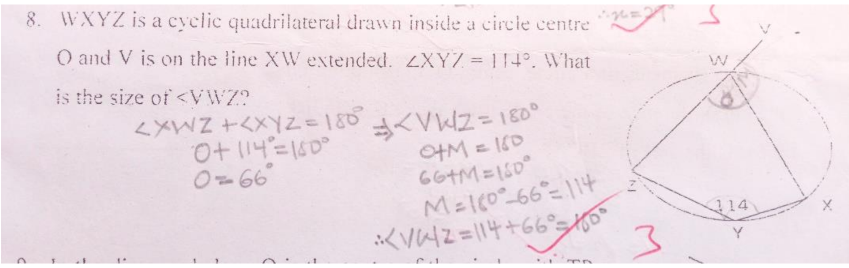

Figure 44: A marked sample of a correctly answered GAT question 8

The respondent used the sum of the interior opposite angles property to find the angle O which was 66°. Straight line angle property was then used to find angle VWX to obtain 114°. The number of respondents who scored all correct were 148 (52.86%) and six (2.14%) scored two marks. Therefore, those who passed for question 8 were 154 (55%). The remaining 126 (45%) respondents could not pass. A sample of a partly answered question 8 is shown in Figure 45.

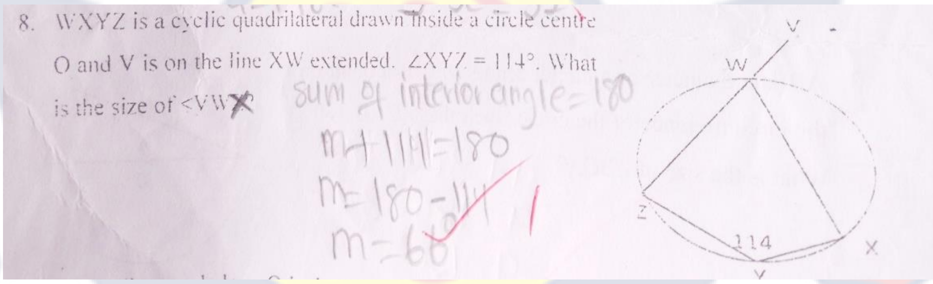

Figure 45: A snapshot of a partly answered GAT question 8

It was deduced that, the respondent had knowledge about the sum of the interior opposite angle property but could not deduce further to find angle VWX.

The question 9 asked the respondent to find the size of angles $x°\ and\ y°$ as shown if Figure 46.

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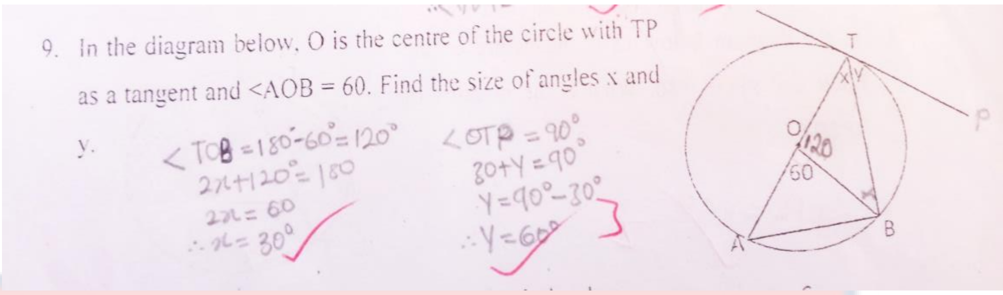

Figure 46: A marked sample of a correctly answered GAT question 9

The number of respondents who scored all correct were 149 (53.21%) and seven (2.5%) scored two marks. Therefore, a total of 156 (55.71%) respondents passed and the remaining 124 (44.29%) failed for question 9. A sample of a partly correct response is shown in Figure 47.

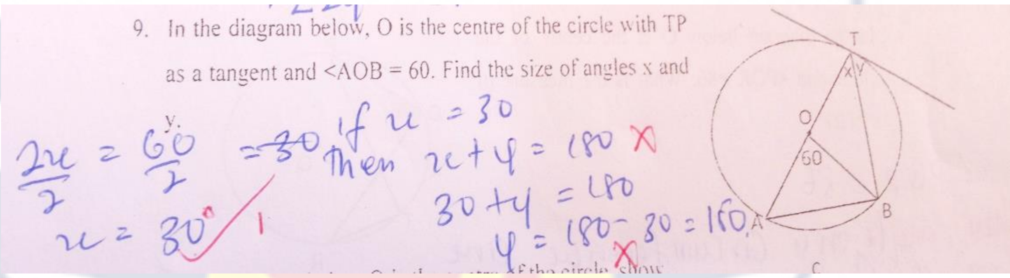

Figure 47: A snapshot of a partly answered GAT question 9

From Figure 47, the respondent was able to use the ‘angle at the centre of a circle is twice any angle at the circumference standing on the same arc’ theorem to find angle $x°$ but could not use the ‘tangent to a radius of a circle theorem’ to find angle $y°$.

The student GAT question 10 asked the respondents to show that $p + q = 90°$. Out of the 280 respondents, 75 (26.79%) were able to solve it correctly and 16 (5.71%) scored two marks out of the three. Therefore, those who passed were 91 (32.5%). This indicates that, majority of the respondents were not able to answer. A sample of a correctly answered question 10 is shown in Figure 48.

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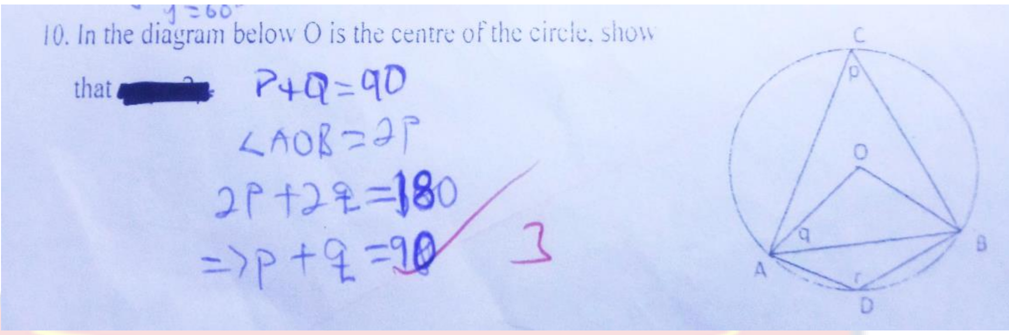

Figure 48: A snapshot of a correctly answered GAT question 10

The number of students who could not pass for answering question 10 were 189 (67.5%). A sample of a partly correct response given is shown in Figure 49.

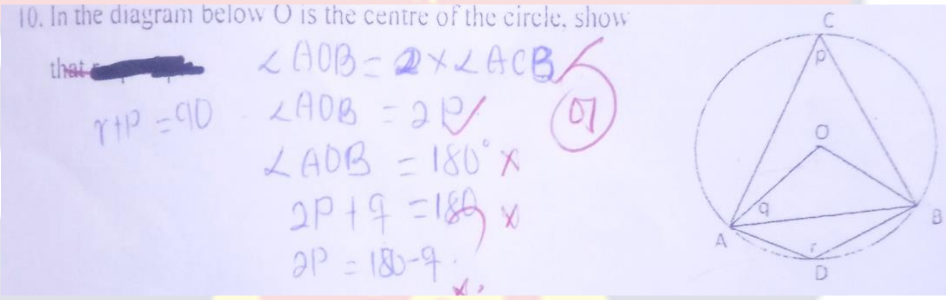

Figure 49: A snapshot of a partly correct answered GAT question 10

Student performance in the VHGT level 4 (Deduction)

This level called for the respondent to understand the role of theorems and postulates as well as their importance. They were required to evince their ability to prove and construct deductive theorems. Thus, students should possess the ability to structure and write geometric proofs with much understanding. The questions answered at this level of geometric thinking were also four and each question carried three marks, making a total of 12 marks. Table 20 exhibits the performance of the students by score with its corresponding percentages at VHL 4.

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Table 20: Frequency distribution for student performance at VHL 4

Score	Frequency	Percent	Cumulative Percent
0	140	50.0	50.0
1	17	6.1	56.1
2	8	2.9	58.9
3	42	15.0	73.9
4	12	4.3	78.2
5	5	1.8	80
6	28	10.0	90
7	7	2.5	92.5
8	1	4	92.9
9	7	2.5	95.4
10	4	1.4	96.8
12	9	3.2	100
Total	280	100.0	

From Table 20, it was realised that exactly 140 (50%) of the students could not answer any of the questions at Level 4. It is also indicating that only nine (3.2%) out of the 280 students were able to respond correctly to all the questions at this level. The frequency of the students who could not pass was very high, as 252 (90%) students failed against only 28 (10%) who passed. This is an indication that the majority of the students were below this level and that only 10% had attained the level of reasoning logically and proving theorems deductively. The remaining 90% of the respondents did not possess the ability to structure and write geometric proofs with much understanding.

For student GAT question 11, the respondents were to deduce and apply the right-angled triangle and isosceles triangle properties to determine the value of x. It was realised that those who could answer correctly were only 62 (22.14%) and 26 (9.29%) scored two out of three marks. Therefore, those who passed for question 11 were 88 (31.43%) out of 280 student respondents. Sample of a correctly answered question 11 is shown in Figure 50.

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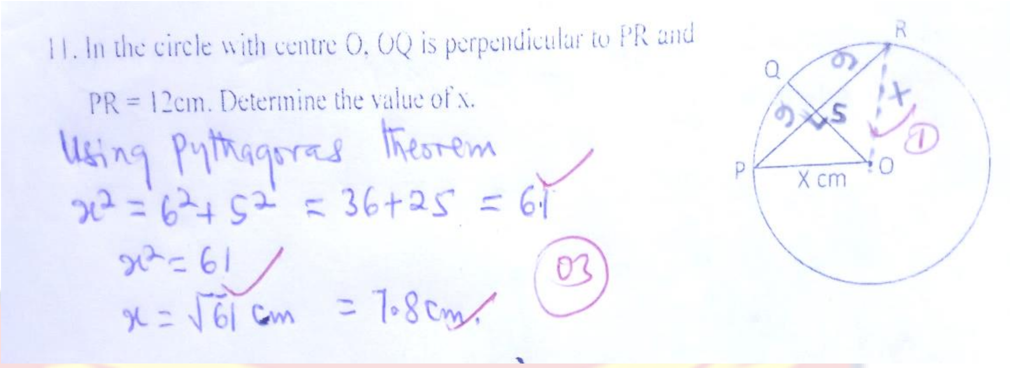

Figure 50: A snapshot of a correctly answered GAT question 11

The respondents who scored one and zero were 18 (6.43%) and 174 (174%) respectively. Therefore, those who did not pass were 192 (68.57%). Sample of a partly correct response is shown in Figure 51.

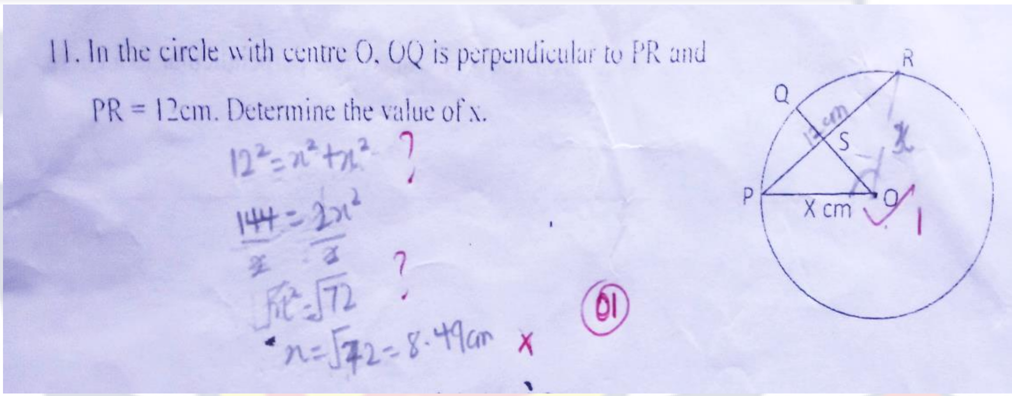

Figure 51: A snapshot of a partly correct answered GAT question 11

The respondent had knowledge about Pythagoras theorem but could not substitute the values correctly.

The question 12 required the respondent to find the values of $< x°$ and $< y°$ from a circle with centre O. The respondents were to use ‘angle at the centre is twice the angle at the circumference theorem’ and right-angled triangle property to guide them answer. An answered response is shown in Figure 52.

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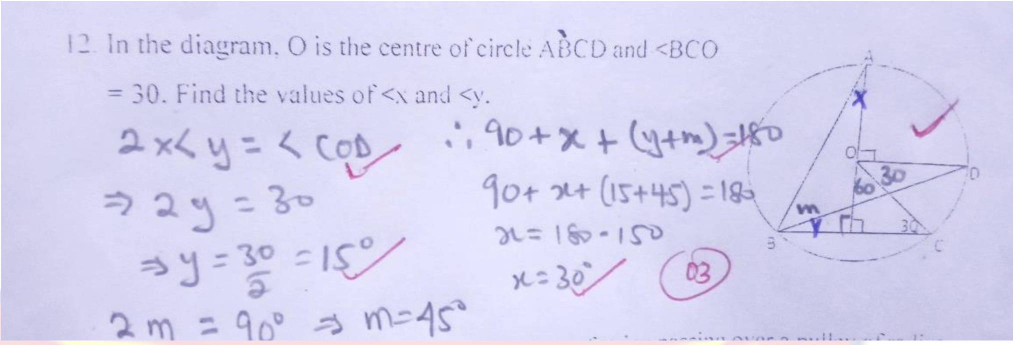

Figure 52: A snapshot of a correctly answered GAT question 12

Out of 280 respondents, 47 (16.79%) scored it correct and 9 (3.2%) scored two marks out of three. A total of 56 (20%) passed. The number of respondents who could not pass were 224 (80%). Therefore, the performance for question 12 was not satisfactory. Sample of an incorrect response is shown in Figure 53.

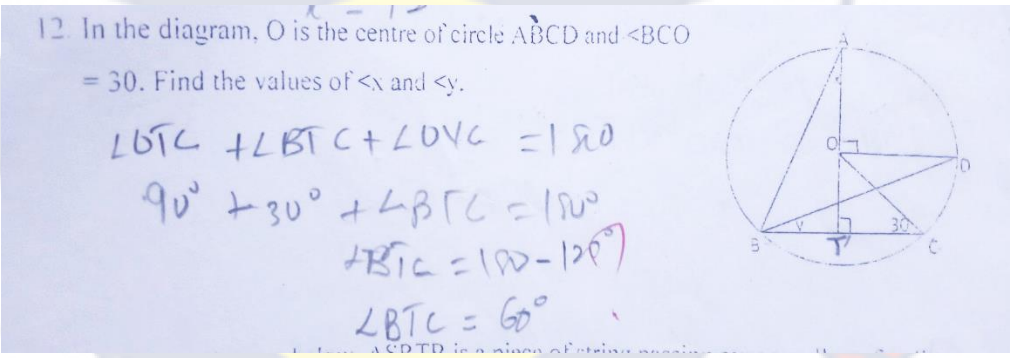

Figure 53: A snapshot of an incorrect answered GAT question 12

The question 13 was a word problem about circle theorem where the respondent was required to ‘two tangents from an external point to a circle are equal theorem’ to find obtuse angle SOT. Out of 280, 27 (9.64%) and 8 (2.86%) scored three and two marks respectively. Therefore, those who passed were 35 (12.5%). A sample of a correct response is shown in Figure 54.

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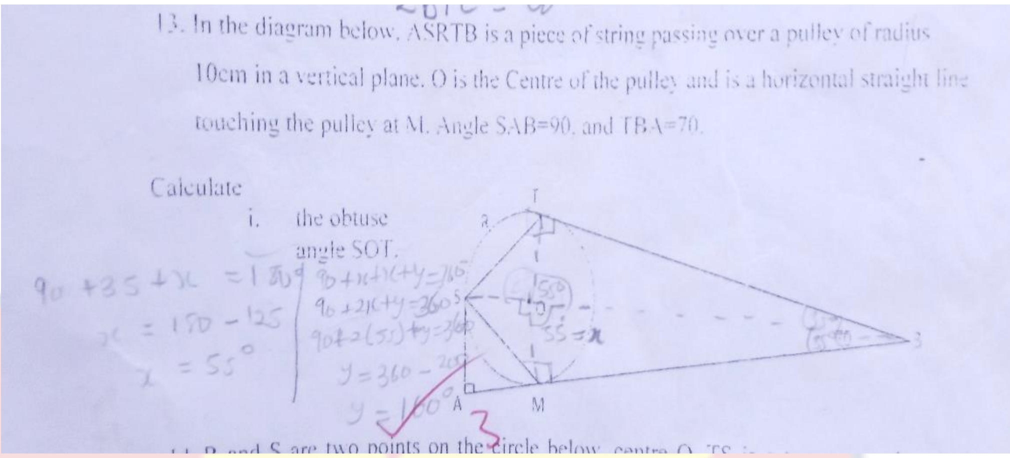

Figure 54: A sample of correct response for GAT question 13

The remaining 245 (87.5%) could not pass question 13. A sample of a student's incorrect response is shown in Figure 55.

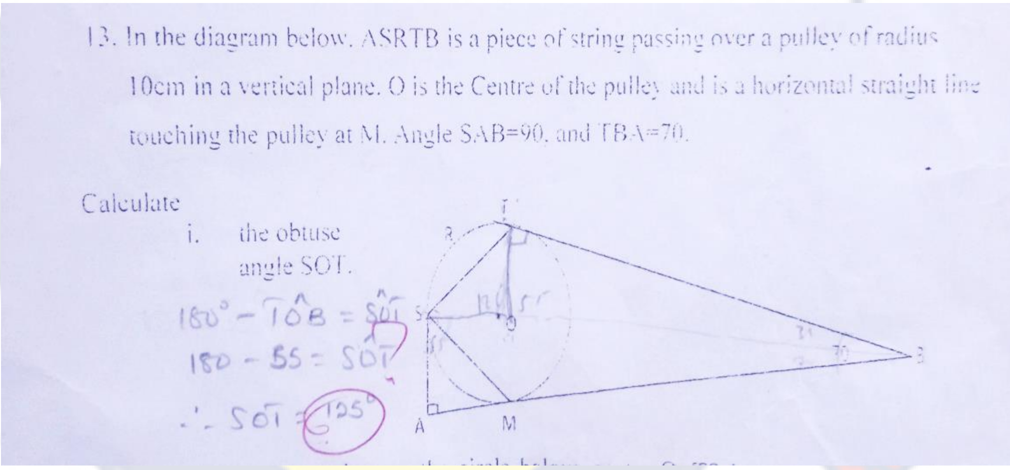

Figure 55: An incorrect student response for GAT question 13

Here, the respondent was not able to deduce the properties that can be used to solve for angle SOT.

The question 14 requested the respondent to use the alternate segment theorem to proof that angle ROS = 2x. The number of student respondents who were able to answer it correctly were 39 (13.93%). Six respondents scored one and the remaining 235 (83.93%) scored zero. Therefore, only 13.93% of the respondents passed.

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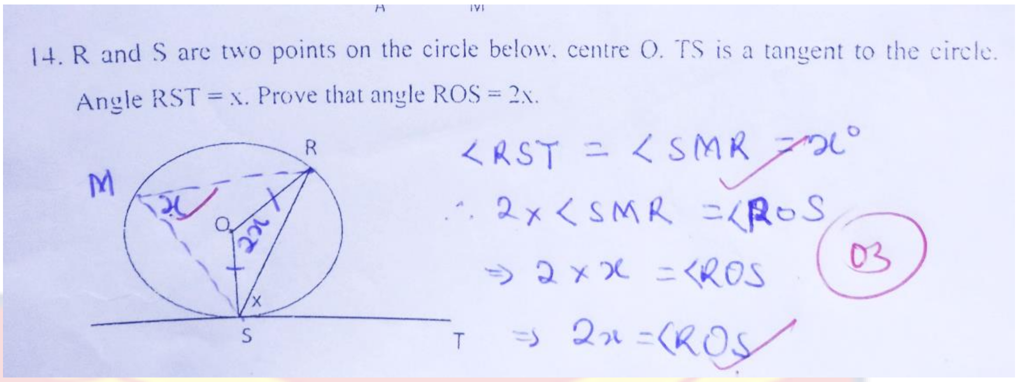

Figure 56: A correct student response for GAT question 14

A sample of an incorrect student response is shown in Figure 57. The respondent tried to use the tangent is perpendicular to the radius at the point of contact theorem and isosceles triangle property to answer but the values were mixed up.

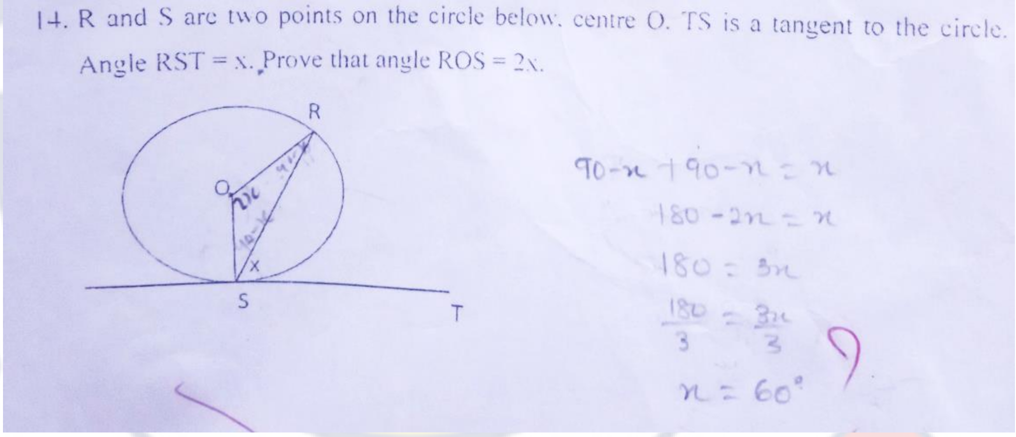

Figure 57: An incorrect student response for GAT question 14

Student performance in the VHGT level 5 (Rigor)

This is the last stage of the levels where the respondent is required to see geometry in the abstract and also be able to analyse and compare theorems. Therefore, a proof question was asked for students to solve. It required the student respondent to understand the need for axioms, theorems, definitions, and proofs to answer.

Table 21 exhibits the performance of the students by score and percentages.

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Table 21: Frequency distribution for students at VHGT level 5

Score	Frequency	Percent	Cumulative Percent
0	257	91.8	91.8
1	7	2.5	94.3
2	2	0.7	95.0
3	2	0.7	95.7
4	12	4.3	100
Total	280	100	

The number of students who could not prove any of the parts of the question was 257 (91.8%), which is low as compared to the only 12 (4.3%) who answered the question correctly. This indicates that the level of reasoning at level 5 is very low. Although some students tried to answer, they could not deduce the proof completely. Thus, 7 (2.5%) could visualise clearly but failed to analyse and only 2 (0.7%) could analyse but could not deduce properly. Therefore, the pass rate at this level was only 5.0%. This implies that only 5% of the respondents could establish and analyse theorems in different postulation systems.

Discussion on Research Question Four

For research question 4, "At what levels of van Hiele's geometric thinking are SHS 2 students operating?" It was discovered that SHS 2 students can operate and demonstrate their geometric content ability up to VHL 3. This implies that, the students have reached the stage in geometric thinking in Circle Theorems where they identify relationships between axioms, theorems, definitions and postulates. They are able to draw conclusions based more on logic than intuition and work with abstract statements concerning geometric qualities. However, they cannot construct geometric proofs and could not find differences between same proofs. They also have low knowledge in

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establishing and analysing theorems. This was reflected in their results, with pass rates of 76.79%, 77.5% and 51.43% for Level 1, Level 2 and Level 3 respectively. At Levels 4 and 5, the performance was below average, and the learners could not pass, as an average of 92.85% failed at these two levels. The percentage pass rate was 10% and 4.3% for Levels 4 and 5 respectively.

The abysmal performance exhibited by the students at Levels 4 and 5 can be attributed to conceptual difficulty in Circle Theorems. A few of the problems the students faced were proof writing, simple deductions and identification of properties of shapes. According to Siyepu (2005), a major goal of geometry curriculum is to teach students how to write proofs, and many high school students find writing proofs in geometry to be one of the most difficult topics. (Hoffer, 1981). Therefore, to achieve this level of geometric thought, more attention must be placed on deductive reasoning and proof finding. This implies that, in the teaching and learning process, all the van Hiele's levels of geometric thinking must be thoroughly considered.

Comparative analysis of the geometric achievement scores at different van Hiele's levels for teachers and students.

A descriptive comparison of the pass marks obtained by both teachers and students after conducting a geometric achievement test are discussed. Table 22 describes the means, pass rate in percentages and the standard deviations at each van Hiele level for the SHS mathematics teachers and their students.

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Table 22: Descriptive statistics of the teachers' and students' van Hiele's level geometric achievement test

		Teachers			Students		
Level	Name	Pass rate (%)	Mean	S. D	Pass rate (%)	Mean	S. D
Level 1	Visualization	81	5.2	1.2	76.79	4.7	1.8
Level 2	Analysis	85	5.1	1.9	77.5	4.5	1.9
Level 3	Abstraction	73	8.0	4.2	51.43	6.3	4.6
Level 4	Deduction	47	5.6	4.3	10	2.4	3.2
Level 5	Rigor	27	1.0	1.6	4.3	0.2	0.9

Table 22 shows that the mean scores obtained by the teachers were higher than those of the students at all the different van Hiele's levels of geometric thought. Which implies that the teachers' performance was higher than that of the students. They both performed admirably, with percentage passes of 81% and 76.79% at Level 1 and 85% and 77.5% at Level 2 respectively. This indicates that both teachers and students have reached the visualization and analysis levels of van Hiele geometric thinking and can thus describe shapes based on their appearance as well as their properties.

At level 3, the mean score of the students of 6.3 and a percentage pass of 51.43% was low as compared to that of the teachers mean score of 8.0 and a percentage pass of 73%. They both passed at this level, but their performance was not very encouraging as compared to that of levels 1 and 2. The outputs showed that more than 50% of the respondents, both teachers and students, have achieved the abstraction level, where they can determine the relationships between shapes and forms and have knowledge of axioms.

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However, the performance of both respondents was below average at Levels 4 and 5. The mean scores of 5.6 and 1.0 for teachers and 2.4 and 0.2 for students were recorded for Levels 4 and 5 respectively. The pass rates for teachers at Levels 4 and 5 were 47% and 27% and the pass rate for students were 10% and 4.3% at Levels 4 and 5. This is an indication that most of the respondents have difficulty making logical deductions and reasoning as they only exhibit minimal knowledge in postulates and theorems. They both performed poorly at level 5, showing that most of the respondents, both teachers and students, have not achieved the level of deducing formal geometric proofs of theorems and axioms.

The pattern of results obtained has a direct bearing on Ball's (1990) assertion on the interest in subject matter knowledge that the teacher's intellectual resources have a significant impact on student learning and the teacher's lack of essential knowledge for teaching mathematics has an impact on the results of the student's output. Clements and Battista (1992) opined that students fail to learn basic geometric problem-solving concepts and are underprepared for more advanced geometric concepts and proofs. Burger & Shaughnessy (1985) also stated that their knowledge of geometric shapes is unimpressive despite the effort to teach them. The output of the teachers' performance revealed that most of the mathematics teachers have very minimal knowledge of advanced geometric concepts and proof. Hence, reflecting on the performance and attitude of the students. Similar to how Van Hiele (1986) claims that many teachers fail to foster their students' conceptual knowledge of the subject because they are unable to align their instruction to their learners' level of thinking in Circle Theorems. Therefore, teachers' lack

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of appropriate pedagogical content knowledge in the subject area such as circle theorem has a direct linkage with the learning difficulty that the students experience in class (van Hiele, 1986; Shulman, 1987; Mji&Makgato, 2006).

It can therefore be deduced that, the performance of the students at van Hiele's Levels 4 and 5 would have been better if the performance of the teacher respondents at the van Hiele's Levels 4 and 5 was better. This is because a higher performance from Level 1 to Level 3 of the teachers showed a higher performance too by the students. A low performance in Level 4 and 5 in the teachers results also resulted in a low students' performance.

Research Hypothesis

There is no statistically significant difference between the mathematics teachers' geometric content knowledge and their students' geometric content knowledge.

An independent-samples t-test was performed by applying the GAT scores obtained from the SHS students and those of the teachers from the selected SHS in order to evaluate the significance of the difference between the teachers' and students' geometric subject knowledge in the Circle Theorems. The two sets of continuous data obtained from both the teacher and the student GAT were analysed. The purpose was to compare the mean scores of these two groups in order to determine if there is any statistical evidence that the two means are significantly different.

To determine the significance, the data of the teacher and the student total scores from the test were checked to ensure that the data can actually be analysed using the independent-samples t-test by passing the assumptions required for a t-test to give a valid result. These assumptions are: test for

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significant outliers, test of normality, variables must be continuous, variables must consist of two independent groups, there must be different participants in these two groups, and test of homogeneity of variances (Pallant, 2005). These assumptions were met since the variables involved in the analysis were continuous, they were obtained from independent groups and the observations were from independent and different participants. The other tests are further explained.

Test of normality

A normality test was conducted to check for the normality of the teacher-student GAT scores. Table 23 shows the Shapiro–Wilk normality test results.

Table 23: Normality test on Teacher to Student GAT scores

Shapiro–Wilk test

	Statistics	Df	Sig.	Skewness	Kurtosis
Teacher GAT scores	.979	104	.095	-.173	-.561
Student GAT scores	.991	280	.091	-.021	-.284

Test is significant at alpha level of 0.05

At an alpha level of 0.05, the sig. value recorded for teacher GAT was .095 and the sig. value recorded for student GAT was .091. This indicates that the data were normally distributed for both test scores. Table 23 also shows that the skewness of the two GAT data was between -0.5 and 0.5, and that both data were negatively kurtosis but significantly normal (-1.96 to 1.96). Figures 58 and 59 show the description of histograms of both teacher and student variables, which shows the collected data was approximately normal.

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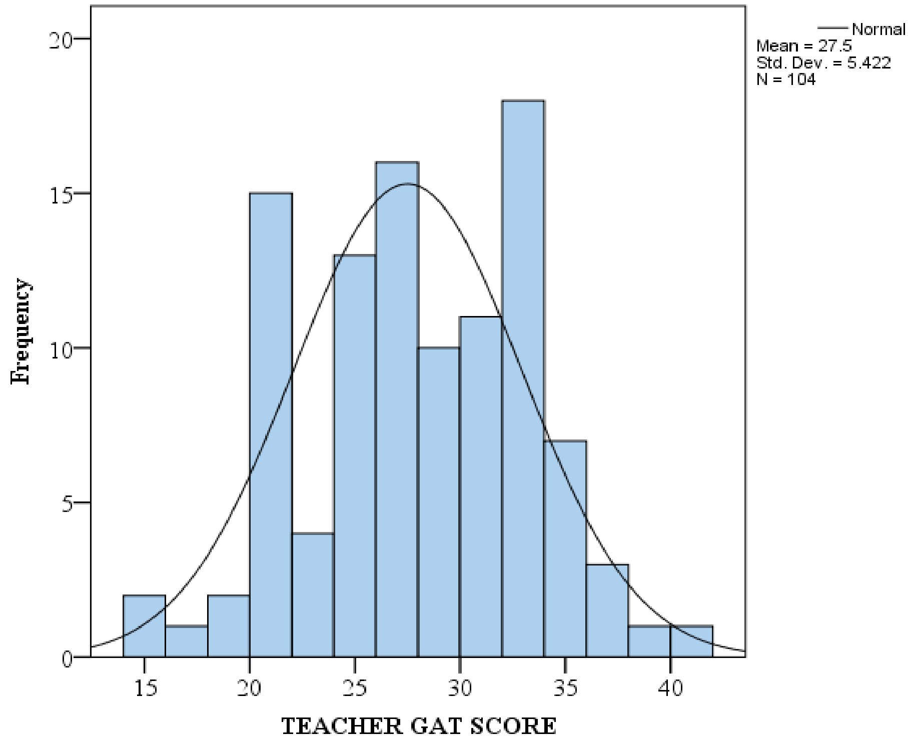

Figure 58: Histogram showing the normal distribution curve of Teacher GAT

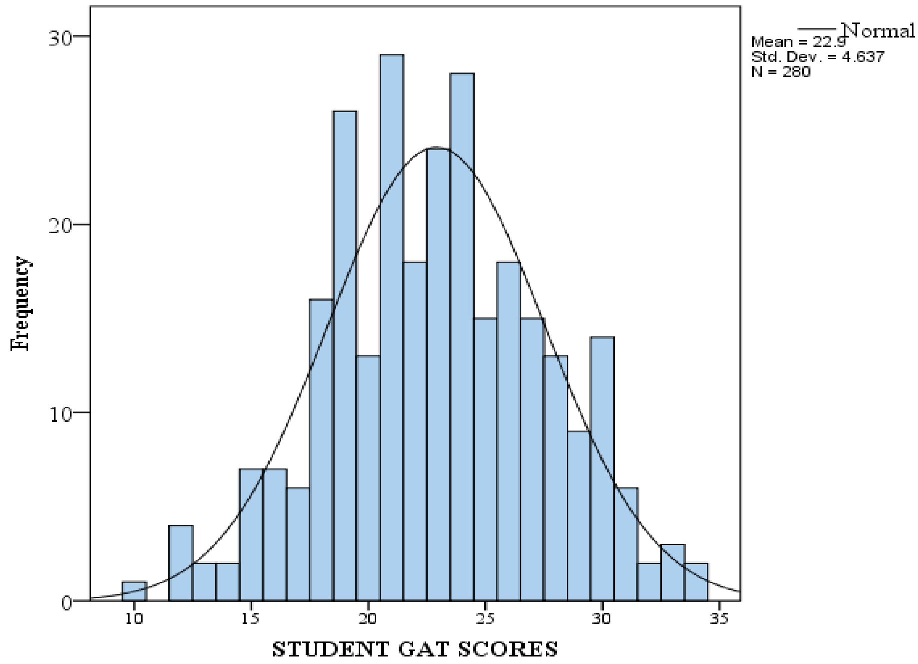

Figure 59: Histogram showing the normal distribution curve of student GAT

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Outliers

The data collected was statistically normal and there were no outliers in both test scores. These can be identified in the box plot in Figure 60 and Figure 61.

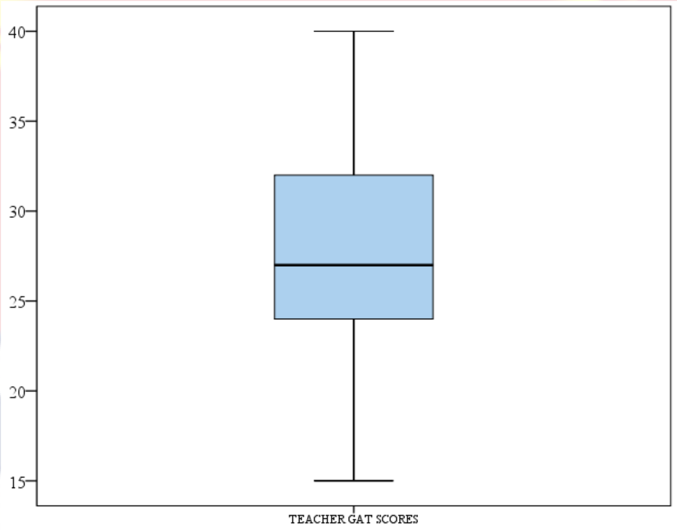

Figure 60: A box plot showing the normal distribution of Teacher GAT scores

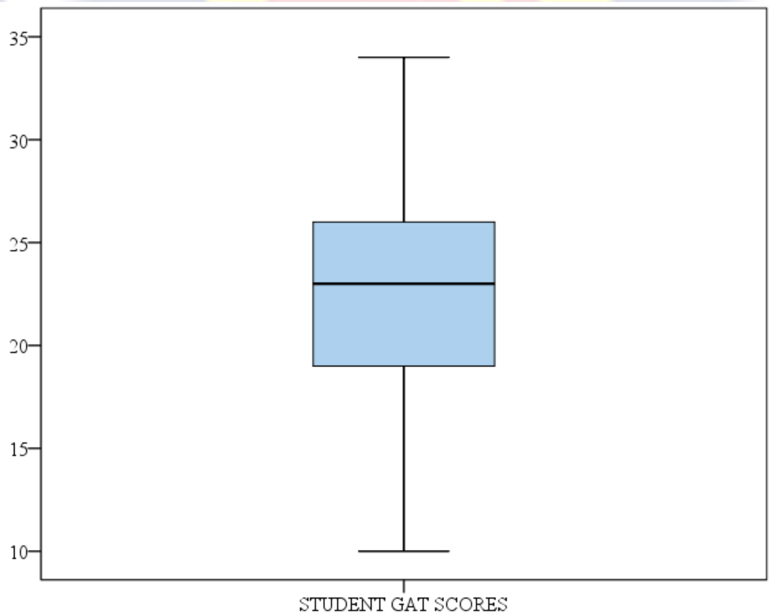

Figure 61: A box plot showing the normal distribution of Student GAT scores

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Test of homogeneity of variance

This assumption is used to determine if samples obtained from populations are of equal variance. This implies that the variability of the obtained scores for each group is similar (Pallant, 2005, p. 198). To test for homogeneity of variance, Levene's test for equality of variance was performed. The output of the analysis is presented in Table 24.

Table 24: Levene's test for equality of variance

	F	Sig.	T	Df	Sig (2 – tailed)
Equal variances not assumed	5.860	.016	7.672	162.148	.000

The output of the SPSS in the test of homogeneity of variance in Table 24 showed that, the mean sig is 0.016 < 0.05. This implies that, the variances of the teacher GAT scores and student GAT scores are homogeneous. Therefore, the two mean scores of the teachers test and that of the students are significantly different. The F value recorded was 5.860.

T - Test analysis of both teachers' and students' total GAT score

An independent-samples t-test is used when you want to compare the mean scores of two different groups of people or conditions (Pallant, 2005). This was conducted to identify the significant difference in the means of the test scores obtained by these two groups.

Table 25 shows the results obtained from the independent samples t–test conducted for both the Teacher and Student GAT scores.

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Table 25: Output of the Independent Samples t-test for equality of means

Groups	N	Mean	Std. D	T	Df	Sig.	Eta squared
Teachers	104	27.50	5.422	7.672	162.148	.000	0.136
Students	280	22.90	4.637				

The mean scores obtained from the two test scores were 27.50 and 22.90 respectively for both teachers and students. The mean scores revealed that the teacher's performance was greater than that of the learners. The independent samples t-test conducted to compare the teacher test scores (M = 27.5, SD = 5.42) and the student test scores (M = 22.9, SD = 4.64) showed that there is a significant difference between these two scores as the p-value [sig (2-tailed)] was 0.00 and t = 7.672. The recorded mean difference had a magnitude of 0.136 which is approximately 0.14. The guidelines for interpreting eta squared values proposed by Cohen (1988) indicated that, a value of 0.14 and above indicates 'large effect' size. This implies that the effect size was large, with a substantial difference between the GAT scores of teachers and that of students.

There is a statistically significant difference between the geometric content knowledge of mathematics teachers and their students, according to the results of the independent-sampled t-test on the outcomes of the teacher-student geometry achievement test. With a large effect size of 0.136, the test results revealed a substantial difference between the mean scores of 27.5 and 22.9 for teachers and students, respectively.

Discussion on Research Hypothesis

Hill, Rowan and Ball (2015) researched on the "effect of teachers' mathematical knowledge for teaching on students' mathematics achievement,"

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using a sample of 1190 and 1773 grade one and three students and 334 and 365 grade one and three teachers respectively. They realised that the teachers' mathematical knowledge had a direct effect on the first- and third-grade students' results, which was in agreement with the findings of the educational production literature. They concluded that the positive effect of teacher content knowledge on students' achievement implies that teacher content knowledge plays a major role even in the teaching and learning of every elementary mathematics content. This research had a true reflection on Hill, Rowan and Ball's summary in the comparative analysis of the teacher and student pass rate at each van Hiele level: the percentage rate of teacher performance appears to influence that of the students. Thus, just as the performance of the teachers declined from van Hiele level 1 to 5, so did that of the students.

According to Mifetu and Amegbor (2019) one of the contributing factors why students have difficulties in answering questions in Circle Theorems was the teacher knowledge and teaching. Due to the extensive geometrical knowledge needed to understand the subject, Luneta (2015) found that teachers find it challenging to teach geometry. The Chief Examiner's report revealed that one of the students' weaknesses was 'difficulty in solving problems involving geometry such as cyclic quadrilaterals, tangent and chord theorem' (WAEC, 2017; 2018; 2019). All these notifications point to the teacher content knowledge. This implies that if the teacher has total control of the content knowledge as stated by WAEC, (2016) that "tuition relating to Circle Theorems should be thorough" it will surely have positive effect on the student content knowledge. Again, teaching students using the van Hiele's

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level of geometric thinking will guide the teacher to assess the student knowledge thoroughly. This research divulged that the teacher content knowledge in Circle Theorems was good but teachers lacked some knowledge in deductions and proofs.

Chapter Summary

This research sought to find the van Hiele assessments of SHS teachers and their students geometric content knowledge in Circle Theorems. The output from both teachers and students GAT was analysed. The findings from the study showed that 93 out of 104 teacher respondents, representing 89.42%, had good content knowledge in Circle Theorems, and the remaining 10.58% did not have good content knowledge in Circle Theorems. Although the pass mark was good, the findings further showed that most of the scores obtained were from questions in levels 1 to 3. This also reflected in the student achievement test, as most of the students were able to answer questions up to the VHL 3. The content knowledge of the students was below average as only 135 (48.21%) passed out of 280 student participants.

The analysis revealed that teachers and the students could not perform well in the higher knowledge achievement questions in levels 4 and 5. This implies that their understanding was at or near Van Hiele level 3. However, the independent sample t - test analysis conducted using the teacher and the student geometric achievement test scores proved that there is a significant difference between the teacher and the student performance as the p-value [sig (2-tailed)] was 0.00 and the effect size was largely significant with eta squared value of 0.136 which is approximately 0.14.

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CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

The research results and relevant recommendations regarding the van Hiele levels of geometric thinking and the content knowledge of Senior high school teachers and their pupils are summarized in this chapter.

Summary

This study was designed to evaluate the content knowledge of SHS core mathematics teachers regarding Circle Theorems and their van Hiele levels of geometric thinking, as well as the content knowledge of SHS students regarding Circle Theorems and their van Hiele levels of geometric thinking. To ascertain whether there is a significant difference between these two groups, the statistical analysis of the two test results from the teacher and student GAT was conducted. A cross-sectional descriptive survey was utilized to get the data. One hundred and four teachers and 280 SHS students were chosen for the study using the purposeful selection methodology, a non-probability sampling strategy. A geometry achievement test (GAT) for teachers and students was given to the respondents to complete as the primary data collection tool for the study.

The quantitative data gathered was examined using van Hiele's theory of geometric thinking. The data were analysed using both descriptive and inferential statistical methods. The analysis of research questions 1 through 4 included frequencies, percentages, averages, and standard deviations. The hypothesis was assessed using the independent-samples t-test.

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Findings

From the study, the findings identified are as follows:

1. The teachers exhibited high content knowledge in Circle Theorems as 89.42% of the respondents passed. Most of the teachers had average to above average content knowledge in Circle Theorems.
2. On the senior high school mathematics teachers van Hiele's levels, it was identified that, though the percentage pass of the teachers was very high, most of the teacher respondents had acquired much content knowledge up to level 3 of van Hiele geometric thinking. Thus, most teachers have not attained the deduction and the rigor level of the van Hiele geometric thinking level. The teacher respondent either had very low marks or could not obtain any mark for level 4 and level 5 of the Van Hiele Geometric Level.
3. It was ascertained that the senior high school students' content knowledge in geometry was at van Hiele level 3 of geometric thinking. They had difficulty in making logical deductions and therefore have a low level of achieving formal geometric proofs.
4. It was also deduced that most of the senior high school students exhibited below average content knowledge in Circle Theorems as only 135 out of 280 representing 48.21% passed the geometry assessment test for students.
5. It was established that, there was a statistically significant difference between the test scores of the teachers as compared to that of the students and the effect size was large.

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Conclusions

This research showed that the core mathematics teachers in this study area do not have absolute control over all the aspects of geometry they teach. It revealed that the core mathematics teachers had circle theorems content knowledge up to the level 3 stage, as described by the van Hiele geometric thinking levels. It can therefore be deduced that; the teachers lack much knowledge in higher thinking order theorems since they performed poorly at the levels 4 and 5.

The SHS students demonstrated content understanding equivalent to van Hiele's level 3 of geometric thinking. Deduction and geometric proofs-based analysis was a challenge for the majority of respondents. Inability to deduce and respond to Circle Theorems is one of the reasons SHS students scored poorly in the WASSSCE. Most frequently, they avoid circle theorem questions, and the few who attempt them show only a lack of understanding in the subject area.

With a large significant effect size, the t-test showed that there was a statistically significant difference between the test mean scores attained by the teachers and the students. This means that, in comparison to the students, the teachers' content understanding was very high.

Recommendations

From the findings of this study, it is recommended that;

1. Teachers should upgrade their knowledge in Circle Theorems by organizing peer teaching, reflecting on their teaching practices as well as experiment with geometry technological tools. This will enhance their teaching skills, strategies and their levels of understanding in the

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teaching of Circle Theorems. Again, core mathematics teachers must pay more attention to deductive reasoning and geometric proofs in Circle Theorems.

2. Circle Theorem lessons must be more practical and activity based by using geometric tools to help students to understand logical deductions and formal geometric proofs to make meaningful logical conclusions. Lesson planning and lesson delivery processes must be based on the van Hiele levels.
3. Student assessment in Circle Theorems must be organised making use of the van Hiele's levels. The assessment scheme will serve as a guide to ascertain the learning progress of the student and inform remediation where necessary.

Suggestions for Further Studies

This study considered the teacher and student knowledge in Circle Theorems at all the various van Hiele levels of geometric thinking. Further studies could be conducted on how to appraise student van Hiele levels of geometric thinking in the geometry classroom.

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APPENDICES

APPENDIX A

GEOMETRIC ACHIEVEMENT TEST FOR TEACHERS

SCHOOL…………………………………………………………………….

SECTION A: DEMOGRAPHIC CHARACTERISTICS

1. Sex:

   Male [ ]

   Female [ ]

2. Age (years)

   20 - 30 [ ]

   31 – 40 [ ]

   41 – 50 [ ]

   51 – 60 [ ]

3. Highest academic qualification

   B.Ed [ ]

   B.A [ ]

   B.Sc [ ]

   M.Ed [ ]

   M.A [ ]

   MSC [ ]

   Mphil [ ]

   PhD. [ ]

   Other (please specify) ……………………………………………….

4. Highest Professional Qualification

   Superintendent [ ]

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Senior Superintendent [ ]

Principal Superintendent [ ]

Assistant Director II [ ]

Assistant Director I [ ]

Other (please specify) …………………………………………….

5. Teaching experience

1 – 5 years [ ]

6 – 10 years [ ]

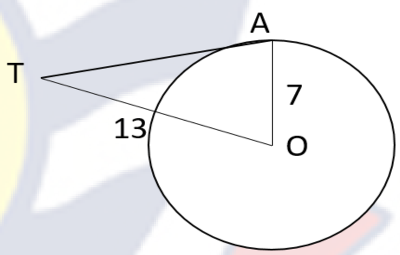
Above 10 years [ ]

SECTION B

INSTRUCTION: *Answer all questions on the question paper provided.*

All calculations must be showed on the question paper.

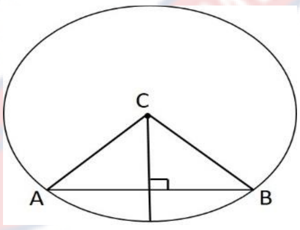
1) In the diagram below AT is a tangent to the circle with Centre O. What is the specific name given to;



c) triangle AOT?

d) angle TAO?

2) In the diagram below, C is the centre of the circle.



a) Why is line AC equivalent to line BC?

b) What is the specific name given to triangle ABC?

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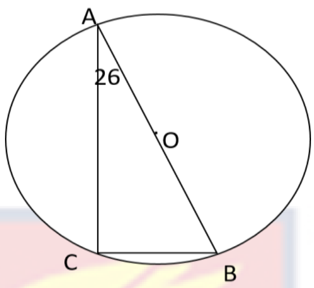
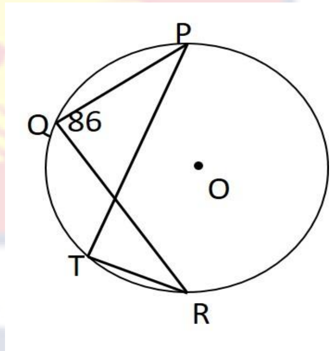
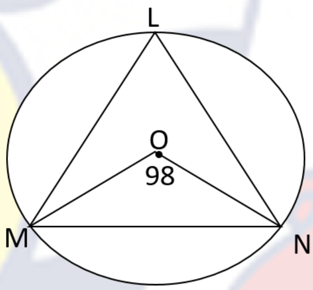
3) AB is a diameter of a circle, centre O. C is a point on the circumference of the circle, such that ∠CAB =26°. What is the size of ∠CBA?



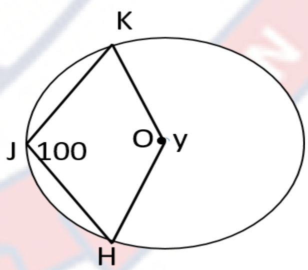
4) In the diagram below O is the centre of the circle and <PQR =86. What is the measure of <PTR?



5) L, M and N are points on the circumference of a circle, centre O. ∠MON = 98°. What is the size of ∠MLN?



6) Given that O is the centre of the circle, determine the value of the angle y.



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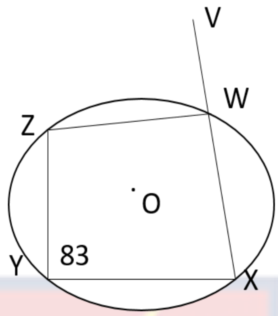
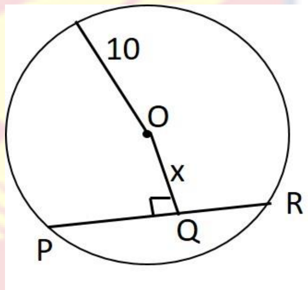
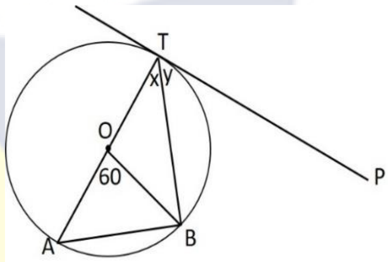
7) WXYZ is a cyclic quadrilateral drawn inside a circle centre O and V is on the line X extended. ∠XYZ = 83°. What is the size of <VWZ?



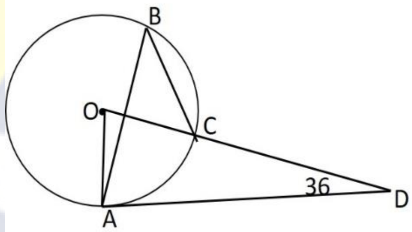
8) In the circle with centre O and radius = 10cm, OQ is perpendicular to PR and PR = 12cm. Determine the value of x.



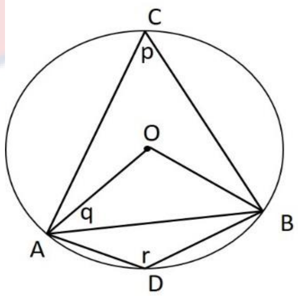
9) In the diagram below, O is the centre of the circle with TP as a tangent and <AOB = 60º. Find the sizes of angles x and y.



10) The diagram below shows a circle centre O. A, B and C are points on the circumference and DA is a tangent to the circle. Angle ADO = 36º. What is the size of angle ABC?



11) In the diagram below O is the centre of the circle, show that p + q = 90º.



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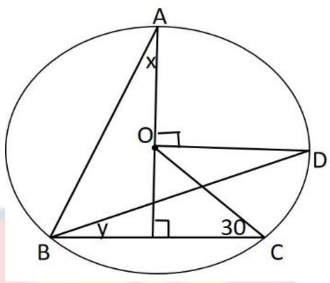
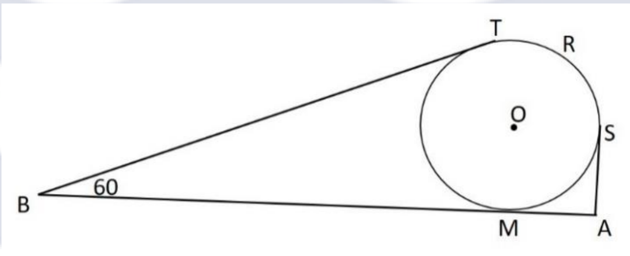
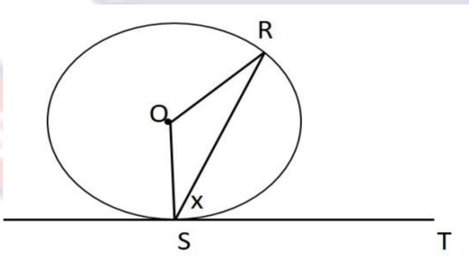
12) In the diagram, O is the centre of circle ABCD and <BCO = 30^o. Find the values of <x and <y.



13) In the diagram below. ASRT is a piece of string passing over a pulley of radius 10cm in a vertical plane. O is the Centre of the pulley and is a horizontal straight line touching the pulley at M. Angle SAB=90^o, and TBA=60^o. Calculate the value of the obtuse angle SOT.



14) R and S are two points on the circumference of a circle with centre O. TS is a tangent to the circle. Angle RST = x. Prove that angle ROS = 2x.



15) Prove the theorem: a line drawn from the centre of a circle perpendicular to a chord bisects the chord.

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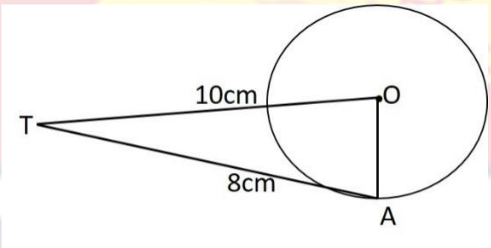
APPENDIX B

GEOMETRIC ACHIEVEMENT TEST FOR STUDENTS

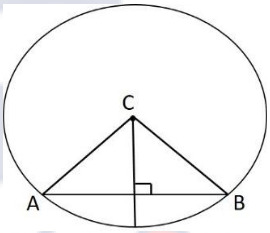
SCHOOL……………………………………………………

INSTRUCTION: answer all questions on the question paper provided.

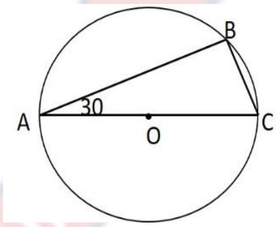
1. In the diagram below line AT is a tangent to the circle with Centre O. Give the special name of triangle AOT and angle OAT.



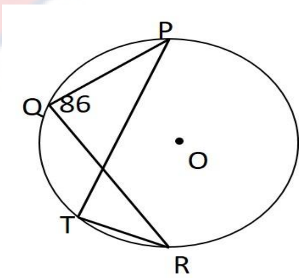
2. In the diagram below C is the centre of the circle.
   a) Why is line AC equivalent to line BC?
   b) What is the specific name of triangle ABC?



3. AB is a diameter of a circle, centre O. C is a point on the circumference of the circle, such that ∠CAB =30°. What is the size of ∠BCA?



4. In the diagram below O is the centre of the circle and <PQR =86. What is the measure of <PTR?



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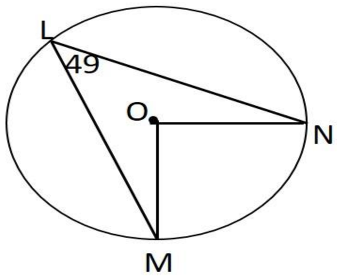
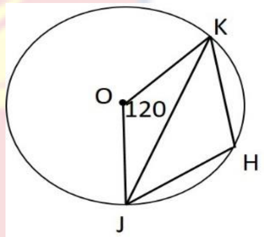
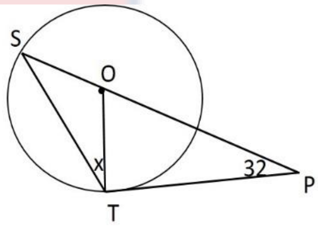
5. L, M and N are points on the circumference of a circle, centre O. ∠MLN = 49°. What is the size of the acute ∠MON?



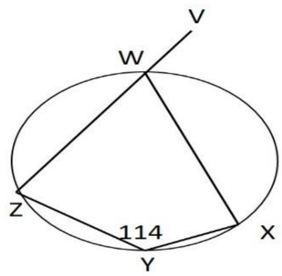
6. Given that O is the centre of the circle, determine the value of the angle KHJ.



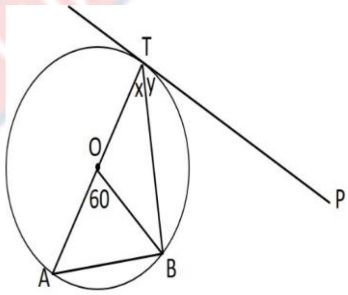
7. The diagram below shows a circle centre O. S and T are points on the circumference and PT is a tangent to the circle. Angle OPT = 32°. What is the size of angle marked x?



8. WXYZ is a cyclic quadrilateral drawn inside a circle centre O and V is on the line XW extended. ∠XYZ = 114°. What is the size of <VWZ?



9. In the diagram below, O is the centre of the circle with TP as a tangent and <AOB = 60°. Find the size of angles x and y.



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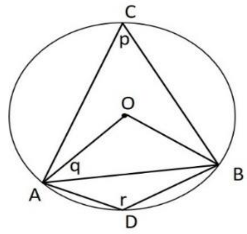
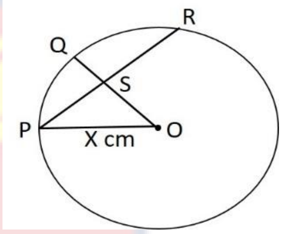
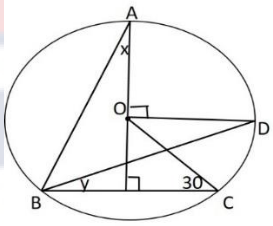
10. In the diagram below O is the centre of the circle, show that r - p =2p.



11. In the circle with centre O, OQ is perpendicular to PR and PR = 12cm. Determine the value of x.

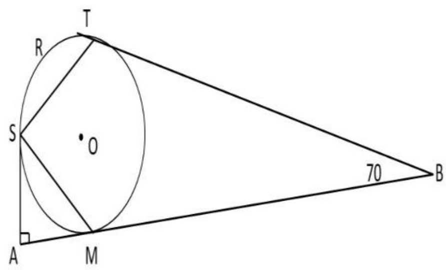


12. In the diagram, O is the centre of circle ABCD and <BCO = 30^o. Find the values of <x and <y.



13. In the diagram below. ASRTB is a piece of string passing over a pulley of radius 10cm in a vertical plane. O is the Centre of the pulley and is a horizontal straight line touching the pulley at M. Angle SAB=90^o, and TBA=70^o.

Calculate the value of the obtuse angle SOT.



14. R and S are two points on the circle below, centre O. TS is a tangent to the circle. Angle RST = x. Prove that angle ROS = 2x.

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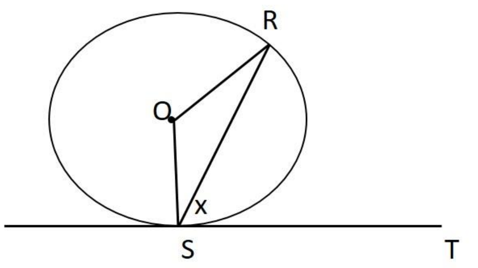

15. Proof the theorem: a line drawn from the centre of a circle perpendicular to a chord, bisect the chord.

The questions were adapted from Dongwi (2012)

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APPENDIX C

INFORMED CONSENT FORM FOR MATHEMATICS TEACHERS

PART I: INFORMATION SHEET

Title: Senior High School Mathematics Teachers and student's Content Knowledge in Circle Theorems and Their Van Hiele Levels Of Geometry Thinking.

Principal Investigator: Frederick Quarshie

Address: P. O. Box 27, Tarkwa – Western Region.

General Information about Research

Frederick Quarshie, is a Master of Philosophy (MPhil) student in Mathematics Education at the University of Cape Coast. He is conducting a study on "Senior High School Mathematics Teachers and students' Content Knowledge in Circle Theorems and Their Van Hiele Levels of Geometry Thinking". The study involves research which seeks to collect data in order to assess the content knowledge of the mathematics teachers and their students in circle geometry at the SHS and their van Hiele levels. You are invited as a potential participant because the researcher believe that your participation will contribute to the success of this research. Your participation will take a day if you accept to participate in the study.

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Procedures

Please to find answers to some of these questions, I invite you to take part in this research project. If you accept, you will be required to answer an achievement test which will be provided by researcher (Frederick Quarshie) and will be collected by him. The test will last for 120 minutes. If you do not wish to answer any of the questions in the test, you may skip them and move on to the next question. Each participant is entitled to answering only one achievement test and no one else except I the researcher will be present. The information received is considered confidential, and no one else except I and my supervisor will have access to the information documented.

Possible Risks and Discomforts

This study is not designed to change your teaching and learning style, but to assess the content knowledge in that area of study.

Possible Benefits

The study will help improve researcher's academic skills in academic writing as well as helping the researcher in partial fulfilment of the award of Master of Philosophy (MPhil) degree at the University of Cape Coast. The study will also help to inform stakeholders if the SHS mathematics teachers have in-depth knowledge in Circle Theorems.

Confidentiality

I will protect information about you to the best of my ability. You will not be named in any reports and that the information you will give will be used solely for study purpose but not any other issues. The information provided by you will not be made known to other people for any reason except my research supervisor and to ensure secrecy of your identity and the information

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you will give; names will not be attached to the information you will provide in the reporting of this study but rather codes.

Compensation: There will be no compensation, payment, or reimbursement for participating in this study but the head of departments of the participated schools will be given recharge cards to make calls to coordinate their teachers.

Voluntary Participation and Right to Leave the Research

Participation is voluntary and that you can withdraw without any penalty. If you choose not to participate, it will not affect your current or future relations with your administration, teachers, the researcher, or the University of Cape Coast. There is no penalty for not participating or discontinuing your participation.

Contacts for Additional Information

At any time, if you have any concerns or questions, you may contact Mr. Frederick Quarshie (the researcher) at 0244141644/0278823791 or k.quarshiefred@gmail.com or my thesis adviser Dr. F. D. Ntow at 0507352711, University of Cape Cost or fntow@ucc.edu.gh

Your rights as a Participant

This research has been reviewed and approved by the Institutional Review Board of University of Cape Coast (UCCIRB). If you have any questions about your rights as a research participant you can contact the Administrator at the IRB Office between the hours of 8:00 am and 4:30 p.m. through the phone lines0558093143/0508878309 or email address: irb@ucc.edu.gh.

PART II: VOLUNTEER'S AGREEMENT

I have read the above document describing the benefits, risks and procedures for the research titled "Senior High School Mathematics Teachers and

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students' Content Knowledge in Circle Theorems and Their Van Hiele Levels of Geometry Thinking" and have also been given an opportunity to ask any question about the research and this has been answered to my satisfaction. I also understand that I reserve the right to change my mind and withdraw at any time without giving a reason and without cost. I therefore, agree to participate as a volunteer in the study.

Volunteer's Name: ……………………………………...

Volunteer's Mark/Thumbprint ……………………….

Date: **……………………………………………………**

NOBIS

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APPENDIX D

INFORMED CONSENT FORM FOR STUDENTS

PART I: INFORMATION SHEET

Title: Senior High School Mathematics Teachers and Students' Content Knowledge In Circle Theorems and Their Van Hiele Levels Of Geometry Thinking.

Principal Investigator: Frederick Quarshie

Address: P. O. Box 27, Tarkwa – Western Region.

General Information about Research

Frederick Quarshie, is a Master of Philosophy (MPhil) student in Mathematics Education at the University of Cape Coast. He is conducting a study on "Senior High School Mathematics Teachers and Students' Content Knowledge in Circle Theorems and Their Van Hiele Levels of Geometry Thinking". The study involves research which seeks to collect data in order to assess the content knowledge of the mathematics teachers and their students in circle geometry at the SHS and their van Hiele levels. You are invited as a potential participant because the researcher believe that your participation will contribute to the success of this research. Your participation will take a day if you accept to participate in the study.

Procedures

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and will be collected by him. The test will last for 120 minutes. If you do not wish to answer any of the questions in the test, you may skip them and move on to the next question. Each participant is entitled to answering only one achievement test and no one else except I the researcher will be present. The information received is considered confidential, and no one else except I and my supervisor will have access to the information documented.

Possible Risks and Discomforts

There are no direct benefits of participating in the study. This study is designed to assess the content knowledge of students in that area of study. This will not affect your academic performance in class and in the foreseeable future.

Possible Benefits

The study will help improve researcher's academic skills in academic writing as well as helping the researcher in partial fulfilment of the award of Master of Philosophy (MPhil) degree at the University of Cape Coast. The study will also help to inform stakeholders if the SHS mathematics teachers have in-depth knowledge in circle theorem.

Confidentiality

I will protect information about you to the best of my ability. You will not be named in any reports and that the information you will give will be used solely for study purpose but not any other issues. The information provided by you will not be made known to other people for any reason except my research supervisor and or the University of Cape Coast. to ensure secrecy of your identity and the information you will give, names and identifiers will not be attached to the information you will provide in the reporting of this study.

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Compensation: No compensation, payment, or reimbursement for participating in this study.

Voluntary Participation and Right to Leave the Research

Participation is voluntary and that you can withdraw without any penalty. If you choose not to participate, it will not affect your current or future relations with your performance, the school, teachers, the researcher, or the University of Cape Coast. There is no penalty for not participating or discontinuing your participation.

Contacts for Additional Information

At any time, if you have any concerns or questions, you may contact Mr. Frederick Quarshie (the researcher) at 0244141644/0278823791 or k.quarshiefred@gmail.com or my thesis adviser Dr. F. D. Ntow at 0507352711, University of Cape Cost or fntow@ucc.edu.gh

Your rights as a Participant

This research has been reviewed and approved by the Institutional Review Board of University of Cape Coast (UCCIRB). If you have any questions about your rights as a research participant you can contact the Administrator at the IRB Office between the hours of 8:00 am and 4:30 p.m. through the phone lines0558093143/0508878309 or email address: irb@ucc.edu.gh.

PART II: VOLUNTEER'S AGREEMENT

I have read the above document describing the benefits, risks and procedures for the research titled "Senior High School Mathematics Teachers' Content Knowledge in Circle Theorems and Their Van Hiele Levels of Geometry Thinking" and have also been given an opportunity to ask any question about the research and this has been answered to my satisfaction. I also understand

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that I reserve the right to change my mind and withdraw at any time without giving a reason and without cost. I therefore, agree to participate as a volunteer in the study.

Volunteer's Name: ……………………………………...

Volunteer's Mark/Thumbprint ……………………….

Date: **……………………………………………………**

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