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UNIVERSITY OF CAPE COAST

FLEXIBLE BAYESIAN METHODS FOR INFLATION MODELLING IN GHANA

BY

MOHAMMED FREMPONG

Thesis submitted to the Department of Statistics of the School of Physical Sciences, College of Agriculture and Natural Sciences, University of Cape Coast, in partial fulfilment of the requirements for the award of Master of Philosophy degree in Statistics.

MAY 2023

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DECLARATION

Candidate's Declaration

I hereby declare that this is the result of my own original work and that no part of it has been presented for another degree in this university or elsewhere.

Candidate's Signature: ……………………… Date: ………………

Name: Mohammed Frempong

Supervisors' Declaration

We hereby declare that the preparation and presentation of the thesis were supervised in accordance with the guidelines on supervision of thesis laid down by the University of Cape Coast.

Principal Supervisor's Signature ……………………… Date………………

Name: Dr. David Kwamena Mensah

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ABSTRACT

Inflation data may exhibit structural instability that might have been engineered by informative macroeconomic variables. Failure to include this relevant economic information in the statistical modelling procedure may produce disingenuous information leading to wrong conclusion. In view of this, this thesis proposed Bayesian-Gaussian process regression, GPR methods based on compound covariance function for modelling inflation in Ghana. The approach model inflation as a mean zero Gaussian process in terms of the observation time with a compound covariance function designed to account for the short-, medium-, and long-term structural characteristics of the inflation process. Macroeconomic variables that drive inflation are incorporated into the model via the covariance using moment-based statistics as alternative macroeconomic predictors. The moment-based macroeconomic predictors serve as transformed predictors and were built based on the existing interrelationships among the variables such that they allow automatic control of the interrelationships, autocorrelation and outliers. This allows Bayesian GPR to be applied to macroeconomic data in which there exist interrelationships. MCMC inference methods were built for the developed GPR models and experimented using real macroeconomic data from Bank of Ghana (BoG) website. Results show that Bayesian GPR models with moment-based macroeconomic predictors outperform their original data predictors in fitting inflation on food and non- food data.

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KEY WORDS

Bayesian Inference

Compound Covariance Functions

Gaussian Process Regression

Inflation

Markov Chain Monte Carlo Method

Time series modelling

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DEDICATION

To my late father, Adam Kofi Antwi, mother, Akua Halima, all my siblings; my wife, Jannat Bint Zakariyya and two daughters.

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LIST OFABBREVIATIONS

AfDB	African Development Bank
AIC	Akaike Information Criteria
AICc	Corrected Akaike Information Criteria
AR	Autoregressive
ARCH	Autoregressive Conditional Heteroscedasticity
ARFIMA	Auto Regressive Fractionally Integrated Moving Average
ARIMA	Autoregressive Integrated Moving Averages
ARMA	Autoregressive Moving Averages
BIC	Bayesian Information Criteria
BMA	Bayesian Model Averaging
BoG	Bank of Ghana
BVAR	Bayesian Vector Auto Regressive
CPI	Consumer Price Index
CUSUM	Cumulative Sum Control Chart
EB	Empirical Bayes
EGARCH	Exponential Generalized Autoregressive Conditional Heteroscedasticity
EGX20	An equity index for the Egyptian stock market that measures performance according to market capitalization and liquidity of the 20 most active businesses.
EGX30	Is an equity index on Egyptian stock market of most actively traded 30 listed stocks

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EGX70	Price index of the next 70 active companies of the Egyptian stock market, after the first 30 most active companies
EGX100	Price index on Egyptian stock market that tracks performance of 100 active companies, including EGX 30 and EGX 70
EM	Expectation-Maximization
ES	Exponential Smoothing
FARIMA	Fractional Autoregressive Integrated Moving Average
FIMA	Fractionally Integrated Moving Average
FMA	Frequentist Model Averaging
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
GDP	Gross Domestic Product
GP	Gaussian Process
GSS	Ghana Statistical Service
IMF	International Monetary Fund
IT	Inflation Targeting
LSTAR	Logistic Smooth Threshold Autoregressive
MA	Moving Averages
MCMC	Markov Chain Monte Carlo
MH	Metropolis-Hastings Algorithm
OECD	Organization for Economic Cooperation and Development
PPI	Producer Price Index
QLR	Quandt Likelihood Ratio
RBF	Radial Basis Function

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RMSE	Root Mean Squared Error
SAR	Simple Auto Regressive
SARIMA	Seasonal Autoregressive Integrated Moving Averages
SETAR	Self-Exiting Threshold Autoregressive
SSA	Sub-Saharan Africa
SVAR	Structural Vector Auto Regressive
TVP	Time-Varying Parameter
VAR	Vector Autoregressive
VECM	Vector Error Correction Model

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CHAPTER ONE

INTRODUCTION

This chapter provides a comprehensive exposition on inflation and how it has evolved over years. It seeks to review literature in areas such as background issues pertaining to inflation, statement of problem of the research, objectives, purposes and significance of the study, delimitations and limitations of the study.

Background to the Study

There is rich literature on the evidence supporting the claim that many macroeconomic and financial time series exhibit structural instability (Stock & Watson, 1996). It is likely that this structural instability might have been engineered by intrinsic economic features. These features may be informative about the underlying dynamics of the macroeconomic times series under consideration. For instance, the inflation process, its persistence, and possible subtle changes in the inflation process may purely be due to some important economic features. Inability to incorporate these features in statistical model specification may lead to generating models with low predictive power resulting in misleading conclusions (Hendry & Clement, 2001; Koop & Potter, 1998).

The inflation process and its properties such as dynamics (possible changes) and persistence have attracted much attention in literature recently, due to its importance in topics of mainstream macroeconomics and monetary policy. As a result, various attractive methodologies for modelling inflation processes have been advanced for consideration in research works. Ruud and Whelan (2007) provide a comprehensive assessment of recent research based on the use of

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models with rational expectations and sticky-prices. Conventional approaches to modelling inflation are based on standard time series models. Cogley and Sargent (2002), and Primiceri (2005), proposed a time varying parameter estimation model in the context of multivariate time series. They observed that inflation persistence exhibits some dynamic behaviour, showing an increased trend in the early 1970s, remained high for about 10 years and then declined. Stock (2001) considered univariate methods to investigate inflation dynamics. His approach leads to the realization that inflation persistence showed high and constant trend over the past 40 years. All these illustrate the ambiguity associated with empirical evidence about the properties of inflation persistence in literature.

Though these approaches are appealing, they are limited in terms of incorporation of inflation specific covariates which may be informative about the dynamics of the inflation process as well as its properties. In addition, how these features are related may also be of interest and the question is how to capture such information in the model specification. An attractive approach that addresses the above limitation is the Bayesian method. The Bayesian formalism provides a principled approach to incorporation of relevant covariate information. In the Bayesian framework, Jochmann (2015) developed a sticky infinite hidden Markov model to study U.S inflation dynamics. An infinite hidden Markov model is a nonparametric Bayesian model that extends the usual hidden Markov model. A nonparametric Bayesian model can simply be viewed as a probability model characterized with infinitely many parameters. This project explored Bayesian

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Gaussian process regression modelling to develop appropriate flexible methods for modelling the inflation process in Ghana.

Macroeconomic indicators especially inflation, gross domestic growth, public deficit and unemployment among others stand central in economic governance and development. Apart from policy makers who utilize them to assess their economies' health, the citizenry also evaluates key managers of the economy and politicians' performance using economic indicators as yardsticks (Mügge, 2016). In general, economic indicators may be described as pieces of information, characteristics or attributes measured on or pertaining to economic data generated from macroeconomic variables, usually from governmental and non-governmental organizations. These data are used to assess and judge the overall strength of an economy thereby making estimates about current and future fortunes of investment opportunities of that economy. Economic indicators as pointed out earlier, is made up of innumerable variables of which indicators such as; stock market, unemployment, price of crude oil, gross domestic product (GDP), and consumer price index (CPI) are not exception. Consumer price index is one of the principal indicators that strongly relates to inflation. Several economic indices of which notable ones such gross domestic product (GDP), producer price index (PPI) and consumer price index (CPI) are used to calculate the rate of inflation. However, the most commonly-used formal measure of inflation in spite of its few draw backs is the CPI.

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Scholars Point of View on Inflation

Economists in general, explain CPI as an economic indicator that measures the change in income required by a consumer to sustain the same standard of living over time. Basically, the CPI calculates the aggregate price level in an economy and thus measures the purchasing power of a country's unit of currency. The weighted average of the prices of goods and services that approximates an individual's consumption patterns is used to compute the CPI.

Judging from the above consideration, it is obvious that inflation is strongly linked to CPI and hence in discussion of the subject matter of inflation, two words "CPI and inflation" are inseparable. Many scholars of economics have expressed several opinions about inflation (Crowther, 1941; Hawtrey, 1950; Kemmerer, 1942) only to mention a few . In spite of the diverse views, one common phenomenon that is fundamental to their explanation of inflation is expressed in the following quotation: "quantity of money in circulation exceeding the total amount of goods and services which leads to an extraordinary increase in prices" (Mandal, Gupta, & Gupta, 2008, p. 412). This phenomenon which gives rise to general level of prices continues largely due to persistent administration of stimulus by incommensurate rising demand against insufficient physical supplies.

Murali (2004) revealed that the word inflation owes its origin to the Latin word inflare, which literally means "to blow into", from flare, "to blow". This description accurately reflects the current understanding of inflation as an unsubstantiated increase in prices that does not reflect in changes in relative scarcity. Researching more on the history, origin and global trend of inflation as

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traced from Taylor (2019), it was discovered that the 20th century produced the worst (high) record of inflation in the history of mankind.

The following events revealed that in the third Century, Roman Rulers caused inflation by debasing their coins; in the 14th Century, Chinese Emperors exchanged coins with paper currency leading to inflation; Europe and other parts of the biosphere were adversely hit by inflation in the 16th Century after dealing in gold and silver; while an upward turn in inflationary values was witnessed during American and French Revolutions. In the 20th Century every single nation of the globe suffered a severe inflation. This account about inflation is further elaborated by (Harl, 1996; Dobson, 2002). They opined that throughout history of mankind, the swiftness with which amount of money or overall supply of money in diverse forms for transactions have occurred in different societies of the world. They further illustrate that, in the ancient days, when gold was used as currency in some countries, governments produced more coins from the alloy derived from a mixture of copper, silver or lead and the gold from exiting gold coins, thereby increasing the stock of coins without incurring extra cost in gold. Governments profited from this strategy as the seigniorage – what the government gets as revenue, which is the difference between the value of the coin as a metal and the value of the coin as a legal tender – increased. This customary way of transaction led to a devaluation of the coins but ensured a growth in the money supply. Thus, one needed to pay extra for the same services and goods, leading to a hike in prices or what is otherwise described as inflation.

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Some authors have expressed varied opinions about inflation. In one dimension, inflation could be inferred as upward growth in prices of goods and services triggered by excess money in circulation and hence inflation caused by increase in money supply. One principal phenomenon underpinning this concept of inflation is that the rate of increase in national income is lower than corresponding supply of money. Economic scholars who hold this view are described as monetarists. Advocates of this view of inflation include but not only; Milton Friedman, Karl Brunner, Anna Schwartz and Phillip Cagan (Monetarism, 2021).

In another breadth, there is another group of intellectuals of economics who hold a contrary view about increase in money supply completely-driven idea about inflation. To them, apart from increase in money supply which leads to inflation, issues such as existence of idle resources, a fall in productivity, increase in population and involvement of many middlemen in the distribution process may result in inflation. In effect, they strongly believe that inflation may arise as a result of improper adjustments of institutional structures of the business community. They are described as structuralist economists. These include proponents such as Raul Prebisch, Celso Furtado and John Maynard Keynes (Celso Furtado, 2021).

Notwithstanding the divergent opinions about these two schools of thought about inflation, many researchers share the thought that inflation is the overall increase in prices or increase in the cost of living in a country, or how much more expensive the relevant set of goods and or services has become over a

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period of generally one year. Based on the foregoing discussion, inflation could be defined as persistent and appreciable increase in the general price levels of goods and services and factors of production over time (Odoom, 2015).

According to Odoom (2015), apart from other forms of inflation, inflation is classified into two main types. These are demand-pull, and cost push inflation. In demand-pull inflation, the rising prices of goods and services are caused by excess demand that has been created in the economy hence “pulling” the prices to increase. Such inflation is further characterized by aggregate demand exceeding aggregate supply leading to volume of purchases incessantly surpassing production or supply of commodity. Consumers in this circumstance have more purchasing power causing aggregate demand without appropriate increase in supply or output. Its causes include but not limited to; incommensurate money supply by government without corresponding increase in productivity, low productivity of goods and services, increase in minimum wage that is unmatched with productivity, high population growth that does not correspond with productivity, embezzlement and corruption, unnecessary expenditure by government, smuggling and hoarding etcetera (Odoom, 2015).

On the contrary, in cost-push inflation, rising prices of goods and services are caused by increase in cost of production. Levels of price generally, shoot up as a result of higher cost of production. Underlying factor accounting for this type of inflation is scarcity in supply of raw materials resulting from high cost of inputs of production. Corporate establishments, and other relevant goods and service providers increase the price of their goods and services to offset their increase

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cost of production in order to keep them operational. Phenomena that mark this kind of inflation are causes such as alternatives, competition, natural disasters, man-made crises, government regulation, wage inflation, monopoly, exchange rate among others (Amadeo, 2020).

Levels of Inflation by International Monetary Fund (IMF)

Having considered two principal classifications of inflation, we briefly discuss three levels of inflation as described by the International Monetary Fund (IMF) in a 2004 report of Reserve Bank of Fiji, three levels of inflation are low-to moderate, galloping, and hyperinflation. In low-to-moderate inflation, the values of goods and services increase gradually over time. In view of this, individuals and corporate entities are motivated and encouraged to enter into long-term agreements, save their money because the value of their investments is not eroded by high inflation (Bank of Fiji., 2004). This is further marked by moderate rise in price and annual rate of inflation of at most 10%.

In galloping inflation, prices increase at double- or triple-digit rates such as 30 % or 100% per annum. As a result, money loses its value at rapid rate. This level of inflation adversely affects the economy by discouraging consumers from savings, reducing their purchasing power because they will have to buy goods and services before prices rise again. Also, individuals and corporate organizations will not be willing to engage in long-term contracts because of unstable and rapid change in prices of goods and services. Investors undertake investment in ventures that will recoup dividend that are above the inflation rate (Bank of Fiji., 2004).

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The next level of inflation is hyperinflation. Hyperinflation generally communicate the incidents when the monthly inflation rate exceeds 50 percent. However, an arbitrary limit is used in its measurement (Salemi, 1976). This is characterised by severer case of increase in prices of goods and services. In this circumstance, price increase occurs at the rate of a 1000%, 1000000% or even 1000000000% per year. The price increase is so rapid that value of goods and services can be rising even during the day. This phenomenon of inflation is catastrophic to economy because it devalues the worth of money in honouring payment of goods, services, and other transactions (Bank of Fiji., 2004). The report further added that consumers engage in extravagant expenditure which ultimately push inflation upwards. In general, one primary cause of this phenomenon is printing of money to finance excessive government deficits. By so doing, more currency is put into circulation which create rapid price rise in response to demand for goods and services. For instance, Germany experienced hyperinflation between 1921 and 1923 during the Weimar Republic, while countries, such as Argentina, Brazil and Peru in a decade (1989-1999) suffered hyperinflation.

Analysis of Inflation from Global Dimension

The next section describes modelling of inflation from international perspective. Gattini et al., (2012), in their analysis of inflation and its propagation channels for five decades (1960-2010) from a global perspective revealed that global money demand shocks affect global inflation and also global commodity prices, which eventually influence inflation; positive house price shocks exert a

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significant influence on inflation. Also, according to Bohl and Siklos, (2018), globally, the record of inflation over the past 50 years indicates considerable diversity in inflation performance around the world with lowest inflation occurring in advanced economies and maximum inflation in the emerging and developing world. This inflation rates are volatile and exhibit sharp movements over time. In connection with high inflation occurring in emerging and developing world of which many African countries are not exception, a confirmation is evidenced in the following quotation: “Three African countries have witnessed hyperinflation in the period 1960-2015, and these are Angola, the Democratic Republic of Congo and Zimbabwe. Other countries have moderately sized inflation levels” (Franses & Janssens, 2017, p. 4). It was further found that “many countries have grappled with high inflation and in some cases hyperinflation, 1,000 percent or more a year. In 2008, Zimbabwe experienced one of the worst cases of hyperinflation ever, with estimated annual inflation at one point of 500 billion percent” (IMF, 2017, p.30) of high inflation, in some situations compelled affected countries to resort to embarking on stringent policy decision to recover from the adverse consequences of inflation. Such decisions at times end up giving up their state currency.

Additionally, a review report on inflation about African continent revealed that inflation dynamics across Sub-Saharan Africa countries (SSA) are mainly driven by domestic supply shocks, although the contribution of these shocks to inflation has substantially declined in contemporary times. As the region becomes more integrated with the global economy, however, the role of global oil and food

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shocks as well as inflation spill overs from other countries have increased (Nguyen et al., 2015). Yaya, Akintande, Ogbonna, and Adegoke (2019), in their analysis of inflation data of some 16 African countries including Ghana, covering a period of 47 years (i.e., almost five decades) revealed that the dynamics of inflation pertaining to these nations are volatile and non-linear and for that matter require pragmatic policy intervention to revert inflation to its original trend. They suggested that careful selection of estimation method is used in computation of inflation, especially in countries where non-linearities were found.

Investigation of Inflation in Ghana

Turning our attention to review on macroeconomics data about Ghana Alagidede, Baah-Boateng, and Nketiah-Amponsah (2013), in giving an overview of the Ghanaian Economy revealed in their research findings that a number of factors, including inflation and poor economic choices made to finance domestic borrowing, led to unfavourable growth that was quite phenomenal during the 1970s and the early 1980s. Additional investigation into Ghana's economic progress by OECD/AfDB(2002)**,** revealed that after 20years of Structural Transformation and Economic Recovery Programme which touted Ghana as one of Africa's giants of economic growth, the basics of macroeconomic variables of the country have weakened. This situation led to decline in fiscal and monetary positions, poor domestic policies and external constraints, high rise in rate of inflation and cost of borrowing, and massive depreciation of the foreign exchange of the local cedi which in all cumulatively translated in real GDP growth of 3.7 per cent in 2000, the lowest level in about a decade (OECD/AfDB, 2002).

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Ghana's performance in terms of inflation after independence has been quite remarkable. While it experienced inflationary rate of 0.09% in 1960, inflation rose to 11% in almost two decades (i.e., in 1979) after 1960. In the next four years (1983), unfortunately, the country experienced one of the severest (123%) inflation. Inflation was estimated at 17% in 1999 and it shot up to 40.8% in 2000. Inflation declined to 11.5% in 2007 (Odoom, 2015). Again, inflation fluctuated to 6.7% in 2010, 11.7% in 2013, 17.5% in 2016. In 2018, inflation rate for Ghana was 9.8%. Though, Ghana's inflation rate fluctuated considerably in recent years, it tended to decrease through 1999-2018 ending at 9.8% in 2018 (O'Neil, 2023). In spite of the vivid picture about the state of Ghana's inflation, Odoom (2015) further averred that some economics experts and other financial or policy institutions hold contrary view about these inflationary statistics. They argue that these statistics may be fabricated figures intended to give the public a bright picture of the economy.

In spite of divergent opinions on rates of inflation in Ghana's economy, the country continues to experience inflation in each blessed year (Odoom, 2015). Price stability is one of the principal aims of every government as it is an important economic indicator that governments, politicians, economists and other players of the economy use as basis of argument when deliberating on state of the economy (Nasiru & Sarpong, 2012a). Judging from the literature surveyed, it is indisputable fact that inflation is a significant economic indicator that ought to be studied with all attention and seriousness it deserves especially in terms of; its causes, effects, and estimation. Ghana's economy being one of the emerging and

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fastest growing (among the top 10 in Africa) in the global environment Adegoke (2018) must be examined. The next section is dedicated to methods that some researchers have used in measuring inflation in Ghana. This is discussed succinctly in statement of the problem which motivated the research work.

Statement of the Problem

Inflation is one of the major economic indicators that has gained a lot of attention and recognition nationally and globally in the management of economy, since its discovery to contemporary world of today. Because of its main role in the management of economy, many countries including Ghana have shown much commitment in the measuring and monitoring of inflation. Such commitment is motivated by the fact that its causes, effects and estimation have both positive and adverse impact on the economy**.** Its adverse effects include but not limited to inflation framework being distorted, exchange rate depreciation, volatility of prices of goods and services, distributional effect of wealth, distortion of economic behaviour, breakdown of function of money, investment and allocation of resources, government finances, foreign trade and expectation etcetera (UCC - ECO 201, 2010). Growing markets, healthy profits, and general climate of business confidence, other factors based on Keynesian principles and theory are some positive effects of inflation (UCC - ECO 201, 2010).

There is a growing literature on inflation in Ghana. In many of such reports, researchers used the traditional time series only to mention a few in modelling the inflation for Ghana (Alnaa & Abdul-Mumuni, 2005; Owusu, 2010). One major defect of this approach is the model's inability to factor in variables

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(features) of uncertainty (e.g., covariates information) related to the macroeconomic variables being studied and consequently may provide misleading conclusions due to poor predictions. This has been realized by some researchers such as: Hendry and Clement (2001); and Koop and Potter (1998).

Further, a voluminous literature has established that structural instability is present in a wide variety of macroeconomic time series (Canova, 1993; Cogley & Sargent, 2002; Koop & Potter, 2007)**,** among many others of which the nature of inflation data for Ghana is not an exception. Again, according to Stock and Watson (1996), there is a concrete literature on the evidence supporting the claim that many macroeconomic and financial time series exhibit structural instability**.** Are the instabilities in the form of fluctuations (troughs and crests) displayed on time series graphs attributable to factors such as dollar currency, prices of crude oil on the global market etcetera?

One popular approach to model this structural instability is via a time-varying parameter (TVP) model in which the parameters in the conditional mean can evolve gradually over time. A related literature has emphasized the importance of allowing for time-varying volatility in macroeconomic time series, where the heteroscedastic errors are typically modelled using a stochastic volatility specification (Primiceri, 2005; Timothy & Sargent, 2005). For macroeconomic forecasting, D'Agostino, Gambetti, and Giannone (2013), found that both features are crucial in producing accurate forecasts**.** Moreover, additional question that arises is: does low inflation rate result from commitment to developing good model for measuring and forecasting inflation? If so, was

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inflation measured through the use of traditional time series methods? Or alternative methods were applied. It is imperative to be cognizant of diverse measures of inflation because the rate of inflation has significant implications for monetary policy of the nation.

According to Antwi, Gyamfi and Kyei (2019), several research works have been conducted about inflation modelling in Ghana. In these investigations, non-linear models were applied which failed to account for conditional heteroscedasticity in the model. Countries such as US and some advanced economies of Europe have successfully applied these models in which real evidence pertaining to their relative performance was given. However, in Ghanaian setting, it appears not much research work or virtually no work has been conducted. This shows a gap in literature in this area that ought to be filled because stakeholders are challenged with which of these models is best in modelling macroeconomic and financial data such as inflation about emerging economies such as Ghana. Antwi et al. (2019), further proposed the use of threshold models as opposed to traditional Box and Jenkins models for future modelling about inflation in Ghana because the former is able to capture heteroscedasticity, variability persistence in monthly rate of inflation and therefore able to produce more accurate estimates.

Last but not least, as the density of inflation data increases, application of usual statistical methodologies becomes difficult in flexibility in modelling and incorporation of relevant available features informative about the inflation process for robust inflation forecasting. In light of this, this research project aims to

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develop flexible methods in the Bayesian framework, that allow easy integration of inflation process specific covariates for robust analysis of inflation persistence and forecasting in Ghana.

Purpose of the Study

The purpose of this research work is to develop flexible methods in the Bayesian framework that allow easy integration of inflation process specific covariates (features) for robust analysis of inflation persistence and forecasting in Ghana. Again, to examine how these features are related and how to capture such information in the model specification. Further to this, issues such as failure of usual statistical methodologies and lack of flexibility in modelling as a result of increase in density of data will be addressed.

Research Objectives

The research seeks to develop flexible methods in the Bayesian framework that allow easy integration of inflation process specific covariates for robust analysis of inflation persistence and forecasting in Ghana. In accomplishing this goal, the research seeks to address the following specific objectives:

1. To develop flexible statistical model for modelling inflation data;
2. To explore incorporation of available features or inflation specific covariates informative about the inflation dynamics;
3. Develop Bayesian computational methods for inference;
4. To apply the developed methods to real inflation data in Ghana.

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Research Questions

The objectives above are translated into the following research questions which guided the study:

1. Which flexible statistical model can be developed to model inflation data in Ghana?
2. What useful inflation dynamics covariates must be integrated into our model for modelling inflation in Ghana?
3. Which computational method (e.g., Bayesian and Classical) will be appropriate for inference about Ghana's inflation data?
4. Can the developed method be applied to real inflation data in Ghana?

Significance of the Study

Inflation process and its properties such as dynamics in terms of possible changes and persistence have attracted much attention in literature recently due to its importance in mainstream macroeconomics and monetary policy. Further to this, inflationary problems properly modelled will enable us to undertake proper planning to avoid unexpected shocks in production cost leading to insolvency of enterprises, unrealistic demands by labour unions as a result of high cost of living and instability of prices of goods and services. Also, stakeholders will be able to take on strategic transformation programmes that will bring diversity into their economies thereby enhancing living standards of the citizenry.

Delimitations

This research project considered secondary empirical data set. In particular, inflation data of varied types. For instance, inflation on food, non-food items

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etcetera in Ghana. These datasets covering a period of ten years (January, 2007 - November, 2017) were downloaded from the Bank of Ghana website

Limitations

Although, this project or research was conducted using macroeconomic data in Ghana, its generalization to the entire inflation process in Ghana comes with some little challenges that could be viewed as the main limitation of the study. The numeration of the above limitations can be summarized as follows:

1. **Data limitation:** The macroeconomic data utilized for implementation of the developed methods was a cross sectional data. It did not include all the observations on macroeconomic variables considered till date;
2. **Technical limitation**: The implementation and experimentation of the developed MCMC inference methods was delayed unduly due to the lack appropriately high-specification computers in the computer laboratory of the Department of Statistics for Graduate research work. This made the researcher to consider other means of running such algorithms leading to delays;
3. **Other limitations** can be seen in the frequent light outs. This affected the running of the algorithms in many ways.

Organization of the Study

The research work is organized into five main sections as follows: Chapter One discusses the introduction. In this section, background issues pertaining to inflation, statement of the problem, purposes and significance of the study, delimitation and limitation are discussed. Chapter Two considers review of

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literature. This area deliberates on some relevant literature and provides support for the research work. In this respect, extensive review of literature using time series methods in modelling macroeconomic, and other financial data is firstly pursued. The second section analyses literature on Gaussian process and its application in regression in the Bayesian paradigm.

Next to review of literature, is methodology. In this chapter, thorough review of Bayesian-Gaussian models that generate appropriate approximate flexible methods jettisoning time series approach to modelling inflation were explored. This is the methodology development. Issues such as basic theoretical parts of the statistical and computational methods are constructed. Matters pertaining to data, modelling framework, and inferential methods are discussed. The fourth chapter considers results and discussion. In this situation, implementation of the developed computational methods in simulation and real data application are pursued. Also, discussion on the data analysis and its implication, and are considered. Chapter Five is the summary, conclusion, recommendation. References and Appendices mark the end of the research work.

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CHAPTER TWO

LITERATURE REVIEW

Introduction

There are two major sections in this chapter of the thesis. The first part is made up of review of literature pertaining to modelling of macroeconomic, and other financial data using traditional time series approaches, and other methods that are incapable of factoring in relevant covariates informative about inflation dynamics. The second section analyses literature on Gaussian process methods within the Bayesian paradigm appropriate for addressing the objectives of the research. To this end, literature on modelling inflation using traditional time series and other methods that are unable to capture relevant covariates that provide useful information about inflation dynamics from both global and Ghanaian contexts are surveyed.

Time Series Analyses of Inflation and Some Macroeconomic data from Global Perspective

According to Fischer (1993), many research works have revealed evidence of non-linearity and instability in inflation and growth nexus using several estimation techniques. He further reported that research works contained in the reports of Ajide and Olukemi (2012), Fabayo and Olubanjo (2006) are country-specific studies, while panel studies were employed in others such as (Bick, 2010; Ibarra & Trupkin, 2016; Khan & Senhadji, 2001; Seleteng et al., 2013). Giot and Sebastien (2003) averred that it is both statistically inaccurate and logically erroneous to model data and generate models that are dependent on the

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assumption of constant variance over some period in situations when the resulting series progress over time. In the case of some macroeconomic and other financial data, large and small errors occur in clusters which implies that large returns are followed by more large returns and small returns are also followed by further small returns.

By extending this idea to inflation data, we could equally infer that periods of high inflation might usually be preceded by further periods of high inflation while low inflation is likely to be followed by further periods of low inflation (Amos, 2010). He analysed financial time series data from South Africa, covering a period of one and half decade (1994-2008) by using two sets of time series methods namely the autoregressive integrated moving averages (ARIMA) with extension to the Seasonal ARIMA and described it as (SARIMA) model. The other time series model used was the autoregressive conditional heteroscedasticity (ARCH) with extensions to the generalized ARCH called the (GARCH) model. In conclusion it came to light that that GARCH derivative model outperformed the SARIMA model. This was attributable to fluctuating mean and variance that characterized the data.

Awogbemi and Ajao (2011) modelled monthly Consumer Price Index (CPI) of five selected commodities of Nigerian market that cover a period of one decade (1997–2007). They compared conventional Box and Jenkins ARMA models with advanced time series approaches such as ARCH and GARCH models to study the presence or otherwise of volatility in the data. The outcome revealed that ARCH and GARCH models outperformed conventional Box and

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Jenkins ARMA models. Judging from the above, we could infer that the use of time series methods in modelling macroeconomic and other financial time series data appear to produce results that are less accurate hence might lead to wrong conclusion.

Ezzat (2012) examined daily stock earnings on the Egyptian stock market during the political uproar in 2011 to model issues of uncertainty resulting from the financial turbulence. Huge shocks resulting from such turbulence produce grounds for examining mechanism and dynamics of volatility during periods of severe fluctuations. One major goal of this study was to compare volatility in both periods of extreme fluctuations and volatility during peaceful periods. Daily prices of four Egyptian stock market indices; EGX 30, EGX 70, EGX 100, and the EGX 20, covering a period of inception of each index to 30th of June 2012 were analysed. Pre-and post-Egyptian uprising where marked volatile fluctuations occurred in stock returns constitutes sampled period for this research. After using the advanced time series model, EGARCH, to compare the volatility of the two eras, it was found that all the examined indices experienced more volatility during the revolution with, EGX 70 seeing the most volatility. It came to light that though, long memory was more apparent during the pre-revolution period, the leverage effect was more apparent during the revolution era.

Mia, Nabeen and Akter (2019) modelled CPI data in Bangladesh through econometric models. They used yearly time series of CPI data covering a period of a little over three decades (1986-2018). Their analyses and five-year (2019-2025) projection revealed that Bangladesh could continue to experience an

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upward growth in CPI with passage of time. They declared that although, relatively few researches have been conducted by modelling and forecasting CPI by different econometric models, most of these research studies were carried out using a group of time series methods such as moving averages (MA), autoregressive (AR), exponential smoothing, ARIMA, vector autoregressive (VAR), ARCH, and GARCH models of which ARIMA model was ranked as highest most frequently used time series method. Adams, Awujola and Alumgudu (2014), examined three decades (1980-2010) quarterly data of CPI of Nigeria's rate of inflation. They identified ARIMA (1, 2, 1) as the best-fit model that could forecast Nigeria's CPI for its following five years.

The use of ARIMA model propounded by Box and Jenkins (1970) for modelling and forecasting time series data on short-term inflation, has received several applications in wide range of literature. For instance, Meyler, Kenny, and Quinn (1998) examined Irish inflation data and appraised the performance of the ARIMA model in terms of its forecasting. Additional research works in similar perspective includes but not limited to Salam, Salam and Feridun (2007) in Pakistan.

Literature on Vector Autoregressive and ARIMA and Other Models on Global Scale

Assessment of VAR model and ARIMA models for modelling and forecasting inflation is seen in research works of (Kelikume & Salami, 2014). In this investigation, monthly inflation data generated from CPI obtained from the National Bureau of Statistics, and Central Bank of Nigeria covering the periods of

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January, 2003 to June, 20012 was examined**.** The outcome revealed that both VAR and ARIMA models tracked real inflation values for June, 2012 to September, 2012. Also, the VAR model was adjudged the best closest estimate to inflation in Nigeria from which the model forecasted June, 2012 inflation value at 11.06%. In spite of the appealing nature of the VAR model exhibited in testing conditional predictability, the ARIMA model is quite robust in producing short term estimate, however, it has poor prediction power at turning points. Although this approach appeared attractive, the research centred on two models at the expense of neural network analysis. Based on this, the use of broad range of procedures for modelling inflation was recommended for future research. The application of ARIMA methods in examining inflation data is widely reported in literature. Researchers such as Adebiyi, Abeng, Adenuga, Omanukwe, and Ononugbo (2010); Samad, Ali, and Hossain (2002); Stockton (1987), and Valle (2002) are just a few examples of those who have published in this field. Contrarily, Shaibu and Osamwonyi (2020), revealed in their research report that the use of least squares regression estimation methods was widely reported as opposed to Box-Jenkins (ARIMA) procedure where no evidence was found in examining inflation dynamics of inflation data from Nigeria.

In spite of wide application and high recognition of ARIMA models in modelling and forecasting CPI time series data, one significant defect in its usage is its inability to deal with fundamental problem of erratic and unclear seasonality. To explore and address this shortfall, Box, Jenkins and Reinsel (1994) constructed a derivative of the time series model to SARIMA models. The efficiency of

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SARIMA was lauded by Junttila (2001); Pufnik and Kunovac (2006); Saz (2011) after modelling and forecasting Finish, Croat, and Turkish inflationary data respectively. In a similar development, Akhter (2013) modelled Bangladeshi monthly CPI data, ranging from the entire period (2000-2013) using SARIMA. Three sets of assessment namely; Cumulative sum control chart (CUSUM), Quandt likelihood ratio (QLR) and Chow tests performed suggested structural breaks in the time series data which occurred at February, 2007 and September, 2009. However, after analysing a condensed form of the data covering September, 2009 to December, 2012, the study predicted an upward growth of inflation over the period 2013 from which it was suggested to the Government of Bangladesh and its Central Bank to institute suitable economic and monetary interventions to avert such inflationary shocks.

Several research have also been done to model and predict inflation. In these studies, researchers evaluated the relative accuracy of the various inflation modelling and forecasting techniques. Among the models employed in inflation prediction, a few of them are: Simple Auto Regressive (SAR), Random Walk, Bayesian Vector Auto Regressive (BVAR), Auto Regressive Fractionally Integrated Moving Average (ARFIMA), VAR, Structural Vector Auto Regressive (SVAR), and SARIMA (Owusu, 2010).

In a related development pertaining to inflation modelling, the following additional inferences were made: Going beyond a single model such as the Phillips curve and including a wide set of potential explanatory variables, produce concrete modelling leading to better model in terms of forecast precision (Stock &

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Watson, 1999). Over 100 countries were used by Barro (1995) to conduct a thorough investigation into the connection between inflation and economic growth over a three-decade (1960-1990) span. His findings established existence of negative relation between economic growth and inflation. In other words, inflation and economic growth were inversely related. He ultimately concluded that marked long-term inflation slows down growth of economy. Faria and Carneiro (2001) modelled one and half decade (1980-1995) yearly data from Brazil via VAR and established that as the rate of inflation becomes high, economic growth retards temporarily. However, this phenomenon did not adversely affect economic expansion in the long-term.

The use of traditional methods such as ARIMA and its related methods in modelling and forecasting time series data containing covariates that might affect response variable appears to be wide spread in literature with its challenges. Such limitations are expressed in the opinion of Assis, Amran and Remali (2010, p. 208) in the following quotation: *Time series forecasting is a major challenge in many real-world applications such as stock price analysis, palm oil prices, natural rubber prices, electricity prices, and flood forecasting. This type of forecasting is to predict the values of a continuous variable (called as response variable or output variable) with a forecasting model based on historical data.*

Popularity of Time Series Methods in Modelling

Nkwatoh (2012), averred that time series data in general, are known to exhibit a trend such that their consecutive values are not independent of each other or not uncorrelated. In modelling such data, one principal goal that

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researchers seek to achieve is to unearth and capture fundamental phenomenon underpinning the situation using the observed time series to give broader and clear picture into the future. The most frequently used technique in modelling and forecasting is traditional ARIMA and its extensions. Wold (1938) produced mathematical model called ARMA model and justified that it could be applied to model any stationary time series which has precise specific number of AR terms (p), and the appropriate number of MA terms (q). By extension, he indicated that any series can be modelled as a linear combination of its former time value and a finite number of its previous errors. In spite of this mathematical discovery, ARMA models failed to model actual time series until computer technology became accessible and economical in mid-1960s where usage of this technique became relatively high (Makridakis & Hibon, 1979).

The popularity of ARMA models was engineered and enhanced by Box and Jenkins (1970) who established principles for stabilizing series stationary in its mean and variance. Similar modelling using ARIMA models was further reiterated by (Makridakis & Hibon, 1979).

The use of this approach received high recognition in several disciplines of science because of the advancement of new statistical procedures coupled with more powerful computers that are compliant in the management of larger data sets. According to Nkwatoh (2016), this methodology was severally applied in 1970s. For instance, Cooper (1972); Elliot (1973); Nelson (1972), among others; McWhorter (1975); Narasimham, Castellino and Singpurwalla (1974) and in the early twenty first 21st Century (Ahmad & Latif, 2011; Ghosh, 2008; Lee et al.,

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2012; Mandal, 2005; Pei, Shih-Huang, Hui-Hua, Ching-Tsung, & Mei-Rong, 2008; Proietti, 2001; Rachana, Suvarna, & Sonal, 2010). However, as it is common with modelling and forecasting methods, some defects of Box-Jenkins' methodology for ARIMA models are difficulty in interpretation of autocorrelation functions on the part of inept researchers, and also susceptibility of models to outliers leading to inaccurate conclusions ensuing from erroneous results.

Moreover, application of Box-Jenkins approach in examining economic variables, especially macroeconomic data is vividly expressed in research works of (Assis et al., 2010; Kahforoushan et al., 2010). Nkwatoh (2016), opined that currently, many data prognosticators have adopted alternative data analysis procedure such as multiple regression analysis, Box-Jenkins ARIMA and other methods to model and forecast observations due to the fact that traditional approach of modelling and forecasting time series data are usually rigorous, time-consuming, and above all, require arduous iterative techniques. He discovered that although, Box-Jenkins procedure for modelling econometric data has been adopted and used widely in literature, econometric techniques other than ARIMA models have also been applied in modelling of macroeconomic variables as asserted by Fatimah and Roslan (1986), Nkwatoh (2012), and Purna (2012) among others. For instance, Nasir, Hwa and Huzaifah (2006) only to mention are few evidence. In his view, regression method appears to be dominating in contemporary inspiring research works such as Abdul-Rahman, Noor, and Khalid (2005), Lee et al., (2012), Taylor (2008) among others.

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On the contrary, one main defect of regression method is stringent assumptions that govern it hence leading to ranking Box-Jenkins methodology widely over regression approach in contemporary times (Floros, 2005; Kamil & Noor, 2006; Purna, 2012). Two notable conditions that affect regression analysis adversely are constant variance of residuals (homoscedasticity) and independence of residuals (no autocorrelation). This assertion is expressed in the following citation: *Violation of these two assumptions may make the regression estimates meaningless* (Bourbonnais, 2002; Greene, 2003; Gujarati & Porter, 2010; Nanda, 1988). *Another key assumption of regression analysis is the independence of explanatory variables (multicollinearity) and its violation which leads to a singular matrix (determinant equals to zero) thus, making it impossible to obtain regression estimates* (Nkwatoh, 2016, p. 157).

Next section discusses literature survey in the Ghanaian perspective.

Time Series Analyses of Inflation and Other Macroeconomic Data in Ghana

This section presents review of materials pertaining to modelling and forecasting of macroeconomics, and other financial time series data about Ghana using classical time series methods. Mbeah-Baiden (2013), used three ARCH family type of models originating from traditional time series models comprising the ARCH, GARCH, and the Exponential GARCH (EGARCH) models to examine monthly inflation data about Ghana, covering a period of almost five decades (1965-2012). He disclosed that although there was the presence of irregular effects cumulating from quite unstable monthly rates of inflation, there was an absence of leverage effects as positive shock increased the volatility in

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monthly rate of inflation more than a negative shock of equal magnitude. In view of this, the use of heteroscedastic models in modelling and forecasting monthly rate of inflation is recommended since this approach lends more credence to research of this kind by capturing volatility of monthly rate of inflation (Mbeah-Baiden, 2013).

Empirical studies about inflation modelling and forecasting in Ghana reveals that these researches attempted to model inflation using models that failed to capture the conditional heteroscedasticity of the time series inflation data. This is evidenced in research works of (Aidoo, 2010; S. Alnaa & Ahiakpor, 2011; Nasiru & Sarpong, 2012b). Gujarati (2004) in his research work declared that the fundamental characteristic of most financial time series is the demonstration of random walk in their level form. In this situation, upcoming issues cannot be projected on the basis of past records hence in the context of stock market, it implies that short-run changes in stock prices are erratic leading to the production of non-stationary series.

To put the literature survey in more elaborative perspective for complete exposition of the subject matter under discussion, research studies of Antwi et al. (2019), is examined. Data comprising monthly rate of inflation of about three and half decades, spanning a period (1981-2016) were used in the modelling and forecasting exercise. The main aim was to model the inflation data using Keenan and Tsay tests of non-linear methods. The outcome of the test established non-linearity in terms of relation among monthly rate of inflation. Consequently, threshold models were used and compared with standard linear AR models in

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terms of their fitness and forecasting performance. It was concluded that, though the simple linear AR models performed better than the non-linear models in terms of forecasting, Self-Exiting Threshold Autoregressive (SETAR) and Logistic Smooth Threshold Autoregressive (LSTAR) models were appropriate fit for the data. They further averred that many research works have been conducted about inflation modelling in Ghana where the researchers used non-linear models that failed to address and account for conditional heteroscedasticity in the model. In their research, it further came to light that empirical evidence with respect to relative performance of threshold models in economies of advanced countries such as US and some European countries are documented in literature. It was suggested that stakeholders interested in modelling future rate of inflation in Ghana, choose threshold models over traditional Box and Jenkins models since the threshold models have advantage of capturing heteroscedasticity in the model thereby accounting for variability persistence in the monthly rates of inflation and hence providing more accurate estimates.

Alnaa and Abdul-Mumuni (2005), in their research studies about assessment of two models of VAR and ARIMA models using the Root Mean Squared Error (RMSE) revealed that cointegrating VAR model appeared to have lower RMSE than the ARIMA model and hence might be more efficient than the ARIMA model in modelling and forecasting inflation in Ghana. Based on this judgement, it was concluded that VAR model was more effective in modelling and predicting variables that spark-off inflationary process, and the extent to which these variables exert inflation. Again, it follows from analyses of their

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research that variances associated with VAR and ARIMA models for the last period of the years under review are (0.00829921) and (0.008937244) respectively. These values are far lower than the calculated value of (69,107) from the entire data set. The lower values are indicative of prediction of lower volatility of inflation by the two models with VAR model exhibiting more stability in predicting inflation. The higher value of (69,107) computed from the data demonstrates the unstable nature of Ghana's inflation under the period of review (2001-2003).

In addition to the above, the use of Box-Jenkins ARIMA method is seen in research work of (Owusu, 2010). Data set comprising monthly rate of inflation for the entire period of 1990 to 2009 culminating to 251 months from which 251data points were sourced from the Bank of Ghana and the Statistical Service. An assessment on the first half (from 1990 to 2000) and second half (2001 to 2009) of the data was conducted out of which models were developed and compared. It was established that rate of inflation during the second half period (2001-2009) was higher than inflation during the first period (1990-2000). Added to this is further evidence by Magnus and Fosu (2011). Their research report disclosed that the Bank of Ghana in their assessment of inflation expectation in Ghana applied alternative inflation forecasting models as opposed to the use of customary macroeconomic models that analyze the relations between variables, and the use of ARIMA and VECM models.

Though, these approaches appeared appealing, they were unable to take into account relevant covariates fundamental to examining inflation dynamics.

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Asanewa (2018), conducted stochastic monitoring into macroeconomic variables in Ghana. She used monthly inflationary data of almost 10 years (2009-2017) and GDP data covering a period of almost six decades (1961-2017). After a thorough assessment of the data, she established that ARIMA (1,1,2) was the best model for the inflation time series data while ARIMA (1,1,1) was adjudged the top-most model for the GDP time series data. She suggested that in future studies, researchers use other non-linear models in modelling and forecasting these macroeconomic variables. Again, we may infer that this investigation failed to incorporate relevant covariates informative about inflation dynamics into the modelling process.

Similarly, the use of ARIMA models and its derivative such as SARIMA in examining the nature of behaviour of long-term monthly inflation of Ghana is reported in research work of (Aidoo, 2010). In his work, appropriate use of the method, strength of the model in terms of performance with other models, its applicability to wide range of situations, and challenge(s) are documented.

Application of advanced ARIMA extensions such as ARCH, GARCH, and EGARCH models in exposition of subject matter of inflation is further documented by (Abdul-Karim et al., 2019). In their research, ARIMA derivatives were applied to almost two decades (2000-2018) inflation data obtained from the Ghana Statistical Service (GSS) to determine the nature of volatility of Ghana's inflation. The outcome revealed upward growth in prices of goods and services in 2018 and beyond, tendency of the economy to experience instability in 2018 and 2019, although increasing volatility was being forecasted, higher order models

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were required to accurately clarify Ghana's inflation volatility. Based on this, stakeholders of the economy were advised to institute appropriate measures to curtail the expected inflation. According to Abdul-Karim et al. (2019, p. 931), it is an established fact that "traditional time series models assume that the conditional variance is constant. However, most economic and financial time series data are heteroscedastic (non-constant variance) in nature". This suggests that time series models that defy the assumption of constant variance in assessment and analysis of data are likely to produce poor results leading to wrong conclusion. Chinomona (2010) in his opinion considered the construction of time series models that will be able take-into account challenge associated with data with non-constant variance. To this end, ARCH model Bera and Higgins (1993); Engle (1982, 1983) and its extension to GARCH model Bollerslev (1986) and EGARCH model Nelson (1991) have been proposed to model the non-constant volatility of such series (Abdul-Karim et al., 2019). Nortey, Mbeah-Baiden, Dasah and Mettle (2014), identified researchers such as Abledu and Agbodah (2012); Nasiru and Sarpong (2012), in their effort to examine inflation in Ghana used models that failed to account for conditional heteroscedasticity.

Moreover, it is worthy to examine literature on methods that are incapable of factoring covariates useful about inflation dynamics. This is discussed in the ensuing section.

Analyses of Incorporation of Inflation-Related Variables

This part of the research work reviews literature pertaining to application of methods that are unable to factor-in appropriate covariates informative about

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inflation dynamics and then explores materials relating to the incorporation of auxiliary variables about inflation modelling. These procedures are espoused copiously in the succeeding section.

According to Cooley and Dwyer (1998), VAR model emerged as an important tool in the empirical analysis of macroeconomic time series in the early 1980s. Žiković (2016), also asserted that one principal feature of VAR design is stationarity exhibited by the variables being studied. One generally-known method of examining and assessing stationarity of variables is application of Phillips-Peron test (PP) and ADF unit root test. Since in general, macroeconomic time series data display instability, one notable way to stabilize non-stationary variables for possible incorporation in the model development is to use transformation. The transformation is carried by differencing values of the non-stationary variables. In spite of the embracing nature of this procedure, one key setback of the VAR model is the differencing. In the opinion of Žiković (2016), differencing fails to capture useful information pertaining to dynamics underlying phenomena such as presence of cointegration among variables, and also, inability to improve on the efficiency of the estimated autoregression models. Additional defect posed by differencing effect of VAR model is removal of long-term information and acknowledging of only short-term relationship between variables.

Though many economic and financial series in general are non-stationary, it is likely to have stationary linear blend of integrated variables described as cointegrated variables. As comprehensively expressed in this quotation: “If two variables are cointegrated, in other words tend to reach a long-term equilibrium,

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the causality must exist at least in one direction" (Žiković, 2016, p. 519). In this situation the proper means to address these variables is to use the vector error correction model**,** VECM, because it offers better understanding of non-stationary variables, improves longer term forecasting. Žiković and Vlahinić-Dizdarević (2011), distinguished between long and short term relationships among variables, and above all, identify sources of causality that cannot be detected by the usual Granger causality test (Žiković, 2016).

Further, according to Tumala, Olubusoye, Yaaba, Yaya and Akanbi (2017), the application of conventional Phillips curve and some other procedures in examining inflation have come under strong criticisms on the grounds that it is unable to properly track the trajectory and patterns of inflation. In many situations, decision ought to be taken based on single model because these models do not accommodate adequate variables to be used as regressors. Therefore, in exploring future trajectory of inflation in Nigeria, they applied Bayesian model averaging (BMA) and Frequentist model averaging (FMA) techniques to inflation data comprising numerous sets of variables, from which they concluded that both in-sample and out-of-sample forecasts were reliable. Other studies emphasizing inflation dynamics is seen in research works of (Hudu et al., 2016; Odusanya & Atanda, 2010). Ayinde, Olatunji, Omotesho and Ayinde (2010) used preliminary statistics and cointegration techniques to examine factors that affect inflation in Nigeria. In conclusion, they established that, variations existed in the trend pattern of inflation rates and some significant variables such as annual total import,

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annual CPI for food, annual agricultural output, interest rate, annual government expenditure, exchange rate and annual crude oil export**.**

Similarly, according to Ayinde et al. (2010), inflation unquestionably remains as one of the leading and most dynamic macro-economic indicators that adversely affects nearly all emerging and advanced economies globally. Its ability in distorting economies has rendered it a strong force to reckon with. Additional empirical finding pertaining to inflation dynamics is contained in the report of (Hudu et al., 2016). They found Saz (2011) to have expressed modelling inflation dynamics in the following quotation: *Some econometric models have been used to describe inflation rates, but they are restrictive in their theoretical formulations and often do not incorporate the dynamic structure of the data and have tendencies to inflict improper restrictions and specifications on the structural variables* (Hudu et al., 2016, p. 98). The above citation justifies the fact that many researches in inflation modelling have been conducted without integration of covariates informative to inflation dynamics. This defect is further evidenced in the work of (Mordi et al., 2007). In their report about best models for forecasting Nigeria's future inflation with minimal error, established and proposed, consideration of new ARIMA models, VAR models and a P-Star model amongst others (Mordi et al., 2007).

As discussed in earlier sections, ARIMA models and its extensions appeared to have been widely applied in modelling and forecasting inflation globally and nationally as opposed to other methods. In comparative evaluation of these models encountered in the literature surveyed, other methods in general,

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seemed to take precedence in terms of their flexibility and accuracy over ARIMA models and its extensions. However, a contrary observation about ARIMA models is opined by (Saz, 2011). In his perspective, absence of restriction on the model affords it the advantage of short-term forecast, and flexibility to capture dynamic features of inflation. On this account, he used SARIMA to model and forecast inflation data from Turkey. Rutasitara (2004) assessed the effect of key determinants of inflation with much emphasis on essence of exchange rate policy changes of inflation data of Tanzania from a period described as more liberalized government (1986) to contemporary time (2002). The conclusion of his report revealed that the parallel exchange rate had a stronger effect on inflation.

Another empirical literature on the use of models other than ARIMA and its allied extensions for inflation modelling and possible examination of inflation dynamics is evidenced in the work of Omekara, Ekpenyong and Ekerete (2013). They applied Periodogram and Fourier Series Analysis to model data comprising all-items of monthly inflation rates of Nigeria covering a nine-year period (2003-2011). The research sought to identify inflation cycles, fit an appropriate model to the data and make estimates for future values. In conclusion, they established that inflation cycle within the period under review was 51 months which corresponded to two administrations within the period. Fourier series model comprising the trend, seasonal and error components was suitable for data. The developed model was used to forecast 13-months inflation rates from which both estimates and actual values were consistent for the 13-months period (Omekara et al., 2013). The study of inflation and its dynamics to facilitate exposition of macroeconomics

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and monetary policy in both emerging and advanced economies has been widely reported globally and nationally. One famous research study that espouses academic research on inflation dynamics across Sub-Saharan Africa (SSA) is Oulatta (2018). Some relevant research works about inflation dynamics he identified include "(Blavy, 2004; Coleman, 2010; Diouf, 2007; Durevall & Ndung'u, 1999; Kinda, 2011; Klein & Kyei, 2009; Moriyama, 2008; Moser, 1994; Nassar, 2005; Sacerdoti & Xiao, 2001; Ubide, 1997)" (Oulatta, 2018, p5). Also, Ngailo, Luvanda, and Massawe (2014) observed that inflation dynamics and evolution could be examined by applying stochastic modelling technique that captures the time reliant structure contained in the time series inflation data.

Having explored literature extensively on inflation modelling in areas of time series and other methods, we now undertake review of literature about Gaussian Processes (GPs) and Bayesian framework pertaining to the study objectives.

Gaussian Process

Gaussian processes are non-parametric statistical models that are made up of collection random variables, any finite number of which have joint Gaussian distributions (Rasmussen & Williams, 2006). The non-parametric perspective stems from the fact that the Gaussian process places a prior on the space of functions f directly, without parameterizing f. The process is fully specified by a mean function and a covariance function, represented respectively by $m(x)$ and $c(x,x')$ which cumulatively leads to the function $f \sim GP\left(m(x), c(x,x')\right)$. The argument x of the function, $f(x)$ defined in the process,

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$$f(x) \sim GP\left(m(x), c(x, x')\right) \tag{2.1}$$

plays a principal role of the index.

The management and operation of the Gaussians process after finite number of data points is observed, are regulated and administered by fundamental rules of multivariate Gaussian distributions. Alternatively, a random process X_t is a GP if for all n and for $(t_1, t_2, \ldots t_n)$ the random variables $\{X(t_1), X(t_2), \ldots X(t_n)\}$ have a jointly function, which may be expressed mathematically as

$$f(x) = \frac{1}{(2\pi)^{n/2}\left[\det(\Sigma)\right]^{1/2}} \exp\left[-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)\right]$$

where,

$X \sim N(\mu, \Sigma)$

$X = \left[X(t_1), X(t_2), \ldots, X(t_n)\right]^T$: n random variables

$\mu = E(X)$: mean value vector

$\Sigma = C(X, X')$: n $\times$ n covariance matrix

In this case, the covariance matrix, Σ is non-singular. In some instances, such as environmental applications, the subscript t will typically denote a point in time, or space, or space and time. The mean and covariance functions are denoted respectively by $\mu(t) = E(X_t)$ and $\gamma(s,t) = \text{cov}(X_s, X_t)$.

Further to this, the group of GPs is one of the most extensively used families of stochastic processes for modelling dependent data observed over time, or space, or time and space (Davis, 2014). According to him, the extensive application of GPs is due to the fact that it has two important properties namely

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the mean and covariance functions, and relative ease with which it solves prediction problem. The possession of mean and covariance functions uniquely eases in model fitting as only the first- and second-order moments of the process require specification. The wide range of application of GPs from archaeology to zoology is driven by tractability, well approximation to diverse situations. For instance, the use of GP in estimating covariances between financial assets plays an important role in risk management (Nirwan & Bertschinger, 2018).

The basic description that characterizes the GP is that all finite dimensional distributions display a multivariate normal (or Gaussian) distribution. Specifically, the distribution of each observation under consideration must exhibit normal distribution. Inferring from above consideration, it is established that mean, $m(x)$ and covariance, $c(x, x')$ functions are indispensable elements in the GPs. According to Ofori (2020), any finite sub-collection of function values has a multivariate Gaussian distribution with mean vector, $\boldsymbol{\mu}$ and variance-covariance matrix**,** $\boldsymbol{C}$ between any combined observations. This distribution is characterized by the kernel function.

$$\left[\left[f(x_1), f(x_2), L\ldots, f(x_T)\right]^T \sim N(\mu, C)\right]$$

$$\mu = \left[m(x_1), m(x_2), \ldots, m(x_T)\right]^T$$

$$C_{ij} = c\left(x_i, x_j\right)$$

The mean vector indicates and describes the expected value of the distribution, with its components describing the mean of the respective dimension. The variance-covariance matrix examines along each dimension and assesses the correlation among the different random variables. In order to satisfy the Gaussian

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density function, the variance-covariance matrix must be symmetric and positive semi-definitive. In other words, the matrix must display non-singularity (Hazewinkel, 1994). Further to this, Rasmussen and Williams (2006), averred that with the exception of covariance functions that are non-positive semi-definitive, covariance functions of characteristics of non-stationarity (e.g., in which case the length-scale may depend on the values of x), functions formed from sum or product of other covariance functions are desirable for Gaussian process.

The characteristic of $f(x)$ in equation (2.1) such as cyclicity or periodicity, and smoothness are determined by the kernel function $c(x, x')$. Since the kernel defines similarity between the values of our function, controls and dictates the likely form or shape that might be generated or assumed from a fitted function, we must ensure that in the choice of kernel, we follow suitable properties of the covariance matrix (Görtler et al., 2019). Also, the broad use of kernel in machine learning is due to its advantage in the measure of similarity over the use of standard Euclidean distance, possession of adaptable framework capable of accommodating the input points into a higher dimensional space in which they then measure the similarity.

There are two basic categories for kernels: stationary and non-stationary kernels, or non-stationary and stationary covariance functions. The covariance of two points depends on their relative positions for stationary kernels like Radial Basis Function (RBF) kernel or the periodic kernel because they are invariant to translations. Contrarily, non-stationary kernels, like the linear kernel, depend on absolute position and do not have this requirement. The squared exponential (SE)

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kernel which is specified in equation (2.2) below is a well-known kernel that has garnered attention and widespread usage in the field of machine learning.

$$c(x, x') = \sigma^2 \exp\left(-\frac{\|x - x\|^2}{2\ell^2}\right) \tag{2.2}$$

which is parameterized by a length scale ℓ and a scale factor σ. Functions generated by a GP with a SE kernel are smooth owing to the fact that the kernel function is unboundedly differentiable (Rasmussen & Williams, 2006).

The distribution changes over the function values for changes in the input x sanctioning a particular smoothness is determined by the length scale ℓ. In view of the versatileness of SE, its application is diverse and enormous in the generation of density or distribution functions with the derived functions within our desired bounds or conditions. In other words, SE is able to produce results that fall within our expectation. Occasional functions, for instance, are not well modelled by an SE kernel, but rather captured by a periodic kernel

$$c(x, x') = \sigma^2 \exp\left(-\frac{4\sin^2\left(\frac{\pi\|x - x'\|}{p}\right)}{2\ell^2}\right) \tag{2.3}$$

where p is the period of the function. When modelling financial time series, the SE kernel or the periodic kernel are often used in combination to capture the unknown source-specific smoothness and periodicity of the trajectories of covariates (Dürichen et al., 2015; Stegle et al., 2008).

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The Process and the Weight-Space View of Gaussian Processes

In standard linear regression, we have

$$y_n = w^T X_n \tag{2.4}$$

where the predictor $y_n \in R$ is a linear combination of the covariates $X_n \in R^D$ for the n sample out of N observation. In this case, we can make this model more tractable by using M fixed basis functions,

$$f\left(X_n\right) = W^T \Phi\left(X_n\right) \tag{2.5}$$

$$\Phi\left(X_n\right) = \left[\Phi_1\left(X_n\right)..., \Phi_M\left(X_n\right)\right]^T \tag{2.6}$$

It is worthy to note that equations (2.5) and (2.6) appropriately hold for the fields, $X_n \in R^D$ and $X_n \in R^M$ respectively (Gundersen, 2019).

Now consider a Bayesian treatment of linear regression that places prior on w,

$$p(w) = N\left(w/o, a^{-1} I\right) \tag{2.7}$$

where $a^{-1}I$ is a diagonal precision matrix. According to Bishop (2006), for a given value of w equation (2.6) defines a particular function of X. The probability distribution over w defined by equation (2.7) therefore induces a probability distribution over functions $f(x)$. This means that if w is random then $W^T \Phi\left(X_n\right)$ is also random. The above deliberation indicates how we can consider Bayesian linear regression as distribution over functions. In this case, we can describe a random variable w or a random function, f induced by w. Fundamentally, f could be thought of as infinite dimensional function since we

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can imagine countless data and an infinite number of basic functions. Although, this is practical perspective, in practice, we seek to identify a finite set of functions that have joint distribution over our actual data.

Let $y = \begin{bmatrix} f(X_1) \\ \vdots \\ f(X_N) \end{bmatrix}$ and ϕ be a matrix such that $\phi_{nk} = \varphi_k(X_n)$. Then we can rewrite w as

$$y = \phi w = \begin{bmatrix} \phi_1(X_1) \cdots \phi_M(X_1) \\ \vdots \quad \ddots \quad \vdots \\ \phi_1(X_N) \cdots \phi_M(X_1) \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_M \end{bmatrix}$$

If $z_1, \ldots z_N$ are independent Gaussian random variables, then the linear combination $\alpha_1 z_1 + \alpha_2 z_2 + \ldots + \alpha_N z_N$ is also Gaussian for all $\alpha_{1,} \alpha_{2,} \ldots, \alpha_N \in R$. We describe $z_1, z_2, \ldots, z_N$ as being jointly Gaussian since each component of y_n is a linear combination of independent Gaussian distributed variables $(w_1, \ldots, w_M)$, making the components of y jointly Gaussian.

Further to this, by computing the mean vector and covariance matrix, we will be able to state the distribution of y.

$$E(y) = 0$$

$$Cov(y) = \frac{1}{\alpha} \Phi \Phi^T$$

If we define K as $\operatorname{cov}(y)$, then we can describe K as Gram matrix such that

$$K_{mn} = \frac{1}{\alpha} \phi(X_n)^T \phi(X_m) \cong k(X_m, X_n)$$

where $k(X_m, X_n)$ is called a covariance or kernel function $k: R^D \times R^D \rightarrow R$.

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Stationary Processes

In GP, mean and covariance functions play a key role in the modelling process. The natural and usual form at which the mean and covariance functions exist in general, makes it difficult to model physical phenomena. To reduce if not completely eliminate the difficulty associated with these computations, we introduce certain assumptions in terms of simplification to enable us carry out the intended mathematical computations. This leads us to stationarity. The philosophy underpinning the issue of stationarity is expressed in the following excerpts: “The assumption of stationarity frequently provides the proper level of simplification without sacrificing much generalization. Moreover, after applying elementary transformations to the data, the assumption of stationarity of the transformed data is often quite plausible” (Wolfert, 2002, p. 840).

Using Wolfert (2002), it is explained that a Gaussian time series X_t is said to be stationary if the following conditions are satisfied:

i. $m(t) = E(X_t) = \mu$ is independent of t,

ii. $\sigma(t+h,t) = \operatorname{cov}(X_{t+h}, X_t)$ is independent of all h.

In the stationary process setting, it is ideal to write the covariance function σ as a function on T instead of on $T \times T$. Thus, we express $\sigma(h) = \operatorname{cov}(X_{t+h}, X_t)$ as the autocovariance function of the process.

Further to this, for stationary Gaussian processes, X_t we have

$x \sim N(\mu, \sigma(0))$, for all t $(X_{t+h}, X_t)'$ follows bivariate normal distribution having covariance matrix for all t and h. $\begin{bmatrix} \sigma(0) & \sigma(h) \\ \sigma(h) & \sigma(0) \end{bmatrix}$

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In the view of (Wolfert, 2002), a process is described as weakly or second-order stationary if the stationary stochastic process, X_t fulfil conditions: $m(t) = E(X_t) = \mu$ is independent of t, and $\sigma((t+h),t) = \operatorname{cov}(X_{t+h}, X_t)$ is independent of all h. In this situation, the processes are invariant with respect to time translations.

On the other hand, Wolfert (2002) explained that a stochastic process, $\{X_t\}$ is strictly stationary if the distribution of $(X_{t_1},\ldots,X_{t_n})$ is the same as $(X_{t_1+s},\ldots,X_{t_n+s})$ for any s. This implies that under the same translation, the distributional features of the time series are the same. In the Gaussian time series, the principles and concepts of weak and strict stationarity go together. The consequence of this is due to the fact that weakly stationary processes, $(X_{t_1},\ldots,X_{t_n})$ and $(X_{t_1+s},\ldots,X_{t_n+s})$ have the same mean vector and the same covariance matrix. Since each of the two vectors $(X_{t_1},\ldots,X_{t_n})$ and $(X_{t_1+s},\ldots,X_{t_n+s})$ has a multivariate normal distribution, they must be identically distributed.

Stationary Covariance Functions

As mentioned in the previous section, the covariance function is a crucial and an important factor in the Gaussian paradigm owing to its ability to encode the assumptions about the function being studied. A covariance function is described as stationary if it is of the form $\tau = x - x'$. This means that the function is invariant to translation in the input space. In other words, the covariance

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functions solely take into account the inputs' Euclidean distance, $r = \|x - x'\|$, or in some instances stationary covariance function may be described as a function of single argument $c(\tau)$. This function may be described as positive measure in the domain of the Fourier transform (Rasmussen & Williams, 2006). In the complex-valued perspective, Buchner's theorem defined it as function c on $\mathbb{R}^D$ is the covariance function of a weakly stationary mean square continuous complex-valued random process **on** $\mathbb{R}^D$ **if** and only if it can be represented as

$$c(\tau) = \int_{\mathbb{R}^D} e^{2\pi i s \tau} d\mu(s) \tag{2.8}$$

where μ is a positive finite measure."

We then consider examples of some selected stationary covariance functions based on some desirable characteristics such as: differentiability and continuity, smoothness, and above all, ability to encode the assumptions underlying the function being examined. These are Squared Exponential (SE), Matérn, Ornstein-Uhlenbeck Process and Exponential, γ-exponential, Rational Quadratic, and Piecewise Polynomial.

Squared Exponential Covariance Function

The squared exponential (SE) covariance function is defined mathematically as

$$c_{SE} = \exp\left(-\frac{r^2}{2\ell^2}\right) \tag{2.9}$$

where ℓ and r are the characteristic length-scale and Euclidean distance respectively. The squared exponential covariance function is continuously differentiable. In other words, when all conditions are satisfied, GP with this

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covariance function has mean square derivatives and is accordingly smooth. The SE kernel has a spectral density, $S(s)$ given by

$$S(s) = \left(2\pi\ell^2\right)^{\frac{D}{2}} \exp\left(-2\pi^2\ell^2 s^2\right) \tag{2.10}$$

According to Rasmussen and Williams (2006), SE appeared to be most widely-utilized covariance function in the machine learning paradigm, however, in the opinion of (Stein, 1999), assumptions of strong smoothness about SE are unrealistic and will fail to reveal many physical phenomena and therefore suggested the application of Matérn's class of covariance functions for identifying such phenomena. Halvorsen (2018), explained Matérn covariance function as generalized Gaussian radial basis function that contains complete exponential kernel which produces different results. This class of covariance functions is able to capture many physical phenomena because of its finite differentiability for finite v. Also, its urge over other covariance functions is its ability to produce results that are relatively accurate, and also generate a prediction model that is less susceptible to numerical errors (Abusnina et al., 2014). Below are some Matérn class of covariance functions.

Mathematically, $C_v(d) = \sigma^2 \frac{2^{1-v}}{\Gamma(v)}(\sqrt{2v}\frac{d}{\rho})^v K_v\left(\sqrt{2v}\frac{d}{\rho}\right)$

where d is a distance function (such as Euclidean distance), Γ is the gamma function, K_v is the modified Bessel function of the second kind, ρ and v are positive parameters. Practically, v is workable for values of $v = \frac{3}{2}$ and $\frac{5}{2}$ (Halvorsen, 2018).

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The spectral density of the Matérn class of covariance functions is given by $S(s)$ in D dimensions where,

$$S(s)=\frac{2^{D}\pi^{\frac{D}{2}}\Gamma\left(\upsilon+\frac{D}{2}\right)(2\upsilon)^{\upsilon}}{\Gamma(\upsilon)\ell^{2\upsilon}}\left(\frac{2\upsilon}{\ell^{2}}+4\pi^{2}s^{2}\right)^{-\left(\upsilon+\frac{D}{2}\right)} \tag{2.11}$$

In this set of covariance functions, as $\upsilon\to\infty$ we choose a scale that generate squared exponential function $\exp\left(\frac{-r^{2}}{2\ell^{2}}\right)$. As mentioned earlier, Matérn class of covariance functions is feasible when υ is half-integer; that is $\upsilon=p+\frac{1}{2}$ where $p\geq 0$. This viability, is seen in the values of $v=\frac{3}{2}$ and $\frac{5}{2}$. In this situation, the kernel is a result of an exponential and a polynomial of order p.

On this account, Abramowitz and Stegun (1965), formulated a general equation shown in (2.12) as follows:

$$c_{\upsilon=p+\frac{1}{2}}(r)=\exp\left(-\frac{\sqrt{2\upsilon r}}{\ell}\right)\frac{\Gamma(p+1)}{\Gamma(2p+1)}\sum_{i=0}^{p}\frac{(p+i)!}{i!(p-i)!}\left(\frac{\sqrt{8\upsilon r}}{\ell}\right)^{p-i} \tag{2.12}$$

The next covariance function for discussion is Ornstein-Uhlenbeck (OU**)** process and exponential covariance function.

Ornstein-Uhlenbeck (OU) process and Exponential Covariance Function

This is a special case of Matérn class of covariance function for which $v=\frac{3}{2}$. The cumulative exponential kernel generated in this situation is $k(r)=\exp\left(\frac{-r}{\ell}\right)$. The relating process is Mean Square (MS) continuous, however

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is not MS differentiable. The Ornstein-Uhlenbeck (OU) process kernel is found in $D=1$. According to Rasmussen and Williams (2006), the Ornstein-Uhlenbeck technique serves as a scientific representation of the speed of a molecule undergoing Brownian motion. However, its application goes beyond finance and other disciplines.

The γ-exponential Covariance Function

This group of kernel integrates both exponential and squared exponential functions. The mathematical equation is shown in equation (2.13) below**.**

$$c(r) = \exp\left(-\left(\frac{r}{\ell}\right)^{\gamma}\right) \quad \text{for } 0 < \gamma < 2 \tag{2.13}$$

When $\gamma = 1 \text{ and } 2$, this leads to Ornstein-Uhlenbeck (OU) process and squared exponential functions respectively. The γ-exponential covariance function has hyperparameters that are comparable with number of parameters of the Matérn class of covariance functions, nonetheless it appears to be less compliant (Stein,1999). This is a result of the comparing process not being MS differentiable with the exception of $\gamma = 2$.

Rational Quadratic Covariance Functions

The Rational Quadratic (RQ) covariance function, where $\tau = x - x'$

$$c_{RQ}(r) = \left(1 + \frac{r^2}{2\alpha\ell^2}\right)^{-\alpha} \tag{2.14}$$

with $\ell, \alpha > 0$, can be viewed as a scale mixture (infinite sum) of SE covariance with various characteristics length-scales (Rasmussen & Williams, 2006).

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Piecewise Polynomial Covariance Functions with Compact Support (PPCF)

According to Rasmussen and Williams (2006), this group of functions provide unique set of covariance functions characterized by zero covariance between the points when their distance surpass a certain bound. This circumstance produces sparse covariance matrix leading to more favourable computational advantages. Although the design of these functions is appealing and computation-compliant, one setback of this approach is non-guarantee of positive definiteness. Generally, the PPCF are not positive definite for all input dimensions however, their validity is confined to some extreme dimension (D). They gave an example of covariance function $C_{ppD,q}(r)$ that are positive definite in $\mathbb{R}^D$ as:

$$c_{ppD,0}(r) = (1-r)_+^j$$

$$c_{ppD,1}(r) = (1-r)_+^{j+1}\left((j+1)r+1\right),$$

$$c_{ppD,2}(r) = (1-r)_+^{j+2}\left(\left(j^2+4j+3\right)\right)$$

$$c_{ppD,3}(r) = (1-r)_+^{j+3}\begin{pmatrix} \left(j^3+9j^2+23j+15\right)r^3 + \\ \left(6j^2+36j+45\right)r^2 + (15j+45)r+15 \end{pmatrix} \tag{2.15}$$

where $j = \left(\dfrac{D}{2}\right) + q + 1$.

After review of some contextual literature of the GPs, stationary covariate functions, our next point of discussion focuses on Gaussian process for regression.

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Gaussian Process Regression

This section describes some fundamental issues about the application of GP in regression.

In this regard, the Bayesian inference of the standard linear regression model using Gaussian noise is presented. Consider the standard linear regression model

$$f(z) = z^T w \tag{2.16}$$

$$y = f(z) + \varepsilon \tag{2.17}$$

where z is the input vector, w is a vector of parameters of the linear model, f is value of the function while y is the observed value. It is possible to add a bias weight nevertheless this can be realized by increasing z using an extra component whose value is constantly one, we do not clearly incorporate it in the presentation here. Let us assume that y vary from the functional term $f(z)$ due to the noise added. The added noise is also assumed to follow an independent, identically distributed (iid) Gaussian distribution whose mean is zero and variance σ_n^2 (Rasmussen & Williams, 2006). That is;

$$\varepsilon \sim \mathrm{N}\left(0, \sigma_n^2\right) \tag{2.18}$$

Based on (2.16) and (2.18), the probability density function of the observations taking into account the parameters which is factored over cases in the training set is expressed as;

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$$P(y \mid z,\, w) = \prod_{k=1}^{n} P(y_k \mid z_k, w) = \prod_{k=1}^{n} \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left(-\frac{(y_k - zw)^2}{2\sigma_n^2} \right)$$
$$= \frac{1}{(2\pi\sigma_n^2)^{\frac{n}{2}}} \exp\left(-\frac{1}{2\sigma_n^2} \left| y_k - z^T w \right|^2 \right) = N\left(z^T w, \sigma_n^2 I\right) \tag{2.19}$$

where $|z|$ is the Euclidean distance of vector z. In the Bayesian paradigm, one has to determine a prior over the parameters, indicating our beliefs pertaining to the parameters before utilizing the data. Consider a zero mean Gaussian prior with covariance matrix, Σ_p, on the weights

$$w \sim N\left(0, \Sigma_p\right) \tag{2.20}$$

Inference for the Bayesian linear model is made via the posterior distribution over the weights using the Bayes' rule.

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{marginal likelihood}}$$
$$P(w \mid y, K) = \frac{P(y \mid z, w) P(w)}{P(y \mid z)} \tag{2.21}$$

where the marginal likelihood (the normalizing constant) does not depend on the weights and is shown as

$$P(y \mid z) = \int P(y \mid z, w) P(w)\, dw \tag{2.22}$$

Equation (2.22) above combines both the prior and likelihood and takes into account all that we know about the parameters. We can re-write the numerator of equation (2.21) (the part of the posterior made up of the prior and the likelihood) of the posterior that depends on the weights, and completing squares of the exponent in w gives

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$$P(y \mid z, y) \propto \exp\left(-\frac{1}{2\sigma_n^2}\left(y - z^T w\right)^T \left(y - z^T w\right)\right)\exp\left(-\frac{1}{2} w^T \Sigma_p^{-1}\right)$$

$$\propto \exp\left(-\frac{1}{2}\left(w - \bar{w}\right)^T \left(\frac{1}{\sigma_n^2} zz^T + \Sigma_p^{-1}\right)\left(w - \bar{w}\right)\right) \tag{2.23}$$

where $\bar{w} = \sigma_n^{-2}\left(\sigma_n^{-2} zz^T + \Sigma_p^{-1}\right)^{-1} zy$, the posterior distribution can now be seen as a Gaussian whose mean is $\bar{w}$ and covariance matrix A^{-1} given by

$$p(w \mid z, y) \sim N\left(\bar{w} = \sigma_n^{-2} A^{-1} zy,\ A^{-1}\right) \tag{2.24}$$

such that $A = \sigma_n^{-2} zz^T + \Sigma_p^{-1}$.

It should be noted that the mean and the mode of a Gaussian posterior are the same and can also be called the maximum a posterior (MAP) estimate of w.

Gaussian Process Approximations

Rasmussen and Williams (2006) described Gaussian process as one of the powerful tools that is used for modelling distributions over non-linear functions. This superiority of the GP in modelling distributions is due to its robustness to over-fitting, well-laid down processes to harmonize hyperparameters. In addition to this, there are limits on the results that are essential for tasks that require non-linear functions like regression, reinforcement learning, density estimation, and others. These limits create further uncertainty (Brochu et al., 2010). According to Gal and Turner (2015), in spite of the advantages of the GP, its manipulation is challenged in areas such as high cost of computation, and large matrix inversion. For example, if there is a high density of data points N in a given situation, the model will require a $O\left(N^3\right)$ level of time complexity. In order to reduce the model's time complexity, a new GP was designed. In this design, a small number

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of "inducing inputs and outputs" are used which was described as "sparse pseudo-input" approximations (Quinonero-Candela & Rasmussen, 2005). Quinonero-Candela and Rasmussen (2005), proposed a design where the new GP was anticipated to generate outcomes that are more accurate than the original GP but it failed to do so in terms of modelling highly complex models in general.

Further to this, Lázaro-Gredilla, Quiñonero-Candela, Rasmussen and Figueiras-Vidal (2010), suggested an alternate approximation procedure to GP. To this end, they decomposed a stationary covariance function of a GP into Fourier series. They then approximated the resulting infinite series to finite series, adjusted the series frequencies to reduce the deviation from the complete GP. The approach termed "sparse spectrum" approximation was also in concordance with design by (Rahimi & Recht, 2007). Although, both designs mentioned above agreed, in the latter method, frequencies were sampled from some distribution as opposed to optimized procedure in the former. Alluding to the above designs, Wilson, Gilboa, Nehorai and Cunningham (2014) declared that both designs are capable of capturing globally multifaceted behaviour, nevertheless, direct optimization of diverse quantities usually results in some kind of over-fitting.

Gal and Turner (2015), proposed the utilization of variational inference for the sparse spectrum approximation as a solution to the issue of over-fitting. This approach enables the user to evade over-fitting. Again, it captures well, globally complex behaviour. In this regard, they designed finite approximation generated from a finite random variable approximation which is a product of Monte Carlo integration, to substitute the stationary covariance function. When this finite

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random variable was conditioned on the dataset, it generated inflexible posterior. This intractable posterior continues to undergo approximation until a stable covariance function that fits the data is achieved. The prior derived from the GP model prevents the approximating distribution from excessively fitting to the data.

The next section deliberates on Sparse Spectral design in the perspective of Bayesian Model of GPs

Bayesian Model of GPs using Sparse Spectral Representation

This section discusses Sparse Spectral concepts which addresses limitations of covariance functions in its approximation precision by constructing adaptable groups of kernel typical of all stationary kernels in contrast to simple spectral representation condition (Yves-Laurent & Roberts, 2015). According to Ofori (2020), Buchner's theorem offers a proper way to re-develop the covariance function of a given process by considering the origin of the process in the frequency domain, by utilizing the spectral density of the related covariance if it exists. Using a stationary covariance function of the form $C\left(t,t^{T}\right)$ it can be proved that the covariance is expressible as $C\left(t-t^{T}\right)$ for all $t,t^{T}\in\mathbb{R}^{d}$. According to Stein (1999) interpretation of Buchner's theorem, any stationary covariance function $C\left(t-t^{T}\right)$ has the ability to be represented as a positive finite measure of Fourier transform. Therefore, the covariance function $C\left(t-t^{T}\right)$ can be expressed as the Fourier transform of a finite amount $\sigma^{2}f\left(s\right)$.

$$
\begin{aligned}
C\left(t-t^{T}\right) &= \int_{\mathbb{R}^{d}} \sigma^{2} f\left(s\right) e^{-2\pi i s^{T}\left(t-t^{T}\right)} ds \\
&= \int_{\mathbb{R}^{d}} \sigma^{2} f\left(s\right) \cos\left(2\pi s^{T}\left(t-t^{T}\right)\right) ds
\end{aligned}
\tag{2.25}
$$

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for a real-valued $C\left(t,t^{T}\right)$ and probability density function

$$f\left(s\pi(\theta \mid x)=\frac{f(x \mid \theta)g(\theta)}{\int f(x \mid \theta)g(\theta)d\theta} \propto f(x \mid \theta)g(\theta)\right).$$

The next section considers literature survey in Bayes theorem.

Literature of Methods in Bayesian Paradigm

This aspect considers review of literature in areas of the Bayes' theorem, and the elements of Bayesian statistics. According to Schoot et al. (2013), three principal issues that govern Bayesian statistics first outlined by T. Bayes in 1774 Bayes and Price (1763), Stigler (1986) are the background knowledge on the parameters of the model being tested, information contained in the data itself, and the third factor based on combination of the first two issues, which is called posterior inference. In the first principal issue, all background knowledge available before accessing the data are contained in what is described as prior distribution (Schoot et al., 2013). For instance, either normal distribution or any other distribution. They added that the level of uncertainty of the population parameter of interest is measured by variance of the prior distribution. The overall judgement of uncertainty is expressed as the larger the variance, the more uncertain we are and vice versa. The prior variance is an indicator that measures precision. This is basically, inverse of the variance. The lower the prior variance, the higher the precision, and the more confident that the prior mean reflects the population mean (Schoot et al., 2013).

Next to the first issue is the information contained in the data itself. This is described as observed evidence in a form of the likelihood function of the data

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given the parameters. Not only does this stage considers observed evidence but also seeks to solicit answers to the probability of the data under consideration given a set of parameters such as the mean, variance, proportion etcetera. The third stage or ingredient is obtained from the two earlier stages which is described as posterior inference. The first two stages mentioned earlier are combined through Bayes' theorem to the posterior distribution. This is seen as concession of prior knowledge and observed evidence (Schoot et al., 2013). The above three stages basically constitute Bayes' theory which is expressed in the following citation: "our updated understanding of parameters of interest given our current data depends on our prior knowledge about the parameters of interest weighted by the current evidence given those parameters of interest" (Schoot et al., 2013, p. 4).

We can incorporate one's belief or previous knowledge about the parameter of interest to be factored into the study using the Bayesian technique, which is utilized in statistical inference. Contrary to the frequentist perspective, which sees the parameter as having a fixed but unknown value, in this context, the parameter under study is viewed as a random variable, population is assumed to have only one true population parameter such as, one true mean or one true regression coefficient, population proportion etcetera. The posterior distribution, $\pi(\theta \mid x)$ of the parameter, $\theta = (\theta_1, \theta_2, \theta_3 \ldots \theta_p)$ given the evidence in a form of the data $x = \{x_1, x_2, x_3, \ldots, x_n\}$ is of utmost significance in the Bayesian approach. The posterior distribution is a description of the current state of knowledge that is produced when the prior distribution, and data representing observed evidence in a form of likelihood function are combined (Gelman et al., 2003; King et al.,

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2009). If $g(\theta)$ and $f(x|\theta)$ are the prior distribution and the likelihood functions respectively, then mathematically Bayes' theorem gives the equation below.

$$\pi(\theta|x) = \frac{f(x|\theta)g(\theta)}{\int f(x|\theta)g(\theta)d\theta} \propto f(x|\theta)g(\theta) \qquad (2.26)$$

Expression (2.26) constitutes the fundamentals of Bayesian inference (Carlin & Louis, 2009; Gelman, Carlin, Stern & Rubin, 2004; King, Morgan, Gimenez & Brooks, 2010).

As it is generally important to undertake successful research by considering relevant information by learning from previous research findings and incorporating information from these research findings into our current studies, so it is with Bayesian methodology. The essence of this incorporation is to facilitate the choice of appropriate theoretical or conceptual framework, research designs and variables to be examined. In Bayesian approach, our prior beliefs regarding the parameter are dictated by the real data under consideration (Kaplan & Depaoli, 2013).

Prior Specification

As pointed out earlier, one principal issue that determines our prior beliefs regarding parameters of interest is data that is accessible to us. The choice of a prior is based on the quantity and quality of information that we believe to possess prior to the data collection. Basically, there are two circumstances. The first one considers some instances where we may not have adequate prior information that will enable us draw meaningful inference about the posterior distribution. In

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Bayesian consideration, absence of such information is relevant and ought to be integrated into our statistical modelling. Thus, it is imperative to measure both our ignorance and overall understanding of the task under consideration (Kaplan & Depaoli, 2013). Further, our level of certainty or uncertainty about prior parameter of interest such as mean, median, proportion etcetera is stated by prior precision. This precision is measured by the prior variance. The changes, and other alteration of this indicator and its corresponding remarks have been explained earlier in previous section. The following excerpts summarize prior information in terms of precision, certainty and variance:

The more certain we are, the smaller we can specify the prior variance and, as such, the precision of our prior will increase. Such a prior distribution encodes our existing knowledge and is referred to as a subjective or informative prior distribution (Schoot et al., 2013, p. 6).

However, if a low precision is specified, it is often referred to as a low-informative prior distribution.

In a nutshell, the salient characteristics that are worth considering about prior distribution is as captured by Baah (2014, p. 30) in the following summary of the observation of (Carlin & Louis, 2008; Jean-Michel & Christian, 2007): An important consideration in specifying a prior distribution is its adequacy in accounting for the prior information available about the parameter one is investigating, before the application of the observed data. Also, of importance when choosing the prior distribution is the tractability of the posterior distribution in terms of its derivation and the computations that come with it when simulating

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from the distribution. Thus, one could choose a prior that results in the posterior and the prior belong to the same distribution and differing only in terms of their parameters. In this case the prior is referred to as the conjugate of the likelihood. Baah (2014) cites the example of $Ga(\alpha,\beta)$ being conjugate to Exp(θ), given that the independently and identically distributed (i.i.d) data $x=\{x_1,x_2,x_3,\dots,x_n\}$ from an exponential distribution Exp(θ) leads to a posterior of $Ga(\alpha+n,\beta+n\bar{x})$.

Citing Carlin and Louis (2008), he further averred that a prior which does not provide adequate information on the parameter under investigation is called a non-informative prior. This non-informative prior usually have almost negligible influence on the inference as it encodes very little information a priori. For instance, if $\theta=(\theta_1,\theta_2,\theta_3,\dots,\theta_n)$ is the only information available on the discrete parameter θ,a prior of the discrete Uniform distribution, $p(\theta_i)=\frac{1}{n}, i=1,2,3,\dots$n is a non-informative one as all potential values of θ attract an equal probability value of $1/n$, which provides no further knowledge on θ (Carlin & Louis, 2008).

Furthermore, a cascade of priors (referred to as a hierarchical prior) usually made up of two sets of distribution in series could be specified. In this case the prior parameters (hyperparameters) also have prior distributions (hyperpriors), This arrangement tend to dissipate the effect any prior assumption is likely to have on the posterior (Jean-Michel & Christian, 2007; King et al.,

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2009). For the Pois(λ) i.i.d data $x = \{x_1, x_2, x_3 \ldots, x_n\}$, a hierarchical prior could be specified as $\lambda \sim Ga(a,b)$ with $a \sim Exp(\theta_a)$ and $b \sim Inv - Exp(\theta_b)$ (Baah, 2014).

In sum, the sequence of steps outlined below detailed the key issues about prior. The prior depicts the knowledge about parameter of interest in our model before observing the data. If the researcher chooses not to specify prior knowledge, then he can replace this by non-informative prior instead. In this regard the overall result will not be affected. The technique of using non-informative prior to generate the posterior results in Bayesian statistics context is described as objective Bayesian statistics (Press, 2003). Schoot et al. (2013) assert that specifying low-informative prior in general provides results that are hardly influenced by the specification of the prior, especially for large samples. According to them, the more prior information is included, the more informative priors we obtain, therefore we will be able to integrate information from earlier research findings into our analyses and also construct smaller credible Bayesian intervals for our parameter. It is worthy to note that in Bayesian statistics, a distribution is assumed for each parameter including variances and covariances, therefore a prior distribution for each corresponding parameter in the model is equally required.

Observed Evidence and Posterior Distribution

The next point worthy of consideration after stating the prior distribution is the observed evidence. This is achieved by considering some information on the variable for a certain number of cases or observation. Then with the aid of appropriate software, we construct or generate a data set with parameters such as

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the mean, standard deviation, proportion etcetera which were specified manually. As indicated earlier, in Bayesian study, the observed evidence in a form of data is purported to contain the parameter such as sample mean, variance and etcetera of the variable under study. This evidence is condensed by the likelihood function containing the information about the parameters given the data set. The likelihood function depicts the most likely values of the unknown parameters, given the data. (Schoot et al., 2013). They added that it is important to note that alternatively, a non-Bayesian method such as the maximum likelihood estimation method can be used to estimate the likelihood function. In the opinion of Schoot et al. (2013), prior distribution and the data under consideration are then combined by means of Bayes' theorem to generate the posterior distribution. To add, Bayesian inference utilizes the information or evidence usually in a form of statistics produced by observed data about the parameter or a group of parameters develop to posterior state of beliefs about the set of parameter(s) (Ravenzwaaij et al., 2018).

In this regard, we can apply our prior information in measuring the population parameters such as mean, variance, proportion and other features of the distribution for the sample under study.

Chapter Summary

This chapter reviewed literature in areas of modelling of macroeconomic and financial data using time series and other techniques that were unable to capture covariate informative about inflation dynamics both in global as well as Ghanaian context. Additional area of the literature exploration included materials on methods in the Gaussian-Bayesian setting, suitable for addressing the

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objectives of the research. The Gaussian-Bayesian paradigm of the literature spans GP and its definition, stationary process, stationary covariance function such as squared exponential covariance functions and Matérn's class of covariance functions. The review further delved into GP regression, Gaussian approximations, Gaussian model of the GP using sparse representation. In Bayesian method, prior specification, observed evidence and posterior distribution were discussed.

The salient outcome of the literature survey is that although traditional time series methods and its extensions have been widely employed in modelling macroeconomic and financial data, its use in examining inflation data is questionable. This is as result of assumption of constant variance about the data, inability to account for non-time dependent covariates relevant to inflation dynamics, rigorous and time-consuming nature, and above all, require arduous iterative techniques. In some instances, these researches attempted to model inflation using models that failed to capture conditional heteroscedasticity of the time series data of the inflation. The cumulative effect of the above challenges is likely to produce poor results leading to wrong conclusion.

Further to this, some researchers who used methods other than time series and its derivative procedures, suggested threshold models, need for standard models, inclusion of a wide set of potential explanatory variables in modelling inflation will in the long run capture heteroscedasticity thereby accounting for variability persistence hence providing more accurate estimates. Also, challenges such as short-term forecasting, inflexibility in the modelling procedure, and

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failure to incorporate appropriate variables were envisioned to be addressed by other methods. This is further buttressed by the opinion of Mbeah-Baiden (2013) that the use of heteroscedastic models in modelling of inflation is recommended to researchers and other stake holders because it lends more credence to research results by capturing volatility. Added to this, VAR models were also suggested in the literature. However, some challenges with this method are inability to establish presence of cointegration among variables, removal of long-term information and recognizing only short-term relationship between variables, and inability to enhance the effectiveness of the predicted autoregression models. These challenges were ascribed to the differencing approach used in the VAR models.

Further, Tumala et al. (2017) averred that the use of conventional Philips curve and some methods have been criticized because they are unable to properly track the trajectory and patterns of inflation as these models do not accommodate adequate variables to be used as regressors.

In respect of the Gaussian-Bayesian paradigm, it follows from analyses of the literature that Gaussian processes are non-parametric statistical models, composed of collection of random variables, any finite number of which have joint Gaussian distributions. The process is uniquely characterized by mean and covariance functions. These two exclusive features have given rise to its extensive application due to: its tractability in diverse situations, relative ease with which it solves prediction problems, its robustness to over-fitting, well-laid down processes to harmonize hyperparameters, and above all, the ability of the

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covariance function to encode the assumptions about the function being studied. Also, to model physical phenomena, the mean and covariance functions in general, ought to be made stationary to empower us conduct the proposed mathematical computations. The philosophy underlying stationarity is to provide proper level of simplification without foregoing much generalization after which reliability of the data is assumed. In addition, squared exponential covariance function appeared to be most widely-utilized function, however, Stein (1999), proposed the use Matérn's class of covariance functions to identify and model such phenomena.

Moreover, in the Bayesian paradigm, three key issues namely; background knowledge on the parameters of the model being tested, information contained in the data itself, and the combination of the first two items to achieve what is called posterior inference were reviewed.

In sum, it came to light that in one breadth, traditional time series methods and its extensions were broadly used to model macroeconomic and financial data while in another scope, other methods dominated the modelling procedure. This dichotomy in terms of methodology may be attributable to the associated strengths and weaknesses. Judging from this, it evident that a new methodology that minimizes if not absolutely addresses these challenges should be explored, hence the research topic "flexible Bayesian methods for inflation modelling in Ghana". The next chapter discusses research design and reviews appropriate theory of statistical methods of Empirical Bayes techniques solely in Gaussian-Bayesian paradigm, and Markov Chain Monte Carlo Methods (MCMC).

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CHAPTER THREE

METHODOLOGY

Introduction

This chapter explains procedures and principles that were followed in conducting the research work. On this note, research design and review of appropriate theory of statistical methods and inferential methods applicable within the Gaussian-Bayesian framework that seek to address the objectives were reviewed. These areas include detailed description of some basic elements of research design, and Gaussian process regression models for inflation, robust covariate information extraction, and performance evaluation under theory of statistical methods.

Research Design

Just as many activities are governed and regulated by a plan, so is research. In the research enterprise, the plan that dictates and controls activities is the research design. Many researchers appear to agree that research design provides appropriate framework that details theoretical account about the problem or issue under investigation. This entails a concise description of principal processes comprising identification or formulation of the research problem and its objectives, extensive literature reviewing, data collection and analysis from which a conclusion is reached. In the view of Ofori (2020), research design may be described as a plan, or outline, that regulates the research study in such a way that maximum control is exercised over factors that could adversely affect the validity of the research results. He further added that research design is the researcher's

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overall plan for getting answers to the research questions that guide the research study. Also Kumar, Leone, Aaker, and Day (2018), opined that the principal aim of research design is to offer the required framework that provides support and drive for the research work. One key issue in this process is the research approach that will help to elicit relevant information which will eventually help to attain the goals or objectives of the research. Although this process is made up of multiple events, these activities are not carried in isolation but rather are interconnected. The next section considers a review of EB method in the context of Gaussian-Bayesian setting.

Empirical Bayes Method

According to "Empirical Bayes Methods" (2020), EB methods are described as procedures in the statistical inference paradigm where use is made of the data in estimating the posterior distribution. Unlike the standard Bayesian methods where the posterior distribution is fixed before any data sets are observed, EB methods may be seen as guesstimate to fully Bayesian perspective of consideration of a hierarchical model in which parameters at the highest level of the hierarchy are set to their most likely values, instead of being integrated out. In the view of Braun (1988), estimation of parameter in the EB setting, interpretations regarding the parameters that we seek to achieve are conditioned on the observed data and the MLEs of the parameters of the prior. By using the evidential information or applying Expectation-Maximization (EM) algorithm through the marginal distribution, we can generate the MLEs of the parameter of the priors. Braun (1988), further added that in a right Bayesian investigation, fully

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specified hyperpriors for these parameters would instead be proposed and a standard Bayesian solution would be designed to solve such problem.

Referring to Rubin (1981), it has been reasoned and rationalized that customarily, EB solution in usual sense characterizes right estimation to fully Bayesian procedure on condition that likelihood function for the prior parameters is almost symmetric about a point in the interior of the parameter space and that non-informative hyperpriors are employed. Further to this, Casella (1985), described EB methods as one of the powerful data-analysis tools that has witnessed much usage in contemporary times. This high patronage in recent times stems from the fact that it is more resourceful and regularly generate higher quality estimates of parameters as opposed to classical or ordinary Bayes model.

In a similar description, EB methods are considered as a collection of methods and procedures in the computation, estimation and updating of parameters pertaining to prior probability distribution. The procedure which agrees with the traditional Bayesian statistical model, however transforms development of estimating initial assumptions (prior probability) and eventually reduce it into a two-step procedure ("Empirical Bayes Methods", 2020). According to "Empirical Bayes Methods" (2020) in situation when more than one parameter is known, but is known to be insufficient to generate a fixed-point probability distribution without subjective guesswork, EB methods are preferred to Entropy Principle ("Empirical Bayes Methods", 2020). Several estimation methods are available for assessing each kind of probability distribution in statistical inference framework, however, they all appear to share the same

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fundamental characteristics. The sequence of steps include construction of adaptable and regulating parameters called hyperparameters that produce a model with optimal output in a form of probability distribution as opposed to finding fixed values for each parameter in prior assumption, followed by performing test of prior probability on a sample data, which transforms the hyperparameters into an estimate value for each parameter. Having considered prior probability on sample of data, we then use this new prior assumption, which is technically considered and described as posterior probability, as a prior probability when running the model on the full data set in situations where hyperparameters and the prior probability parameters use different distributions. One superiority and benefit of this procedure is that we can continuously have mutual relation between conjugate prior and posterior ("Empirical Bayes Methods", 2020).

To digest the subject matter completely and give full exposition of EB methods utterly, views expressed in a summary of Klebanov et al. (2018), appeared to concisely describe the basic concepts underlying the EB methods. Klebanov et al. (2018) described EB methods to perform statistical inference in two parts. According to Klebanov et al. (2018), the first part consists of estimation of the prior using all the appropriate measurements x_m, followed by the application of Bayes' rule with that prior for each x_m separately (Berger & Bernardo, 1991). Using the EM algorithm developed by Dempster, Laird and Rubin (1977) as a standard tool, we then maximize the likelihood function, $L(\pi)$. Although major issues under discussion pertain to Bayesian statistics, EB methods may be described as combination of Frequentist and Bayesian statistics.

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They added that though the prior is chosen by MLE, the real individual parametrizations are inferred using Bayes' rule. Shedding more light on EB methods, Si, Dyk and Hippe (2015) asserted that we compute MLE or a maximum a posteriori probability (MAP) of hyper-parameters by optimizing their likelihood function and then fit group-level parameters in a traditional Bayesian paradigm. The mathematical perspective of the parameters and algorithms of the EB methods in the inflation modelling process are reviewed in the section below.

A vital aspect of statistical inference is modelling of observed data based on appropriate statistical modelling paradigm. There exist many paradigms in statistics for example, we have the Bayesian paradigm, classical, parametric, semi-parametric and non-parametric etc. Most importantly, the parametric, semi-parametric and non-parametric methods can be considered within both the Bayesian and classical paradigms. This chapter of the thesis focuses on the development of appropriate statistical models, computational and inference methods for observed inflation dataset.

Flexible Inflation Model with covariates

Suppose inflation dataset $\{y(t_i), t_i\}_{i=1}^{m}$, where t_i denotes the time stamps over which the inflation $y_i(t)$ was observed. Given that there exist covariates informative on the dynamics of inflation that are either time dependent or otherwise but can impact on the behaviour and dynamics of the inflation process. Suppose we denote such covariates by $x = (x_1, x_2, \ldots x_p)$. Then, we assume that the inflation process evolves through the Bayesian-Gaussian process model given by

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$$y(t) = Z(t) + \varepsilon, \varepsilon \sim \mathrm{N}\left(0, \sigma_\varepsilon^2 I_n\right) \tag{3.1}$$

where $y(t) = \left[y(t_1), y(t_2), \ldots, y(t_m)\right]$ and $t = [t_1, t_2, \ldots, t_m]$. The entire process $Z(t)$ in equation (3.1) is modelled with a Gaussian process (Rasmussen & Williams, 2006b).

Mathematically, the entire process with corresponding parameters and hyperparameters is displayed in model (3.2) below;

$$Z(t) \sim GP\left(m_z(t), \tau\left(\delta, \sigma_\tau^2, \lambda_1, \lambda_2, \lambda_3, \theta_1, \theta_2\right)\right), \delta = |t - t'| \tag{3.2}$$

where $\delta = |t - t'|$ is Euclidean distance between the inputs.

$$\tau\left(\delta, \sigma_\tau^2, \lambda_1, \lambda_2, \lambda_3, \theta_1, \theta_2\right) = \tau_1(\lambda_1, \sigma_\tau, \delta) + \tau_2(\theta_1, \lambda_2, \delta) + \tau_3(\theta_2, \lambda_3, \delta)$$

$$\tau_1\left(\lambda_1, \sigma_\tau^2, \delta\right) = \sigma_\tau^2 \exp\left(-\lambda_1^2 \delta^2\right) \tag{3.3}$$

$$\tau_2(\theta_1, \lambda_2, \delta) = \theta_1^2 \left(1 + \frac{\sqrt{5}\delta}{\lambda_2} + \frac{5\delta^2}{3\lambda_2^2}\right) \exp\left(-\frac{\sqrt{5}\delta}{\lambda_2}\right) \tag{3.4}$$

$$\tau_3(\theta_2, \lambda_3, \delta) = \theta_2^2 \left(1 + \frac{\sqrt{3}\delta}{\lambda_3}\right) \tag{3.5}$$

The hyperparameters are modelled with inverse gamma and normal models as follows.

$$\begin{aligned}
&\lambda_1 \sim N\left(x'\beta, \sigma_{\lambda_1}^2\right), \beta \sim N\left(\mu_\beta^0, \textstyle\sum_\beta^0\right) \\
&\theta_1 \sim IG\left(a_{\theta_1}^0, b_{\theta_1}^0\right), \theta_2 \sim IG\left(a_{\theta_2^0}, b_{\theta_2^0}^0\right) \\
&\lambda_2 \sim IG\left(a_{\lambda_2}^0, b_{\lambda_2}^0\right), \lambda_3 \sim IG\left(a_{\lambda_3}^0, b_{\lambda_3}^0\right) \\
&\sigma_\tau^2 \sim IG\left(a_\tau^0, b_\tau^0\right), \sigma_\varepsilon^2 \sim IG\left(a_\varepsilon^0, b_\varepsilon^0\right),
\end{aligned}$$

where $x = \left[x_1, x_2, \ldots x_p\right]$. The designed vector x can be viewed as transformation of the usual $n \times p$ design matrix in various regression modelling frameworks. The choice of the appropriate transformation or statistics or feature extraction

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from design matrices is crucial in Gaussian process regression tasks as the choice can affect the predictive performance of the model. This is because the presence of outlying observations cannot be ignored in any statistical data. As a result, there is the need to consider methods which can either control for their presence automatically or automatically delete them. In this thesis, it is proposed that outlying observations at the predictor level be allowed whiles their overall impact is controlled automatically through the use of novel methods.

Probability distribution -based covariate information

Covariates are very useful in any modelling task since they can inform the choice of appropriate direction for analysis and eventually the appropriate methodology that best fits the data at hand. Usually, there exist intrinsic relationship whether linear or nonlinear between the underlying response and some covariates if available. In other words, every regression problem is generated by some mechanism that exhibits some hidden relationship among competing variables.

The application of covariate information can present some challenges in model specification as the covariate space may come with complexities in terms of size, type etc. Regarding size, large covariates may present large design matrix which may lead to computational and storage issues. In some cases, the adopted modelling framework does not permit the use of design matrix but a design vector. This normally happens in hierarchical modelling in which many singleton parameters are modelled hierarchically. Furthermore, there can exist some outlying covariates which may pose extra issues in both modelling and

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estimation of parameters if not controlled. The above discourse suggests that some pragmatic attention and treatment ought to be given to covariates and their inclusion in statistical models at the modelling phase. Available covariates should be incorporated into a given model in a principled fashion to encode modelling assumptions, a modeller wants to consider properly, in order not to have drastic impact on the parameter estimation. As a result, interest in appealing approaches for handling covariate information hierarchically within the Gaussian process regression framework has been revived recently. Mensah, Ofori, and Howard (2020) proposed the use of robust covariates via the Orthogonalized Gnanadesikan-Ketterning (OGK) statistics (Maronna et al., 2019) extraction in the application of Gaussian process regression in systolic blood pressure monitoring.

Automatic Outlier-Resistant Predictors for Gaussian Process Regression

It is possible for predictors to be designed to control outliers automatically so that the presence of outliers cannot affect modelling and parameter estimation. The design of such predictors can be based on statistics that naturally has the ability to offer control of outlier so that such property can be inherited by the adopted statistics automatically.

Probability Distribution-based Predictors

Proposals for extracting design predictor vectors from design predictor matrices that have the potential to offer automatic control for outlying predictors are considered here. The fundamental idea hinges on the application of probability concept. In particular, the probability density function of a random

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variable. The probability density function is an important concept in statistics as basic data structures and other interesting features, for example the quantiles can be derived easily. Adopting the probability density concept coupled with the concept of moments, novel predictor selection schemes as well as varied design vector extractors with the ability to retain the original data features whiles controlling for the impact of extreme predictor values of models can be developed. In this thesis, proposals for predictors with both single-level and double-level controls were considered. Formal exposition on the above proposals is provided in brief next.

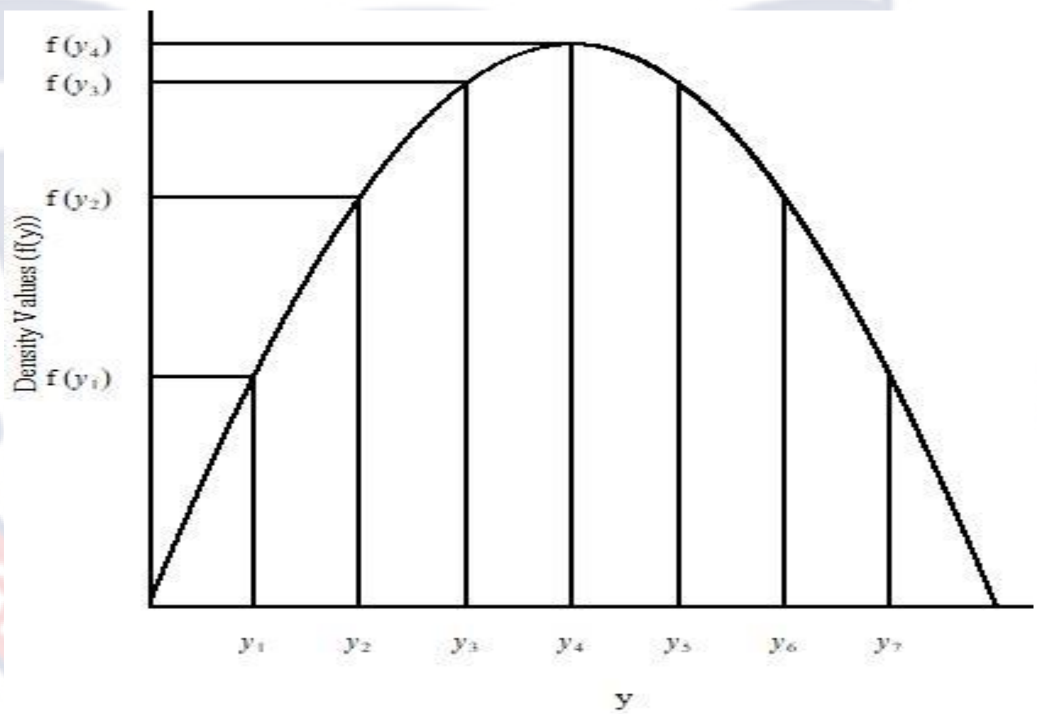

Figure 1: Probability Density Function for the Random Variable *y*

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Predictors with Single-Level Outlier Control

The moments of random variables present a unique way for understanding key structures of the distribution of a random observation generated by random variables. In particular, the non-central moments (i.e., moments about the origin) can serve as a good source of assessing the contribution of each out of sample observation to the common measure of central tendency generated by the sample. Considering the first moment about the origin, of a random variable, say, X following the population with probability density function (pdf), or probability mass function (pmf), $f(y,\theta)$, where $\theta > 0$ denotes a population characteristic. Writing for $f(x,\theta)$, suppressing the dependence on θ, for notational convenience, it is straightforward to define non-central moments of the random variable X. The k^{th} moment of X about the origin can be defined as;

$$\mathrm{E}\left[X^k\right] = \begin{cases} \int_{-\infty}^{\infty} x^k f(x)\,dx & \text{if } X \text{ is continuous} \\ \sum_x x^k f(x) & \text{if } X \text{ is discreet} \end{cases} \tag{3.6}$$

Note that with the above notation, it can be assumed for example the nature of $f(x)$ is as shown in Figure 1 above for better appreciation and understanding of the concept of moment. Let $C(x)$ denote the statistic that defines the contribution of each possible candidate predictor observation, x to the moment $\mathrm{E}\left(X^k\right)$. Then, it is clearly evident from Figure 1, that the random variable (statistics) $C\left(x^k\right)$ based on (3.6).

$$C\left(x^k\right) = x^k f(x) \tag{3.7}$$

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can be observed as a transformation or mapping of the original X unto the probability density space, allowing an easy analysis of the effective impact of each possible candidate of X. Thus, it can serve as an appealing source for developing vital predictor statistics for various modelling purposes, particularly, in cases where continuous predictors are involved. Setting $k=1$ in (3.7) gives;

$$C(x) = x\,f(x) \tag{3.8}$$

the expected value contribution statistic which can be viewed as the contributions from each observed predictor value to the common centre. Basically, $f(x)$ in (3.8) acts as one of the natural data-driven weights. Thus, weights the x according to their position under the probability density function space. With this perspective, it is straightforward to see that values that are extreme in relation to the common centre are highly weighted such that their contributions are somewhat with no negative effect on the centre. This property will be preserved when these contribution-based predictors are utilized in regression tasks instead of the original predictors. In addition, the extraction of statistics from the $C(x)$'s comes with the intrinsic ability to preserve the underlying features thereby offering automated control for presence of outlying observations. The following statistics are proposed for use as probability density function-based predictors with single-level ability for controlling outliers automatically. Consider a statistical model with p continuous predictors say, $X=\left[X_1, X_2, \cdots X_p\right]$, where $X_j=\left[X_{j1}, \cdots X_{jn}\right]'$, $j=1,2,\cdots,p$. The mean, median and the highest contribution statistics are defined as

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$$X_{c_j} = \frac{1}{n}\sum_{j=1}^{n} C\left(x_j\right), C\left(x_j\right) = \left[C\left(x_{j1}\right), C\left(x_{j2}\right), \cdots, C\left(x_{jn}\right)\right] \tag{3.9}$$

$$\tilde{X}_{c_j} = median\left[C\left(x_{j1}\right), C\left(x_{j2}\right), \ldots, C\left(x_{jn}\right)\right] \tag{3.10}$$

and

$$\tilde{X}_{cj} = \max\left[C\left(x_{j1}\right), C\left(x_{j2}\right), \cdots C\left(x_{jn}\right)\right] \tag{3.11}$$

Predictors with Double-level Outlier Control

For probability density function based predictors derived from the contribution statistics $C(x)$, with the capability to provide double-level automatic control for outlying predictors, Orthogonalized Gnanadesikan-Kettenring (OGK) statistic Maronna et al. (2019), computed from the $C(x)$ predictors and the median contributors statistics are considered. The OGK predictor for the j^{th} predictor variable X_j is computed using

$$X_{c_j}^{o} = \frac{\sum_{j=1}^{n} C\left(x_j\right)\omega\left(z_j\right)}{\sum_{j=1}^{n}\omega\left(z_j\right)}, z_j = \frac{C\left(x_j\right) - \tilde{\mu}_0}{\tilde{\sigma}_0} \tag{3.12}$$

where $\tilde{\mu}_0$ denotes the median of $C(x)$ and $\tilde{\sigma}_0$ is the median absolute deviation (MAD) of $C(x)$. The double-level control ability of the above proposal is seen in the following ways:

1. Firstly, the use of the contribution statistics, $C(x)$ which has automatic self-control based on the effect of the density weights $f(x)$, and
2. Secondly, the derivation of robust statistics from the $C(x)$.

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Empirical Bayes Inference for Gaussian Process Model

Let $f(y \mid \xi)$ be the likelihood function generated by a Bayesian Gaussian process model with a set of free parameters ξ and data (y,t). With the assumption that the parameters are random but not fixed, let ξ follow distribution of the form $g(\xi)$ before the data is either observed or used. Using the data, the distribution, $g(\xi)$ can be updated into another distribution, $f(\xi \mid y)$, termed posterior distribution, using conditional probabilities

$$f(\xi \mid y) = \frac{f(y \mid \xi) g(\xi)}{f(y)} \tag{3.13}$$

where $f(y) = \int f(y \mid \xi) g(\xi) d\xi$. From equation (3.13), it means the following distributional statements are true.

$$f(\xi \mid y) \propto f(y \mid \xi) g(\xi) \tag{3.14}$$

$$f(\xi \mid y) \propto \frac{1}{f(y)} \tag{3.15}$$

It can easily be inferred from (3.15) the usual term normalizing constant or marginal likelihood given to $f(y)$ within the statistical community. It also follows from (3.13) that the marginal likelihood can be expressed as

$$f(y) = \frac{f(y \mid \xi) g(\xi)}{f(\xi \mid y)} \tag{3.16}$$

The standard information provided by equation (3.16) is that the marginal likelihood is proportional to the product of data likelihood and the assumed joint prior distributions of the parameters. Also, it is inversely proportional to the true joint posterior distribution.

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$$f(y) \propto f(y \mid \xi) g(\xi) \tag{3.17}$$

$$f(y) \propto \frac{1}{f(\xi \mid y)} \tag{3.18}$$

The standard way for making parameter inference for ξ is through the use of the posterior distribution defined in (3.13). Bayesian inference for parameters, say, ξ based on equation (3.17) yields the empirical Bayes approach (Chang et al., 2019). In empirical Bayes methods, usually the natural logarithm of the marginal likelihood is optimized for parameter inference. This is because, the logarithm of the marginal likelihood and the true marginal likelihood attain the same maximum.

The natural logarithm of $f(y)$ is of the form

$$\log f(y) \propto \log f(y \mid \xi) + \log g(\xi) \tag{3.19}$$

Maximization of equation (3.19) with respect to ξ yields the empirical Bayes estimates for making parameter inference. For more details on empirical Bayes approach and its application to Gaussian regression, see for examples Carlin and Louis (2000)**,** and Shi and Choi (2011). It can be observed the empirical Bayes approach avoids the computation of complex high dimensional integrals.

Though, the Empirical Bayes methods avoids the computation of the normalization constant, the source of intractability in the derivation of the true joint posterior, the optimization of the optimization function here can also be complex for models with large parameters. This is due to the construction of the second order partial derivative matrix (Hessian). Although the Empirical Bayes method can be used, it is not attractive for the model under consideration due to

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the complexity of the Hessian matrix. As a result, we take into account the exact inference approach using MCMC (Markov Chain Monte Carlo Methods).

A Gaussian Process Model with Exact Bayesian Inference

This section considers posterior inference for the developed Bayesian fitting models. The posterior distribution defined by most Gaussian process regression models are complex leading to intractable marginal likelihoods. For such statistical model, alternative approaches are considered for posterior inference. One of such methods that is widely used in the Bayesian paradigm is the MCMC methods. MCMC methods are constructed using marginal posterior distributions termed as the full conditionals in the MCMC literature. A full conditional for a given parameter is derived by considering all factors in the joint distribution of all parameters and variable when expressed conditional distributions. By this technique, it is easy to see that the MCMC method is a general-purpose method applicable to all statistical models that adopts the Bayesian modelling framework. In addition, the nature of the marginal posteriors is key to the choice of typical MCMC method for posterior inference. The fundamental idea that the MCMC hinges on is the construction of a Markov Chain. A Markov chain say, $\{W^{(t)}\}$ is sequence of dependent random variable

$$W^{(0)}, W^{(1)}, W^{(2)}, W^{(3)}, \ldots, W^{(t)}, \ldots$$

such that the probability distribution of the current random variable, $W^{(t)}$ given the past variables depend only on the most current past, $W^{(t-1)}$ (Robert et al.,

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2010). The resulting conditional distribution is known as the transition kernel or Markov kernel in the MCMC literature. Illustratively, it can be written for $W^{(t)}$ as

$$W^{(t+1)} \mid W^{(0)}, W^{(1)}, W^{(2)}, W^{(3)}, \ldots, W^{(t)} \sim K\left(W^{(t)}, W^{(t+1)}\right).$$

For illustrative purpose, consider a simple random walk Markov chain satisfying

$$W^{(t+1)} = W^{(t)} + \eta_t,\ \eta_t \sim \mathrm{N}(0,1)$$

independently of $W^{(t)}$. The corresponding Markov kernel $K\left(W^{(t)}, W^{(t+1)}\right)$ is the normal density $N\left(W^{(t)}, 1\right)$.

Markov Chain Monte Carlo Methods is based on the working principle that can be easily described as follows. Given a target probability density, say, $g(w)$, a Markov kernel K is constructed with stationary distribution g and a Markov chain $\left\{W^{(t)}\right\}$ is then generated based on this kernel such that the limiting distribution of $\left\{W^{(t)}\right\}$ is g. Posterior inference can then be based on the resulting Markov chain $\left(W^{(t)}\right)$ (Robert et al., 2010). Construction of a kernel K that is related to any arbitrary probability density g can be a challenging task in MCMC. Interestingly however, several methods exist for building theoretically valid Markov chain for any probability density g. Gibbs sampling algorithm is the simplest MCMC methods for generating Markov chain for posterior inference for Bayesian statistical models that generates full conditional (marginal posterior) distribution with standard forms. However, for Bayesian models with full conditionals not belonging to any known or identifiable standard form, standard

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method for constructing a Markov chain for posterior inference are the Metropolis-Hasting and Metropolis algorithms (Hastings, 1970; Metropolis et al., 1953). It is sometimes possible to blend two or more standard MCMC sampling steps to construct a Markov chain for a complex statistical model within the Bayesian framework.

This is motivated by the nature of the full conditionals corresponding to the assumed Bayesian model. When two or more standard MCMC methods are combined to construct Markov chain for simple Bayesian statistical model a hybrid MCMC method results. Hybrid methods for building MCMC samplers belong to the advanced MCMC methods. Andrieu and Thoms (2008); Gelman et al. (2013); Givens and Hoeting (2013), provide comprehensive introduction to the advanced MCMC methods with illustrative implementation outlines. Now applying the idea to the Bayesian Gaussian process regression model (3.1), it is easily seen that the nature of the model will allow a both closed form and non-closed form full conditionals for the model parameters. In particular, the parameters $Z_{(t)}, \beta$ and σ_{ε}^{2} generate full conditionals that are identifiable.

Specially, $Z_{(t)}$ and β generate multivariate normal full conditional distributions while σ_{ε}^{2} yields inverse gamma as full conditional distribution. These standard full conditionals suggest the use of Gibbs sampling steps for building Markov chain for the above parameters. The remaining parameters namely, $\theta_1, \theta_2, \theta_3, \lambda_1, \lambda_2, \lambda_{3,}$ and σ_{τ}^{2} all generate full conditionals that cannot be identified to have any known distribution forms.

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The set of full conditionals suggest the use the of Metropolis-Hasting sampling algorithm for constructing appropriate Markov chain for posterior inference. Combining the Gibbs sampling steps and the Metropolis-Hasting steps appropriately yield an appealing hybrid MCMC algorithm for joint posterior inference for all parameters in the Gaussian process regression model (3.1). The corresponding hybrid MCMC inference algorithm for model with covariate is outlined in Appendix A2. The derivations of the full conditional distributions utilized in the Algorithm 2 are detailed in Appendix B. Note that we have written *Z(t)* as *Z* in the algorithm to avoid conflict with the index of iteration *t*.

Performance Evaluation

A comprehensive exposition on how the proposed methods were implemented and assessed is considered in this section of the thesis. The models are considered adequate for modelling the financial data based on some well-known statistical model fitting performance measures such as the Standard Mean Absolute Fitted Error (SMAFE), Mean Absolute Fitted Error (MAFE) and Mean Squared Fitted Error (MSFE) (Amewonor, 2022). These measures are defined as follows:

$$\text{SMAFE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i - \hat{y}_i}{y_i}\right|$$

$$\text{MAFE} = \frac{1}{n}\sum_{i=1}^{n}\left|y_i - \hat{y}_i\right|$$

and

$$\text{MSFE} = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2, y_i = y\left(t\right)_i$$

$\hat{y}_i$ and y_i are respectively the fitted and true observations on a given financial

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variable. The utility of the proposed methods was evaluated using real financial dataset which spans the period January, 2007 to November, 2017 from the Bank of Ghana website. The dataset is multivariate dataset covering observations on five financial variables namely Interest Rate, Exchange Rate, Inflation on food, Inflation on non-food and Monetary Policy Rate. The data is of dimension

131×5.

Implementation of Proposed Methods

A comprehensive exposition on how the proposed methods were implemented is considered in this section of the thesis. The developed algorithms were implemented using the R statistical package. The proposal for the generating alternative predictors using the non-central moment-based statistics requires the estimation of the probability density function $f(y)$. The implementation adopted the non-parametric estimation method for $f(y)$. The non-parametric density estimation is based on the kernel density estimation procedure. The kernel density estimator of $f(y)$ is of the form

$$\hat{f}(y) = \frac{1}{n}\sum_{i=1}^{n} K_h(y, y_i), K_h(y, y_i) = \frac{1}{h} K\left(\frac{y - y_i}{h}\right) \tag{3.20}$$

for a symmetric kernel function $K_h(\cdot)$ and smoothing parameter, h. This was implemented in R using the package ks (Duong, 2007). The parameter h was taken to be cross-validation estimator implemented in the above package. Details on the kernel density estimation theory is presented in (Scott, 2015). Also, for its practical implementation for both univariate and multivariate dataset, see for example, (Duong, 2007). In particular, all codes were written using Tinn-R and run on an Intel (R) Core (TM) i7, 6700 processor Windows PC 3.40 GHz workstation. Four different applications were considered using the real datasets

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leading to four different models being fitted. The first line of application was made on the inflation on food and inflation on non-food as responses and in each case with the remaining variables considered as predictor or covariates. The application here considered the alternative covariates. On the other hand, the second line of application utilized the responses mentioned above but the predictors were in the original state. That is, some appropriate statistics from the predictors other than the proposal discussed earlier were used. In particular, the mean, median and OGK statistics calibrated as $X, \tilde{X}$, and X^o respectively were computed directly from the predictor data.

Choice of proposal distribution and parameter initialization

On the choice of proposal distributions, the following were considered based on the restrictions on the parameters. Basically, the proposal distributions came from the inverse gamma and the normal family of distributions with;

$$g_1 = IG\left(2, \theta_1^{[t]}\right), g_2 = IG\left(2, \theta_2^{[t]}\right), g_3 = N\left(\lambda_1^{[t]}, 0.05\right)$$
$$g_4 = IG\left(2, \lambda_2^{[t]}\right), g_5 = IG\left(2, \lambda_3^{[t]}\right), g_6 = IG\left(2, \theta_\tau^{[t]}\right)$$

MCMC algorithms require starting values for parameters of interest as can be seen in Appendix A2 since it is iterative in nature. Parameter initialization utilized for all the data applications assumed the following settings.

$$\theta_1^{[0]} = \theta_2^{[0]} = \lambda_2^{[0]} = \lambda_3^{[0]} = 2$$
$$\lambda_1^{[0]} = \sum X_i, \sigma_\tau^{2[0]} = \sigma_\varepsilon^{2[0]} = 1, \beta^{[0]} = 0$$

With the above settings, $Z^{[0]}$ was initialized with a random draw from its posterior, $N\left(\mu_Z^{[0]}, \Sigma_Z^{[0]}\right)$,

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$$\Sigma_Z^{[0]} = \left[\sigma_\varepsilon^{2-[0]} I_n + K_0^{-1} \right]^{-1}, \mu^{[0]} = \sigma_\sigma^{2-[0]} \Sigma_Z^{[0]} y$$

$$K_0 = \tau\left(\delta, \sigma_\tau^{2[0]}, \lambda_1^{[0]}, \lambda_2^{[0]}, \lambda_3^{[0]}, \theta_1^{[0]}, \theta_2^{[0]}\right)$$

The hyperparameters settings for the priors considered the following;

$$a_\varepsilon^0 - a_{\theta_1}^0 - a_{\lambda_1}^0 - a_{\lambda_2}^0 - a_{\lambda_3}^0 - 1$$

$$b_\varepsilon^0 = a_\tau^0 = b_{\lambda_2}^0 = b_{\theta_2}^0 = 2$$

$$b_{\lambda_1}^0 = b_{\lambda_3}^0 = b_\tau^0 = 3$$

$$\Sigma_\beta^0 = 1000 I_p, \mu_\beta^0 = 0.$$

The convergence of the algorithm was assessed via the assessment of representative chain samples using trace plots.

Chapter Summary

Chapter three is the core of the thesis. It focuses on the development of the Bayesian theoretical framework for the research. Based on the developed Bayesian framework, the Gaussian process regression models for the inflation process are developed. The modelling process was developed in such a way that it examines the nature of the variables defining the inflation structure to assess the degree of dependences existing among them in order to determine the appropriate modelling direction. Basically, the highly dependent variable among the set of variables is modelled in the terms of the remaining such that the natural dependence structure is preserved.

In view of this, a generic inflation process is modelled as Gaussian process regression in terms of its observation time, with mean and covariance functions treated randomly. The random mean and covariance functions are further modelled. However, the mean function was assumed to be deterministic with

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value 0 for simplicity, whiles the covariance was modelled. A three-component compound covariance function comprising one Gaussian covariance function and two Martin Class covariance functions of different orders was considered with varied hyperparameters depending on the complexity of the assumed covariance structure. The flexibility of the covariance function allowed multiple ways of encoding relevant modelling assumptions. Thus, the remaining data information was incorporated via the covariance function by modelling an appropriate hyperparameter. Specially, the remaining variables were treated as covariates and subjected to an appropriate transformation to handle the possibility of autocorrelation, inter-correlations, and outliers.

The transformation was based on the probability density function theory and gave rise to different schemes for extracting potential candidate statistics to be used as covariates. The key data from the probability density function transformation were the contribution statistics, thus, the name contribution covariates were adopted for the types of covariates utilized in thesis. The covariate generating scheme yielded covariates that allow automatic control for outliers at single and double levels. These covariates were particularly; mean, median, maximum and OGK, which served as single and double levels of controlling outliers respectively. These covariates were incorporated in the Gaussian process regression model leading several models for the inflation process.

Furthermore, Bayesian inferential methods were developed fitting and inference for the developed models. In particular, a hybrid Markov Monte Carlo

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(MCMC) inference algorithm. Statistical measures, namely, SMAFE, MAFE and MSFE, for assessing the developed methods empirically were also developed. The chapter ends with an outline for implementing for the inference methods in which some, direction for choosing appropriate initial values and proposal distributions were discussed.

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CHAPTER FOUR

RESULTS AND DISCUSSION

Introduction

This Chapter of the thesis presents and discusses the results obtained from the implementation of the proposed methods using the financial dataset. The presentation starts with the derivation of alternative covariates from the original covariate information and ends with results from the fitting of the Gaussian process models using both the alternative and original covariates.

Alternative covariate extraction

The information extracted from the original covariates as alternative covariates for modelling the financial data using Gaussian process regression techniques are presented in this section. As stated earlier, the alternative covariate proposal hinges on probability density functions underlying the original covariate data for which their estimation is crucial. The implementation in this thesis made use of the Kernel density estimation approach which has been highlighted under the implementation section. First, the nature and the appropriateness of the dataset for implementation is examined before giving exposition on the fitting performances of the developed methods. Figure 2 shows a plot of the observation on the macroeconomic variables defining the financial data against time (month). The time was taken to be the sequence with which the observations were observed and coded and scaled to be within the interval [0, 2]. The varying dynamics of the pattern underlying the macroeconomic variables is evident. Table 1 reports some

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key statistics computed from the raw data. The statistics considered include the minimum and maximum variable-specific observations, the out of sample mean, median and variance. The differences in the information contained in the financial data across the macroeconomic variables can be clearly seen. Most importantly, as expected of financial variables, the variation in variability associated with the macroeconomic variables is outstanding. It can be observed that Interest rate (y_1) exhibits the highest variability, followed by Inflation on non-food (y_4) with Inflation on food (y_3) and Monetary Policy (y_5) reporting moderate variability, while Exchange rate (y_2) recorded the least variability.

The nature of the probability density function-based features extracted from the original data as alternative covariates for fitting Gaussian process regression using macroeconomic variables with the ability to handle both auto-correlation and inter-correlations among variables is reported as shown in Table 1. It is clear that the contribution statistics can inherit the underlying features of the data via the data pattern. This is usually done via the underlying probability density function of the random variable that defines the data. Also, since the density function constitutes the building block of the contribution statistics, there is the natural likelihood to be passed on certain key features to any statistics developed out of the contribution statistics. Furthermore, the kernel density estimator considered as estimator for the probability density function used in the contribution statistics handles macroeconomic specific auto-correlations. This allows the contribution statistics to ensures that the variable specific auto-correlations are taken in account automatically as these are vital in financial

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variables. Elaborating on how the auto-correlations are handled by the density function, it can be observed that the estimator (3.20) comprises a kernel function of the form $K\left(\frac{y-y_i}{h}\right)$. This kernel function obviously, considers all possible distances each observation has with all observations with a composite measure utilized in the estimation. By this, it can be seen that all possible levels of lags are automatically handled. In addition, repeated observations which are inevitable in financial data are not an issue with the use of such statistics as they are also automatically handled.

The typical statistics that can be utilized as alternative covariates extracted from the contribution statistics is reported in Table 2. It is not difficult to see differences among the key statistics. Using these key statistics $\left(X_c, \tilde{X}_c, X_c^o, X_c^m\right)$ as the information content of the data as statistics are meant to summarize the information in the given data, it can be observed that y_1 stands out, followed by y_5, and y_4. However, y_2 and y_3 record modest information. Putting all together, the following relative arrangement can be used for the information content of the macroeconomic data in terms of the overall contribution statistics.

$$C(y_2) < C(y_3) < C(y_4) < C(y_5) < C(y_1)$$

This arrangement informed the underlying contribution statistics may be attributed to the inter-correlations existing among the macroeconomic variables. Table 3 reports the estimated smoothing parameters utilized for the estimation of probability density functions of the macroeconomic variables. Again, the differences in magnitude and their corresponding functions of the estimated

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probability density functions can be clearly deduced. This also adds to the already established inference that the macroeconomic variable is different in pattern, information content and other features.

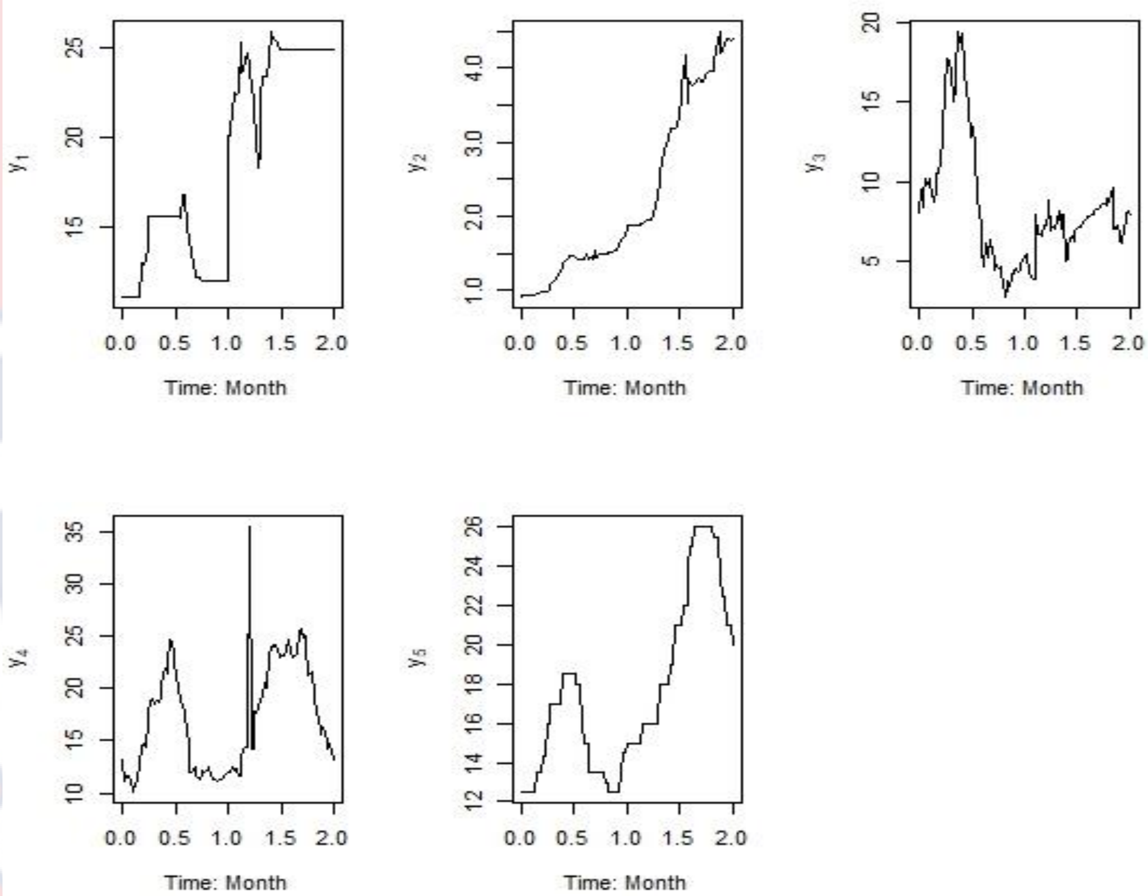

Figure 2: Plot of Macroeconomic Variables against Time

y_1 = Interest rate, y_2 = Exchange rate, y_3 = Inflation on food, y_4 = Inflation on non-food, y_5 = MPR. Time coded and scaled to be within [0, 2].

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Table 1: Key Statistics of Data; (n = 131 × 5)

	Macroeconomic Variables				
Statistic	y_1	y_2	y_3	y_4	y_5
Minimum	11.130	0.912	2.780	10.100	12.500
Maximum	25.870	4.484	19.400	35.430	26.000
Mean	18.756	2.255	8.415	17.110	17.638
Median	18.756	2.255	8.415	17.110	17.638
Variance	31.182	1.374	15.059	25.377	19.056
X	18.756	2.255	8.415	17.110	17.638
$\tilde{X}$	18.756	2.255	8.415	17.110	17.638
X^o	18.712	1.878	7.179	16.689	16.101

Source: (Author's Computations)

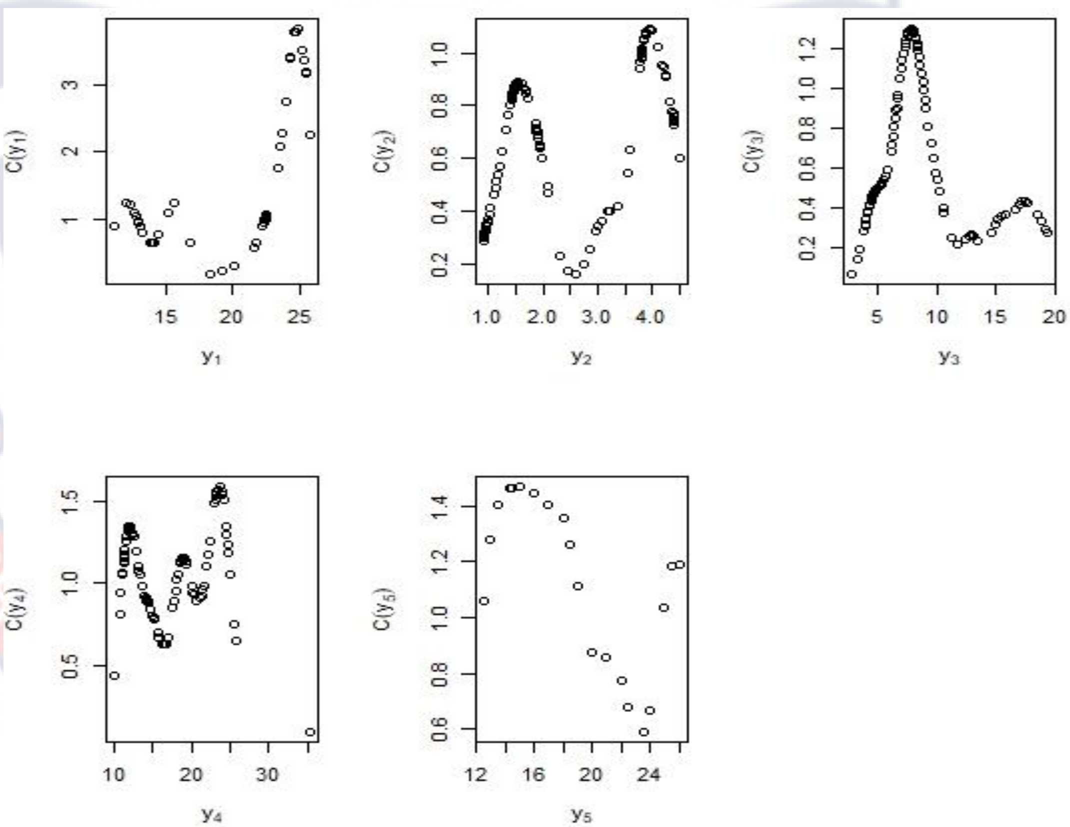

Figure 3: Dynamics of Contribution Statistics, $C\ (y)$ of Macroeconomic Variables

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Table 2: Covariate Information from Original Data

	Macroeconomic Variable				
Covariates	$C(y_1)$	$C(y_2)$	$C(y_3)$	$C(y_4)$	$C(y_5)$
X_c	2.003	0.696	0.757	1.128	1.230
$\tilde{X}_c$	1.242	0.758	0.718	1.152	1.276
X_c^o	1.117	0.755	1.744	1.148	1.278
X_m	3.833	1.090	1.300	1.582	1.466

Source: (Researcher's Output, 2023).

Table 3: Estimated Smoothing Parameters for Financial Variables

	Financial variables				
	y_1	y_2	y_3	y_4	y_5
h	$\hat{h}_1$	$\hat{h}_2$	$\hat{h}_3$	$\hat{h}_4$	$\hat{h}_5$
	0.909	0.231	0.764	1.073	1.164

Source: (Author's Computations)

Results: Inflation on food application

This part presents the results obtained from applying Gaussian process regression (GPR) models with extracted covariates to inflation on food as the response. In particular, for this application, the response y_t was set as the inflation on food while the remaining macroeconomic variables were considered as covariates with their transformed forms in terms of the proposed covariates were used. The fitting performance of the developed methods in terms of the ability to recover the underlying functional trends existing in the response observation was of interest here.

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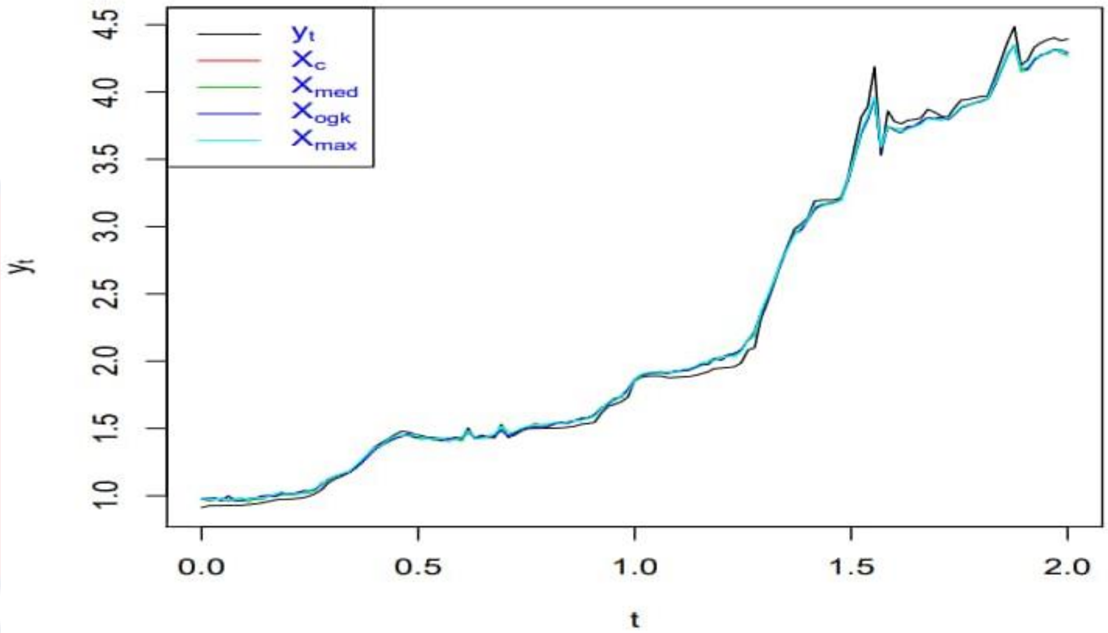

Figure 4: Fitted Dynamics of Inflation on Food using Feature Covariate Information

Figure 4 shows the fitting performance of the Gaussian process regression model with the alternative covariates X_c, $\tilde{X}_c$, X_c^o and X_c^m. The representation of these statistics in the Figure 4 follows $X_c = X_c$, $\tilde{X}_c = X_{\text{med}}$, $X_c^o = X_{\text{ogk}}$ and $X_c^m = X_{\max}$. With regards to the multi-coloured curves, the colour codes are as follows. The black curve is the true inflation trend while the red, green, blue and sea blue curves are the fitted curves corresponding respectively to the GPR with $X_c, \tilde{X}_c, X_c^o$ and X_c^m. The corresponding values of the above statistics used as covariates are as presented in Table 2. It is easily seen that all the alternative covariates ensure better recovery of the underlying true trend in the response observations. Despite, the above reasonable fitting performance achieved by all the alternative covariates, there exists some key differences in the fitting

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performance among the GPR models in terms of the covariates used. This is evident in the differences between the fitted and true curves.

Figure 5 presents the fitting performance of the GPR model based on the original covariates characterized as $X, \tilde{X},$ and X^{o} in Table 1. The true inflation trend is represented by the black curve while the red curve denotes the GPR based on $X = X_c$, green curve denotes the GPR with $\tilde{X}_c = X_{\text{med}}$ and blue curve represents the GPR based on X_{ogk} as covariates. Again, it can be observed that the use of each of these covariates in the GPR allows better recovery of the underlying true trend in the inflation observations with some obvious difference at the level of the information content that the statistics are able to summarize from the covariate data. Furthermore, comparing the fitting performance of the above GPRs with their corresponding alternative covariates counter-parts in Figure 4 above $\left(i.e.\ X_c, \tilde{X}_c, \text{ and } X_{\text{ogk}}\right)$, it can be noticed that some level of differences exists. It is not difficult to notice that GPRs with alternative covariates out perform their original counter-parts by some magnitude.

Also, as seen in Figures 4 and 5, there exist some differences in performances of the GPR models based on the varied covariate types considered, we sort to well differentiate the within and between covariate-level performances. To achieve that and better understand the magnitude of the difference in the fitting performance among the variant covariates, the fitting is quantified in using key statistics usually considered in the Gaussian process regression modelling as discussed in Chapter Three. Table 4 and Table 5 report the overall fitting performances of the GPR models in terms of the performance outlined in Chapter

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Three, namely, the Standard Mean Absolute Fitted Error (SMAFE), Mean Absolute Fitted Error (MAFE) and Mean Squared Fitted Error (MSFE) for the GPRs based on the alternative (transformed) and original covariates. Clearly, all the alternative covariates record low fitted errors with some marginal differences. Also, the GPR models based on the original covariates yield low fitted errors with marginal differences. Nevertheless, comparing the magnitude of the fitted errors among computing models, it can be observed the models using the transformed covariates report lower fitted errors.

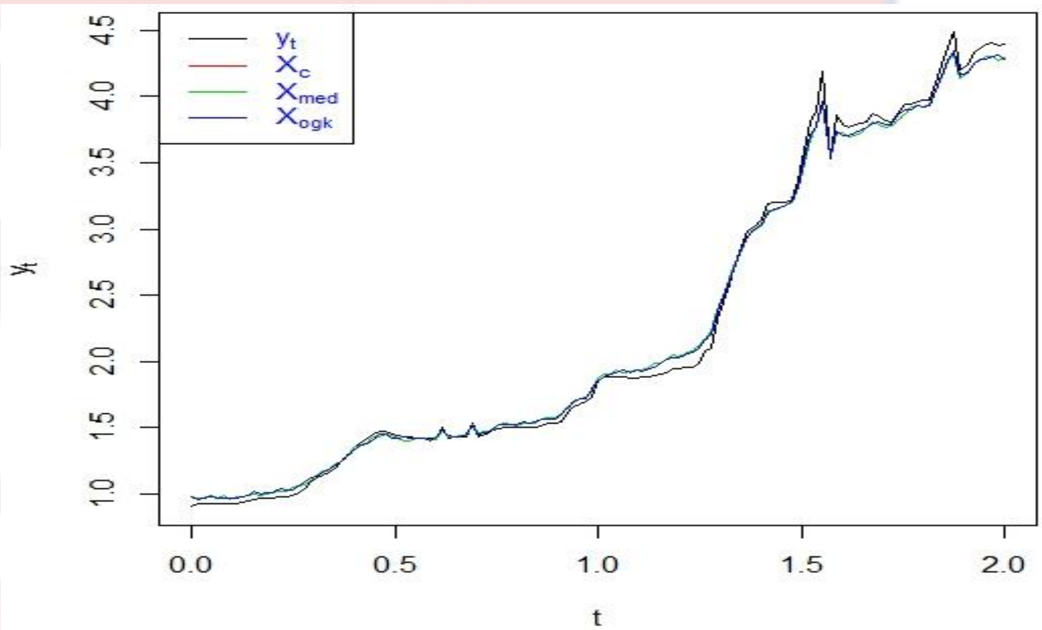

Figure 5: Fitted Dynamics of Inflation on Food using Original Covariate

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Table 4: Fitted Error Statistics: Covariates used

Fitted Error	X_c	X_c^o	$\tilde{X}_c$	X_c^m
SMAFE	0.0210	0.0209	0.0210	0.0211
MSFE	0.0029	0.0030	0.0029	0.0030
MAFE	0.0420	0.0428	0.0420	0.0420

Source: (Author's Computations)

Table 5: Fitted Error Statistics: Original Covariate used

Fitted Error	X	X^o	$\tilde{X}$
SMAFE	0.0231	0.0231	0.0245
MSFE	0.0033	0.0033	0.0039
MAFE	0.0460	0.0460	0.0500

Source: (Author's Computations)

Table 6: Covariates: Posterior Estimates of Key Parameters with their 95% Bayesian Credible Intervals (BCIs)

	X_c	X_c^o	$\tilde{X}_c$	X_c^m
ϑ	$\hat{\vartheta}$	$\hat{\vartheta}$	$\hat{\vartheta}$	$\hat{\vartheta}$
λ_1	45.533 [33.511, 59.615]	45.533 [22.968, 45.600]	47.257 [42.671, 52.395]	43.103 [37.562, 48.521]
λ_2	1.147 [0.409, 3.469]	1.147 [0.369, 2.739]	1.154 [0.410, 3.149]	1.139 [0.411, 3.198]
λ_3	29.680 [5.513, 45.665]	29.680 [5.513, 45.665]	30.501 [8.514, 58.084]	27.744 [7.472, 52.856]
θ_1	0.423 [0.211, 0.886]	0.423 [0.2142, 0.8523]	0.423 [0.2142, 0.8523]	0.436 [0.219, 0.920]
θ_2	0.598 [0.253, 1.418]	0.598 [0.268, 1.555]	0.578 [0.243, 1.319]	0.588 [0.258, 1.348]
β_1	12.537 [−28.698, 54.045]	12.537 [−45.201, 61.195]	11.382 [−41.776, 64.427]	8.038 [−26.593, 40.569]
β_2	4.746 [−54.410, 63.334]	4.746 [−52.839, 61.688]	6.098 [−55.043, 64.983]	2.449 [−58.474, 63.456]
β_3	7.206 [−48.067, 65.654]	7.206 [−44.136, 60.050]	11.227 [−42.807, 65.783]	3.679 [−53.660, 62.222]
β_4	7.305 [−48.256, 62.093]	7.305 [−42.454, 60.847]	12.196 [−38.676, 63.883]	2.595 [−57.330, 61.316]
σ_τ^2	0.143 [0.108, 0.197]	0.143 [0.114, 0.218]	0.1364 [0.103, 0.183]	0.143 [0.103, 0.198]
σ_ε^2	0.100 [0.067, 0.147]	0.099 [0.0638, 0.1365]	0.100 [0.068, 0.147]	0.100 [0.068, 0.148]

Source: (Author's Computations)

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Table 7: Original Covariates: Posterior Estimates of Key Parameters with their 95% Bayesian Credible Intervals (BCIs)

	X	X^o	$\tilde{X}$
ϑ	$\hat{\vartheta}$	$\hat{\vartheta}$	$\hat{\vartheta}$
λ_1	50.075 [42.143, 58.335]	50.075 [56.145, 77.617]	53.831 [47.395, 66.818]
λ_2	1.130 [0.395, 3.088]	1.130 [0.433, 3.204]	1.182 [0.456, 3.108]
λ_3	32.465 [8.869, 61.461]	32.465 [11.661, 82.612]	35.014 [9.654, 67.941]
θ_1	0.421 [0.228, 0.869]	0.421 [0.210, 0.837]	0.432 [0.216, 0.862]
θ_2	0.612 [0.252, 1.593]	0.612 [0.263, 1.491]	0.606 [0.263, 1.550]
β_1	0.681 [−49.5486, 50.482]	0.681 [−49.360, 49.765]	0.756 [−45.846, 48.606]
β_2	0.757 [−59.944, 60.362]	0.757 [−62.168, 62.729]	−0.261 [−60.135, 59.548]
β_3	1.202 [−50.766, 51.542]	1.202 [-59.975, 54.016]	1.365 [−47.981, 51.356]
β_4	0.852 [−51.425, 51.300]	0.852 [−50.822, 51.415]	1.131 [−51.061, 53.913]
σ_τ^2	0.135 [0.098, 0.181]	0.135 [0.094, 0.166]	0.131 [0.098, 0.180]
σ_ε^2	0.101 [0.068, 0.153]	0.101 [0.070, 0.157]	0.103 [0.069, 0.151]

Source: (Author's Computations)

Table 6 reports the 95% credible intervals for the estimates of the posterior of model parameters that generated the fitted characteristics illustrated in Figure 4. The recorded estimates are the point estimates comprising posterior means and their corresponding Bayesian credible intervals. It is not difficult to notice that the parameter estimates are consistent as they exhibit marginal differences in magnitude among some parameters and appreciable differences among others. The differences are dominant in the hierarchical regression estimates, the β s for the smoothness parameter λ_1 . These visible differences may be due the information content of the statistic utilized depending on the sort of statistics as a result of their ability to summarize the information in the covariate dataset. However, the above ability can be seen in the fitting of the λ_1 model. That is the

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estimate of λ_1 should reflect the information content of the statistics used as covariates so that their usefulness can be assessed.

Further to this, one vital aspect of the estimates obtained here is that they are all within the parameter-specific restrictions. Most importantly, for covariance parameters that determine the smoothness characteristics of the Gaussian process, $\lambda_1, \lambda_2,$ and λ_3, the smaller the value of λ_1, the larger the values of λ_2 and λ_3 the better the smoothness of features generated by the compound covariance function. This in turn, results in better smoothness features of the resultant Gaussian process. Also, the estimated Gaussian process variances, $\hat{\theta}_1, \hat{\theta}_2$ and $\hat{\sigma}_\tau^2$ from the four transformed covariates are highly comparable. Finally, the estimated error variance, $\hat{\sigma}_\varepsilon^2$ from the four alternative covariates are also comparable. These comparable parameter estimates obtained from the use of the four transformed covariates resulted in the Gaussian processes with fitting performances that were similar as discussed earlier. Table 7 presents the 95% credible intervals for the estimates of the posterior of model parameters that generated the fitted characteristics illustrated in Figure 5. It can be noticed that similar consistent estimates are obtained for the model parameters $\lambda_1, \lambda_2, \lambda_3, \theta_1, \theta_2, \sigma_\tau^2$ and σ_ε^2 with the use of non-transformed covariates X, X^0 and $\tilde{X}$ as covariates in fitting the Gaussian process regression model. Also, marginal differences can be seen among the estimates for the three covariates. With regards to the estimates for the βs it is clearly seen that estimates similar for X and X^0 but comparing with those of the $\tilde{X}$ some significant differences can be observed. Most importantly, there exist

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change of signs of estimates. This change of signs of estimates is attributable to the presence of high autocorrelation and multicollinearity existing within and among the variables respectively that are not controlled with the use of the above statistics as covariates. It is interesting to note that though median statistics are very useful in the presence of outlying observations, the presence of high repeated observations and multicollinearity can limit its utility. For repeated observations are inevitable in any data generated by a financial variable as result of natural autocorrelation.

Results: Inflation on non-food

The results obtained from the implementation of the Gaussian process regressions (GPR) models built on the inflation on non-food as response with the remaining macroeconomic variables as predictors are presented in this section of the thesis. Note that both original and transformed covariates were considered as in the case of the inflation on food application. We begin the discussion with the examination of the performance in recovering the inflation on non-food pattern underlying the data. Figure 6 presents the performance of the Gaussian process regression model in fitting the inflation on non-food observation using the transformed covariates, calibrated as $X_c, \tilde{X}_c, X_c^o,$ and X_c^m . The black curve denotes the true inflation on non-food observations over the observational time in months coded, the red curve is the fitted inflation on non-food using the GPR model with X_c calibrated as y_c. The green curve denotes the fitted inflation on noon-food using the GPR model with X_c^{med} calibrated on the graph as y_{med} while the blue and sea blue curves are the fitted inflation on non-food pattern using the

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GPR models with $X_c^{\max}$ and X_c^{ogk} respectively corresponding to the colour codes of $y_{\max}$ and y_{ogk}. The values of the statistics utilized as covariates are provided in Table 2. The fitting capabilities of the GPR models are clearly exhibited with the varying recovery abilities at the levels of the information contained in the covariate statistics (statistics used as covariates). Interestingly, it looks like all the covariates boosted the GPRs in fitting the inflation non-food data well. However, a closer examination depicts some marginal differences. This marginal difference may be attributed to the varying potential of the covariate statistics in summarizing the evidence that the original predictor data contains. One particular observation is the red curves not being seen as it is covered by some of the fitted curves, indicating that the corresponding covariates are exhibiting the same performance in fitting the GPR models. Figure 7 shows the corresponding fitting performance of the GPR models with the original covariates. The statistics used as covariates are $X, \tilde{X}, \text{and } X^o$, with the fitted inflation on non-food corresponding to them which are calibrated as $y_c, y_{\text{ogk}}, y_{\text{med}}$. Again, it can be observed that the GPR models with the above covariates exhibited good fitting performance. There exist some significant differences among the fitted curves as well as observed in the GPR models with the transformed covariates but at different magnitudes. Table 8 reports the posterior estimates of the key GPR model parameters using the transformed covariates with their corresponding 95% Bayesian credible intervals. These parameter estimates generated the fitted inflation on non-food underlying pattern presented in Figure 6. It can be observed that the estimates are legitimate as they satisfy the parameter restrictions. Also,

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the obtained estimates are consistent in sign, yielding comparable results for most of the predictors. In particular, it can be observed that the estimates for, X_c, X_c^o, and X_c^m yielding much comparable estimates than that of $\tilde{X}_c$. Table 9 records the posterior estimates of GPR model parameters based on the original covariates $X, \tilde{X}$, and X^o with their corresponding 95% Bayesian credible intervals. The estimates generated the fitted results illustrated in Figure 7. Apparently, some level of sign of inconsistency can be observed among the estimates for the βs as compared with those obtained for the GPR model with transformed covariates. Again, this may be due to the different ways the statistics utilized as predictors handle the information content of the covariate data in terms of existing inter-relationships, presence of outliers, autocorrelation as well as repeated observations. Table 10 and Table 11 report the fitting errors (MAFE, MSFE, SMAFE) associated with the GPR models using the transformed and original covariates respectively. The fitting errors associated with the different covariates (transformed and original) are different in magnitude.

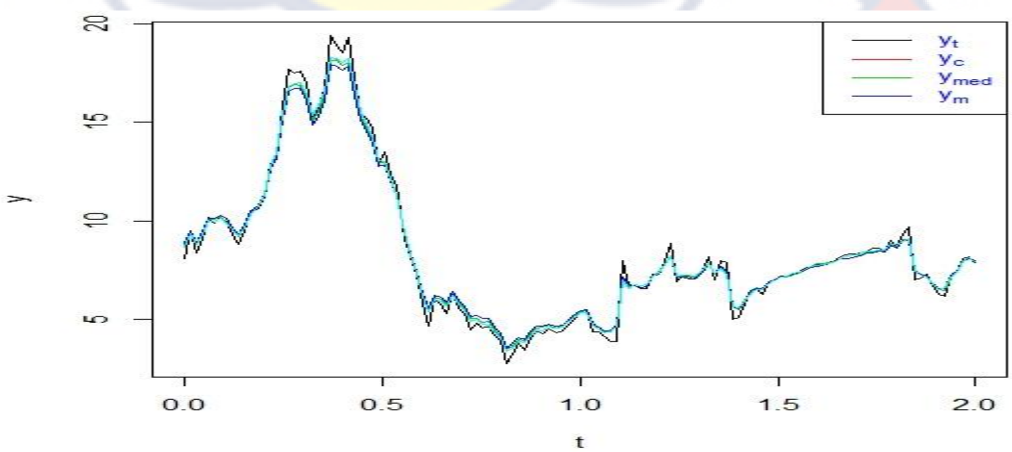

Figure 6: Fitted Dynamics of Inflation on Non-Food using Varied Covariate Information

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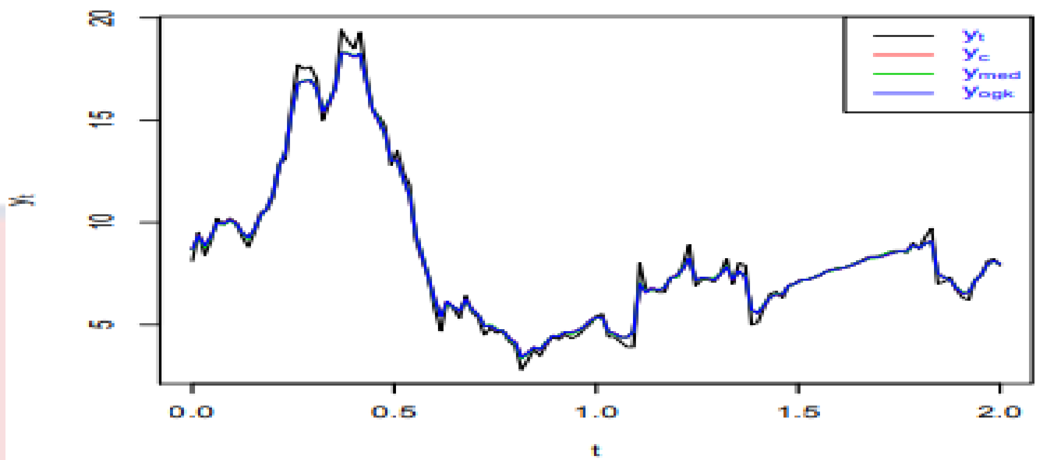

Figure 7: Fitted Dynamics of Inflation on Non-Food using Original Covariate

Table 8: Transformed Covariates: Posterior Estimates of Key Parameters with their 95% Bayesian Credible Intervals (BCIs)

	X_c	X_c^o	$\tilde{X}_c$	X_c^m
ϑ	$\hat{\vartheta}$	$\hat{\vartheta}$	$\hat{\vartheta}$	$\hat{\vartheta}$
λ_1	16.992 [9.620, 24.813]	16.992 [10.566, 19.761]	23.618 [17.159, 28.502]	17.815 [12.221, 22.979]
λ_2	0.954 [0.352, 2.637]	0.954 [0.336, 2.332]	1.012 [0.334, 2.838]	0.956 [0.312, 2.686]
λ_3	11.089 [2.508, 24.779]	11.089 [2.456, 19.611]	15.248 [4.151, 29.990]	11.462 [3.012, 23.361]
θ_1	0.452 [0.220, 0.957]	0.452 [0.220, 0.905]	0.446 [0.224, 0.899]	0.461 [0.220, 0.979]
θ_2	0.583 [0.264, 1.331]	0.583 [0.264, 1.492]	0.572 [0.263, 1.297]	0.583 [0.257,1.333]
β_1	5.318[−33.653, 42.99]	5.318 [−48.180, 55.185]	7.531 [−42.887, 57.915]	3.448 [−28.945, 34.249]
β_2	2.179[−56.160, 60.83]	2.179 [−53.670, 58.142]	3.653 [−53.520, 65.075]	0.665 [−61.461, 59.961]
β_3	1.722[−56.033, 64.31]	1.722 [−54.321, 61.105]	3.227 [−55.494, 61.162]	1.680 [−58.362, 58.409]
β_4	2.853 [−52.684, 54.38]	2.853 [−44.899, 52.124]	7.188 [−41.148, 56.820]	1.152 [−56.417, 60.048]
σ_τ^2	0.229 [0.143, 0.382]	0.229 [0.159, 0.395]	0.189 [0.129, 0.269]	0.215 [0.140, 0.328]
σ_ε^2	0.099 [0.066, 0.148]	0.099 [0.065, 0.138]	0.107 [0.070, 0.164]	0.099 [0.066, 0.146]

Source: (Author's Computations)

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Table 9: Original Covariates: Posterior Estimates of Key Parameters with their 95% Bayesian Credible Intervals (BCIs)

	X	X^o	$\tilde{X}$
ϑ	$\hat{\vartheta}$	$\hat{\vartheta}$	$\hat{\vartheta}$
λ_1	16.991 [12.647, 29.594]	16.991 [10.096, 24.215]	16.473 [11.899, 22.451]
λ_2	0.978 [0.316, 2.535]	0.978 [0.323, 2.523]	0.917 [0.352, 2.619]
λ_3	11.200 [2.649, 28.878]	11.200 [2.38423.716]	10.680 [12.764, 22.611]
θ_1	0.445 [0.225, 0.890]	0.445 [0.237, 0.893]	0.455 [0.222, 0.897]
θ_2	0.627 [0.265, 1.663]	0.627 [0.257, 1.519]	0.583 [0.246, 1.481]
β_1	0.485 [−43.504, 45.119]	0.485 [−41.303, 43.370]	−0.395 [−43.694, 43.131]
β_2	−0.910 [−59.749, 58.746]	−0.910 [−60.016, 59.201]	0.009 [−61.156, 60.341]
β_3	−0.100 [−60.142, 58.526]	−0.100 [−60.350, 59.035]	0.128 [−60.386, 59.825]
β_4	0.612 [−45.168, 49.136]	1.420 [−47.181, 49.859]	1.380 [−46.135, 49.955]
σ_τ^2	0.225 [0.150, 0.348]	0.225 [0.152, 0.381]	0.224 [0.147, 0.345]
σ_ε^2	0.097 [0.066, 0.140]	0.097 [0.065, 0.138]	0.096 [0.067, 0.140]

Source: (Author's Computations)

Table 10: Transformed Covariates: Fitted Error Statistics

Fitted Error	X_c	X_c^o	$\tilde{X}_c$	X_c^m
MAFE	8.4167	8.4167	8.4160	8.4146
MSFE	79.3981	79.3981	79.4919	79.3694
SMAFE	1.0481	1.0481	1.0470	1.0477

Source: (Author's Computations)

Table 11: Original Covariates: Fitted Error Statistics

Fitted Error	X	X^o	$\tilde{X}$
MAFE	0.2355	0.2340	0.2355
MSFE	0.1093	0.1054	0.1093
SMAFE	0.0321	0.0316	0.0321

Source: (Author's Computations)

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The utility of compound covariance function

In this section, we analyse the nature of the covariance function utilized in model (3.2) highlights its usefulness in modelling key features of the data considered. The covariance used was of the form;

$$\tau\left(\delta,\sigma_\tau^2,\lambda_1,\lambda_2,\lambda_3,\theta_1,\theta_2\right)=\tau_1\left(\lambda_1,\sigma_\tau^2,\delta\right)+\tau_2\left(\theta_1,\lambda_2,\delta\right)+\tau_3\left(\theta_2,\lambda_3,\delta\right),$$

$$\tau_1\left(\lambda_1,\sigma_\tau^2,\delta\right)=\sigma_\tau^2\exp\left(-\lambda_1^2\delta^2\right)$$

$$\tau_2\left(\theta_1,\lambda_2,\delta\right)=\theta_1^2\left(1+\frac{\sqrt{5}\delta}{\lambda_2}+\frac{5\delta^2}{3\lambda_2^2}\right)\exp\left(-\frac{\sqrt{5}\delta}{\lambda_2}\right)$$

$$\tau_3\left(\theta_2,\lambda_3,\delta\right)=\theta_2^2\left(1+\frac{\sqrt{3}\delta}{\lambda_3}\right)\exp\left(-\frac{\sqrt{3}\delta}{\lambda_3}\right)$$

The corresponding process variance can be obtained by evaluating the covariance function at $\delta=0$, yielding

$$\begin{aligned}\sigma_p^2&=\tau\left(\delta=0,\sigma_\tau^2,\lambda_1,\lambda_2,\lambda_3,\theta_1,\theta_2\right)\\&=\tau_1\left(\lambda_1,\sigma_\tau^2,\delta=0\right)+\tau_2\left(\theta_1,\lambda_2,\delta=0\right)+\tau_3\left(\theta_2,\lambda_3,\delta=0\right)\\&=\sigma_\tau^2+\theta_1^2+\theta_2^2\end{aligned}$$

Also, the smoothness of the Gaussian process is determined by multiple parameters namely, $\lambda_1,\lambda_2,$ and λ_3 with the requirement that λ_1 should be small whiles λ_2 and λ_3 are large. This compound covariance function is basically a combination of the Gaussian and Martin-Class covariance functions. Thus, allows modelling short-, medium- and long-term variabilities. It seems using the smoothness parameters to discriminate the covariate types would not paint a clear picture. Considering the process variance, it may be simpler to device a way to distinguish the usefulness of the covariates in fitting the macroeconomic data.

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Table 12 reports the estimated process variances, $\hat{\sigma}_p^2$, of the corresponding fitted Gaussian process regression models based on the different predictors proposed for the inflation on food observations. Based on the magnitude of the estimated process variances, the following statements can be established. For the Gaussian process regression models using transformed covariates, $\text{GPRM}\hat{X}_c$ recorded the smallest process variance, followed by $\text{GPRM}X_c^m$, while, $\text{GPRM}X_c$ and $\text{GPRM}X_c^o$ yielded the same process variance but the largest among the competing GPRMs. That is, we can write

$$\text{GPRM}\tilde{X}_c < \text{GPRM}X_c^m < \text{GPRM}X_c = \text{GPRM}X_c^o .$$

Also, for the Gaussian process regression models with original covariates, the process variance- based assessment of performance can be expressed as;

$$\text{GPRM}\tilde{X} < \text{GPRM}X^o = \text{GPRM}X .$$

Considering comparable GPR models across the varied covariate information used, it can be observed that the GPR models based on transformed covariates exhibit small process variances than their original covariates counter-parts. This observation leads to the following statements that $\text{GPRM}X_c < \text{GPRM}X$, $\text{GPRM}X_c^o < \text{GPRM}X^o$, and $\text{GPRM}\tilde{X}_c < \text{GPRM}\tilde{X}$. Table 13 provides the estimated process variances, $\hat{\sigma}_p^2$, for the fitted Gaussian process regression using the various covariates types for the inflation on non-food modelling. Again, using the magnitude of the estimated process variances, the following statements are clearly evident. Gaussian process regression models based the transformed covariates yield smaller process variances than their

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comparable GPR models with original covariates as observed in the case of the inflation on food application. With the above observations, it can be deduced that, $\text{GPRM}\tilde{X}_c < \text{GPRM}X_c^m < \text{GPRM}X_c = \text{GPRM}X_c^o$.

Furthermore, $\text{GPRM}X_c < \text{GPRM}X$, $\text{GPRM}X_c^o < \text{GPRM}X^o$, and $\text{GPRM}\tilde{X}_c < \text{GPRM}\tilde{X}$. Finally, it can be established that, Gaussian regression models based on transformed covariates for modelling macroeconomic variables are more appealing than GPR models with the original covariates.

Table 12: Inflation on Food. Estimate of Process Variances of Fitted GPR Models

$\hat{\sigma}_p^2$	$\text{GPRM}X_c$	$\text{GPRM}X_c^o$	$\text{GPRM}\tilde{X}_c$	$\text{GPRM}X_c^m$
	0.6795	0.6795	0.6602	0.6788
	$\text{GPRM}X$	$\text{GPRM}X^o$	$\text{GPRM}\tilde{X}$	
$\hat{\sigma}_p^2$	0.6868	0.6868	0.6859	

Source: (Author's Computations)

Table 13: Inflation on Non-Food. Estimate of Process Variances of Fitted GPR Models

	$\text{GPRM}X_c$	$\text{GPRM}X_c^o$	$\text{GPRM}\tilde{X}_c$	$\text{GPRM}X_c^m$
$\hat{\sigma}_p^2$	0.7714	0.7714	0.7151	0.7674
	$\text{GPRM}X$	$\text{GPRM}X^o$	$\text{GPRM}\tilde{X}$	
$\hat{\sigma}_p^2$	0.8162	0.8162	0.7709	

Source: (Author's Computations)

Chapter Summary

This chapter of the thesis basically focuses on the presentation of the results obtained upon application of the developed methods using real data collected from macroeconomic variables. Deductions and expositions based on

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the reported results are also provided. Macroeconomic data are time series in nature, thus possess interesting features, for example, interrelationships, autocorrelations, and repeated observations that are vital in practice. This information can suggest important practice directions for addressing certain issues and should be considered in model specification when they are available. In this study, Gaussian process regression approaches for modelling macroeconomic variables that can take into account the above data information in the model specification were explored. Dwelling on the natural ability of the probability density function (pdf) to handle extreme observations and the fact that moments are key features of pdf, appropriate statistics were developed to transform information in predictor data matrix into a vector information. Using the flexibility of the compound function of the Gaussian process model, the vectorised predictor information was incorporated into the GP model for easy development of inferential schemes for parameter inference. Thus, an appropriate MCMC Method was developed and experimented using real macroeconomic data from the Bank of Ghana's website. The utility of the moment-based macroeconomic predictors over original macroeconomic predictors of inflation was clearly evident in terms of the process variance as an overall performance measure.

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CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Overview

This chapter of the thesis summarizes the entire research work, and presents some derived conclusions, recommendations based on the results obtained from the implementation of the developed methods using real macroeconomic data. In addition, some potential directions for extending the work are outlined as future work. The chapter begins with summary, conclusions, followed by recommendation and eventually ends with future work.

Summary

This thesis focused on constructing flexible Bayesian methods that allow easy incorporation of inflation process specific covariates for robust analysis of inflation persistence and forecasting in Ghana. It provides evidence in support of the subject matter under study in area of the background matters relating to inflation where it was established that macroeconomic data display structural instability which might have been caused by inherent economic features that could be relevant to inflation dynamics. Further to this, inflation and its significance in the economy, state of inflation in ancient and contemporary era are discussed from which two schools of thought and definition of inflation emerged. Demand-pull and cost-push inflation are discussed as two main types of inflation. This is followed by low-to moderate, galloping, and hyperinflation as levels of inflation. This constitutes the summary of Chapter One.

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The literature review further examined inflation from national as well as global perspectives. This led to statement of the problem where some researchers were identified to have used several methodologies which are in some instances at variance with one another leading to ambiguities in methods, therefore appropriate procedure in Gaussian-Bayesian method is required. Throwing additional light and putting the research topic in a better perspective, it is established that although traditional time series methods and its extensions among others, have been widely used in modelling macroeconomic and financial data, their use in examining inflation data is questionable because the overall effect of this challenge among others is likely to produce poor results leading to wrong conclusion. This dichotomy in terms of methodology may be attributable to the associated strengths and weaknesses. This constitutes the foundation phase of the problem under study which called for Gaussian-Bayesian approach. This is the synopsis of the second chapter.

The possession of the mean and covariance functions of the Gaussian process in terms: of its docility in various situations, the ability of the covariance function to encode the assumptions about the function under study, relative ease with which it addresses problems of prediction, its robustness to over-fitting, and above all, possession of well-laid down processes to harmonize hyperparameters, affords it the strength over other methods pointed out earlier. Although squared exponential covariance function appeared to be most widely-utilized function, Stein (1999), ranked Matérn's class of covariance functions to be more appropriate in identifying and modelling such phenomenon as inflation.

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Moreover, the three foremost components of the Bayesian concept comprising the background knowledge about the parameters of the model under test, information contained in the data itself, and a blend of the first two to provide posterior inference were explored.

The pivotal part of the research work that dictates and drives all the ideas is contained in Chapter Three of the thesis. It begins with development of Bayesian framework that necessitated the GPR models for the inflation process. The modelling was designed to examine the variables constituting the inflation structure such that the direction of the model was provided based on degree of dependences among the variables. The intrinsic dependency structure of the highly correlated dependent variable within the set of variables is preserved by modelling with respect to the other variables.

A three-component compound covariance function made up of one Gaussian covariance function and two Martin Class covariance functions of different orders was considered with varied hyperparameters depending on the complexity of the assumed covariance structure. The flexibility of the covariance function enabled the incorporation of data information by modelling an appropriate hyperparameter. With this, the remaining variables were considered as covariates and subjected to probability density function theory to cater for the possibility of autocorrelation, inter-correlations, and outliers. The key data from the probability density function transformation were the contribution statistics or covariates. The covariates generated, thus mean, median, maximum and OGK statistics that provided single and double levels of controlling outliers

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respectively, were incorporated in the GPR model generating several models for the inflation process.

Bayesian inferential methods of a hybrid MCMC inference algorithms were developed for parameter estimation and posterior inference. As a result, statistical metrics namely, SMAFE, MAFE and MSFE, were constructed to evaluate the methods empirically.

The outcomes that were achieved when the methodologies outlined in Chapter Three were applied to actual macroeconomic data were copiously expounded in Chapter Four. The implications of the findings are also given. As time-time dependent as they are, macroeconomic variables exhibit fascinating properties including autocorrelations, repetitions and interrelationships that are crucial in real-world situations. These attributes are key to identifying solutions to specific challenges in practice when taken into consideration during model specification. This is exactly what was done when the GPR approaches outlined in Chapter Three were used to explore the aforementioned data. Capitalizing on the probability density function's (pdf) innate capacity to manage extreme observations and given that moments are essential components of pdfs, suitable statistics were derived to convert data from predictor data matrices into vector data. The GP model was able to easily create inferential techniques for parameter inference by using the vectorized predictor information, using the flexibility of the compound function of the Gaussian process model. This is how, using actual macroeconomic data from the Bank of Ghana's website, a suitable MCMC Method was created and tested. Considering the process variance as an overall

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performance metric, it was evident that the moment-based macroeconomic predictors were much more useful than the original macroeconomic predictors of inflation.

Conclusions

We have proposed and implemented a novel perspective for modelling inflation on food and non-food in terms of other macroeconomic variables that affect inflation via Gaussian process regression. The inflation trend is modelled as a Gaussian process with mean zero and a compound covariance function in terms of observation time. Related macroeconomic variables are considered as predictors and allowed to enter the Gaussian process regression model by affecting the covariance function of Gaussian process. Due to some key features of the predictors such as predictor-specific autocorrelations, predictor inter-relationships etc., moment-based statistics are developed to compress the matrix information into a vector for easy inclusion in the main Gaussian process regression model.

The moment-based statistics inherit the automatic outlier control capabilities of the associated probability density functions underlying the predictor variables. This allows a formal way for constructing alternative predictors at varying levels of outlier controls. Combining this with the Gaussian process regression technique allowed development of various Gaussian process regression models within the Bayesian framework for modelling inflation. MCMC methods were developed for the built models and experimented using real macroeconomic data which contained inflation on food and non-food from the

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BoG website. The results from the implementation show moment-based macroeconomic predictors boost Gaussian process regression models for both inflations on food and non-food than the ordinary statistics from the original predictor data.

Recommendations

Some possible recommendations based on the results realized from the implementation of the developed approaches for application of Bayesian Gaussian process regression to macroeconomic data for which inflation on food and non-food are inclusive. The following recommendations are considered. We recommend:

1. The use of Bayesian Gaussian process regression models with macroeconomic predictor statistics boosters for fitting inflation data in Ghana.
2. The use of moment-based linear macroeconomic boosters with the naturally capability of handling within variable-specific inter-relationships (autocorrelations) and between variable inter-relationships in Bayesian Gaussian process regression for modelling inflation data. Due to the inter-correlations that naturally exist among macroeconomic variables, modelling inflation in terms of such variables one can utilize the dependence structure to convert the design matrix into a design vector and incorporate the resultant model into the Gaussian process regression model via the mean function or covariance function for computational and memory (storage) savings.

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3. The use of composite (compound) covariance functions in Gaussian process regression for macroeconomic variables for modelling short-, medium- and long-term dynamic patterns.
4. The application of Gaussian process variance as a simple assessment tool for the assessment of the appropriateness or performance of moment-based linear macroeconomic boosters.

Also, this project can be extended for future work in the following ways:

1. The model selection approach adopted for distinguishing the various covariate extraction methods proposed was based on the Gaussian process variance. Formal model selection methods can be developed within the Bayesian paradigm for example, Bayesian Information Criteria, Variational Bayes Information Criteria etc., in that regard.
2. Consider Gaussian process regression methods with non-linear covariate extraction methods. The proposed methods for extracting appropriate covariates from other available macroeconomic variables were based on linear correlations as inter-relationships existing among the variables. In the case of non-linear correlations, proposed methods cannot handle this information well.
3. Extension of Gaussian process regression principle to building conditional models, using the highly correlated macroeconomic variables.

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APPENDICES

APPENDIX A: MCMC ALGORITHMS

A1. MCMC Algorithm of Model with Covariates

1: Initialization: Set MCMC sample size, M.

2: Set $t=0$ and set starting values $Z^{[0]}, \beta^{[0]}, \lambda_1^{[0]}, \lambda_2^{[0]}, \lambda_3^{[0]}, \theta_1^{[0]}, \theta_2^{[0]}, \theta_3^{[0]}, \sigma_\varepsilon^{2[0]}, \sigma_\tau^{2[0]}$

3: Sample $\theta_1^{[t+1]}$ via Metropolis-Hasting step

1. Sample θ_1^* from $g_1\left(\theta_1^* \mid \theta_1^{[t]}\right)$
2. Take

$$\theta_1^{[t+1]} = \begin{cases} \theta_1^* \text{ with probability } \rho\left(\theta_1^{[t]}, \theta_1^*\right) \\ \theta_1^{[t]} \text{ with probability } 1-\rho\left(\theta_1^{[t]}, \theta_1^*\right) \end{cases}$$

where $\rho\left(\theta_1^{[t]}, \theta_1^*\right) = \min\left\{\dfrac{p\left(\theta_1^* \mid \text{rest}\right) g_1\left(\theta_1^{[t]} \mid \theta_1^*\right)}{p\left(\theta_1^{[t]} \mid \text{rest}\right) g_1\left(\theta_1^* \mid \theta_1^{[t]}\right)}, 1\right\}$

4: Sample $\theta_2^{[t+1]}$ via Metropolis-Hasting step

1. Sample θ_2^* from $g_2\left(\theta_2^* \mid \theta_2^{[t]}\right)$
2. Take

$$\theta_2^{[t+1]} = \begin{cases} \theta_2^* \text{ with probability } \rho\left(\theta_2^{[t]}, \theta_2^*\right) \\ \theta_2^{[t]} \text{ with probability } 1-\rho\left(\theta_2^{[t]}, \theta_2^*\right) \end{cases}$$

where

$$\rho\left(\theta_2^{[t]}, \theta_2^*\right) = \min\left\{\frac{p\left(\theta_2^* \mid \text{rest}\right) g_2\left(\theta_2^{[t]} \mid \theta_2^*\right)}{p\left(\theta_2^{[t]} \mid \text{rest}\right) g_2\left(\theta_2^* \mid \theta_2^{[t]}\right)}, 1\right\}$$

5: Sample $\lambda_1^{[t+1]}$ via Metropolis-Hasting step

1. Sample λ_1^* from $g_3\left(\lambda_1^* \mid \lambda_1^{[t]}\right)$
2. Take

$$\lambda_1^{[t+1]} = \begin{cases} \lambda_1^* \text{ with probability } \rho\left(\lambda_1^{[t]}, \lambda_1^*\right) \\ \lambda_1^{[t]} \text{ with probability } 1-\rho\left(\lambda_1^{[t]}, \lambda_1^*\right) \end{cases}$$

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where

$$\rho\left(\lambda_1^{[t]},\lambda_1^{*}\right)=\min\left\{\frac{p\left(\lambda_1^{*}\mid \text{rest}\right)g_3\left(\lambda_1^{[t]}\mid\lambda_1^{*}\right)}{p\left(\lambda_1^{[t]}\mid \text{rest}\right)g_3\left(\lambda_1^{*}\mid\lambda_1^{[t]}\right)},1\right\}$$

6: Sample $\lambda_2^{[t+1]}$ via Metropolis-Hasting step

1. Sample λ_2^{*} from $g_4\left(\lambda_2^{*}\mid\lambda_2^{[t]}\right)$
2. Take

$$\lambda_2^{[t+1]}=\begin{cases}\lambda_2^{*} \text{ with probability } \rho\left(\lambda_2^{[t]},\lambda_2^{*}\right)\\ \lambda_2^{[t]} \text{ with probability } 1-\rho\left(\lambda_2^{[t]},\lambda_2^{*}\right)\end{cases}$$

where

$$\rho\left(\lambda_2^{[t]},\lambda_2^{*}\right)=\min\left\{\frac{p\left(\lambda_2^{*}\mid \text{rest}\right)g_4\left(\lambda_2^{[t]}\mid\lambda_2^{*}\right)}{p\left(\lambda_2^{[t]}\mid \text{rest}\right)g_4\left(\lambda_2^{*}\mid\lambda_2^{[t]}\right)},1\right\}$$

7: Sample $\lambda_3^{[t+1]}$ via Metropolis-Hasting step

1. Sample λ_3^{*} from $g_5\left(\lambda_3^{*}\mid\lambda_3^{[t]}\right)$
2. Take

$$\lambda_3^{[t+1]}=\begin{cases}\lambda_3^{*} \text{ with probability } \rho\left(\lambda_3^{[t]},\lambda_3^{*}\right)\\ \lambda_3^{[t]} \text{ with probability } 1-\rho\left(\lambda_3^{[t]},\lambda_3^{*}\right)\end{cases}$$

where

$$\rho\left(\lambda_3^{[t]},\lambda_3^{*}\right)=\min\left\{\frac{p\left(\lambda_3^{*}\mid \text{rest}\right)g_5\left(\lambda_3^{[t]}\mid\lambda_3^{*}\right)}{p\left(\lambda_3^{[t]}\mid \text{rest}\right)g_5\left(\lambda_3^{*}\mid\lambda_3^{[t]}\right)},1\right\}$$

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A2: MCMC Algorithms for Model without Covariates

$1:$ Sample $\sigma_\tau^{2[t+1]}$ via Metropolis-Hasting step

(a) Sample σ_τ^{2*} from $g_6\left(\sigma_\tau^{2*} \mid \sigma_\tau^{[2t]}\right)$

(b) Take

$$\sigma_\tau^{2[t+1]} = \begin{cases} \sigma_\tau^{2*} \text{ with probability } \rho\left(\sigma_\tau^{2[t]}, \sigma_\tau^{2*}\right) \\ \sigma_\tau^{2[t]} \text{ with probability } 1\text{-}\,\rho\left(\sigma_\tau^{2[t]}, \sigma_\tau^{2*}\right) \end{cases}$$

where

$$\rho\left(\sigma_\tau^{2[t]}, \sigma_\tau^{2*}\right) = \min\left\{ \frac{p\left(\sigma_\tau^{2*} \mid \text{ rest}\right) g_6\left(\sigma_\tau^{2[t]} \mid \sigma_\tau^{2*}\right)}{p\left(\sigma_\tau^{2[t]} \mid \text{ rest}\right) g_6\left(\sigma_\tau^{2*} \mid \sigma_\tau^{2[t]}\right)}, 1 \right\}, \sigma_\tau^{*} = \sigma_\tau^{2*}, \sigma_\tau^{[t+1]} = \sigma_\tau^{2[t}$$

$$2:\ \sigma_\varepsilon^{2[t+1]} \sim IG\left(\frac{\eta}{2} + a_\varepsilon^0, b_\varepsilon\right), b_\varepsilon = \frac{1}{2}\left[\left(y - Z^{[t]}\right)'\left(y - Z^{[t]}\right)\right] + b_\varepsilon^0$$

$$3:\ \beta^{[t+1]} \sim N\left(\mu_\beta^{[t+1]}, \Sigma_\beta^{[t+1]}\right),$$

$$\Sigma_\beta^{[t+1]} = \left[\sigma_\varepsilon^{2-[t+1]} xx' + \Sigma_\beta^{0-1}\right]^{-1}, \mu_\beta^{[t+1]} = \Sigma_\beta^{[t+1]}\left[\sigma_\varepsilon^{2-[t+1]} \lambda_1 x + \Sigma_\beta^{0-1} \mu_\beta^0\right]$$

$$4:\ Z^{[t+1]} \sim N\left(\mu_Z^{[t+1]}, \Sigma_Z^{[t+1]}\right),$$

$$\Sigma_Z^{[t+1]} = \left[\sigma_\varepsilon^{2-[t+1]} \mathrm{I}_n + K_\varepsilon^{-1}\right]^{-1}, \mu^{[t+1]} = \sigma_\varepsilon^{2-[t+1]} \Sigma_Z^{[t+1]} y$$

$$K_\varepsilon = \tau\left(\delta, \sigma_\tau^{2[t+1]}, \lambda_1^{[t+1]}, \lambda_2^{[t+1]}, \lambda_3^{[t+1]}, \theta_1^{[t+1]}, \theta_2^{[t+1]}\right)$$

$5:$ If $t < M$, set $t = t+1$ and return to step 2. Otherwise stop.

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APPENDIX B: DERIVATION OF FULL CONDITIONALS

FULL CONDITIONAL FOR $Z(t)$

$$P(Z(t)\,|\,\text{rest}) = P\left(y\,|\,Z(t),\sigma_\varepsilon^2\right)P\left(Z(t)\,|\,\lambda_1,\lambda_2,\lambda_3,\theta_1,\theta_2,\sigma_T^2\right)$$

$$= N\left(y;Z(t),\sigma_\varepsilon^2 I_n\right)N\left(Z(t);0,K_\varepsilon\right)$$

$$= N\left(\mu_Z^*,\Sigma_Z^*\right),$$

$$\Sigma_Z^* = \left[K_\varepsilon^{-1} + \frac{1}{\sigma_\varepsilon^2}I_n\right]^{-1}$$

$$\mu_Z^* = \frac{1}{\sigma_\varepsilon^2}y'\Sigma_Z^*$$

FULL CONDITIONALS FOR λ_2

$$P(\lambda_2\,|\,\text{rest}) = P\left(Z(t)\,|\,\lambda_1,\lambda_2,\lambda_3,\theta_1,\theta_2,\sigma_T^2\right)\times P(\lambda_2)$$

$$= N\left(Z(t);0,K_\varepsilon\right)IG\left(\lambda_2;a_{\lambda_2}^o,b_{\lambda_2}^o\right)$$

$$\times\left|K_\varepsilon\right|^{-1/2}e^{-1/2\left[Z'(t)K_\varepsilon^{-1}Z(t)\right]}\times$$

$$(\lambda_2)^{-a_{\lambda_2}^o-1}e^{-\left(b_{\lambda_2}^o/\lambda_2\right)}$$

FULL CONDITIONALS FOR λ_3

$$P(\lambda_3\,|\,\text{rest}) = P\left(Z(t)\,|\,\lambda_1,\lambda_2,\lambda_3,\theta_1,\theta_2,\sigma_T^2\right)$$

$$\times P(\lambda_3)$$

$$= N\left(Z(t);0,K_\varepsilon\right)IG\left(\lambda_3;a_{\lambda_3}^o,b_{\lambda_3}^o\right)$$

$$\propto\left|K_\varepsilon\right|^{-\frac{1}{2}}e^{-\frac{1}{2}\left[Z'(t)K_\varepsilon^{-1}Z(t)\right]}$$

$$\times(\lambda_3)^{-a_{\lambda_3}^o-1}e^{-\left(b_{\lambda_3}^o/\lambda_3\right)}$$

FULL CONDITIONALS FOR σ_ε^2

$$P\left(\sigma_\varepsilon^2\,|\,\text{rest}\right) = P\left(y\,|\,Z(t),\sigma_\varepsilon^2\right)P\left(\sigma_\varepsilon^2\right)$$

$$= N\left(y;Z(t),\sigma_\varepsilon^2 I_n\right)IG\left(\sigma_\varepsilon^2;a_\varepsilon^o,b_\varepsilon^o\right)$$

$$\propto\left(\sigma_\varepsilon^2\right)^{-\frac{n}{2}}e^{-\frac{1}{2\sigma_\varepsilon^2}\left[(y-Z(t))'(y-Z(t))\right]}$$

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$$\times\left(\sigma_\varepsilon^2\right)^{-b_\varepsilon^o-1} e^{-\left(b_\varepsilon^o / \sigma_\varepsilon^2\right)}$$

$$\propto\left(\sigma_\varepsilon^2\right)^{-\left(\frac{n}{2}+a_\varepsilon^o\right)-1} e^{-\left(\frac{1}{2}\left[(y-Z(t))'(y-Z(t))\right]+b_\varepsilon^o\right)}$$

$$= IG\left(\frac{n}{2}+a_\varepsilon^o, \frac{1}{2}\left[(y-Z(t))'(y-Z(t))\right]+b_\varepsilon^o\right)$$

FULL CONDITIONALS FOR λ_1

$$P(\lambda_1 \mid \text{rest}) = P\left(Z(t) \mid \lambda_1, \lambda_2, \lambda_3, \theta_1, \theta_2, \sigma_T^2\right) \times P(\lambda_1 \mid \beta) \propto e^{-\frac{1}{2}\left[Z'(t)K_\varepsilon^{-1}Z(t)\right]-\frac{1}{2\sigma_{\lambda_1}^2}(\lambda_1-x'\beta)^2}$$

FULL CONDITIONALS FOR σ_T^2

$$P\left(\sigma_T^2 \mid \text{rest}\right) = P\left(Z(t) \mid \lambda_1, \lambda_2, \theta_1, \theta_2, \sigma_T^2\right) \times P\left(\sigma_T^2\right)$$

$$= N\left(Z(t); 0, K_\varepsilon\right) IG\left(\sigma_T^2; a_T^o, b_T^o\right)$$

$$\propto |K|^{-\frac{1}{2}} e^{-\frac{1}{2}\left[Z'(t)K_\varepsilon^{-1}Z(t)\right]} \times e^{-a_T^o-1} e^{-\left(b_T^o / \sigma_T^2\right)}$$

FULL CONDITIONALS FOR θ_1

$$P(\theta_1 \mid \text{rest}) = P\left(Z(t) \mid \lambda_1, \lambda_2, \lambda_3, \theta_1, \theta_2, \sigma_T^2\right) P(\theta_1)$$

$$= N\left(Z(t); 0, K_\varepsilon\right) IG\left(\theta_1; a_{\theta_1}^o, b_{\theta_1}^o\right)$$

$$\propto |K|^{-\frac{1}{2}} e^{-\frac{1}{2}\left[Z'(t)K_\varepsilon^{-1}Z(t)\right]} \times (\theta_1)^{-a_{\theta_1}^o-1} e^{-\left(b_{\theta_1}^o / \theta_1\right)}$$

FULL CONDITIONALS FOR θ_2

$$P(\theta_2 \mid \text{rest}) = P\left(Z(t) \mid \lambda_1, \lambda_2, \lambda_3, \theta_1, \theta_2, \sigma_T^2\right) \times P(\theta_2)$$

$$= N\left(Z(t); 0, K_\varepsilon\right) IG\left(\theta_2; a_{\theta_2}^o, b_{\theta_2}^o\right)$$

$$\propto |K_\varepsilon|^{-\frac{1}{2}} e^{-\frac{1}{2}\left[Z'(t)K_\varepsilon^{-1}Z(t)\right]} \times (\theta_2)^{-a_{\theta_2}^o-1} e^{-\left(b_{\theta_2}^o / \theta_2\right)}$$

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