UNIVERSITY OF CAPE COAST

PRESERVICE MATHEMATICS TEACHERS' CONTENT AND PEDAGOGICAL KNOWLEDGE FOR TEACHING LOGICAL REASONING IN SENIOR HIGH SCHOOL

ABRAHAM KOJO ABAKAH

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BY

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Thesis submitted to the Department of Mathematics and ICT Education, Faculty of Science and Technology Education, College of Education Studies, University of Cape Coast, in partial fulfilment of the requirements for the award of Master of Philosophy in Mathematics Education

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DECLARATION

Candidate's Declaration

I hereby declare that this thesis is the result of my own original research and that no part of it has been presented for another degree in this University or elsewhere

Candidate's Signature.. Date...................................

Candidate's Name: Abraham Kojo Abakah

Candidate's Number: ET/MDP/20/0010

Supervisor's Declaration

I hereby declare that the preparation and presentation of the dissertation were supervised in accordance with the guidelines on supervision of thesis laid down by the University of Cape Coast.

Supervisor's Signature.................................... Date……………………........

Supervisor's Name: Mr. Benjamin Yao Sokpe

ABSTRACT

The purpose of the study was to investigate the content knowledge and pedagogical knowledge of preservice Mathematics teachers for teaching Logical reasoning in senior high school. The research design used was a cross-sectional survey. Target population was 184 and accessible population was 138 level 400 preservice Mathematics teachers from the Department of Mathematics and ICT Education, University of Cape Coast participated in the study. Questionnaire and Achievement were data collection instrument employed in the study. The questionnaire covered sections testing the pedagogical knowledge and Achievement test also covered the section test content knowledge of preservice Mathematics teachers on Logical reasoning. The data collected was analysed using mean, standard deviation, and independent sample t-test. The study revealed that preservice Mathematics teachers possess much content knowledge in the sub-strands on Logical reasoning except compound statements. Also, female preservice Mathematics teachers possess more content knowledge on Logical reasoning than their male counterparts. However, there is no significant difference in the knowledge of strategies for teaching Logical reasoning possessed by West Africa Senior Secondary Certificate Examination and Diploma in Basic Education applicants. Also, difference in knowledge of teaching strategies in Logical reasoning possessed by male and female preservice Mathematics teachers is not significant. Content and methods courses mounted by the DMICTE have a positive impact on preservice Mathematics teachers' strategies for teaching Logical reasoning. It was concluded that preservice Mathematics teachers are most likely to avoid teaching compound statements when they are deployed to senior high school. It was recommended that Departments of Mathematics education at the university should ensure that preservice Mathematics teachers are given more exposure to compound statements in their course content.

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DEDICATION

To Karin Westra, Wim Levens, Jayden Abakah and Aba Tawiah

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LIST OF ACRONYMS

DMICTE	Department of Mathematics & ICT Education
SHS	Senior High School
CKT	Content knowledge theory
CK	Content Knowledge
PK	Pedagogical Knowledge
PCK	Pedagogical Content Knowledge
ALT	Academic Learning Time

CHAPTER ONE

INTRODUCTION

Mathematics is one of the most useful and fascinating subjects in the world, but many people have misconceptions about it. Some people believe that the concepts of Mathematics learned in school have no use or relevance to the problems we face in our personal and professional life (Hatfield, Edward, & Bitter 1997). As a result, many students do not take their study of Mathematics seriously enough. The performance of students is influenced by a many of factors, and a teacher's knowledge cannot be exempted. Wilmot (2009) and Yarkwah (2017) assert that teachers are vital to students' performance in a certain construct. Recent conceptualisations of teachers' knowledge for teaching stemmed from the argument that the knowledge needed to teach is different. Wilmot (2009) revealed that some of these conceptualisations made it difficult to fairly measure teachers' knowledge as they exhibited instructor knowledge as a domain-specific construct (Shulman, 1987).

Background to the Study

Logical reasoning and mental rigour are supported by Mathematics, which provides an efficient method of developing mental discipline. Unfortunately, students' achievement in Logical reasoning as a topic is not very encouraging at the secondary school level of education in Ghana.

British Columbia's New 2016 Curriculum K-9, in Canada expressly mentions that Logical reasoning is part of its main aims. Additionally, according to the Monash University School of Mathematical Sciences, mathematics will iprove your ability to think clearly, logicaly and using a variety of problem-solving techniques. The School of Mathematics and Statistics at the University of Sydney (Australia) introduced a course related to Logical reasoning. The objectives and skills of the course include "enhancing students' problem-solving skills." This is part of their studies in first-year courses to develop logical thinking in second-year studies, and it will help their further studies on "analyzing and constructing logical arguments" as part of the course in the third year. The University of Cambridge's Faculty of Mathematics, UK, provides a dedicated document, "Transferable Skills in the Mathematical Tripos" as part of its undergraduate Mathematics course information, which again lists "analytic ability, creativity, initiative, logical, and methodical reasoning" (Polya, 2004).

Yawa (2009), attested that students' performance is affected by the kind of environment they find themselves in, the teacher's experience, and their mastery of content. It is essential that, for effective teaching to occur in Mathematics both content and pedagogical knowledge are needed. Wayne and Youngs (2003) report that teachers' content knowledge influences students' performance in Mathematics. Janeiro (2012) in his study, found a relationship between student performance and teachers' content knowledge. What counts in this teacher's content knowledge and how it relates to student performance has not been much researched in the past years.

Pedagogy is an important variable that may influence how well students perform in Mathematics. Pedagogy is the term used to describe teaching methods (Ogunboyede, 2011). The type of principle and teaching methods that are used in instruction are referred to as pedagogy. The information or skill the teacher wants to deliver will depend on the kind of principal and teaching methodology used for instruction. Depending on the information or skill the teacher is conveying, teachers have many different teaching methods that vary from one another (Kearney & Garfield, 2019). In order to ensure that every student has an equal opportunity to learn, a range of tactics and strategies are employed. However, if the teaching strategies do not improve students' understanding of concepts, it will have an impact on their performance.

Numerous studies have shown that knowledge is a dominant, individualised, and flexible force that plays a significant role in education (Alexander, 1996). There is much discussion on the types of knowledge that teachers need to possess in order to be effective in the classroom. This is due to the fact that while most observers agree that effective teachers utilise specialised knowledge in their work with students during teaching, it has proven to be challenging and contentious to identify and quantify this knowledge in the field of education. Even with extensive subject-matter knowledge, a teacher must possess the necessary skills for teaching. Pedagogy is the precise term for these abilities. Pedagogy includes communication, classroom management, motivation, and student participation in instruction. The majority of a teacher's ability to convey information to students depends on the pedagogical strategies used in the classroom. For example, students's understanding of the subject matter is greatly impacted by the teaching strategies that teachers adopt. Bosu (2010) asserts that a teacher who possesses pedagogical knowledge but is not a subject matter specialist is just as incompetent as a teacher who possesses pedagogical skills but lacks content knowledge. This emphasises how subject matter and pedagogical knowledge are essential for teachers to be effective in the teaching profession.

Based on a growing body of research, teachers' content and pedagogical knowledge have a greater impact on students' performance than do students' prior academic record or the school they attend (Ishola & Udofi, 2017). As stated by Baumert et al. (2010) and Hill, Rowan, and Ball (2005), the teachers' content and pedagogical knowledge are largely responsible for the academic achievement of the classroom lesson. Because of this, a lot of programmes put on by education stakeholders primarily focus on developing strong content knowledge and pedagogical knowledge to cater to the different requirements of students in the classroom. The discourse surrounding the content and pedagogical knowledge of teachers has garnered significant attention in recent times from many agents of change within the education sector. It is common knowledge that every country whose government aspires to greatness should assign students to highly trained teachers who possess extensive understanding of the subject matter as well as pedagogical knowledge, ethics, and other skills. Research findings indicate that teachers' intellectual resources have a major impact on students' learning experiences (Odumosu, Olusesan, & Abel, 2016), which has caused teachers to concentrate on subject matter knowledge.

Ogar (2006) also emphasized that students understand concepts of Mathematics very well when they are assigned to effective Mathematics teachers who have acquired much content knowledge and pedagogical knowledge. It is generally accepted that a teacher needs to be knowledgeable about both pedagogy and content in order to teach effectively. When employed effectively, teachers' knowledge of the topics to be covered in the syllabus and the subject matter they are expected to teach is crucial for fostering effective teaching and learning. A teacher with extensive knowledge of pedagogy is able to comprehend how students form mental habits and adopt a positive outlook on learning, in addition to acquiring content and pedagogical knowledge. The research conducted by Sidhu, Fook, and Kaur (2011) demonstrates that most teachers focus on lecturing over teaching students to develop critical thinking skills across topic areas and apply what they have learned to situations in the real world.

One of the most crucial learning concepts to master is Logical reasoning, which Serna (2015) describes as the reasonable process by which people arrive at accurate conclusions using their brains. It is attained through the growth of reasoning skills and a logical relationship between all of the factors involved in every individual situation. The ability to use inference processes with precise terms and to organise and formulate logical procedures are the key components of Logical reasoning. According to Nigel (2002), Logical reasoning may be used to create persuasive arguments that address problems. In Mathematics, everything is held together by Logical reasoning. Thus, Logical reasoning helps students solve the majority of Mathematics problems using the appropriate approach. Smith (1992) states that Logical reasoning should be a topic that will develop students' understanding more than common sense, but some people also accumulate common sense experiences that help them create inferences in Logical reasoning. However, using a Logical reasoning approach must result in a similar conclusion. Smith emphasises that logic cannot serve as the primary concept guiding all judgements and conclusions about the world, but Baron (2007) also makes the argument that it would be more advantageous if logical reasoning played a more significant part in daily life, that is, if it stimulated the brain in a way that made it helpful. Students can make the connection between logic and everyday conversations by using basic logical relations to express some basic statements. Most teacher education institutions have overloaded educational courses that teachers need to master, before going to the classroom but they exclude courses on Logical reasoning (Niemi, 2002). Students study a lot of Mathematics and notions that have no real-world application, which undermines the goal of teaching them Logical reasoning. In such a way, students possess the ability to solve basic problems and repeat tasks, but they do not learn the reasoning necessary to logically address problems that are a little challenging (Van, Kirschner, & Kester, 2003).

Preservice Mathematics teachers are needed to help address the issue of student performance in Logical reasoning. Preservice Mathematics teachers are being trained with the required skills and mastery of Mathematical content. Among the matters to be taken into account in teacher preparation are the methodology required for teaching Logical reasoning and the required skills for teaching Logical reasoning (Jorgensen, Schuh, & Nisbet, 2005). Tebabal and Kahssay (2011) declared that bringing about a fundamental change in the student is the primary goal of instruction at all educational levels. This change can be accomplished if preservice Mathematics teachers have a comprehensive understanding of Logical reasoning in Mathematics.

In order to determine gender differences, Bessoondyal (2005) also conducted research in Mathematics. According to his research, boys outperformed girls by a significant margin. Gonzales et al. (2004) also found that there were gender differences in the TIMSS data, and girls performed better than boys. Research comparing girls to boys revealed that girls lacked confidence, perceived Mathematics as a field for men, and experienced anxiety when it came to Mathematics (Kearney & Garfield, 2019). It was discovered that males and girls have different levels of self-confidence in Mathematics (Nagar, 2008). The findings indicate that girls were more likely to think logically in Mathematics than boys. As a result, there is a need to investigate the content knowledge and pedagogical knowledge of male and female preservice Mathematics teachers.

In Ghana, One of the fundamental subjects to be studied in senior high school is Mathematics, which appears to have a prominent place on the curriculum. Due to the significance of Mathematics, the government of Ghana is committed to making sure that the pre-tertiary educational system in the country is staffed with highly qualified Mathematics teachers. Some universities, such as the University of Ghana (UG) and the University of Cape Coast (UCC) are offering Critical Thinking and Practical Reasoning as a course in their curriculum. For example, at the University of Cape Coast, the Critical Thinking and Practical Reasoning course emphasizes deductive and inductive reasoning. In addition, the Department of Mathematics and ICT Education (DMICTE) in the Faculty of Science and Technology Education at the University of Cape Coast offers two mathematics education courses that include topics meant to expose their preservice Mathematics teachers to

Logical reasoning. These courses are Nature of Mathematics-(EMA 201) and Advanced Algebra and Calculus- (EMA 202). The aim is for preservice Mathematics teachers to acquire more skills in Logical reasoning. The Ghanaian high school Mathematics consists of core Mathematics and elective Mathematics. The Mathematics content areas that students will study in high school are covered by each curriculum. All high school students are required to take Core Mathematics, which is designed to give them the knowledge and skills they will need to solve problems on a daily basis and during the vacation (Ministry of Education, 2010). The deeper mathematical knowledge, skills, and competency that students get from elective Mathematics subjects, which are demands for study, serve as the basis for the requirements of subsequent studies in Mathematics programmes at the tertiary level of education. In all instances, the content is structured to cover three years.

The core Mathematics major contents areas are "Numbers and Numeration, Plane Geometry, Mensuration, Algebra, Statistics and Probability, Trigonometry, Vectors and Transformation in a Plane and Problem solving and application (mathematical processes)" (Ministry of Education, 2010a, p. iii). The major contents areas of the elective Mathematics curriculum are also categorized under the following areas: "Algebra, Coordinate Geometry, Vectors and Mechanics, Logic, Trigonometry, Calculus, Matrices, Transformation, Statistics and Probability" (Ministry of Education, 2010b, p. ii). Problem-solving is not an independent subject in these two mathematics curricula; rather, it permeates every topic. Therefore, rather than having students memorise algorithms, teachers are urged to include problems that will prompt their mathematical thinking.

Logical reasoning is a topic in SHS 3 core Mathematics and elective Mathematics, but more expanded in elective Mathematics than core Mathematics. Currently, it seems in Ghana that not much emphasis is placed on the teaching and learning of Logical reasoning in Mathematics. This is evident in WAEC reports (2013 & 2021) that many senior high school students do not attempt to answer Logical reasoning questions and many of those who attempt do not perform well.

Statement of the Problem

Many researchers have investigated the factors that affect student performance in Mathematics. Wilmot et al. (2018) and Yarkwah (2017) agree with stakeholders in education to address all possible factors affecting student performance in Mathematics. One of the factors affecting student performance is the teacher's knowledge of the subject matter. According to Mewborn (2003), teachers play a critical role in making sure that students acquire the necessary knowledge and skills to master Mathematics in order to excel in their future academic and professional endeavours. This makes the teacher an important asset in helping students advance in their careers, and all of this can be worthwhile if the teacher possesses both content knowledge and pedagogical knowledge.

According to Shulman (1987), teachers must have a thorough understanding of the subject matter and how it will affect students' knowledge in order to teach all students in accordance with today's standards. The ability of teachers to teach will be based on their knowledge of the learning challenges that their students encounter and the most effective strategies to address those challenges (Buabeng, Yeboah, Cobbinah, et al., 2019).

According to research by Olfos Goldrine, and Estrella (2014), Mathematics teachers may not possess the necessary content knowledge to effectively teach the subject. It is likely that teachers will teach Logical reasoning poorly or not at all if their knowledge of the topic is weak and yields attempts to introduce inconsistencies into it (Stylianides, Stylianides, & Philippou, 2007). The important components of human intelligence include the ability to reason, problem-solve, and make decisions in daily situations (Holyoack & Morrison, 2005).

In Ghana, West African Senior Secondary Certificate Examination (WASSCE) results reveal that students' performance on Logical reasoning still needs improvement. Students' performance in Logical reasoning has been very poor according to the chief examiner's reports in core Mathematics (WAEC, 2013 & 2021). There have been gaps concerning students' ability to interpret mathematical statements logically without overgeneralizing or undergeneralizing (WAEC, 2013 & 2021). This indicates weaknesses in Logical reasoning. The set of questions on Logical reasoning was within the general coverage of the syllabus and was of a standard that matched the abilities of the candidates (WAEC, 2013 & 2021). Logical reasoning is a fundamental skill that serves as a basis for many courses at the Unversity. It involves using reasoning argmmetation, and evidence- based thinking to evaluate informaton, make informed decisions, and solve complex problems. Courses that rely on Logical reasoning include: Mathematcs Computer Scence, Philosophy, Law, Engineering, and Economcs etc. In critical thinking, Logical reasoning helps students to develop critical thinking skills which enable them to evaluate information, identify biases, and make informed decision. Logical reasoning helps students to articulate their thoughts, arguments, and conclusions clearly and persuasively. Logical reasoning assists individuals in making sense of the world. Logical reasoning is one the top skills required for success in the modern workforce (WEF, 2020). It is crucial to develop these skills in school. It is easy to train preservice teachers in order to be good at Logical reasoning before they deploy them to the senior high schools. These remarks highlight the necessity for a study to investigate preservice Mathematics teachers' content knowledge and pedagogical knowledge in Logical reasoning.

Purpose of the Study

The main purpose of the research is to investigate content knowledge and pedagogical knowledge of preservice Mathematics teachers for teaching Logical reasoning in senior high school. The study specifically seeks to find out:

i. Preservice Mathematics teachers' content knowledge (CK) in Logical reasoning.
ii. Preservice Mathematics teachers' knowledge of strategies for teaching Logical reasoning.
iii. The difference in the content knowledge of male and female preservice Mathematics teachers in Logical reasoning.
iv. Difference in the knowledge of strategies for teaching Logical reasoning possessed by WASSCE and DBE applicants.
v. Difference in the knowledge of male and female preservice Mathematics teachers about strategies for teaching Logical reasoning.

Research Questions and Hypotheses

Research Questions

The study sought answers to the following questions:

1. What are preservice Mathematics teachers' content knowledge in Logical reasoning?
2. What are preservice Mathematics teachers' knowledge of strategies for teaching Logical reasoning?

Research Hypotheses

The following null hypotheses guided the study.

1. $H_{0:}^{1}$ There is no significant difference in the content knowledge of male and female preservice Mathematics teachers in Logical reasoning.
2. $H_{0:}^{2}$ There is no significant difference in the knowledge of strategies for teaching Logical reasoning possessed by WASSCE and DBE applicants.
3. $H_{0:}^{3}$ There is no significant difference in the knowledge of strategies for teaching Logical reasoning possessed by male and female preservice Mathematics teachers.

Significance of the Study

The findings of this study will highlight the content knowledge and pedagogical knowledge of preservice Mathematics teachers. Many researches have shown that teachers' knowledge of teaching Mathematics has a key influence on students' performance in Mathematics. Shulman (1987) also pointed out that teacher effects on student performance were also driven by teachers' pedagogical knowledge and content knowledge to carry out the tasks of teaching the subject. Preservice teachers involved in this study have taken relevant content courses covering Logical reasoning. The result from this study will therefore help unveil the level of knowledge in Logical reasoning preservice Mathematics teachers possess. This again, will inform the Department of Mathematics and ICT Education to ensure preservice teachers are well equipped with both the relevant content knowledge and how to deliver this content. That will help to improve students' performance in teaching Logical reasoning.

Furthermore, the study will inform the Department of Mathematics and ICT Education, to decide whether to mount Logical reasoning as a full course. At level 200, the Department offers two courses where Logical reasoning is treated as a topic in each. Senior high school students are expected to have a mastery of content in Logical reasoning before they write WASSCE. However, the chief examiner's reports indicate that students perform poorly on Logical reasoning questions. Hence, there is a need to undertake this study to investigate content knowledge and pedagogical knowledge of preservice Mathematics teachers for teaching Logical reasoning in senior high school.

Delimitation

The purpose of this study was to nvestigate content knowledge and pedagogical knowledge of preservice Mathematics teachers for teaching Logical reasoning in senior high school. The scope of the study is limited to Logical reasoning alone. Furthermore, only level 400 tertiary mathematics students pursuing Mathematics education programmes were used in this study. This cohort of preservice teachers was chosen because, at the time of the study, they had completed the necessary methodology and content courses in Mathematics, including a couse on Logical reasoning, which is a prerequisite for teaching Mathematics in high school.

Limitation

The limitation of this study is that it's possible that respondents felt awkward acknowledging their lack of knowledge of content knowledge and pedagogy knowledge for teaching Logical reasoning. This factor may have affected responses, which would have misrepresented the study's findings. It was anticipated that, in spite of this possible limitation, the study's conclusions would apply to preservice Mathematics teachers at the Department of Mathematics and ICT Education, University of Cape Coast, in the Central region of Ghana.

Definition of Terms

1. Preservice Mathematics teachers: In practical terms, this refers to students studying Mathematics educational programmes at Level 400.
2. Off-Campus Teaching Practice: This is the name given to the one-semester required internship that preservice teachers at the university are required to complete during their final year of study. The preservice teachers choose a senior high school of their choosing during this time, and they teach Mathematics under the supervision of a university faculty member and a mentor who has been assigned to the school.

Organisation of the Study

This study is structured into five chapters. Chapter One, captioned Introduction, covers the Background to the Study, Statement of the Problem, Purpose of the Study, Research questions and Hypotheses, Significance of the Study, Delimitation, Definition of terms, and Organisation of the Study.

Chapter Two looks at the review of Literature relevant to the study. It talks about the theoretical framework that goes along with the conceptual framework that the study was developed around. Specifically, the concepts of content knowledge and pedagogical knowledge were exhaustively explained. It also covered research on teachers' knowledge for teaching Logical reasoning and the impact of the field teaching experience on preservice teachers' knowledge. The research methods employed in the study were covered in Chapter Three. It describes the research design, population, sample and sampling procedures, data collection procedures, validity and reliability of the instruments, and data analysis procedures.

Chapter Four is the apportionment of the results and discussion of the findings concerning the research questions and hypotheses. The last chapter, Chapter Five, includes a summary of the research process and key findings, conclusions derived from the findings, and recommendations.

CHAPTER TWO

LITERATURE REVIEW

The review of related literature is presented under theoretical framework, conceptual framework, and empirical review. The theoretical review is rooted in mental model theory while the empirical review covers content knowledge and pedagogical knowledge of preservice Mathematics teachers. It also covers the knowledge of preservice Mathematics teachers specifically focusing on Logical reasoning, the impact of field teaching experience on preservice teachers' content knowledge and pedagogical knowledge.

Theoretical framework

A Logical reasoning test evaluates our capacity for Logical reasoning. Logical reasoning tests often check abilities that are nonverbal. You have to draw conclusions, structures, and rules using logical and abstract reasoning. Then, you have to apply these conclusions to narrow down a list of possible responses to the correct one. The mental model theory is the theory that deals with Logical reasoning.

Mental Model Theory

A representation of a possibility with a structure and content that captures what is common to the many ways that the possibility might occur is called a mental model theory. Johnson-Laird (2004) stated that individuals initially construct a mental model that is consistent with the premises. Individuals with low working memory capacity struggle to create enough models to accurately evaluate a conclusion's validity.

Nevertheless, this translation can be done easily by the effectiveness of teachers' content knowledge and pedagogical knowledge. For example, when individuals understand conjunction such as "There is a triangle and there is a circle," they represent its meaning (its intention), from which they are able to create a picture of what it alludes to (its extension). Δ symbolises a triangle and O represents a circle in the mental model that serves as the representation of the extension. The model specifies nothing about the objects' sizes, spatial relationships, or other relevant details, but rather what is common to all scenarios involving a triangle and a circle. Nonetheless, two mental tokens with attributes that match those of the two objects are used to represent both of them.

Thus, developing models from descriptions is an aspect of verbal knowledge; the specifics of this process have been well discussed in various contexts (Garnham, 1987; Johnson-Laird, 2004; Stevenson, 1993). According to the concept, as reasoners create mental models based on their understanding of the premises and any relevant knowledge, reasoning is a semantic process as opposed to a formal one. By making sure there are no models of the premises in which the conclusion is false, they may ensure the validity of the conclusion they construct in these models is true.

Many cognitive scientists believe that deductive reasoning is fundamental to human intellect, which is why standard deduction has been the subject of so much research. On these tests, people frequently make logical mistakes and show systematic biases in their findings, which caused a great deal of debate over human rationality (Evans, 2002). Nonetheless, the majority of area psychologists concur that untrained reasoners do exhibit some basic logical ability. For instance, their ability to differentiate between true and false conclusions is far higher than chance rates. However, explaining such competence gave rise to an ongoing, occasionally unpleasant, and unresolved discussion between two theoretical groups. The discussion is on deductive reasoning, which is possible through theory or mental models (Evans, 2002). The concept of "natural deduction," which is a formalisation of logic that is intuitively plausible and postulates rules of inference for every logical term, is the source of rule-based theories. The systems developed by Rips (1994, 2001) have been shown to be the most resilient of these. There are two reasons, nevertheless, why this hypothesis has been put forth. First, mental models and theorists have mostly focused on propositional reasoning and have said very little about syllogistic inference in their theoretical and experimental work. Second, the published mental model or theory can account for essential but impractical inferences since it only offers guidelines for legitimate inferences.

Byrne, Schaeken, and Johnson-Laird (1991) developed the mental model theory account of human deduction. With this method, valid inference rules are not used to syntactically verify conclusions. Instead, their deductions come from a knowledge of the semantic principle, which states that a conclusion is true if no model of the premises can make it false. Psychological restrictions found in the model theory do not apply to formal semantic techniques like truth-table analysis. Specifically, it is assumed that individuals selectively depict and concentrate on scenarios where the premises would be true. Originally, the mental model theory was developed to explain syllogistic thinking (Johnson-Laird & Bara, 1984). Mental model theorists have conducted a vast experimental programme of study in an effort to bolster their theory's predictions (Johnson-Laird, Byrne, & Schaeken 1991). For instance, Johnson-Laird and Bara (1984) used a production task technique to conduct extensive research on syllogistic reasoning. After presenting every potential pair of premises in the standard syllogistic form, students were asked to draw a conclusion. Depending on whether these premise pairings were consistent with one, two, or three models, the researcher analysed them. Error rates were greater on multimodal issues, as expected. This demonstrates the psychological principle that thinking about multiple models places a strain on their mental capacity. Evans (2002) disputes many specific findings that have been asserted to support the model theory. Evans (2002) asserts that because neither theory is entirely described, it is quite difficult to choose between the two methods in the majority of these arguments. Because of this, he attempted to test a few predictions that flow from the model theory's most basic ideas alone, without the need for any additional presumptions to explain the particular jobs. They also don't rely on any presumptions regarding the representation of certain quantifiers or connectives. These predictions deal with people's actions when asked to decide between necessity and possibility.

Teacher's Knowledge

The current policies that now govern the teaching profession require teachers to possess a high degree of competency in the many areas where the teaching-learning processes are integrated. In order to support their own and their students' emotional, linguistic, intellectual, and social growth, teachers must rise to the challenges encountered (Hudelson, 2001). This means that teachers must cover a wide range of topics in their work. Many definitions and justifications have been proposed for the term "teacher's knowledge," also known as "teacher's knowledge base," which has long been the focus of much research. The term was mostly used to refer to the fundamental skills needed to educate. It made reference to the implementation of pedagogical strategies and subject matter knowledge (Pineda, 2002). Many academic works have also reported the first process-product studies on teachers' knowledge, which began in the first half of the twentieth century (Brophy & Good, 1984; Gauge, 1978). The purpose of the process-product research paradigm was to establish a connection between student achievement and the methods teachers used in the classroom. As a result, scholars such as Gauge (1978) and his colleagues developed a number of "teacher should" statements that focused on certain teacher actions in the hopes that these behaviours, when implemented in the classroom, would result in high student performance.

Many critiques were levelled at the process-product research paradigm (Gauge & Needels, 1989; Solomon, 1979). The critiques were divided into four main categories: productivity, conceptualization, interpretation-application, and methodology. From a methodological standpoint, the process-product research paradigm faced criticism due to its implausible association between the behaviours of instructors and the academic accomplishment of pupils in different subject areas and at different times. The conception was criticised mostly for defining expected classroom actions for teachers while ignoring their primary objectives or intentions for a given lesson. The remaining complaints focused on the paradigm's predictive potential and how research funding was used to create instructional guidelines.

As a result of these objections, a new study design emerged that modified the process-product research paradigm (Berliner, 1979; Peterson & Sweing, 1982) by introducing the Academic Learning Time (ALT) variable. Berliner (1979) defined ALT as the amount of time students spend working on a task assigned by the teacher within a specific instructional period. He contends that if students are only given simple assignments, their academic performance will not significantly improve. Similarly, if students are working on more challenging assignments, they won't have enough time to learn additional algorithms, ideas, and abilities that will help them do well. Berliner (1979) goes on to say that the ALT is crucial because it acts as a gauge of students' learning and has a direct relationship between the actions of teachers and students' performance. Berliner's (1979) ALT failed to: 1) specify the kind of knowledge that teachers need to have in order to assess students' work fairly; and 2) provide the proper time for a teacher to transition to a new idea.

When scholars such as Peterson and Swing (1982) and Putman (1987) entered this field of study, they contended that it would be wise to place the teacher's mental faculties at the forefront of the investigation. Putman (1987) said that teachers, especially those with more experience, develop mental models of the students they are teaching because of their knowledge of previous students. He went on to say that experienced teachers have this model of their students, which gives them the ability to choose when to assign a new task, give extra practice on a particular type of difficulty, and assess students' knowledge through activities and replies. These researchers believe that by placing the teacher's mental faculty at the centre of the study, it will be possible to properly examine how teachers apply their knowledge to their instructional strategies. According to Freeman and Johnson (1998), "usually in the form of general theories and methods that were assumed to apply to any teaching context," this is among the discrete information that teacher education programmes should impart to teachers.

Content Knowledge (CK)

Content knowledge (CK) has to do with being knowledgeable about the subject matter. Content knowledge is the teacher's understanding of the subject matter being taught or learned, as well as the content to be covered at school according to the curriculum. Stronge (2018) came out that, this understanding would include knowledge of concepts whereas Erickson (2002) states that teachers can use specific strategies to teach students skills they need to think conceptually and to solve complex problems. According to Howard and Milner (2021), students' performance in Mathematics is a function of the teacher's knowledge of the subject matter of Mathematics. In the context of teaching, content knowledge is what teachers teach. But content knowledge is generic. This study specifically focuses on mathematical knowledge in Logical reasoning possessed by preservice teachers who will be deployed to teach Mathematics at senior high schools.

Preservice Teachers' Content Knowledge

A practice-based theory that characterises the content knowledge used in teaching a subject is called content knowledge theory (CKT) (Shulman, 1986). The theory derives from a type of job analysis in which specific instances of recurring content-based teaching activities are found by analysing the work of teachers. After that, CKT is deduced by examining the content demands that teachers must meet in order to complete these tasks. Phelps and

Schilling (2004) state that when a logical case can be made for a direct application of topic knowledge to teaching practice, CKT is the key component to keep in mind. Knowledge of specific topics and general knowledge of the subject matter are the two types of content knowledge. The uses and invariants of the concept constitute the knowledge of specific. (for example, knowledge of Logical reasoning), and general knowledge of subject matter is how things function or are justified, encompassing procedures for testing. The knowledge that connects concepts with various modes of representation is included in the representation of knowledge. The usage of outlines, sketches, and images, as well as the use of materials like ribbons, paper, and cards, are a few indicators of these. This argument is also examined in the study, as it supports the idea that students' academic achievement is mostly dependent on their subject matter knowledge.

Pedagogical Knowledge (PK)

Pedagogical knowledge is the second category. It can be characterised as a variety of methods and strategies that a teacher needs to carry out the teaching task. It gives an in-depth knowledge of teaching and learning processes and practices, including educational goals, techniques, purposes, values, and others. It also comprises knowledge of classroom procedures and approaches, the nature of students' assessment, understanding of how students learn, and knowledge of methods of teaching and learning. Specifically, this study looks at strategies preservice teachers possess to enable them to teach Logical reasoning at SHS.

Preservice Teachers' Pedagogical Knowledge

According to Rodger and Raider-Roth (2006), a teacher must be knowledgeable about their subject matter without continually being able to break it down so that students can understand it. A teacher with expertise in pedagogy may "decompose" subject matter knowledge in a unique way. According to Shulman (1986), pedagogical knowledge is any idea or belief a teacher has about teaching and the learning process that has an impact on the way they teach. According to Risko et al. (2008), pedagogical knowledge may be changed throughout fieldwork and university education coursework, and it is crucial for teaching.

Hudson (2007) investigated the final-year preservice Mathematics teachers from the universities. According to Hudson's research, teachers who work well with their students in the classroom are able to have excellent teaching experiences. The study demonstrates that during undergraduate studies, mentors, fieldwork, and coursework all have a significant impact on instructional knowledge. Acquiring instructional knowledge may also be achieved through experience. According to Gatbonton's (2008) research, novice and experienced teachers had equivalent pedagogical knowledge, but experienced teachers had greater in-depth understanding, particularly when it came to the attitudes and actions of their students. According to his research, fieldwork and college courses are more beneficial in enhancing the pedagogical expertise of aspiring teachers.

A teacher's in-depth understanding of the procedures, methods, and approaches used in teaching and learning is known as pedagogical knowledge. It includes goals, ideals, and objectives related to education. This broad knowledge pertains to lesson design, general classroom management, student assessment, and comprehension of how students learn. A teacher with extensive training in pedagogy has knowledge of students' understanding, their construction of information and skills, their development of mental habits, and their attitude towards learning. Therefore, knowledge of cognitive, social, and developmental theories of learning and how they relate to students in the classroom is necessary for pedagogical knowledge.

Pedagogical Content Knowledge (PCK)

Shulman (1987) defined Pedagogical Content Knowledge as “the most useful form of content representation, the most powerful analogies, illustration, examples, explanations and demonstration, the ways of representing and formulating the subject to make it comprehensible to others…”(p. 9). Effective instruction is the result of the combination of teachers' content knowledge and pedagogical content knowledge, according to Shulman (1987, 1986). He said that the emphasis on teachers' content knowledge and pedagogical knowledge was seen as a mutually exclusive area in research on these domains. According to Shulman, PCK comprises a teacher's knowledge of what makes a particular topic simple or difficult to learn, as well as the ideas and assumptions that students from various backgrounds and ages bring to the study of those most commonly taught subjects and courses. This definition reveals that pedagogy and content are blended to create a knowledge of how certain topics, problems, or challenges are offered for instruction while being organised, represented, and adapted to the various interests and abilities of learners. This highlights the significance of pedagogical knowledge and content knowledge, as well as the links between them.

Shulman (1986) highlighted that a teacher must have in-depth knowledge of the content in order to effectively translate it into a form that students are able to understand. Shulman contended that while subject matter knowledge and general pedagogical strategies are important, they are insufficient to completely represent the knowledge of effective teachers. He suggested that PCK should address the teaching process, including “the ways of representing and formulating the subject that make it comprehensible to others” in order to characterise the various ways in which teachers think about how specific information should be taught. Teachers must embody the aspects of content most germane to its teaching ability (Shulman, 1986, p. 9) in order to successfully address both pedagogy and content difficulties at the same time.

Knowledge of Instructional Strategies

Teachers' knowledge of instructional strategies may be divided into two primary categories in this aspect of pedagogical content knowledge: knowledge of subject-specific strategies and knowledge of topic-specific strategies. The range of effectiveness is what separates both of these strategies from one another. While subject-specific strategies are tailored to the teaching of a particular subject, they are generally applicable. In other words, the strategies mentioned here reflect broad approaches to or overarching plans for implementing education in a certain subject. As a result, teachers must be aware of the different strategies used in the subjects they teach. It can be claimed that subject-matter knowledge and student understanding may play a role in a teacher's ability to use a subject-specific technique.

However, topic-specific strategies have a limited application; they are limited to teaching certain topics within a subject's area. Teachers who are knowledgeable about topic-specific strategies might aid students in understanding particular subject topics. Peace, Fuentes, and Bloom (2018) state that there are two types of topic-specific techniques in scientific teaching. There are activities and representations. Teachers employ representations to help students understand certain concepts or principles, and they are also aware of the relative advantages and disadvantages of different representations (Peace, Fuentes, & Bloom, 2018). Examples, models, analogies, and illustrations can all be classified as representations. It is crucial to remember that having little understanding of topic-specific representation might negatively affect both the subjects and the teaching strategy that is used. In a given teaching scenario, an excellent teacher must also assess what kind of depiction would help students understand and go beyond it.

Students can be assisted in understanding certain ideas or relationships through the use of activities like experiments, simulations, puzzles, demonstrations, and investigations. Teachers that possess this kind of pedagogical content knowledge are those who are aware of an activity's conceptual power, or "the extent to which an activity presents signals or classifies important information about a specific concept or relationship." Two empirical subdomains, pedagogical knowledge and content knowledge, were developed by Desimone (2009) to evaluate Mathematics teacher competence in teaching these tasks. To facilitate student learning in Mathematics, teachers must possess both pedagogical and content knowledge. Clermont, Borko, and

Krajcik (1994) reported that more experienced teachers than less experienced teachers appeared to be familiar with several ways to demonstrate Mathematics tasks for lessons.

Knowledge of Students' Understanding

This facet of pedagogical content knowledge examines how well students understand the lessons that are taught in the classroom. Teachers need to have the prerequisite knowledge in order for students to master specific concepts. This knowledge includes understanding the abilities and skills that students may require. Teachers should be aware of the various reasons why students find learning difficult, since there are many factors that might contribute to this. Students may find certain topics challenging since the ideas are highly abstract and have little influence on the experiences of the students as a whole. Teachers must be aware of the topics that fit into this category as well as the parts of these topics that students find most difficult to understand (Peace, Fuentes, & Bloom, 2018).

Teachers are better able to respond to the diverse needs of their students when they possess pedagogical content knowledge. Some teachers are not able to predict what their students may already know, the questions they may find challenging, their possible responses to instructions, or the questions they may pose. As a result, teachers struggle to modify representations to fit the demands of students (Zembal, Starr, & Krajcik, 1999). Teachers must employ instructional procedures that support arguments for such ideas (Smith, 1999). An additional type of pedagogical content knowledge for teaching the topic is teachers' knowledge of students' syntactic concepts and strategies for assisting them in developing a more nuanced understanding of the subject matter. However, this study focuses on the content knowledge and pedagogical knowledge of preservice Mathematics teachers for teaching Logical reasoning in senior high school rather than the PCK of preservice Mathematics teachers for teaching Logical reasoning in senior high school.

The Conceptualization of Teachers' Knowledge

In the twentieth century, there was a lot to discuss in an attempt to investigate the teachers' knowledge. Coleman (1968) reported in a written debate on the subject of an idea that supports equality of educational opportunities in the field of education. The findings indicated that school-related variables, such as the teacher and the family background, which have been associated with greater changes in students' performance, could only account for one-tenth of the variance in students' performance. Following the publication of Coleman (1968) findings, a great deal of study was done to determine whether or not school-related variables affect students' academic ability. Research from this era and beyond has shown that school-related factors influencing students' academic progress include teacher qualifications, their instructional strategies, and their efficient use of instructional materials (Enu et al., 2015; Farooq et al., 2016; Mji & Makgato, 2006). Furthermore, supporting these conclusions, eminent research has demonstrated that the subject matter knowledge of teachers has a significant impact on the instructional activities they carry out and, in turn, on the academic performance of their students (Howard & Milner, 2021).

But the purpose of these studies was not simply to refute Coleman's (1968) results; it was also to prove that teachers have a certain type of knowledge that is distinct to the teaching profession. Which knowledge is more relevant and distinct to a teacher is still up for debate. Many scholars looked into the distinctive data that comes with teaching, and this led to the discovery of many study opportunities. Shulman (1987) is a prominent researcher whose findings have attracted the interest of educationists. Seven strands were used by Shulman (1987) to conceptualise teachers knowledge. These seven knowledge types are "content knowledge, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge, knowledge of learners and their characteristics, knowledge of educational contexts, and knowledge of educational ends, purposes, and values".

PCK is important among the knowledge strands that Shulman (1987) conceptualised. According to Shulman (1987), the knowledge that distinguishes teachers as professionals from content area experts is PCK. PCK is the teacher's knowledge base, which enables them to translate their subject-matter knowledge into compelling presentations and visuals that effectively and efficiently help students understand the material.

Conceptual Framework

Teachers' content knowledge and pedagogical knowledge are the areas that are grounded in the conceptual framework, which forms the foundation of the conceptual framework that guides the study. According to Pineda (2002), the framework adopts a comprehensive view of teachers' knowledge, which includes both subject matter knowledge and the implementation of pedagogical strategies. The framework presented in Figure 1 provides a visual representation of the variables under study.

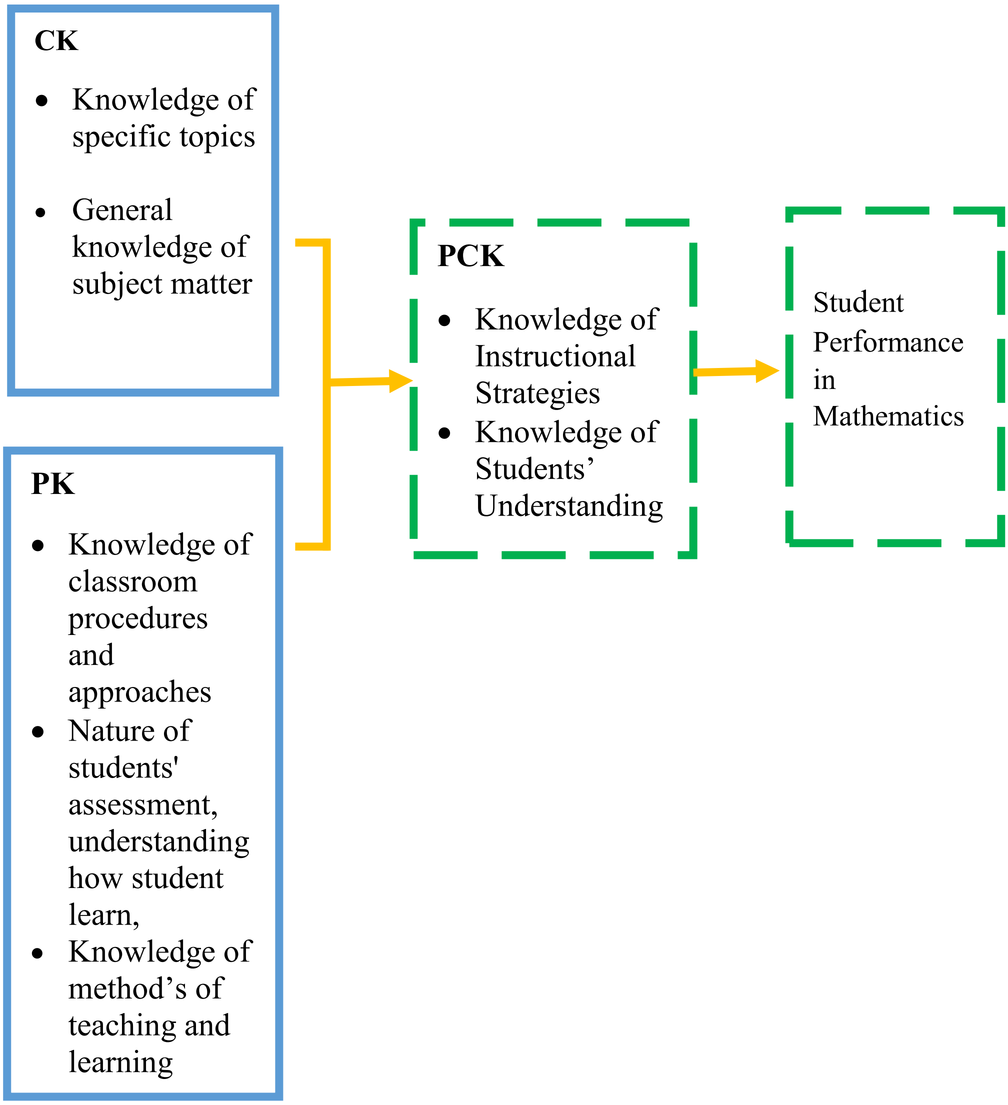


Figure 1: Conceptual Framework

Source: Author's construct (2023)

The study focused on the content knowledge and pedagogical knowledge of preservice Mathematics teachers at the DMICTE. Content knowledge is categorized into Knowledge of a specific topic and General knowledge of the subject matter while Pedagogical knowledge is also categorized into Knowledge of classroom procedures and approaches, Nature of students' assessment, understanding how students learn and Knowledge method of teaching and learning. Content knowledge and pedagogical knowledge influence the pedagogical content knowledge of preservice

Mathematics teachers which then determines students' performance in Mathematics. Data was not collected on pedagogical content knowledge and students' performance in Mathematics which were shown in broken lines. For this study, data was collected on only the content knowledge and pedagogical knowledge of preservice Mathematics teachers.

The impact of field teaching experience on preservice teachers' knowledge

It makes intuitive sense that a person would perform better in his or her field of specialisation the more experienced they have become in it. According to Darling Hammond (2000), a teacher's years of experience in the classroom might serve as an indicator of their ability. A number of study findings support Darling Hammond's (2000) assertion that teachers with more experience are more successful in terms of student achievement. (Harrison & Sass, 2011; Kane et al., 2008; Clotfelter et al., 2007).

Research has indicated that teachers of Mathematics acquire knowledge through teaching experience (Klecker, 2002; Roseholtz, 1987). Putman (1987) further categorised this by claiming that experienced teachers possess internalised knowledge of both the distinctive characteristics of each student and the teaching strategies they employ, allowing them to adapt their education to fit the needs of each student and disprove any misunderstandings they may have. Therefore, teaching in a classroom setting not only improves the teacher's profession but also gives them access to new knowledge that may not have been included in their training; it also gives them the opportunity to advance their pedagogical knowledge. Field teaching has become an essential part of the curriculum for the majority of teacher-training institutes as a result of the recognition of the advantages of teaching experience and the need to guarantee that aspiring teachers have a complete awareness of the classroom environment. Depending on the requirements of the institution's curriculum, these prospective teachers will either complete a semester or a full year of teaching experience, which can be done full-time or in addition to their education. During this time, these teachers have to plan, prepare, and conduct classes in addition to taking part in extracurricular activities.

Different perspectives have been used in studies on the effects of field teaching experience on preservice teachers. The influence of preservice teachers' field teaching experience on their efficacy as teachers has been the subject of continual research (Al-Awidi & Alghazo, 2012; Cheong, 2010; Logerwell, 2009; Moseley, Reinke, & Bookout, 2002; Yilmaz & Cavas, 2008). According to this research, field experience for preservice teachers improves their effectiveness as teachers. In contrast to the beneficial impact that field experience has on the efficacy of preservice teachers, Moseley et al. (2002) discovered that the efficacy of preservice teachers was high prior to the teaching experience but considerably decreased following seven weeks of teaching. Given that it indicates preservice teachers' confidence in their ability to carry out the teaching act, the assessment of the impact of field experience on the attitudinal construct of teaching effectiveness is valuable. The challenge is the possibility that preservice teachers' teaching efficacy may not accurately represent their subject matter and teaching methodology knowledge.

Research on the impact of teaching practicum on the knowledge of preservice Mahematics teachers has produced conflicting findings (Philipp et al., 2007; Strawhecker, 2005). Strawhecker (2005) investigates how various teacher training programmes affect students' subject matter knowledge in Mathematics. There was no discernible difference in the content knowledge of Mathematics between preservice teachers enrolled in any of the following four programmes: concurrently taking a method course, content course, and weekly field teaching experience (CMF group); taking Mathematics methods courses and weekly field teaching experience (MF group); taking Mathematics methods courses only (M-only); and taking Mathematics content courses only (C-only). This was determined by an analysis of variance (ANOVA) and a post-hoc test of the post-test. The CMF group and the M-only group, the MF group and the M-only group, and the MF group and the C-only group were shown to vary significantly from one another. Additionally, it seemed that the CMF and MF groups knew the same amount about this concept. The significant difference in PCK found between those who participated in field teaching and those who took either topic or technique courses demonstrates that teaching has the ability to improve preservice teachers' PCK.

Additionally, this study aims to determine how field teaching experience influences perservice Mathematics teachers' pedagogical knowledge of Logical reasoning in the Ghanaian context. This is due to the recent conceptualization of teachers' knowledge (Ferrin-Mundy et al., 2005; McCrory et al., 2012; Wilmot et al., 2018) and the advent of describing teachers' knowledge for teaching as a measurable construct.

Mathematics Teaching Knowledge

This is the knowledge of Mathematics that is relevant in the classroom but is not covered in a regular secondary or university Mathematics programme. Nonetheless, Mathematics teachers may receive this knowledge as part of their formal education so that they may learn how to teach Mathematics through practicing. It comprises imparting pure Mathematics knowledge within the confines of Shulman's conceptualization of PCK (McCrory et al., 2012). It consists of knowledge that has become increasingly familiar and easily accessible to teachers in practice throughout the period. This kind of knowledge is comparable to the specialised subject knowledge described in the conceptualization of Mathematical knowledge of education by Howard and Milner (2021). Since it is not taught in classes dedicated to teaching only Mathematics, this type of mathematical knowledge is unique to the teaching of Mathematics. Moreover, it is the kind of knowledge that separates a Mathematics teacher from a mathematician.

Preservice teachers' knowledge for teaching high school Mathematics

According to research on teacher education programmes, course design and structure affect teacher knowledge and practices, which consequently affect students' learning (Howard & Milner, 2021; Quinn, 1997; Vacc & Bright, 1999). The emphasis on Mathematics Content Knowledge and Pedagogical Knowledge of Shulman's (1986) conceptualization of teachers' knowledge differs greatly between nations, according to a cross-national comparative study into the structures and organisation of Mathematics teacher preparation programmes (Tatto et al., 2010). Tatto et al. (2010) state that while pedagogical knowledge and Mathematics content knowledge were also prioritised in high school teacher education programmes, Mathematics content knowledge was given more emphasis in high school teacher education programmes. The most important thing is that preservice teachers who go through these frameworks have a strong basis in the subject matter they teach and other education-related courses. This basis influences their classroom practice, which in turn affects students' achievement.

Goos (2013) studied the connection between Pedagogical Knowledge and Mathematics Content Knowledge. Pedagogical Knowledge (PK) and Mathematics Content Knowledge (MCK) are the result variables for a large number of stepwise regressions that have been conducted. While PK and prior level of Mathematics experience were the predictors of MCK, MCK was the single predictor of PK. According to these results, MCK and PK are two essential categories in Mathematics education, and students enrolled in teacher education programmes may enhance both of them at the same time. Furthermore, the finding that MCK is the sole predictor of PK gives credence to Byrne's (1983) claim that a teacher's effectiveness in the classroom is derived from both their use of the right pedagogical procedures and their understanding of the subject matter. As a result, content knowledge and pedagogical knowledge as they are presented in teacher preparation programmes have to be regarded as distinct concepts.

Understanding students mathematical knowledge is a basis for evaluating a teacher's knowledge in the Mathematics (Howard & Milner, 2021). The tertiary-level educational goals for preservice teachers are to possess Pedagogical Knowledge (PK) and Mathematics Content Knowledge (MCK). Perservice Mathematics teachers had low MCK and PK, based on an analysis of data from these studies (Depaepe et al., 2015; Leong, Chew, & Rahim, 2015). Depaepe et al. (2015) evaluated the MCK and PK on rational numbers using preservice Mathematics teachers from elementary and lower secondary schools. Items measuring pedagogical knowledge (PK) assessed preservice teachers' knowledge of fraction and decimal numbers, as well as the algorithmic operations (addition, subtraction, multiplication, and division) associated with these concepts. These items were developed using Shulman's (1987) conceptualization of pedagogical knowledge and content knowledge.

Leong et al. (2015) reported that 6.9% of preservice Mathematics teachers in secondary school in Malaysia who were enrolled in the Teacher Education Study in Mathematics (TED-M) fell within the higher level of MCK, while 57.1% of them fell within the lower level. The preservice Mathematics teachers in secondary school had a mean score of 493 on items, which was lower than the global mean of 530. On the pedagogical knowledge (PK) items, the same participant group's mean score was 472, whereas the global mean was 520. These studies suggest that preservice teachers are more competent in MCK than in PK, despite the fact that the results indicate that preservice Mathematics teachers have limited understanding of both MCK and PK. This implies that, in contrast to the PK, teacher training institutions can provide preservice teachers with the required MCK.

Theory of Performance

Elger (n.d.) introduced the theory of performance, which builds upon and connects fundamental ideas related to sex in order to develop a framework for understanding performance and enhancing it. As stated in the Cambridge Advanced Learner's Dictionary (2007), to perform is to carry on an action or task in order to produce results. In other words, to perform is to carry out a difficult set of tasks that combine knowledge and abilities to yield a useful outcome. Therefore, an individual or a group of individuals working together is a performer. It is possible for students to do exceptionally well in their academic careers. Every single day in the classroom, students perform at a remarkable level.

Academic performance is defined by Ankomah (2011) as the output that students exhibit at the end of a set of tests. According to Tetteh (2011), achieving one's goals or reaching one's potential through effort may also be considered academic success. Furthermore, academic success is viewed as a process where students demonstrate their desire to complete assignments. Therefore, after completing a period of thoroughly planned instruction, students might need to be assessed. This will make it possible for teachers to evaluate and group students' work in order to determine their performance.

Gender Performance in Mathematics

According to Ramaswamy (1990), the importance of gender in academic performance and achievement has brought up important issues for educational scholars, such as what factors influence student performance and how gender disparities in academic achievement show up. Students knowledge and skills in school courses are referred to as their academic performance. This applies to both male and female students. According to Sinha (1970), students who achieve high marks in their academic performance are successful. In contrast, students who do poorly on their prior examination and have low divisions on their examination are viewed as having failed in their attempts.

Ajai and Imoko (2015) report that there is no statistically significant difference between the performance of males and females in mathematics at SHS. Findings of some research studies have reported no statistical difference between the performance of males and females in mathematical tasks; some reported differences in favour of females and vice-versa. For example, Aiyedun (2000), Jahun and Momoh (2001), and Abiam and Odok (2006) reported non-significant statistical differences in the performance of male and female senior high school students in mathematical tasks.

A study conducted by Armah, Akayuure and Armah (2020 on Mathematics achievement among male and female distance learning students. The results indicate that male students perform better than female students. Also, Wilson (2007) conducted a study on performance in Core Mathematics by sex from 2001 to 2005 in senior secondary school. Wilson (2007) uses chi-square test statistics to analyse performance in Core Mathematics by sex. The result indicates that performance in Core Mathematics in the SSSCE for the period 200I to 2005 is dependent on the sex of the candidate. Male students performed better than their female counterparts. Also, Reyes and Stanic (1988) reported that male students achieved a higher level of mathematics in primary and middle school than female students. Awoniyi (2016) observed that male candidates performed better, relative to female candidates in subjects requiring quantitative ability. He said males show superiority in science, statistics and accounting.

Raimi and Adeoye (2002) in their research on gender differences among college students as determinants of performance in integrated science revealed that there is a significant difference between the performance of males and females in integrated science which is in favour of males. It is very necessary to compare students' performance in Logical reasoning based on gender to know if male students' performance in Logical reasoning is better than that of female students. Hence, the a need to investigate male and female preservice Mathematics teachers' content knowledge in Logical reasoning.

The need to focus on Logical reasoning

Mathematics has been categorized into different content areas. Major categories are Algebra, Geometry, Trigonometry, Calculus, and Logical reasoning. In Ghana, there are two Mathematics curricula (Core Mathematics and Elective Mathematics) at the senior high school level. Elective Mathematics covers Algebra, Coordinates geometry, Trigonometry, Logic, Vectors and Mechanics, Calculus, Matrices and Transformation, Statistics and Probability. The Core Mathematics curriculum also covers Numbers and numeration, Plane geometry, Mensuration, Algebra, Statistics and Probability, Trigonometry, and Vectors and Transformation, in a plane (Ministry of Education, 2010a; 2010b).

The syllabus is structured to cover the three years of senior high school (SHS). Each year's work has been divided into units. For core mathematics, SHS 1 has 13 units; SHS 2 has 12 units while SHS 3 has 4 units of work. (Ministry of Education, 2010a).The SHS 3 units include; Constructions, Mensuration II, Logical reasoning and Trigonometry II. Sub-trands of Logical reasoning in core Mathematics include the statements, implications etc. Logical reasoning is a topic in both Core Mathematics and Elective Mathematics at SHS, the content of Logical reasoning at SHS serves as the prerequisite knowledge for some courses at the tertiary level. Curriculum developers of high school Mathematics of Ghana esteemed the importance of Logical reasoning and its proficiency to students in other areas of studies and towards national development. For this reason, Logical reasoning is considered to be in both curricula even though they differ in the scope of the content.

Logical reasoning has many definitions but for this study, it is defined as selecting and interpreting information from a given context, making connections, verifying and drawing conclusions based on the information provided, and interpreting the information with the associated rules. The contents of Core Mathematics and Elective Mathematics for Logical reasoning in senior high school include Statements, Negation, Implications, Compound statements, Disjunction, Conjunction, and truth tables (MOE, 2010).

Students are assessed based on the content of Logical reasoning during their West African Secondary School Certificate Examination (WASSCE). The Mathematics chief examiner of WAEC reports that students have weaknesses in answering Logical reasoning questions during summative examinations. The Chief examiners' reports specifically show that students performed very poorly in Logical reasoning (WAEC, 2013 & 2021). The Chief examiner over the years has highlighted that most students do not attempt Logical reasoning questions, and also many of the few who attempt it had it wrong. It shows that the area of Logical reasoning was neglected by the candidates, or probably was not taught by the teachers. Many researchers have also indicated that understanding proof, particularly Logical reasoning, is a challenge to students of all ages, including preservice teachers (Bell, 1976; Healy & Hoyles, 2000; Stylianides & Stylianides, 2022). In particular, studies conducted on college students' Logical reasoning reported that undergraduates in general, and preservice elementary teachers in particular, were not able to interpret disjunctive and logical statements well (Eisenberg & McGinty, 1974). Moreover, Vest (1981) also noted that college students did not have a deep understanding of disjunction and conjunction statements. Because of those difficulties, certainly, secondary school students are not able to reason logically and this does not develop their critical thinking abilities autonomously.

Focusing on Logical reasoning is an essential goal of Mathematics educators in Ghana at the tertiary level. Mathematics educators found out that, students do not possess the required knowledge in Logical reasoning at the secondary school level therefore they consider it best to start running courses that are related to Logical reasoning. For instance, the University of Cape Coast is running "Critical Thinking and Practical Reasoning" as a compulsory course for its students. Besides this, the DMICTE at the University of Cape Coast also runs courses that are embedded with Logical reasoning for their students to acquire more skills. Logical reasoning is believed to facilitate students' ability towards solving most problems in Mathematics correctly. It serves as the basis for the approach used to solve Mathematics problems (NCTM, 2000). The National Council of Teachers of Mathematics (NCTM) recommends that all high school Mathematics programmes should focus on Logical reasoning. This will help students to develop connections between new learning and their existing knowledge, increasing their likelihood of understanding and retaining the new information.

Statements

A statement can be defined in this context as an expression or sentence that can be either true or false, but not both (Suppes & Hill, 1992). Any time you say a proposition that can be labelled as either true or false, you are making a statement. For instance, “a triangle has three sides”. This is a statement because it can be true or false. But whenever a proposition is neither true nor false then it is not a statement. For example, “How many sides does a triangle have?” This cannot be considered to be true or false, because this expression requires the number of sides of a triangle. The example, “the number x^2 is always positive” can be considered to be true or false. Statements in general can be about anything at all. When you have two statements and you want to combine them, you can add either an 'and' or an 'or' between the two statements. Each has a different meaning.

Disjunction

One important form of connective logic is disjunction. A disjunction is a statement involving an “or” to combine two simple statements. For example, suppose we have two statements, P and Q.

P: 29 is a composite number

Q: 49 is a prime number

These two statements from the disjunction, “29 is a composite number or 49 is a prime number”. It can be written as P or Q and denoted as P ∨ Q. Either P or Q can be a true or false statement. From the given statements, P is a false statement and Q is also a false statement. Therefore, the disjunction is false. A disjunction is considered true if either one or both of the statements is true. In other words, if either P or Q is true, then the disjunction is true. Let us consider a second example; suppose we have the following two simple statements P and Q.

P: The sum of the interior angles of a pentagon is 180°

Q: The sum of angles on a straight line is 180°

These two statements form the disjunction, "The sum of interior angles of a pentagon is 180° or the sum of angles on a straight line is 180°". From the given statements, P is a false statement while Q is a true statement. Hence, the resulting disjunction is true, since one of the statements is true.

Conjunction

A conjunction is a statement formed by adding two simple statements with the connector "and". The symbol for conjunction is "∧" which is read as "and". When two statements p and q are joined to form a new statement, the conjunction will be expressed symbolically as $p \wedge q$. If both of the simple statements are true, then the resulting conjunction will be true. That is, the conjunction will only be true if both the combining statements are true; otherwise, it is false. Suppose we have P: "27 is a composite number" and Q: "49 is a prime number" as our two simple statements. We combine them with 'and' to make the compound statement $P \wedge Q$: "27 is a composite number and 49 is a prime number". Statement P is true but statement Q is false. Hence the resulting conjunction is false. A conjunction is true only when the two statements are true.

Implications

The concept of logical implication encompasses a specific logical function, a specific logical relation, and the various symbols that are used to denote this function and this relation. To define the specific function, relation, and symbols in question it is first necessary to establish a few ideas about the connections among them. Close approximations to the concept of logical implication are expressed in ordinary language using linguistic forms like the following;

“p implies q” and “if p then q”.

Here p and q are propositional variables that stand for any propositions in a given Mathematics statement. In a statement of the form “if p then q”, the first term, p is called the antecedent and the second term q is called the consequent. Assuming that the conditional statement is true, then the truth of the antecedent is a sufficient condition for the truth of the consequent, while the truth of the consequent is a necessary condition for the truth of the antecedent. In Logical reasoning “if” is represented by $\Longrightarrow$, “Only if” is represented by $\Longleftarrow$ and “If and Only if” is also represented by $\Leftrightarrow$. Examples of implications that consist of compound statements are shown in the following problems.

1. Consider the following statements:

   X: All policemen wear uniforms.

   Y: Civil servants do not wear uniforms.

   If P = {policemen}, U = {people who wear uniform} and C = {civil servants};

   a) Draw a Venn diagram to illustrate X and Y.

   b) Use the Venn diagram to determine whether each of the following implications is a valid or not valid conclusion from X and Y.

      i. Adu wears a uniform $\Rightarrow$ Adu is a policeman.

      ii. Ofei is a policeman $\Rightarrow$ Ofei is not a Civil Servant.

Suggested solution

(a) Let U = {people who wear uniform}

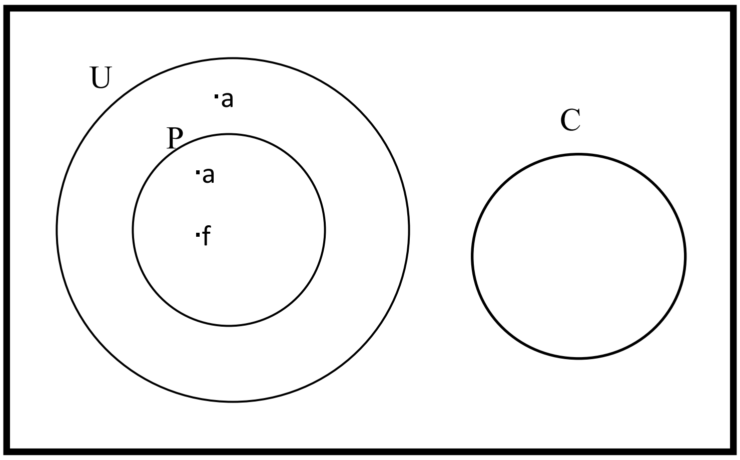
P = {policemen}

C = {civil servants}

·a = Adu

·f = Ofei

From statements X and Y, we can write that $P \subset U$ because all policemen wear a uniform. U and C are disjoint sets because civil servants do not wear uniform. This interpretation guides the drawing of the Venn diagram.



b(i) If Adu wears a uniform then he is a policeman.

"Adu wears a uniform" means Adu is inside set U. There are two possible positions: Adu can be located inside P or outside P but all in U. Adu could be a policeman and wears uniform, or he could be a person who wears uniform but not a policeman ($a \in P$ or $a \in U$). Hence the statement, "If Adu wears a uniform then he is a policeman" is not always true and not valid.

b(ii) "Ofei is a policeman" means Ofei is inside P, and not in C. But P and C are disjoint. Hence the statement, "Ofei is a policeman then he is not a Civil servant" is always true and so valid.

2. Consider the following statements:

   Q: All students are hardworking.

   R: No hardworking person is careless.

   a) Draw a Venn diagram to illustrate the above statements.

   b) State whether each of the following implications is a valid or not valid conclusion from Q and R.

      i. Kwesi is a student $\Rightarrow$ Kwesi is not careless.

      ii. Asiedu is hardworking $\Rightarrow$ Aseidu is a student.

      iii. Efua is careless $\Rightarrow$ Efua is not a student.

Suggested solution:

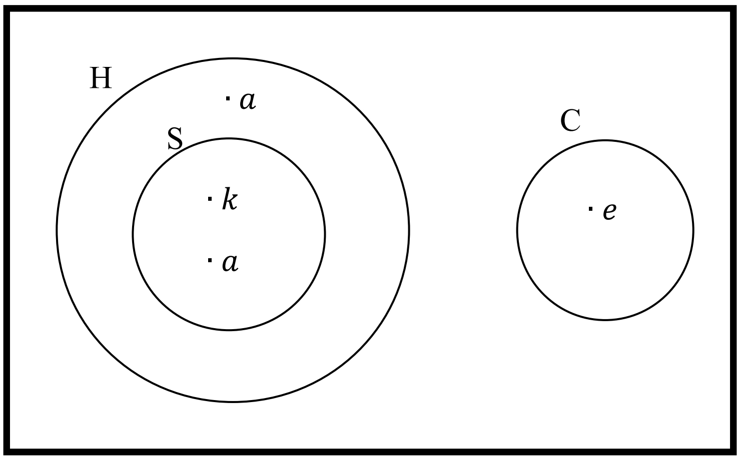
a) Let S = {students} H = {Hardworking people} and C = {careless people}.

$\cdot\, k$ = Kwesi

$\cdot$a = Aseidu

$\cdot$ e = Efua

From the statements R and Q, we can write that $S \subset H$ because all students are hardworking and H and C are disjoint sets (no hardworking person is careless). P and Q are illustrated in the Venn diagram below.



bi) Valid

"Kwesi is a student" means Kwesi is inside S. That is k $\in S$. Therefore, he cannot be inside C. Hence the statement if Kwesi is a student then Kwesi is not careless is always true and so valid.

bii) Not valid

"Aseidu is hardworking" means Aseidu is either inside H or in S but outside H. Hence the statement if Asiedu is hardworking then Aseidu is a student is not always true and so not a valid deduction.

biii) Valid

"Efua is careless" means Efua is inside C. Therefore, she cannot be inside S. Hence the statement if Efua is careless then Efua is not a student is always true and so valid.

Truth Table

Truth table is used to perform logical operations in Mathematics. It is used to check whether the given propositional expression is true or false, as per the input values. This is based on Boolean functions. It consists of columns for one or more input values, say, p and $q,$ and one assigned column for the output results. The output which we get here is the result of the binary operation performed on the given input values. Some examples of binary operations are "if-then", "and", "or", etc. For instance, below is the truth table for the "if-then", "and", "or", of two simple statements P and Q.

P	Q	$P \Rightarrow Q$	$P \wedge Q$	$P \vee Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	F	F	F

Use the truth table above to show whether each of the following statements is true or false.

(a) P: 10 is divisible by 5

Q: 10 is a multiple of 2.

Suggested solution:

Because both P and Q are true statements, the conjunction $P \wedge Q$ is true and the disjunction $P \vee Q$ is also true.

(b) P: A triangle has three sides

Q: A parallelogram has three sides.

Statement P is true while statement Q is false (because not both statements are true). Hence $P \wedge Q$ is false and $P \vee Q$ is true (because one of the statement is true).

Chapter Summary

This chapter extensively reviews related literature on the theory that underpinned the study and conceptual framework with its accompanying concepts. The concept of Content Knowledge (CK) and Pedagogical Knowledge (PK) of preservice Mathematics teachers were extensively discussed. Gender performance of preservice Mathematics teachers was reviewed. The performance of students in WASSCE questions on Logical reasoning was also reviewed**.**

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CHAPTER THREE

RESEARCH METHODS

The main purpose of this research is to investigate the content knowledge and pedagogical knowledge of preservice Mathematics teachers for teaching Logical reasoning in senior high school. This chapter specifically takes a look at the research design, study area, population, sampling procedure, data collection instruments, data collection procedures, and data processing and analysis procedures.

Research Design

The purpose of this study was to investigate content knowledge and pedagogical knowledge of preservice Mathematics teachers for teaching Logical reasoning in senior high school. A cross-sectional survey design was used. A cross-sectional research design was regarded as the most appropriate for this research based on its ability to give a "snapshot of the outcome" (Levin, 2006, p. 24) and request and describe the characteristics of the respondents of the research within a short period of time (Cohen, Manion, & Morrison, 2002; Creswell, 2012). This research gathered information on the kinds of knowledge that define preservice Mathematics teachers' content knowledge and pedagogical knowledge for teaching Logical reasoning. Participants responded to true or false items and open-ended types of questions. The questionnaire used for this research was aimed at investigating content knowledge and pedagogical knowledge of preservice Mathematics teachers that are expected to possess for teaching Logical reasoning at the senior high school level. Also, the research design is efficient in that it permits the collection of current information on a large number of preservice high school teachers' content knowledge and pedagogical knowledge for teaching Logical reasoning during the period (Creswell, 2012).

Many researchers have used this design to extend a large sample size, which ultimately contributes to the generalisation of the findings. In addition to making responses that are simple to code, this design gives respondents an opportunity to respond to the items in the questionnaire at a time and place that's convenient for them (Gay, 2013). A cross-sectional survey design has the ability to provide inferential, descriptive, and explanatory evidence that might be used to establish correlations and associations between the variables being investigated (Cohen, Manion, & Morrison, 2010, p. 169).

According to Sarantakos (2013), a cross-sectional survey also offers a consistent and reliable procedure, and participants are not impacted by the researcher's presence or demeanour. While there are advantages to this research design, Sarantakos also points out certain disadvantages, such as the survey's inability to ask probing questions or follow up with respondents' observations. Also, it cannot ascertain the circumstances under which the participants responded to items on the questionnaire as well as their capability to come up with a remarkable number of responses.

The results of this study are highly subject to change when the same information is collected multiple times, which is a disadvantage of the cross-sectional design. Therefore, the result of the research cannot account for the changes that will take place in preservice teachers' knowledge of teaching Logical reasoning at senior high school after the course of the research. Despite the cross-sectional design's shortcomings, it was concluded that it was the most appropriate design for this research.

Study Area

The research was carried out from Depatment of Mathematics and ICT Education at University of Cape Coast. The University of Cape Coast was selected for the study because they do more courses involving Logical reasoning, level 200 students at the University of Cape Coast all do Critical Thinking and Practical Reasoning. Also, students from Depatment of Mathematics and ICT Education at University of Cape Coast offer courses such as Nature of Mathematics (EMA 201) and Advance Algebra and Calculus (EMA 202) which involve Logical reasoning.

Population

The population for this study was all level 200 to level 400 preservice Mathematics teachers from the Department of Mathematics and ICT Education at the University of Cape Coast. The target population was made up of all 184 (Level 400) preservice Mathematics teachers pursuing a Bachelor of Education in Mathematics at the University of Cape Coast. In the second semester of their third year, preservice Mathematics teachers at these training universities had completed their On-Campus Teaching Practice (On-CTP). Preservice teachers use the On-CTP platform to practice their newly acquired pedagogy and methods for teaching senior high school Mathematics courses. Additionally, it provides the opportunity for preservice teachers to demonstrate their knowledge in other education-related courses that cover classroom management and strategies for motivating and reinforcing students positively. The preservice Mathematics teachers are separated into groups during the On-CTP period, where each group member has the opportunity to teach fellow students once a week under the supervision of a minimum of one faculty member. Thus, the aim of On-CTP is to get preservice teachers ready to begin teaching in senior high schools.

Preservice teachers are assigned to senior high schools across Ghana during the first semester of their final year to participate in what is known as "Off-Campus Teaching Practice" (Off-CTP), where they teach Mathematics for the entire academic semester. They teach Mathematics under the regular supervision of internal supervisors, mostly heads of the department for Mathematics in the various schools. At the various senior high schools of practice, faculty members from the training university frequently supervise preservice teachers.

Accessible population was 138 level 400 preservice Mathematics teachers sampled from the Department of Mathematics & ICT Education at the University of Cape Coast. The reason Level 400 were sampled from Bachelor of Education in Mathematics (level 200 to level 400) students from the Department of Mathematics and ICT Education study was that 1) they learned Logical reasoning as a part of these courses: Nature of Mathematics, Advanced Algebra and Calculus, and Critical Thinking and Practical Reasoning. 2) having been taken through courses such as the Secondary School Mathematics Curriculum, which addresses theories specific to the topics covered in the Senior High School Mathematics Curriculum, such as Logical reasoning. Therefore, it is presumed that students possess a solid understanding of senior high school Logical reasoning. 3) They have taken courses on methods of teaching and had at least one semester of classroom experience.Data was collected in thier final semester. Hence, it is expected that these preservice teachers will be conversant with the understandings and misunderstandings of students in addition to their practical application of Logical reasoning.

Sampling Procedure

All preservice Mathematics teachers who took part in the research study were selected from the Department of Mathematics and ICT Education at the University of Cape Coast. A multi-stage sampling procedure was employed to choose the participants. Two sampling techniques were used in the present study. Specifically, purposive and census sampling techniques were used.

According to Crossman (2020), a purposive sample is a non-probability sample that is chosen based on the characteristics of a population and the objective of the study. Purposive sampling was used to select the Level 400 preservice Mathematics teachers at Department of Mathematics and ICT Education (DMICTE) of the University of Cape Coast because as at the time of data collection they have done more content and methods of teaching courses. The Department of Mathematics and ICT Education at the University of Cape Coast is one of the departments that helps their students take more Mathematics course content in senior high school and those Mathematics course content addresses knowledge in Logical reasoning.

The census technique was used since all level 400 preservice Mathematics teachers selected in Department were included in the study. A census survey is used because a large sample gives better judgement than smaller ones, provided such large samples are accessible and readily available. (Borg & Gall, 1993). The population size was all the level 200 to level 400 preservice Mathematics teachers at the Department of Mathematics and ICT Education (DMICTE). The sample size was level 400 preservice Mathematics teachers from the Department of Mathematics and ICT Education, University of Cape Coast.

Data Collection Instrument

The instrument for the data collection was a questionnaire and achievement test. Questionnaire was Adapted from Kobylarek et al.,(2022). Kobylarek et al.,(2022) questionnaire was about strategies of teaching critical thinking. Questionnaire was adapted to fit the strategies for teaching Logical reasoning. There are two main sections to this questionnaire. That was section A and C. In Section A, respondents' information on demographics is requested. Section C seeks preservice Mathematics teachers' knowledge of strategies for teaching Logical reasoning in senior high school. Section C had 14 strategies on knowledge of teaching Logical reasoning. They were structured using a 5-point Likert scale labelled: Never true (a value of 1), Rarely true (a value of 2), Neutral (a value of 3), Sometimes true (a value of 4) and Always true (a value of 5). Achievement test was Adopted from Wassce 2013 and 2021 questions. The questions were within the content of the syllabus. Section B is an achievement test that contains questions that seek respondents' content knowledge on Logical reasoning. Section B contains fifty questions, with question 4 to 38 being true or false item types and the remaining three questions being open-ended types.

Validity and Reliability of the Instrument

The instrument was put through validity and reliability tests since a research instrument's content validity is crucial to any study. Sarantakos (2013) asserts that it is one of the fundamental principles of social science research. Because one of the means for achieving content validity is judgement from experts (Gay 2013; Borg & Gall 1989), the questionnaire was handed over to my supervisor for review. The expert supervisor also looked at face validity, which is the process of determining if a test "looked valid on the face" (Lehmann & Mehrens, 1991).

Cronbach's alpha was used to examine the questionnaire's reliability, and the results showed a coefficient of 0.891. According to Fraenkel and Wallen (2000), "for research purposes, a useful rule of thumb is that reliability should be 0.70 and preferably higher" (p. 17), the questionnaire was considered reliable.

Data Collection Procedure

The instrument was administered to the final year preservice Mathematics teachers and this process was done in their final semester. The researcher first obtained an ethical clearance from the Institutional Review Board (IRB), UCC. A letter of introduction was then obtained from the Head of the Department of Mathematics & ICT Education. Upon approval to take data from preservice Mathematics teachers from the department, the researcher asked permission from some of the lecturers in the department to administer the instrument to the students during their lecture time. Since the preservice Mathematics teachers were all in one class, the researcher met them at the agreed lecture time and venue with their lecturer. Before the instrument was administered, students were informed of the purpose of the study and its significance at the meeting, and they subsequently gave their consent. When the time allocated to the questionnaire was up, the researcher collected the completed instruments from the students.

Data Processing and Analysis

Every study requires some level of inspection to transform raw data collected in the field into knowledge that is both meaningful and pertinent for making decisions. Sharp, Peter, and Howard (2002) state that the processing of raw data may include ordering and shaping the data generated from the research to yield understanding. Grove Burns & Gray (2012) also found out that processing raw data helps minimise errors by arranging the voluminous data collected and analysing it to come up with findings. The items under content knowledge were assigned true or false. During the data entry, a wrong response was scored 0 and a correct response was scored 1 as coded data on Statistical Package for the Social Sciences (SPSS). For pedagogical knowledge, the five-point Likert scale items were also assigned numbers. The responses were coded as follows: Always true was scored 5 points, Sometimes true was scored 4, Neutral was scored 3, Rarely true was scored 2 and Never true was scored 1 point. Data from this study were solely quantitative. Quantitative data allows us to extrapolate the findings from a sample group to the general population. The first research question was, “What are the preservice Mathematics teachers' content knowledge in Logical reasoning?” Responses gathered from preservice Mathematics teachers to the items in Section B of the research questionnaire were used. A bar graph was used to show the general performance of preservice Mathematics teachers on content knowledge. The bar graph was used to display the various outcomes of the sub-strands on Logical reasoning. It shows various performance in the sub strands on Logical reasoning where red represent “fail” and blue represent “pass”. The second research question was, “What are preservice Mathematics teachers' knowledge of strategies for teaching Logical reasoning?" This item was to investigate the pedagogical knowledge of preservice Mathematics teachers in teaching Logical reasoning. Responses to the items in Section C of the questionnaire were used to analyse this question. The data collected from respondents on pedagogical knowledge were analyzed using descriptive statistics, specifically mean and standard deviation. Graphs and tables were used to help understand the meaning of the analysed data. Mean was used to describe the centre position of a distribution for a data set. Standard deviation was used to describe the spread of the data set. From the scoring guide, a mean score of 3.5 and above showed the "Always true" of the respondents to the statement, while a mean of 2.4 to 3.4 meant the respondents were "Neutral" (not decided) on the statement. However, a mean of 2.4 and below showed "Never true" of the respondents to the statement.

Research hypothesis one was "There is no significant difference in the content knowledge of male and female preservice Mathematics teachers in Logical reasoning." Responses to items in Section B of the questionnaire on content knowledge in Logical reasoning were analysed to answer this research question. An independent sample t-test was used to compare the mean achievement scores of male and female preservice Mathematics teachers. The test was to investigate the significance of any difference in the content knowledge of male and female preservice Mathematics teachers in Logical reasoning.

Research hypothesis two: There is no significant difference in the knowledge of strategies for teaching Logical reasoning to WASSCE and DBE applicants. The purpose is to find out which applicants perform better in the knowledge of strategies for teaching Logical reasoning. To answer this hypothesis, responses from WASSCE and DBE applicants to items in Section C were used. An Independent sample t-test was used to compare the mean scores of WASSCE and DBE applicants.

Research hypothesis three was, "There is no significant difference in the knowledge of strategies for teaching Logical reasoning possessed by male and female preservice Mathematics teachers." This research hypothesis aims to investigate the difference in the knowledge of strategies for teaching Logical reasoning possessed by male and female preservice Mathematics teachers. Data from responses to Section C was organized by sex and used to answer the research hypothesis. Independent sample t-test was used to compare the mean performance of male and female preservice Mathematics teachers and to investigate the significance of any observed difference. Each hypothesis was tested at a significant level of .05.

Chapter Summary

The main aim of this study was to investigate content knowledge and pedagogical knowledge of preservice Mathematics teachers for teaching Logical reasoning in senior high school. To achieve this, cross-sectional design was considered the most appropriate for this study because it has the potential to provide a lot of information from the preservice Mathematics teachers in level 400 within a short period. The study mainly used a quantitative approach.

The target population of the study was all level 400 preservice Mathematics teachers at the Department of Mathematics and ICT Education, University of Cape Coast. Purposive sampling was used to select all level 400 students only because they have done more courses covering the topics under study and have also gone through both on-campus and off-campus teaching practices at the university. The instrument that was used to collect the data was a questionnaire. The data was analysed using descriptive statistics (frequency, percentages, mean, and standard deviation) and inferential statistics (independent sample t-test).

CHAPTER FOUR

RESULTS AND DISCUSSION

Overview

This chapter presents the results from the analysis of data as well as a discussion of the results. The purpose of the study is to investigate content knowledge and pedagogical knowledge of preservice Mathematics teachers for teaching Logical reasoning in senior high school. To answer the research questions, quantitative data was collected from preservice Mathematics teachers at the Department of Mathematics and ICT Education (DMICTE), University of Cape Coast. The data collected were analysed with the use of descriptive statistics (percentages, frequencies, mean, and standard deviation) and inferential statistics (Independent sample t-test). A total of 138 preservice Mathematics teachers responded to the instrument.

Demographic Characteristics of Respondents

The demographic characteristics of the respondents presented in this section include gender and age. The type of certificate respondents used to apply for the programme of study was also considered. These demographic characteristics were considered important because they could help the analysis of the research hypothesis formulated. Additionally, they would improve our understanding of the group of participants who were responders. Frequency and percentages were used to evaluate the data on the respondents' demographic attributes. The results are presented in Table 1.

Table 1: Characteristics of preservice Mathematics teachers

Demography	Frequency	Percentage
Sex		
Male	107	77.5
Female	31	22.5
Age		
20-24 years	70	50.7
25-29 years	46	33.3
30-34 years	20	14.5
35 and above years	2	1.4
Type of certificate used to apply		
WASSCE	113	81.9
DBE	25	18.1

N=138

Source: Field survey (2023)

It is seen from the table that, one hundred and seven (77.5%) of the respondents were male, while thirty-one (22.5%) of them were female. This shows that there is a gender disparity in the preservice Mathematics teachers used for the study. Table 1 also shows that 50.7% of the respondents were in the age range of 20–24years, and 1.4% of them were 35 years and older. In all, as many as 116 (84%) of the respondents were in the age range of 20–29 years.

From the table, 81.9% of the respondents used WASSCE results to apply for the programme while 18.1% of them used a Diploma in Basic Education (DBE) certificate to apply for the programme. This indicates that the Department of Mathematics and ICT Education trains more students with WASSCE certificates than those with a Diploma in Basic Education Certificate.

Preservice Mathematics Teachers' Content Knowledge in Logical reasoning

This research question targeted preservice Mathematics teachers who were at level 400 during data collection. The purpose of the research question was to investigate preservice Mathematics teachers' content knowledge in Logical reasoning. To achieve this purpose, quantitative data were obtained from preservice Mathematics teachers on their content knowledge in Logical reasoning. An achievement test, which was part of the questionnaire, was conducted in Logical reasoning to investigate their content knowledge. The test questions were formulated based on sub-strands in Logical reasoning in both Core Mathematics and Elective Mathematics. The sub-strands in Logical reasoning include statement, disjunction, implication, truth table, the "or" table, "if, only if" and "if and only if" table and compound statements. To answer research question 1, the sub-strands in Logical reasoning were analysed based on the percentage that failed or passed the various sub-strands. The distribution of the pass rate on various sub-strands is presented in Figure 2.

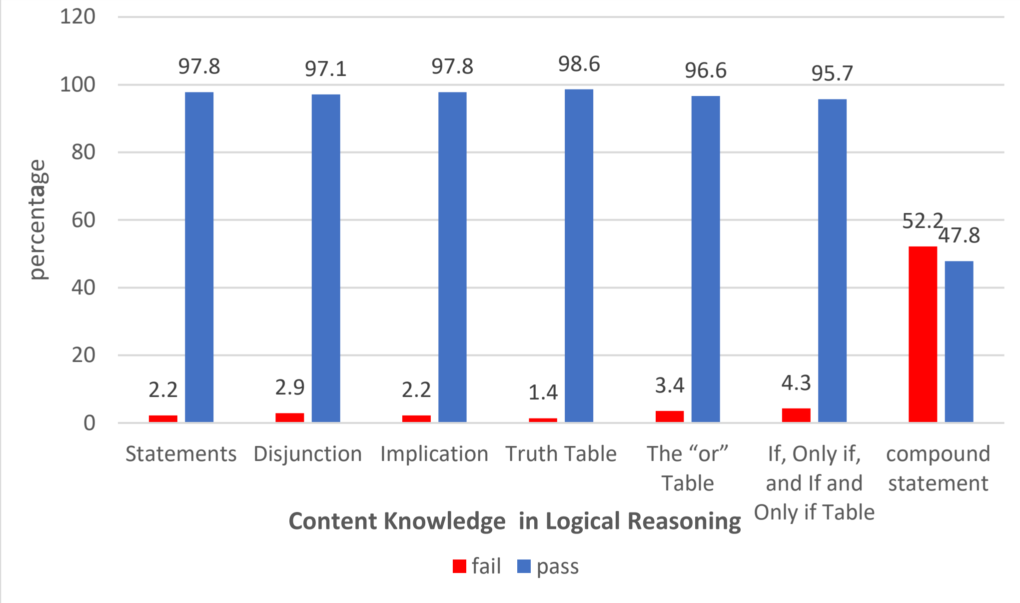


Figure 2: Content knowledge in various subtopics under Logical reasoning

Figure 2 shows the performance of preservice Mathematics teachers on the sub-strands under Logical reasoning in both Core Mathematics and Elective Mathematics. There were 50 items on seven sub-strands of Logical reasoning, which consist of four items on statements, four on disjunction, six on implications, five on the truth table, five on the "or" table, eleven on the “if”, “only if”, and “if and only if” tables, and fifteen on compound statements. It can be seen that preservice Mathematics teachers possess acceptable knowledge in the various sub-strands except compound statements. The pass rates are 97.8% for simple statements, 97.1% for disjunction, and 97.8% for implications. The 'or' table had a pass rate of 96.6%, while the “if, “only if,” and “if and only if” had a 95.7% pass rate. The figure further shows a pass rate of 98.6% for “truth table,” indicating that preservice Mathematics teachers possess more knowledge in truth table under Logical reasoning than any other strand. The lowest pass rate was 47.8% for compound statements.

This shows that preservice Mathematics teachers possess quite below-average content knowledge in compound statements under Logical reasoning.

Some of the responses from the preservice Mathematics teachers on compound statements under Logical reasoning are presented as snapshots in Figures 3, 4 and 5.

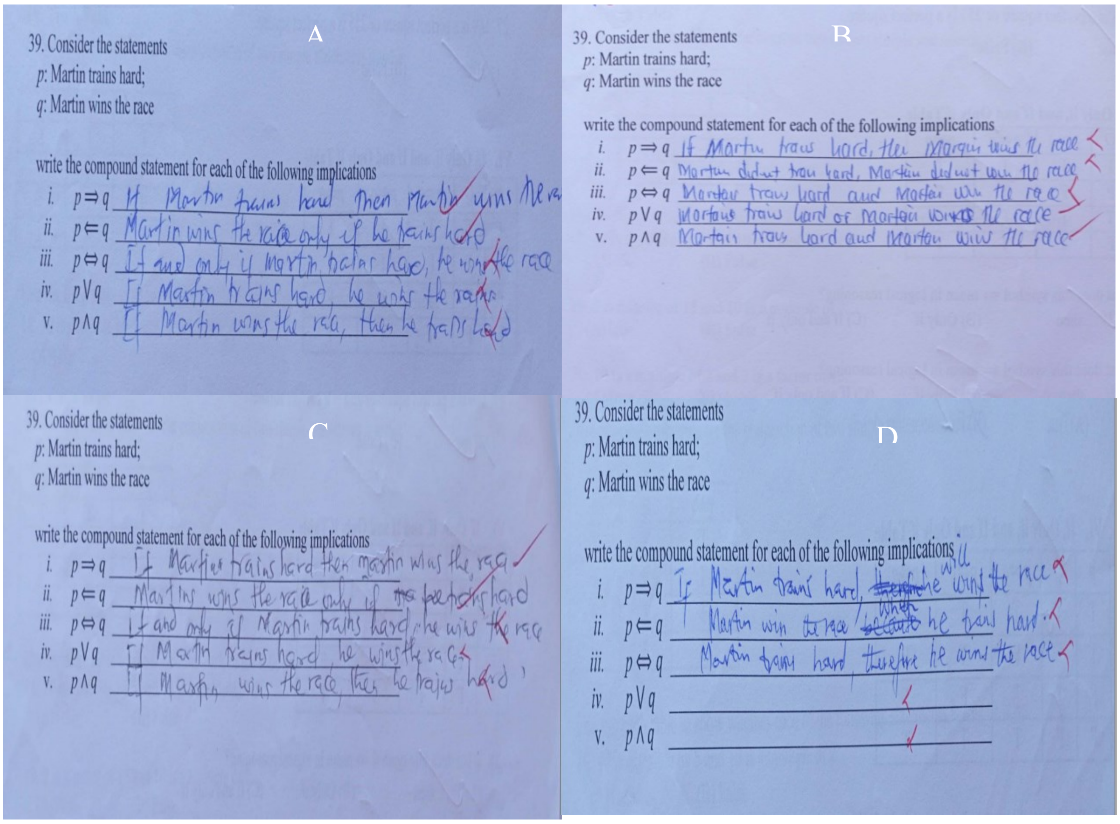
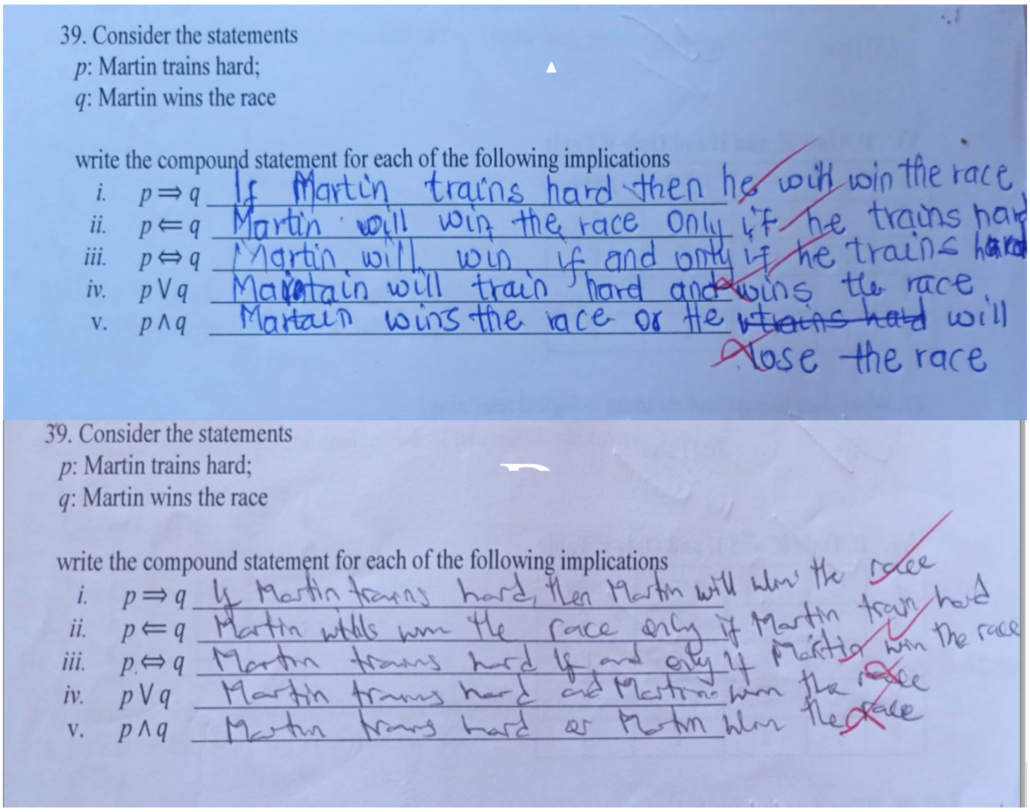


Figure 3: Content knowledge on compound statements ("If ... then, only if, if and only if")

Figure 3 displays some of the responses from the preservice Mathematics teachers on "If ... then," "only if," and "if and only if" under compound statements. The figure shows that some of the preservice Mathematics teachers were unable to use the correct notation to form compound statements. A number of them confused the basic symbols in Logical reasoning and how to interpret them, causing many of them to form compound statements using "therefore" and "comma (,)". For instance, the responses in Fig. 3A and Fig. 3B used "comma" to indicate some of the meanings of the symbols used in the Logical reasoning. Also, the response in Fig. 3D used "therefore" instead of "if and only if."

Figure 4 shows some of the responses from the preservice Mathematics teachers on disjunction and conjunction in compound statements under Logical Reasoning.



39. Consider the statements
p: Martin trains hard;
q: Martin wins the race

write the compound statement for each of the following implications
i. $p \Rightarrow q$ If Martin trains hard then he will win the race
ii. $p \Leftarrow q$ Martin will win the race Only if he trains hard
iii. $p \Leftrightarrow q$ Martin will win if and only if he trains hard
iv. $p \vee q$ Martain will train hard and wins the race
v. $p \wedge q$ Martain wins the race or he ~~trains hard~~ will lose the race

39. Consider the statements
p: Martin trains hard;
q: Martin wins the race

write the compound statement for each of the following implications
i. $p \Rightarrow q$ If Martin trains hard, then Martin will win the race
ii. $p \Leftarrow q$ Martin will win the race only if Martin train hard
iii. $p \Leftrightarrow q$ Martin trains hard if and only if Martin win the race
iv. $p \vee q$ Martin trains hard and Martin win the race
v. $p \wedge q$ Martin trains hard or Martin win the race

Figure 4: Content knowledge on compound statements (disjunction and conjunction)

The results show that some of the preservice Mathematics teachers were confused with the symbol (⋁) for disjunction (or) and the notation (⋀) for conjunction (and). Some of the respondents interchanged the meaning of the symbol. For instance, Figure 4A and Figure 4B show that the respondents use (⋀) for "or" instead of "and" and (⋁) for "and" instead of "or." The interchanging of the symbols affects their performance on the compound statement.

Figure 5 shows some of the responses from the preservice Mathematics teachers on the use of Venn diagram to illustrate compound statements under Logical reasoning.

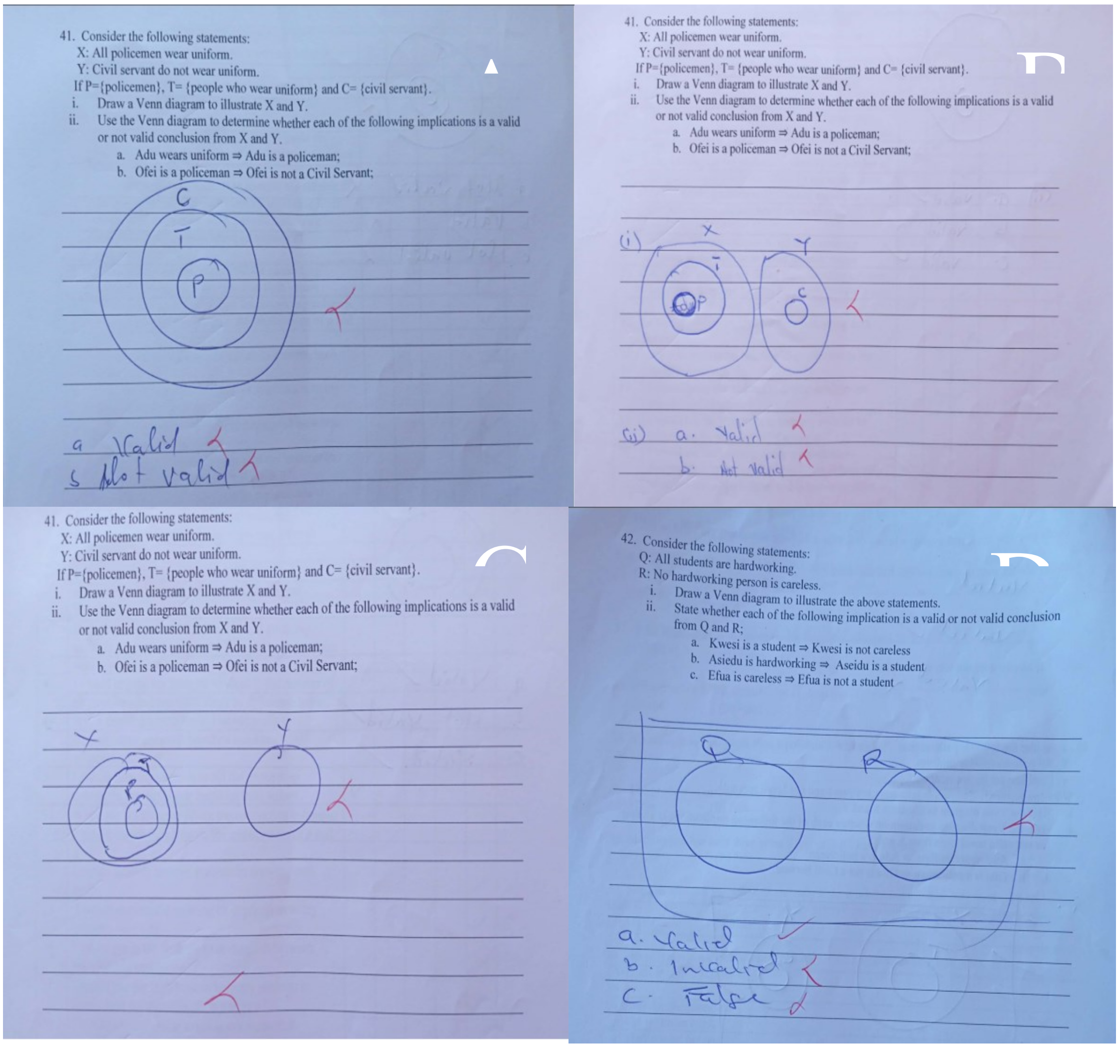


Figure 5: Content knowledge on compound statements (Venn diagram)

The results showed that some of the preservice Mathematics teachers were not able to use the information given to draw a correct Venn diagram. For instance, the responses in Figure 5A, Figure 5B and Figure 5C show that preservice Mathematics teachers were drawing intersecting circles without the Universal set. Figure 5A indicates that some of the respondents could not interpret “do not” and “no” in the given statements to draw a correct Venn diagram. Also, the response in Figure 5D reveals that respondents could not interpret “all” of the given statements. Most of the preservice Mathematics teachers were therefore unable to use the Venn diagram to answer the subsequent questions on compound statements. Some of them wrote “invalid” and “false” instead of “not valid.” It was observed that some of the preservice Mathematics teachers were guessing their responses because even the valid conclusions they drew did not agree with the Venn diagrams. This contributed to their poor performance on the compound statements in Logical reasoning.

Table 2 shows the level of content knowledge in Logical reasoning possessed by preservice Mathematics teachers. The marks indicate the overall performance of preservice Mathematics teachers on all sub-strands in Logical Reasoning. The purpose was to gather information about the overall performance of preservice Mathematics teachers’ content knowledge in Logical Reasoning.

Table 2: Distribution of test scores in Logical Reasoning

Marks	Frequency	Percentage
0-4	1	0.7
5-9	0	0
10-14	0	0
15-19	2	1.4
20-24	2	1.4
25-29	20	14.5
30-34	42	30.4
35-39	52	37.7
40-44	19	13.8
Total	**138**	**100**

N=138

Source: Field survey (2023)

The minimum score of the test is 2 out of 50, and the maximum score is 44. From the table, only 13.8% of respondents had very high scores in the range of 40–44 out of the maximum score of 50, while 114 (82.2%) of respondents had scores within the range of 25–39. This implies there are as many as 133 (96.4%) respondents who had 50% or more (i.e., 25–44) of the maximum score for the test. The mean score was computed to be 34.8. Table 2 indicates that preservice Mathematics teachers possess adequate content knowledge in Logical reasoning for senior high schools. At the time of data collection, preservice Mathematics teachers had taken the required courses at the university on Logical reasoning. Preservice Mathematics teachers are therefore expected to exhibit a high level of content knowledge in these items.

Preservice Mathematics Teachers' Knowledge of Strategies for Teaching Logical reasoning.

This research question sought to investigate preservice Mathematics teachers' knowledge of teaching Logical reasoning. The purpose of research question 2 was to find out preservice Mathematics teachers' knowledge of teaching strategies on statements, implication, disjunction, conjunction, and truth tables under Logical reasoning. Quantitative data, using a questionnaire was collected on preservice Mathematics teachers' knowledge of strategies for teaching Logical reasoning. The data collected from respondents for this research question was analysed using mean and standard deviation. A mean score of 3.5 to 5.0 was interpreted as "always true" about the respondents, while a mean score of 2.4 to 3.4 was interpreted as respondents being neutral (not decided) about the statement. A mean score below 2.40 was interpreted as "never true" by respondents to the statement. There were six items on statements and implications. The results obtained from the preservice Mathematics teachers about their knowledge of teaching strategies on statements and implications are presented in Table 3.

Table 3: Knowledge of strategies for teaching statements and implications

Item Number	Strategies	Mean	SD
43	I would explain the meaning of Mathematical statements very well to my students.	4.16	1.18
44	I would guide my students to know the difference between sentences that are statements and those that are not.	4.38	1.06
45	I would make sure my students represent the Mathematical statements by an uppercase letter or variable.	4.12.	1.09
46	I would give enough class exercises, tests and assignments when I am teaching Mathematical statements.	4.25	1.08
51	When my students are filling the connecting columns, I would ensure they are based on the implications of *p* and *q*.	4.10	1.15
54	I would guide the students to distinguish between premise and conclusion.	4.28	1.10

Source: Field survey (2023)

From Table 3, preservice Mathematics teachers' knowledge of the need to guide their students to know the difference between sentences that are statements and those that are not (item 44) recorded the highest mean (M = 4.38, SD = 1.06). Also, the statement that preservice Mathematics teachers would guide their students to distinguish between premise and conclusion (item 54) recorded a mean score of 4.28 with S.D = 1.10. The statement that preservice Mathematics teachers would explain the meaning of Mathematical statements very well to their students (item 43) has a mean score of 4.16 with S.D = 1.18. Furthermore, item number 51 had the minimum mean (M = 4.10, SD = 1.15).

The mean scores of all six items were more than 3.5. This implies that the statements on knowledge of strategies for teaching statements and implications under Logical reasoning are "always true" about preservice Mathematics teachers. The standard deviation values in the table show that preservice Mathematics teachers did not have very diverse opinions on the Likert scale items in the questionnaire. The responses were a bit more diverse for item number 43 with the highest standard deviation of 1.18 and for item number 51 with a standard deviation of 1.15. The results indicate that preservice Mathematics teachers possess the requisite pedagogical knowledge for teaching statements and implications at the senior high school level.

Table 4 gives the output of the descriptive statistics (mean and standard deviation) of preservice Mathematics teachers' knowledge of strategies for teaching disjunction and conjunction. Preservice Mathematics teachers responded to seven items on strategies for teaching disjunction and conjunction, with four items on disjunction and three on conjunction.

Table 4: Knowledge of strategies for teaching disjunction and conjunction

Item Number	Strategies	Mean	SD
57	I would discuss the representation of conjunction with the students.	4.27	1.02
58	I would guide my students to identify which proposition is false when teaching conjunction.	4.13	1.10
59	I would assign more exercises in conjunction with my students	4.24	0.90
62	I would define disjunction and let my students know the symbol used to represent disjunction.	4.16	1.06
63	I would guide my students to identify the false statement in the disjunction.	4.22	1.08
64	Using the truth table, I would guide my students to deduce from the disjunction that, if any of the propositions is true then the statement is true.	4.15	1.06
65	I would guide my students to identify from the truth table that in disjunction, if both propositions are false, then the statement is false.	4.16	1.20

Source: Field survey (2023)

From Table 4, preservice Mathematics teachers' responses to the statement that they would guide their students to identify which proposition is false when teaching conjunction (item 58) has the lowest mean score (M = 4.13, SD = 1.10). Preservice Mathematics teachers' knowledge of the need to discuss the representation of conjunction with the students (item 57) has the highest mean score (M = 4.27, SD = 1.02). From the table, preservice Mathematics teachers' responses to the knowledge of assigning more exercises in conjunction to their students (item 59) recorded the second highest mean of 4.24 but with the lowest standard deviation of 0.90. This indicates that the responses of preservice Mathematics teachers on this item are not very diverse. The responses to item number 65 recorded a mean score of 4.16, with the highest standard deviation of 1.20.

Over all, Table 4 shows that preservice Mathematics teachers' responses to strategies of teaching disjunction and conjunction under Logical reasoning can be classified as "always true" about them. This indicates that preservice Mathematics teachers possess adequate knowledge of strategies for teaching disjunction and conjunction.

Table 5 shows preservice Mathematics teachers' knowledge of strategies for teaching truth tables. The purpose of the data displayed in Table 5 was to investigate the level of knowledge possessed by preservice Mathematics teachers on strategies for teaching truth tables. The table shows the mean and standard deviation of the responses for each of the eight items on the truth table.

Table 5: Preservice Mathematics teachers' knowledge of strategies for teaching truth table

Item Number	Strategies	Mean	SD
47	Before drawing the truth table, I would guide my students to break the argument into parts and determine the number of rows needed.	4.16	1.09
48	I would engage my students to identify the number of propositions and represent them in uppercase letters.	4.10	1.13
49	When drawing a truth table, I would make sure students draw columns based on the number of propositions.	4.01	1.12
50	I would help my students to fill the connecting column in the truth table based on truth values.	4.09	1.17
52	I would actively involve students in drawing truth tables.	4.36	1.06
55	I would use the truth table to help my students to understand the meaning of "if *p* then *q*."	4.34	1.09
56	I would use the truth table to guide my students to discover that, in conjunction, a conclusion is false only when *p* is true and *q* is false and is true in all other situations.	4.07	1.17
60	With the help of the truth table, I would guide my students to discover that, in conjunction if either proposition is false, then the entire statement is false	4.02	1.14
61	Using the truth table, my students would be able to conclude that both propositions must be true for the conjunction to be true.	4.20	1.08

Source: Field survey (2023)

From Table 5, preservice Mathematics teachers' knowledge of actively involving students in drawing truth tables (item 52) has the highest mean scores of 4.36 and a standard deviation of 1.06. In addition, preservice Mathematics teachers' knowledge of making sure their students draw columns based on the number of propositions (item 49) has the lowest mean (M = 4.36, SD = 1.06). The mean scores of all items were more than 3.5, which suggests that the statements on knowledge of strategies for teaching truth tables under Logical reasoning are "always true" about preservice Mathematics teachers. This shows that preservice Mathematics teachers possess the essential knowledge of strategies for teaching truth tables.

Content Knowledge of male and female Preservice Mathematics Teachers in Logical reasoning.

This hypothesis was aimed at investigating preservice Mathematics teachers' content knowledge by sex. The independent sample t-test was run to compare the mean achievement scores of male and female preservice Mathematics teachers. The hypothesis was tested at a 5% level of significance. The descriptive statistics of the achievement of male and female preservice Mathematics teachers are displayed in Table 6. The table also includes the summary statistics for the independent sample t-test.

Table 6: Independent sample t-test statistics of content knowledge of male and female

Sex	N	Mean	Standard deviation	t	Df	Sig. (2-tailed)
Male	107	34.57	5.44	-2.16	136	.032
Female	31	36.83	3.83			

Source: Field survey (2023)

Table 6 indicates that the mean score of female preservice Mathematics teachers (N = 31, M = 36.83, SD = 3.83) is relatively higher than that of the mean score of male preservice Mathematics teachers (N = 107, M = 34.57, SD = 5.44). This indicates that the female preservice Mathematics teachers performed relatively better than their male counterparts on content items in Logical reasoning. Also, the standard deviation of female preservice Mathematics teachers indicates that they had closer raw scores as compared to male preservice Mathematics teachers. The p-value of .032, which is less than .05 indicates that there is a significant difference between content knowledge of male and female preservice Mathematics teachers in Logical reasoning. It is thus concluded that female preservice Mathematics teachers significantly outperformed male preservice Mathematics teachers in content items on Logical reasoning.

Knowledge of Strategies for Teaching Logical reasoning of WASSCE and DBE applicants.

This hypothesis sought to find out the pedagogical knowledge level of preservice Mathematics teachers. The mean and standard deviation of preservice Mathematics teachers' pedagogical knowledge were computed. Table 7 shows the descriptive statistics of the scores of the preservice

Mathematics teachers by entering qualifications. The independent sample t-test was run to compare the mean scores of WASSCE and Diploma in Basic Education (DBE) applicants. The hypothesis was tested at a 5% level of significance. Table 7 also displays the summary statistics for the independent sample t-test.

Table 7: Independent sample t-test statistics on pedagogical knowledge of WASSCE and DBE applicants

Entering qualification	N	Mean	Standard deviation	T	Df	Sig.(2-tailed)
WASSCE	113	4.264	0.872	0.954	136	.342
DBE	25	4.066	0.937			

Source: Field survey (2023).

Table 7 shows that the mean score of WASSCE applicants (M = 4.254, SD = 0.872, N = 97) is relatively higher than the mean score of DBE applicants (M = 4.066, SD = 0.937, N = 21). The results imply that WASSCE applicants had a relatively higher pedagogical knowledge in Logical reasoning than Diploma in Basic Education applicants. The standard deviation of WASSCE applicants indicates that their scores on the items of strategies for teaching Logical reasoning are less dispersed than those of DBE applicants.

The p-value of .342, resulting from the independent sample t-test indicates that there is no significant difference in the knowledge of teaching Logical reasoning among preservice Mathematics teachers in terms of their entering qualifications. It is therefore concluded that WASSCE applicants and DBE applicants had equal pedagogical knowledge of strategies for teaching Logical Reasoning.

Knowledge of Strategies for Teaching Logical reasoning possessed by male and female Preservice Mathematics Teachers.

This hypothesis focuses on the knowledge of teaching strategies in Logical reasoning possessed by male and female preservice Mathematics teachers. To compare the mean achievement scores of male and female preservice Mathematics teachers, an independent sample t-test was used. A significance level of 5% was used to test the hypothesis. Table 8 displays the descriptive statistics of the achievement of male and female preservice Mathematics teachers. The summary statistics for the independent sample t-test are also shown in the table.

Table 8: Independent sample t-test statistics of teaching strategies of male and female

Sex	N	Mean	Standard deviation	T	Df	Sig.(2 tailed)
Male	107	4.1633	0.951	-1.404	136	.163
Female	31	4.4273	0.603			

Source: Field survey (2023).

According to Table 8, the mean score of female preservice Mathematics teachers (M = 4.4273, SD = 0.603, N = 31) is comparatively higher than the mean score of male preservice Mathematics teachers (M = 4.1633, SD = 0.951, N = 107). This suggests that, compared to male preservice Mathematics teachers, their female counterparts had more adequate knowledge of teaching techniques. In addition, the female preservice Mathematics teachers' standard deviation shows that their raw results were closer than those of the male preservice Mathematics teachers. The independent sample t-test showed a p-value of 0.163, which suggests that there is no statistically significant difference between teaching strategies of Logical reasoning among male and female preservice Mathematics teachers.

Discussion

The results on the age distribution of respondents show that as high as 84% of preservice Mathematics teachers admitted by DMICTE for Bachelor of Education Mathematics in the 2019/2020 academic year are in the 20-29 years age bracket (see Table 1). This age distribution is located within what Prensky (2001) describes as a "digital native." According to Prensky, people born after 1984 fall within this description. The assumption is that the respondents within this age category may possess more content knowledge and pedagogical knowledge. Consequently, this would help them become effective teachers in the 21st century. The youthful age of respondents is an ideal age for the teaching profession, as they would bring much energy and commitment to the teaching and learning process (Biesta, 2015).

Also, the results from Table 1, indicate that there were a relatively larger number of males (77.5%) than females (22.5%). The relative number of male respondents led credence to the general assumption that more males than females are admitted to offer Mathematics (Athene & Owusu-Ansah, 2013). This can be attributed to the general perception that males are better at Mathematics than females.

The results on content knowledge of preservice Mathematics teachers as shown in Figure 2 show that over 95% of respondents scored high on six of the sub-strands of Logical reasoning but had a low pass rate of 47.8% on the seventh sub-strand (compound statements). This reveals that preservice

Mathematics teachers have low performance in compound statements on Logical reasoning. This shows the potential of preservice Mathematics teachers' inability to handle the topic effectively in the classroom when they take up a teaching career. Johnson-Laird (2004) stated that individuals construct a mental model that is consistent with the premises. People with a weak working translation may have a poor ability to evaluate the validity of a conclusion. Nevertheless, this translation can be done easily by the effectiveness of teachers' content knowledge. Teachers need to know which topics fall into this category and what aspects of these topics students find most difficult (Peace, Fuentes & Bloom, 2018).

The results that 96.5% of preservice Mathematics teachers scored 50% or more are an indication that, the respondents possess adequate content knowledge in Logical reasoning (see Table 2). Studies have revealed that teachers' subject matter knowledge is a prime influence on their instructional activities and students' achievement on the subject matter (Howard & Milner, 2021). Stronge (2018) agreed that every effective teacher must integrate content knowledge into an understanding of how particular topics, problems, or issues are organised, represented, and adapted to the diverse interests and abilities of learners.

Preservice Mathematics teachers' responses to items on pedagogical knowledge on teaching the sub-strands under Logical reasoning. The mean value for the responses were above 3.0 which could be classified under the scale "always true." This shows that preservice Mathematics teachers possess adequate teaching knowledge to teach the sub-strands on Logical reasoning. Stylianides and Stylianides, (2022) indicate that understanding proof, particularly Logical reasoning, is a challenge for students of all ages, including preservice teachers. Studies conducted by Buchbinder and McCrone, (2020) on college students on Logical reasoning also showed that undergraduates in general, and preservice elementary teachers in particular, were not able to interpret disjunctive and logical statements well. Studies conducted by Depaepe et al., (2015) show that prospective Mathematics teachers have limited MCK and PK. Also, the result of Leong et al., (2015) shows that preservice Mathematics teachers are more competent in MCK than in PK. The results of this study are in contrast to these studies. Preservice Mathematics teachers in this study exhibited adequate knowledge of Logical reasoning and they can also interpret disjunction and logical statements very well. The results show that field teaching practice has a positive effect on the teaching efficacy of preservice Mathematics teachers for teaching disjunction and statement.

Building upon previous research, Ajai and Imoko (2015) report that there is no statistically significant difference between the performance of males and females in mathematics at SHS. This study confirms that of Armah, Akayuure and Armah (2020) who found that, there is a significant difference between males and females in Mathematics content knowledge. Mathematics performance between males and females at the college level indicates female preservice Mathematics teachers perform better in Logical reasoning than males (Malik, Farooq & Tabassum, 2016). The finding of this study about preservice Mathematics teachers at the university level agrees with that of Malik, Farooq and Tabassum (2016). Female preservice Mathematics teachers significantly outperformed male preservice Mathematics teachers as far as content knowledge in Logical reasoning is concerned (see Table 6).

This study concluded that WASSCE applicants and DBE applicants possess equal levels of pedagogical knowledge of strategies for teaching Logical reasoning. McCrory et al., (2018) stated that field teaching experience has improved preservice Mathematics teachers' knowledge across the various knowledge types. These courses have sharpened the preservice Mathematics teachers' strategies for teaching Mathematics. According to Berliner (as cited in Entsie, 2021), Academic Learning Time (ALT) is essential because it serves as a direct link between teachers' behaviour and students' performance. And it also serves as an indicator of students' learning. Berliner's ALT does not indicate the nature of knowledge teachers must possess to judge the difficult level of tasks given to students and when it will be appropriate for a teacher to move to a new concept. Preservice Mathematics teachers being experienced in ALT has helped them improve respondents' performance.

Philipp (2007) found that Mathematics knowledge of preservice Mathematics teachers has a direct impact on their teaching strategies. Pitman (1987) expected diploma in basic education (DBE) applicants to have possessed more teaching knowledge than WASSCE applicants, since DBE applicants have done methods of teaching courses at the college of education level. However, the methods of teaching Mathematics at the basic level are different from the methods of teaching Mathematics at the senior high school level. This could have accounted for the result of this study being in contrast with that of Pitman (1987). This confirms that the methods of teaching Mathematics courses mounted by DMICTE at UCC are yielding positive results. Both groups of applicants were exposed to methods of teaching courses such as methods of teaching senior high school Mathematics, On-campus Teaching Practice and Off-campus Teaching Practice. These courses have improved preservice Mathematics teachers' knowledge for teaching Mathematics at the senior high school. Both groups possessed the required mastery of teaching strategies of Logical reasoning at the senior high school.

Preservice Mathematics teachers possessing good strategies for teaching Logical reasoning will promote alternative ideas for students during lessons (Smith, 1999). Preservice Mathematics teachers having good knowledge of strategies for teaching Logical reasoning, is an indication that they are likely to identify and handly topics their students find most inaccessible (Peace, Fuentes & Bloom, 2018). Preservice Mathematics teachers will be able to anticipate what topics students already know and what topics they find difficult. Prominent studies have revealed that teachers' subject matter knowledge is a prime influencer of teachers' instructional activities and consequently students' achievement on the subject matter (Howard & Milner, 2021).

Chapter Summary

This chapter highlighted the analysis of data and discussion of the findings of the study. The study revealed that preservice Mathematics teachers have good content knowledge in sub-strands under Logical reasoning except for compound statements. Besides this, preservice Mathematics teachers demonstrated good knowledge of strategies for teaching Logical reasoning in senior high school. The findings indicate a statistically significant difference in content knowledge (CK) favouring female preservice Mathematics teachers over male preservice Mathematics teachers. The results show no significant difference in the knowledge of strategies for teaching Logical reasoning possessed by WASSCE and DBE applicants.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

This chapter presents the summary, conclusions, and recommendations of the study. Suggestions for future research are also given in this chapter.

Summary

The purpose of the study was to investigate preservice Mathematics teachers' content knowledge and pedagogical knowledge for teaching Logical reasoning at the senior high school. Two research questions and three research hypotheses were developed to guide the study. The study employed a quantitative approach. This was because the quantitative approach would be adequate to exhaustively answer the research questions and test the hypotheses. Cross-sectional research was considered suitable for this study due to its ability to provide a "snapshot of the outcome" and also solicit the characteristics of the respondents of a study within a short period. Purposive sampling was used to select level 400 preservice Mathematics teachers from the Department of Mathematics and ICT Education (DMICTE) at University of Cape Coast because level 400 preservice Mathematics teachers have read more Mathematics course contents of senior high school, and advanced Mathematics course contents that address knowledge in Logical reasoning and also teaching method courses. A census technique was also adopted to select all level 400 preservice Mathematics teachers at the DMICTE during data collection. Questionnaire and Achievement test were the instrument used for data collection.

Descriptive statistics such as mean and standard deviation were used to describe each of the data sets. Independent sample t-test statistics were used to test the significance of the three research hypotheses. That is, find out if there was any significant difference in the content knowledge of the male and female respondents, in the knowledge of strategies for teaching Logical reasoning possessed by WASSCE and DBE applicants, and in the knowledge of teaching strategies in Logical reasoning possessed by male and female preservice Mathematics teachers.

Key Findings

Preservice Mathematics teachers had a high score in sub-strands of Logical reasoning such as; simple statements, disjunction, implication, truth table, the "or" table and "if, "only if", and "if and only if" table While they performend below the average on compound statements in Logical reasoning. They could not use the Venn diagram to draw a valid conclusion.

Preservice Mathematics teachers possess required pedagogical knowledge for teaching the sub-strands on Logical reasoning.

There was significant difference in the content knowledge of female and male preservice.

The difference in pedagogical knowledge for teaching Logical reasoning exhibited by both WASSCE and Diploma in Basic Education certificate applicants is not significant.

The difference in the knowledge of strategies for teaching Logical reasoning possessed by male and female preservice Mathematics teachers is not significant.

Conclusion

Based on the findings of this study, the following conclusions are drawn:

Preservice Mathematics teachers have exhibited adequate content knowledge in the sub-strands of Logical reasoning except for compound statements and the use of Venn diagrams to draw valid conclusions for given implications. One implication is that preservice Mathematics teachers are most likely to encounter challenges in teaching compound statements when they are deployed to the senior high school.

In terms of pedagogical knowledge, preservice Mathematics teachers posssess require strategies for teaching the sub-strands on Logical reasoning, it shows that the content and methods courses mounted by the DMICTE and the field experience have a positive impact on preservice Mathematics teachers' knowledge of strategies for teaching Logical reasoning.

The existence of a significant difference in the content knowledge in Logical reasoning between male and female preservice Mathematics teachers in favour of females is an indication that when females are encouraged to read Mathematics they can perform very well.

Perservice Mathematics teachers who entered the university with the WASSCE and Diploma in Basic Education certificates do not differ significantly in their knowledge of strategies for teaching Logical reasoning. This shows that irrespective of the entering qualification preservice Mathematics teachers benefited equally from the Mathematics education courses they were exposed to at the Department of Mathematics and ICT Education at the University of Cape Coast. These courses were meant to equip them with basic teaching strategies for teaching senior high school Logical reasoning.

Recommendations

From the findings of this study, the following recommendations are made:

Departments of Mathematics education at the universities should ensure that preservice Mathematics teachers are given more exposure to compound statements under Logical reasoning in their course content, which will improve their content knowledge in compound statements.

Departments of Mathematics education at the university should make intentioned efforts to put measures in place that will improve the performance of male preservice Mathematics teachers in Logical reasoning. The departments can visit senior high schools and educate female students on their ability to do Mathematics education and encourage them to apply for the Mathematics programme.

Much more emphasis needs to be placed on strategies for teaching, specifically compound statements and the use of the Venn diagram in concluding given implications during the methods of teaching Mathematics courses at the universities. This is to boost the competence of preservice Mathematics teachers when they later take up the teaching of Mathematics at senior high school.

Suggestions for Further Studies

This study was targeted at preservice Mathematics teachers at the Department of Mathematics and ICT Education, University of Cape Coast, specifically to investigate their content knowledge and pedagogical knowledge for teaching Logical reasoning in senior high school. It is suggested that new research, targeting senior high school student's performance in Logical reasoning focus on compound statements and drawing conclusions using the Venn diagram.

Further research could look into the content knowledge of in-service senior high school Mathematics teachers and whether they have challenges with the teaching of compound statements and their implications under Logical reasoning.

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APPENDICES

APPENDIX A

UNIVERSITY OF CAPE COAST

COLLEGE OF EDUCATIONAL STUDIES

DEPARTMENT OF MATHEMATICS AND ICT EDUCATION

QUESTIONNAIRE FOR PRESERVICE MATHEMATICS TEACHERS
(Time: one hour)

*I am **ABRAHAM KOJO ABAKAH,** a Master of Philosophy (Mathematics education) student undertaking research on **PRESERVICE MATHEMATICS TEACHERS' CONTENT AND PEDAGOGICAL KNOWLEDGE FOR TEACHING LOGICAL REASONING IN SENIOR HIGH SCHOOL.** I would be very grateful if you could provide appropriate responses to the questions below. The information you provide will be used solely for academic work and will be treated with the utmost confidentiality it deserves.*

SECTION A: Demographic Data
This section seeks to find the demographic characteristics of the respondents. Please tick (√) the most appropriate as it pertains to your sex, age and type of entry certificate

1. Sex A. Male [ ] B. Female [ ]

2. Age: A. 20 – 24 years [ ] B. 25 – 29 years [ ] C. 30 – 34 years [ ]
D. Above 35years [ ]

3. Indicate the type of certificate you used to apply for the degree
 A. WASSCE [ ] B. Diploma in Education [ ]

SECTION B: Content Knowledge on Logical Reasoning
Circle the correct answer

I. Simple Statements
State whether the following statements are true or false

4. All primes are odd numbers.
 (A)True (B) False
5. The number x^2 is always positive.
 (A)True (B) False
6. $5 \equiv 3 \mod 3$.
 (A)True (B) False
7. The square of an odd number is even.
 (A)True (B) False

II. **Disjunction**

State whether the following disjunctions are true or false

8. 81 is a perfect square or 81 is a prime number.
   (A)True (B) False

9. None of the three sides of an equilateral triangle are equal or all four sides of a rectangle are equal.
   (A)True (B) False

10. 29 is a composite number or 49 is a prime number.
   (A)True (B) False

11. The sum of the interior angles of a pentagon is 540° or the sum of angles on a straight line is 180°.
   (A)True (B) False

II. Implication

State whether the following implications are true or false

12. If $x^2 > 4$, then $x < 2$.
   (A)True (B) False

13. If a number is a perfect square, then it is positive.
   (A)True (B) False

14. $x = 4 \Rightarrow x^2 = 16$.
   (A)True (B) False

15. $x < 0 \Rightarrow x^2 > 0$.
   (A)True (B) False

16. $x > 0 \Rightarrow x^2 > 0$.
   (A)True (B) False

17. $x^2 > 0 \Rightarrow x > 0$.
   (A)True (B) False

III. Truth Table

Below is the truth table for the conjunction of two simple statements p and q.

P	Q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Use the truth table to show whether each of the following statements is true or false.

18. 10 is divisible by 5 and 10 is a multiple of 2.
    (A)True (B) False

19. 2 is a factor of 15 and 10 is a multiple of 2.
    (A)True (B) False

20. 10 is a multiple of 2 and 2 is a factor of 15.
    (A)True (B) False

21. 10 is not a multiple of 2 and 2 is a factor of 15.
    (A)True (B) False

22. 4 is a perfect square and four is the square root of 16.
    (A)True (B) False

IV The "or" Table

Below is the truth table for the disjunction of two simple statements *p* or *q*.

P	Q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Use a truth table to show whether each of the following statements is true or false.

23. 2 is either an odd number or a factor of 15.
    (A)True (B) False

24. A triangle has three sides, or a parallelogram has three sides.
    (A)True (B) False

25. The sum of the interior angles of a triangle is less than 100°, or an octagon has seven vertices.
    (A)True (B) False

26. 83 is an odd number or 38 is a composite number.
    (A)True (B) False

27. 169 is a perfect square or 255 is a perfect square.
    (A)True (B) False

IV. If, Only if, and If and Only if Table

P	Q	$p \Rightarrow q$	$p \Leftarrow q$	$p \Leftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

28. What does this symbol $\Rightarrow$ mean in logical reasoning?
(A) If … then (B) Only if (C) If and only if

29. What does this symbol $\Leftarrow$ mean in logical reasoning?
(A) If … then (B) Only if (C) If and only if

30. What does this symbol $\Leftrightarrow$ mean in logical reasoning?
(A) If … then (B) Only if (C) If and only if

Use the truth table to show whether the statement is true or false.

31. If 2 is an odd number then 2 is not a prime number.
(A) True (B) False

32. If the number 21 is divisible by 10, then 10 is divisible by 2.
(A) True (B) False

33. If the number 121 is divisible by 11, then 5 is divisible by 3.
(A) True (B) False

34. If a triangle is a polygon, then a polygon is a triangle.
(A) True (B) False

35. The product of two numbers is even if and only if both numbers are even.
(A) True (B) False

36. My polygon has only three sides if and only if I have a triangle.
(A) True (B) False

37. The quadrilateral is a square if and only if the quadrilateral has four congruent sides and angles.
(A) True (B) False

38. The polygon is a quadrilateral if and only if the polygon has only four sides.
(A) True (B) False

39. Consider the statements;
p: Martin trains hard.
q: Martin wins the race.

Write the compound statement for each of the following implications

a) $p \Longrightarrow q$ ____________________

b) $p \Longleftarrow q$ ____________________

c) $p \Longleftrightarrow q$ ____________________

d) $p \vee q$ ____________________

e) $p \wedge q$ ____________________

40. From question 39, if $p \Rightarrow q,$ state whether or not the following statements are valid or not valid:

a. If Martin wins the race, then he has trained hard.
____

b. If Martin does not train hard then he will not win the race.
____

c. If Martin does not win the race then he has not trained hard.
____

41. Consider the following statements:
X: All policemen wear uniform.
Y: Civil servants do not wear uniform.
If P= {policemen}, T= {people who wear uniform} and C= {civil servants}.
a) Draw a Venn diagram to illustrate X and Y.
b) Use the Venn diagram to determine whether each of the following implications is a valid or not valid conclusion from X and Y.
   i. Adu wears a uniform ⇒ Adu is a policeman.
   ii. Ofei is a policeman ⇒ Ofei is not a Civil Servant.

____

____

____

____

____

42. Consider the following statements:

Q: All students are hardworking.

R: No hardworking person is careless.

a) Draw a Venn diagram to illustrate the above statements.

b) State whether each of the following implications is a valid or not valid conclusion from Q and R.

i. Kwesi is a student ⇒ Kwesi is not careless.

ii. Asiedu is hardworking ⇒ Aseidu is a student.

iii. Efua is careless ⇒ Efua is not a student.

SECTION C: KNOWLEDGE OF TEACHING LOGICAL REASONING

Below are items on the teacher's knowledge of teaching logical reasoning.

Please tick (√) the most appropriate as it pertains to you using the following keys.

1=Never true 2=Rarely true 3=Neutral 4=Sometimes true 5=Always true

S/N	Strategies	1	2	3	4	5
43	I would explain the meaning of Mathematical statements very well to my students.					
44	I would guide my students to know the difference between sentences that are statements and those that are not.					
45	I would make sure my students represent the Mathematical statements by an uppercase letter or variable.					
46	I would give enough class exercises, tests and assignments when I am teaching Mathematical statements.					
47	Before drawing the truth table, I would guide my students to break the argument into parts and determine the number of rows needed.					
48	I would engage my students to identify the number of propositions and represent them in uppercase letters.					
49	When drawing a truth table, I would make sure students draw columns based on the number of propositions.					
50	I would help my students to fill the connecting column in the truth table based on truth values					
51	When my students are filling the connecting columns, I would ensure they are based on the implications of p and q.					
52	I would actively involve students in drawing truth tables.					
53	I would explain the meaning of implication to my students very well.					
54	I would guide the students to distinguish between premise and conclusion.					
55	I would use the truth table to help my students to understand the meaning of "if p					

	then *q*."					
56	I would use the truth table to guide my students to discover that, in conjunction, a conclusion is false only when *p* is true and *q* is false and is true in all other situations.					
57	I would discuss the representation of conjunction with the students.					
58	I would guide my students to identify which proposition is false when teaching conjunction.					
59	I would assign more exercises in conjunction with my students.					
60	With the help of the truth table, I would guide my students to discover that, in conjunction if either proposition is false, then the entire statement is false.					
61	Using the truth table, my students would be able to conclude that both propositions must be true for the conjunction to be true.					
62	I would define disjunction and let my students know the symbol used to represent disjunction.					
63	I would guide my students to identify the false statement in the disjunction.					
64	Using the truth table, I would guide my students to deduce from the disjunction that, if any of the propositions is true then the statement is true.					
65	I would guide my students to identify from the truth table that in disjunction, if both propositions are false, then the statement is false.					

APPENDIX B

ETHICAL CLEARANCE (DEPARTMENT OF MATHEMATICS AND I.C.T EDUCATION)

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION
DEPARTMENT OF MATHEMATICS AND I.C.T EDUCATION

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University Post Office
Cape Coast, Ghana

Your Ref:

Our Ref: DMICTE/P.3/V.3/111

Date: 19th October, 2022

The Chairman
Institutional Review Board
University of Cape Coast
Cape Coast

Dear Sir/Madam,

REQUEST FOR ETHICAL CLEARANCE

The bearer of this letter, **Abraham Kojo Abakah** with registration number ET/MDP/20/0010 is an MPhil (Mathematics Education) Student of the Department of Mathematics and ICT Education, College of Education Studies, University of Cape Coast.

In fulfilling the requirements for submission for ethical clearance, I would like to indicate that the Department is aware and has approved **Abraham Kojo Abakah's** research topic "**PRESERVICE MATHEMATICS TEACHERS' PEDAGOGICAL AND CONTENT KNOWLEDGE FOR TEACHING LOGICAL REASONING IN SENIOR HIGH SCHOOL.**" The Department is in full support of the submission of his proposal to your outfit for further action.

I would be grateful if you could give him the necessary assistance he may need.

Counting on your usual co-operation.

Thank you.

Yours faithfully,

Dr Forster D. Ntow
HEAD

APPENDIX C

ETHICAL CLEARANCE (INSTITUTIONAL REVIEW BOARD)

UNIVERSITY OF CAPE COAST

INSTITUTIONAL REVIEW BOARD SECRETARIAT

TEL.: 0558093143 / 0508878309
E-MAIL: irb@ucc.edu.gh
OUR REF: IRB/C3/Vol.1/0125
YOUR REF:
OMB NO: 0990-0279
IORG #: IORG0011497

25TH APRIL 2023

Mr Abraham Kojo Abakah
Department of Mathematics and I.C.T Education
University of Cape Coast

Dear Mr Abakah,

ETHICAL CLEARANCE – ID (UCCIRB/CES/2022/167)

The University of Cape Coast Institutional Review Board (UCCIRB) has granted Provisional Approval for the implementation of your research On **University of Cape Coast Preservice Mathematics Teachers' Pedagogical and Content Knowledge for Teaching Logical Reasoning in Senior High School.** This approval is valid from **25th April 2023** to **24th April 2024**. You may apply for a renewal subject to the submission of all the required documents that will be prescribed by the UCCIRB.

Please note that any modification to the project must be submitted to the UCCIRB for review and approval before its implementation. You are required to submit a periodic review of the protocol to the Board and a final full review to the UCCIRB on completion of the research. The UCCIRB may observe or cause to be observed procedures and records of the research during and after implementation.

You are also required to report all serious adverse events related to this study to the UCCIRB within seven days verbally and fourteen days in writing.

Always quote the protocol identification number in all future correspondence with us in relation to this protocol.

Yours faithfully,

Kofi F. Amuquandoh
Ag. Administrator

ADMINISTRATOR
INSTITUTIONAL REVIEW BOARD
UNIVERSITY OF CAPE COAST