UNIVERSITY OF CAPE COAST

SENIOR HIGH SCHOOL MATHEMATICS TEACHERS' PERCEPTIONS AND USE OF PROBLEM-SOLVING AS A PEDAGOGICAL TOOL

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BY

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DECLARATION

Candidate's Declaration

I hereby declare that this thesis is the result of my original research and that no part of it has been presented for another degree at this University or elsewhere.

Candidate's Signature: ……………….……… Date: ……………….…

Name: Gabina Susuoroka

Supervisors' Declaration

We hereby declare that the preparation and the presentation of the thesis were supervised in accordance with the guidelines on supervision of thesis laid down by the University of Cape Coast.

Principal Supervisor's Signature: ………………….… Date: …………...…

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Co-Supervisor's Signature: ……………………… Date: ……………..

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ABSTRACT

The study investigated the mathematics teachers' perception and use of problem-solving as a pedagogical tool in the Upper West Region. This study employed the sequential explanatory mixed-method design. Questionnaires were administered to 151 mathematics teachers from 21 senior high schools for data collection purposes for this research, after which a sample of 7 out of the 151 mathematics teachers were interviewed followed by an observation of their lessons. The sampling technique adopted for this study was multi-stage in nature; simple random and purposive sampling. This technique was adopted to obtain the various samples for this study. Both inferential (correlation) and descriptive statistics (mean, median and standard deviation) were used in analyzing quantitatively the data obtained. Analysis of the qualitative data from the open-ended items was done using the thematic coding technique. This study revealed that mathematics teachers have positive perceptions about the use of problem solving as a pedagogical tool. It was also found that about 91% of the senior high school mathematics teachers professed knowledge of use in teaching about problem solving and teaching for problem solving rather than teaching through problem solving. Teachers reported practices they could not demonstrate in their actual lesson. Their observed practices in their classroom had a lower score than what they reported. The study recommends that in-service senior high school mathematics teachers should be given professional development training on problem solving instructional approach and how it is used in the classroom.

KEY WORDS

Problem Solving

Pedagogical tool

Perception

Knowledge

Mathematics

Instruction

SHS

Mathematics

Teachers

Use

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DEDICATION

To my entire family, I wish to dedicate this project

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INTRODUCTION

1.0 Overview

Teaching reforms cannot occur unless teachers' strongly held opinions and beliefs about teaching and learning mathematics shift in a way that is consistent with the policy documentation. Every new approach to teaching mathematics is essentially dependent on the views, attitudes, and understanding of the instructor regarding the nature of mathematics. The way teachers perceive things has a direct impact on how they teach. The study's background, problem statement, goal, objectives, and research questions are all presented in this chapter along with the study's importance, delimitation, and definitions of words used throughout.

1.1 Background to the Study

Mathematics is a vital tool in human logic and thought. It also has a significant impact on human endeavours to understand and impact the cosmos and its galaxies (National Curriculum Framework, 2018). Every civilization believes that science and technology are built on mathematics. Its necessity in numerous aspects of life is the reason for this (Kusure & Basira, 2012).

Furthermore, it is believed that mathematics is inextricably linked to daily activities since it is crucial to how man creates strategies to control his environment (Colgan, 2014; Mpofu & Mpofu, 2019; Tun, 2015). Mathematics has provided people with an effective way to acquire mental rigour, the ability to solve problems using logic and reasoning, and the development of mental discipline, according to study by Mpofu and Mpofu (Mpofu & Mpofu, 2019).

In addition to the opinions expressed by Mpofu and Mpofu (2019) and the NCF (2018), mathematics knowledge is vital to understanding the material covered in other academic areas, including the social sciences, sciences, and arts. The primary goals of education are to help kids become self-sufficient and capable of supporting themselves. Mathematics is the most important topic in order to accomplish this goal. It aids in preparing students for careers in technology and other fields involving the use of mathematics. For instance, arithmetic skills are necessary for employment in engineering, architecture, accountancy, banking, business, agriculture, tailoring, carpentry, surveying, and office work.

The crucial stage of life that is most influenced by place, time, people, and circumstances is morality. Since mathematical knowledge aids in the formation of character and personality, mathematics as a discipline can contribute to students' moral growth (Colgan, 2014). It cultivates every attribute that a person of strong character needs to have. A child acquires attributes of reality and cleanliness. The primary application of mathematics in this context appears to be in the development of reflective skills and, for those who are more open-minded, a sense of the aesthetic beauty of a solution. Solving mathematical puzzles is enjoyable, particularly when the solution provides the right answer. Every child feels content, assured, and independent at that point.

A devoted "mathematics hater" could not see the beauty in a well-designed solution. As a result, the child receives motivation, fulfilment, and joy when they accomplish outstanding goals. As a result, mathematics fosters their artistic sense, satisfies a variety of interests, and aids in making the most of their free time (Tun, 2015). This aids in the student's comprehension of mathematics' role in the advancement of civilization and culture. She or he is now better equipped to comprehend the significance of mathematics in the beautiful arts and in elevating human existence (Sunita, 2019). A critical element in the advancement of humankind as a whole, mathematics plays a vital and distinctive role in human cultures.

The mathematical understanding of space-time, or the physical world and its natural patterns, and the capacity for computation, which is linked to the strength of technology and social organisation, demonstrate the role that mathematics plays in the formation of a society (Sunita, 2019). Humans are the constituents of society; they form the government and allocate natural resources for the construction of infrastructure. It is the human race that advances society. Man can more accurately interpret his ideas and conclusions with the aid of mathematics. It is the aspect of human life and knowledge that deals with numbers and calculations. It now has a significant impact on our daily lives and is essential to the advancement of the modern world (Sunita, 2019).

Mathematics is a key component of the educational system that shapes young people's future probabilities. The goal of education is to help a person grow as a person, become independent, wise, and contribute to society. In our current educational system, mathematics is required for practically every subject we study in school and college, including physics, chemistry, life science, economics, business and accounting, geography, history, psychology, architecture, designing, computations, statistics, and commerce. Additionally, mathematical expertise is required in a number of vocations, including farming, beauticians, tailors, carpet installers, cooks, and athletes. Basic mathematical ideas are used in even the professions of conductors, shopkeepers, drivers, musicians, magicians, cashiers, etc. (Sunita, 2019).

The study of mathematics is essential to modern civilization. It offers the essential foundation for understanding economics. In the physical sciences, technology, business, financial services, and many other ICT domains, it is indispensable. It is also becoming more and more significant in many social sciences, biology, and medicine. The majority of scientific and commercial research and development is based on mathematics. Modern complex systems and structures are becoming more and more understood solely by mathematical means, and a large portion of high-tech system design and control is dependent on mathematical inputs and outputs. (Bazzini, Gellert, and Aldon, 2017)

There is no other subject in the curriculum that stimulates students' minds as much as mathematics, thus teaching it is crucial for their intellectual growth (Mupa, 2015). Solving problems promotes the growth of mental abilities. Math problems require mental labour to solve. When a child encounters a mathematical problem, their brain starts working to find a solution. Such a series exists in every mathematical problem and is essential to the creative and constructive process. In this sense, mathematics helps children develop all of their mental capacities. Strong willpower, patience, and self-reliance are developed, as well as the faculty of invention and discovery (Awinyam, 2018). According to UNICEF (2018), "life skills" are defined as psychosocial skills required for adaptive and positive behaviour, which enable people to deal with the demands and challenges of daily life. Most people would agree that 21st century skills are those that are or will be needed to succeed in the workplace and in this century. They fall into three main types of skills: interpersonal skills for successful communication and engagement with others; cognitive skills for information analysis and utilisation; and personal skills for the development of personal agency and self-management.

In a recent report, the World Economic Forum (2019) outlined the talents that, by 2020, employers will find vital. The following are thought to be the most crucial ones: resolving complicated issues, critical thinking, creativity, managing human resources, collaborating with coworkers, emotional intelligence, making judgements and decisions, being socially and professionally oriented, negotiating, and cognitive flexibility. Workplace teams are growing more and more diverse, which means we must become more aware of how we interact with one another and how we contribute to problem-solving, which calls for group involvement. Thus, it is imperative to reconsider the worth and significance of existing abilities at the expense of learning new things. The way that traditional education is divided into smaller, modular information packages that may be merged and modified to tailor the learning process to the needs of individual students is a new trend that is becoming more and more popular. Thus, even though this may initially seem a little depressing and controversial for teachers, teachers must assist students in developing their creativity, communication skills, problem solving skills, emotional intelligence, and critical thinking abilities in addition to teaching the disciplines. In order to better anchor our students in the reality around them and to gain a better understanding of the benchmarks that the didactic activity is based on, it is also imperative to capitalise on the elements of community identity within the school (Scridon & Ilovan, 2015, 2016). (Dulamă, Ilovan, Bagoly-Simó, & Magdaș, 2019).

In order to achieve greater autonomy in the classroom and in society, problem-solving skills are always required, not only in the short and medium term but also in the long run for professional integration. From a professional standpoint, workers are constantly placed in one of three comparable scenarios: either they find a solution for a client (internal or external), assist others in discovering solutions for problems they have found in the work process, or they find new problems on their own that require attention.

Additionally, having the capacity to solve issues quickly and creatively is just as crucial for a happy personal life as it is for peaceful interpersonal and professional connections. Primary education should not be viewed as an early stage in which to emphasise behaviours that aid in the development of problem-solving abilities because children naturally desire autonomy and independence, even in the absence of extraordinary life experiences (Michelle & Immaculate, 2010).

The importance of the essentials we want our kids to know, be able to accomplish, and comprehend by the time they graduate from high school has increased as a result of the recent push for national education standards. Decades of discussion in particular about mathematics have centred on what it means for children to acquire the lifelong mathematical understandings. Five stages are necessary for learning and comprehending mathematics, according to the National Council of Teachers of Mathematics, the movement's leading proponent of national mathematics standards. The conceptual understandings, knowledge, and skills that should be incorporated into P–12 curricula are outlined in these requirements for school mathematics. Problem solving is one of those process strands, and it is the subject of this investigation (NCTM, 2010).

Not just in mathematics but in all aspects of life, problem-solving abilities are crucial. Society requires people who can solve problems. Nowadays, a lot of basic calculations can be completed by technology such as computers and calculators, freeing up our time to work on more difficult problems. Future technological innovation and mastery will require a strong mathematical foundation (Badan Standar Nasional Pendidikan, 2006).

According to the Badan Standar Nasional Pendidikan (2006), the goals of studying mathematics in primary and secondary education units are for students to be able to solve problems that involve understanding the problem, creating a mathematical model, finishing the model, and interpreting the solutions they have learned; communicating ideas using symbols, tables, diagrams, or other media to clarify situations or problems; and having an attitude to recognise the value of mathematics in everyday life. In maths classes, addressing problems is appropriate and crucial. This is due to the importance of problem-solving learning objectives and processes in meeting societal demands. The study discovers evidence that, in certain exceptional circumstances, the problem-solving techniques acquired in mathematics classes can be applied or translated to real-world problem-solving. According to Indriati Hartono and Hiltrimartin (2009), older students who are used to solving mathematical problems when faced with challenges tend to respond quickly and creatively.

Regretfully, Ghana's kids perform poorly when it comes to answering problem-solving challenges on an international scale. According to the TIMSS 2011 study, only 2% of Ghanaian students were able to solve problems requiring advanced thinking or questions. While the international median for knowing skills was 75%, only 43% of them were able to respond to the low-level questions (knowing type questions) (Mullis, et al., 2012, p. 114). Students in Ghana are limited to using simple algorithms, calculations, and procedures. It is shown that Ghanaian students perform quite well when it comes to regular problem solving, but they perform very poorly when it comes to tasks involving justification or proof, problem solving, drawing conclusions or conjectures, and determining the relationship between the facts or data. The unsatisfactory outcome raises questions about whether Ghana should modify its curriculum (Wilmot, Davis & Ampofo 2015; Davis & Intsiful 2017). According to Shadiq (2004), problem-solving learning is an activity carried out by the instructor to inspire their pupils to take on challenges and guide them through the problem-solving process. This suggests that the way teachers approach problem-solving becomes a decisive determinant in how problem-solving learning is implemented.

Badan Standar (2006) defines perception as the capacity of an individual to arrange an observation, including the capacity to distinguish, categorise, and concentrate information for a subsequent interpretation. As a result, two people may perceive the same object differently. This is conceivable because of potential discrepancies in knowledge and points of view. According to (Suryadi, 2011), perception is a cognitive process in which meaning is assigned to the outside world.

The process of supplying information about the significance of problem solving derived either from knowledge instructors have ever learned or from their experience teaching is referred to in this study as the mathematics teacher's view of issue solving. Examples of the information include definitions of problems, traits of problem-solving difficulties, problem-solving examples, how to separate problem-solving problems from ordinary tasks, and other topics pertaining to problem-solving. According to Andesta (2012), a maths teacher in a Palembang high school has a positive outlook on students learning how to solve mathematical problems.

A little over 91.4% of respondents had a good attitude towards problem components and problem solving; a little over 8.58% had a negative attitude. Teachers may still believe that problems involving problem-solving are application-related, nevertheless. However, application problems don't help students solve problems because they only teach them how to convert a problem's scenario into a mathematical problem. Furthermore, regular questions rather than problem-solving questions were typically employed in the process of teaching problem-solving (Pratiwi, 2012). The aforementioned research findings suggest that teachers who have a favourable attitude towards problem solving during instruction can help students become more adept at solving problems. But why teachers still use routine issues and application problems instead of problem solving is debatable. This occurs as a result of disparities in the ways that maths teachers see problem solving (Munzar etal, 2021).

Many definitions exist for issue solving; they have been constrained for the sake of this research. According to Hiebert et al., a problem is "any task or activity for which students have no prescribed or memorised rules or methods, nor is there a perception by students that there is a specific correct solution," which is how Lovin and Van de Walle define it. Kantowski provides another definition that was employed for the study's objectives: If completing a task necessitates combining previously known data in a novel way, it is considered a difficulty.

It's an exercise, a normal task, or a routine task if he can identify the steps required to finish it right away (Nagy, 2019).

In order to participate in the mathematics that they are intended to learn through interactions, students should be working on assignments set by the teacher that allow them to "use their ideas and their strategies" when they are having difficulty with the material. The success of at-risk children has also been linked to a more practical, project-based curriculum that offers more flexibility, choice, and linkages to students' daily life, (Rosiyanti et al, 2021).

There have been numerous discussions among Ghanaian math instructors and scholars over the low math performance of children, especially in national and standard exams. Various authors propose alternative explanations, such as inadequate math background knowledge of teachers (Mupa, 2015), inadequate pedagogies for teaching mathematics (Kusure & Basira, 2012), and the calibre of math teachers hired to teach math in schools (Nagy, 2019; Obomanu and Adaramola, 2011).

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pedagogies for teaching mathematics (Kusure & Basira, 2012), and the calibre of math teachers hired to teach math in schools (Nagy, 2019; Obomanu and Adaramola, 2011). Large class sizes were also mentioned by Shamin and Kuchah (2016) as issues (factors) influencing maths instruction in schools.

In his research, Singh (2017) found that a significant portion of maths teachers lack the skills necessary to efficiently implement cutting-edge problem-solving, cooperative learning, problem-based learning, and many other constructivist teaching techniques for their students.

As a result, numerous worries and inquiries about the calibre of Ghanaian mathematics teacher preparation programmes and the degree to which graduates are equipped to teach mathematics are made on a regular basis (Bature 2014). According to Nagy (2019), many countries have hired teachers without any formal mathematics training to teach mathematics since there is a shortage of trained maths teachers. There is evidence that instructors from different specialisations are still teaching mathematics, even while nations like Australia are strengthening regulations and compliance for all math teachers to ensure uniformity of delivery and student accomplishment (Nagy, 2019).

Students experience a variety of circumstances throughout their academic careers that can either positively or negatively impact their ability to solve mathematical problems. Research has demonstrated that while teaching students to solve mathematical word problems, they need to grasp more than just computation and rote memory (Krawec & Montague 2014). For instance, the word problem categories routine and complex (Boonen & Jolles, 2015), the reading and visual imagery methods (Björn, Aunola, & Nurmi, 2016), and the working memory and metacognition cognitive processing skills (Swanson, 2015).

These elements play a significant role in how well children learn to solve mathematical word problems as they progress from simple, one-step problems to complex, multi-step problems.

Studies have indicated that a component of mathematical word problem solving involves students' comprehension of the many kinds of word problems and the procedures involved in arriving at a solution (Bayazit, 2013; Dewolf et al., 2014). Voyer (2011) used a mixed-method approach to investigate the understanding and arithmetic skills of 750 sixth grade elementary children using real-world mathematical word problems. The students were tested on a developmental mathematics examination. The findings showed that students did better on word problem questions that included scenarios from real life—that is, scenarios that the students could identify with.

This is comparable to the results of Bayazit's (2013) study on the methods used by 116 students in the seventh and eighth grades to solve non-routine word problems from the real world, which call for the use of different techniques, creative and critical thinking abilities, and alternative approaches. The findings demonstrated that students were rigid in their interpretation of word problem scenarios and did not integrate knowledge from the real world. Rather of using the standard word problem solving techniques, students employed result-oriented approaches (i.e., arithmetic operations, rules, and factual knowledge) to solve the questions. How to help pupils become more proficient in solving non-routine word problems has been the subject of other studies. Dewolf et al. (2014) investigated in two studies whether the presence of an illustration (images depicting the problem content) or warning (clear instructions or directions) would improve the performance of fifth-grade pupils in solving non-routine word problems.

Simple (S-item) mathematical operations alone cannot address non-routine (P-item) problems; instead, judgement based on presumptions and practical knowledge is needed. Ten difficult (P-item) non-routine word problem questions with four possible outcomes were given to students in the first study: (a) with an illustration and no warning, (b) warning only, (c) with an illustration and warning, and (d) without an illustration or warning. In the second study, there were no S-item word problems and the students were given 16 P-item word problems with the same four criteria. The results of both experiments demonstrated that warnings and illustrations had no influence on upper elementary students' solutions to non-routine problems, but that children benefited when warnings and illustrations were used to answer routine problems.

The development of creative thinkers in schools is essential to the advancement of our civilization. The curriculum needs to be changed to facilitate student learning through problem-solving because components of traditional classroom approaches do not support the abilities required for these changes. Traditional classroom approaches must be modified to accommodate these new skills and provide students with opportunities to acquire them if schools are to help students become 21st-century learners (Brush, 2017).

The primary obstacle to the effective application of problem-solving learning is the change in teaching methodology from traditional methods to problem-solving designs, which enable professors to take on the role of facilitators while students take charge of their education. With a whole new teaching approach and a greater degree of student-directed influence over the classroom, this change may be difficult for teachers (Brush, 2017).

In order to effectively facilitate a problem-solving learning activity in the classroom, teachers must possess a number of foundational abilities, such as motivators, class management, and cross-curricular expertise. Unlike traditional methods, the problem-solving learning structure lacks a structured plan, which means that teachers may not be able to anticipate every scenario that may arise during a problem-solving task because it would be challenging to predict the direction in which students would take their research (Brush, 2017).

This represents a significant departure from conventional instructional methods. Gaining insight into how educators view and apply problem solving as a teaching technique will help us understand how instructors are interpreting this change and responding to it.

A particular difficulty associated with the transition from a traditional classroom model to a design that emphasises problem-solving is a lack of knowledge regarding the role that teachers play in this process. It will directly affect their instruction if teachers lack a firm grasp of their responsibilities to assist the students. Lack of knowledge about what goes into designing a problem-solving activity is another difficulty (Brush, 2017). Instructors might know what is meant by the term "problem solving" and may even have some idea of what it involves.

However, implementation will suffer if teachers are unclear about how to create a problem-solving activity.

Regarding those points, a deeper comprehension of teachers' perspectives on problem-solving and its application as an educational tool will shed more light on how educators are interpreting and reacting to this change. This information can then be utilised to more effectively support the shift and the practice of problem-solving in an environmentally friendly way. For this reason, the mathematics curriculum for high schools in Ghana places a strong focus on problem-solving strategies. "Allow all Ghanaian young people to acquire the mathematical abilities, insights, attitudes, and values that they will need to be successful in their chosen occupations and daily lives" is the stated goal of the country's senior high school mathematics curriculum (CRDD, 2010).The success of developing capable newcomers to science and technology depends on the calibre of mathematics teaching given to secondary school students who are more likely to pursue studies in Science, Technology, Engineering, and Mathematics (STEM) in the near future.

1.2 Statement of the Problem

Problem solving has become one of the primary methods for teaching and learning mathematics during the past 20 years (Akkus, 2016; NCTM, 2010). Because it requires students to learn rules passively and without a clear goal, and because it limits their opportunities to work collaboratively while ignoring their own needs, the traditional method of teaching mathematics creates the impression in the minds of students that the subject is irrelevant and boring (Nardi and Stewart, 2003). It is necessary to approach problem solving in a different way in order to suit the needs of students studying mathematics in the twenty-first century.

Teachers need to have a thorough understanding of mathematical problem solving in order to fulfil these educational goals. It is unknown how Ghanaian mathematics teachers feel about problem solving and using it as a teaching tool in the classroom, despite the fact that there are many benefits to this approach. The teacher's view of (believe in) the mathematical concepts they teach and mathematics education in general is one of the most significant elements influencing the quality of mathematics teaching and learning (Lutovac, 2019).

In this study, the use of problem solving as a pedagogical tool will be investigated from the perspective and understanding of senior high school mathematics teachers. This is due to the emphasis placed on problem-solving techniques in the Ghanaian high school mathematics curriculum. The purpose of the Ghanaian senior high school mathematics curriculum is to "allow all Ghanaian young people to acquire the mathematical abilities, insights, attitudes, and values that they will need to be successful in their chosen occupations and daily lives" (CRDD, 2010). The success of developing capable newcomers to science and technology depends on the calibre of mathematics teaching given to secondary school students who are more likely to pursue studies in Science, Technology, Engineering, and Mathematics (STEM) in the near future.

With the growing importance of STEM education in Ghana, it is seen to be important to teach mathematics using a methodology that is consistent with the curriculum's goals. Since it has the potential to encourage students' mathematical learning, the problem-solving method to education should be the focus of mathematics instruction, according to the numerous mathematics educators and researchers (Cai, 2003; O'Shea & Leavy, 2013; Matlala, 2015). Due to the significance placed on problem solving and learning through problem solving, numerous scholars have conducted in-depth study and studies on this subject in recent years. The majority of these studies concentrated on mathematics teachers' knowledge of problem-solving (see Wilmot, Davis, & Ampofo, 2015, for example), how problem-solving improves students' academic performance (Atteh, Appoh, Obeng-Denteh, Okpoti, & Amoako, 2014), the assessment methods used by employed teachers in problem-solving (Nabie, Akayuure, & Sofo, 2013), and the significance and effectiveness of the problem-solving approach (Brhane, 2012; Alkhater, Alkhatar & Abaidullah 2015; Lester, 2013).

The various studies on problem solving in Ghana looked at things including students' achievement, teachers' subject-matter expertise, teachers' evaluation techniques used during problem-solving sessions, and students' mistakes made when solving problems. In studies on problem-based learning, primary school teachers or aspiring teachers were frequently included. These researches did not look into how problem solving is seen and understood by teachers as a teaching strategy. On their mathematics lectures, 79.9% of teachers indicated that they employ the investigative approach and problem-solving approaches, according to Nabie, Akayuure, and Sofo (2013). However, their research emphasised that the teachers were not observed in the classroom during the instructional times to see the problem-solving strategies used during their instruction. The use of the problem-solving method to instruction and teachers' perceptions of it has not been the subject of any studies conducted in Ghana's senior high schools as far as I know.

In light of the background, the gaps in the literature, and the desire to contribute to the body of knowledge on teaching through problem solving, this study investigates senior high school mathematics teachers' perceptions and use of the problem-solving as a pedagogical tool in the Upper West region of Ghana. In the upper west region, the Ghana education sector analysis indicates that students' performance in the WASSCE mathematics has not been encouraging since 2015 as compared to their counterpart in the Ashanti, Accra, Central, Northern and the Upper East region (MoE, 2022).

1.3 Purpose of the Study

The purpose of this study was to explore teachers' perceptions and use of problem solving as a pedagogical tool. The study explored the perception Senior High School mathematics teachers hold about problem-solving, the factors that influence Senior High School mathematics teachers' perception of problem-solving, the extent to which senior high school mathematics teachers use problem-solving as a pedagogical tool in the classroom, the kind of problem-solving practices utilize by Senior High School mathematics teachers in teaching and the relationship between mathematics teachers' perception of problem-solving and its use as a pedagogical tool. This study presents a strategy formulated for best practices in teaching and learning mathematical problem solving for students of secondary schools in Ghana. This research will specialize in problem-solving tasks.

Main objective

Teaching through problem-solving has been positioned in literature as promoting critical thinking and creative thinking among learners Cheeseman, J. (2019). The main objective of this was therefore to explore the deposition and use of problem-solving as a pedagogical tool among senior high school mathematics teachers.

1.4 Objectives of the Study

The objectives of the study were to:

1. Investigate the perception Senior High School teachers hold about problem-solving;
2. Determine mathematics teachers' knowledge on the use of Problem Solving as a pedagogical tool
3. Determine the factors that influence Senior High School mathematics teachers' perception about problem-solving;
4. Determine how Mathematics Teachers teach through Problem-Solving
5. Explore the kind of problem-solving practices utilise by Senior High School mathematics teachers in teaching, and
6. Establish whether there exists a relationship between mathematics teachers' perception of problem-solving and its use as a pedagogical tool

1.5 Research Questions

The following research questions were formulated to guide the study:

1. What perception do senior high school mathematics teachers hold about Problem-solving?

2. What is mathematics teachers' knowledge on the use of Problem Solving as a pedagogical tool?
3. What factors influence mathematics teachers' use or not use of Problem Solving as a pedagogical tool?
4. How do Mathematics Teachers teach through problem-solving?
5. What kind of problem-solving practices are utilised by senior high school mathematics teachers in teaching?

1.6 Hypothesis

The following null hypothesis guided the study, and was tested at 0.05 level of significance:

H_0: There is no statistically significant relationship between teachers' perception of problem solving and its use as a pedagogical tool

1.7 Significance of the Study

The results of this study have the potential to question the dominant school of thought in Ghanaian mathematics education today and to drive appropriate changes in policy and pedagogy. Such knowledge will be essential at this time, particularly as Ghana tries to reform its mathematics and science curriculum to achieve an international level.

This study will be helpful to senior high school mathematics instructors in Ghana in particular because it will enable them to determine how they feel about problem solving and how that feeling relates to how they use it as a teaching strategy. This study will highlight how to employ problem solving as a pedagogical tool, which will be of interest to mathematics educators, especially those working with high school students. In addition, the findings may warrant additional study by scholars in the field of mathematics education. The findings might provide fresh, unorthodox insight into the learning process.

This study can offer insightful data that might influence the creation of curricula. To satisfy the demands of 21st-century learning, many curriculum designers incorporate a problem-solving activity. The educators' concept of problem solving may really be the source of where and what teachers consider problem solving to be, which may also have an impact on how the assignment is designed for inclusion in the curriculum. Teachers' judgments of what makes a well-designed problem-solving task will also be impacted if the curriculum itself presents problem-solving challenges that are not in line with research best practices. If the definition of a well-designed problem-solving activity is further investigated, it is feasible that educators will alter the curriculum because of their changes.

The study will describe precise criteria that can be applied to a problem-solving activity in addition to defining what best practice in problem-solving looks like. These criteria, together with teacher perceptions, can be used to modify the curriculum as needed. Teachers and policymakers may start to change their curricula as more information about problem solving and its results becomes available. For instance, districts can reevaluate the time allotted for problem-solving teaching in class structures and schedules if instructors are effectively employing problem solving as a pedagogical tool and it is producing excellent outcomes. This might prompt teachers to consider changing the curriculum, the length of the class, or even incorporating problem solving into the curriculum as a whole. Numerous studies have been conducted that help us understand problem solving and the beneficial outcomes it produces. Several researches have looked into the structure of problem-solving assignments and the components it should have.

The goal of the study is to add to the body of literature and information regarding the educational use of problem-solving by mathematics teachers. The degree to which math teachers comprehend and apply the problem-solving method to instruction in their classrooms will also be brought to the attention of educators by this study. It also serves to update the National Council for Curriculum and Assessment (NaCCA) on the extent to which teachers are using problem-solving to teach mathematics and how some of the curriculum's objectives are being met.

1.8 Delimitation

The data collected from the data sources can be applied to other situations with comparable features to the sample requirements. Another delineating factor is location; the study was limited to the Upper West region of Ghana; it is plausible that other locations have distinct mandates that may potentially enhance problem-solving skills and produce entirely different outcomes. In the classroom, issue solving may be taking place regardless of the location. Therefore, the results may be applied to other similar instances given a similar situation.

To further refine the sample requirements, the study will only concentrate on teachers working in schools located in the Upper West region. Nonetheless, the results might apply to other senior high schools across the nation. Finally, whereas the sample consisted solely of senior high school mathematics instructors, the results could still be useful to teachers of other subjects or at the same grade level since the criteria used in the findings are similar.

1.9 Definition Terminologies

Senior High School as used in this study **is** refered to the second cycle institution that provides students with general, vocational, or technical education after completing their basic education (Junior High School, JHS). The SHS system in Ghana plays a critical role in preparing students for higher education, professional training, or direct entry into the workforce.

Problem solving in this study is the methods that students employ to resolve both straightforward and challenging mathematical task (Huang, Liu, & Chang, 2012).

Pedagogical tools are described as classroom procedures and tactics used by teachers to instruct pupils according to Krawec, Huang, Montague, Kressler, and Melia de Alba (2012),

Perception as used in this study is any insight or intuitive judgment that implies unusual discernment of fact or truth‖ (Lunenburg, 2011).

1.10 Organization of the Study

The literature on how mathematics teachers conceive and employ problem solving as a teaching technique was reviewed in the second chapter of this study. It examines the theoretical foundation of van de Walle's idea of problem-based learning. In addition, it examined an empirical assessment of studies that used the problem-solving approach to instruction, teachers' perceptions of the use of this technique, teachers' understanding of applying the problem-solving approach to instruction, and the idea of problem-solving-based mathematics instruction.

The third chapter discusses the methods and procedures used in this study. The research design, population, sample, and sampling strategies, data collection tools, and data collection processes are some of the subjects discussed. It also provides the methods and approaches the researcher used to analyse the data collected.

The study's analysis of the data, and discussions of the results are all included in the fourth chapter. The final chapter, Chapter Five, offers a review of the research, its findings, the conclusions drawn from them, as well as comments and ideas.

CHAPTER TWO

LITERATURE REVIEW

In the classroom, problem solving can yield beneficial results for students' social and academic development (Brush 2017; Saptura, 2019). In order to optimise student learning, Ghanaian schools are starting to include elements of problem-solving in their curricula, (Seidu & James (2015). But just including problem-solving in the curriculum does not mean it will be successful. To address this issue, this chapter will give the background of how problem-solving can be used as a pedagogical tool in the mathematics classroom. This chapter of the study examines the literature on the use of problem-solving as a teaching strategy. The chapter covers conceptual framework, theoretical review and empirical reviews. It then captures issues concerning teacher perception, knowledge, and the concept of mathematics instruction via a problem-solving approach.

2.0 Conceptual Framework

This conceptual framework explores the relationships between key constructs influencing senior high school mathematics teachers' perceptions and use of problem-solving as a pedagogical tool. The framework integrates theories of teaching, learning, and instructional strategies to analyze how perceptions shape and guide the adoption of problem-solving techniques in the classroom.

2.1 Problem Solving

A problem in mathematics is “any task or activity for which the learners have no prescribed or memorized rules or methods, nor is there a perception by learners that there is a specific ‘correct’ solution method”

(Hiebert, et. al., 1997). One of the most crucial mathematics concepts to teach is problem-solving; when students master this skill, they will be better able to apply mathematics to real-world scenarios and solve issues in the real world. It is also employed to gain a greater comprehension of mathematical ideas. Acquiring "mathematics facts" is insufficient; pupils also need to understand how to apply these facts to further their critical thinking abilities.

As per NCTM (2010), mathematical tasks that offer intellectual difficulties to augment students' mathematics comprehension and advancement are referred to as "problem-solving" tasks. What comes to mind when you hear the term "problem-solving" for the first time? Word problems or story problems? It's possible that story difficulties are too "problematic" and restricted. For instance, you could give the pupils the task of determining a rectangle's area given its length and width. This kind of task is a computational exercise that can be finished mindlessly without any knowledge of area.
Problems that are genuinely troubling and have the ability to give pupils context for their mathematical progress are considered worthwhile. In the study of mathematics, problem-solving is especially crucial (Wilson, Fernandez, and Hadaway, 2011). The primary objective of teaching mathematical problem-solving to pupils is to help them acquire a general skill in using mathematics in practical settings and addressing real-world problems. It can also be applied as a teaching strategy to help students grasp things more deeply. The ability to solve mathematical problems successfully depends on a variety of abilities and attributes.

The fact that a student needs a variety of abilities in order to be an effective issue solver makes this one of the hardest problems to learn.

Teaching problem-solving is also one of the most difficult subjects to teach because of these elements and abilities (Guzman, 2018). Natural and artificial environments are quantified numerically and spatially through the use of mathematics. In addition to being used to solve issues, it has aided in the advancement of social, economic, and technical fields (Guzman, 2018). Although learning mathematical facts and concepts is vital, it is insufficient. Pupils must to be taught how to apply this information to enhance their critical thinking abilities when addressing issues.

It is widely acknowledged among math educators (Guzman, 2018) that one of the most crucial elements of any math curriculum or programme is the development of actual mathematical problem-solving skills. Students who solve mathematical issues can enhance and strengthen their conventional problem-solving skills, reasoning and critical thinking abilities, obtain a thorough comprehension of subjects, collaborate with others in groups, and engage with them (Wilson et al., 2011).

In particular, it may increase someone's motivation to attempt to analyse mathematical problems as well as their determination and self-concepts regarding their problem-solving abilities; it may also increase someone's awareness of problem-solving techniques, the importance of approaching problems methodically, and the fact that many problems can be solved in multiple ways; and it may enhance someone's capacity to accurately apply solution strategies, choose the best solution strategies, and obtain the right answers to problems (Hoon, Kee, and Singh, 2013).

A heuristic is a methodical approach to addressing mathematical problems that is developed outside of context. Furthermore, by looking at particular examples, creating a diagram, focusing on the solution, and generalising the solution, a heuristic method can promote the connection of mathematical ideas (Hoon, Kee, and Singh, 2013). It is related to non-routine mathematics tasks, such thinking ahead or backward. Numerous investigations were carried out to enhance pupils' proficiency in resolving mathematical puzzles. Hoon, Kee, and Singh (2013) looked into how students responded while using the heuristics technique to solve mathematical problems as well as how proficient they were at using it. Teaching for problem solving, teaching about problem solving, and teaching via problem solving are the three approaches to issue solving.

Learning a skill is the first step in teaching problem solving. For instance, the narrative problems you choose are multiplication problems because children are learning how to multiply a two-digit number by a one-digit number. Make sure the activities you choose or create for problem-solving instruction can support the growth of mathematical comprehension.

Suggested problem-solving strategies are the first step in teaching problem solving. Take "draw a picture," "make a table," etc. as examples. Teachers' classrooms may have posters promoting the "Problem-Solving Method" with images like these: 1) Study the issue, 2) Make a strategy, 3) Address the issue, and 4) Review your work. There is scant or no evidence that teaching kids about problem solving enhances their problem-solving skills. Pupils will view a word problem as a distinct task and pay more attention to the procedures than the mathematical concepts. Furthermore, rather than emphasising sense making, students will frequently rely on trial and error (Hoon, Kee, and Singh, 2013).

Students' attention is drawn to concepts and sense-making while developing mathematical practices while teaching through problem-solving. Problem-solving instruction helps pupils build on their talents and boosts their confidence. Students can collaborate with one another and become more involved in their own education thanks to it.

There are three basic parts to any problem: operations, goals, and givens. The givens are the details or facts that are provided to characterise the issue. The intended solution to the issue is the objective. Operations are the steps that must be taken to accomplish the intended outcome. Based on the way the problem and goal are stated, problems are classified as either well-defined or ill-defined. Ill-defined problems are those that have several solutions or complex representations. Well-defined problems have discrete representations and finite aims.

There is a continuum that separates well-defined from ill-defined problems according to the cognitive load and complexity of the problem. There are two types of conceptual knowledge related to problem solving. Declarative knowledge is the awareness of a fact. It is the understanding of events, theories, facts, and objects. Knowing how to perform something is known as procedural knowledge. It consists of cognitive, motor, and cognitive strategy skills. When solving problems, declarative and procedural information are both triggered in working memory. Declarative and procedural knowledge are two different but related types of knowledge, according to psychologists who make this distinction. When solving problems, declarative and procedural knowledge interact in a number of ways.

The proposition is a fundamental declarative knowledge unit in the human information processing system. This articulates or suggests the connections between ideas. For example, the sentence "the man fixed the tyre" expresses a whole concept. A relation and one or more arguments are the two components that make up every proposition.2. Expert and novice problem solvers can be distinguished from each other using three common features. These include domain-specific tactics, fundamental, automated skills, and conceptual understanding.

These qualities and their application in differentiating expert from novice problem solvers are described in the ensuing paragraphs. Conceptual knowledge encompasses both the content and structure of the information stored in memory. Schema theory, which holds that knowledge is stored in memory as frameworks or structures that, once instantiated, give a lens through which to perceive, is strongly associated with conceptual thinking. The way senior high school mathematics teachers view and employ problem-solving as a teaching method usually consists of these important elements:

Pedagogical Tool: Problem solving is recognized as a method that engages students in active learning, encourages critical thinking, and enhances their understanding of mathematical concepts beyond rote memorization.

Teachers' Perceptions: This includes how mathematics teachers view problem-solving in terms of its effectiveness, challenges, and alignment with educational goals. Teachers' beliefs about the importance of problem-solving influence their instructional strategies and classroom practices.

Use in Teaching Practice: This aspect examines how teachers incorporate problem solving into their teaching. It includes the types of problems selected, the instructional strategies employed, and the extent to which problem solving is integrated into the curriculum.

Impact on Learning Outcomes: Researchers often investigate how the use of problem-solving affects students' mathematical achievement, problem-solving skills, attitudes towards mathematics, and overall engagement in learning.

Contextual Factors: These include school policies, curriculum guidelines, teacher training, and the availability of resources that support or hinder the implementation of problem solving in mathematics education.

2.2 Perception

The process of organising and interpreting sensory impressions to provide context for one's surroundings is known as perception. (Stephen Robbins and others, 2009) The psychological process of forming an internal representation of the external world is called perception. (Mero and Tosi, 2003). Nervous system impulses, which are produced when the sense organs are physically stimulated, are the basis for all perception.

For instance, pressure waves are used in hearing, smell is transmitted by odour molecules, and vision is the result of light striking the retinas of the eyes. Furthermore, everyone attempts to make decisions quickly. We refer to these short cuts as perceptual bias. Three steps make up the perception process: selection and stimulation of the senses, organisation, and interpretation. Our perceptions of the world around us are shaped by these stages, even though we are rarely aware of passing through them clearly.

Danish psychologist Harald Hoffding questioned the idea that vision is so elementary that it only requires associationism—the linking of what one sees and what one remembers. James J. Gibson, a well-known and contentious theorist who challenged associationism, lived from 1904 until 1980. The information in our sensory receptors, including the sensory context, is all we need to perceive anything, according to Gibson's theory of direct perception. This perspective is also known as ecological perception since the environment provides us with all the information we require for perception. Put another way, there is nothing more that we require than higher cognitive processes to act as a mediator between our perceptions and our sensory experiences.

Perception can occur without certain preexisting ideas or sophisticated inferential cognitive processes. According to Gibson, there is typically enough contextual information available in the real world for people to make perceptual decisions. He asserted that the explanation of perception does not require an appeal to higher order cognitive processes. According to Gibson (1979), humans make immediate use of this contextual knowledge. Essentially, our bodies are programmed to react to it. Gibson claims that humans interpret texture gradients as indicators of distance and depth. These cues help us directly perceive an object's or an object's part's relative proximity or distance.

Consequently, Gibson's model is occasionally referred to as an ecological model, as previously mentioned (Turvey, 2003). Gibson's interest in perception in natural settings (the ecological environment) as opposed to lab settings, where contextual information is less readily available, is what led to this reference. When attempting to understand the feelings and intentions of others in interpersonal situations, direct perception may also be important (Gallagher, 2008).

Ultimately, we are able to identify emotion in faces in and of themselves; we do not see facial expressions that we subsequently attempt to reconstruct into a representation of an emotion (Wittgenstein, 1980). Mirror neurons fire when an individual performs an action and when they witness someone else conduct the same action. Furthermore, research suggests that the processing of shape, colour, and texture in objects occurs along distinct neural pathways (what pathways) in the lateral occipital lobe.

A cognitive understanding (perception) of a stimulus is constructed by the perceiver in constructive perception. He or she builds the perception employing information from several sources while utilising sensory data as the cornerstone for the building. Because it highlights the importance of learning in perception and asserts that higher-order thinking plays a significant role in perception, this point of view is also known as intelligent perception (Fahle, 2003).

According to some researchers, perception is shaped by the environment, yet perception itself also shapes the reality we perceive (Goldstone, 2003). These concepts originate from Immanuel Kant's philosophy. Put differently, our perspective and the world we live in are mutually dependent. The reality as we experience it both influences and is influenced by perception.

Marr referred to this component of his theory of visual perception as "computational theory." It is very clear that the phrase does not refer to a theory that is merely "computer-related." Rather, it articulates the precise and potent notion that the first step towards comprehending perception is to determine what data a perceiver needs from the environment and what consistent aspects of the environment can be included into procedures for gathering that data.

Put differently, before trying to comprehend how a visual system executes its calculations, we must be aware of the computations that it must perform. Examples of Marr's application of computational theory to issues like object recognition, depth perception, and surface edge detection will be shown in later chapters. The methodology has had a significant impact; we examined the idea that cells in the visual cortex function as filters adjusted to statistical regularities in pictures of natural environments in Chapter 3 (p. 57). In fact, there is some overlap between Gibson's theory and Marr's thanks to the computational approach.

By taking into account the information that an animal need from light to direct its activities, computational theories of perception can be extended not only to human vision but to the vision of other species. According to Vagle's (2009) primacy of perception, everyone is perceiving. He clarified that human perceptions are a natural phenomenon that arises from our ideas, perspectives, and experiences on issues over time rather than being based on our intellectual reasoning. He pointed out that thinking and perception are not the same thing. According to Wasike, Ndurumo, and Kisilu (2013), perception is the outcome of connections made with information stored in our memory. He argued that our psychological and physical state, experiences, and knowledge interpret our perceptions. For teachers, the pre-instructional process, the instructional process, and all other things related to instructions are affected by perception. Cheeseman (2019) noted that, to some teachers, their inability to adopt investigative tasks in the classroom is a result of their view of how young children learn mathematics. As teachers' perceptions of effective teaching are influenced by other factors like students' characteristics and teacher experiences likewise the choice of an instructional strategy to use for a lesson is also influenced by teachers' perceptions.

This diagram shows how teachers' perceptions influence their use of problem-solving as a pedagogical tool, which in turn affects student outcomes. It also highlights the role of professional development and support in shaping teachers' perceptions. Teachers' perceptions of problem-solving play a crucial role in determining how frequently and effectively it is used in pedagogy (Blömeke and Kaiser, 2014). Positive perceptions lead to more frequent and innovative use of problem-solving techniques, enhancing student engagement and learning outcomes. Conversely, negative perceptions hinder the integration of problem-solving, limiting its potential benefits for students

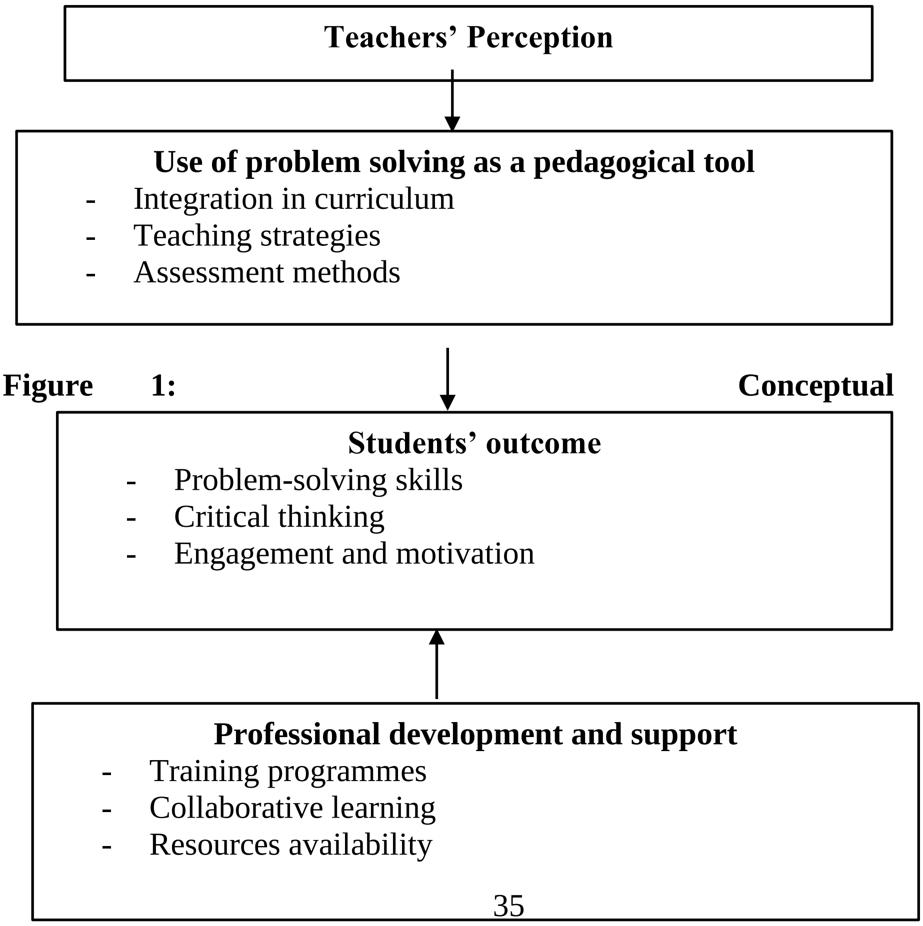

Figure 1: Conceptual

Framework of Mathematics Teachers' Perceptions and Use of Problem-Solving as a Pedagogical Tool

2.3.0 Teachers' Perceptions of Problem-Solving

Teachers generally perceive problem-solving as a critical skill for students to develop. It is seen as essential not only for academic success but also for real-life situations. The importance placed on problem-solving often stems from its ability to foster critical thinking, creativity, and independence among students (Manderfeld and Siller, 2019). Teachers' confidence in teaching problem-solving can vary significantly. Factors that influence this confidence include:

Training and Professional Development: Teachers who have received specific training in problem-solving strategies and methodologies typically feel more confident.

Experience: More experienced teachers often feel more capable of incorporating problem-solving into their teaching.

Subject Matter: Teachers of certain subjects, such as mathematics and science, may feel more confident in teaching problem-solving due to the nature of the content.

Standard Alignment: Ensuring that problem-solving activities align with educational standards and learning outcomes.

By integrating problem-solving into the curriculum, employing diverse teaching strategies, and utilizing varied assessment methods, educators can effectively develop students' problem-solving skills, preparing them for both academic and real-world challenges (Hannula, 2012).

2.3.1 Adequate Training and Resources

Adequate training and resources play a crucial role in shaping teachers' perceptions of problem-solving and their confidence in teaching it effectively (Giaconi et al., 2016).

Enhanced Knowledge and Skills: Professional development programs that focus on problem-solving strategies and methodologies equip teachers with the necessary knowledge and skills. This empowerment increases their confidence in teaching problem-solving.

Best Practices: Exposure to best practices through workshops, seminars, and collaborative learning communities helps teachers understand effective problem-solving instruction. Teachers who are well-informed about these practices feel more competent and prepared.

Continuous Learning: Ongoing professional development opportunities ensure that teachers stay updated with the latest educational trends and research. Continuous learning reinforces their confidence and competence.

2.3.2 Positive Perceptions of Problem-Solving

Value Recognition: Training programs that highlight the importance and benefits of problem-solving in the curriculum help teachers recognize its value. This understanding positively influences their perceptions.

Efficacy Beliefs: When teachers receive adequate training and resources, they develop stronger beliefs in their ability to teach problem-solving effectively. This self-efficacy translates into more positive perceptions of problem-solving as a pedagogical tool.

Resource Utilization: Access to high-quality resources, such as instructional materials, technological tools, and problem-solving frameworks, supports teachers in implementing problem-solving activities confidently. Adequate resources also reduce the perceived burden of teaching problem-solving.

Support Structures

Support structures, including mentorship, collaboration, and administrative support, further enhance teachers' perceptions and confidence. Experienced mentors and coaches provide guidance, feedback, and support, helping teachers refine their problem-solving instruction. This personalized support boosts teachers' confidence and encourages positive perceptions. Professional learning communities and collaborative networks allow teachers to share experiences, strategies, and resources. Collaboration fosters a sense of community and collective efficacy, enhancing confidence and perceptions of problem-solving (Giaconi et al., 2016). Supportive school leadership that prioritizes professional development and provides the necessary resources and time for teachers to engage in training positively impacts teachers' perceptions and confidence.

Research studies have shown that teachers who participate in targeted professional development programs focused on problem-solving report higher confidence levels and more positive perceptions (Giaconi et al., 2016). For example, a study might reveal that teachers who received training in problem-solving techniques and had access to relevant resources were more likely to implement problem-solving activities effectively in their classrooms. Specific case studies of schools or districts that have invested in professional development for problem-solving can illustrate the positive impact on teachers' confidence and perceptions. These examples provide concrete evidence of the benefits of adequate training and support.

In summary, adequate training and resources are fundamental in improving teachers' confidence and perceptions of problem-solving. When teachers feel well-prepared and supported, they are more likely to embrace problem-solving as an essential part of their instructional practices, ultimately benefiting student learning and development.

Positive Perceptions

When teachers have positive perceptions of problem-solving, it significantly influences their teaching practices, leading to more frequent and effective use of problem-solving techniques. Teachers who view problem-solving positively are more likely to incorporate it regularly into their lessons. They see it as a valuable tool that enhances student learning and engagement. Positive perceptions encourage teachers to use problem-solving across various subjects and contexts, not just in mathematics or science. This interdisciplinary approach helps students see the relevance of problem-solving in different areas of study.

2.3.3 Effective Implementation:

Teachers with positive perceptions are more open to experimenting with innovative problem-solving strategies, such as project-based learning, inquiry-based learning, and collaborative problem-solving activities (Blömeke and Kaiser, 2014). These teachers are more likely to adopt student-centered approaches, where students take an active role in identifying, analyzing, and solving problems. This leads to deeper understanding and retention of knowledge. Positive perceptions drive teachers to differentiate their instruction, providing appropriate challenges and support to meet the diverse needs of their students. This tailored approach enhances the effectiveness of problem-solving activities.

2.3.4 Engagement and Motivation:

Teachers who believe in the value of problem-solving tend to create more engaging and motivating learning environments. They emphasize real-world applications and authentic problems, making learning more relevant and interesting for students. When teachers convey positive attitudes towards problem-solving, students are more likely to develop confidence in their own problem-solving abilities. This self-efficacy encourages them to take on challenging tasks and persist in the face of difficulties (Wang et al., 2022).

Conversely, negative perceptions of problem-solving can hinder its integration into teaching practices, limiting its potential benefits for students. Teachers with negative perceptions are less likely to incorporate problem-solving into their lessons. They may view it as time-consuming, difficult to manage, or not aligned with curriculum standards. These teachers might restrict problem-solving to specific subjects or types of problems, rather than integrating it across the curriculum. This narrow approach limits students' opportunities to develop and apply problem-solving skills in diverse contexts.

Teachers with negative perceptions may rely on traditional, teacher-centered approaches, avoiding innovative problem-solving strategies. This can lead to less engaging and less effective instruction. These teachers may not provide the necessary scaffolding and support for students to succeed in problem-solving tasks. Without appropriate guidance, students may struggle and become frustrated, reinforcing negative attitudes towards problem-solving. When teachers view problem-solving negatively, they may convey these attitudes to their students, resulting in lower motivation and interest. Students may perceive problem-solving as a tedious or irrelevant activity (Blömeke and Kaiser, 2014). Negative perceptions can also affect students' confidence in their own problem-solving abilities. If teachers do not believe in the value of problem-solving, students may internalize these beliefs and doubt their own capabilities.

Research consistently shows that teachers with positive attitudes towards problem-solving are more likely to implement it effectively in their classrooms. For example, a study might find that teachers who receive training and support in problem-solving techniques report higher usage and greater student engagement. Observational studies can provide concrete examples of how positive or negative perceptions influence teaching practices. For instance, classrooms where teachers have positive perceptions may exhibit more dynamic, interactive problem-solving activities, while those with negative perceptions may show a reliance on rote learning and teacher-centered instruction. When problem-solving strategies are effectively integrated into pedagogy, they can significantly enhance student outcomes in terms of both skills and motivation.

Critical Thinking: Effective problem-solving instruction encourages students to analyze situations, identify problems, and devise solutions, which sharpens their critical thinking skills.

Creativity: Engaging with open-ended problems allows students to think creatively and explore multiple solutions, fostering their ability to innovate.

Collaboration: Group problem-solving activities teach students to work together, share ideas, and build on each other's strengths, improving their collaborative skills.

Communication: Explaining their thought processes and solutions helps students develop strong verbal and written communication skills.

Resilience: Facing and overcoming challenges in problem-solving builds resilience and perseverance, teaching students to view failures as learning opportunities.

Increased Motivation

Real-world problems make learning more relevant and interesting for students, increasing their intrinsic motivation to engage with the material. Allowing students to take ownership of their learning through problem-solving activities fosters a sense of autonomy and empowerment. Interactive and dynamic problem-solving tasks are more engaging than traditional rote learning methods, keeping students actively involved in their education (Wang et al., 2022). Successfully solving problems provides a sense of accomplishment, boosting students' confidence and motivation to tackle further challenges. Conversely, ineffective use of problem-solving strategies can negatively impact student outcomes, leading to poor performance and disengagement. Without proper guidance and support, students may become confused and frustrated with problem-solving tasks, leading to poor performance and a lack of progress. Ineffective problem-solving instruction that focuses on quick solutions rather than deep understanding can result in surface learning, where students memorize procedures without truly grasping underlying concepts. Ineffective differentiation in problem-solving tasks can leave struggling students behind, exacerbating learning gaps and inequities in the classroom.

Poorly designed problem-solving activities that do not challenge or interest students can lead to boredom and disengagement.

Lack of Relevance: If students do not see the relevance of problem-solving tasks to their lives or future goals, they may become disengaged and unmotivated.

Negative Attitudes: Repeated negative experiences with problem-solving can lead to the development of negative attitudes towards learning and school in general.

Research studies have consistently shown that effective problem-solving instruction leads to improved student outcomes. For example, a study might find that students who participate in problem-solving activities demonstrate higher critical thinking skills and better academic performance compared to those who do not. Observational studies can provide concrete examples of how effective and ineffective problem-solving instruction impacts student outcomes. Classrooms where problem-solving is effectively integrated often exhibit high levels of student engagement, collaboration, and achievement, while those with ineffective practices may show signs of student frustration and disengagement. The use of problem-solving as a pedagogical tool can significantly impact student outcomes, both positively and negatively. When implemented effectively, problem-solving strategies enhance critical skills and motivation, leading to improved academic performance and a more engaging learning experience. Conversely, ineffective use of problem-solving can result in poor student performance and disengagement, highlighting the importance of proper training, support, and resource allocation for teachers to ensure the successful integration of problem-solving into their instructional practices.

2.4 Beliefs

A person's beliefs are a component of their affective domain, which affects how they learn (Manderfeld and Siller, 2019). Gaining insight into the emotional aspects and attitudes of young pupils regarding mathematics, as well as teachers' perspectives on teaching and problem-solving in mathematics, is a challenging endeavour (Giaconi et al., 2016). On mathematical assumptions, numerous study frameworks have been constructed (Hannula, 2011, 2012). The teaching beliefs of mathematics teachers include their approaches to the subject, which include viewpoints on instructional activities, students' cognitive processes, and the goal of mathematics (Wang et al., 2022). As of right now, international assessments like TEDS-M and other research frequently employ the transmissive and constructive taxonomy of teaching beliefs (Blömeke and Kaiser, 2014). Regretfully, there isn't much of a framework for how teachers view and teach problem-solving.

2.5 Theoretical Framework

The terminology used by the Connected Mathematics Project (Lappan et al. 2014) to represent the steps involved in participating in mathematics, including activating prior knowledge, thinking mathematically about a task, and extracting and summarising key mathematical ideas, effectively captures the same ideas as Van de Walle's (2004) three-step model. The foundation of the three-step model is the idea that problem solving is a valid and effective way to teach mathematics. In other words, the arithmetic problems that the pupils tackle ought to be challenging. It ought to demand that they use their minds, reason, solve issues, make sense of the world, speculate, and assess. The parts that follow go into greater detail on each of these stages.

2.5.1 Launch

The teacher has the chance to involve pupils in the task's context and mathematical concepts during the lesson's launch phase. It is crucial to immerse pupils in the activity's context; this makes the task more relatable to them personally and enables them to see the variety of applications mathematics may have in their own and others' lives. In order for pupils to acquire a general comprehension of the problem they are to solve; it is also crucial to involve them in the mathematical concepts of the assignment.

Students can be engaged in the context of an activity in a variety of ways. Asking, "What do you know about... (e.g., Election Day)?" may be the easiest. Using a broad approach like this will help determine the students' current position within the problem's context. It might clear up misconceptions, but it might also reveal family customs that can be discussed to help students grow in their capacity to appreciate and comprehend the experiences of others. Other inquiries that could be made are as follows:

a. Has anyone here ever been …?
b. How many of you like to …?
c. How many of you celebrate …?

Sharing students' beliefs and experiences provide the teacher an opening to add to the conversation, providing more information about the holiday or seasonal event to further connect students to the frame of reference.

Of course, depending on location, there may be very limited experience with the context of a task. If students have never seen snow before, it may be challenging to interest them in a situation about sledding or snowfall. If students have lived their entire lives in an urban setting, they may not bring prior experiences in camping or farming. If this is the case, it is still an excellent occasion to expand students' horizons. Perhaps some students have experience with these less common activities that they can share with the rest of the class. Online pictures and videos are good sources for context development.

Beyond clarifying the context of the task, the Launch is an ideal time to make sure that students understand the problem in which they are about to engage. To do this, students must employ the first Standard of Mathematical Practice, make sense of problems and persevere in solving them. With teacher support, students should establish what they know about the problem. This may include knowledge gleaned from the problem itself, such as, "Avery has five friends," or inferences based on the information provided in the problem, such as "Six people will be making valentines." Students may be inclined to dive right into problems, performing operations on the numbers provided without thinking much about the problem itself before doing so. Asking them, "What do you know about this problem?" and listing their responses on a visual display requires them to think about the problem before jumping in.

Students should also be asked to determine what they want to know. Their initial focus might be on the answer to the problem. However, there may be questions that emerge as they make sense of the problem that they should be encouraged to recognize as important. Students' identification of these questions helps them realize that the problem solving will not be automatic; they may also be more metacognitive aware of some mental processes they are using while they work toward a solution.

The Launch is also an opportunity for students to develop a tentative plan for solving the problem. This can be tricky to negotiate; students sharing their plans can sometimes funnel other students' thinking at the cost of their own problem-solving strategies. Asking students to share their tentative plans with an elbow partner may alleviate this challenge. Voicing their plans may also help students identify places where their understanding of the task is still limited. Therefore, concluding the conversation with "Does anyone have questions about this problem?" provides a final occasion to clarify the context or the task before they set out to work.

Finally, it is important that students understand how the Explore portion of the lesson will progress. A variety of classroom materials should be made available for their use (e.g., rekenreks, Unifix cubes, color counters, etc.). Although some students may choose not to use manipulatives, these tools offer students who are reasoning less abstractly an entry point to the problems. Students should be assigned to partners or small groups, and be clear on the format expected for a final product.

It is important to note that the Launch portion of the task is not the place where the teacher does a similar problem with students or demonstrates how to solve the problem at hand. Doing either of these may drastically reduce the cognitive demands of the task, students' willingness to engage in the challenge, and the chance to learn important mathematics. While engaging students in preliminary processes for problem-solving is necessary, both for tackling the task at hand and developing mathematical practices they can apply to any problem situation, this portion of the lesson should be limited to a meaningful ten minutes that effectively involves students in the context of the problem and the processes that will allow each student access to the problem itself.

2.5.2 Explore

It is the teacher's responsibility to offer the proper scaffolding while allowing the kids to complete these chores, without taking away from their learning opportunities. How can an educator accomplish this? Make inquiries. Pay close attention. Evaluate a child's comprehension of the issue and identify the parts that require additional work. Encourage kids to discuss their approaches to problem-solving, including what has and has not worked.

The Explore portion of a lesson can be the most challenging for teachers who are not accustomed to teaching mathematics through problem solving. As teachers, it is our tendency to want to "help," make the path easier for our students and reduce their struggle. Although we do not want students to get to the point of unproductive frustration, we also need to be cautious about our "helpful" tendencies. Van de Walle's recommendation for this section needs to be taken to heart: "Let go!" (Van de Walle, Karp, & Bay-Williams 2013, p. 49).

It is necessary to give the kids several minutes to start working on the task before letting go of them. These student groups or partners should be allowed time to continue processing the assignment, communicate their preliminary plans, and start investigating these plans for the first few minutes of the lesson's Explore section. Working together will provide them more exploratory ideas and insights into how specific mathematical concepts may be applied to the problem. The instructor should go around the classroom at this time, keeping an open mind and just listening to the students' immediate problems and revelations.

Though there might be themes that emerge from group to group, giving them the space to talk without interruption might also provide them the chance to work out misunderstandings and disagreements. Short sessions with each of the groups can help students continue to advance as they solve more problems. Pose them the following queries to find out more about their logic:

a. Can you tell me why you decided to do this?
b. What does this represent?
c. What do you think your next step might be?
d. What does this number mean in relation to the problem you are solving?

However, it is important to keep in mind when considering this section of the lesson that the teacher's task is to identify and understand students' mathematical thinking in relation to the task as well as their misconceptions and challenges. This is not possible without close and careful listening to students' discourse. Task-specific questions or additional support may be necessary for groups that fail to find access to a task. A note of caution, however: This support should not be provided prematurely. Students learn through struggle, and as long as this is not unproductive frustration, they are probably grappling with important ideas and challenges.

Teachers may feel a similar temptation to rescue students who have gotten a wrong answer or are heading down an incorrect path. Again, this struggle is worthwhile, and as long as teachers are open and honest about honoring and respecting the learning that occurs through cognitive dissonance and/or mistakes, then children will see these as valuable learning opportunities as well. This may require a shift in what is honored and emphasized in the mathematics classroom. Processes, strategies, and the longitudinal development of mathematical concepts must be at the core, and mistakes must be valued as learning opportunities by both teachers and students. Teachers may find it helpful to circulate with a clipboard throughout the Explore portion of the lesson so that misconceptions, challenges, insights, and strategies can be recorded. In many problem-solving lessons, a particular order for sharing strategies in the Summarize portion of the lesson is appropriate. Thus, knowing this order and being able to attend to the students who have reasoned in particular ways is worthwhile for leading students in a productive discussion of the important mathematical ideas related to each task.

2.5.3 Summarize

The teacher has the chance to involve the entire class in synthesising key mathematical concepts during the Summarise section of the course. This has to go beyond just exchanging strategies. While this is crucial, it is even more crucial that students have the opportunity to talk about the mathematical concepts that are established in each assignment, connect different tactics, recognise generalisations when it is appropriate, and offer new challenges.

Students should be exposed to the strategies that were used throughout the classroom community. There are multiple ways to arrange this exposure.

Gallery walks, in which students circulate throughout the classroom to observe the work of others is one effective way to share strategies. Sometimes, it may be appropriate for students to present their strategies to the whole class. At times, it may be important to provide time for everyone to share. Generally, however, the choices for sharing should be based on the mathematical ideas and strategies utilized by particular groups, with a long-range view of making sure that all students have opportunities to participate in this way. Even if they might not get the chance to share their ideas during every lesson, students still need to be actively participating in the discussion of the tactics that are taught and drawing parallels between them and their own.

Initially, students may not know how to engage in a community of learners that discusses important mathematical ideas. Although students may focus on nonmathematical ideas at first (e.g., “I like your drawing!”), the teacher can model appropriate probing questions and comments. For example, a teacher might offer comments similar to the following:

- “I’m interested in how you knew that you needed to add these numbers together. Can you explain that?”
- “I see that you used a lot of the same numbers in your problem-solving strategy, although you had a different way of solving the problem. Why do you think we are seeing the same numbers in these places?”
- “How do you think the first group’s use of Unifix cubes is similar to your drawing?”
- “I’m not sure I understand what this picture represents. Could you explain that again?

Students will learn from the teacher's modeling, but there should also be explicit attention to initiating a mathematical discussion. The teacher might ask students to think about the question she just asked and how it helped her clarify her own understanding of the mathematical ideas. The teacher might also consider providing sentence starters to help students' structure appropriate and meaningful questions of their own. Students will learn from the teacher's modeling, but there should also be explicit attention to initiating a mathematical discussion. The teacher might ask students to think about the question she just asked and how it helped her clarify her own understanding of the mathematical ideas. The teacher might also consider providing sentence starters to help students' structure appropriate and meaningful questions of their own.

It is critical that the mathematical ideas associated with problem-solving tasks be elicited and summarized during the discussion. Anticipating specific questions is helpful, but teachers should also experiment with questions that are particular to the strategies and misconceptions that surface in the classroom. Particular attention can be focused on generalizations that arise from students' thinking. What patterns do they notice? What do they expect would happen with a different set of numbers? What rules can be articulated, either informally or formally? These generalizations may lead to opportunities for problem posing. Out of many good questions come more questions! Preparation to follow through with students' questions in subsequent problems or to record and post new problems encourages students to think about how these mathematical ideas extend beyond one problem-solving experience. Similarly, if extensions for students have been provided during the Explore phase, students should share the results of these extensions, making deliberate connections to the original task.

In this final discussion, mistakes and misconceptions should be tackled head-on. The sharing of incorrect answers may discomfort teachers, but this should be an acceptable experience for students. This requires a safe community of learners in which students are comfortable with risk, expect mistakes to be made, and see opportunities for learning from these mistakes. As students engage in more and more problem-solving experiences, they should be encouraged to take on more and more of the classroom discourse. Teachers should guide and facilitate rather than manage and direct. Students should be challenged to ask questions and make connections. Increasing the number of student comments occurring between teacher comments enables students to increasingly guide the discussion. The teacher needs to know the map, but oftentimes, the students are capable of choosing the route.

Of course, engaging in mathematical discourse like this takes time, effort, and patience. Primary children have plenty of ideas but have difficulty articulating them. However, the only way to improve discourse is through discourse, so teachers should use rich tasks to take risks, and as one pre-service teacher described it "embrace the train wreck" that may occur when following students' trains of thought. These "train wrecks" can lead to profound learning experiences! Van de Walle (2004) in dealing with instruction through problem solving developed a three-step model, which serves as a guide for teachers using the problem-solving instructional strategy. According to Van De Walle, mathematics lessons consist of three parts, namely: before, during, and after stages. Lappan, Phillips, Fey, and Friel

(2014) also corroborated Van De Walle's model through the Connected Mathematics Project, bringing the Launch/Explore/Summarize stages for teaching through problem solving. These phases were looked at as a sequential process and any teacher adopting the problem-solving approach to instruction should look at it as such. Presented below is the expected behavior of the teacher and teaching activities in each of the phases of the problem-solving approach to mathematics instruction.

2.5.4 The Before Phase

The before phase is the first stage of the problem-solving approach to instruction. It spells out to teachers the activities and the preparation that should be done before the actual problem-solving activity. The before phase set the tone for the lesson. Van De Walle (2004) & SAIDE (2008) explained that, in the before phase of the lessons, the teacher's agenda must be to study and examine the problem that students will work on. This will help the teacher to anticipate the approach students are likely to use and also clarify their expectations. This is important because at this stage students are mentally prepared for the task ahead of them. He noted that it is important for students to know the meaning of the problem before devising a plan to solve it. Therefore, at the before stage students' understanding of the problem must be clear. In addition, vary the happenings at this stage of the lesson with the task. Students must be engaged in a series of activities related to the problem. The presentation of the problem to be solved may occur at the beginning or the end of the 'before phase'. Van De Walle summarized the before phase of the lesson in terms of teachers' actions:

Activate prior knowledge: start with a simplified version of the project; relate to students' experiences; discuss potential methods or resolutions; determine or foresee whether single-computation tasks are intended to establish a computational process. The teacher must take into account the mathematical concept they want the students to learn when choosing the problem (Van de Walle, 2003 in Selmer & Kale, 2013). Be sure the problem is understood: Have children explain what the problem is asking. Go over vocabulary that may be troubling. Note: This does not mean that you are explaining how to do the problem- just that students should understand what the problem is about.

Establish clear expectations: tell students whether they will work individually, in pairs, or in small groups or if they will have a choice. Tell them how they will share their solutions and reasoning. Other teacher actions suggested by SAIDE (2008) include;

- Start with a simplified version of the task; break it down into more basic words.
- Brainstorm: When a task is not simple, have the students offer alternatives and techniques, which will result in a diversity of solutions.
- Estimate or use mental computation - To develop the computational technique, have the students mentally perform the computation or estimate the result on their own.
- Confirm that the task is understood; failure to do so is not an option. Always check that students comprehend the issue before assigning them a task. Keep in mind that their viewpoint is distinct from your

own. Request that they explain the issue in their own terms to have them reflect on it.

- Specify your expectations. This step is crucial. It is important to communicate expectations to learners. For instance, justify (in writing) why you believe your response is accurate.
- One written explanation from the group should be used when students are working in groups. Share your ideas with a partner and then select the best approach to be presented.
- Describe how you would approach the small version of the assignment to resolve the problem.
- Put this problem as a test for your students. Be mindful of the sophistication of their thinking, building, and manipulations. You could give them a few square tiles.
- What background information would the students need to comprehend the issue? Explain

2.5.5 The During Phase

The important part of the model is the middle phase (during, explore) where students without having been directed by the teacher on how to solve the problem work on a problem or task at hand. Here is where the teacher must “let go” and “allow the students to make mistakes and develop their ideas”. A good lesson must have this time for students to develop ideas while attempting to solve a problem – the during phase (Van de Walle, 2004). Further, students are made to tackle the problem either alone, in pairs, or in small groups. At this time students are not to be told how to handle the problem at hand or how their solution is evaluated rather teachers can find out about the different approaches and illustrations students adopt in their solution, the solutions that are interesting, and concepts that are misrepresented to handle them in the after phase.

It is very important also at this stage that teachers allow the question to be problematic for the students. This is because a teacher seeing students struggle may be asking leading questions that can help students with the solution (Selmer & Kale, 2013) the students must be allowed to struggle to find a solution (Selmer & Kale, 2013) to enable teachers, identify what the students know, how they think and how they are approaching the task given them (Van de Walle, 2004 & SAIDE, 2008). Van de Walle summarized the teacher's agenda in "the during phase" in four subthemes:

Let go: despite the need to jump in and assist, resist the urge and take time to observe and absorb what the pupils are doing instead.

Notice students' mathematical thinking: base your question on the students' work and their responses to you. Use questions like "tell me what you are doing" and "Can you tell me more about your…" i.e. inquire more about students' procedures. Ask questions that might help understand their thinking.

Provide appropriate support: Instead of instructing pupils on how to solve a problem, try to find ways to encourage their thinking. Make sure the students comprehend the issue and ask them about any solutions they have already tried. Tell the pupils that they could try a different tactic. **Provide a worthwhile extension**: Early finishers should be given a challenge relating to the issue they just resolved. Ask possible questions like; "I see you found one way to do this, are there any other solutions? Are any of the solutions different or more interesting than others" and ask good questions to extend students thinking. E.g. “Would the same idea work for...?” Once a teacher is satisfied that students are ready to work on the task, teachers now assume the facilitating role (SAIDE, 2008) by:

- Demonstrating confidence and respect for the learners’ abilities.
- The students get into the habit of working in groups – to indulge in cooperative group work.
- Listening actively – find out what your learners know, how they think, and how they are approaching the task.
- Providing hints and suggestions – when the group is searching for a place to begin when they stumble. Suggest that they use a particular manipulative or draw a picture if that seems appropriate. The teacher must be an active facilitator, helping students select the result to share as a group. Also ask mathematically stimulating questions that still keep the task in its problematic form (Selmer & Kale, 2013).
- Encouraging the testing of concepts Stay away from endorsing their findings or concepts. Remind the students that answers without testing and justification cannot be accepted.
- Suggesting broadenings or generalization. Many of the good puzzles appear to be easy on the surface. The extensions are what really stand out. (SAIDE, 2008).

It is not so difficult to guide students through their solutions; however, teachers frequently step in to help students face a lot of difficulties (Selmer & Kale, 2013).

2.5.6 The After Phase

Van De Walle (2006) opined that; good lessons must have this time for students to develop ideas while attempting to solve a problem. However, if some time is not reserved to culminate it with a rich discussion of students' solutions, then the approach will fail. He also explained that teachers need the skill to develop a classroom environment where students talk to each other, truly discuss ideas and look at others' responses. Hiebert (2003) was of the view that classroom discourse can arise when students' procedures and solution methods used in solving the problem are shared. As students explain the thinking behind their solutions, they also think through the solutions shared by colleagues. Much of the learning then occurs as students individually and collectively contemplate ideas they have explored. Van de Walle (2004) summarized the main agenda of the after phase as:

Promoting a community of learners: Students need to learn what you anticipate from them during this period as well as how to treat others with respect. Role-play suitable ways to respond to each other. The section on orchestrating classroom discourse offers suggestions and methods for facilitating discussions that foster a community of learners.

Listening actively without evaluation: Here, the objective is to draw attention to students' mathematical reasoning and make it apparent to other students. To encourage students to offer their views more freely, refrain from evaluating whether or not a response is correct. Simply asking what other people think about the students' comments can help kids think without evaluating their ideas.

Summarizing the main idea and identifying future problems: Formalize the lesson's central idea while highlighting the relationships between other mathematical concepts or procedures. Additionally, this is the perfect moment to reiterate the proper terminology, meanings, and symbols. Additionally, you might wish to build the groundwork for the next projects and activities.

Certain lesson formats are more conducive to creating problem-solving experiences that promote meaningful mathematics learning. Van de Walle, Karp, and Bay-Williams (2013) advocate for a three-phase lesson format. The three phases include the following:

1. *Getting Ready*: Activate prior knowledge, be sure the problem is understood, and establish clear expectations.
2. *Students Work*: Let go! Notice students' mathematical thinking, offer appropriate support and provide worthwhile extensions.
3. *Class Discussion*: Promote a mathematical community of learners, listen actively without evaluation, summarize main ideas, and identify future problems. (p. 49)

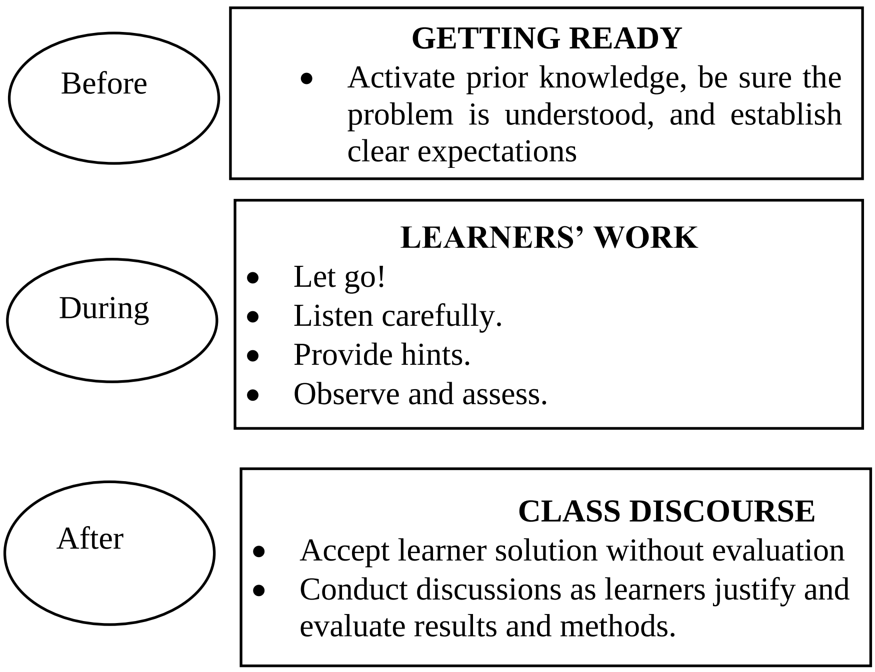

Figure 2: Van de Walle three-step model

Both models follow a clear progression of introducing a problem, facilitating exploration, and concluding with reflection. Teacher facilitation is central during the exploration phase, encouraging students to think critically and creatively. Reflection and discussion are emphasized to deepen learning and reinforce understand

The combination of these models provides a comprehensive framework for teaching through problem-solving across multiple disciplines. Van de Walle's model offers specific strategies for mathematical problem-solving, making it highly relevant for developing critical thinking in mathematics. Lapan et al.'s broader framework allows for general application, supporting critical and creative thinking across diverse learning contexts. Together, they offer complementary perspectives that enrich instructional practices by tailoring approaches to both subject-specific and cross-curricular goals.

Using both models provides a holistic approach to teaching through problem-solving:

Van de Walle's model is well-suited for developing mathematical thinking and problem-solving skills specific to mathematics.

Lapan et al.'s model offers flexibility for applying problem-solving strategies across diverse subjects and real-life situations.

By integrating these approaches, educators can adapt problem-solving instruction to fit both **subject-specific needs** and **interdisciplinary inquiry**, enhancing students' ability to think critically and creatively in varied contexts.

2.6 The use of Problem-Solving Instructional Approach

Davis and Chaiklin (2015) conducted a study that sought to emphasize the importance of the radical-local approach and how it will be relevant in our Ghanaian basic schools. This was exemplified with the concept of measurement in primary four pupils by considering the following questions in measurement:

1. What do we measure?
2. Why do we measure? and
3. What is measured from things we measure?

Davis and Chaiklin used the problem-solving approach to instruction to illustrate how the radical-local approach can be useful in our classrooms. According to them, the radical-local approach is a way of bridging the gap between students out of school and in-school mathematic knowledge. In other words, the radical local approach helps the teacher to duel on students' societal and cultural practices to teach mathematics in a meaningful way to the student in the classroom.

The researchers randomly selected from the Cape Coast Metropolis a rural school. Using the problem-solving instructional method four introductory lessons on measurement were presented to the pupils. Their study found that in using the problem-solving approach to instruction, teachers can use the social and cultural traditions of their pupils in a way that encourages them to participate actively in the lesson and to truly understand the various ideas. They noted that the demands and the success of using such an instructional approach depend greatly on the capability of the Ghanaian schoolteacher. Their research also revealed that the teaching strategy gave students the chance to independently use known cultural objects to highlight various traits rather than just being told by the teacher. The pupils felt like their opinions mattered, which increased their confidence.

They concluded that to improve pupil performance and interest in school mathematics such approaches are practicable ways of doing so. The research backs up the claim that a radical-local strategy may be used for mathematics instruction more generally. Although this study only focused on one topic and one school, it has demonstrated that it is possible to use students' societal and cultural practices in Ghana and other nations with characteristics similar to that of Ghana to teach mathematical concepts in a relevant way in elementary schools.

This study by Davis and Chaiklin (2015) brings to light two major aspects of using the problem-solving method. That is the teachers' capacity and preparedness in using the problem-solving approach and the teachers' knowledge base in terms of the content of the topic to be treated, and the approach to be used. They stated that the only way students on our side of the world can learn meaningful mathematics is to adopt the method we have mistrusted for a very long time. They also emphasize that this kind of strategy can be applied at any level looking at the setting at the level where the method of instruction was exemplified.

Another study is that of Donaldson (2011). She conducted a qualitative study that employed interviews and classroom observation for data collection in Athens, Georgia. The study looked at the practices and beliefs of high school mathematics teachers who used problem-solving methods of instruction. According to Donaldson, for a long time, academics in mathematics education have provided numerous recommendations for problem-solving instruction. They now want to know how these recommendations are actually being implemented. She opined that knowledge of what instruction through a problem-solving approach is about by teachers who wish to use the approach can help them implement practices that support the instructional approach.

Four senior high school mathematics teachers who were well-known for their skillful instruction and who are recognized as problem-solving teachers were used for the study. According to the research, the teachers' fundamental pedagogical philosophies were comparable, but they had different approaches to assisting students in becoming better problem solvers. According to the study, there are numerous approaches to support student's development of their problem-solving skills. They described their practices for teaching through problem-solving to give mathematics teachers a better understanding of what instruction through problem-solving should look like. Along with giving out good problems, they collectively put into practice the following teaching strategies: Problems that conflict with the recommendation to teach through problem-solving contained in the literature on mathematics education. The study concluded that although each teacher was unique in their practices, some were more prevalent than others. However, these teachers as a group regularly engaged in the following: (a) "teaching problem-solving strategies", (b) "modeling problem-solving", (c) "limiting teacher input—for example, having students work in groups", (d) "promoting metacognition, and (e) highlighting multiple solutions".

2.7 Teachers' Perceptions of Mathematical Problem and Problem-solving

According to Ekici (2013), the knowledge levels, perceptions, and views about the problem-solving process teachers are of great importance for them to teach the problems to their learners and use problem-solving skills in their life. Chapman, (2013) concurred with Ekici (2013) in that teachers need to hold knowledge of mathematical problem-solving for themselves as problem solvers and to assist learners to become better problem solvers. Teachers' knowledge and abilities to reason abstractly, make sense of word problems, and progress through problem-solving tasks are critical elements for teachers' mathematical problem-solving teaching success (Yee & Bostic, 2014). In a study conducted by Kaino and Yaqiang (2004), it was reported that mathematics teachers had an average understanding of problem-solving, and both teachers and pupils had low scores in solving a mathematical problem. Andesta (2012) stated that teachers are still likely to have the perception that problem-solving problems are application problems. Lee and Kim (2005) investigated a group of elementary school teacher candidates' perceptions of 'good problems' and found that the majority considered typical routine problems as good and showed strong resistance to some non-routine problems that have atypical characteristics. The same applies to Hiltrimartin (2017) who investigated teachers' perceptions of problem-solving tasks and found out that the question procedures are not usually non-routine.

Cecil and Unsri's (2017) study aimed to know mathematics teachers' perceptions of a problem-solving task. So, they selected two mathematics teachers of a Senior High School in Lahat using the purposive sampling technique as the subject of the study. The semi-structured interview method was used to collect data. The data was analyzed using the content analysis method. The study revealed that the teachers stated that problem-solving task was important to be given to their students to make them see the connection between mathematics and daily life. To be able to solve problems, the teachers stated that their students had to understand mathematics concepts. Polya's problem-solving techniques can be used to help students solve problems. A problem is stated as a problem-solving task only if it is not a routine problem. Not all mathematics problems can be categorized as a problem-solving task. As a matter of fact, the teachers found some difficulties in developing routine problems into the problem-solving task, so problem-solving tasks were found only in particular mathematics topics. This study indicates that problems teachers use in their instruction are very important and can be used as a tool to enhance students' high thinking abilities.

In the field of mathematics education, problem-solving is an innovative teaching strategy that is currently gaining support and attention. It might help students acquire the information necessary to apply concepts in practical settings. It encourages students to recognise the relevance of learning mathematics. In mathematics classrooms where traditional teaching approaches are the norm, it might be impossible to achieve such an instructional efficacy. In the latter, the teacher merely solves problems while talking and demonstrating concepts to the students. In problem-solving, teachers and students collaborate to find solutions to problems as a shared learning experience. (1) Teachers' conceptions of problem-solving; (2) Teachers' articulations of problem-solving, such as creating problem-solving assignments; and (3) Teachers' shared experiences and obstacles in teaching problem-solving were all examined by Nkosikhona (2019).

The purpose of the study was to learn more about how Form 1 teachers at the Eswatini schools perceived problem-solving. In order to maximise the collection of desired data from four Form 1 maths instructors in four high schools in the Manzini region, the study used a case study research technique. Semi-structured interviews and instructional observations were the instruments used for gathering data. Data from interviews were confirmed by lesson observations. The results of the study showed that most participants' ideas about mathematical difficulties and problem-solving in general did not align with definitions found in the literature. One teacher showed that they have the necessary understanding of what a problem is and how to solve one in mathematics. Instructors concurred that applying problem-solving techniques to instruction is beneficial.

They both felt that practicing problem-solving techniques aids in the development of critical thinking abilities and practical problem-solving capabilities. It is not possible to adequately adopt the technique until its values are reflected in the curriculum, textbooks, and evaluation system. Obstacles include teachers' insufficient time, big classes, and lack of problem-solving skills. To fully achieve the curriculum's goals for problem-solving, students must receive the appropriate instruction in mathematics. Research evidence shows that teachers have varied perceptions of solving the problem and that their classroom practices are influenced by these perceptions. Dollah (2006) claimed that accepting a difficult challenge is viewed as a crucial component of problem-solving by pupils. It doesn't always matter if a task is hard or not when it comes to problem-solving as long as the learner views it as a challenge. When a student accepts a challenge, it suggests that they are eager to explore for suitable solutions. In a study of secondary school mathematics teachers' problem-solving strategies, Saleh (2009) found that instructors saw mathematics problem-solving as tough word problems that are demanding and typically connected to real-world issues.

In addition, it calls for the manipulation of symbols and numbers as well as the application of other techniques and abilities. According to Van de Walle (2004), one of the main instructional strategies used to completely engage kids in significant mathematical learning scenarios is problem-solving. It also encompasses daily activities in general and extends beyond the field of mathematics. According to Anderson, Sullivan, and White (2004), children investigate non-routine problems through the process of problem-solving. The explorations entail developing the processes of analysis, reasoning, generalisation, and abstraction in addition to applying a variety of tactics to tackle novel tasks. Children err during their discovery phase and go back. Errors and retracements are inevitable when solving problems.

Henderson (2002) provided evidence in support of this claim, stating that some educators believe that making mistakes and having to take a step back is a normal part of solving problems. He continued by saying that using problem-solving to teach mathematics is an illogical method that relies on trial and error. Children acquire content, skills, and strategies related to mathematics through problem-solving, according to Traiton and Midgett (2001). They also gain a deeper understanding of pedagogy and content when they solve problems and interpret the reasoning behind the solution process.

Reading procedures and resolving mathematical circumstances are two skills that are necessary for solving mathematical problems, according to Goldberg (2003). The Principles and Standards for School Mathematics (NCTM, 2000) define problem-solving as engaging in an activity for which there is no obvious solution. Numerous researches conducted at various times have presented the same idea (Hiebert, 2003; Lambdin, 2003; Van De Walle, 2001, 2003).

2.8 Factors that influence mathematics teachers' perception and use problem-solving as a pedagogical tool

Several factors influence mathematics teachers' perceptions and use of problem-solving as a pedagogical tool. These factors can be categorized into personal, professional, contextual, and external influences (Jiang and Jiang, 2022)

2.8.1 Personal Factors

- **Beliefs and Attitudes**: Teachers' beliefs about the nature of mathematics, teaching, and learning play a crucial role. If they view mathematics as a set of procedures, they may be less inclined to use problem-solving. Conversely, seeing it as a dynamic and exploratory field may encourage problem-solving.
- **Self-Efficacy**: Confidence in their ability to teach problem-solving effectively influences whether and how teachers incorporate it into their practice.
- **Experience and Background**: Previous experiences with problem-solving, both as learners and as teachers, shape perceptions and usage.

Teachers with positive experiences are more likely to value and use problem-solving.

2.8.2 Professional Factors

- **Professional Development**: Access to training and professional development opportunities that focus on problem-solving can enhance teachers' skills and confidence.
- **Knowledge of Pedagogical Strategies**: Understanding different approaches to teaching problem-solving (e.g., scaffolding, collaborative learning) affects how teachers implement it in the classroom.
- **Subject Matter Knowledge**: A deep understanding of mathematics content is essential for effectively facilitating problem-solving activities.

2.8.3 Contextual Factors

- **Classroom Environment**: The overall classroom atmosphere, including the level of support for risk-taking and collaboration, influences the use of problem-solving. A supportive environment encourages students to engage with challenging problems.
- **Student Characteristics**: The abilities, attitudes, and prior knowledge of students can impact how teachers use problem-solving. Teachers may adjust their methods based on their perceptions of students' readiness and responsiveness.
- **Resource Availability**: Access to appropriate materials, technology, and other resources supports the implementation of problem-solving activities.

2.8.4 External Factors

- **Curriculum and Standards**: The alignment of problem-solving with national or regional curriculum standards can either support or hinder its use. Curricula that emphasize problem-solving encourage teachers to incorporate it more frequently.
- **Administrative Support**: Support from school leadership, including encouragement and provision of resources, plays a significant role in facilitating problem-solving approaches.
- **Assessment Practices**: The nature of assessment practices, both formative and summative, influences how teachers approach problem-solving. Assessments that value problem-solving skills can motivate teachers to integrate it into their teaching.
- **Community and Cultural Expectations**: Societal and cultural attitudes towards education and mathematics can influence teachers' perceptions and practices. In communities where problem-solving is valued, teachers may be more inclined to use it.

2.8.5 Interaction of Factors

These factors do not operate in isolation but interact in complex ways. For example, a teacher with strong self-efficacy but limited resources may still struggle to implement problem-solving effectively. Similarly, supportive administration and professional development can enhance teachers' confidence and skills, even if initial self-efficacy is low. Teachers' knowledge of teaching content affects their classroom practice, which involves student learning and achievement (Peterson et al., 1989). Problem-solving is getting from where you are to where you want to be by continuously reformulating the problem until it becomes something you can manage (Kilpatrick, 2016). The cognition of mathematical problem-solving affects mathematical problem-solving and teaching behavior. Teachers' beliefs about mathematics impact their teaching, and teachers with different views about mathematics teach differently (Philipp, 2007; Lester & Cai, 2016). Teachers are central to advancing the effective atmosphere and social interaction of the class (Pehkonen et al., 2016), and their beliefs have a considerable impact on the nature of classroom practice (Kayan Fadlelmula & Cakiroglu, 2008). Therefore, teachers need to have a good understanding of mathematical problem-solving and its teaching.

Pre-service mathematics teachers are prospective teachers who will teach mathematics after graduation (Jiang and Jiang, 2022). Many pre-service teachers complete advanced mathematics courses with a limited interpretation of critical terms, incorrect beliefs about the nature of mathematics, and a failure to recognize that mathematics stimulates analytical thinking and creativity (Paolucci, 2015). They have difficulty raising and solving problems (Mallart et al., 2018). Teacher education can alleviate negative attitudes or beliefs about mathematics and teaching mathematics to college students preparing to become teachers (Looney et al., 2017). Pre-service mathematics teachers with proper training will have better problem-solving and problem-solving teaching performance (Crespo and Sinclair, 2008; Karp, 2010). They need a teacher preparation program that focuses their attention on the learning of the students they are teaching (Kilpatrick, 2016). They must understand mathematics, teaching, and pedagogy (Register et al., 2022). A proper understanding of mathematical problem-solving and its teaching is an essential part of the professional quality of pre-service mathematics teachers and can effectively guide their future teaching of mathematical problem-solving.

Significant advances have been made in understanding the affective, cognitive, and metacognitive aspects of problem-solving in mathematics and other disciplines (Lester and Cai, 2016). Research on the correlation between the use of various problem-solving strategies and problem-solving success has been plentiful over the last century (Schoenfeld, 2007), and there have been many suggestions for teaching problem-solving effectively (Mason, 2016). However, empirical studies of pre-service teachers' understanding of mathematical problem-solving and its teaching are still rare. The big question facing current mathematical problem-solving research and teaching practice is this: How do we make meaningful problem-solving a regular feature of mathematics classrooms (Leong et al., 2016)? It is necessary to train many outstanding mathematics teachers, and a feasible method to achieve this is to pay attention to the education of pre-service teachers. We should understand the mathematical problem-solving and teaching knowledge of pre-service teachers. On the basis of this knowledge, we can develop educational strategies to improve their problem-solving and teaching skills.

Improving the mathematical problem-solving teaching ability of teachers requires understanding the mathematical problem-solving and teaching perception of in-service and pre-service teachers. For high-quality mathematics (problem-solving) teaching, pre-service teachers think that "developing students' thinking ability" and "mathematical communication ability" is more critical. By contrast, in-service teachers think "learning arrangement" and "building connections" are more important (Clooney and

Cunningham, 2017). Age and work experience may shape beliefs related to mathematical problem-solving (Metallidou, 2009). It is helpful for the training of pre-service teachers to understand how in-service teachers view mathematical problem-solving and its teaching. Therefore, comparing the cognition of pre-service and in-service teachers toward mathematical problem-solving and its teaching is necessary. The cognition of pre-service teachers can be better understood by placing them in the background of in-service teachers' perceptions. Issues such as whether their perceptions have something in common, what the differences are, how to narrow the gap, how to further optimize their perceptions, which perceptions can be optimized before they are employed, and which can only be optimized afterward are not only significant for the training of pre-service teachers but also help the continuing training of in-service ones.

The study of teachers' and students' beliefs regarding solving mathematical problems has been motivated by the supposition that there is a relationship between actions and beliefs (Wilson, Fernandez, & Hadaway, 1993). According to research, beliefs have an impact on the mathematical behaviours, attitudes, and performances of both teachers and pupils (Pajares, 1992; Thompson, 1992; Kloosterman & Stage, 1992; Schommer-Aikins, Duell, & Hutter, 2005). As well as measuring "beliefs which are related to motivation and thus achievement on mathematical problem-solving," many measures were created to gauge students' perceptions of the value of mathematics (Fennema & Sherman, 1976; Kloosterman & Stage, 1992, p. 109).

One of the most popular scales is the Indiana Mathematics Belief Scale, created by Kloosterman and Stage in 1992. It consists of thirty items, six of which are in each of the following five sub-scales: “I can solve time-consuming mathematics problems,” “There are word problems that cannot be solved with simple, step-by-step procedures,” “understanding concepts is important in mathematics,” “word problems are important in mathematics,” and “effort can increase mathematical ability” (Kloosterman & Stage, 1992, p. 115). Fennema-Sherman (1976) created another widely used measure with six items measuring beliefs regarding the value of mathematics. According to Schoenfeld (1992), there is a large volume of research on the beliefs of students, a reasonable but growing body of research on the beliefs of teachers, but very little on the beliefs of mathematicians themselves.

Researchers have been examining the beliefs of pre-service and in-service elementary and secondary teachers of many subject areas for the past few decades. The type of classroom environment that teachers produce is determined by their views (Schoenfeld, 1992). According to numerous studies (Thompson, 1992; Cady, Meier, & Lubinski, 2006; Philipp, 2007), teachers' beliefs play a significant role in setting up classroom environments and instructional strategies. They also have an impact on students' beliefs about mathematics and their academic performance (Grouws, 1996; Schoenfeld, 1992; Wilkins & Brand, 2004; Frykholm, 2003; Kayan & Cakiroglu, 2008; Lloyd & Wilson, 1998).

Moreover, empirical research suggests that “the nature of teachers’ experience, background, and knowledge influence their beliefs (Philipp et al., 2007; Thompson, 1992)" (Page 4 of Lui & Bonner, 2016). Pre-service teachers' mathematical experiences, conceptual understanding, and beliefs were investigated by Adnan, Zakaria, and Maat (2012). They created a measurement model for each of these three constructs, and their study showed that there was little correlation between any two of these combinations. Based on the results of their study, they suggested that "more studies should be conducted to investigate mathematical beliefs, conceptual knowledge, and mathematical experience, particularly using a quantitative approach" (p. 1718). Their findings are consistent with those of Willcox-Herzog's (2002) study, which showed that there was no meaningful correlation between teaching methods and beliefs in a mathematics classroom. However, Quillen's study from 2004 demonstrated that there is a strong correlation between one's ideas about mathematics and their knowledge of the subject matter as well as between their experiences with the subject.

When pre-service and in-service primary school mathematics teachers' opinions were evaluated, Lui and Bonner (2016) discovered that both groups supported constructivist views about teaching and learning more than traditional beliefs. However, they found "a negative correlation between constructivist beliefs and overall accuracy in mathematics problem-solving" (Lui & Bonner, 2016, p. 8). Mason (2003) cited many research studies which showed that "teachers have a remarkable influence on students' construction of their beliefs through the ways in which they present the subject matter, the kinds of tasks they set, assessment methods, procedures and criteria (Pehkonen, 1998; Pehkonen & Törner, 1996; Törner, 1998)" (p. 83).

Since teachers' beliefs shape students' beliefs through their actions, in-service teachers should be encouraged to examine their own beliefs, change their convictions, and use alternative methods in the classroom as part of both teacher training programmes and professional development initiatives (Franke, Fennema, & Carpenter, 1997). Furthermore, students may be given the chance to constructively and methodically reflect on their learning process as a result of the emphasis placed in teacher preparation and professional development programmes on effective problem-solving instruction (Wilson, Fernandez, & Hadaway, 1993).

Brown (2003) used the Mathematical Problem-Solving Attitude Scale created by Whitaker (1976) and the Indiana Mathematics Belief Scale developed by Kloosterman and Stage (1992) to examine the attitudes of elementary mathematics teachers towards problem-solving, beliefs about problem-solving, and abilities to solve problems quantitatively. In addition, the researchers qualitatively examined the attitudes by interviewing the participants.

Her study's findings demonstrated that although instructors had low problem-solving abilities, they generally had positive views about the process. In order to better understand middle school teachers' perceptions of problem-solving, reported problem-solving instructional practices, and beliefs about students' capacity for problem-solving, Poetzl (2007) attempted to "examine the effects of participating in a multi-year mathematics professional development programme focused on problem-solving" (p. xiii). Her research revealed that following the professional development initiative, teachers' perceptions of problem-solving changed. Teachers also frequently gave their students extra chances to solve problems, which allowed them to gain greater insight into the skills of their pupils.

2.9 Teachers' Knowledge for Teaching through Problem-Solving

Sakshaug and Wohlhuter (2010) investigated how 41 teachers' problem-solving learning experiences enhanced their teaching of mathematics through an action research study. Sakshaug and Wohlhuter found that “teachers’ successes and challenges occurred in these contexts: a) comfort level with mathematics, b) selection of problems, c) instructional components, d) impact on students and e) beliefs about the process.” (p. 401). The ability of 39% of the teachers to teach complicated word problems was hampered by their lack of familiarity with the concepts, reasoning, and communication required to solve complex word problems. Owing to the discomfort, teachers frequently gave pupils the knowledge and techniques they needed rather than allowing them to grow and create meaning on their own (Sakshaug & Wohlhuter, 2010). Marchis (2011) employed a qualitative approach to investigate the ways in which sixty-two teachers assisted their pupils in solving mathematical puzzles. Students were given challenging problems to tackle by their teachers. The findings indicated problems with the techniques teachers used while instructing students in problem-solving. The teachers read the material more aloud, underlined important ideas, made diagrams, and reworded the text question to match the comprehension level of the pupils. Additionally, according to Marchis (2011), two thirds of the teachers did a poor job of giving their pupils the chance to apply a variety of tactics and explain their solutions.

A series of research led by Schoenfeld (1992) examined the capacity of math specialists to instruct in mathematical problem-solving. Schoenfeld created four unique phases that educators use to assist students in improving their ability to solve mathematical puzzles. In Phase 1, resource knowledge, the instructor evaluates students' prior knowledge to help them apply mathematical concepts successfully. In Phase 2, the instructor gives the class heuristic tactics to help them solve problems quickly.

In order to obtain legitimate answers to a variety of word problem kinds, teachers instruct students on how to apply tactics such symbolising operations, sorting details, working backward, annotating issues, drawing figures, removing details, and marking formulae. In Phase 3, which involves monitoring and processing, the instructor assists students in realising that they are the ones who solve problems. The instructor guides students in the use of visuospatial and relational processing, self-monitoring and self-regulation (a component of metacognition), and other methods to acquire, process, and apply mathematical techniques that are demonstrated and taught for word problem solving.

Teachers' resource otherwise referred to as mathematical knowledge is the basis for any mathematical problem-solving instruction. Literature suggests that there are 7 forms of knowledge possessed by teachers for instruction. According to Susanta (2012), these consist of the following: (a) content knowledge; (b) general pedagogical knowledge; (c) curricular knowledge; (d) pedagogical content knowledge; (e) knowledge of learners and their characteristics; (f) knowledge of educational environments; and (g) knowledge of educational ends. She explained that among these types of knowledge is pedagogical content knowledge, which distinguishes the kinds of knowledge for instruction. Pedagogical content knowledge considers how the subject can be exhibited to the learner to elicit understanding. She explained that this kind of knowledge involves the knowledge and understanding of how particular topics, problems, or issues are organized, represented, adapted to the diverse interests and abilities of learners, and then presented for instruction (Susanta, 2012). She went on to explain these kinds of knowledge by saying that pedagogical content knowledge also includes an awareness of what influences how easy or difficult a topic is to learn: the ideas and preconceptions that students from various backgrounds and ages bring to the study of the most commonly taught subjects and lessons.

The significance of teaching approaches in the teaching and learning process is emphasized by these collections of pedagogical content knowledge. Boz and Boz (2007) write that pedagogical content knowledge has other related aspects like students, curriculum, instruction, and evaluation. Since the main concept of pedagogical knowledge is the delivery of good instruction, teachers should understand that students with misconceptions about topics may not come to the class as blank slates. These teachers need to know the strategies that may be effective in reorganizing the students' understanding. A teacher’s knowledge of instructional strategies according to Boz and Boz (2007) is of two kinds; knowledge of topic-specific strategies, which comprise the teachers’ ability to select appropriate instructional strategies for a topic, and knowledge of subject-specific strategies.

Arbaugh, Chval, Webb, and Jackson (2009) explained the four-domain model of teacher knowledge for teaching by Grossman. They emphasized the teachers’ pedagogical content knowledge in four distinct categories: (a) “the conceptions of purposes for teaching subject matter – this involves knowledge and beliefs about the purposes for teaching a subject at different grade levels”. They stated that mirrored in the teachers’ aims for teaching a subject matter are the primary conceptions of teaching a particular subject. (b) “Knowledge of students' understanding. explaining this concept Arbaugh et al said, to generate appropriate explanation and representations, teachers must have some knowledge about what students already know about a topic and what they are likely to find puzzling". (c) "Curricular Knowledge includes knowledge of curriculum materials available for teaching particular subject matter, as well as knowledge about both the horizontal and vertical curricula for a subject" (d) "knowledge of instructional strategies – Experienced teachers may possess rich repertoires of metaphors, experiments, activities, or explanations that are particularly effective for teaching a particular topic while beginning teachers are still in the process of developing a repertoire of instructional strategies and representations".

Teachers' knowledge of instructional strategies is also subject-specific as explained by Grossman in her fourth domain on pedagogical knowledge. Teaching mathematics requires teachers' knowledge of various instructional strategies. Thus, it requires strategies that will enable the students to acquire the necessary skills and attitudes that the national goals of mathematics education emphasize. Martin (2007) opined that the success of a mathematics teacher depends on the knowledge of students learning and teaching. Special mathematical knowledge for teaching is needed for students' proficiency. Zaslavsky and Leikin (2004) are of the view that mathematics teachers develop knowledge in two ways. That is, ''through learning, as facilitated by a mathematics teacher educator, or through teaching, when they facilitate [mathematics teachers'] learning''.

Ball (1988) came out with a research model on pedagogical knowledge for teaching mathematics. She noted that the justification for adopting a mathematical approach to teaching mathematics is from the subject matter, learners, learning, and the context but the use of this knowledge about instructional strategies rallies on one's opinion of the goals of teaching mathematics. Munter and Correnti (2017) explained that irrespective of the approach, the quality of a teacher's mathematical instruction has been the link between teachers' pedagogical content knowledge in mathematics and the teachers' mathematical instruction. They further stated that teachers may have the same Pedagogical Content Knowledge (PCK) however, this kind of knowledge may only encourage variations in the instructional strategies.

Teachers value and find the best way an instructional strategy could be used when they know and understand the underlying theories behind those instructional strategies (Liu, Jones & Sadera, 2010). Though teachers can acquire their PCK and the best methods for teaching specific topics in mathematics through experiences with the use of such strategies, the basic source of a teacher's mathematical knowledge on instructional strategies has been their professional development programs. And the inability of their previous teacher education programs to adequately prepare them for teaching using the various instructional strategies, like the problem-solving approach, is one of the difficulties most teachers face (Buschman, 2004; Matlala, 2015) concerning the use of their pedagogical content knowledge. Matala asserts that most times teachers tend to teach in the manner they were also taught during their professional training programs. However, they are expected to teach in a manner they did not experience from their teachers. Elaboration on this aspect of teachers' knowledge for instructional practice Matala stated that teachers possess the theoretical knowledge of the problem-solving instructional approach; however, they lack the ability to implement such knowledge in the classroom.

2.10 Mathematics teachers' perceptions and use of Problem-Solving as a pedagogical tool

Awinyam (2018) indicated that the sort of improvement and the pedagogical choices of teachers for effective instruction was the result of their perception. Teachers' perceptions and instructional practices are most times subject-specific. Since perception influences human behavior, it is very essential to look at the perception of mathematics teachers (Awinyam, 2018). The mathematics teachers' perception of mathematics instruction is influenced by the effectiveness of an instructional strategy and students' performance. Liu, Jones, and Sadera, (2010) hold the view that a mathematics teacher's possibility of using an instructional strategy like the problem-solving instructional strategy is strongly based on the teacher's perception of the instructional practices.

Teachers who do not use this instructional strategy cite factors like knowledge of problem posing and also organizational and managerial factors (Cheeseman, 2019). In support of Cheema's claim, Akhter, Akhtar, and Abaidullah (2015,) stated that teachers indicated ample space in the classroom, and good and sufficient materials and environments are needed to implement the problem-solving method. Further on reasons for not using the problem-solving teaching method, Xiuping (2002) also argued that due to time constraints, the number of resources needed and the huge number of students in the class, the problem-solving approach cannot be adopted every time during mathematics instruction. These reasons identified, exemplify the perception of teachers when it comes to the use of the problem-solving approach.

Akhter, Akhtar, and Abaidullah (2015) indicated that though teachers have a good perception of the problem-solving methods in terms of their importance, many teachers believe that the use of the problem-solving instructional approach should be done taking into consideration the students' class size and level. They further expressed that teachers find it difficult to use the problem-solving method due to teachers' low level of understanding of the process involved in using the approach and also teachers' inability to plan, manage and pose questions to the class when using the problem-solving instructional method. This point out that knowledge of a concept also affects one's perception.

Some teachers adopt different teaching strategies to help children develop an understanding of mathematical ideas. These teachers hold the perception that children create their knowledge through the learning of mathematics. This is done by using their mental activities and engagement with the environments (Awinyam, 2018; Cathcart, Pothier, Vance, & Bezuk, 2001). For a teacher to adopt an instructional approach the teacher needs to have a positive perception which is based on both theory and experience in the practice of the importance and effectiveness of such an approach.

2.11 Mathematical Instruction through Problem-Solving

Teaching is the art of interaction between students and teachers to bring about the achievement of the desired learning outcome based on certain content. This process involves writing a specific objective and following through with a defined plan. Based upon the initial evaluation of student skills, teachers assess the level of student's knowledge of a previous lesson to present a preceding instruction (Arafeh, 2008; Swanson, 2001). As a result, there have been many suggested approaches to effective mathematical instructional delivery since individual teachers have their views of effective instruction taking into consideration societal needs, teachers' context, and student characteristics (Cho, 2014).

Susanta (2012) noted that the development of students' understanding of mathematics is the aim of mathematics instruction. Explaining the concept of understanding in mathematics, she maintained that understanding connotes the ways of knowing the procedures, skills, facts and the sort of thinking needed in mathematics. Using an appropriate approach during mathematics instruction leads to the achievement of the goals of mathematics instruction. One most important of such approaches is the problem-solving approach to teaching. There are three basic aspects of problem-solving instruction. Instruction for problem-solving, instruction about problem-solving, and instruction through problem-solving. Masingila, Olanoff, and Kimani (2017) addressed three issues related to mathematics instruction through problem-solving. They included: teaching about problem-solving in which students are taught problem-solving heuristics Polya (1957), teaching for problem-solving in which the focus is on ''ways in which the mathematics being taught can be applied to both routine and non-routine problems'' and teaching via problem-solving in which ''problems are valued not only as a purpose for learning mathematics but also as a primary means of doing so'.

Brhane (2012) defined the problem-solving approach as a method of teaching mathematics through problem-solving contexts and inquiry-oriented environments characterized by the teacher 'helping students to construct a deep understanding of mathematical ideas and processes by engaging them in doing mathematics: creating, conjecturing, exploring, testing, and verifying. Also, Donaldson (2011) explained mathematics teaching through problem-solving as a technique of instruction in which teachers use problem-solving as the fundamental way to convey mathematical concepts to students in order to aid in the integration of their mathematical knowledge. Most mathematicians and researchers in mathematics education have recommended this method of instruction and they have advocated for its use in mathematics classrooms. There has been other research on problem solving and revisions of curricula by educators and trainers to assist students in using their higher-order thinking and problem-solving abilities (Cai, 2003). It is in this view that the use of a problem-solving instructional approach has now become the focus of mathematical instruction in the curriculum of many countries and national standards.

The advocacy for the use of the problem-solving approach to instruction is grounded in the fact that this approach is the current basic method needed by today’s learners (Brhane, 2012). He stressed that Problem-solving should be the central focus of the mathematics curriculum. He continued that; mathematics instruction must be such that the subject will be envisaged as problem-solving by students. Brhane, (2012), maintained that: teachers should concentrate on problem-solving approach to instruction due to its numerous benefits to the student. The adoption of a problem-solving approach to instruction is based on the assumption that by solving problems, students can enhance their understanding, and develop and extend their knowledge (Donaldson, 2011).

Thus, the main import of using a problem-solving instructional strategy is not only to make learners problem solvers but ultimately to enable students to gain a deepened understanding of mathematics (Donaldson, 2011). In the concept of mathematics, an indispensable and decisive pedagogy is the problem-solving approach to instruction (Brhane 2012). In the medium term, the underlining mathematical concept is what instruction through problem solving directs students' attention (Van de Walle, 2003). The adoption and use of the problem-solving instructional approach as enshrined in the curriculum is a decision by the teachers.

Donaldson (2011), said that teaching through problem-solving is not assured just by handing over to the teacher a problem-based curriculum. Donaldson explained that due to inadequate research, most teachers are unable to ascertain what the method of teaching through problem-solving looks like. The decision not to utilize the problem-solving approach to instruction can be influenced by other reasons such as the impression of teachers to complete the content of the textbook within a specified period, teachers' awareness, and lack of dedication to use the approach (Alemayehu, 2010) and teachers' misinterpretation of the problem-solving instructional methodology (Brhane 2012). Akhter, Akhtar and Abaidullah (2015) also noted that those who adopt the problem-solving instructional approach encounter problems with classroom management, planning and questioning. Samuel (2002) in Akhter, Akhtar and Abaidullah (2015) stated that these difficulties arise due to the teachers' lack of expertise and difficulty in using the problem-solving instructional method for the existing mathematics curriculum.

The study by Donaldson (2011) showed that the problem-solving instruction approach looks differently for every teacher so teachers who wish to implement the approach should know what the approach looks like. The problem-solving instructional strategy is implemented differently by different teachers as a result Donaldson advised that beginning teachers should be cautious so as not to copy how other teachers implement the instructional approach. But how successfully teachers use this instructional method as they modify their method of teaching is characterized by the support and encouragement they get from other teachers and associates (Cai, 2009). However, there are recommended principles and roles teachers follow when using the problem-solving strategy. In the problem-solving strategy, the lesson starts with posing a good problem for students to solve using various strategies whiles the teacher serves as a guide.

Teaching through problem-solving involves selecting a task appropriate for the lesson, allowing the students to work out a solution and organizing a classroom discussion that will unearth the underlining concept of the lesson (Selmer & kale, 2013; Cai, 2009). Donaldson (2011) added that teachers should avoid intervening too frequently and finishing too much of the mathematical work too quickly if they want to allow mathematics to be a challenge for their students. This highlights how crucial it is to have a conversation about the duties and behaviours of teachers while using this strategy.

Literature has it that, implementing the problem-solving strategy to reduce students' difficulty during their problem-solving process, teachers must choose the sort of challenging questions to ask, the part of the work to emphasize and the kind of support to give the students so as not to think for them. Another role of the teacher in teaching through problem-solving is by teaching problem-solving strategies (NTCM, 2000, Cai 2009). Donaldson (2011) gave 7 guiding principles to help the roles and actions of the teacher in teaching through problem-solving. They include; Giving many problems, providing "quality" problems, and imparting either specialised or generic problem-solving techniques (including heuristics), Promoting metacognition by modeling problem-solving, limiting teacher involvement—for example, by having students work in small groups—and fostering reflectiveness and the highlighting of multiple options.

While certain researches do identify the traits of maths teachers in addressing problems, these studies are insufficiently comprehensive. There are numerous shortcomings in problem solving among maths teachers. Özgen and Alkan (2012) found that the majority of math teachers have poor problem-solving abilities. For example, when it comes to questioning techniques, math teachers make seven different kinds of mistakes when they pose questions concerning fractional splitting (I sık and Kar, 2012). Some maths teachers find it challenging to include real-world examples in their questions, to make them appropriate for a particular educational level, and to allow pupils to self-correct (Mallart et al., 2018).

Although they can utilise a variety of approaches, maths teachers are not always able to solve problems through strategy and problem solving (Avcu and Avcu, 2010). Similarly, they struggle to articulate mathematical procedures in language (Özdemir and Çelik, 2021). They therefore require improved knowledge of problem solving and how to teach it through practical experience.

According to Temur (2012), math teachers find it challenging to identify unique mathematical issue scenarios and to select particular mathematical problems. As a result, there are a lot of issues with problem-solving instruction for maths teachers that require careful discussion. Zsoldos-Marchis (2015) investigated how well cooperative problem solving could influence primary school teachers' perceptions of mathematics. They discovered that there were statistically significant improvements in students' enjoyment of mathematics when they employed cooperative learning strategies.

The students' perception of mathematics' value increased, and they began to favour solving unusual problems. Other researchers examined the process of getting a credential for math teachers enrolled in middle school math techniques courses, emphasising the necessity for more study on these kinds of courses (Gómez, 2009). Although the research is far from comprehensive, some researchers have attempted to provide an overview of problem-solving in Chinese mathematics education, highlighting the expertise of Chinese math teachers in problem-solving and teaching (Cai and Nie, 2007). In summary, there is a need for study on problem-solving and problem-solving instruction for maths teachers. There is substantial evidence from frontline instructors, students, and classrooms.

Through solitary assignments and class discussions, learners engage in self-learning and use problem-solving techniques. A good approach makes the issue solver consider the meaning of the problem statement as well as the mathematical equation (Aydogdu, 2014: 54). Working backward, taking on various perspectives, identifying patterns, sketching out a solution, tackling a simpler or comparable problem, taking into account extreme scenarios, making educated guesses and testing (approximations), outlining all potential outcomes, employing logical reasoning, and organising data are some examples of problem-solving techniques. Students apply the problem-solving techniques they have learned in the problem-solving mathematics classroom when they work through challenges during group projects or self-study. The problem solver solves the problem by taking deliberate actions, such as following leads that are productive and giving up on dead ends (Schoenfeld, 1992).

According to Hatfield, Edwards, Bitter, and Morrow (2007), learners can advance in their ability to solve increasingly difficult and challenging situations by using problem-solving strategies.

The ideas of teachers about problem solving, how they articulate it (e.g., by creating tasks for students to do), and their shared experiences and difficulties in teaching issue solving were all examined by Nkosikhona C. N. (2019). He discovered that the majority of respondents had opinions about mathematical problem solving that did not align with definitions provided in the literature. Their idea of problem-solving involved posing a question to students, having them consider it carefully, and having them use previously taught methods and strategies to solve mathematical problems with clear-cut solutions. Just two of the four educators surveyed said they were familiar with problem solving.

They agreed with the literature concept of problem solving in mathematics when they defined it as "giving learners a question, requiring them to think, without an obvious way to get to the solution" in their definitions. Nkosikhona Calvin Nhlabatsi draws the conclusion that teachers contended that the methodology could not be appropriately applied unless the curriculum, textbooks, and evaluation system acknowledged its benefits. Other obstacles include teachers' insufficient time, huge class sizes, and lack of problem-solving skills.

It was also observed that every responder believed that in order to successfully carry out the curriculum's objectives for problem-solving, they needed in-person instruction in mathematics. Although these significant difficulties are brought up in Nkosikhona Calvin Nhlabatsi's work, he fails to investigate the effect that teachers' perceptions have on students' performance.

The problem-solving techniques and professional development requirements of maths instructors in Ghanaian junior and senior high schools were investigated (Ampadu, 2019). 114 maths instructors from 28 junior high schools (JHS) and senior high schools (SHS) in the Cape Coast Metropolis were chosen by him to participate in the research. A semi-structured questionnaire was used to gather both quantitative and qualitative data. The study's findings demonstrate that while educators recognise the value of problem-solving in enhancing mathematics instruction, they have not received the necessary problem-solving training to assist them in this role.

Thus, the researcher contends that even with all of the benefits that come with using the problem-solving approach in teaching and learning, the most important thing we can do to support children in developing problem-solving abilities is to provide instructors with ongoing professional development opportunities. Students in Ghana's JHS and SHS may find themselves at a disadvantage while competing in worldwide comparison exams against their peers from other nations if our teachers aren't provided with the necessary resources to become skilled in using problem-solving techniques in the classroom.

2.12 The extent to which senior high school mathematics teachers use problem solving as a pedagogical tool in the classroom

According to Petersen (2016), problem solving is defined as handling routine tasks that require the solver to apply their analytical and predictive abilities. In this case, the problem solver's tools of choice for solving problems are analysis and prediction. Solving problems is a crucial part of studying mathematics. It gives students the chance to participate in meaningful mathematical conversation, which includes evaluating different representations and arguments for their answers. By using problem-solving techniques in the classroom, teachers may actively participate as learners and students can take an active role in the learning process.

In this regard, Bay (2000) describes problem-solving instruction as a way for maths teachers to deliver more meaningful instruction. The author continues to support his position by stating that teaching through problem-solving entails applying mathematical information through the use of appropriate tools and problem-solving techniques. Through problem-solving, students expand, deepen, and enhance their understanding (Hieber & Wearne, 2003). Students who learn through problem-solving are better equipped for able-minded lives.

Additionally, it fosters students' confidence in their ability to handle problems and encourages them to take calculated risks (Tratton & Midgett, 2001).

Furthermore, Van de Walle (2007) notes that using a problem-solving approach in mathematics instruction enables teachers to fully engage their pupils in the fundamentals of the subject. This suggests that problem solving is a fundamental component of all mathematical tasks and that it is a general talent that calls for independent thought and critical examination of problems, both of which are crucial for lifelong learning. Therefore, it is ideal for pupils to start developing mathematical problem-solving abilities in elementary school as a sign of readiness for both the workforce and lifetime learning. In Ghana, there is still a problem with teachers' lack of expertise in applying efficient and successful problem-solving strategies when teaching mathematics (Mereku, 2015).

Mereku contends that in Ghanaian basic schools, educators are not doing enough to support pupils' growth as problem solvers. Teachers must include the fundamental principles of teaching via problem solving into Polya's model in order to enhance the strategies. Using this approach, math teachers can deliver more engaging math instruction. The twentieth-century mathematician George Polya (1887–1985) was credited with bringing classical analysis to problem solving. By answering issues, students should expand, deepen, and enhance their learning using this approach (Hieber, 2003). Students gain confidence and master mathematical concepts as a result of this.

Teaching through problem solving, according to Cai and Lester (2012), fosters conceptual comprehension, increases reasoning and mathematical communication skills, and piques students' enthusiasm and curiosity in the subject of mathematics. Polya's paradigm views problem-solving as an activity that requires a range of talents. The goal of problem-solving is to be a learnt and applied practical art. It has to do with the child's capacity to adapt prior information to novel circumstances (Polya, 1945).

2.13 Teachers engaging students in Problem-Solving at the senior high schools' level

Action is the foundation of student participation, which is acknowledged as a complicated and multidimensional concept (Bond et al., 2020; Kahu, 2013; Reschly & Christenson, 2012). It represents the time and effort that students devote to their education when seen within the context of a course (Borup et al., 2020; Fredricks et al., 2016; Skinner & Pitzer, 2012). A definition proposed by Fredricks et al. (2004) based on a qualitative literature analysis is cited by numerous authors (Bond et al., 2020; Christenson et al., 2012; Fredricks et al., 2019; Kahu, 2013; Lawson & Lawson, 2013; Manwaring et al., 2017; Schindler et al., 2017). This definition is frequently used to characterise the phenomenon as a multidimensional psychosocial process. These writers define the three connected characteristics of student engagement behavioral, emotional, and cognitive.

In a course, behavioural engagement among students is defined as participation in assignments and following rules or expectations. Consequently, emotional reactions to classmates, the instructor, and activities as well as a sense of community within the course are all seen as components of a student's emotional engagement. Finally, learning or metacognitive strategies used by students and their mental commitment to tasks meant to help them acquire complex information are measures of their cognitive engagement. According to Christenson et al. (2012), in their overview of student engagement (p. v), "Engaged students do more than attend or perform academically; they also put forth effort, persist, self-regulate their behaviour towards goals, challenge themselves to exceed,".

Because pedagogy is changeable, student engagement may be impacted by instructors' methods, or the things they do to encourage participation from their students in their lectures (Fredricks et al., 2004, 2019; Kahu, 2013; Lawson & Lawson, 2013). Studies on teachers' actions and motivations in problem-solving settings are scarce, nevertheless (Taylor et al., 2019; Torrisi-Steele & Drew, 2013; Smith & Hill, 2019). Very few studies (Halverson & Graham, 2019; Halverson et al., 2014; Jeffrey et al., 2014; Manwaring et al., 2017; Siemens et al., 2015; Miller et al., 2018) examine how educators foster student participation. For example, while research indicates that synchronous and asynchronous BL modes should be carefully combined to maximise student engagement (Garrison & Vaughan, 2008; McGee & Reis, 2012), a number of authors noted that there aren't many specific suggestions available in this area (Graham et al., 2014; Manwaring et al., 2017; Siemens et al., 2015; Taylor et al., 2018). The limited body of research on instructors' methods for encouraging student participation in big learning (BL) covers the topic in a variety of ways, from general tactics to activities to particular digital technology apps.

The primary focus of the literature review that follows is educators' perspectives on strategies for promoting student involvement in BL. With the exception of course case studies, Vaughan (2014) examined the use of online collaborative learning tools to enhance student achievement and engagement in regular BL courses for first-year undergraduates. Using a mixed technique that involved eight instructor interviews, the author came to the conclusion that these kinds of applications raised student interest.

However, he recommended that future studies look into whether using digital tools directly increases student participation in BL or if they act as a mediator for a broader tactic like active and collaborative learning. Through teachers' narratives (n = 3), Montgomery et al. (2015) also looked at how digital technology promoted student engagement in three traditional BL courses for students studying education.

Teachers stated that in order to promote student engagement, a variety of resources (such as texts and videos) were used to engage students asynchronously online at initially. During the ensuing synchronous sessions, experiential learning was occasionally used to maintain student interest through active learning. Subsequently, digital materials and individual or group initiatives aimed at expanding comprehension of the subject matter were used to bolster student participation online. Lastly, in order to increase student engagement, the writers emphasised the significance of student-content interactions. Although a number of teachers' tactics for encouraging student participation in problem solving were mentioned in this paper, there was only one course offered, and there was no comprehensive study of the strategies that the three courses shared. These factors could be construed as methodological limitations.

2.14 Summary of Literature Review

The varied literature examined touched on instructors' perceptions of problem-solving instruction and instruction through problem solving as well as their understanding of teaching problem solving. The focus of the various writers was on teachers' understanding of mathematics instruction and their ability to teach particular mathematical concepts. Very little precise information has been provided by research on mathematics teachers' perceptions and use of problem-solving as a pedagogical tool. Teachers' expertise of the problem-solving teaching technique has not been extensively covered in the literature. The literature evaluation fell short of addressing discussions and concerns regarding teachers' perceptions and use of problem-solving as a pedagogical tool, particularly in senior high school. This study indicates Senior High School mathematics teachers' perceptions and use of problem-solving as a pedagogical tool.

CHAPTER THREE

RESEARCH METHODS

In this chapter the methods that were used to explore the research problem are provided. Specifically, the research design, the population, sample and sampling procedures that were used are provided. Also presented in this chapter are the study area and participants, instruments of data collection procedures, validity and reliability of instruments and instruments of data analysis". The chapter ends with the relationship between research questions and data analysis procedure

3.0 The research design

The mixed methods explanatory research design was used in investigating senior high school mathematics teachers' perception and use of problem solving as a pedagogical tool. The design was used to collect both quantitative and qualitative data from a cross-section of senior high school mathematics teachers (Creswell, 2014). This helped the researcher to address the research problem in chapter one. Literature suggests that pragmatist philosophies drive mixed methods designs. Pragmatism research philosophy accepts concepts to be relevant only if they support action. Pragmatists recognise that there are many different ways of interpreting the world and undertaking research, that no single point of view can ever give the entire picture and that there may be multiple realities. According to pragmatism research philosophy, research question is the most important determinant of the research philosophy. Pragmatics can combine both, positivist and interpretivism positions within the scope of a single research according to the nature of the research question (Subedi, 2016).

The pragmatic paradigm was deemed appropriate for this study because the research problem and questions informed the methodology for this research study (Davis, 2010). The mixed methods research design uses the pragmatist philosophies to explain inferences, interpretations, research questions, hypotheses, and experiments. These were followed by qualitative transcripts and experiences of the models (Naidu, 2013; Subedi, 2016).

In addition, the pragmatist philosophies of mixed methods research design systematically, scientifically, and logically helped to carry out the hypotheses, research questions, data collection, data analysis, deductions, and inferences (Creswell, 2012). The pragmatist philosophies allow for flexible approaches to solving practical problems regardless of their objective truths or subjective perceptions (Harwell, 2011). It is the best way of exploring data whose respondents are novices in the area of concern and lack the requisite knowledge and skills to provide complete and comprehensive data (Subedi, 2016).

Furthermore, according to Cohen, Manion, and Morrison (2011), the main purposes of data mixing are triangulation, complementarity, development, initiation, and expansion. In this study, triangulation helped to control threats, complementarity helped to assess similarities and differences, a development helped to refine instruments and expansion helped to add richness to findings. And even though any of the four main strategies of data-mixing, namely transformation, topology development, extreme case analysis, consolidation, and merging were suitable. More specifically, the researcher employed the sequential explanatory mixed methods research design to explore the senior high mathematics teachers' perception and use of problem-solving as a pedagogical tool. A mixed methods sequential explanatory design is one that employs quantitative data collection in the first phase. The purpose of the first phase is to explore the statistical significance of quantitative variables. After analysis of the quantitative results, the researcher employs qualitative data collection and analysis. The purpose of the second phase was to enhance, complement, confirm, and corroborate the quantitative results (Creswell, 2014). In the first phase, the researcher collected the quantitative data through a questionnaire. In the second phase, an interview guide was used by the researcher to collect the qualitative data.

Figure 3 shows the mixed methods research design that was used to carry out the study. The model was used for the questionnaire, interview, and observation of the study.

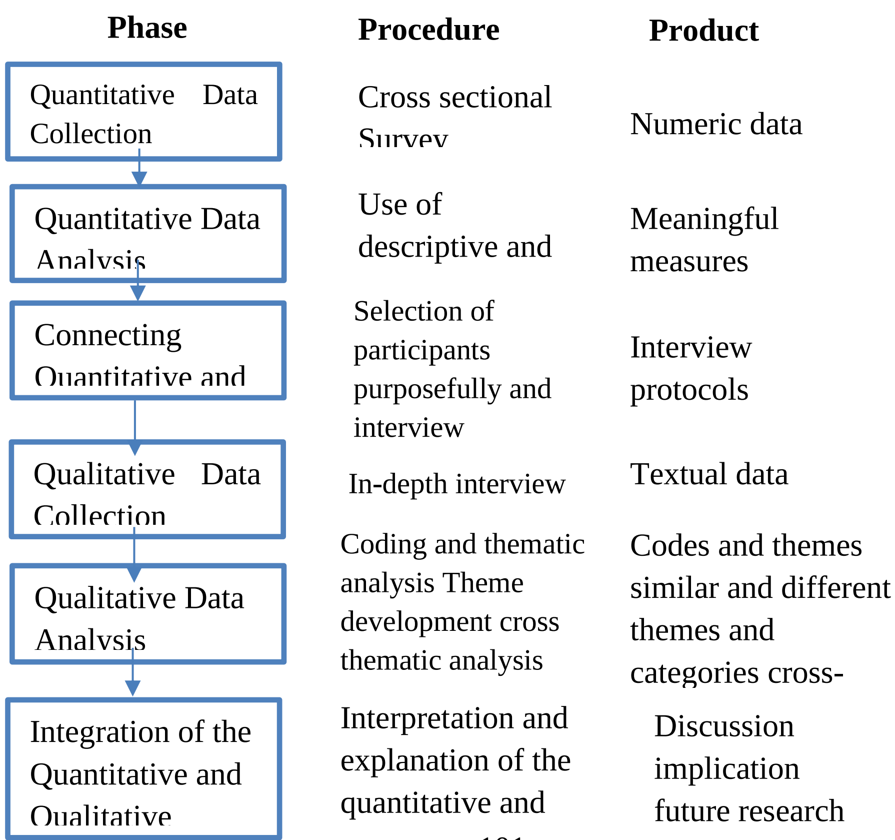

Figure 3: Explanatory Mixed Method Design

The Study area

3.1 Description of Upper West Region

This study was conducted in the Upper West region of Ghana. The region shares international boundaries with two countries, Burkina Faso to the north and Ivory Coast to the west. It has a population of 901,502 (Ghana Statistical Service, 2020). The region covers a geographical area of 18,476 sq km, constituting 12.7% of the total land area of Ghana. There are eleven districts in this region: Wa Municipal; Wa West; Wa East; Dafiama-Busie-Issah; Nadowli-Kaleo; Lawra Municipal; Jerica Municipal; Nano; Lambusie-Kani; Sissala-West and Sissala-East Municipality. These districts are made up of rural and urban communities and are mostly considered to be deprived areas. The majority of the people in the region are peasant farmers and traders. Each district has its own unique challenges such as dilapidated school blocks, lack of infrastructure, and inadequate teaching and learning materials when it comes to education. By the national boundaries, it is located to the west of the Upper East and to the south of the Savana Region. According to the Ghana Statistical Service (2012), Upper West Region is classified as one of the poorest regions in Ghana. Data available from the Ministry of Education in 2022 shows that there were about 39 senior high schools in the Upper West Region.

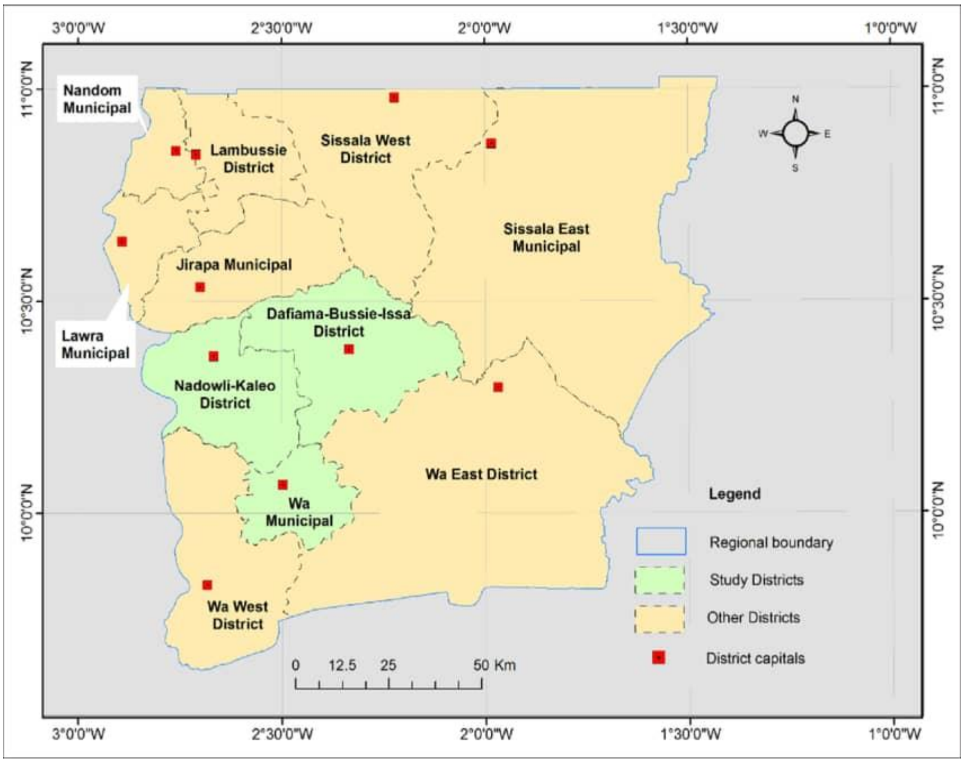

Figure 4: Map of Upper West

3.2 The population of the Study

All mathematics teachers at public senior high schools in a region in the northern part of the country made up the study's population. The Ghana Education Service (GES, 2023) statistics indicate that there exist thirty-nine (39) public senior high schools within the region. There are eleven (11) Districts, Municipal and Metropolitan Assemblies where these schools in the region are found. The population of mathematics teachers within the region is estimated to be 500. Analysis of students' performance in West African Senior School Certificate Examinations (WASSCE) shows that the 'Upper West performed poorly as compared to other regions in Ghana (MoE, 2022). There is therefore the need to explore better ways of teaching Mathematics in schools in the region in order to make the subject very meaningful and attractive to learners. The population for this study, therefore, consisted of all

500 senior high mathematics teachers (both core and elective) in the Upper West region of Ghana.

3.3 Sample and Sampling Procedure

The sample and the sampling procedures adopted for this research were multi-stage sampling procedures. First, purposive sampling technique was used to select the region, the simple random sampling procedure was used in selecting 35 senior high schools within the region. According to Van Dalen (1979), at least ten (10) to fifteen (15) per cent of the target population is a good sample for a study. Therefore 89.7 per cent of the 39 schools were used for the study. The names of the various schools were recorded down on paper and then sequentially drawn with replacement (table of random numbers). In all, 151 mathematics teachers from the sampled schools within the region participated in the study.

The researcher chose the teachers at random to represent the Grade A, Grade B, Grade C, and Grade D senior high schools using simple random sampling (table of random numbers) techniques. With approval from the regional education directorate, heads of school, heads of mathematics departments, and mathematics teachers, the participating schools and teachers were chosen overall. This made it easier for the researcher to understand how senior high school mathematics teachers view and employ problem solving as a teaching technique.

Once again, mathematical formula tables were used to calculate the sample size. Walliman (2011) demonstrates in the mathematical table that the proportions drawn are bigger for smaller populations. Therefore, the sample size grows at a decreasing pace with population growth and stays steady at little more than 380 cases. As a result, studies with 100 participants need between 80 and 100 percent, whereas studies with 1,200 participants only need a sample of 25 percent.

The sampling techniques that were used to determine the sample size, were probability (simple random sampling) and non-probability (purposive sampling). Probability sampling ensures non zero chance of selecting 151 teachers, while non-probability sampling restricts participation in interviewing and observation procedures.

3.4 Sampling Procedures for the Interview and Observation Participants

Purposive sampling technique was used to select participants from those who indicated that they use problem-solving as a pedagogical tool for both the observation and interview. Purposive sampling, also known as judgmental, selective, or subjective sampling, is a non-probability sampling technique where the researcher selects participants based on their knowledge, experience, or judgment about which individuals or groups will be most useful or representative for the research study. This method is often used in qualitative research where specific criteria are required to meet the research objectives. Subjective Selection (Participants are chosen based on specific characteristics or criteria defined by the researcher), Specific Purpose (The sampling targets a particular purpose or need in the research), Expert Judgment (The researcher relies on their expertise to decide who will provide the most relevant and rich data) and Diversity and Depth (It aims to capture a wide range of perspectives within the selected criteria to gain a deep understanding of the phenomenon)

3.4.1 Instruments

The instruments used in gathering data for this study were questionnaires, interview guide, and observation guide. The questionnaires and interviews were used to elicit teachers’ perceptions, knowledge base, and the procedures adopted by teachers when using the problem-solving approach to instruction. The observation checklist was used to observe teachers during their mathematical instruction to find out how they use the problem-solving instructional approach. (See Appendices).

3.4.2 Questionnaire

The questionnaire was divided into four parts. It was structured into both open and close-ended questions. The first part inquired about the biographical data of the respondents. The second part measured mathematics teachers’ perceptions of the use of the problem-solving instructional approach. The questions were close-ended questions on a five-point Likert scale. Open-ended questions were posed in the third part to assess the knowledge of the mathematics teachers in the utilization of the problem-solving approach to instruction. The last part of the questionnaire elicited information on mathematics teachers' practices and activities when using problem-solving instructional approaches in the classroom. The questions in part four had four-point Likert scale questions which reflected Van De Walle’s steps and activities for instruction through problem-solving. (See Appendix D).

3.4.3 Interviews guide

The interview items were open-ended, value- and opinion-driven, construct-forming, and interpretive. But the primary goal was to get responses that were close to being transparent, easy to understand, profound, and sincere. To fit the research problem and objectives, the research questions and hypothesis, the instructors' perspectives, and their use of problem solving as a teaching technique, the replies were paraphrased and transcribed. Such open items were contained in Appendix E

Once more, the interview questions varied from formal to semi-formal to wholly casual, so the researcher assumed a supporting role and let the teachers handle the conversations and debates among themselves. The specific attributes of the interview guide are listed by Creswell (2012) as the "life world to teachers' lives," Relevant to the topic at hand, qualitative in terms of the knowledge and experiences of the respondents, candid in the way that the interview items were described, detailed in terms of the actions and viewpoints of the necessary things, and narrowly focused on certain themes. The in-person interviews were costly and time-consuming, but they were helpful in gathering more detailed information about how math teachers view and use problem-solving as a teaching tool, how these views and uses relate to one another, what influences math teachers' perceptions of problem-solving, and what types of problem-solving techniques math teachers employ in the classroom. Additionally, they made sense when it came to instructors who had specific needs (Creswell, 2014).

The interviewers/observers for this study were experienced professionals with 6 years of relevant experience or specific qualifications in educational research, mathematics education, or pedagogy. Each interviewer/observer underwent training or briefing session details to ensure consistency and reliability in data collection. Their expertise included familiarity with problem-solving pedagogy, qualitative/quantitative research methods, etc., which allowed them to effectively analyze and interpret teachers' perceptions and practices.

3.4.4 Observations guide

Van de Walle's guide to teaching through problem-solving was used in drawing the lesson observation checklist to observe how the teachers taught mathematics through problem-solving. The observation checklist was a Likert scale-type checklist that consisted of four scales. Namely; not observed, ineffective, somewhat effective, and effective. The observation checklist reflected the Van De Walle scale for Problem-solving Instructional Strategy. The model divides the problem-solving approach to instruction into three main stages; the Before Problem Solving stage, During Problem-Solving, and After Problem-Solving activities. Each of these three stages is further divided into either 4 or 5 subdivisions depending on the activities expected to be carried out at a particular phase they include; activating prior knowledge, Ensuring Understanding of the Problem, Establishing Clear Expectations, Providing Appropriate Support, Providing worthwhile Extension, Promotion of community of learners, listening without evaluation and Summarizing the main idea. Summary (See Appendix F).

3.4.5 Validity and Reliability

Though the questionnaire was adapted, it was still crucial for a professional to determine whether the instrument measured what it was intended to measure (Polit & Beck, 2012). To ensure the instruments elicited valid responses, they were given to my supervisor who is a professor and an expert in the field of problem-solving instruction, for inspection and judgment. In order to ascertain the validity of the content and its face validity, he assessed the quality of the questionnaire. The quality of data emanates from the quality of the instrument used in obtaining the data. According to Mohajan (2017), to ensure the quality of research instruments, it is necessary to conduct a validity test. In order to confirm that the instruments were accurate and dependable, a pilot test was carried out in two pilot districts different from the district in which the main study was carried out. These pilot districts were selected because schools within these districts have common characteristics with the districts where the main study was carried out. The piloting allowed the researcher to modify and restructure the instruments for the collection of good data. The researcher administered questionnaires and also interviewed a sample of teachers from the two selected districts. The responses of the respondents in the pilot test helped to refine and check the suitability of both the questionnaires and the semi-structured interview scheme.

According to Shweta, Bajpai, and Chaturvedi (2015), this reliability value provides sufficient statistical evidence of consistency and approval for use. Abubakar, Wimmer, Bereznicki, Dwan, Black, and Bezabhe, (2020) also were of the view that a reliability value of 0.70 is acceptable evidence that the results obtained from the two instruments are comparative and hence measure similar constructs. Nahid (2003) also remarks that reliability in research can only be a matter of degree since it is not possible to get a perfect score. To increase the validity of the instruments and make them appropriate for this study, based on the comments from my supervisor and the results from the pilot test the necessary suggestions, and modifications were made. The questionnaire, the interview guide and the observation guide proved reliable for gathering data, as it achieved Cronbach's alpha reliability coefficients of 0.86, 0.75 and 0.80 respectively.

3.5 Data Collection Procedure

Prior to the collection of data on the field, I sought ethical clearance. After Institutional Review Board (IRB) UCC granted the ethical clearance, an introductory letter from the department of mathematics and ICT education was obtained and submitted to the schools where data was collected. The letters were submitted to the heads and the school's administration in an effort to get their approval to carry out the study at that institution. After permission was granted, the researcher met the heads of the mathematics department in those schools and discussed with them the nature of the project and how data collection will be done. The researcher then met and explained to teachers the nature of the project and the need for them to participate. The researcher administered the questionnaires. In schools where not all sampled teachers were available during the visit of the researcher, the head of the mathematics department did the administration of the questionnaire at the appropriate times.

Data from the questionnaires were gathered over a three weeks period, and data from interviews and observations were also gathered over the course of another two weeks. Each interview session took between 30 and 40 minutes to complete. The interviews were successful with the help of a semi-structured interview guide. Seven teachers in total were interviewed for this study. The observation was also done using an observation checklist. Each of the 7 teachers was observed for one period (50 minutes to 1 hour) in their classroom. All teachers participating in this study answered a questionnaire, however, a sample of teachers who indicate "yes" for use of the problem-solving approach to instruction were interviewed and observed in their classrooms at their various schools. The questionnaires were administered first, after which the observation and the interview were carried out. The observation and interview were done after the collection and analysis of the data from the questionnaire. The observation and interview were administered on the same day.

3.6 Ethical Issues

This study was carried out taking into consideration all provisions of accepted ethical regulations on the conduct of academic research. Ethical clearance was sought from the Institutional Review Board (IRB), UCC. All the necessary ethical protocols were observed during and after the conduct of the study. Data collected are secured to avoid third-party access or use. All information relating to the subjects are under lock. All soft copies of data are secured on a laptop under a password. Permission was sought from the school management authority and teachers before the conduct of the study. Participants were at liberty to withdraw from the study at any given time as and when they became disinterested in participating in the study. Before administering the questionnaire, a consent form was given to each of the teachers for them to give their consent to participate in the study. Respondents were assured of their anonymity in participating in the study. All authors cited were also duly referenced (see references)

3.7 Data analysis Procedure

Data collected from the questionnaires and the interviews were analyzed separately and the results were put together for final discussion. First, the quantitative data which involved the questionnaires consisting of five-point Likert scale-type questions were analyzed. The scales on the questionnaire range from Strongly Disagree as the lowest to strongly agree as the highest. Positively framed items on the questionnaires were coded from the lowest to the highest as follows; Strongly Disagree -1, Disagree - 2, Uncertain-3, Agree – 4, and Strongly Agree - 5. Negatively framed items on the questionnaires were coded from the lowest to the highest as follows; Strongly Disagree - 5, Disagree - 4, Uncertain -3, Agree – 2, and Strongly Agree – 1.

In addition, with the open-ended items, the various salient and general ideas that emerged from the responses given were put into themes and then coded. Data from the questionnaires were also inputted into SPSS and analyzed. Descriptive and inferential statistical tools were used to analyze the data obtained from the closed-ended questions. Percentages, frequencies, means, and standard deviation were the tools utilized to present the results for the descriptive statistics. The tools used to present the inferential statistics results were Pearson product correlation and the Wilcoxon sign rank test. The data from the interview and observation were also analyzed using thematic coding processes and methods. The responses from the interviews were transcribed. The common ideas that emerged were used as the themes for the various topics covered in the interview.

Table 1: Relationship between research questions and data analysis procedure

Research Question	Data Source	Data Analysis Strategies
RQ1: The perception Senior High School teachers hold about problem-solving	Questionnaire Interview	- Previous Codes: unstructured, practical, and cooperative - Code of description/emergency: cross-disciplinary
RQ2: Factors that influence SHS teachers' perception about problem-solving	Questionnaire Interview	- Priorit codes include critical thinking, creativity, cooperation, and communication. - describing/emergent code: interdisciplinary

Table 1 Continued

Research Question	Data Source	Data Analysis Strategies
RQ3: Mathematics teachers use of problem-solving as a pedagogical tool in the classroom	Questionnaire Interview observation	- Previous Codes: instructor-driven, facilitation - Code of description/emergency: cross-disciplinary
RQ4: Teachers' problem-solving practices	Questionnaire Interview observation	- Previous Codes: instructor-driven, facilitation - Code of description/emergency: cross-disciplinary Previous Codes: instructor-driven, facilitation
RQ5: the relationship between mathematics teachers' perception of problem-solving and its use as a pedagogical tool	Questionnaire Interview	- Code of description/emergency: cross-disciplinary

CHAPTER FOUR

RESULTS AND DISCUSSION

4.0 Overview

The study's results are reported and discussed in this chapter. The results are presented based on the research questions that were put forth to guide the research. The presentation of the results and the findings from this study are organized under the headings; respondents' biographical data, teachers' perception, teachers' knowledge and practices of teachers with the use of the problem-solving instructional approach. Also presented in this chapter is the discussion of the results.

4.1 Respondents Biographical Data

In this study, the biographical information of the respondents was collected through the data collection. These included; sex, age, academic background, professional background, and years of teaching. Table 1 shows the summary of results of biographical information of the teachers.

Table 1: Respondents' Biographical Data

Item	Categories	Frequency	Percentage
Sex	Male	133	88.1
	Female	18	11.9
	20-29 year	22	14.6
Age	30-39 years	93	61.6
	40-49 years	30	19.9
	50-59 years	6	3.9

	BEd. Mathematics	80	53.0
	BSc. Mathematics	38	25.2
Academic Qualification	MEd. Mathematics	8	5.3
	MPhil. Mathematics	5	3.3
	MSc. Mathematics	4	2.6
	Other	16	10.6
Training Background	Professional	144	95.4
	Non- Professional	7	4.6
Years of Teaching	Less than 4 years	9	6.0
	4-8 years	43	28.5
	9-12 years	56	37.1
	13-16 years	16	10.5
	Above 16 years	27	17.9

Source: Field Data, 2024

Findings from Table 1 indicate that the senior high schools that took part in this study had more male mathematics teachers than female mathematics teachers. The majority (76.2%) of the teachers were below the age of 40. This suggests that the majority of the mathematics teachers were youthful. This also implies that any rolled-out programme that seeks to enhance mathematics teachers' methods of teaching mathematics, especially using problem solving, as a pedagogical tool will be useful for the schools since these teachers have more years to apply those strategies and to gain experiences from them. On mathematics teachers' academic qualifications, 90 (59.6%) of the respondents had a background in mathematics education (B.Ed. Mathematics, M.Ed. Mathematics, MPhil. Mathematics education). This implies that the majority of the senior high school mathematics teachers have had training on the methods and strategies for mathematics instruction. Few of these mathematics teachers do not have a professional background.

However, there are still mathematics teachers in senior high schools whose academic and professional qualifications are not mathematics-related. This implies that some mathematics teachers teach mathematics without any training on how mathematics should be taught. The data on teachers' teaching experience shows that 142(94.04%) respondents had more than 4 years of experience in teaching at the senior high school level. This suggests that more than half of the mathematics teachers in the senior high schools have gained enough knowledge of the various teaching strategies including teaching via problem-solving. It also suggests that many of the more experienced mathematics teachers might have left from the senior high school level.

4.2 Research Question 1: What perceptions do Senior High School mathematics teachers hold about problem solving?

This question sought to examine how teachers perceive the use of problem solving as a pedagogical tool in mathematics instruction. Questions, relating to perception were posed to respondents for them to indicate whether they disagree, agree, or are neutral about the statements. A five-point Likert scale which had Strongly Disagree (SD = 1), Disagree (D = 2), Uncertain (U= 3), Agree (A = 4), and Strongly Agree (SA = 5) were used. Any of the variables that obtain a mean score of 3.5 or more is regarded as a positive perception of the statement but a score of 2.4 or less is regarded as a negative perception. A score between 2.5 and 3.5 is an indication that those teachers are inclined to a positive perception. To find out about teachers' general perception of the use of the problem-solving approach to instruction, an open- ended question was also posed. (See appendix A). Table 2 shows a summary of teachers' response.

Table 2: Teachers' Perceptions on the use of Problem Solving as a Pedagogical tool

S/N	Statements	SD (%)	D (%)	U (%)	A (%)	SA (%)	Mean	Std.D
1	I am motivated to plan lessons using the problem-solving approach in mathematics	5(3.3)	13(8.6)	7(4.6)	104(68.9)	22(14.6)	3.83	0.90
2	I need enough space, resources, and a conducive environment in the class to use the Problem-solving strategy	2(1.3)	10(6.6)	1(0.7)	74(49.0)	64(42.4)	4.25	0.87
3	I find the problem-solving approach to instruction more supportive of learners of all abilities	5(3.3)	4(2.6)	7(4.6)	81(53.6)	54(35.8)	4.16	0.88
4	It is more difficult to satisfy slow and weak learners when teaching through problem solving	26(17.2)	72(47.7)	11(7.3)	33(21.9)	9(6.0)	3.48	1.18
5	I think the problem-solving approach is useful for mathematical instruction	1(0.7)	2(1.3)	2(1.3)	83(55.0)	63(41.7)	4.36	0.65
6	The problem-solving approach is not suitable for teaching children in developing countries like Ghana even if there is enough time	6(4.0)	14(9.3	7(4.6)	79(52.3)	45(29.8)	3.95	1.04
7	The use of problem-solving method is impossible with large classes	39(25.8)	70(46.4)	10(6.6)	27(17.9)	5(3.3)	3.74	1.13
8	The problem-solving approach develops students' confidence and critical thinking abilities	1(0.7)	3(2.0)	60(39.7)	0(0.0)	87(57.6)	4.52	0.67
9	The use of problem-solving approach in teaching mathematics is time wasting	77(51.0)	72(47.7)	2(1.3)	0(0.0)	0(0.0)	4.50	0.53
10	The use of problem-solving approach will not help teachers to cover all the topics in the syllabus	69(45.7)	81(53.6)	1(0.7)	0(0.0)	0(0.0)	4.45	0.51
	Overall						**4.12**	

The findings in Table 2 suggest that mathematics teachers had positive perceptions regarding the problem-solving approach to teaching and how it is used in the classroom (Overall Mean 4.12 and Standard Deviation 0.6). This overall mean of almost 4.0 denotes strong support and a favorable opinion of the employment of problem-solving techniques in teaching mathematics in the classroom. From the Table 2, it can be seen that items like the problem-solving approach develop students' confidence and critical thinking abilities and problem solving is useful for mathematics instruction, over 90% of the respondents agreeing with those statements. Again, items such as I am motivated to plan lessons using problem-solving approach" and "I find problem-solving approach to instruction more supportive of students of all abilities had more than 70% of the mathematics teachers responding positively to them. None of the items had less than 60% of teachers supporting it. This evidence suggests that mathematics teachers have a very positive perception of the problem-solving approach to instruction.

Teachers were also asked to give their perceptions about the use of the problem solving as a pedagogical tool. Their responses were coded and summarized into themes. Table 3 shows the results from the data

Table 3: Teachers' perception about Problem Solving as a pedagogical tool to Instruction

Question	Theme	Frequency	Percent	Examples of teachers Responses
What is your view about the problem-solving approach to mathematics instruction	Best instructional Approach for Math	39	28.7	1. 1. It is the most effective way to teach mathematics to improve comprehension. 2. It is the most effective method for teaching mathematics. 3. The most effective method for teaching mathematics is the problem-solving technique. However, it is less advantageous to employ in our classroom due to the quantity of topics and type of the evaluation.
	Students' critical thinking, confidence and understanding	43	31.6	1. In my opinion, teaching mathematics using problem-solving methods helps pupils become more self-assured and critical thinkers. 2. This method of instruction must be applied when teaching mathematics since it fosters students' creativity and critical thinking. 3. Students gain confidence in the subject and comprehend both the theoretical and practical aspects of it.
	Time factor, Class size, Resources	16	11.8	1. Although problem-solving instruction is a highly recommended method of teaching mathematics, it takes time, particularly in big classes and with students who struggle. 2. Although it is a fantastic strategy, using it is challenging due to time constraints, a lack of resources, and large class sizes. 3. If there had been more time allotted for mathematical instruction, the problem-solving approach would have been the most effective way to teach mathematics.

Table 3:Cont.

Teachers Knowledge and Skills	2	1.5	1. It is very good to have adequate knowledge in problem solving approach to teach mathematics at the SHS level
Others	30	22.1	The problem- s o l v i n g approach is pupil centered in mathematical instruction unlike the traditional approach, which is teacher, centered. It is the ability to solve problems in an effective and timely manner without any impediments. the problem-solving approach to mathematical instruction helps both the teacher and students to do more research and also know the purpose of they are studying

The results from Table 3 indicate that the teachers' responses did not deviate from their responses in the likert scale questions. The teachers' responses indicated that most mathematics teachers appreciate the effectiveness of the quality of the use of problem solving as a pedagogical tool on students' achievements. From Table 3, over 60% of the teachers expressed that using problem solving, as a pedagogical tool is the best approach that can help develop students' critical thinking, confidence and understanding in mathematics. This indicates that a large number of senior high school mathematics teachers are familiar with the problem-solving approach and understanding its benefits.

4.3 Research Question 2: What is mathematics teachers' knowledge on the use of Problem Solving as a pedagogical tool?

This question sought to find out about teachers' knowledge on the use of Problem Solving as a pedagogical tool (PSPT). Four open-ended questions were posed to teachers for them to express their knowledge on the various aspect of problem-solving instruction. Teachers were asked if they knew about the use of problem solving as a pedagogical tool. The summary of teachers' response is indicated in Table 4.

Table 4: Teachers who know or not the use of Problem Solving as a pedagogical tool

Item	Knowledge of PSPT	Frequency	Percent (%)
Do you know about the use of problem Solving as a pedagogical tool	YES	137	90.7
	NO	14	9.3

The results in Table 4 indicates that majority of senior high school mathematics teachers had knowledge about the problem-solving approach. This suggests that mathematics teachers had read, studied, or implemented the problem-solving approach to teaching in some capacity during their classes Respondents were then asked; what it means to teach through problem solving. The responses obtained from teachers were analysed and summarised. The themes that emerged from the various responses are summarized in Table 5.

Table 5: Teachers responses on what it meant to teach through Problem Solving

Item	Response Types	Frequency	Percent	Examples of Teachers Responses
What does it mean to teach through Problem Solving?	Solving Practical or Real-life Problems	34	24.8	1. From what I understand, it refers to instructing using real world and practical examples. *(Teacher 21)* 2. Teaching through problem solving involves using previously learned information to address real-world issues that have mathematical solutions *(Teacher 31).*
	Using strategies	16	11.7	1. Teaching through problem solving is a method in which the instructor helps students use techniques like role playing, heuristics, and brainstorming to understand mathematical concepts. *(Teacher 21)* 2. Using heuristics to solve problems is known as teaching through problem solving. *(Teacher 31)*
	Using intellectually challenging Problems	29	21.2	1. It entails giving students cognitively demanding problems, questions, or projects that encourage mathematical thinking. *(Teacher 21)* 2. The lesson starts with an issue that has to be solved, and students attempt to solve it by using reflective thinking. (*Teacher 31)*
	Discovery Learning	24	17.5	1. Teaching through problem solving entails teaching theory to practice *(Teacher 21).* 2. We impart knowledge by going from the unknown to the known. *(Teacher 31)* 3. Learn about facts and knowledge through guided inquiry *(Teacher 11)*
	Solving word problems	12	8.8	1. In this instance, questions are presented as word statements so that students can read them and use them to help them solve the problem. *(Teacher 21)*

Others	22	16.0	1. The goal of problem-solving instruction is to enable students of all skill levels to provide individualised explanations for their responses to problems *(Teacher 21).* 2. To inspire students to approach challenges in a more logical, realistic, and thought-provoking manner. *(Teacher 31)*

From Table 5 it can be seen that most mathematics teachers said that teaching through problem solving involves helping students solve real-world problems in a practical setting or utilising cognitively demanding mathematics tasks that engage students in deep mathematical thinking to help them learn the subject. Teachers' shared responses provide insight into how they view the use of problem solving as a pedagogical tool. The teachers were further asked about whether or not they use the problem solving as a pedagogical tool for their mathematics lessons. Table 6 shows the responses provided by teachers.

Table 6: Teachers use of problem solving as a pedagogical tool (PSPT)

Item	Response	frequency	Percent
Do you use the problem solving as a pedagogical tool in teaching?	YES	127	84.1
	NO	24	15.9

Table 6 shows that more than half of mathematics teachers said they use the problem solving as a pedagogical tool for their lessons. This makes it clear that not all teachers who know about using problem solving as a pedagogical tool use it in the classroom. After giving an affirmative response to the question regarding using problem solving as a pedagogical tool (PSPT), teachers were asked to list the procedures they use when instructing students through problem solving. The responses from the teachers were examined and contrasted with standard and current problem-solving models of mathematics education, such as Van De Walle's. Table 7 displays their answers. Teachers who responded in the affirmative about using Problem Solving as a pedagogical tool (PSPT) were then asked to write the steps or the processes they use when teaching through problem solving. The answers provided by teachers were analyzed and compared with regular and existing models of mathematics instruction through problem solving like that of Van De Walle's. Their responses are shown in Table 7.

Table 7: Steps for teaching through Problems Solving

Item	Response type	Frequency	Percent	Extract from teachers Responses
Steps teachers follow to apply the problem-solving instructional strategy	Polya's Strategy	51	33.8	I typically use Polya's four-phase framework: Assist pupils in understanding the issue, 2. Create a plan, and 3. Carry out the plan 4. Lastly, we evaluate the process by looking back with the students. *(Teacher 21)*
	Inductive-Deductive Approach	15	9.9	Post an issue on the board invite students to suggest solutions, gather their thoughts, summarise their answers in front of the class, and then create a rule or formula that can be applied to tackle similar or broader problems. *(Teacher 31*
	Problem Solving Approach	21	13.9	We start with a simplified form of the problems, brainstorm, as a class, and I make sure that students comprehend them. I also assist students in connecting the work to others and provide support without taking on the more difficult aspects of the problem. *(Teacher 11)*
	Algebraic Word Problems Strategy	23	15.2	Carefully read the problem, represent the unknown quantities with variables, create a mathematical statement using the variables and the question's conditions, then solve or simplify your expression or statement. *(Teacher 9)*
	Others	41	27.2	Introduction by updating a well-known and relevant topic, connecting the old topic to the new topic by following the steps in answering that specific question, varying the questions and the procedures in the needed solution, and assessing students on the developed concept for assessment. *(Teacher 13)*

The findings in Table 7 reveal that most of the mathematics teachers are familiar with Polya's problem-solving method and believe it to be the approach for teaching mathematics through problem solving. However, only three out of the 127 teachers who indicated they use the problem solving as a pedagogical tool could enumerate most of the steps, which are grounded in literature for teaching mathematics through problem solving. These responses further illustrate the low level of teachers' knowledge about using the problem-solving approach.

4.4 Research Question 3. What factors influence mathematics teachers' use or not use of the Problem Solving as a pedagogical tool?

The purpose of this question was to inquire from the mathematics teachers, the various factors that influence them to use or not use the PSPT. Two open-ended question were posed to ascertain whether teachers have adopted the PSPT or not and why do they adopt or do not adopt the PSPT in their mathematics lessons.

On the question of if, mathematics teachers have adopted the problem solving as a pedagogical tool or not, Table 8 shows a summary of teachers' responses.

Table 8: teachers' adoption of problem solving as a pedagogical tool (PSPT)

Item	Response	Frequency	Percent
Adopt	Adopt	57	37.7
Not Adopt	Do not adopt	94	62.3

The results from Table 8 indicate that, the majority of the teachers (62.3%) said they do not use PSPT in their mathematics lesson delivery. Respondents were asked to explain the reason why they have adopted or have not adopted the PSPT as their teaching approach. The responses that were provided for adopting the PSPT are presented in Table 9.

Table 9: Teachers Reasons for adopting the PSPT

Reasons for Adopting PSPT	Time Factor	Critical Thinking and Confidence	Resources and Class Size	Difficult Method	Active Class	Others
Adopt PSPT	3(5.3%)	21(36.8%)	2(3.5%)	1(1.8%)	11(19.3%)	19(33.3%)

The results in Table 9 revealed that most mathematics teachers who adopt the problem-solving approach to instruction do so due to its benefit on students' mathematical achievements. Surprisingly, from the results, as many as 33.3% of teachers gave reasons which are not exactly derived from the benefit of using the problem-solving instructional approach. Table 10 shows the results on why teachers do not adopt the problem-solving approach.

Table 10: Teachers Reasons for adopting or not the PSPT

Reasons for not Adopting PSPT						
	Time Factor	Critical Thinking and Confidence	Resources and Class Size	Difficult Method	Active Class	Others
Do not Adopt PSIA	41(43.6)	4(4.3%)	15(16.0%)	7(7.4%)	8(8.5%)	19(20.2%)

Results from Table 10 shows that time factor, others and resources are the major deciding factors for most teachers who do not adopt the problem-solving approach.

Table 11: Extract from Teachers Responses on Why they Adopt PSPT

Item	**Response Types**	**Adopt PSPT**	**Examples of Teachers Responses**
		Adopt	
Reasons for Adopt PSPT	Time Factor	3	I manage the time of lesson properly.
	Critical Thinking and Confidence	21	I employ problem-solving instruction because it helps pupils become more self-assured and capable of critical thought. I make the approach more adaptable.
	Resources and Class Size	2	I consider the class size in getting sufficient teaching resources
	Difficult Method	1	It is challenging to operate.
	Active Class	11	I decide to center the lesson on the student.

	Others	19	I employ problem solving because it allows for students' training and growth. For students who perform below average, it is ineffective.

Table 11 presents extract from teachers' responses on the various factors for adopting the PSPT. The reasons assigned by teachers for adopting of the problem-solving as a pedagogical tool from Table 11 suggest that majority of the teachers have very similar reasons that encourages them to adopt problem-solving as a pedagogical tool. Some of these reasons might be personal with most teachers.

Table 12: Extract from Teachers Responses on Why they do not PSPT

Item	**Response Types**	**Do Not Adopt PSPT**	**Examples of Teachers Responses**
		Do Not Adopt	
Reasons for Adopt Not PSPT	Time Factor	41	The problem-solving teaching approach takes a lot of time, so I do not use it.
	Critical Thinking and Confidence	4	It needs proper planning and executing
	Resources and Class Size	15	Since the resources are not available, I do not employ the procedure. If the teacher could make the approach more adaptable and the class size is appropriate with sufficient teaching resources, I could definitely use it.
	Difficult Method	7	It is challenging to operate.
	Active Class	8	I decide to center the lesson on the student.

Others	19	It allow me to cover much as it's so demanding

The reasons assigned by teachers for not adopting the problem-solving approach to instruction from Table 12 suggest that the teachers have similar reasons that discourage them not to adopt the problem-solving as a pedagogical tool.

4.5 Research Question 4: How do mathematics teachers teach through problem-solving?

Research Question 4 sought to establish how Mathematics teachers engaged pupils in Problem-Solving. The response of teachers on the extent of students' engagement in problem-solving during the instructional process is presented in table 12: To accomplish this goal, mean scores were calculated under the following criteria: The mean of less than 2.50 indicated rarely engaged, between 2.50 and3.50 indicated occasionally engaged and above 3.50 indicated always engaged.

Table 13: Descriptive Statistics on students Engagement in using Problem-Solving as a pedagogical tool

Kinds of engagements	Mean	Std. dev.	Levels of engagement
Use of manipulative	3.72	0.51	Always engage
Active engagement of learners	3.45	0.79	Occasionally engage
Teacher as a facilitator	3.19	0.82	Occasionally engage
Building lessons on learners RPK	3.15	0.75	Occasionally engage
Motivation	3.15	0.83	Occasionally engage
Application of knowledge	2.92	0.82	Occasionally engage
Assessment	2.60	1.00	Occasionally engage
Curiosity	2.60	0.90	Occasionally engage
Perseverance	2.19	0.85	Occasionally engage
Total level of engagements	**3.00**	**0.80**	**Rarely engage**

Results in Table 13 showed that generally, the mathematics teachers occasionally (M=3.00, SD=0.80) engaged learners in all the kinds of engagement outlined in the study as problem-solving instructional techniques. They present students with real-world problems that are relevant to their lives or future careers (this can increase their interest and motivation to solve the problems), Encourage group work and collaborative projects (working in teams allows students to share ideas, learn from each other, and develop critical thinking skills), Pose open-ended questions that require more than rote memorization (these questions should challenge students to think critically and creatively) and Utilize interactive tools and technology, such as simulations, online problem-solving platforms, and educational games (these tools can make problem-solving more engaging and interactive).

According to Van de Walle (2007) and Florence (2012), Mathematics manipulatives can help engage students for a longer period by helping them stay focused on particular tasks. Florence feels that while lecture-based teaching might be tedious, manipulatives allow students to participate actively in their learning. The interviewees conveyed their practices on how they engage students in the following ways:

Question Posing and Student Capability

- **Tailored Questioning:** Teachers tailor their questions to match students' capabilities, aiming to pose questions that students can handle and successfully solve.

  "I pose questions to them that I think they can handle, and they deliver" (TR 21), "I give them the questions I think they can handle."

Collaborative Problem Solving

- **Group Work and Collaboration:** Teachers use group work to encourage collaboration among students, helping them share ideas and find solutions together.

  "I help students solve problems by grouping them so they can collaborate and share ideas to find solutions" (TR 31).

Challenges and Student Disengagement

- **Student Dislike and Fear of Mathematics:** Some students have a strong dislike for mathematics, and difficult questions can discourage them from attending school.

"children dislike Mathematics and at times difficult questions might scare them from even coming to school and as you know, our system too" (TR 11).

Encouraging Procedural Learning

- **Following Procedures:** Teachers emphasize the importance of following specific procedures to solve problems, encouraging students to mimic the teacher's methods.

"I give them questions and I encourage them to follow the procedures I used to solve it," "I urge them to use good reasoning even if it is challenging."

Independent Problem Solving

- **Autonomy in Learning:** Teachers encourage students to attempt problem-solving on their own, fostering independence and critical thinking skills.

"By allowing pupils to attempt problem-solving questions on their own" (TR 15).

Balancing Challenge and Support

- **Encouraging Reasoning:** Teachers motivate students to use good reasoning, even when the problems are challenging, supporting them through the difficulty.

"I urge them to use good reasoning even if it is challenging, and occasionally some students will figure it out" (TR 22).

The thematic analysis highlights the strategies teachers use to engage students in mathematics, balancing the need to challenge students with the support required to prevent disengagement. Key approaches include tailoring questions to student capabilities, encouraging collaboration, emphasizing procedural learning, and fostering independent problem-solving skills. The challenges related to student dislike and fear of mathematics are also acknowledged, indicating the need for thoughtful and supportive teaching practices (Florence, 2012).

4.6 Research Question 5: What kind of problem-solving practices are utilized by Senior High School mathematics teachers in teaching?

This question sought to find out how teachers apply the PSPT in their mathematics lessons based on the knowledge they possess on the teaching approach. To answer this question, close ended consisting of 22 items which reflected the Van De Walle scale for PSPT was administered to find out how teachers use the PSPT in their classrooms. Teachers were also observed in their classroom, using an observation checklist, which is also in the domain of the Van De Walle scale for PSIA. This was done in order to ascertain how teachers' knowledge and perception of the PSIA reflect their actual classroom practices. The Van De Walle model and guide for teaching through problem was structured into a 4-point likert scale (1=SD = Strongly Disagree, 2= D = Disagree, 3= A = Agree and 4= SA= Strongly Agree) for the teachers who indicated they teach through problem solving. The model divides the problem-solving approach to instruction into three main stages; the Before Problem Solving stage, During Problem Solving and After Problem Solving activities. These three stages are further divided into; activating prior knowledge, Ensuring Understanding of Problem, Establishing Clear Expectation, Providing Appropriate Support, providing worthwhile Extension, Promotion of community of learners, listening without evaluation and Summarizing main idea. Summary of teachers reported practices on their use of the PSIA in their classroom is presented in Table 14.

Table 14: Teachers responses on their Problem-Solving Instructional Practices (PSIP)

Item	Activity	SD (%)	D (%)	A (%)	SA (%)	Mean	SD
Activating prior knowledge	The teacher starts lesson with an easy version of the task	1(0.7)	5(3.3)	90(59.6)	55(36.4)	3.32	0.570
	The teacher brainstorms with students' various approaches or solution strategies	0(0.0)	4(2.6)	94(62.3)	53(35.1)	3.32	0.523
	The tasks provided builds on students' prior knowledge of mathematics	0(0.0)	0(0.0)	78(51.7)	72(47.7)	3.74	3.255
Ensuring understanding of the problem	Teacher ensures learners understood the problem before setting them to work	0(0.0)	7(4.6)	77(51.0)	67(44.4)	3.40	0.578
	Help students to relate the task with other problems solved in the past	0(0.0)	3(2.0)	92(60.9)	56(37.1)	3.35	0.519
Establishing clear expectations	Teacher tell students whether they are working in small groups or individually on the task given	0(0.0)	4(2.6)	89(58.9)	58(38.4)	3.36	0.533

	Teacher establishes with students how their solutions and reasoning will be shared	0(0.0)	17(11.3)	86(57.0)	47(31.1)	3.20	0.624
Letting go	The teacher demonstrates confidence and respect for students' abilities by allowing them to work	0(0.0)	1(0.7)	81(53.6)	69(45.7)	3.45	0.512
Noticing students mathematical thinking	Listens to students attentively and motivates them intrinsically	0(0.0)	1(0.7)	74(49.0)	76(50.3)	3.50	0.515
Providing appropriate support	Encourage students to consider many strategies	0(0.0)	2(1.3)	86(57.0)	63(41.7)	3.40	0.519
	Encourages students to test their ideas	0(0.0)	2(1.3)	99(65.6)	50(33.1)	3.32	0.495
	Provides hints and suggestions to students	0(0.0)	6(4.0)	97(64.2)	48(31.8)	3.28	0.531
Providing worthwhile extension	Support students without removing the challenging aspect of the problem	8(5.3)	13(8.6)	100(66.2)	30(19.9)	3.01	0.707
	Allot appropriate amount of time to the task	0(0.0)	6(4.0)	96(63.6)	49(32.5)	3.28	0.534
	Patient to listen to students' reflection without interruption	0(0.0)	4(2.6)	94(62.3)	53(35.1)	3.32	0.523
Promotion of community of learners	Allow students to listen and respect the ideas of others	0(0.0)	1(0.7)	83(55.0)	67(44.4)	3.44	0.511
	Allows students to defend their answers, and the open the discussion with the class	1(0.7)	2(1.3)	88(58.3)	60(39.7)	3.37	0.549
Listening without	Ask questions that show an interest in students' ideas	2(1.3)	5(3.3)	93(61.6)	51(33.8)	3.28	0.591

evaluation	Encouraging students to focus in developing thinking skills and strategies in solving problems rather than on obtaining one right answer	0(0.0)	3(2.0)	76(50.3)	72(47.7)	3.37	0.549
Summarizing the main idea	Presses students to forward their solutions and strategies	1(0.7)	2(1.3)	88(58.3)	60(39.7)	3.28	0.591
	Ask for an extension of the problem	5(3.3)	2(1.3)	51(33.8)	93(61.6)	3.46	0.538
	Summarises the main points which are anticipated to challenge students	10.7	42.6	8153.6	6543.0	3.39	0.577

The responses of teachers in Table 14 indicate that mathematics teachers in the senior high schools said they use problem-solving as a pedagogical tool in their classroom. Each of the construct in the model for teaching through problem solving had more than 85% of teachers supporting its usage in the classroom during mathematics instruction. The overall mean of 3.4, shows that the mathematics teachers agree to the use of the Before, During and After phases for mathematics instruction through problem solving. This represents a good indication of teachers' practicing the use of the PSPT.

A 4-pointobservation scale, having 0 = Not Observed, 1 = Ineffective, 2 = somewhat effective and 3 = Effective as their key, was used to observe the lesson of teachers who indicated they adopt and use the PSPT.10 of such teachers were sampled; 4 of them from the urban schools and 6 from the rural schools. Table 15 presents the summary of teachers' actual practices of the PSPT as observed in their classrooms.

Table 15: Summary of Mathematics Teachers Observed Lesson on Problem Solving as a Pedagogical Tool (PSPT)

Item	Activity	Not (%)	Ineff (%)	S.E (%)	Eff (%)	Mean	SD
Activating prior knowledge	The teacher starts lesson with an easy version of the task	0(0.0)	7(70.0)	3(30.0)	0(0.0)	1.30	0.483
	The teacher brainstorms with students' various approaches or solution strategies	1(10.0)	5(50.0)	4(40.0)	0(0.0)	1.30	0.675
	The tasks provided builds on students' prior knowledge of mathematics	1(10.0)	7(70.0)	1(10.0)	1(10.0)	1.20	0.789
Ensuring understanding of the problem	Teacher ensures learners understood the problem before setting them to work	0(0.0)	5(50.0)	5(50.0)	0(0.0)	1.50	0.527
	Help students to relate the task with other problems solved in the past	0(0.0)	5(50.0)	5(50.0)	0(0.0)	1.50	0.527
Establishing	Teacher tell students whether they are working in small groups or individually	1(10.0)	7(70.0)	2(20.0)	0(0.0)	1.10	0.568

clear expectations	on the task given						
	Tr establishes with students how their solutions and reasoning will be shared	1(10.0)	8(80.0)	1(10.0)	0(0.0)	1.00	0.471
Letting go	The teacher demonstrates confidence and respect for students' abilities by allowing them to work	1(10.0)	6(60.0)	3(30.0)	0(0.0)	1.20	0.632
Noticing students mathematical thinking	Listens to students attentively and motivates them intrinsically	3(30.0)	6(60.0)	1(10.0)	0(0.0)	0.80	0.632
Providing appropriate support	Encourage students to consider many strategies	2(20.0)	4(40.0)	4(40.0)	0(0.0)	1.20	0.789
	Encourages students to test their ideas	0(0.0)	8(80.0)	2(20.0)	0(0.0)	1.20	0.422
	Provides hints and suggestions to students	2(20.0)	6(60.0)	2(20.0)	0(0.0)	1.00	0.667
Providing worthwhile extension	Support students without removing the challenging aspect of the problem	2(20.0)	4(40.0)	4(40.0)	0(0.0)	1.20	0.789
	Allot appropriate amount of time to the task	1(10.0)	6(60.0)	3(30.0)	0(0.0)	1.20	0.632
	Suggest extensions or generalizations	0(0.0)	6(60.0)	4(40.0)	0(0.0)	1.40	0.516
	Patient to listen to students' reflection without interruption	1(10.0)	3(30.0)	6(60.0)	0(0.0)	1.50	0.707
Promotion of community of learners	Allow students to listen and respect the ideas of others	0(0.0)	8(80.0)	2(20.0)	0(0.0)	1.20	0.422
	Allows students to defend their answers, and the open the discussion	1(10.0)	3(30.0)	6(60.0)	0(0.0)	1.50	0.707

	with the class						
Listening without evaluation	Ask questions that show an interest in students' ideas	2(20.0)	6(60.0)	2(20.0)	0(0.0)	1.00	0.667
	Encouraging students to focus in developing thinking skills and strategies in solving problems rather than on obtaining one right answer	2(20.0)	5(50.0)	3(30.0)	0(0.0)	1.10	0.738
Summarizing the main idea	Presses students to forward their solutions and strategies	3(30.0)	5(50.0)	2(20.0)	0(0.0)	0.90	0.738
	Ask for an extension of the problem	2(20.0)	6(60.0)	2(20.0)	00.0	1.00	0.667
	Summarises the main points which are anticipated to challenge students	1(10.0)	4(40.0)	5(50.0)	00.0	1.40	0.699

The results in Table 15 show almost all the teachers could not effectively activate students' prior knowledge. However, it was observed that teachers presented students with a task to be solved after explaining the topic and concept of the lesson. The results further show that 60% of the teachers (6 out of the 10) listened to students' ideas without interruption and also encouraged their students to listen to the ideas of other colleagues. The overall mean of 0.6 connotes that most of the constructs teachers claim they apply in their classroom were not actually practiced by teachers during their lessons. Teachers practice very little of the activities and procedures for instruction through problem solving. Teachers' performance on the various subscales in the reported practices and the observed practices were compared to find out if there exist differences in their performance. Figure 1 gives a summary of the results of the PSPT scores delineated by the teachers' expressed reported practices and observed teaching practices through PSPT.

Based on these results, it very apparent that the reported practices recorded high scores for PSPT and all its dimensions. The high scores were noticing students mathematical thinking 3.77, summarizing main idea 3.70 and listening without evaluation 3.57 whiles establishing clear expectations, providing worthwhile extensions and activating prior knowledge obtained low average scores. However, the observed practices had a highest score of 0.92 on ensuring understanding of problem, and noticing mathematical thinking with a score of 0.71. Providing appropriate support had the lowest score of 0.28.

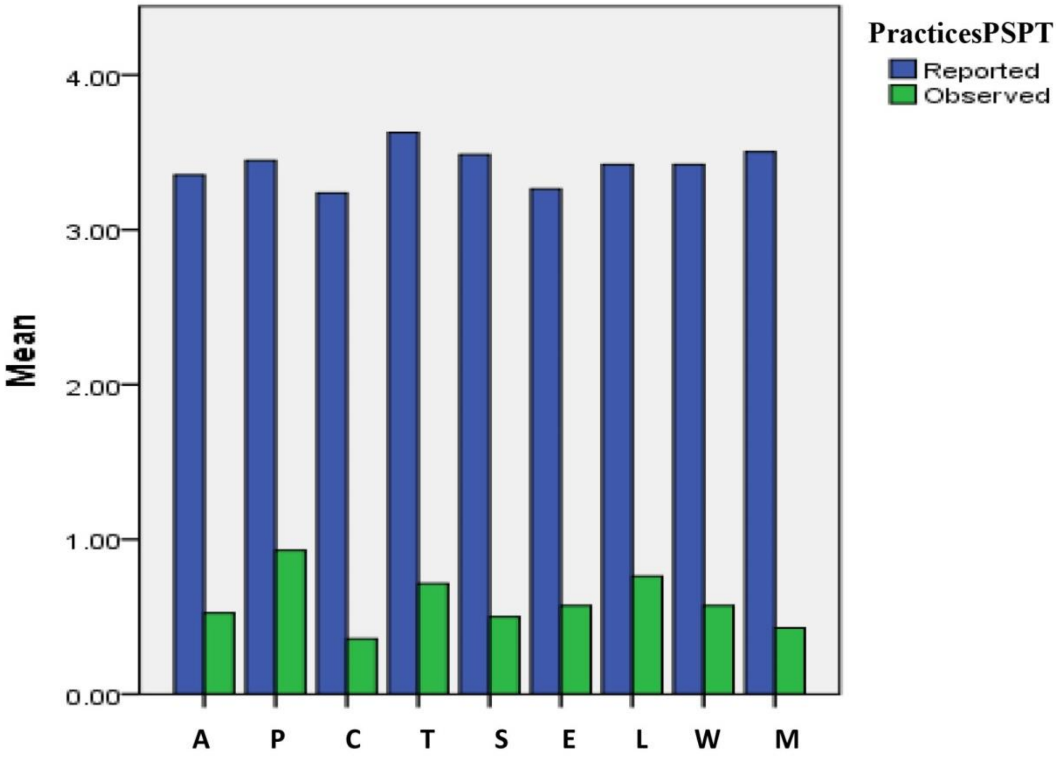

Figure 4: **Graphical summary of teachers reported**

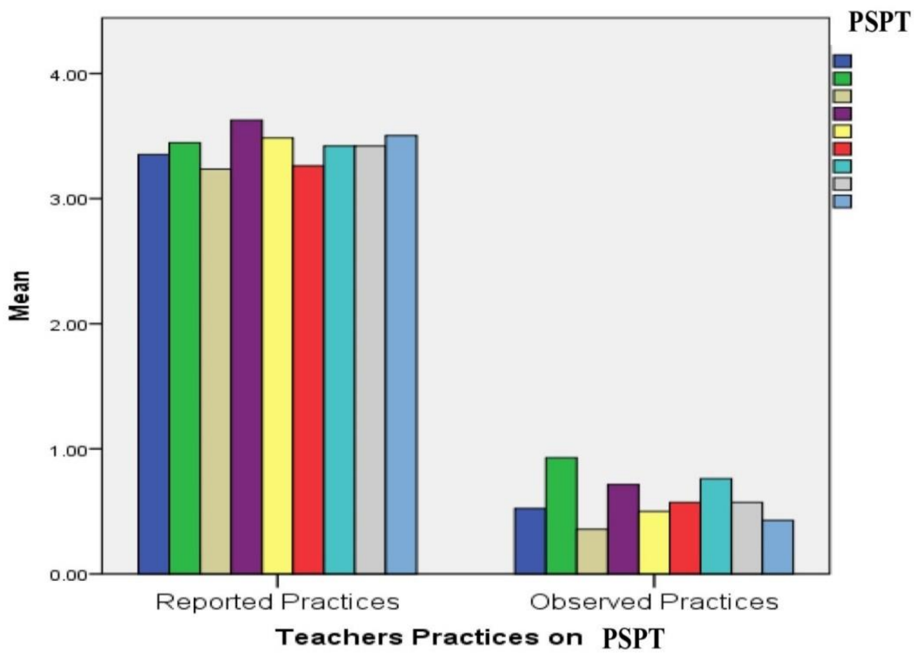

Figure 5: **Observed practices of PSPT**

The figures show the disparities that exist between the reported and the observed data.

The teachers reported practices and observed practices in applying the problem-solving instructional approach is shows that the reported practices indicated by teachers had higher means scores as compared to their corresponding means for the observed lessons. Activation of prior knowledge, which included the presentation of the problem, had a reported mean of 3.35 but a mean of 0.52 in the observed lesson. This implies teachers did not sufficiently activate the students' knowledge about the problem to be solved. Most teachers during the observation stage did not even present students with problems to be solved in the beginning of the lesson.

The study also found out if there exist difference in the averages of the reported and the observed practices. A Wilcoxon Signed Rank Test -being the non-parametric alternative of t-test- was used to analyze the differences in scores for the reported and the observed lesson data.

Table 16 shows the results from the Wilcoxon sign rank test of the two data set.

Table 16: Results from Wilcoxon Sign Rank Test

Number	Sig. test	Standard Error Statistic	Standardized
7	0.019	5.926	-2.332

The results from the sign ranked test in Tables 16 shows that statistically there exist a significant difference in the averages of the teachers reported practices and their observed practice with a sig value of 0.019 at alpha level of 0.05.

After analyzing the quantitative data, a qualitative data was then collected to obtain further explanation of teachers' practices on the use of the problem-solving as a pedagogical tool. They were asked to clarify the steps and processes they used when teaching using problem solving as a pedagogical tool. Some of the responses provided are presented in italics. Qualitative analysis of the provided teacher statements reveals various approaches and philosophies towards using problem-solving as a pedagogical tool. Here are the key insights from each teacher's perspective:

Real-World Relevance and Free Thought

Teacher 21

> *"As I mentioned in my opening statement, in order for students to truly comprehend that problem solving is useful and can be connected to everyday activities, I need to provide them with a task or work example if the problem is relevant to a real-world scenario. If not, I will have to assign them an assignment that will facilitate their free thought. In order to prepare, I get them if they have any instances or queries that pertain to our everyday activities. I also make an effort to connect what I teach to real-world situations."*

Main Points:

Real-World Connection: Teacher 21 emphasizes the importance of connecting problem-solving tasks to real-world scenarios to make the learning process meaningful for students.

"*I need to provide them with a task or work example if the problem is relevant to a real-world scenario.*"

Facilitating Free Thought: If a direct real-world connection is not available, the teacher assigns tasks that promote independent thinking.

"I will have to assign them an assignment that will facilitate their free thought."

Student Engagement: The teacher actively engages students by asking them for examples or questions related to everyday activities, making learning more relevant and interactive.

"I get them if they have any instances or queries that pertain to our everyday activities."

Effort to Connect Learning: A continuous effort is made to link teaching content to real-world situations, reinforcing the practical application of learned concepts.

"I also make an effort to connect what I teach to real-world situations."

Reflective Learning and Diverse Techniques

Teacher 31

> *"At the end, we just make an effort to look back or reflect on what we have done in the classroom. I attempt most of the time to use Polya's four important areas to help us work around that. Let us approach something this way, then. When I enter the classroom, for instance, I start teaching a series of three problems. Upon arriving at the classroom, my approach is to have the students reach out to the provided topic and then do our best to experience it—that is, to try to see the subject's reality in order to make it seem less abstract. As a result, we attempt to experience the subject and comprehend it as simply as possible before attempting, say, to attack or solve the*

question. We use a combination of techniques to solve the problem. I offer the kids the chance to find solutions. At the end of the day, you realise that the kids are approaching problems in different ways, but we persevere and eventually figure out which technique best fits the situation. Next, we examine which of the two solutions provided addresses the given problem".

Main Points:

Reflective Practice: Teacher 31 values reflection at the end of lessons to understand and consolidate what was learned.

"At the end, we just make an effort to look back or reflect on what we have done in the classroom."

Use of Polya's Framework: The teacher applies Polya's four-step problem-solving process to guide students through problem-solving activities.

"I attempt most of the time to use Polya's four important areas to help us work around that."

Experiential Learning: The teacher encourages students to experience and understand the subject matter in a practical context before tackling problems, making abstract concepts more tangible.

"We attempt to experience the subject and comprehend it as simply as possible before attempting, say, to attack or solve the question."

Diverse Problem-Solving Techniques: Students are allowed to explore different methods for solving problems, promoting a variety of approaches and critical thinking.

"You realize that the kids are approaching problems in different ways, but we persevere and eventually figure out which technique best fits the situation."

Conceptual Understanding and Practice

Teacher 11

> *"You start by going over the idea with them. You provide them with concept-related exercises to complete after you have explained the idea. Because without understanding the notion, individuals are unable to solve difficulties. Thus, you present the idea to them first, and then you present them with related difficulties to solve"*

Main Points:

Foundational Knowledge: Teacher 11 focuses on ensuring that students have a solid understanding of concepts before moving on to problem-solving.

"You start by going over the idea with them."

Concept-Related Exercises: After explaining concepts, the teacher provides exercises directly related to those concepts to reinforce understanding.

"You provide them with concept-related exercises to complete after you have explained the idea."

Sequential Learning: The approach is sequential, starting with conceptual explanation followed by related problems to ensure comprehension and application.

"Thus, you present the idea to them first, and then you present them with related difficulties to solve."

The qualitative analysis demonstrates that teachers adopt diverse strategies to facilitate problem-solving in mathematics. Teacher 21 prioritizes real-world connections and independent thought, Teacher 31 focuses on reflective practice and diverse techniques, and Teacher 11 emphasizes a strong conceptual foundation followed by practical exercises. Together, these approaches highlight the multifaceted nature of effective teaching methodologies aimed at enhancing student problem-solving skills.

4.7 Hypothesis H_o: There is no statistically significant relationship between teachers' perception of problem solving and its use as a pedagogical tool.

The hypothesis sought to find out if there is a significant correlation between teachers' perception of the PSIA and their practices for using of the PSIA in the classroom. A correlation was carried out between the two variables to establish the relationship. Table 17 shows the results.

Table 17: Correlation between Teachers Perception and Observed Practices on PSIA

Item		Observed
	Correlation coefficient (r)	0.0490
Perception	Sig.	0.264
	Number	10

From Table 17, it could be seen that there exist a weak but positive correlation between teacher perception and their observed practices. However according to the data there is no statistically significant relationship between the two variables, at an alpha level of 0.05.

4.8 Discussion of Results

The purpose of this study was to explore teachers' perceptions and use of problem solving as a pedagogical tool. The study explored the perception Senior High School mathematics teachers hold about problem-solving, the factors that influence Senior High School mathematics teachers' perception of problem-solving, the extent to which senior high school mathematics teachers use problem-solving as a pedagogical tool in the classroom, the kind of problem-solving practices utilize by Senior High School mathematics teachers in teaching and the relationship between mathematics teachers' perception of problem-solving and its use as a pedagogical tool. This study presents a strategy formulated for best practices in teaching and learning mathematical problem solving for students of secondary schools in Ghana. This research will specialise in problem-solving tasks. Five research questions were posed to guide the whole study.

According to the interviewees, teachers learned about the idea of problem solving and how to teach through it, but the programmes they attended were not sufficiently in-depth to give them the necessary knowledge and abilities to use problem solving as a teaching tactic. According to Matlala (2015), a significant obstacle facing the majority of mathematics teachers is that their prior professional development programmes did not effectively shape them to employ the problem-solving approach. The findings, however, indicate that most teachers believe the problem-solving approach is a useful instructional strategy to apply because of the nature of the problem-solving talks and the application of the problem-solving approach to instruction. Based on their comments, teachers appear to have a highly positive opinion of the problem-solving approach to instruction and how it is used in the classroom.

The data showed positive perception of mathematics teachers about the use of the problem-solving instructional strategy. The views expressed by teachers about the use of the problems solving as a pedagogical tool suggest that the different mathematics teachers have very different perceptions about mathematical instruction via problem-solving approach. In relation to this finding, Donaldson (2011) expressed that the problem-solving instruction approach looks differently for every teacher. The data indicated that the different views about the problem-solving approach were based on their understanding of what problem solving and instruction through solving is all about. Some view the approach based on its importance to mathematics instruction. Therefore, a greater number of teachers agreed and suggested that problem solving approach to mathematics instruction is the most suitable approach for mathematics instruction that builds on students' confidence, critical thinking and understanding. Though Akhter, Akhtar and Abaidullah (2015) support this notion, that problem solving is the most suitable approach to use; there are those whose view of the concept of mathematics instruction through problem solving is in terms of its usage. The data shows that teachers hold the perception that the problem-solving approach to mathematics instruction is based on enough time, resources and smaller class size, consistent with the views of Hiebert et al. (1997).

The perception that effective problem-solving approaches in mathematics instruction require sufficient time, resources, and smaller class sizes is supported by various studies and educational theories. Here's a discussion on why teachers might hold these perceptions: Problem-solving approaches often require in-depth exploration of mathematical concepts. This means spending more time on fewer topics, allowing students to develop a thorough understanding and proficiency. Problem-solving is an iterative process where students might need multiple attempts to find a solution. This trial-and-error process demands more classroom time compared to traditional lecture-based teaching. Extended engagement with problem-solving tasks leads to deeper understanding and retention of mathematical concepts (Hiebert & Grouws, 2007).

Effective problem-solving instruction often requires a variety of instructional materials, such as manipulatives, visual aids, and technology tools. These resources help make abstract mathematical concepts more concrete and accessible. Access to diverse instructional materials and technology enhances students' problem-solving skills and engagement (Clements & Sarama, 2007). Smaller class sizes allow teachers to give more individualized attention to each student. This is crucial in problem-solving instruction, where students may need personalized guidance to overcome challenges and misconceptions. Managing a classroom where students are actively engaged in problem-solving can be challenging. Smaller class sizes make it easier for teachers to facilitate group work, monitor student progress, and provide timely feedback. Smaller class sizes improve student outcomes, particularly in problem-solving contexts where individualized instruction and support are critical (Krueger, 2002).

The perception among teachers that effective problem-solving instruction requires sufficient time, resources, and smaller class sizes is well-founded. These factors create an environment conducive to deep learning, individualized support, and active engagement, which are essential for developing strong problem-solving skills in mathematics. Addressing these needs can help optimize the effectiveness of problem-solving approaches in the classroom. One contradictory view expressed by teachers was that, the

Problem-Solving Approach (PSPT) to mathematics instruction is suitable for all abilities however; they also suggest that the PSPT is not suitable for slow and weak learners. This contrasting perception, presupposes that mathematics teachers appear not to be sure whether this approach is for all students.

It could also be argued that since mathematics teachers hold the perception that the problem-solving approach to instruction is not suitable for large classes and weak or slow learners, they may feel reluctant to use the strategy for their mathematics instructions as pointed out by Attom (2017) that human behaviour is influenced by perception. It could be said based on the data that mathematics teachers generally have a good perception of the problem-solving method of teaching with respect to its benefits. This resonates with the finding of Akhter, Akhtar and Abaidullah (2015). The finding from the data obtained in this study about teachers' knowledge of the PSPT shows that 87.3% of the teachers indicated they possess knowledge about the problem-solving approach. Literature indicates that one of the sources of teachers' knowledge of the problem-solving approach emanates from teachers' professional development programmes (Matlala, 2015).

However, it was evident from the results of the study that unlike problem solving approach, the majority had limited knowledge about teaching through problem-solving. The majority of teachers indicated that teaching through problem solving meant solving real-life or practical problems. Others think of the problem-solving approach as using strategies to solve problems. These views do not agree with the concept of teaching through problem solving from the various literature which indicates that teaching through problem solving implies that important math concepts are learnt by the student via solving math problems using their strategies and their existing cognitive tools kids (Brhane, 2012; Matlala, 2015). The data suggest that mathematics teachers in senior high school possess an inadequate understanding of the use of the problem-solving teaching approach.

This also buttresses the assertion of Matlala (2015) that improving understanding with the problem-solving approach poses many challenges for teachers having a limited level of knowledge in the use of the problem-solving approach. If teachers' source of knowledge of the problem- solving instructional approach is from their pre-service training, then it can be inferred that teachers' pre-service training did not equip them with adequate knowledge of the concept of mathematics instruction through problem solving (Wilmot et al, 2015). This was confirmed when the teachers were interviewed about the training on instruction through problem solving that they received during their teacher education programmes. Teachers were asked whether they had formal professional training on the problem-solving approach to instruction, and if yes, what did the training entail?

Teacher 31 said "Naturally, I will answer "yes." I'll say that the training came from my master's programme at the university. I had to watch lectures since I earned an M.Ed. Since we finished the entire course on problem solving, I believe I am qualified to make this assertion. It took a lot of work. We were informed that these are not your normal word puzzles. In the classroom, where the teacher attempts to write a word problem on the board and we attempt to solve the solutions, it does not operate that way. It is quite involved". (Field interview with Teacher 31; 2024). Teacher 11 said "I was trained in problem-solving throughout my initial teacher training, and I received more problem-solving training when I went abroad for further study. It has to do with how lessons are taught to kids so they fully comprehend. Dissecting ideas so that kids can comprehend them.” However, teacher 21 expressed a different view on his training "Well, I'll admit that we didn't do much in school. However, after we graduated, I had the chance to participate in three different workshops, and even though they did not discuss problem solving specifically, we were talking about it here. Instead, we took a topic or approached a topic in a way that might make it easier for students to solve problems in that. Since this workshop and this specific meeting are focused on issue solving, I really can't say that.”

Given this, Chapman (2015) proposed that teachers should possess knowledge of instructional practices for strategies and metacognition to gain proficiency in problem-solving instructional approach. This data also implies that there are teachers who said they used the problem-solving instructional method in their lesson but could not enumerate the steps and processes they follow through for the lesson. Naturally, a lack of knowledge and understanding of a teaching method does not encourage the use of such a method. However, whereas over 50% of respondents indicated they use the problem-solving approach for their lessons only 5% of such teachers could demonstrate some knowledge of the processes of teaching through problem solving which are grounded in the theories of Van de Walle (2004) and Lappan et al. (2014). Most teachers indicated George Polya’s strategy as their method for teaching mathematics through problem solving.

However, Masingila, Olanoff and Kimani (2017) explained that Polya’s (1957) problem-solving heuristics explains teaching about problem solving rather than teaching through problem solving. This implies that teachers possess inadequate knowledge of the process and steps for implementing the problem-solving teaching strategy. It is not much surprising that other teachers indicated the inductive-deductive and algebraic word problem strategies as problem-solving instructional approaches, though it can be said that these approaches are embedded in the PSIA. The results suggest that mathematics teachers could not differentiate between teaching for problem solving, teaching about problem solving and teaching through problem solving and thereby confuse the previous two for the latter. Teachers' ability to differentiate between problem solving and the problem-solving approach to instruction depends largely on their knowledge.

The findings from the study suggest that the problem-solving instructional approach comes with some challenges for teachers as indicated by Buschman (2004). Teachers cited problems such as time factors, inadequate resources and large class size as their hindrances for not adopting the problem-solving strategy are largely also due to the nature of the setting of the senior high school system in Ghana. During the interview, some teachers explained that, because of the nature of the senior High school mathematics curriculum, the number of topics to be treated within a semester and the high emphasis on students passing the West Africa Senior High School Certificate Examination (WASSCE) in Ghana, it was not possible to adapt the problem-solving approach to instruction. To them, the problem-solving approach requires the use of adequate resources, time, small class size, and a good classroom environment. Though this view of teachers is consistent with that of Anderson, White, and Sullivan, (2005), Samuel (2002) as cited in Akhter, Akhtar and Abaidullah (2015) and Xiuping (2002) other literature indicates that it behooves on the teachers to decide on how to put in place the necessary protocols in order to be abreast with the use of the problem-solving teaching approach in mathematics (Liu, Jones and Sadera 2010). They indicated that the teachers' ability to adopt the problem-solving method of teaching is based on the teacher's willingness to implement new instructional practices.

The results also demonstrate that teachers' lack of confidence in using the problem-solving approach in their mathematics lessons is due to their inadequate knowledge of the problem-solving instructional approach. Researchers have suggested that mathematics teachers' pre-service education are usually unable to equip them with skill to teach using the problem-solving approach to mathematics instruction (Artzt, Armour-Thomas, & Curcio, 2008; Matlala 2015). The teachers' knowledge is one of the major resources needed for mathematics instruction through problem-solving however teachers in this study could not exhibit the possession of enough knowledge on the problem-solving approach and therefore did not reflect in their readiness and ability to adapt to the PSPT in their lesson. The interview with some of the teachers with respect to their practices and how they apply the PSPT in their classroom further buttressed the point of teachers' inadequate knowledge on the use of the PSPT.

Teacher 21's explanation in connection with how he applies the problem-solving strategy contradicts his self-reported activities during problem-solving approach to instruction. He rather affirmed the majority view that mathematics instruction through problem solving is about presenting problems that are practical to students. However, during his implementation of the concept of teaching through problem-solving in the class, it was observed that the steps of his instruction were not the problem-solving approach but rather his concept problem-solving approach was about the type and the nature of questions he poses to his students after teaching the topic. Teacher 31 rather explained the concept of teaching about problem solving and he tried demonstrating the same during his teaching. He taught (three set problem) concepts and asked students to solve similar problems. After a while, he discussed with students how the ideal solution would be by asking students questions about how they solved the question given using the Polya strategy. Teacher 11 also implemented his explanation of his teaching through problem-solving during the observation section. Nevertheless, the type of problems given to students was not challenging enough for students to deal with. Using the observation checklist, his teaching did not conform to the steps for implementing mathematics lessons via problem solving.

Teachers' explanation during the interview further confirms the disparities in the means between the reported practices and their actual practice. It was evident that teachers' explanations and the report did not conform to what they practice in the classroom which supports the findings from Agyei and Voogt, 2015; Matlala, 2015. The results suggest that teachers overestimated their knowledge and practice with respect to applying the problem-solving approach to mathematics instruction. Teachers' responses seem that they misconstrued helping students with problem solving skills for teaching through problem solving. The data imply that mathematics teachers seem to possess theoretical knowledge about the instruction via problem solving but lack in-depth knowledge about how the teaching approach is applied in practice and the ability to practice it in their mathematics classrooms (Akhter, Akhtar & Abaidullah, 2015).

Teachers must come to accept the fact that their modes of mathematics instruction through problem solving do not conform to what theories on problem-solving instruction suggest and thereby do not help to achieve the mathematics curriculum goals which are with the use of the problem-solving approach to teaching. Teachers must come to accept the fact that their modes of mathematics instruction through problem-solving do not conform to what theories on problem-solving instruction suggest and thereby do not help to achieve the mathematics curriculum goals which are with the use of the problem-solving approach to teaching. It can be noticed that teachers' approach to instruction is likely to create an inconsistency with respect to teachers' classroom practices and what the mathematics curriculum requires from mathematics teachers (Brodie, Lelliott, & Davis, 2001; Matlala, 2015). Most teacher attributes their inability to adopt mathematics instruction through problem solving to the nature of the curriculum and the current demands in Ghana's educational system which looks out for students passing rate in the exam. It is also clear that most teachers find it difficult to change their current modes of mathematics instruction for a new strategy because their current approach to mathematics instruction looks very easy for them to implement.

Findings on teachers' perceptions and actual classroom practices demonstrate that teachers' perceptions have little influence on those their practices. The balance between theoretical knowledge and practical application is a critical consideration in education, training, and professional development. Over-emphasizing theories at the expense of practicum can lead to several issues. These are: over-emphasizing theories can result in a lack of practical skills. Individuals may understand concepts but struggle to apply them effectively, theoretical knowledge without practical application can become abstract and disconnected from real-world challenges and solutions and learners might find purely theoretical education less engaging, leading to reduced motivation and interest in the subject matter.

In conclusion, while theoretical knowledge is essential for understanding and adaptability, practical application is crucial for skill development and real-world problem-solving. Striking a balance between the two ensures that individuals are well-prepared for their professional roles and can effectively apply their knowledge in practical scenarios. This finding is consistent with the finding of Akhtar, Akhter and Abaidullah (2015) who reported that, though teachers have a very good perception of the problem-solving approach to instruction, the majority are not interested in using the approach due to the fact many of them have the view that that the curriculum and assessment systems do not support the implementation of the problem-solving approach in their classrooms.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The Summary of findings, conclusions, and recommendations are presented in this chapter.

5.0 Summary

The purpose of this study was to explore teachers' perceptions and use of problem solving as a pedagogical tool. The study explored the perception Senior High School mathematics teachers hold about problem-solving, the factors that influence Senior High School mathematics teachers' perception of problem-solving, the extent to which senior high school mathematics teachers use problem-solving as a pedagogical tool in the classroom, the kind of problem-solving practices utilize by Senior High School mathematics teachers in teaching and the relationship between mathematics teachers' perception of problem-solving and its use as a pedagogical tool. This study presents a strategy formulated for best practices in teaching and learning mathematical problem solving for students of secondary schools in Ghana. This research will specialise in problem-solving tasks. A hypothesis and five research questions were posed to guide the whole study. These were as follows:

1. What perception do senior high school mathematics teachers hold about Problem-solving?
2. What is mathematics teachers' knowledge on the use of Problem Solving as a pedagogical tool?
3. What factors influence mathematics teachers' use or not use of Problem Solving as a pedagogical tool?
4. How do Mathematics Teachers teach through problem-solving?

5. What kind of problem-solving practices are utilised by senior high school mathematics teachers in teaching?

H_o: There is no statistically significant relationship between teachers' perception of problem solving and its use as a pedagogical tool

The sequential explanatory mixed method design was used in this study. Questionnaires, interviews, and observation protocols were used to collect the data to address the research questions and the hypothesis. The participants in this study were 151 senior high school mathematics teachers selected through a multi-stage sampling strategy. The stages of sampling included simple random sampling and purposive sampling. A sample of 7 mathematics teachers who claimed to use the PSPT in their lessons were interviewed using a semi-structured interview guide to find out more about the teachers' understanding of the application of the problem-solving approach to education. The interview responses and the observation results from the teachers served as evidence to bolster the quantitative data collected. The study used both descriptive and inferential statistics to present its findings. Teachers' perceptions of the use of the PSPT, their reported practices of the use of Problem Solving as a pedagogical tool, their observed practices of the PSPT in their mathematics classroom, and their knowledge of the PSPT and the factors that encouraged their adoption of the approach or not were all reported using frequency counts, percentages, means, and standard deviations.

To determine the relationship between these variables, a correlation was done between teachers' perceptions and their observed practices when using the PSPT. A Wilcoxon sign rank test was used to compare the medians or means of the observed practices and the stated practices of the teachers in order to look for any differences.

5.1 Key Findings

This study revealed that mathematics teachers possess a positive perception of the problem-solving approach to mathematics instruction, however this perception has little bearing on their practice in using problem-solving as a pedagogical tool. Their perceptions of the problem-solving instructional approach also influence their thinking on how their mathematics instructions through problem solving should be done. Majority of the teachers indicated they do not use the PSPT for their mathematics instructions. They gave reasons such as time factor, resources, class size, as some of the major reasons for not adopting such a good approach to instruction (Ibrahim,2022). Few of them claimed they use the approach for their mathematics instruction due to students' critical thinking, confidence and active participation in class. However, these few teachers could not demonstrate and outline how instruction through problem solving is done.

Again, the study found a weak positive correlation between teachers' reported practices and their observed practices on PSPT in the classroom but this relationship was statistically not significant (Ibrahim,2022). A little over 91% of senior high school mathematics teachers claimed to have experience teaching both problem solving and problem solving for teaching, as opposed to teaching through problem solving Teachers' engagement with learners in using problem-solving as a pedagogical tool was occasional and moderate. The moderate engagement allowed teachers and pupils to enjoy mathematics lessons in the classroom.

5.2 Conclusions

The following conclusions were made in light of the findings;

1. The Mathematics teachers possess a positive perception of the use of problem-solving as a pedagogical tool. However there seem to be a gap between their perception and their use of the problem-solving approach to mathematics instruction. This may be due to inadequate training on the use of the problem-solving approach to mathematics instruction.
2. The Senior high school mathematics teachers claimed to have experience in teaching both problem solving and problem solving for teaching, as opposed to teaching through problem solving but there was a gap between their knowledge of use and actual practice of the Problem Solving as a Pedagogical Tool (PSPT).
3. Teachers reported practices for instruction through problem solving were very good. This implies they are aware of the kind of practices they are expected to do but due to some reasons such as time factor, resources, class size and other factors, they do not actually use them.
4. There was a weak positive correlation between the teachers' perception and their observed classroom practices on PSPT but the correlation was not statistically significant.
5. Teachers do not often engage their learners/students when using problem solving as a pedagogical tool because of time factor, resources, class size and other factors

5.3 Recommendations

1. The following recommendations are given based on the findings of the study. To meet the demands of Ghana's current educational trend, which is focused on teaching Science, Technology, Engineering, and Mathematics (STEM) and emphasises the use of the PSPT, in-service mathematics teachers should be given sufficient professional development training on the problem-solving instructional approach. Develop and implement comprehensive professional development programmes focused on problem-solving techniques and strategies.
2. Ghana Education Service and NaCCA should ensure that problem-solving is systematically integrated into the mathematics curriculum, with clear guidelines and expectations for teachers.
3. GES should provide Continuous Professional Development (CPD) by Organizing regular workshops, seminars, and in-service training programs to help teachers translate their theoretical knowledge of problem-solving into practical classroom strategies. These sessions should include hands-on activities, real-world examples, and guided lesson planning.
4. Allocate resources to provide teachers with the necessary tools, such as instructional materials and access to collaborative platforms, to facilitate problem-solving activities.
5. Encourage school administrations to foster a supportive environment by promoting collaborative teaching practices and providing necessary support for problem-solving initiatives.

6. Ghana Education Service should continually assess the effectiveness of problem-solving as a pedagogical tool and gather feedback from teachers to refine and improve the teaching strategy.
7. GES should allow teachers who are not ready to use problem solving as a pedagogical tool to Participate in workshops, seminars, or online courses focused on problem-solving in mathematics. Training sessions often provide ready-made resources, hands-on experience, and frameworks for implementation.
8. Teacher training institutions should **i**ncorporate Problem-Solving Modules in Pre-Service Training by developing dedicated courses or modules that focus on problem-solving strategies in mathematics education. These modules should include theoretical foundations, practical applications, and opportunities for trainees to design and implement problem-solving tasks during their training.

5.4 Suggestions for Further Studies

It is advised that a study be conducted to Assess the role of teacher training and professional development programs in enhancing teachers' confidence and competence in using problem-solving. It is also advised that a study is done to explore how novice versus experienced teachers perceive and use problem-solving methods differently.

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APPENDICES

APPENDIX A

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
OFFICE OF THE PROVOST

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UNIVERSITY POST OFFICE
PRIVATE MAIL BAG
CAPE COAST, GHANA

4th September, 2023

The Chairman
Institutional Review Board
University of Cape Coast
Cape Coast

Dear Sir/ Madam,

REQUEST FOR ETHICAL CLEARANCE

The bearer of this letter, Mr. Gabina Susuoroka, with registration number ET/DME/20/0002 is a PhD (Mathematics Education) student of the Department of Mathematics and ICT Education, College of Education Studies, University of Cape Coast.

As part of the requirements for the award of doctorate's degree, he will require an ethical clearance from your outfit to undertake a research visit to selected senior high schools in the Upper West Region of Ghana. Mr Susuoroka's topic is **"SHS MATHEMATICS TEACHERS' PERCEPTIONS AND USE OF PROBLEM SOLVING AS A PEDAGOGICAL TOOL".**

I would be grateful if you could give him the necessary assistance to commence his data collection.

I count on your cooperation

Yours faithfully,

Prof. Ernest Kofi Davis
(Provost/Main Supervisor)

APPENDIX B

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION
DEPARTMENT OF MATHEMATICS AND I.C.T EDUCATION

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Cape Coast, Ghana

Your Ref:

10th September, 2023

Our Ref: DMICTE/P.3/V.3/160

The Chairman
Institutional Review Board
University of Cape Coast
Cape Coast

Dear Sir,

REQUEST FOR ETHICAL CLEARANCE

The bearer of this letter, **Mr. Gabina Susuoroko**, with registration number ET/DME/20/0002 is a PhD (Mathematics Education) Student of the Department of Mathematics and ICT Education, College of Education Studies, University of Cape Coast.

In fulfilling the requirements for submission for ethical clearance, I would like to indicate that the Department is aware and has approved **Mr. Gabina Susuoroko's** research topic **"SENIOR HIGH SCHOOL MATHEMATICS TEACHERS' PERCEPTIONS AND USE OF PROBLEM SOLVING AS A PEDAGOGICAL TOOL"**.

The Department is in full support of the submission of his proposal to your outfit for further action.

I would be grateful if you could give him the necessary assistance he may need.

Counting on your usual co-operation.

Thank you.

Yours faithfully,

Dr Forster D. Ntow
HEAD

APPENDIX C

INSTITUTIONAL REVIEW BOARD SECRETARIAT

TEL: /
E-MAIL:
OUR REF: IRB/C.3/Vol.1/0626
YOUR REF:
OMB NO: 0990-0279
IORG #: IORG0011497

20TH FEBRUARY, 2024

Mr Gabina Susuoroka
Department of Mathematics and ICT Education
University of Cape Coast

Dear Mr Susuoroka

ETHICAL CLEARANCE – ID (UCCIRB/CES/2023/199)

The University of Cape Coast Institutional Review Board (UCCIRB) has granted Provisional Approval for the implementation of your research **Senior High School Mathematics Teachers' Perceptions and use of Problem-Solving as a Pedagogical Tool.** This approval is valid from **20th February 2024 to 19th February 2025.** You may apply for an extension of ethical approval if the study lasts for more than 12 months.

Please note that any modification to the project must first receive renewal clearance from the UCCIRB before its implementation. You are required to submit a periodic review of the protocol to the Board and a final full review to the UCCIRB on completion of the research. The UCCIRB may observe or cause to be observed procedures and records of the research during and after implementation.

You are also required to report all serious adverse events related to this study to the UCCIRB within seven days verbally and fourteen days in writing.

Always quote the protocol identification number in all future correspondence with us about this protocol.

Yours faithfully,

Kofi F. Amuquandoh
Ag. Administrator

APPENDIX D

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
DEPARTMENT OF MATHEMATICS AND ICT EDUCATION
MATHEMATICS TEACHERS' PERCEPTION AND USE OF PROBLEM- SOLVING AS A PEDAGOGICAL TOOL

TEACHERS' QUESTIONNAIRE

This study is being used to learn more about mathematics teachers' knowledge and perceptions of the use of problem-solving as a pedagogical tool in public senior high schools. The data collection is done in support of a Ph.D. thesis. Therefore, it is solely intended for academic purposes. Realizing the purpose of this research is strongly dependent on the data you provided, I will be delighted if you participate in the study by providing truthful answers to the questions. Please know that we will treat your information with the utmost confidentiality. Many thanks.

Directions:

1. Not important to write your name
2. Please give brief and clear answers to open-ended questions

PART A: GENERAL INFORMATION

1. Name of school ……………………………………….

For items 2 – 7, tick [√] in the appropriate space for the response that applies to you.

2. Sex: Male ☐ Female ☐
3. Age: Below 20 [ ] 20-29 [ ] 30-39 [ ] 40-49 [ ] 50-59 [ ]
4. What is your academic background?

   a. B.Ed. Mathematics [ ] b. BSc. Mathematics [ ] c. M.Ed.

Mathematics [ ] d. MPhil. Mathematics [ ] e. MSc.

Mathematics [ ] f. Others [ ]

5. Training background

☐ Professional ☐ Non-Professional

6. How long have you been a teacher?

Less than 4 years [ ] 4-8years [ ] 9-12 years [ ] 12-16 years [ ] above 16 years [ ]

PART B: PERCEPTION OF SHS MATHEMATICS TEACHERS ON THE USE OF PROBLEM-SOLVING AS A PEDAGOGICAL TOOL

Please respond to items given below by putting a tick [√] in the appropriate space using the following scale: 1 = Strongly Disagree (SD), 2 = Disagree (D), 3 = Uncertain (U), 4 = Agree (A) and 5 = Strongly Agree (SA)

No	Statements	SD	D	U	A	SA
1	I am motivated to plan lessons using the problem-solving approach in mathematics					
2	I need enough space, resources, and a conducive environment in the class to use the Problem-solving strategy					
3	I find the problem-solving approach to instruction more supportive of learners of all abilities					
4	It is more difficult to satisfy slow and weak learners when teaching through problem-solving					
5	I think the problem-solving approach is useful for mathematical instruction					
6	The problem-solving approach is not suitable for teaching children in developing countries like Ghana even if there is enough time					
7	The use of problem-solving method is impossible with large classes					
8	The problem-solving approach develops students' confidence and critical thinking abilities					

9	The use of problem-solving approach in teaching mathematics is time wasting					
10	The use of problem-solving approach will not help teachers to cover all the topics in the syllabus					

11. What is your view about the problem-solving approach to mathematical instruction?

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....

PART C: TEACHERS' KNOWLEDGE OF PROBLEM-SOLVING INSTRUCTIONAL APPROACH

1. Do you know about the problem-solving instructional strategy? If your answer is NO kindly move to 3a.

☐ YES ☐ NO

2. What does it mean to teach through problem-solving?

....

....

....

....

....

....

3a. Do you use the problem-solving instructional strategy in teaching?

A YES b.NO

3b. If your answer in 3a above is YES, kindly list the steps you follow to teach through problem-solving in your class.

………

………

………

………

………

………

………

4. Explain why you adopt the problem-solving teaching method

……………………………………………………………………………………………………

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……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

5. List some of the challenges you encounter using the problem-solving approach in your mathematics class.

6. If your answer to 3a is No, explain why you do not adopt the problem-solving teaching method

………

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7. What is your view about the use of the problem-solving instructional approach?

8.

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....

9. Teachers' workload (number of periods per week)

PART D:__PRACTICES AND PROCEDURES TEACHERS FOLLOW WHEN TEACHING MATHEMATICS IN THE CLASSROOM.

Direction: Put a tick (√) in the column next to the comment that most accurately reflects how you are incorporating these concepts into your mathematics lessons.

NB: 1= Strongly Disagree, 2= Disagree, 3= Agree and 4= Strongly Agree

Item (Teacher's activities in the classroom)		1	2	3	4
1.	I begin with a simple version of the task				
2.	I brainstorm with students, the various approaches or solution strategies				
3.	The tasks provided build on students' prior knowledge of mathematics				
4.	I ensure learners understand the problem before setting them to work				

5.	I help students to relate the task to other problems solved in the past.				
6.	I tell students whether they are working in small groups or individually on the task given.				
7.	I establish with students how their solutions and reasoning will be shared				
8	I demonstrate confidence and respect for students' abilities by allowing them to work				
9	Listens to students attentively and motivates them intrinsically				
10	Encourage students to consider many strategies				
11	Encourages students to test their ideas				
12	Provides hints and suggestions to students.				
13	I support students without removing the challenging aspect of the problem				
14	I allot appropriate amount of time to the task				
15	I patiently listen to students' reflections without interruption				
16	I allow students to listen and respect the idea of others				
17	I allow students to defend their answers, and then open the discussion to the class				
18	Ask questions that show an interest in students' ideas				
19	Encourage students to focus on developing thinking skills and strategies for solving problems rather than for obtaining one right answer				

APPENDIX E

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES DEPARTMENT OF MATHEMATICS AND ICT EDUCATION

Interview Guide for Teachers

1. Could you throw more light on your perceptions about teaching mathematics through problem-solving and why?
2. Have you received any professional training in the use of problem-solving as an instruction instructional tool? If yes, briefly explain what went into the training and how it is helping your teaching.
3. What is your understanding of the problem-solving method of teaching in mathematics?
4. How do you apply the problem-solving method of teaching in your classroom?
5. Kindly throw more light on the challenges you often encounter using problem-solving as an instructional tool.
6. What do you consider as a problem, when adopting the problem-solving instructional method?
7. What motivates you to use the problem-solving approach to teach?

APPENDIX F

UNIVERSITY OF CAPE COAST

COLLEGE OF EDUCATION STUDIES

DEPARTMENT OF MATHEMATICS AND ICT EDUCATION

Observation Checklist on Mathematics Instruction through Problem Solving

This checklist is designed for teachers who are going to be observed as they teach mathematics through problem-solving in the classroom. This checklist was designed using the Van De Walle framework for teaching through problem-solving; before problem-solving, during problem-solving, and after problem-solving. Teachers' lessons are observed in the window of this framework with some specific items under each stream to be considered throughout the lesson observation.

This observation checklist is developed to collect data on the implementation of mathematics instruction through a problem-solving approach in public senior high schools in the central region of Ghana.

General Information

1. Name of School…………………………..
2. Number of students presented in the class………………

Date…………………………… Lesson Duration……………………

Grade level……………………… Topic………………………………

NB: 0 = Not Observed 1= ineffective, 2= somewhat effective and 3= Effective

Stage		No	Item (Teacher's activities in the classroom)	0	1	2	3
Before phase of a problem solving instructional strategy	Activating Prior Knowledge	1.	The teacher starts the lesson with an easy version of the task				
		2.	The teacher brainstorms with students various approaches or solution strategies				
		3.	The tasks provided build on students' prior knowledge of mathematics				
	Ensuring Understanding of the Problem	4.	The teacher ensures learners understood the problem before setting them to work.				
		5.	Help students to relate the task with other problems solved in the past				
	Establishing clear expectations	6.	The teacher tells students whether they are working in small groups or individually on the task given				
		7.	Teacher establishes with students how their solutions and reasoning will be shared				
During	Letting go	8.	The teacher demonstrates confidence and respect for				

			students' abilities by allowing them to work				
	Noticing students mathematical thinking	9	Listens to students attentively and motivates them intrinsically				
	Providing appropriate support	10.	Encourage students to consider many strategies				
		11.	Encourages students to test their ideas				
	Providing worthwhile extension	12	Provides hints and suggestions to students.				
		13	Support students without removing the challenging aspect of the problem				
		14	Allot appropriate amount of time to the task				
		15	Suggest extensions or generalizations.				
After the Phase of problem-solving	Promotion of community of learners	16	Patient to listen students' reflection without interruption				
		17	Allow students to listen and respect the idea of others				
		18	Allows students to defend their answers, and then open the				

			discussion with the class.				
	Listening without evaluation	19	Ask questions that show an interest in students' ideas				
		20	Encouraging students to focus in developing thinking skills and strategies in solving problems rather than on obtaining one right answer				
	Summarizing the main idea	21	Presses students to forward their solutions and strategies				
		22	Ask for an extension of the problem				
		23	Summarizes the main points which are anticipated to challenge students				

Provide a summary of your observation of the lesson

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