UNIVERSITY OF CAPE COAST

A COMPARISON OF TWO METHODS OF TEACHING COMPOUND SUBTRACTION

(DECOMPOSITION VERSUS BASE-COMPLEMENT ADDITION)

FRANCIS AKPANYI

2010

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BY

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DISSERTATION SUBMITTED TO THE DEPARTMENT OF SCIENCE AND MATHEMATICS EDUCATION, FACULTY OF EDUCATION, UNIVERSITY OF CAPE COAST IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR AWARD OF MASTER OF EDUCATION DEGREE, IN MATHEMATICS EDUCATION

JUNE 2010

DECLARATION

Candidate's Declaration

I hereby declare that this dissertation is the result of my own original research and that no part of it has been presented for another degree in this university or elsewhere.

Candidate's Signature: ………………………….. Date: ………………...

Name: ……………………………………………………………………….....

Supervisor's Declaration

I hereby declare that the preparation and presentation of the dissertation were supervised in accordance with the guidelines on the supervision of thesis laid down by the University of Cape Coast.

Supervisor's Signature: ……………………………. Date: ………………..

Name: ………………………………………………………………………

ABSTRACT

The study focused on comparing the efficiency of the Decomposition (conventional method) and Base Complement Addition (non-conventional method) using two randomized groups of pupils. The comparison was done on measures of speed, accuracy and retention.

In all, sixty pupils (thirty from each school) were selected from St. Peter's and Jesus Cares Preparatory Schools at Dunkwa Offin in the Upper Denkyira East Municipality in the Central Region of Ghana. The researcher used simple random sampling to select primary three pupils for the study.

Pretest-posttest design was used in the study. The t-test and median test (all tested at 0.05 level of significant) were the statistical tools used in the study. The Statistical Package for the Social Sciences (SPSS) was used to analyse the measures of speed, accuracy and retention of the pupils of Decomposition and Base Complement Addition groups.

The analysis revealed that Base Complement Addition group performed better than their counterparts in the Decomposition group on measures of speed, accuracy and retention. These differences were significant at 0.05 significant levels. It was recommended that students should be encouraged to conduct further research on the topic in different geographical areas, the sample size must be increased and the intervention period must be extended.

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Lastly, I offer my profound gratitude to all those who supported me in diverse ways to make this dissertation a success.

DEDICATION

I dedicated this work to my dear wife Mrs. Priscilla Akpanyi for her support and word of advice which encouraged me to pursue this programme.

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CHAPTER ONE

INTRODUCTION

Overview

This chapter consists of the background to the study, statement of the problem, purpose of the study, research questions, research hypotheses, significance of the study, delimitations, limitations and definition of terms.

Background to the study

In a paper, delivered at the Third International Conference on Mathematical Education, Krygowska (1976) maintained mathematics holds an important position as a compulsory subject or at least an optional subject during the final years of pupils at school.

From the above discussion, it is obvious that mathematics plays a leading role in our lives. This shows that the placing of mathematics on the school curriculum is useful. Gyening (1993) is of the view that subtraction is one of the essential topics in social mathematics and manifest itself as follows;

(i) in buying service we use money that exceeds the price of the item we purchase, the need for a change involves the concept of subtraction.

(ii) the separation of rotten items from the good ones by farmers involves subtraction.

Despite the fact that subtraction plays an immense role in our daily lives, the topic has been creating uneasiness for both the learner and teacher when it involves regrouping (Grossnickel & Brueckner, 1963; Seville, 1964). Available literature has it that teaching children how to subtract has been considered as a problem area in mathematics for ages (Winch, 1920; Thorndike, 1921). According to Ballard (1941) there is a public outcry over pupils' inability to subtract with that ease and accuracy which ordinary life demands. Again, Ballard (1941) was of the view that the concept of subtraction is weak among pupils and that the root cause of the weaknesses lies in the method (i.e. decomposition). Perhaps, these are the considerations that have attracted much more research in the area of arithmetic (Seville, 1964).

The controversy in relation to the teaching of compound subtraction is to find the method that will achieve better results. It is doubtful whether any of the methods have been proved significantly superior to the other when judged on speed and accuracy (Rheins & Rheins, 1955; Grossnickel & Brueckner, 1963). Evidence gathered indicates that the Decomposition (DEC) method is used in Britain, United States of America and Canada whereas the Equal-Addition (EA) is popular in many European countries (Mueller, 1964).

Results on the two conventional methods have not been definitive in favour of any of the methods. Early research works on compound subtraction were almost in favour of EA (Murray1941; Johnson, 1938) and as if that is enough it continues to earn the support of recent researchers (Ohlsson, Ernest & Rees, 1992). The

DEC has also attracted a myriad of favour from other researchers (Brownell, 1947; Brownell & Moser, 1949; Trafton, 1979; Sherill, 1979)

Generally, it has been observed that individuals use the DEC more than the EA method owing to the numerous advantages associated with this method. The advantages include the following;

(i) the method is popular owing to its ability to demonstrate regrouping procedure (Seville, 1964; Kennedy and Tipps, 1988).

(ii) the method helps pupils to learn our decimal numeration better (Thorndike, 1921).

The DEC also has disadvantages and these are;

(i) the algorithm's inability to deal with zeroes in the minuend (Seville, 1964).

(ii) the use of crutch by this algorithm demeans the concept of place value (Gyening, 1993).

Carpenter et al (1975) relying on records of responses of pupils in the National Assessment of Educational Progress (NAEP,1972-1975) Mathematics Assessment noticed that forty-five percent (45%) of nine-year-olds could not compute two digit subtraction problems involving re-grouping. The fourth NAEP (1988) reported that over seventy percent (70%) of the seventh and eleventh graders who took part indicated that they thought arithmetic was important for securing a job.

The issue of lack of computational skills in both children and adults has attracted a myriad of concern worldwide. In view of this, in 1976 the then British

Prime Minister Calaghan called for a probe into the circumstances surrounding the falling standard in simple computation in pupils. As a result of this development, a committee under the chairmanship of W. H. Cockcroft (1982) was charged with the responsibility of looking at the teaching of mathematics in primary and secondary schools in England and Wales with particular reference to the mathematics required in higher and further education, employment and adult life generally and subsequently make recommendations. Some of the findings of the committee are that the

(i) study of mathematics is regarded by most people as being essential that it will be difficult to live a normal life in many parts of the world without making use of mathematics of some kind.

(ii) usefulness of mathematics is perceived in different ways. For many it is seen in terms of arithmetical skills which are needed for use at home or in the office or workshop. Others also see mathematics as the basis of scientific development and technology.

The DEC method is widely used in teaching subtraction in Ghanaian schools. Despite the fact that subtraction is introduced early (in late Basic One) with the complex ones following later as one progresses through the learning stages, pupils performance at the basic level leaves much to be desired. For instance, the Chief Examiner's Report (2001) on the Basic Education Certificate Examination stated categorically that the general performance of pupils has gone down slightly although some pupils' performance was excellent (Chief Examiner's Report, 2001).

In 1993, the Ministry of Education published a report on Primary Education Programmes (PREP) 1992 Criterion Reference Test for primary six pupils in Mathematics and English; the report indicated that only one point one percent (1.1%) of the pupils had over fifty- five percent (55%) passed in Mathematics paper and about sixty percent (60%) of them were able to answer compound subtraction questions involving two- digit numbers correctly (PREP, 1992).

The 2003, Trends in International Mathematics and Science Study (TIMSS) results for eighth -grade students showed that Ghana came 47th out of forty-nine (49) participating countries with an average score of 276 representing 40% (International Association for the Evaluation of Education Achievement (IEA) Trends in Mathematics and Science Study, TIMSS; 2003). Highlights from TIMSS 2007, revealed that in the ranking participating countries based on the average scores of eighth- grade students; Ghana took the 47th position out of 48 countries.

These results suggest that there is something wrong with the teaching and learning of mathematics and science.

From the foregoing observation, it seems the conventional methods Decomposition (DEC) and Equal Addition (EA) are not appropriate in handling compound subtraction problems. Therefore the need arises for a more efficient method. It is against this background that Gyening (1993) presented a paper at a departmental seminar titled “Facilitating compound subtraction”. Thereafter, a consensus was reached that the method was comparable to the conventional methods on the basis of theoretical considerations. Gyening (1993) was of the view that the BCA method has a number of advantages. These advantages are enumerated below as follows;

(i) it can be objectified like the DEC method;

(ii) it has a potential for accuracy;

(iii) it preserves the place value concept;

(iv) it induces speed;

(v) it avoids the use of crutch so working involving it looks tidy and

(vi) it ensures understanding as the DEC method.

It is worthy to note that the DEC method (conventional method) lend itself easily to the use of manipulative and therefore is much easier to teach and learn than other methods. This method is the reverse of the "carrying" procedures use in solving addition problems involving renaming. In solving the subtraction problem 61-36, the DEC method looks at it as follows:

61-36 = (6 tens +1 ones) - (3 tens + 6 ones)

= (5 tens + 1ten + 1 one) - (3 tens + 6 ones)

= (5 tens + 10 ones + 1 ones) – (3 tens + 6 ones)

= (5 tens + 11 ones) – (3 tens + 6 ones)

= (5 tens – 3 tens) + (11 ones – 6 ones)

= 2 tens + 5 ones

= 25

The contracted form of the above method is illustrated below

$$\begin{array}{r} 6\,1 \\ -3\,6 \\ \hline \end{array} \qquad \begin{array}{r} \cancel{6}^{1}\cancel{1}^{11} \\ -\ \underline{3\,6} \end{array} \qquad \begin{array}{r} \cancel{6}^{1}\cancel{1}^{11} \\ -\ \underline{3\,6} \\ \underline{2\,5} \end{array}$$

The explanation for the DEC method for solving 61 minus 36 is as follows:

One ten is taken from the six (6) tens of the minuend (61) and converted into ten (10) ones. The ten ones were added to the one (1) one of the minuend to obtain five (5) tens and eleven (11) ones. Two subtractions are then carried out: (a) 6 ones from 11 ones yielded 5 ones and (b) 3 tens from 5 tens yielded 2 tens which results in the final answer 25.

The BCA which is non-conventional is seen as an adaptation and an improvement over the EA method. It is difficult to teach the EA algorithm to the mentally immature primary pupil since the method cannot be objectified or rationalized. Using the principle of compensation, the BCA transforms compound subtraction problem to a simple subtraction problem by adding the same complement of ten to both the minuend and subtrahend, where an impasse occurs before subtraction is carried out. For instance, in finding the difference between sixty – one and thirty – six (61 – 36), the impasse is associated with the ones. The complement of six (6) in base ten is four (4) and therefore four (4) is added to the minuend and the subtrahend before subtraction is carried out; yielding twenty – five (25). The BCA algorithm to the problem is illustrated below as follows

61 – 36 = (6 tens + 1 ones) – (3 tens + 6 ones)

= (6 tens + 1 ones + 4 ones) – (3 tens + 6 ones + 4 ones)

= (6 tens + 5 ones) – (3 tens + 10 ones)

= (6 tens + 5 ones) – (3 tens + 1 tens + zero ones)

= (6 tens + 5 ones) – (4 tens + zero ones)

= (6 tens – 4 tens) + (5 ones – zero ones)

= 2 tens + 5 ones

= 25

The contracted form of the above problem is illustrated as follows

$$\begin{array}{rcrcr} 61 & & (64+4) & & 65 \\ -\underline{36} & & -\ \underline{(36+4)} & = & -\underline{40} \\ & & & & \underline{} \end{array}$$

Below is the explanation for the BCA method for solving the problem 61-36.

Owing to the fact that 1-6 has no solution on the set of whole numbers it is expedient to resolve the impasse at the ones column. In order to do this effectively, the base (base ten) complement of 6 which is 4 is added to both the subtrahend and minuend to transform the compound subtraction (61-36) into a simple subtraction problem (65-40). Two subtractions are carried out as follows: (a) 5- 0 = 5 (i.e. ones column) and (b) 6 – 4 = 2 (i.e. tens column). These two subtractions give the final answer to the given subtraction problem as 25.

It is obvious that the concept of addition with “carrying” is inherited from the BCA method. One can therefore conclude that the BCA could be related to addition with carrying which is taught earlier than subtraction.

As a result of the claims of the proponents of the DEC and BCA methods, the researcher conducted this study to find the relative efficiency of the two methods on measures of speed, accuracy, retention and understanding with primary three pupils at Dunkwa Offin.

Statement of the Problem

Results of Trends in Mathematics and Science Study (TIMSS, 2007) revealed that in the ranking of participating countries based on the average scores of eighth- grade students; Ghana took the 47^{th} position out of 48 countries.

Research studies on Decomposition and Equal Addition method have not produced definitive results (Winch, 1921; Thorndike,1961; Grossnickel & Brueckner, 1964) .The need therefore arises for researchers and educators to come out with a non- conventional method that is more reliable, accurate and easier for pupils to learn and use. Consequently the Base-Complement Addition method was introduced in this study and compared with the Decomposition method, which is the conventional method.

Purpose of the Study

The purpose of the study is to replicate earlier studies on the BCA method in other to review or support what was known about the method.

The Significance of the Study

The findings of the study will be used to improve upon the teaching methods to counteract the poor performance of pupils in subtraction at the basic level. Besides, it may enhance the knowledge of researchers and shareholders in the development of relevant policies and models for the teaching and learning of Mathematics in our schools. Finally, the current study might bring to bare valuable information regarding possible strength and weaknesses which exist in each algorithm.

Research Questions

The research is designed to answer the following questions:

(i) Which group (DEC or BCA) will have more compound subtraction problems correct?

(ii) Which group (DEC or BCA) will be able to remember the knowledge gained in the treatment of the two methods?

(iii)Which group (DEC or BCA) will be able to use the knowledge gained in the treatment of the two methods to solve word-problems correctly?

(iv)Which group (DEC or BCA) will finish compound subtraction problems faster?

Research Hypotheses

Based on the above research questions the following research hypotheses were formulated for testing, all at 0.05 level of significance:

Hypotheses I:

H_0: There is no significant difference between mean scores of the DEC and BCA group on the post-test with respect to accuracy.

H_1: There is a significant difference between mean scores of the DEC and BCA group on the post-test with respect to accuracy.

Hypotheses II:

H_0: There is no significant difference between mean scores of the DEC and BCA group on the retention test with respect to accuracy.

H_1: There is a significant difference between mean scores of the DEC and BCA group on the retention test with respect to accuracy.

Hypotheses III:

H_0: There is no significant difference between mean scores of the DEC and BCA group on the understanding test with respect to accuracy.

H_1: There is a significant difference between mean scores of the DEC and BCA group on the understanding test with respect to accuracy.

Hypotheses IV:

H_0: There is no significant difference between the median finishing times of the DEC and BCA group on the post-test.

H_1: There is a significant difference between the median finishing times of the DEC and BCA group on the post-test.

Hypotheses V:

H_0: There is no significant difference between the median finishing times of the DEC and BCA group on the retention test.

H_1: There is a significant difference between the median finishing times of the DEC and BCA group on the retention test.

Delimitation

The study involved sixty primary pupils aged between seven and thirteen years selected from two basic schools in the Upper Denkyira Municipality. Literature has it that teaching children to subtract has been considered as problem area in mathematics (Winch, 1920; Thorndike, 1921). According to the ministry of education programmes (CRDD, 2007) Criterion Reference Test for primary six pupils 1.1% of the pupils had over 55% pass and about 60% of them were not able to answer compound subtraction questions involving two-digit numbers correctly. With these two reasons in mind, researcher deemed it necessary to have an in-depth study on compound subtraction. Again, class three was chosen because compound subtraction is taught in that class (mathematics syllabus for primary schools, 2007)

Limitation

That the researcher would have extended the study to cover more basic school, however, owing to financial and time constraints, lack of means of transport as well as limited resources the study was limited to only four basic schools.

Assumptions

The assumptions made in the study are stated as follows:

(i) that the subjects of both groups have equal level of ability.

(ii) that the subjects of both groups had been exposed to the DEC algorithm.

(iii) that the test items in each test had the same level of difficult and for that reason instruments were reliable and valid.

(iv) that the mean ages of the subjects of both groups were almost the same.

(v) that the researcher could teach effectively

(vi) that experimental mortality was to be negligible

Definition of Terms

Below are the operational definitions of terms that have been used in the study.

Accuracy: This refers to correct responses to the test items used in the study.

Algorithm: Algorithm is a well defined and systematic series of procedure leading to finding an answer for a given example.

Base Complement: The Base- Complement of a number, *n* to the base, *b* is the positive number, *p* that adds to *n* to give the number *p*. Let *p* be the complement of *n*, then $n + p = b$. In base ten numeration computation algorithm the base- complement of 3, 4 and 6 are 7, 6 and 4 respectively.

Column Impasse: This is the situation whereby there is an encounter of a column difference for which there is no entry in the set of basic subtraction facts. Consider the following.

6 1

$-\underline{\overline{3\ 6}}$

In the above problem, there is a column impasse in the ones since when 6 is taken from 1 the answer is not defined on the set of whole numbers.

Compensation: This means that if the same number is added to both minuend and subtrahend the difference remains unchanged. For instance $61 - 36 = 65 - 40 = 25$

Compound Subtraction: This is used when the need arises to regroup the larger number to subtract as demonstrated in the following example (Grossnickle and Bruekner, 1959)

61 = (five tens + one ten + one ones) = (5 tens + 11 ones)

$-$36 = (three tens + six ones) = (3tens + 6 ones)

Conventional Methods: These are the two most widely used methods namely; Decomposition and Equal- Additions.

Crutch: This is a symbolic learning aid specific to each algorithm (Brownell and Moser, 1949).

Decomposition: This a rule or procedure for solving compound subtraction problem with the set of whole numbers involving renaming the minuend; the minuend is decomposed by increasing ones place value by 10 ones and decreasing the tens place value by one ten (Martin, 1992)

Efficiency/ Speed: This is the ratio of the time of completion to computation accuracy.

Experimental Mortality: This refers to the differential loss of subjects from the different groups that are being compared as a result of illness, family relocation, lost of interest, truancy, etc.

Retention: This is a period of two weeks during which groups work on different topics (other than compound subtraction) under the supervision of group leaders.

Subtrahend: In the example, 61- 36; 36 is the subtrahend.

Understanding: This is the ability to depend on an existing knowledge preferably of a lower task to execute a higher task. For instance, transfer of learning or understanding occurs when pupils are able to use the learning experience gained in solving a two- digit compound subtraction problem to solve a word problem involving three- digit compound subtraction.

Organistation of the Rest of the Study

The study is classified under five main chapters. The first, second, third, fourth and fifth chapter look at the introduction of the study; the review of related literature; methodology; results and discussion; summary, conclusions and recommendations respectively.

Chapter one covers introduction, background to the study, statement of the problem, purpose of the study, the significance of the study, questions, research hypotheses, delimitation, limitation, assumptions, definition of terms and organization of the rest of the study.

Chapter two focuses on the early works in subtraction, methods of doing compound subtraction (the decomposition method, equal additions method), non-conventional method of doing compound subtraction (base-complement additions method), controversy over the superiority of the decomposition and the equal additions methods, empirical evidence, the Brownell-Moser study (1949), recent research efforts, the decomposition versus base-complement method and a summary of the major findings of the literature review.

The third chapter presents the research design, population and sampling, data collection instruments (pre-test, posttest, test of understanding and retention test), procedure, the instructional sequence for the period of intervention, administration of tests, scoring of test items and analysis of data.

The fourth chapter begins with the results of the study. The chapter also highlights speed tests and discusses the results.

The fifth chapter looks at the summary, conclusions and recommendations made by the researcher.

CHAPTER TWO

REVIEW OF RELATED LITERATURE

Introduction

This chapter consists of five main sections. The chapter discusses; the origin of the two most widely used methods of teaching and learning of compound subtraction, the outline of events that resulted in the widespread of the DEC algorithm in the United States of America and other parts of the world, the nature of the three algorithms (DEC, EA and BCA), the controversy over the superiority of the two conventional methods (DEC and EA), their merits and demerits as well as empirical evidence.

Early Work in Subtraction

Through the years, there have been many different procedures used to solve subtraction problems. This development has been of great concern to both investigators and educators. Winch (1920) rightly remarked: “No methods give more trouble and are less successful than those of teaching subtraction” (p. 207). According to Thorndike (1921) the controversy of how children should be taught subtraction has to do with either the use of “subtractive” or “additive” method. This readily brings to mind the comparison of the DEC and EA methods. Smith (1925) minced no word in pointing out that the terminology “subtraction” had “varied greatly and is not settled even now”. For example, terms like “subtract 3 from 6" or "take 3 from 6" or "6 minus 3" are used to talk about subtraction. Grossnickle and Brueckner (1959) were of the view that subtraction had attracted more investigation than any topic in arithmetic. Martin (1992) rightly stated that more than sixteen studies and a number of articles investigating the relative merits of various procedures of doing compound subtraction were published in the twentieth century. In brief, the above discussions point to the fact that studies on compound subtraction dates back to the early 1900s.

Methods of Doing Compound Subtraction

There are a number of methods of doing compound subtraction. These methods could be classified under two main headings namely; the conventional and non-conventional methods. The most widely used conventional methods are the DEC and the EA. Examples of the non-conventional methods are the Austrian, Base-complement additions, Complementary and Residue methods.

The Decomposition Method

The decomposition method, which is widely used today, is also known as the borrowing method. However, the term borrowing may be a misnomer since it suggests that something needs to be returned, which is not clearly seen in the decomposition method. When this method was first introduced in the United States around 1821 (Johnson, 1938), the word "borrow" was used.

The technique used in the DEC method renames the minuend by making use of the concept of place value. For instance in solving 64 – 27, 64 is renamed as 5 tens and 14 ones. Thereafter two subtractions are carried out to solve the above example.

i 14 ones – 7 ones = 7 ones and

ii 5 tens – 2 tens = 3 tens

This consequently gives 3 tens and 7 ones which could be expressed in symbols as i.e. $64 - 27 = 37$. This can be expressed vertically as follows:

	TENS	ONES		T	O		T	O
64	6	4	=	5	14	=	$^{6}5$	14
−27	− 2	7		− 2	7		− 2	7
							3	7

The proponents of this method claimed that it is associated with a number of advantages. These include:

i that it has the ability to demonstrate the regrouping procedure using bundle sticks and the Dienes multi- base arithmetic blocks.

ii that it helped children to learn our system of decimal numeration better than other methods.

iii that it is easier to rationalize and generalize beyond whole numbers to mix or denominate numbers.

iv that with regrouping, the process of making subtraction is seen as the reverse of addition.

Despite the fact that the decomposition method has a number of advantages it has some drawbacks. The most obvious drawback of the method becomes evident when successive zeros occur in the minuend.

The Equal Additions Method

The Equal- Additions method can be traced back to the fifteenth and sixteenth centuries (Johnson, 1938). It was also called the borrow and repay method. The term borrow closely fits this method than the decomposition method, since in this method, a power of ten is borrowed and added to the digit in minuend where the impasse occurs and repaid by adding the same power of ten to the subtrahend. The Equal- Additions method is based on the principle of compensation (Suydam and Desert, 1976). Thus, the difference remains unchanged if the same number is added to both the minuend and subtrahend. It is worthy to note that in equal– additions method, each time the need arises to add a number to the minuend; a compensation addition is made in the subtrahend. Copeland (1976) looks at the the Equal- Additions method under two sub- headings:

i the Subtract- Equal Addition method.

ii the Add- Equal Addition method.

In using the general EA method, 10 is added to both the minuend and subtrahend. It is interesting to note that these additions are carried out in the ones and tens column of the minuend and subtrahend respectively. For example, when the Subtract-Equal Additions method is used to solve 64 – 27, the minuend becomes 6 tens and 14 ones whereas the subtrahend turns to be 3 tens and 7 ones. Two subtractions are then carried out as follows:

i. 14 ones – 7 ones = 7 ones.

ii. 6 tens   3 tens = 3 ones.

Thus the answer is 3 tens and 7 ones which is 37. Vertically, it can be expressed as follows:

$$\begin{array}{cc} 6 & 4 \\ -2 & 7 \end{array} \qquad \begin{array}{cc} T & O \\ & 10 \\ 6 & 4 \\ -2^{1} & 7 \end{array} \quad = \quad \begin{array}{cc} T & O \\ & 14 \\ 6 & \cancel{4} \\ -\cancel{2}^{3} & 7 \\ \hline 3 & 7 \\ \hline \end{array}$$

The school of thought which has unflinching support for the EA method is of the view that it results in faster and accurate computations as well as solving the difficulties associated with the decomposition method when successive zeros occur in the minuend. It has been demonstrated (Olhsson, Ernest and Rees, 1992) that the learning of DEC method conceptually required almost twice as much effort as any other method. Even though the proponents of the EA method gave a number of advantages, it has some disadvantages. One distinct disadvantage is that, it is difficult to objectify and rationalize. According Gagg (1954) the Equal-Additions method is not as logical as others to children. They will realize that adding 10 to each does not change the result (p. 30). Despite the above stated shortcoming, the author went the extra mile to endorse the method by rightly stating that “ this is just one of the rare occasions when many children will just have to believe what you say about the method being accurate” (Gagg, 1954)

Another disadvantage of the EA method is the problem with nine or successive nines in the subtrahend. Martin (1992) minced no word in pointing to the fact that experience shows that it should not be overlooked. In solving a problem like 125- 97, there appears to be a place value problem with the nine in the subtrahend. According to Martin (1992) this difficulty can be surmounted if it is carefully solved during instruction time as demonstrated in her work.

Non- Conventional Methods of doing Compound Subtraction

The difficult nature of the topic subtraction as stated earlier has been the headache of mathematics educators over the years. In view of this, several studies have been carried out to arrive at a more appropriate method of solving compound subtraction that demands a minimum number of memories. Some of these non-conventional methods include the Austrian, Base- Complement Addition, Residue, Complementary and Colton.

A subtraction algorithm known as Colton method was introduced in 1980. This method is sometimes referred to as subtraction without borrowing. Colton (1980) made the following strong points in attempt to comment on the potency of the method:

i. that only one adjustment is necessary in any subtraction problem.
ii. that only subtraction facts up to 9 are required and that they are the ones used frequently.
iii. That the same technique is employed if the minuend contains zeros.

For instance, in solving $50045 - 35867$ we have:

$$
\begin{array}{rcr}
50045 & = & 49999 + 46 \\
-\underline{35867} & & -\underline{35867} \\
 & & 14132 \\
 & & \underline{+\,46} \\
 & & \underline{14178}
\end{array}
$$

In the above example, 50045 is expressed as $49999 + 46$ by renaming $10{,}000$ as 9999 + 1 and 1 is added to 45.

Vance (1982) decried the Colton method on the ground that it made use of borrowing or regrouping. Vance not only criticized the Colton method but also came out with an algorithm known as the Residue method. Vance claimed that the residue method did not make use of borrowing; this observation was made while supervising his students who were using the method in solving compound subtraction.

Vance used residue synonymously as subtrahend. For instance:

(i)

$$
\begin{array}{ccl}
6\;1 & & \;\;6\;\;1 \\
\underline{-3\;8} & = & \underline{-3\;\;8} \\
 & & \;\;\underline{2\;\;3} \\
 & & \;\;\downarrow\;\;\downarrow \\
 & & \;\;\downarrow\;\;(10-8)+1 \\
 & & 5-3
\end{array}
$$

(ii) $\begin{array}{r} 738 \\ -\underline{397} \end{array}$ $\begin{array}{r} 738 \\ -\underline{397} \\ \underline{341} \end{array}$ -

$8-7$

$(10-9)+3$

6 -3

Despite his claim, Vance (1982) was convinced by a colleague to accept the fact that both methods (Colton and Residue) employed some level of borrowing.

The Austrian method is the additive form of the Equal- Additions method. The method was initially introduced by Buteo in 1559. It is has been demonstrated (Martin, 1992) that the Austrian method is widely used in North-Eastern United States of America as well as recent immigrants to the United States.

The Complementary method is another example of the non- conventional methods. Copeland (1976) suggested that the complementary method follows almost the same principles as the residue method.

According to Copland (1976) the rationale for the complementary method is based on the idea of the complement of a number. He defined complement of a number as the difference between that number and the next higher power of ten (10). For example, the complement of 7 is $10 \quad 7$ or 3, the complement of 72 is $100-72$ or 28 and the complement of 825 is $1000-825$ or 175. In using the complementary method to solve a subtraction problem, we add the complement of the subtrahend to it. For example, in solving the problem 78- 56, we add the complement of 56 which is 44 to the minuend and subtract 100 from the result. The procedure is illustrated below:

Thus – 56 is (+ 44- 100)

It implies that

$$\begin{array}{lcl} 78 & & 78 \\ -56 & = & (+44-100) \end{array}$$

(122- 100) = 22

In the above example, cancelling the last digit to the left is the same as subtracting 100 from 122.

$$\begin{array}{rcr} 78 & & 78 \\ -56 & & +\underline{44} \\ & & \underline{\cancel{1}22} \end{array}$$

The procedure in which the last digit of the subtrahend is taken away from 10 and the remaining (numbers under tens column, hundred column, thousand column, etc.) from 9 before the result is added to the minuend is demonstrated in the example below;

$$\begin{array}{r} 321 \\ -\underline{176} \end{array}$$

3 2 1   6 from 10 is 4 and 4 + 1 = 5

9 9 10

<u>- 1 3 6</u>   3 from 9 is 6 and 6 + 2 = 8

1 8 5   1 from 9 is 8 and 8 + 3 = 11

Copland (1976) maintained that it is easy to learn the complementary method, however, it is difficult to rationalize it.

The Base- Complement Additions Method

The non- definitive nature of the results of studies conducted on the two conventional methods (DEC and EA) as well as the inherent limitations of these algorithms have prompted a number of people to research for a more efficient and reliable method that can help pupils to solve subtraction problems easily.

The BCA is a modification of the EA method (Gyening, 1993). As a variant of the EA method, the BCA is based on the principle of compensation.

Brykit (1988) is of the view that if subtraction by restoration is practiced to the same extent as subtraction by borrowing, it could be observed to be generally faster and more accurate. This idea earned the support of Armar and Brown (1971).

Under the Base- Complement Addition method, the same number is added to both the minuend and subtrahend. The motive behind it is to transform the compound subtraction problem to a simple subtraction problem. As the name implies, it makes use of the complement system. For instance, in solving 52- 37, 3 (i.e. the base ten complement of 7) is added to both the minuend and subtrahend. This consequently transforms the initial problem into 55- 40 which is more or less a simple subtraction problem.

The above process is demonstrated vertically as:

$$\begin{array}{r} 5\ 2 \\ -\underline{3\ 7} \end{array} = \begin{array}{r} (5\ 2 + 3) \\ -\underline{(3\ 7 + 3)} \end{array} = \begin{array}{r} 5\ 5 \\ -\underline{4\ 0} \\ \underline{1\ 5} \end{array}$$

Also, the BCA method can be used to solve 416 – 238 and the procedure is as follows:

First, under the ones column, 2 (i.e. the base ten complement of the subtrahend, 8) is added to both the minuend and the subtrahend. The original problem is then transformed to 418 – 240. A careful study of the tens column indicates that there is another column impasse. In order to overcome the impasse, 6 (i.e. the base ten complements of 4) is added to both the minuend and subtrahend in the form of 6 tens. This transforms the problem to 478 – 300 (i.e. a complete simple subtraction problem). Thereafter, three subtractions are carried out as:

(i) 8 ones – 0 = 8 ones

(ii) 7 tens – 0 = 7 tens

(iii) 4 hundreds – 3 hundreds = 1 hundred

The final result is; 1 hundred, 7 tens and 8 ones (i.e. 178). The above process can be expressed vertically as follows:

$$\begin{array}{r} 4\ 1\ 6 \\ -2\ 3\ 8 \\ \hline \end{array} = \begin{array}{ccc} H & T & O \\ 4 & 1 & (6+2) \\ -2 & 4 & (8+2) \\ \hline \end{array} = \begin{array}{ccc} H & T & O \\ 4 & 1 & 8 \\ 2 & 5 & 0 \\ \hline \\ \hline \end{array}$$

$$\begin{array}{ccc} H & T & O \\ 4 & (1+6) & 8 \\ -2 & (4+6) & 0 \\ \hline \end{array} = \begin{array}{ccc} H & T & O \\ 4 & 7 & 8 \\ -3 & 0 & 0 \\ \hline 1 & 7 & 8 \\ \hline \end{array}$$

The following are some of the advantages associated with the BCA method:

i. it could be meaningfully taught using concrete objects or materials (i.e. unlike the EA, the BCA can be objectified or rationalized easily).
ii. it requires fewer additional subtraction facts. (i.e. the nine base ten complements).
iii. it does not flout the normal place-value concept in our numeration system.
iv. it is much easier to do compound subtraction using the BCA method when the problem is expressed horizontally.

Common Inherent Limitations of the DEC and EA Methods

The common inherent limitations of the DEC and EA methods include the following:

i. that there is the need to learn thirty- six additional subtraction facts. This impinges cognitive stress on the learner.
ii. that cancelling some digits in the original problem and substituting them with “crutches” makes working untidy.
iii. recording of the working is at variance with the numeration system where a single numeral has a place value in addition to its face value.

Controversy over the Superiority of DEC and EA Methods

The controversy over the superiority of the two methods dates back to the early 1900s. In 1918, McClelland compared the Equal- Additions method and Decomposition method and concluded: that the method of Equal-Additions appears superior in speed, accuracy, and adaptability to new conditions, while the method of decomposition is superior after long practice (Osburn, 1927, p. 239).

Carpenter (1981) rightly indicated that computation is important and since computation is invariably based on algorithms, what is needed are algorithms that student will remember and use rapidly and accurately to solve routine problems. In view of this, it is obvious that both DEC and EA have immense roles to play.

According to Bruner (1965) the BCA method could be taught right from the enactive stage through the iconic to the symbolic stage. Gyening (1993) in support of Bruner, made mention of how easily and efficiently the BCA method could be demonstrated using concrete materials. He demonstrated the few steps involved in the method, thus, leading to accuracy of computation and speed. Also the method helps pupils to have high rate of retention.

Empirical Evidence

Early Research Efforts.

According to Brownell (1947) available literature has it that most studies conducted point to the fact that the EA method was superior to the DEC method. The research studies discussed below buttress the above assertion.

Ballard (1914) conducted a research based on a timed arithmetic test involving 18, 600 subjects ranging from 8 to 14 years. The results indicated that the EA subjects were far better than DEC subjects on measures of speed and accuracy. He also made it known that the subjects of the DEC were at disadvantage where there were zeros in the minuend.

Winch (1920) conducted two studies on subtraction. In his first study he used two classes of older girls (girls in standard V and V1b who are aged between 12 and 14 years) conversant with only the DEC method. The subjects were put into two equal and parallel groups on the basis of their proficiency in subtraction. One group was given eight short lessons in equal additions and the other the same number of lessons in decomposition. The result of the study was significant in favour of the EA method on measures of accuracy and speed. The second study involving eight and nine year olds confirmed the findings of his first study.

Johnson (1938) carried out studies involving several methods of subtraction in 1924 and 1931 respectively. The subjects of these studies were college students, adults and fifth through eighth graders. These studies yielded significant results in favour of the EA method on measures of accuracy and speed. Johnson conducted his third study in 1938 with 1046 third graders through eighth graders and 43 adults. The study compared performance efforts using the DEC, EA and the Austrian method to subtraction on measures of accuracy and speed. The summary of the conclusions drawn by Johnson (1938) is as follows:

all other things being equal, the decomposition method in subtraction of whole numbers is, by its own intrinsic nature, the poorest method to employ from the stand point of accuracy and speed. When compared with the Equal- additions method, the decomposition method produces 18 percent more errors and requires 15 percent more speed.

Murray (1941) studied speed and accuracy of over 3,000 children comparing the DEC, EA and Austrian methods of subtraction. In this study, Murray used 1662 pupils who were being taught the compound subtraction for the first time and 1675 pupils, whose ages ranged from 10 to 11years and already familiar with subtraction. Murray came to a conclusion that the DEC subjects were significantly inferior to subjects of both the EA and Austrian methods. As a result of the influence of his findings on the committee of primary school subjects, it was recommended that the EA method be adopted as the sole method to be used in the schools of Scotland.

The Brownell- Moser Study (1949)

Brownell and Moser used 1400 third grader pupils from four urban areas of North Carolina. Owing to differences in arithmetic background, three of the centres were organized with their results analyzed separately. Classes at respective centres were divided into two in order to ensure that each group (DEC or EA) had the same number of subjects. Thereafter, these groups were subdivided so that each group using each method learnt it meaningfully and the other mechanically. The four experimental sections were; decomposition taught mechanically, decomposition taught rationally, equal- additions taught mechanically and equal- additions taught rationally.

At the end of the study, the researchers came out with the following major conclusion:

i. rational decomposition was better than mechanical decomposition or equal- additions method on measures of understanding and accuracy.

ii. rational equal-additions were significantly better than mechanical equal-additions on measures of understanding and accuracy.

iii. mechanical decomposition was not as effective as either equal-additions method or decomposition method. It was concluded that whether to teach the equal-additions or the decomposition method depends on the desired outcome.

In addition, there were other specific findings of the Brownell- Moser study (1949). The focus of earlier research on speed and accuracy was too limited to provide a basis to determine the relative merits of DEC and EA. Meaningful or rational EA was significantly better than the mechanical EA on measures of understanding.

i. Teachers reported that the EA approach is difficult to teach meaningfully. Teachers of the DEC did not report any such difficulty.

ii. In connection with evidence gathered regarding the benefits of using a crutch, teachers indicated that with DEC crutch was easily discarded without much difficulty by most of the children.

iii. The longer route of teaching rationally provided a better result over the duration of the experiment, supporting Brownell’s hypothesis that meaningful learning produces better result than the mechanical approach. In the researcher’s words, the advantage in understanding of the decomposition subjects who were taught rationally "actually functioned" six weeks later (Brownell, 1947: 169).

Weakness of the Brownell- Moser Research

The following are the weaknesses of the Brownell- Moser study:

i. in both methods, one has to recall thirty-six additional subtraction facts when solving compound subtraction problem (Gyening,1993). This put strains on the mind of the learner.

ii. the cancellation of some digits in the minuend and subtrahend and substituting them with crutches make the work appear untidy.

iii. both methods cannot be expressed in a horizontal form as the crutches make the problem assumed different form.

In summary, Seville (1964) was of the view that the results as to which method is superior were inconclusive.

Recent Research Effort

The belief that there is always the best way of doing everything has urged researchers to find an algorithm flexible enough in handling subtraction problems, especially in the wake of the controversy over the superiority of the decomposition and equal additions methods. Some of the recent research efforts are discussed in the subsequent paragraphs.

Sherrill (1979) carried out a study on comparison of subtraction algorithms (DEC and EA) involving pupils in an elementary school. Based on results from a forty (40) item computation test, he came to a conclusion that the DEC was superior to EA on the measure of accuracy. In addition, he found out that EA subjects made more digit reversal errors. According to Sugai and Smith (1986) the major flaw of this study was the absence of pre-test.

In the maiden study of Sugai and Smith (1986), the researchers expressed doubt about the certainty that children understand the role of place value in regrouping. The researchers dwelled on the efforts of Johnson (1958) who was of the view that the concept of place value was equally necessary for the EA method. Sugai and Smith (1986) deduced that the EA may be an effective method for teaching subtraction involving regrouping and concluded that the performance of EA subjects was better than that of the DEC subjects.

Sugai and Smith (1986) conducted another study with seven learning disabled elementary aged children. First, all the subjects were instructed with the DEC algorithm. Thereafter, a modeling technique was used to teach the EA algorithm. A significant increase in performance was recorded only after the introduction of EA. Again, the EA method was associated with generalization of the procedure to some untrained problems for the seven subjects.

Ohlsson, Ernest and Rees (1992) came out with the following conclusions after conducting two computer simulation experiments.

Conception instruction caused more cognitive work than procedural instruction. This view holds for both subtraction methods, however, the procedural instruction was far better.

Learning DEC procedurally takes almost twice as much effort as the other. The results of their study pointed to the fact that there was no significant difference in the level of difficulty of performing the two methods, once they are learnt. In addition it was observed that it is more difficult to learn the regrouping method, especially, when taught conceptually rather than procedurally. In the nutshell, the researchers maintained that EA was easier to learn under the procedural condition than the conceptual condition. It is worth noting that the conclusions of this study contradict that of Brownell (1947) as well as Brownell and Moser (1949).

Cheryl Martin (1992) investigated into two methods of subtraction involving 178 second and third grades subjects. Not only that the investigation was limited to the comparison of DEC and EA but also it was extended to cover the attitudes of children towards the study of mathematics in connection with these methods. In the study, Martin gave forty minutes lessons in DEC and EA methods that lasted for two weeks. The major findings of the study are enumerated below as follows:

i. there were no significant differences in efficiency between the DEC and EA groups among the subjects. However, differences which occurred were in favour of the DEC group.

ii. the DEC produced better computation accuracy among the second and third graders than the EA method. The difference was greater in the case on of the third graders than that of the second graders.

iii. it was concluded that either method produces significant transfer results for third graders; however, the DEC method produces better transfer results in second graders.

iv. concerning the retention measures in her study the conclusion was that the immediate posttest and delayed posttest mean of the DEC group was higher than that of the EA group in the third grade, but the reverse was true for second grade.

v. besides the results of computation retention are mixed. Information gathered from the data indicated that no conclusions were made regarding a preferred method for retention of computation accuracy. The means at both grade levels in the delayed posttest were higher than the EA means.

vi. concerning transfer in immediate posttest and delayed posttest, it was concluded that the DEC was preferred to the EA method for retaining transferability. This was true for the second graders. With respect to the third graders it was still found that DEC is preferred to EA for retention of transferability on immediate posttest and delayed posttest.

The major drawback of this study was that Cheryl Martin's qualification of teaching subtraction was limited to the decomposition algorithm. Therefore she employed the services of an implementer and spent 45 minutes daily discussing the instructional sequences with the implementer throughout the two-week instruction period. This development affected the study since the implementer raised concern about presenting the EA method as effectively as the DEC method.

The Decomposition Method versus Base-Complement Additions Method

In recent times, studies have been conducted to arrive at a more appropriate and reliable method of solving compound subtraction problems that demand minimum stress on the memories of subjects. One of such methods is the Base- Complement Additions method (BCA).

McCarthy (1994) conducted a research into relative efficiency of BCA method of solving compound subtraction as compared to the DEC on measures of accuracy. The subjects were eight year olds from University of Cape Coast Primary School in Cape Coast Municipality. The researcher used seven days in experimental teaching. Results from this study indicated that the mean scores of the subjects of the BCA group were higher than that of the DEC group. However, statistical analysis showed that there was no significant difference between the two groups on the measure of accuracy. Even though the researcher wanted to determine efficiency, the study was however silent over speed.

Essel (2000) carried out a similar study with 64 primary three pupils of seven schools (six public and one private) all at Breman Asikuma. Experimental mortality reduced the number of subjects to 36. The study which lasted for four weeks focused on the measures on speed, accuracy, retention and understanding all tested at 0.05 level of significance. The researcher used t-test and the median tests for the statistical analysis. In this study, speed was measured as the ratio of time spent on test items to scores. Even though subjects of the BCA group scored higher marks on the posttest, the results were not significant on measures of speed, accuracy, retention and understanding.

Appiah (2001), researched into a comparative study on the BCA method and DEC method with 59 primary four pupils at Ajumako on measures of speed, accuracy, retention and understanding, all tested at 0.05 significant level. The study lasted for four weeks. The researcher employed the analysis of covariance and the median test. In this study, speed was measured as the ratio of time spent on test items to scores. The results of the study revealed that there were significant difference on measures of speed, accuracy, retention and understanding.

The researcher made the following observations based on the foregoing studies.

i. These studies were conducted at different levels and geographical areas.

ii. The subjects of these studies were either randomized or taken as intact group.

iii. The mean scores were higher and most often than not significant in favour of the BCA group (McCarthy, 1994; Essel, 2000; Appiah, 2001).

iv. In all the studies except Essel (2000) and Appiah (2001), speed was measured without relating the finishing times to the respective scores. This was not good enough since the fastest child could do so with poor score.

Summary of the Major Findings of the Literature Review

A brief overview of the related literature indicated that subtraction algorithms had been subjected to a spectrum of studies.

Early research works were in favour of the EA method (Johnson, 1938; Brownell, 1947; Brownell and Moser, 1949; Sherill, 1979). The Brownell (1947) study recommended the teaching of subtraction through meaningful development of the DEC procedure. This made it popular and consequently became the most widely used subtraction algorithm in the United States of America. However, the Brownell- Moser (1949) study left a number of questions unanswered. The introduction of EA and the controversy over the DEC and EA raised a lot of concern.

Furthermore, this controversy urged researchers to find a more reliable and efficient method that can help pupils to solve subtraction problems with little or no difficulty. The search finally gave birth to the BCA method which has made appreciable gains over the DEC on measures of speed, accuracy, retention and understanding (Essel, 2000; Appiah, 2001).

CHAPTER THREE

METHODOLOGY

This chapter is made up of the following subsections: research design, population and sampling, instruments, data collection and data analysis procedures.

Research Design

The study was experimental in nature. The researcher used pretest-posttest design. The study allowed for the control of relevant variables while permitting the examination of the effect of some definite variables (i.e. one teacher taught both the experimental and control groups). A computational pretest was administered to the subjects of both groups (DEC and BCA) prior to the two weeks intervention. This was done to gather information on measures of speed and mechanical accuracy. Thereafter, the researcher taught each group one method of compound subtraction for ten working days and each lesson lasted for one hour. The intervention was followed by immediate posttest and retention test a day and two weeks respectively after the intervention. The purpose of these tests was to gather information on measures of speed, accuracy, retention and understanding. Finally, the test of understanding (delayed posttest) was administered to the two groups a fortnight after the retention test to gather information on measures of speed, accuracy and retention of the knowledge gained during the period of intervention.

Population and Sampling

The population of the Upper Denkyira East Municipality is about two hundred thousand (data from the Municipal Assembly-Planning and Statistics). It shares the northern boundary with Amansie West District, southern with the Twifo-Hemang Lower District, western with Upper Denkyira West District and eastern with the Assin South Municipal Assembly. The seat of the head of administration is situated at Dunkwa-on-Offin. There are 74 primary and 68 Junior High Schools in the Municipality. The inhabitants are mostly farmers, traders, business men, unlicensed gold miners and government workers.

The target population of the study was primary three pupils and the accessible population comprises pupils from Jesus Cares and St. Peters Preparatory Schools. The teachers of these two classes are untrained teachers. The schools which are 1km apart were selected owing to their proximity to the residence of the researcher.

The researcher used 60 pupils of which 31 are boys and 29 are girls. The ages of the pupils range from 8 to 12 years. The pupils are the wards of parents from varied socio-economic background. With this in mind the researcher had the premonition that the level of motivation enjoyed by the subjects varied from pupil to pupil depending on the occupation of the parents.

The sample from St. Peter’s Preparatory School were coded X and those from Jesus Cares Preparatory School were coded Y. Sample X comprised 16 girls and 14 boys also adding up to 30 pupils, while sample Y was made up of 13 girls and 17 boys adding up to 30. The researcher collected information on the ages of the subjects to find out whether they have the same level of chronological age. The result indicated that the subjects of sample X (DEC group) had a mean age of 8.6 years and a standard deviation of 0.81 whereas the subject of sample Y (BCA group) had a mean age of 8.8 years and a standard deviation of 1.21.

The choice of sample was based on purposive sampling technique in order to select subjects who were relevant to the study. The researcher used simple random sampling method to assign subjects to the groups involved to ensure that all units of the target population had an equal chance of being selected. The researcher prepared the lists of the pupils in these classes in alphabetical order and assigned a number to each name on the list (sample frame). He then constructed the table of random numbers. The researcher randomly selected a starting place, went through the table across the rows and listed the numbers as they appeared on the table. Pupils of the class with the selected numbers constituted the sample. Thereafter, numbers (1 and 2) were assigned to the teachers of these two classes. The researcher repeated the process, however, the first number selected was associated with B whereas the second was associated with D (where B and D represented Base-complement addition and Decomposition algorithm respectively). Based on the outcome of the exercise, pupils selected from St. Peter's Preparatory School and Jesus Cares Preparatory School were assigned to the Decomposition and Base-complement addition method respectively.

Data Collection Instrument

The data collection instrument of the study was the use of test items. The pretest, accuracy test, retention test and test of understanding were made up of twenty items each. The pretest was made up of fourteen simple and six compound subtractions problems. Both posttest and retention tests were made up of one simple and nineteen compound subtraction problems, while the test of understanding comprised twenty compound subtraction problems of which eight are word problems (see Appendix A, B, C and D).

Pretest

The pretest items consisted of twenty subtraction problems (see Appendix A). In all, eighteen problems were presented in the vertical form whereas two problems involving renaming were in the horizontal form.

The subtraction problems were made up of thirteen simple subtraction problems and seven problems involving renaming.

Posttest

a) Accuracy test

The accuracy test was made up of twenty items; four of them were subtraction problems without regrouping (all presented in vertical form) and the remaining were subtraction problems involving regrouping (two presented in horizontal form and the rest in vertical form (see Appendix B).

b) Test of understanding

Under the test of understanding, twenty test items on subtraction were constructed. Of these, twelve were presented in vertical form whereas the remaining eight items were word problems (see Appendix C). The purpose of the test was to find out pupils' ability to transfer knowledge gained to solve complex and word problems.

c) Retention test

The retention test was made up of twenty test items on subtraction involving regrouping. All of test items were arranged in vertical form except one (see Appendix D).

Piloting the instruments, the researcher administered the pretest, posttest, retention test and the test of understanding items at Dunkwa Offin Presbyterian Primary School (primary three) to find the reliability of the instruments. This was, however, done after the instruments had been prepared and the face and content validity had been ascertained by the supervisor of the researcher.

The piloting was carried out in March, 2009. The responses of the tests were scored dichotomously (one (1) mark for a correct response and zero (0) mark for an incorrect response). Kuder – Richardson formula (K- R -21) was used to calculate the coefficient of reliability. The reliability coefficients of the pretest, posttest and retention tests were 0.78, 0.81, and 0.79 respectively. The results pointed to the fact that the reliability of the data collecting instruments was high and for that reason the data instruments were reliable.

Procedure

The researcher sent a letter from his supervisor to the Upper Denkyira Municipal Director of Education. The Municipal Director indicated at the bottom of the letter that all concerned headmasters should give the researcher the necessary assistance. The researcher visited head teachers of Presbyterian Primary School, St. Peter's Preparatory School and Jesus Cares Preparatory School to inform them about the researcher's intention to conduct the study in their schools. The heads consequently introduced the researcher to the class teachers. Thereafter, the class teachers were briefed on the nature of the data collection exercise in their individual schools. Next, the pupils for the study were selected and interviewed and a pretest was conducted the following day. This was followed by an intervention which lasted for four weeks. An accuracy test was administered a day after the intervention period. The retention test and test of understanding were administered two weeks and fifteen days respectively after the accuracy test.

The researcher met the pupils of the two groups four times a week at their school premises and each instructional period lasted for an hour. More exercises were assigned to the BCA group in the sense that it is a new method. This was done to assist the BCA group to be at par with the subjects of DEC group who had already received tuition from their class teacher. Messages were sent to parents through the pupils by the researcher to allow them study on their own during the treatment period. This decision was taken in order to ward off contamination. In order to monitor the pupils effectively, the researcher prepared attendance sheet for each group that helped in checking their attendance. The experimental teaching was conducted from 3rd – 17th March, 2009.

Instructional Sequence for the Period of Intervention

The main features of the instructional activities for the DEC group are stated below as follows:

1. Revision of simple addition of whole numbers using concrete materials.
2. Revision of simple subtraction of whole numbers using concrete materials.
3. Doing simple subtraction without using concrete materials.
4. Doing addition with renaming or regrouping using concrete materials.
5. Doing addition with renaming or regrouping without using concrete materials.
6. Representation of compound subtraction involving two- digits number with concrete materials.
7. Representation of compound subtraction involving three-digit number with concrete materials.
8. Iconic representation of compound subtraction involving `two and three-digit number.
9. Symbolic representation of compound subtraction.
10. Solving compound subtraction problems with the aid of crutches.
11. Revision of compound subtraction problems involving two-digit and three- digit numbers (see Appendix G).

The principal features of the instructional activities for BCA group are as follows:

1. Revision of addition of wholes numbers; simple addition and addition with regrouping.
2. Concept of base ten complement and identification of base ten complement.
3. Concrete representation of compound subtraction; two-digit minuend and one-digit subtrahend as well as two-digit minuend and two-digit subtrahend.
4. Transformation of compound subtraction involving two- digit subtrahend into simple subtraction problem.
5. Solving two-digit compound subtraction problems at the concrete stage.
6. Solving two and three digit compound subtraction problems.
7. Solving compound subtraction problems.
8. Symbolic representation of compound subtraction.
9. Revision of compound subtraction problems involving two-digit and three -digit numbers (see Appendix F).

Administration of Tests

The researcher printed all the four sets of questions and on each question paper spaces were created for pupils to write their name, class, age and the occupation of their parents or guardians. The sharing and collection of question papers were done by the researcher and the research assistants (trained class teachers). The following time intervals (in minutes); 1.00- 1.04, 1.05-1.09, 1.10-1.14, 1.15- 1.19, 1.20-1.24, etc were used to tally and record the finishing time of the pupils of the BCA and DEC groups. A stop watch was used to time the pupils in each of the test conducted; the start button was pressed at the commencement and end of a test. Pupils were also informed to raise up their hands when they finished their work for the researcher to tally and record the finishing time.

Scoring of Test Items

All the test items were scored dichotomously, that is, either correct or wrong. The marking schemes for the test items were prepared by the researcher and each correct response attracted one point. The researcher marked all the scripts.

Analysis of Data

The researcher gave the same achievement test on pre-test, accuracy test, test of understanding and retention test to the two groups. The respective means, standard deviations and variances were used for statistical analysis. The independent t-test was used to determine whether there was a significant difference between the mean scores of the two groups on the pre-test, accuracy test and retention test. In order to satisfy the condition under which one could use this test, the variances of the test scores of the two groups on pretest were subjected to F_{max} test. The method of pooled variance was used after the confirmation of homogeneity of variance. The researcher used median finishing test to analyze the speed of the pupils of these groups at 0.05 significance level. This test offered information as to whether it was possible that the independent groups were chosen from populations with the same median (Field,2000). The

Statistical Package for the Social Sciences (SPSS) was used in the analysis of the data.

The study was conducted on 60 pupils at primary three. This was made up of two groups namely BCA and DEC, and each group was made up of thirty pupils. The groups received different treatment and achievement test (pre-test, posttest and retention test) which was conducted for the pupils before, during and after the intervention. The results were presented in tables and used to test the hypothesis formulated for the study. The decision to accept or reject the hypotheses was solely based on the tables.

Research Hypotheses

Based on the above research questions the following research hypotheses were formulated for testing, all at 0.05 level of significance:

Hypotheses I:

H_0: There is no significant difference between mean scores of the DEC and BCA group on the post-test with respect to accuracy.

H_1: There is a significant difference between mean scores of the DEC and BCA group on the post-test with respect to accuracy.

Hypotheses II:

H_0: There is no significant difference between mean scores of the DEC and BCA group on the retention test with respect to accuracy.

H_1: There is a significant difference between mean scores of the DEC and BCA group on the retention test with respect to accuracy.

Hypotheses III:

H_0: There is no significant difference between mean scores of the DEC and BCA group on the understanding test with respect to accuracy.

H_1: There is a significant difference between mean scores of the DEC and BCA group on the understanding test with respect to accuracy.

Hypotheses IV:

H_0: There is no significant difference between the median finishing times of the DEC and BCA group on the post-test.

H_1: There is a significant difference between the median finishing times of the DEC and BCA group on the post-test.

Hypotheses V:

H_0: There is no significant difference between the median finishing times of the DEC and BCA group on the retention test.

H_1: There is a significant difference between the median finishing times of the DEC and BCA group on the retention test.

In order to make the analyses of the hypotheses easy and effective the mean ($\bar{x}$), standard deviation (S), the computed t- statistic and t- critical for all the achievement tests were provided. The chapter also discusses the 2×2 contingency tables. The 2×2 contingency tables demonstrate frequencies above and below the common median finishing times of the pupils on the measure of speed. However, the researcher considered the 2×2 contingency tables of the pre- test, posttest and retention tests. This is because the hypotheses of the study were based on these achievement tests. Besides, information on the median finishing times, chi-square and p-values on the measure of speed and common median finishing times have been provided in the tables on the achievement tests.

CHAPTER FOUR

RESULTS AND DISCUSSION

The chapter focuses on presentation of results and discussion on the tables for the entire achievement test (pre-test, posttest and retention test) conducted in the study.

The summary of the components are given in tables 1, 2, 3 and 4. For the detailed information about these tables see appendices I, J, K, L and M.

Posttest

Table 1 shows the mean, the standard deviation (S) and the t-values of the posttest scores.

Table 1

The Mean, the Standard Deviation and the t-Values of the Posttest Scores

				Computed		Critical	
Group	Mean Score	Sd	N	t	p	t	p
BCA	10.86	3.94	29	0.835	0.705	0.710	0.05
DEC	10.00	3.98	30				

The table shows 59 pupils wrote the posttest. Of these, 29 of the pupils were subjects of BCA group, while the remaining 30 were subjects group of DEC. The minimum scores of the BCA and DEC group are 1 and 8 whereas the maximum scores of the BCA and DEC group is 20. The mean score and standard deviation of the subjects of BCA and DEC groups were 10.86 and 10.00, and 3.94 and 3.98 respectively. Using 0.05 significant level, t-computed yielded 0.835 and t-critical was 0.710 with 0.705 and 0.05 as the respective values of p-computed and p-critical. The null hypothesis was rejected and it was concluded that a significant difference exists between the two groups – BCA and DEC in favour of BCA group.

Retention test

Table 2 shows the mean, the standard deviation (S) and the t-values of the retention test scores.

Table 2

The Mean, the Standard Deviation and the t-values of the Retention Test Scores

				Computed		Critical	
Group	Mean Score	Sd	N	t	p	t	p
BCA	9.51	3.97	29	-2.161	0.045	0.035	0.05
DEC	6.54	5.07	28				

The table shows that the retention test was taken by 57 pupils; the pupils of the BCA and DEC groups were 29 and 28 respectively. The minimum scores of the BCA and DEC group is 1 whereas the maximum scores of the BCA and DEC group are 16 and 15 respectively. The mean score and the standard deviation of the BCA and DEC groups are 9.51 and 6.54, and 3.97 and 5.07 respectively. Using 0.05 significant level, t-computed yielded -2.161 and t-critical was 0.035 with 0.045 and 0.05 as the respective values of p-computed and p-critical. The null hypothesis was rejected and it was concluded that a significant difference exists between the two groups – BCA and DEC in favour of BCA group.

Test of Understanding

Table 3 shows the mean, the standard deviation (S) and the t-values of the test of understanding scores.

Table 3

The Mean, the Standard Deviation and the t-Values of the test of understanding scores

				Computed		Critical	
Group	Mean Score	Sd	N	t	p	t	p
BCA	6.00	3.93	28	-2.855	0.085	0.006	0.05
DEC	3.26	3.12	27				

The table shows that 55 pupils took the test of understanding. The BCA and DEC groups were made up of 28 and 27 pupils respectively. The minimum scores of the BCA and DEC groups were 2 and 0 whereas the maximum score of the BCA and DEC group were 14 and 11 respectively. The BCA and DEC had a mean score of 6.00 and 3.26, and a standard deviation of 3.93 and 3.12 respectively. Using 0.05 significant level, t-computed yielded -2.855 and t-critical was 0.006 with -0.085 and 0.05 as the respective values of p-computed and p-critical. The null hypothesis was rejected and it was concluded that a significant difference exists between the two groups – BCA and DEC in favour of BCA group.

Speed

Table 4

A Contingency Table on the Frequencies Above and Below the Median Finishing Time for the Two Groups on Posttest

	BCA	DEC	Total
Above	15	13	28
Below	14	17	31
Total	29	30	59

Table 4 shows that 15 and 13 pupil of the BCA and DEC respectively were above the median finishing time, while 17 and 16 pupils of the BCA and DEC respectively were below the median finishing time.

Table 6 illustrates a 2×2 contingency table for the frequencies above and below the median finishing time for the two groups on retention test.

Table 5

A Contingency Table for the Frequencies Above and Below the Median Finishing Time for the Two Groups on Retention Test

	BCA	DEC	Total
Above	21	10	31
Below	8	18	26
Total	29	28	57

Table 5 shows that 21 and 10 pupils of the BCA and DEC respectively were above the median finishing time, while 8 and 18 pupils of the BCA and DEC respectively were below the median finishing time.

Pre-test speed

Table 7 illustrates the median finishing times of the BCA and DEC groups, the common median finishing time and the corresponding chi- square and p-values for the pretest speed.

Table 6

The Median Finishing Times, Corresponding Chi- Square and p-Values for the Posttest Speed

Median Finishing Time (Minutes)		Common Median Finishing Time (Minutes)	Computed		Critical	
BCA	DEC		χ^2	p	χ^2	p
11.44	21.05	21.05	0.17	0.971	3.84	0.05

Table 6 shows that the common median finishing time of the two groups was 21.05 minutes. Also, the median finishing time of pupils of the BCA and DEC groups were 11.44 minutes and 21.05 minutes respectively. The frequencies in table 4 were computed for chi-square with one degree freedom χ^2. The chi-computed yielded 0.17 and chi-critical was 3.84 with 0.971 and 0.05 as the respective values of p-computed and p-critical. The conclusion was that no significant difference exists between the two groups of pupils in terms of median finishing time.

Table 7

The Median Finishing Times and Corresponding Chi- Square and p -Values for the Retention Test Speed

Median Finishing Time (Minutes)		Common Median Finishing Time (Minutes)	Computed		Critical	
BCA	DEC		χ^2	p	χ^2	p
22.00	29.00	26.00	6.33	1.00	3.84	0.05

Table 7 shows that the common median finishing time of the two groups was 26.00 minutes. In addition, the median finishing time of pupils of the BCA and DEC groups were 22.00 minutes and 29.00 minutes respectively. The frequencies in Table 5 were computed for chi-square with one degree of freedom χ^2. The chi-computed yielded 6.33 and chi-critical was 3.84 with 1.00 and 0.05 as the respective values of p-computed and p-critical. There was therefore, no significant difference between the two groups in terms of median finishing time.

Discussion of Results

This section discusses the posttest, retention test and the median finishing times of the groups with respect to the formulated hypotheses.

Posttest

The scores of the pupils of the BCA group ranged from 2 to18 and that of the DEC ranged from 3 to 20 (see Appendix H). The mean score and standard deviation of the pupils of BCA and DEC groups were 10.86 and 10.00, and 3.94 and 3.98 respectively. Using 0.05 significant level, t-computed yielded 0.835 and t-critical was 0.407 (see Table 2) which implies that there is a significant difference between the mean score of the two groups on the posttest (accuracy). Perhaps the excellent performance of the pupils of the BCA could be due to the fact that the BCA (a variant of the EA method) can be objectified through the use of concrete materials (Gyening, 1993). This result seems to confirm the higher mean score in the studies involving the BCA and DEC methods in which the puipls of the BCA exhibited an impressive performance (MarcCarthy, 1994; Essel, 2000; Appiah, 2001). There was vast difference in means between the pre-test and posttest of both groups (BCA and DEC). It shows that pupils have difficulty in handling subtraction problems involving regrouping. This result seems to confirm that subtraction is a problem area in mathematics (Winch, 1920; Thorndike, 1921)

Retention Test

The retention test was taken by fifty-seven pupils; the pupils of the BCA and DEC groups were 29 and 28 respectively. The scores of pupils of the BCA ranged from 1 to 16 and that of the pupils of DEC group ranged from 1 to 15 (see Appendix I). The mean score and the standard deviation of the BCA and DEC groups are 9.51 and 6.54, and 3.97 and 5.07 respectively. Using 0.05 significant level, t-computed yielded -2.161 and t-critical was 0.035 (see Table 3). It was concluded that there is a significant difference between the mean score of the two groups on retention test.

Unlike the current study, the study of McCarthy (1994) and Essel (2000) did not produce significant results. The impressive performance of the BCA group was commendable. This is because they had to unlearn the DEC algorithm and learn the BCA algorithm at the same time.

Test of Understanding

The scores of 30 pupils of the BCA group ranged from 1 to 20, while that of the DEC ranged from 8 to 20 (see Appendix F). In addition, most of the pupils had test item 13- 20 wrong. This pointed to the fact that subjects lack the concept of regrouping. The BCA and DEC had a mean score of 16.10 and 16.40, and a standard deviation of 3.81 and 2.19 respectively. Using 0.05 significant level, t-computed yielded 0.374 and t-critical was 0.710 (see Table 1) which implies that there was a significant difference in the performance of the two groups as at the time the pretest was conducted. This confirms that the entry level of the two groups was not the same at the pretest stage.

The Speed Test

The speed of pupils of both groups (BCA and DEC) was computed for the pretest, posttest and retention test. Two hypotheses were formulated to measure the speed of the BCA and DEC groups on the posttest and retention test.

Posttest Speed

The 2×2 contingency table (Table 5) indicated that 15 and 13 pupils of the BCA and DEC respectively were above the median finishing time, while 17 and 16 pupils of the BCA and DEC respectively were below the median finishing time. The common median finishing time of the two groups was 21.05 minutes. Also, the median finishing time of pupils of the BCA and DEC groups were 11.44 minutes and 21.05 minutes respectively. The frequencies in table 5 were computed for chi-square with one degree of freedom. The chi-square values led to the conclusion that there is no significant difference between the two groups on the measure of speed on the posttest. The slow pace of pupils of the DEC was due to lack of practice. The finding of the study supports that of McCarthy (1994) and Essel (2000) where there was no significant difference between the two groups on measure of speed on the posttest. However, the finding of the study was in sharp contrast with that of Appiah (2001) where there was significant difference between the two groups on measure of speed on the posttest.

Retention Test

The 2×2 contingency table (Table 7) indicated that 21 and 10 pupils of the BCA and DEC respectively were above the median finishing time, while 8 and 18 subjects of the BCA and DEC respectively were below the median finishing time. The common median finishing time of the two groups was 26.00 minutes. In addition, the median finishing time of subjects of the BCA and DEC groups were 22.00 minutes and 29.00 minutes respectively. The frequencies in table 5 were computed for chi-square with one degree of freedom The values revealed that there was a significant difference between the two groups on the measure of speed on the retention test. This result confirms the findings of Appiah (2001). On the contrary, this result was different from that of McCarthy (1994) and Essel (2000).

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The chapter summarizes the study and offers suggestions and recommendations for further research work in future.

Summary

The controversy over the superiority of the BCA and DEC methods as well as pupils' inability to handle compound subtraction problems effectively as reported by National Assessment of Educational Progress (1988) and Criterion Reference Test became an issue of great concern to all stakeholders. In 1993, Gyening of University of Cape Coast, in a paper presentation at a departmental seminar enumerated the advantages of the Base-Complement Addition which is a variant of the Equal Additions Method. In this study, the researcher compared two methods of solving compound subtraction (BCA and DEC) on measures of speed, accuracy and retention.

The researcher used pupils of St. Peter's and Jesus Cares preparatory schools (All in Upper Denkyira East Municipality) for the study. Subjects of the groups were taught for nine working days in their respective schools. Pretest, posttest, test of understanding and retention test were conducted a day before, a day after, three days and two weeks respectively after the intervention period.

The t-test and median test were used to compare the two groups on posttest and retention test on measures of accuracy and speed at 0.05 significant levels.

The major findings of the study were as follows:

1. There was a significant difference between the two groups on the measure of speed in favour of the BCA group as far as the posttest and retention test were concerned.
2. The subjects of the BCA group performed significantly better than their counterparts in the DEC group on accuracy.
3. There was a significant difference between the two groups on the measure of retention. It was established the BCA method could promote better retention than the DEC method.

Conclusion

Based on the results of the study, it could be deduced that the BCA method made appreciable gains over the DEC method. The following conclusions were drawn:

The Base-Complement Additions group had a significantly less speed than the Decomposition group. This implies that the BCA method induces much speed in children than the DEC method.

This finding also shows that the BCA method has the potentials to assist pupils to give more accurate answers to compound subtraction problems than the DEC method. The findings on the retention test revealed that pupils using the BCA method have better ability to retain or consolidate the concept of subtraction; more especially those involving regrouping than pupils using DEC method. The BCA method being a variant of the EA can be taught meaningfully to pupils by using enactive, iconic and symbolic instructional activities (see appendix F). It corrects all deficiencies in the EA method and for that reason the BCA is superior to the EA method. Therefore, it is not surprising that the results of the present study show that the performance of pupils of the BCA group was better than pupils of the DEC group.

In brief, the impressive performance of subjects of the BCA group on the measures of speed, accuracy and retention suggests that the BCA method could be more reliable, efficient and easier for pupils to use than the DEC method.

Recommendations and Suggestions

In view of the findings of the study which indicates that the BCA method made appreciable gains over the DEC method on measures of speed, accuracy and retention the following recommendations were made:

1. The sample size should be adjusted upwards and the study should be conducted in different geographical areas in order to make the findings of a study more valid and reliable.
2. Students should be encouraged to conduct further research on the BCA method so as to verify the potency of the BCA method.
3. The intervention period should be extended. Nine lessons of which each lasting sixty minutes was woefully inadequate for the BCA group since that was their first time they were introduced to the base-complement additions method.

4. The method should be subjected to investigation at all levels of the first cycle institution.
5. Seminars, workshops and conferences should be organized for teachers, policy makers and curriculum developers on the base-complement additions method

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APPENDICES

APPENDIX A

DEPARTMENT OF SCIENCE AND MATHEMATICS EDUCATION

FACULTY OF EDUCATION

UNIVERSITY OF CAPE COAST

EXPERIMENT ON COMPOUND SUBTRACTION

PRETEST

NAME: ………….....………………………………………… CLASS: ………….

AGE: ………… PARENTS/ GUARDIANS OCCUPATION: ………………

1. $\begin{array}{r} 45 \\ \underline{32} \\ \underline{\quad\;} \end{array}$
2. $\begin{array}{r} 38 \\ \underline{21} \\ \underline{\quad\;} \end{array}$
3. $\begin{array}{r} 456 \\ \underline{323} \\ \underline{\quad\;} \end{array}$
4. $\begin{array}{r} 46 \\ \underline{42} \\ \underline{\quad\;} \end{array}$
5. $\begin{array}{r} 585 \\ \underline{427} \\ \underline{\quad\;} \end{array}$
6. $\begin{array}{r} 57 \\ -\underline{26} \\ \underline{\quad\;} \end{array}$
7. $\begin{array}{r} 443 \\ -\underline{235} \\ \underline{\quad\;} \end{array}$
8. $\begin{array}{r} 246 \\ -\underline{23} \\ \underline{\quad\;} \end{array}$
9. $\begin{array}{r} 265 \\ -\underline{47} \\ \underline{\quad\;} \end{array}$
10. $\begin{array}{r} 468 \\ -\underline{353} \\ \underline{\quad\;} \end{array}$
11. $35 - 18 = \square$
12. $\begin{array}{r} 2456 \\ -\underline{1243} \\ \underline{\quad\;} \end{array}$
13. $\begin{array}{r} 59 \\ -\underline{47} \\ \underline{\quad\;} \end{array}$
14. $\begin{array}{r} 25 \\ -\underline{13} \\ \underline{\quad\;} \end{array}$
15. $\begin{array}{r} 58 \\ -\underline{35} \\ \underline{\quad\;} \end{array}$
16. $\begin{array}{r} 235 \\ -\underline{121} \\ \underline{\quad\;} \end{array}$
17. $\begin{array}{r} 57 \\ -\underline{28} \\ \underline{\quad\;} \end{array}$
18. $\begin{array}{r} 89 \\ -\underline{25} \\ \underline{\quad\;} \end{array}$
19. $73 - 39 = \square$
20. $\begin{array}{r} 483 \\ -\underline{274} \\ \underline{\quad\;} \end{array}$

APPENDIX B

DEPARTMENT OF SCIENCE AND MATHEMATICS EDUCATION

FACULTY OF EDUCATION

UNIVERSITY OF CAPE COAST

EXPERIMENT ON COMPOUND SUBTRACTION

POSTTEST

NAME:……………..……………………………………… CLASS: ..………….

AGE: ………………PARENTS'/ GUARDIANS OCCUPATION: ……………

1. $\begin{array}{r} 69 \\ -28 \\ \hline \\ \hline \end{array}$ 2. $\begin{array}{r} 556 \\ -45 \\ \hline \\ \hline \end{array}$ 3. $\begin{array}{r} 459 \\ -225 \\ \hline \\ \hline \end{array}$ 4. $\begin{array}{r} 59 \\ -25 \\ \hline \\ \hline \end{array}$ 5. $\begin{array}{r} 343 \\ -156 \\ \hline \\ \hline \end{array}$

6. $\begin{array}{r} 513 \\ -384 \\ \hline \\ \hline \end{array}$ 7. $\begin{array}{r} 563 \\ -435 \\ \hline \\ \hline \end{array}$ 8. $\begin{array}{r} 475 \\ -233 \\ \hline \\ \hline \end{array}$ 9. $81 - 48 = \square$

10. $92 - 57 = \square$

11 $\begin{array}{r} 503 \\ 511 \\ \hline \\ \hline \end{array}$ 12. $\begin{array}{r} 920 \\ -151 \\ \hline \\ \hline \end{array}$ 342 13. $\begin{array}{r} 743 \\ -624 \\ \hline \\ \hline \end{array}$ 14. $\begin{array}{r} 408 \\ -459 \\ \hline \\ \hline \end{array}$ 15. $\begin{array}{r} \\ -175 \\ \hline \\ \hline \end{array}$

16. $\begin{array}{r} 834 \\ 581 \\ \hline \\ \hline \end{array}$ 17. $\begin{array}{r} 7864 \\ 352 \\ \hline \\ \hline \end{array}$ 18. $\begin{array}{r} 2130 \\ 1715 \\ \hline \\ \hline \end{array}$ 19. $\begin{array}{r} 5004 \\ 1715 \\ \hline \\ \hline \end{array}$ 20. $\begin{array}{r} 2054 \\ 1856 \\ \hline \\ \hline \end{array}$

APPENDIX C

DEPARTMENT OF SCIENCE AND MATHEMATICS EDUCATION

FACULTY OF EDUCATION

UNIVERSITY OF CAPE COAST

EXPERIMENT ON COMPOUND SUBTRACTION

TEST OF UNDERSTANDING

NAME: …………...………………………………………… CLASS: ………….

AGE: ………………PARENTS'/ GUARDIANS OCCUPATION: ……………...

1. $\begin{array}{r} 2701 \\ -1928 \\ \hline \\ \hline \end{array}$

2. $\begin{array}{r} 1506 \\ -1445 \\ \hline \\ \hline \end{array}$

3. $\begin{array}{r} 40000 \\ -27295 \\ \hline \\ \hline \end{array}$

4. $\begin{array}{r} 359 \\ -285 \\ \hline \\ \hline \end{array}$

5. $\begin{array}{r} 911 \\ -153 \\ \hline \\ \hline \end{array}$

6. $\begin{array}{r} 3513 \\ -1384 \\ \hline \\ \hline \end{array}$

7. $\begin{array}{r} 3563 \\ -1435 \\ \hline \\ \hline \end{array}$

8. $\begin{array}{r} 2475 \\ -1233 \\ \hline \\ \hline \end{array}$

9. $\begin{array}{r} 2183 \\ -2094 \\ \hline \\ \hline \end{array}$

10. $\begin{array}{r} 30001 \\ -14999 \\ \hline \\ \hline \end{array}$

11. $\begin{array}{r} 5003 \\ -3157 \\ \hline \\ \hline \end{array}$

12. $\begin{array}{r} 9020 \\ -5624 \\ \hline \\ \hline \end{array}$

APPENDIX C

13 There are 7028 mangoes in a basket. Ama took 4519 and the remaining mangoes were given to Kwame. How many mangoes did he get?

14 A farmer has 3445 goats. He sells 1999 goats at the market. How many goats does the farmer have left?

15 In a school, there were 2005 pupils. Out of this, 1099 are girls. How many boys are in the school?

16 A man gave his daughter GH ¢ 5,000 to buy some items in the market. What was the man's change, if the daughter spent GH ¢ 2,050?

17 Maame Nyame went to Takyiman to buy 3425 tubers of yam. She later found out that 2115 of them are rotten. Find the number of tubers of yam left.

18 There were 2330 counters in a box. A girl took 1558 counters and the rest were given to Mensah. How many counters did he get?

19 A storekeeper sold 1893 tins of milk out of 25000 in a box. Find the number of tins of milk left.

20 A bag contains 1458 pencils. Of this 743 are blue and the remaining is green. How many of them are green?

APPENDIX D

DEPARTMENT OF SCIENCE AND MATHEMATICS EDUCATION

FACULTY OF EDUCATION

UNIVERSITY OF CAPE COAST

EXPERIMENT ON COMPOUND SUBTRACTION

RETENTION TEST

NAME: …………..………………………………………… CLASS: ………….

AGE: ……………… PARENTS'/ GUARDIANS OCCUPATION: ……………

1. $\begin{array}{r} 97 \\ -28 \\ \hline \\ \hline \end{array}$	2. $\begin{array}{r} 583 \\ -44 \\ \hline \\ \hline \end{array}$	3. $\begin{array}{r} 649 \\ -351 \\ \hline \\ \hline \end{array}$	4. $\begin{array}{r} 60 \\ -29 \\ \hline \\ \hline \end{array}$	5. $\begin{array}{r} 343 \\ -156 \\ \hline \\ \hline \end{array}$
6. $\begin{array}{r} 513 \\ -384 \\ \hline \\ \hline \end{array}$	7. $\begin{array}{r} 563 \\ -435 \\ \hline \\ \hline \end{array}$	8. $\begin{array}{r} 475 \\ -289 \\ \hline \\ \hline \end{array}$	9. $\begin{array}{r} 321 \\ -187 \\ \hline \\ \hline \end{array}$	10. $\begin{array}{r} 415 \\ -334 \\ \hline \\ \hline \end{array}$
11. $\begin{array}{r} 580 \\ -258 \\ \hline \\ \hline \end{array}$	12. $\begin{array}{r} 619 \\ -583 \\ \hline \\ \hline \end{array}$	13. $331 - 94 = \square$	14. $\begin{array}{r} 712 \\ -175 \\ \hline \\ \hline \end{array}$	15. $\begin{array}{r} 510 \\ -342 \\ \hline \\ \hline \end{array}$
16. $\begin{array}{r} 913 \\ -581 \\ \hline \\ \hline \end{array}$	17. $\begin{array}{r} 2861 \\ -352 \\ \hline \\ \hline \end{array}$	18. $\begin{array}{r} 1960 \\ -1785 \\ \hline \\ \hline \end{array}$	19. $\begin{array}{r} 5056 \\ -4728 \\ \hline \\ \hline \end{array}$	20. $\begin{array}{r} 1724 \\ -1266 \\ \hline \\ \hline \end{array}$

APPENDIX E

Computation of the Coefficient of Reliability for Pretest, Posttest and Retention Test, Using Kuder Richardson's Formula ($K - R - 21$)

$$r_{total\ test} = \frac{KS^2 - \bar{X}(K - \bar{X})}{S^2(K-1)}$$

where K is the number of test items,
S^2 is the variance of the test
$\bar{X}$ is the mean of the test

Pretest

$K = 20, S^2 = 16, \bar{X} = 14$

$$r_{total\ test} = \frac{20 \times 16 - 14(20 - 14)}{16(20 - 1)}$$

$$= 0.78$$

Posttest

$K = 20, S^2 = 14, \bar{X} = 16$

$$r_{total\ test} = \frac{20 \times 14 - 16(20 - 16)}{14(20 - 1)}$$

$$= 0.81$$

Retention test

$K = 20, S^2 = 14, \bar{X} = 15.5$

$$r_{total\ test} = \frac{20 \times 14 - 15.5(20 - 15.5)}{14(20 - 1)}$$

$$= 0.79$$

APPENDIX F

DAILY RECORD OF INSTRUCTIONAL ACTIVITIES OF THE BCA GROUP

DAY 1

Topic: Addition of whole numbers.

Sub-Topic: Revision of addition of whole numbers

Instructional Materials: Abacus.

Content Outline:

i. Simple addition

ii. Addition involving regrouping

Instructional Objectives:

i. Pupil will be able to recall basic addition facts

ii. Pupil will be able to work simple and addition involving regrouping.

Relevant Previous Knowledge:

Pupils can recall some basic addition facts and work some addition problems.

Introduction:

Teacher revises pupils' relevant previous knowledge

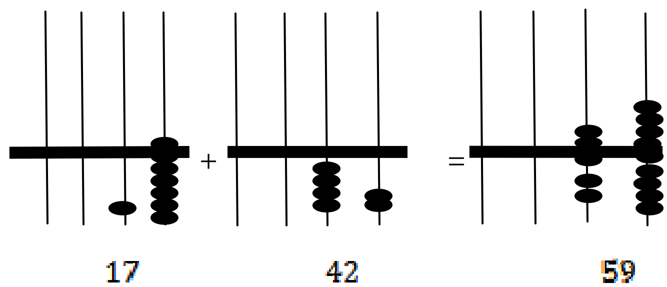
Example: (1) 3 | 4 (2) 2 | 7 (3) 1 | 6 etc

Pupils' work sums: (1) 3 + 4 – 7 (2) 2 + 7 – 9 (3) 1 + 6 – 7

Development:

Step 1: Teacher discusses addition of two-digit numbers with the class

e.g. (1) 17 + 42



Step 2:

Teacher discusses addition of three-digit numbers with the class

Pupils make contributions and ask questions

e.g.

(1)		(2)		(3)	
	174		215		194
	+ 113		+ 207		+ 338
	____		____		____

(1)		(2)		(3)	
	174		215		194
	+ 113		+ 207		+ 338
	287		422		532

Closure: teacher gives pupils exercises to do.

DAY 2

Topic: Subtraction of whole numbers.

Sub-Topic: Revision of simple subtraction.

Instructional Materials: Pieces of paper square grid boards, square slabs and number cards.

Content Outline:

i. Revision of simple subtraction.

ii. The concept of base complement.

iii. Identification of base ten complements.

Instructional Objectives:

i. Pupil will be able to recall basic subtraction facts.

ii. Pupil will be able to mention the complement of 1, 2, 3…... 9 with respect to base ten.

Relevant Previous Knowledge:

i. Pupils can recall the basic subtraction facts.

ii. Pupils can find two whole numbers whose sum is ten.

Introduction

Teacher revises pupils' relevant previous knowledge

Example: (1) $9+5$ (2) $3+7$ (3) $15+8$ etc

Pupils' work sums: (1) $9+5=14$ (2) $3+7=10$ (3) $15+8=23$

Development:

Step 1:

Teacher put pupils into smaller groups of five and gives them pieces of paper square grid boards and square slabs. Teacher allows pupils to have a feel of the materials and assists them to count the number of squares in each column and row on the square board.

Figure 1. Demonstrating the square grid

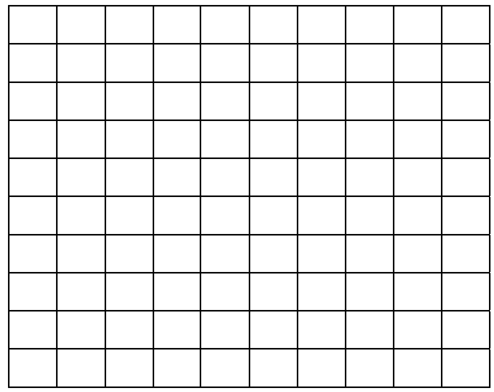


Figure 1.1

Pupils count the number of squares in each column and row on the square board

(Ten squares in each column and row on the square board)

Step 2:

Teacher discusses the idea of base complement of to the class. Teacher helps pupils to arrange one, two, three, four, five and six square slabs in first, second, third, fourth, fifth and sixth column respectively of the square board (the arrangement starts from the left as shown in the diagram below).

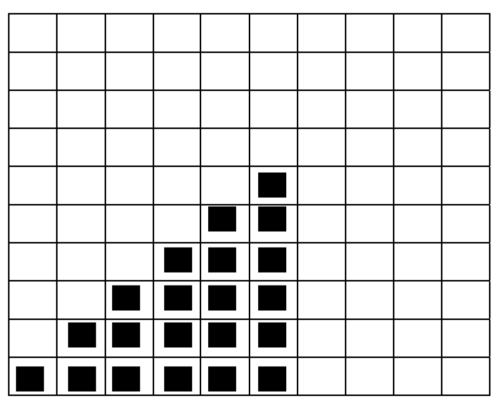


Figure 1.2

Thereafter, teacher asks pupils to count the number of slabs needed to fill the first column. Pupils answer, “Nine”. Teacher explains that the base complement of 1 is 9(nine is the only whole number that can be added to one to obtain 10). This activity continues till pupils answer question on the sixth column.

Step 3:

Next, teacher picks a number card and asks pupils to give the base ten complement of number on the card. Example, find the base complement of $\boxed{7}$

Pupils answer, “Three”.

Closure:

Pupils are given exercises to do. Example, find the complement of 3, 5, 7 and 9.

DAY 3

Topic: Subtraction of whole numbers.

Sub-Topic: representation of compound subtraction with concrete materials

Instructional Materials: Pieces of paper square grid boards and square slabs.

Content Outline:

i. Two-digit minuend and one-digit subtrahend.

Instructional Objectives:

i. Pupil will be able to use the slabs to illustrate the arrangement of the minuend and subtrahend on the square grid board.

ii. Pupil will be able to find the difference between two numbers with the aid of concrete materials.

Relevant Previous Knowledge:

iii. Pupils can find the number of give the number of square slabs needed to fill each column of the square grid board.

Introduction

Teacher revises pupils’ relevant previous knowledge

Example: how many square slabs are needed to fill each column of the square grid board? Pupils answer, “Ten”

<u>Development:</u>

<u>Step 1:</u>

Teacher illustrates how to represent compound subtraction using the square grid and square slabs as shown in the diagram below. Example, $16 - 9$

<u>Figure 2</u>. Demonstrating the arrangement of square slabs on the square grid

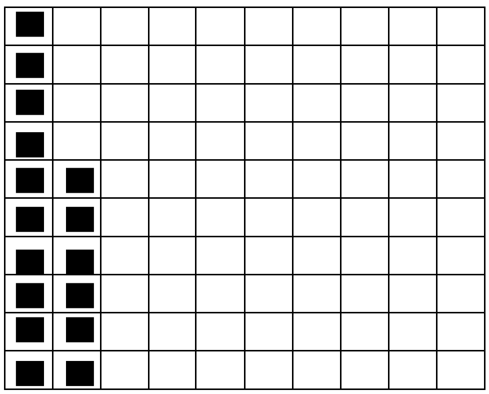


Figure 2.1: Board with 16 slabs

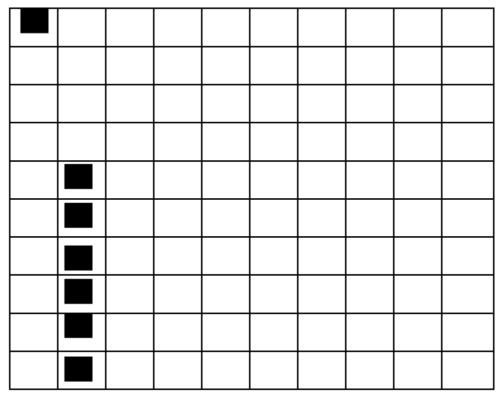


Figure 2.2: Board with 7 slabs

Ten slabs fill the first column of 10 squares on the extreme left while 6 slabs fill six squares in the second column from the bottom. To take away 9, nine slabs were removed in turns starting from the bottom of the column on the extreme left. Figure 2, illustrates the remaining slabs on the square grid board after removing 9 square slabs from the first column. To find the difference, the remaining square slab in the first column and six square slabs in the second column are counted ($i.e \quad 16 - 9 = 7$). Pupils pay attention and ask questions.

<u>Step 2:</u>

Teacher assists pupils to form small groups of five members and gives each group a square grid board and twenty square slabs. After that teacher asks pupils to use the concrete materials to solve the following problems.

(1) 17 8   (2) 18 9

Teacher goes round to supervise pupils' work and help them where necessary.

Step 3:

Teacher asks pupils to identify the compound subtraction illustrated below.

Figure 3. Identification of compound subtraction problems

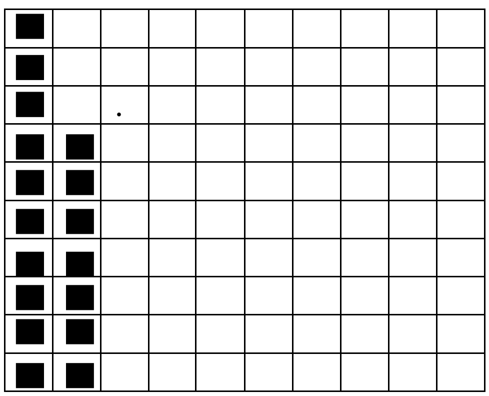


Figure 3.1

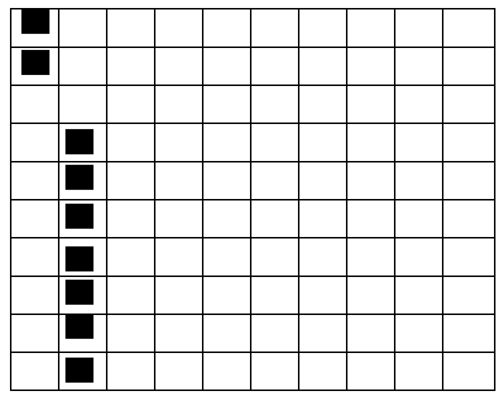


Figure 3.2

Pupils write answer: $17 - 8$

Closure:

Teacher gives exercises to pupils to do in groups. E.g. use square grid board and square slabs to represent the following.

(i) $18 - 9$ (ii) $23 - 8$

DAY 4:

Topic: Subtraction of whole numbers.

Sub-Topic: representation of compound subtraction with concrete materials

Instructional Materials: Pieces of paper square grid boards and square slabs.

Content Outline:

i Two-digit minuend and one -digit subtrahend.

ii Two-digit minuend and two-digit subtrahend.

Instructional Objectives:

Pupil will be able to use the slabs to illustrate the arrangement of the minuend and subtrahend on the square grid board.

Relevant Previous Knowledge:

Pupils can identify compound subtraction problem when illustrated with concrete materials.

Introduction

Teacher revises pupils' relevant previous knowledge

Example:

Teacher asks pupils to identify the compound subtraction illustrated below.

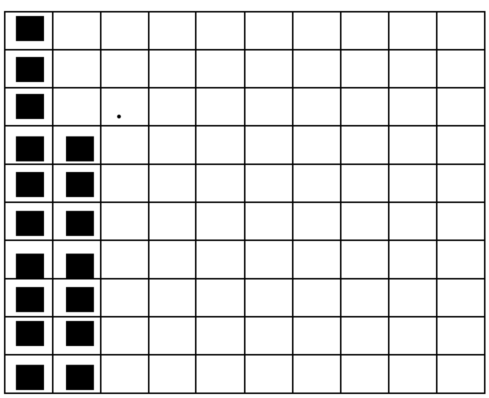


Figure 4.1

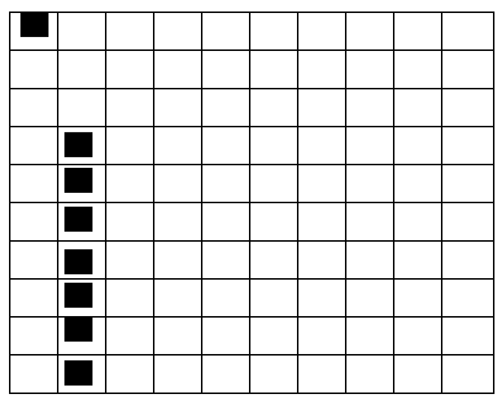


Figure 4.2

Pupils identify the compound subtraction as **17 – 9**

Development:

Step 1:

Teacher illustrates how to represent compound subtraction using the square grid and square slabs as shown in the diagram below.

Example, 45 28

Figure 4. Demonstrating the representation of compound subtraction using square grid and square slab

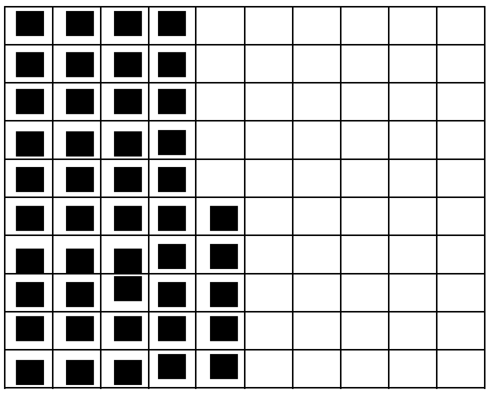


Figure 4.3: 45

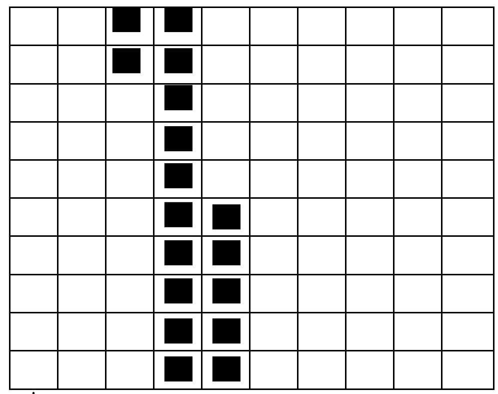


Figure 4.4: 45 – 28

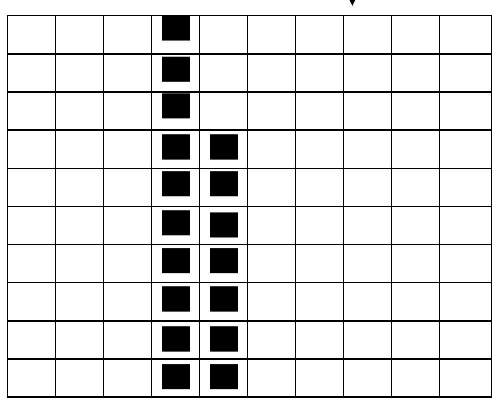


Figure 4.5: 47 – 30

Forty-five square slabs arranged on a square grid board (see figure 4.3). To take away 28, twenty-eight slabs were removed in turns starting from the bottom of the column on the extreme left (see figure 4.4). To transform the compound subtraction problem to a simple subtraction problem, the two square slabs in the 9th and 10th square of the 3rd column of the square grid in figure 4.4 are removed and added to those in the fourth column (see figure 4.5). Teacher guides pupils to count both the number of empty squares and square slabs on square grid board starting from the column on the extreme left to the last square slab on the square grid board (i.e. 47). Thereafter, teacher guides them to count the number of empty squares in the first three columns of the square grid board (i.e. 30). Teacher explains that $45 - 28$ can be written as $47 - 30$. Teacher guides pupils to solve $45 - 28$ and $47 - 30$ and compare answers. Pupils solve the problems: $45 - 28 = 17$ and $47 - 30 = 17$. Teacher also discusses other examples like (i) $53 - 37$ and $41 - 26$ with the class.

<u>Step 2:</u>

Teacher assists pupils to form small groups of five members and gives each group a square grid board and sixty square slabs. After that teacher asks pupils to use the concrete materials to transform the following compound subtraction problems to simple subtraction problems.

$(i)\ \ 52 - 33 \qquad\qquad (ii) \qquad 48 - 19$

Teacher goes round to supervise pupils' work and help them where necessary.

Pupils transform compound subtraction problems to simple subtraction problems.

<u>Figure 5</u>. Demonstrating the transformation of compound subtraction problem to a simple subtraction problem

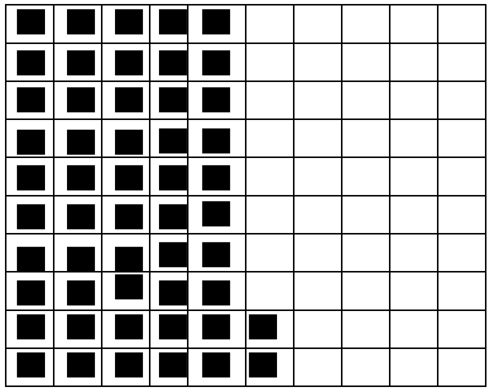


Figure 5.1: 52

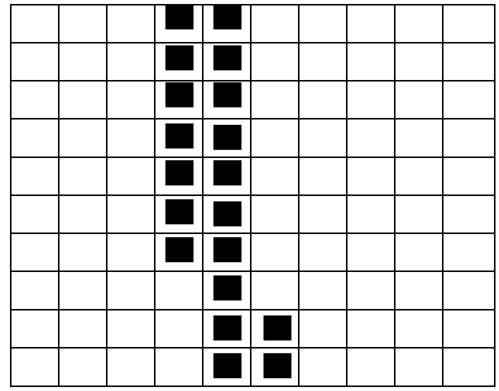


Figure 5.2: 52 – 33

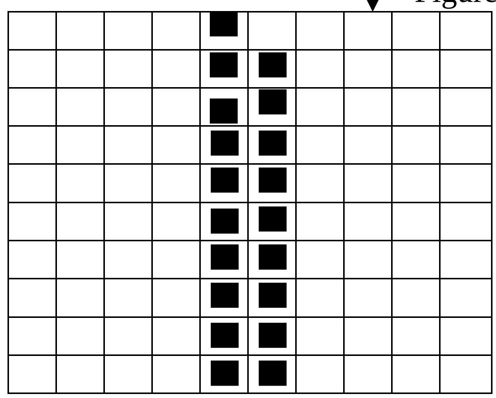


Figure 5.3: 59 – 40

Pupils write their answer:

(i) 52 – 33 = 59 – 40 (ii) 48 – 19 = 49 – 20

<u>Closure:</u>

Teacher asks pupils to write pair of subtraction problems (compound subtraction and its equivalent simple subtraction) in their exercise books.

<u>DAY 5</u>

<u>Topic</u>: Subtraction of whole numbers.

Sub-Topic: Doing two-digit compound subtraction with concrete materials

<u>Instructional Materials</u>: Pieces of paper square grid boards and square slabs.

<u>Content Outline</u>:

Two-digit minuend and two-digit subtrahend.

<u>Instructional Objectives</u>:

Pupil will be able to use the square grid board and square slabs to solve compound subtraction problem.

Relevant Previous Knowledge:

Pupils can transform compound subtraction problem to simple subtraction problem using concrete materials.

Introduction

Teacher revises pupils' relevant previous knowledge by inviting pupils to write a pair of subtraction problems (compound subtraction and its equivalent simple subtraction) on the chalkboard.

Pupils write a pair of subtraction problem on the chalkboard as they are invited to do so.

Development:

Step 1:

Teacher discusses the following problems with the class.

$$(i)\quad \begin{array}{r} 8\ 1 \\ -\ 5\ 6 \\ \hline \\ \hline \end{array} \qquad\qquad (ii)\quad \begin{array}{r} 6\ 4 \\ -\ 4\ 7 \\ \hline \\ \hline \end{array}$$

Figure 6.Transformation of compound subtraction problem to a simple subtraction problem

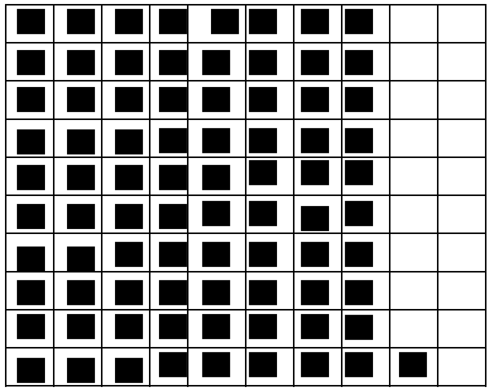


Figure 6.1: 81

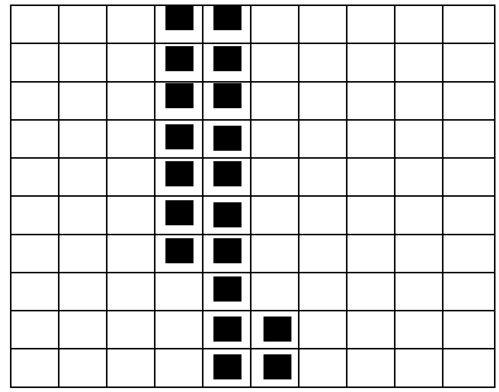


Figure 6.2: $81 - 56$

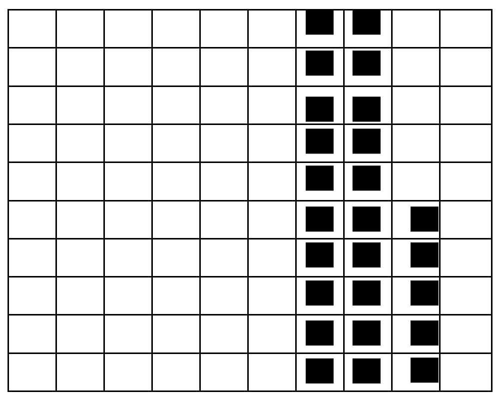


Figure 5.3: $85 - 60$

From the above diagram, we have

$(i)\ 81 - 56 = 85 - 60 = 25$

Pupils pay attention and contribute to the discussion

Step 2:

Teacher asks pupils to break into small groups of five and gives each group a square grid board and hundred square slabs. Thereafter, teacher asks pupils to use the concrete materials to solve the problems below.

$$(i)\quad \begin{array}{r} 93 \\ 78 \\ \hline \\ \hline \end{array} \qquad\qquad (ii)\quad \begin{array}{r} 70 \\ 45 \\ \hline \\ \hline \end{array}$$

Pupils solve the problems as follows: (*i*)

Figure 7.Transformation of compound subtraction problem to a simple subtraction problem

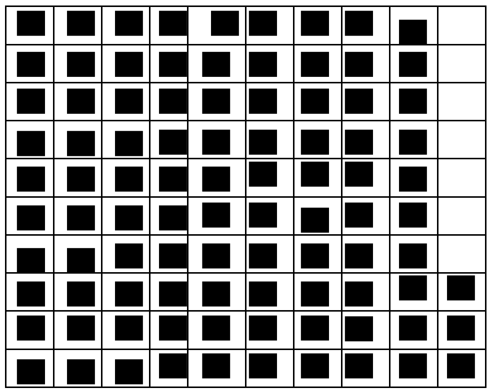


Figure 7.1: 93

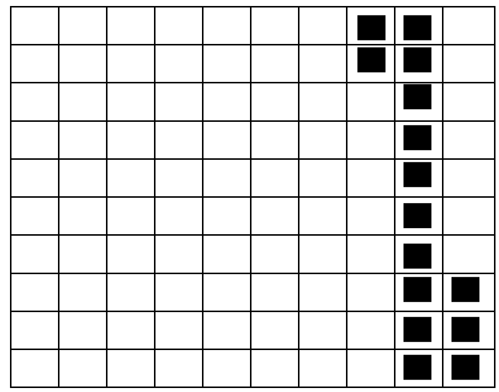


Figure 7.2: 93 – 78

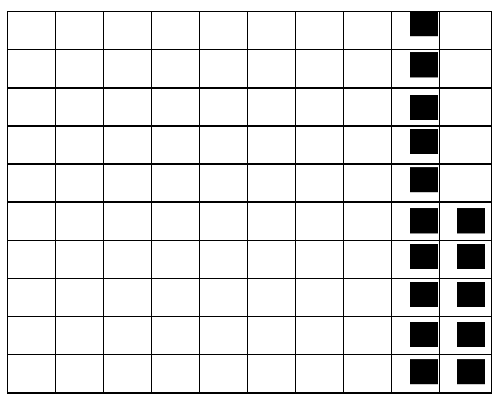


Figure 7.3: 95 – 80

Thus,

$$\begin{array}{r} 9\ 3 \\ -\quad 7\ 8 \\ \hline \\ \hline \end{array} \quad = \quad \begin{array}{r} 9\ 5 \\ -\quad 8\ 0 \\ \hline 1\ 5 \\ \hline \end{array}$$

Closure:

Teacher assigns the following exercises to pupils to practice.

(*i*) $\begin{array}{r} 9\,0 \\ -\quad 7\ 3 \\ \hline \\ \hline \end{array}$ (*ii*) $\begin{array}{r} 7\ 1 \\ -\ 4\ 4 \\ \hline \\ \hline \end{array}$

$$
\begin{array}{r} (iii)\quad 72 \\ -\ 47 \\ \hline \\ \hline \end{array}
\qquad\qquad
\begin{array}{r} (iv)\quad 70 \\ -\ 55 \\ \hline \\ \hline \end{array}
$$

DAY 6:

Topic: Subtraction of whole numbers.

Sub-Topic: Doing two-digit compound subtraction with concrete materials

Instructional Materials: Pieces of paper square grid boards and square slabs.

Content Outline:

i. Two-digit minuend and two-digit subtrahend.

Instructional Objectives:

Pupil will be able to use shading to illustrate compound subtraction problem on a square grid board and solve.

Relevant Previous Knowledge:

Pupils can use square grid board and square slabs to solve compound subtraction problem.

Introduction

Teacher revises pupils' relevant previous knowledge by asking individuals to use the above mentioned concrete materials to solve compound subtraction problem. Pupils go to the chalkboard to solve compound subtraction problem as they are invited to do so.

Development:

Step 1:

Teacher discusses the following problems with the class.

(i) $\begin{array}{r} 60 \\ -\ 43 \\ \hline \\ \hline \end{array}$ (ii) $\begin{array}{r} 71 \\ -\ 44 \\ \hline \\ \hline \end{array}$

Figure 8. Representing and solving compound subtraction problem on a chalkboard

(i)

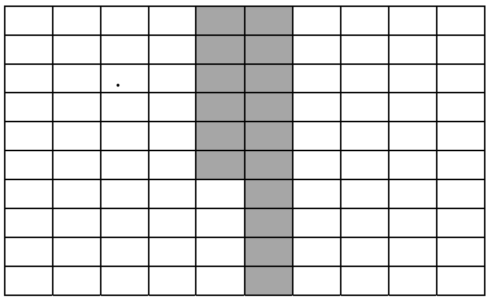


Figure 8.1: 60 – 43

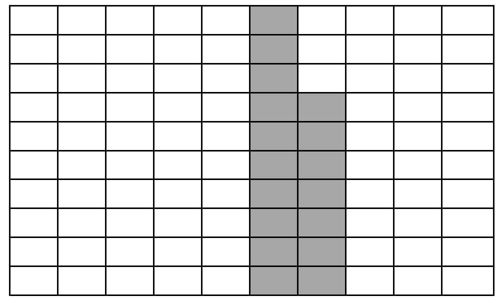


Figure 8.2: 67 – 50

Thus,

$$\begin{array}{r} 60 \\ 43 \\ \hline \\ \hline \end{array} \quad = \quad \begin{array}{r} 67 \\ 50 \\ \hline 17 \\ \hline \end{array}$$

(ii)

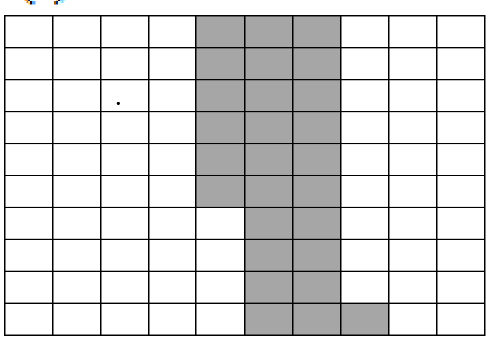


Figure 8.3: 71 – 44

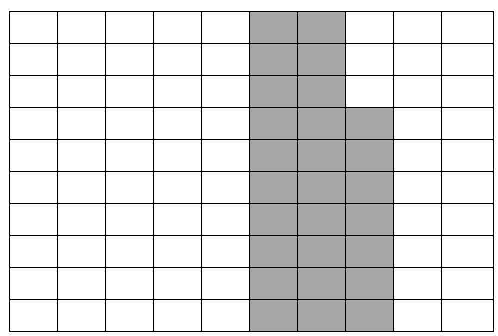


Figure 8.4: 77 – 50

Thus,

$$\begin{array}{r} 71 \\ -\ 44 \\ \hline \\ \hline \end{array} \quad = \quad \begin{array}{r} 77 \\ -\ 50 \\ \hline 17 \\ \hline \end{array}$$

Closure:

Teacher assigns the following exercises to pupils to practice.

$$(i)\quad \begin{array}{r} 90 \\ -\;73 \\ \hline \\ \hline \end{array} \qquad\qquad (ii)\quad \begin{array}{r} 71 \\ -\;44 \\ \hline \\ \hline \end{array}$$

$$(iii)\quad \begin{array}{r} 72 \\ -\;47 \\ \hline \\ \hline \end{array} \qquad\qquad (iv)\quad \begin{array}{r} 70 \\ -\;55 \\ \hline \\ \hline \end{array}$$

DAY 7:
Topic: Subtraction of whole numbers.

Sub-Topic: Algorithm for Base Complement Addition

Instructional Materials: Pieces of paper square grid boards and square slabs.

Content Outline: Algorithm for Base Complement Addition.

Instructional Objectives:

Pupil will be able to use algorithm for base complement addition to solve compound subtraction problem.

Relevant Previous Knowledge:

Pupils can use square grid board to transform compound subtraction problem to a simple subtraction problem and solve it.

Introduction:

Teacher revises pupils' relevant previous knowledge by asking individuals to use the above mentioned concrete materials to solve compound subtraction problem.

Pupils go to the chalkboard to solve compound subtraction problem as they are invited to do so

Development:

Step 1:

Teacher demonstrates the algorithm for the base complement addition using square grid board. For instance, $42 - 27$

Figure 9. Demonstrating the algorithm for Base- complement addition method using the square grid

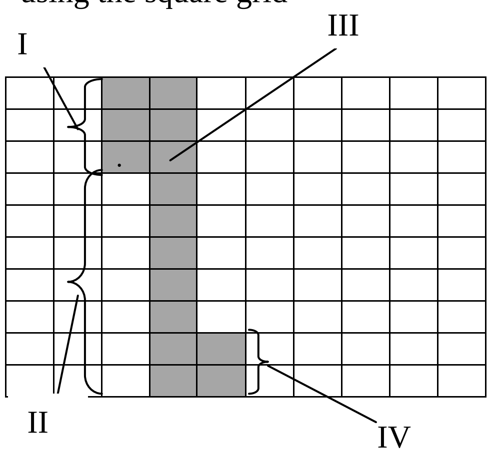


Figure 9.1: $42 - 27$

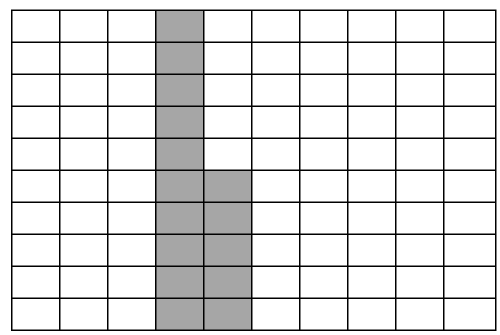


Figure 9.2: $45 - 30$

The above procedure transforms the compound subtraction problem, $42 \quad 27$ to a simple subtraction problem, $49 \quad 30$ (see figure 9.2). Teacher explains to pupils that the base complement of 7 in base ten is 3(i.e. $I + II - 10$, see figure 9.1). Teacher further explains that the compound subtraction problem was transformed to a simple subtraction problem by adding 3 (the complement of 7 in base ten) to both the minuend and the subtrahend to obtain $49 - 30$. The algorithm for the above problem is illustrated below as follows:

$$\begin{array}{rcrcr} 42 & \xrightarrow{becomes} & (42 \mid 3) & \xrightarrow{becomes} & 45 \\ -27 & & -(42+3) & & -\ 30 \\ \hline & & & & \end{array}$$

Pupils pay attention and ask questions. Teacher demonstrates the algorithm for other examples like $53 - 24$ and $72 - 33$ to the class.

Step 2:

Teacher guides pupils to form small groups of five and write the following problems on the chalkboard for pupils to solve them.

$$(i)\quad \begin{array}{r} 91 \\ -\;55 \\ \hline \\ \hline \end{array} \qquad\qquad (ii)\quad \begin{array}{r} 70 \\ -\;49 \\ \hline \\ \hline \end{array}$$

Pupils present solutions as follows:

$$(i)\quad \begin{array}{r} 91 \\ -55 \\ \hline \\ \hline \end{array} \xrightarrow{becomes} \begin{array}{r} (91+5) \\ -(55+5) \\ \hline \\ \hline \end{array} \xrightarrow{becomes} \begin{array}{r} 96 \\ -\;60 \\ \hline 36 \\ \hline \end{array}$$

$$(ii)\quad \begin{array}{r} 70 \\ -49 \\ \hline \\ \hline \end{array} \xrightarrow{becomes} \begin{array}{r} (70+1) \\ -(49+1) \\ \hline \\ \hline \end{array} \xrightarrow{becomes} \begin{array}{r} 71 \\ -\;50 \\ \hline 21 \\ \hline \end{array}$$

Closure:

Teacher assigns the following exercises to pupils to do.

$$\begin{array}{r} 41 \\ -\;25 \\ \hline \\ \hline \end{array} \qquad\qquad (ii)\quad \begin{array}{r} 50 \\ -\;19 \\ \hline \\ \hline \end{array}$$

DAY 8:

Topic: Subtraction of whole numbers.

Sub-Topic: Compound Subtraction at the Abstract Stage

Outline: Doing compound subtraction at the abstract stage

Instructional Objectives:

Pupil will be able to solve compound subtraction problem without using concrete or semi-concrete materials.

Relevant Previous Knowledge:

Pupils can find the complement of 1, 4, 7, 54, 89, etc

Introduction:

Teacher revises pupils' relevant previous knowledge by asking individuals to give 1, 4, 7, 9, etc.

Pupils give the complement of the above numbers as 9, 6, 3,1, etc.

Development:

Step 1:

Teacher discusses problems with the class but this time round verbalizing the algorithms instead of illustrating them.

$$\begin{array}{rr} & 4\ 11 \\ - & 2\,3\,5 \\ \hline & 1\,7\,6 \\ \hline \end{array} \qquad \text{(ii)} \quad \begin{array}{rr} & 5\,2\,0 \\ - & 4\,1\,9 \\ \hline & 1\,0\,1 \\ \hline \end{array}$$

Pupils contribute to the discussion and ask questions.

Step 2:

Teacher discusses compound subtraction problems involving subtrahend with successive nines with the class.

(i) 431 (ii) 720

− 299 − 499

(i) 431

299

(2)

5 3 1

(3)(0)(0)

2 9 9

3 3 2

Closure:

Teacher gives the exercises below to pupils to do.

(i) 315 (ii) 550

137 392

(iii) 3150 (ii) 5003

− 1267 −2899

DAY 9:

Topic: General revision of subtraction of whole numbers.

Instructional Materials: Pieces of paper square grid boards and square slabs.

Content Outline: Discussion on compound subtraction.

Relevant Previous Knowledge:

Pupils have been introduced to the concept of base complement and compound subtraction.

Introduction:

Teacher revises pupils' relevant previous knowledge by asking them to give base complement of some numbers.

Pupils give the base complement of numbers and go to the chalkboard to solve compound subtraction problem as teacher invites the in turns to do so.

Development:

Step 1:

Teacher revises compound subtraction (two-digit minuend and one- digit subtrahend, two two-digit and three-digits) with the class using concrete and semi-concrete materials.

Pupils go to the chalkboard in turns to solve problems as teacher invites them to do so

Step 2:

Teacher discusses word problems on compound subtraction with the class.

Pupils contribute to the discussion and ask questions.

Closure: Teacher gives pupils exercises to try at home.

APPENDIX G

DAILY RECORD OF INSTRUCTIONAL ACTIVITIES OF THE DEC GROUP

DAY 1:

Topic: Addition of whole numbers.

Sub-Topic: Revision of addition of whole numbers

Instructional Materials: Dienes'Multibase Arithmetic Blocks.

Content Outline:

Simple addition

iii. Addition involving regrouping

Instructional Objectives:

iii. Pupil will be able to recall basic addition facts

iv. Pupil will be able to work simple and addition involving regrouping.

Relevant Previous Knowledge:

Pupils can recall some basic addition facts and work some addition problems.

Introduction:

Introduction:

Teacher revises pupils' relevant previous knowledge

Example: (1) $3 + 4$ (2) $2 + 7$ (3) $1 + 6$ etc

Pupils' work sums: (1) $3 + 4 = 7$ (2) $2 + 7 = 9$ (3) $1 + 6 = 7$

Development:

Step 1: Teacher discusses addition of two-digit numbers with the class.

E.g. (1) 17 + 42

17 + 42 = 59

Step 2:

Teacher discusses addition of three-digit numbers with the class

Pupils make contributions and ask questions

e.g.

(1)		(2)		(3)	
	174		215		194
	+ 113		+ 207		+ 338
	____		____		____

(1)		(2)		(3)	
	174		215		194
	+ 113		+ 207		+ 338
	287		422		532

Closure: teacher gives pupils exercises to do.

DAY 2

Topic: Subtraction of whole numbers.

Sub-Topic: Revision of simple subtraction.

Instructional Materials: Dienes'Multibase Arithmetic Blocks.

Content Outline:

i. Revision of simple subtraction facts.

ii. Revision of simple subtraction.

Instructional Objectives:

i. Pupil will be able to recall basic subtraction facts.

ii. Pupil will be able to solve simple subtraction problems involving two one- digit numbers.

Relevant Previous Knowledge:

i. Pupils can recall the basic subtraction facts.

ii. Pupils can solve simple subtraction problems involving two one-digit numbers.

Introduction

Teacher revises pupils' relevant previous knowledge

Example: (1) $9-5$ (2) $8-5$ (3) $7-2$ etc

Pupils' work sums: (1) $9-5=4$ (2) $8-5=3$ (3) $7-2=5$

Development:

Step 1:

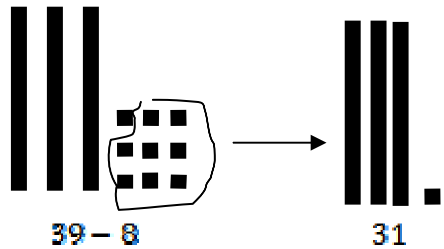
Teacher put pupils into smaller groups of five and gives them Dienes'Multibase Arithmetic Blocks (one flat, ten longs and ten cubes). Teacher allows pupils to have a feel of the materials and assists them to find the number of cubes that make on long and the number of longs that one flat (i.e.$10\ cubes = 1\ long, 10\ longs = 1\ flat$)

Step 2:

Teacher uses Multibase Arithmetic Blocks to discuss the following simple subtraction problems with the class.

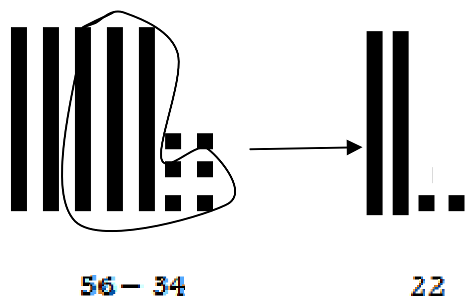
(i) $39 - 8$ (ii) $56 - 34$ (iii) $352 - 140$, etc

i.



To solve $39 - 8$, teacher arranges 3 longs and 9 cubes which represents 39. Thereafter, teacher removes 8 cubes (i.e. $39 - 8$) and the remaining blocks are counted and recorded as the answer of the given problem $(39 - 8 = 31)$.

ii.



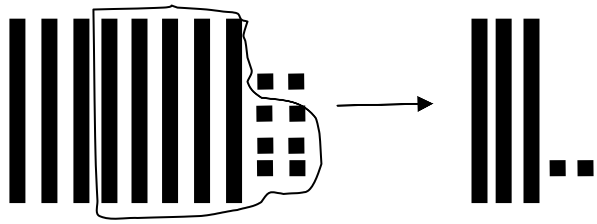
Step 3:

Next, teacher gives the following subtraction exercises to pupils to do in their various groups.

(i) $\begin{array}{r} 88 \\ -\ 56 \\ \hline \\ \hline \end{array}$ (ii) $\begin{array}{r} 69 \\ -\ 47 \\ \hline \\ \hline \end{array}$

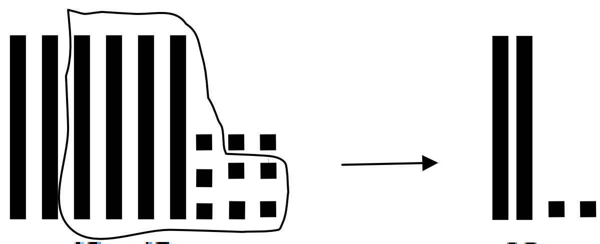
Pupils present their solution as follows:

i.



88 – 56 32

ii.



69 – 47 22

Closure:

Teacher gives pupils the following exercises to do.

(i)
$$\begin{array}{r} 53 \\ -\ 21 \\ \hline \\ \hline \end{array}$$

(ii)
$$\begin{array}{r} 65 \\ -\ 40 \\ \hline \\ \hline \end{array}$$

(iii)
$$\begin{array}{r} 378 \\ -\ 246 \\ \hline \\ \hline \end{array}$$

(iv)
$$\begin{array}{r} 629 \\ -\ 418 \\ \hline \\ \hline \end{array}$$

DAY 3

Topic: Subtraction of whole numbers.

Sub-Topic: Concrete representation of compound subtraction.

Instructional Materials: Dienes'Multibase Arithmetic Blocks.

Content Outline:

i. Concrete representation of compound subtraction involving two-digit minuend and one-digit subtrahend.

ii. Concrete representation of compound subtraction involving two-digit minuend and two-digit subtrahend.

Instructional Objectives:

i. Pupil will be able to represent compound subtraction problems using Dienes'Multibase Arithmetic Blocks.

ii. Pupil will be able to decompose and regroup minuend using Dienes'Multibase Arithmetic Blocks.

Relevant Previous Knowledge:

i. Pupils can represent subtraction problem which does not involve regrouping with Dienes'Multibase Arithmetic Blocks.

Introduction

Teacher revises pupils' relevant previous knowledge

Example: (1) $9 \quad 5$ (2) $18 \quad 10$ (3) $17 \quad 12$ etc

Pupils' work sums: (i) $9 \quad 5 = 4$ (ii) $18 \quad 10 = 8$ (iii) $17 \quad 12 = 5$

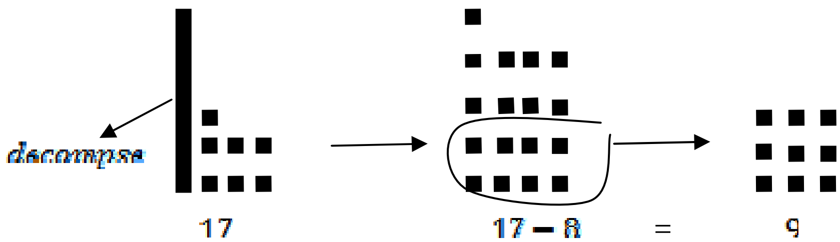
Development:

Step 1:

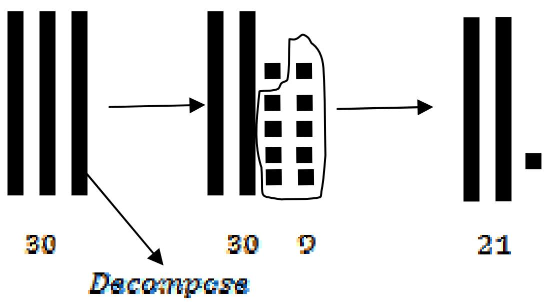
Teacher discusses the representation of compound subtraction involving two-digit minuend and one-digit subtrahend and solves them with the class.

e.g. (i) $17 - 8$ (ii) $30 - 9$ (iii) $42 - 8$

i.



ii.



Pupils pay attention and ask questions.

<u>Step 2:</u>

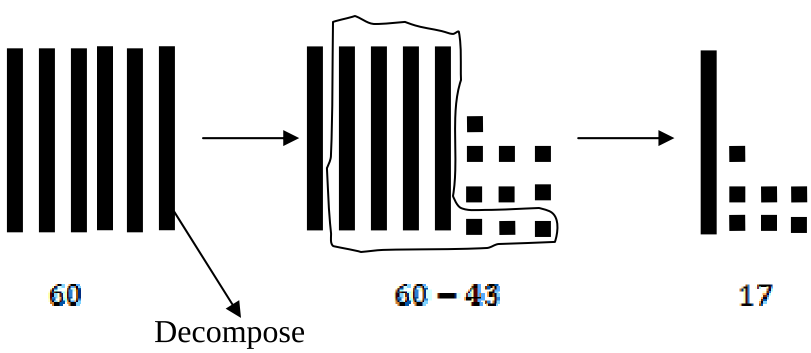
Teacher discusses the representation of compound subtraction involving two-digit minuend and two-digit subtrahend and solves them with the class.

e.g.

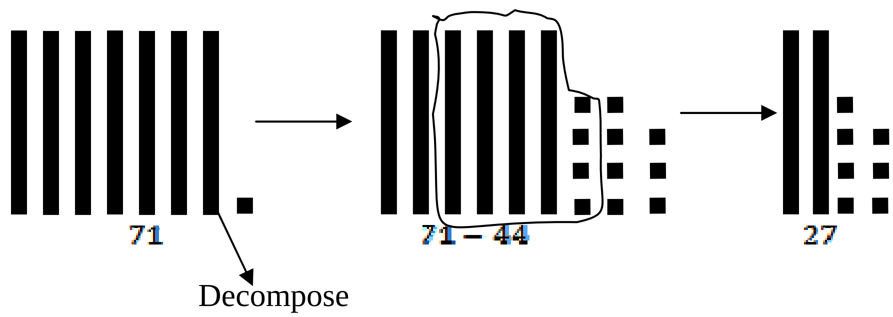
(i)
$$\begin{array}{r} 6\ 0 \\ -\ 4\ 3 \\ \hline \\ \hline \end{array}$$

(ii)
$$\begin{array}{r} 7\ 1 \\ -\ 4\ 4 \\ \hline \\ \hline \end{array}$$

i.



ii.



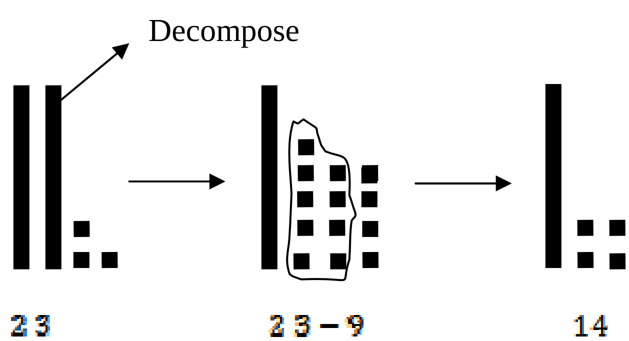
Pupils pay attention and ask questions.

<u>Step 3:</u>

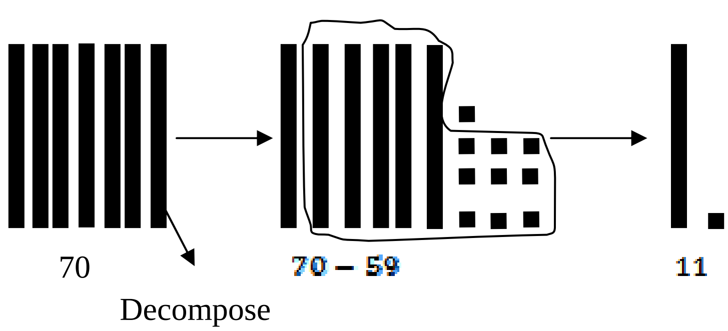
Teacher guides pupils to form small groups of five and gives them Dienes'Multibase Arithmetic Blocks. Thereafter, teacher gives the following exercises to pupils to do.

(i) $\begin{array}{r} 23 \\ -\ \ 9 \\ \hline \\ \hline \end{array}$ (ii) $\begin{array}{r} 70 \\ -\ 59 \\ \hline \\ \hline \end{array}$

i.



ii.



Closure:

Teacher assigns the following exercises to pupils to practice.

$$\text{(i)}\quad \begin{array}{r} 90 \\ -\ 73 \\ \hline \\ \hline \end{array} \qquad\qquad \text{(ii)}\quad \begin{array}{r} 71 \\ -\ 44 \\ \hline \\ \hline \end{array}$$

$$\text{(iii)}\quad \begin{array}{r} 72 \\ -\ 47 \\ \hline \\ \hline \end{array} \qquad\qquad \text{(iv)}\quad \begin{array}{r} 70 \\ -\ 55 \\ \hline \\ \hline \end{array}$$

DAY 4:

Topic: Subtraction of whole numbers.

Sub-Topic: Concrete representation of compound subtraction.

Instructional Materials: Dienes'Multibase Arithmetic Blocks.

Content Outline:

i. Concrete representation of compound subtraction involving three-digit minuend and two-digit subtrahend.

ii. Concrete representation of compound subtraction involving three-digit minuend and three-digit subtrahend.

Instructional Objectives:

i. Pupil will be able to represent compound subtraction problems using Dienes'Multibase Arithmetic Blocks.

ii. Pupil will be able to decompose and regroup minuend using Dienes’Multibase Arithmetic Blocks.

<u>Relevant Previous Knowledge</u>:

ii. Pupils can represent subtraction problem which involves regrouping with Dienes’Multibase Arithmetic Blocks.

<u>Introduction</u>

Teacher revises pupils’ relevant previous knowledge

E.g.: (i) $\begin{array}{r} 22 \\ -\ \ 7 \\ \hline \\ \hline \end{array}$ (ii) $\begin{array}{r} 60 \\ -\ 39 \\ \hline \\ \hline \end{array}$

Pupils use concrete materials to represent the above compound subtraction problems and solve them.

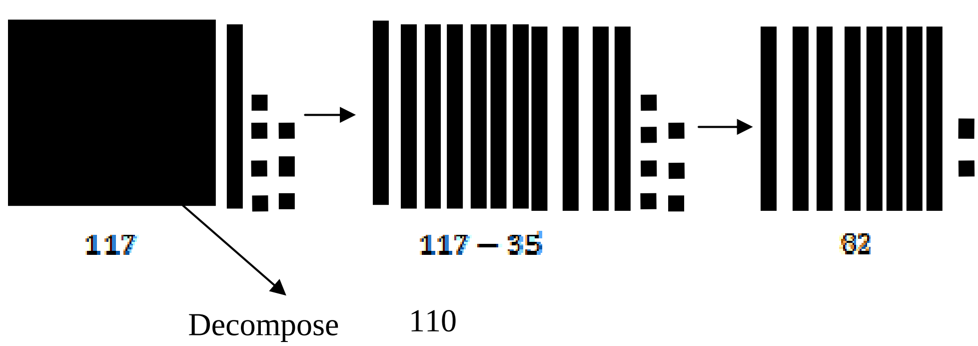
<u>Development:</u>

<u>Step 1:</u>

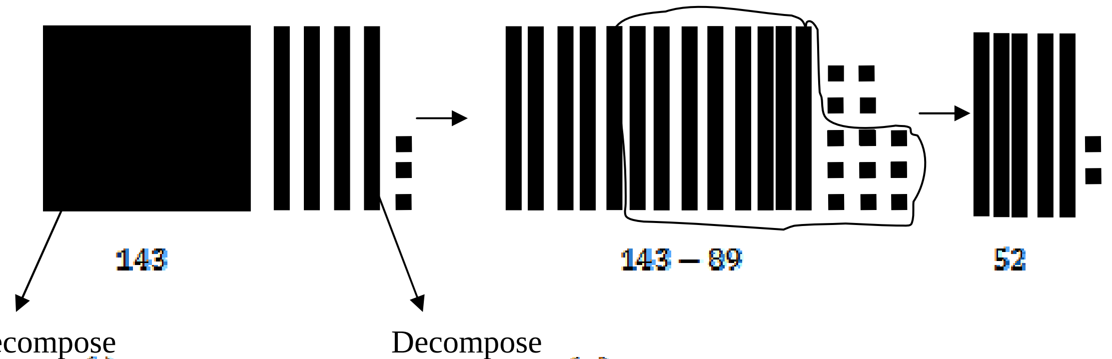
Teacher discusses the representation of compound subtraction involving three-digit minuend and two-digit subtrahend and solves them with the class.

e.g. (i) $\begin{array}{r} 117 \\ -\ \ 35 \\ \hline \\ \hline \end{array}$ (ii) $\begin{array}{r} 143 \\ -\ \ 98 \\ \hline \\ \hline \end{array}$

i.



ii.



(i)

	1 1 7
–	3 5
	8 2

(ii)

	1 4 3
–	9 8
	52

<u>Step 2:</u>

Teacher discusses the representation of compound subtraction involving three-digit minuend and three-digit subtrahend and solves them with the class.

e.g.

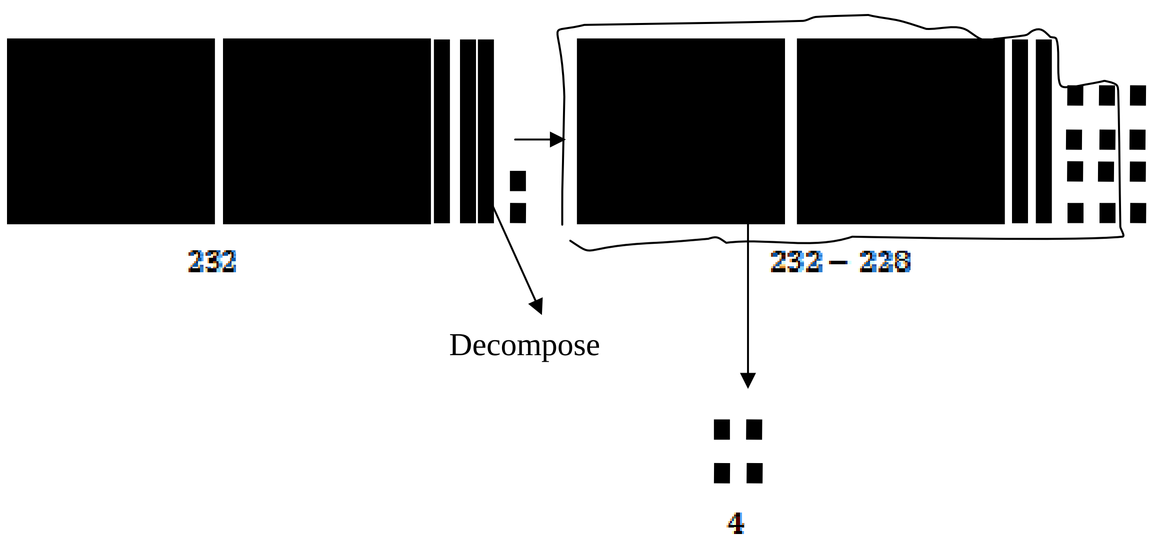
(i)

	2 3 2
–	2 2 8

(ii)

	2 6 0
–	1 2 7

i.



$$\begin{array}{lr} (i) & 232 \\ - & 228 \\ \hline & 4 \\ \hline \end{array} \qquad \begin{array}{lr} (ii) & 260 \\ - & 127 \\ \hline & 133 \\ \hline \end{array}$$

Step 3:

Teacher guides pupils to form small groups of five and gives them Dienes'Multibase Arithmetic Blocks. Thereafter, teacher gives the following exercises to pupils to do.

$$\begin{array}{lr} (i) & 272 \\ - & 225 \\ \hline & \\ \hline \end{array} \qquad \begin{array}{lr} (ii) & 280 \\ - & 157 \\ \hline & \\ \hline \end{array}$$

Closure:

Teacher assigns the following exercises to pupils to practice.

$$\begin{array}{lr} (i) & 290 \\ & 173 \\ \hline & \\ \hline \end{array} \qquad \begin{array}{lr} (ii) & 351 \\ & 265 \\ \hline & \\ \hline \end{array}$$

DAY 5

Topic: Subtraction of whole numbers.

Sub-Topic: Doing compound subtraction at the iconic stage.

Instructional Materials: Diagrams and pictures.

Content Outline:

i. Using semi-concrete materials (diagrams) to solve compound subtraction problems.

Instructional Objectives:

i. Pupil will be able to represent compound subtraction problems involving at most two two-digit numbers with diagrams.

ii. Pupil will be able to decompose and regroup minuend using with the aid of diagrams.

Relevant Previous Knowledge:

i. Pupils can represent subtraction problem with Dienes'Multibase Arithmetic Blocks.

Introduction

Teacher revises pupils' relevant previous knowledge.

Example: (i) $17 - 5$ (ii) $46 - 29$ (iii) $31 - 12$ etc

Pupils do the following exercises: (i) $17 - 5 = 12$ (ii) $48 - 29 = 19$

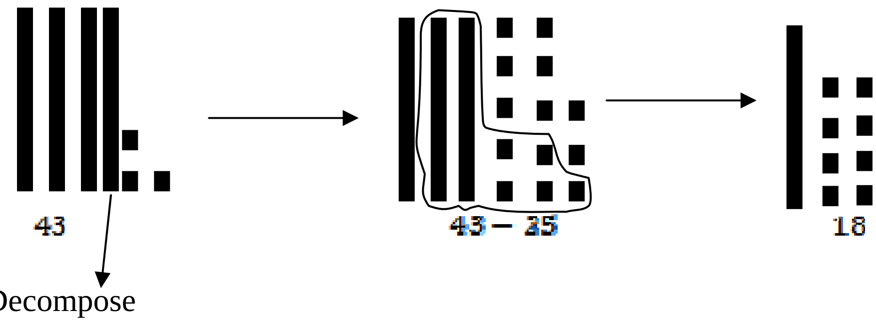
(iii) $31 - 12 = 19$

Development:

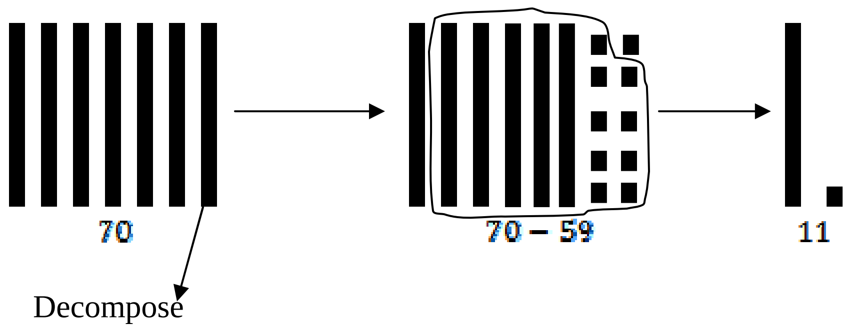
Step 1:
Teacher discusses the representation of compound subtraction involving two-digits with the class.

$$(i)\quad \begin{array}{r} 43 \\ -25 \\ \hline \\ \hline \end{array} \qquad\qquad (ii)\quad \begin{array}{r} 7\ 0 \\ -\ \ 5\ 9 \\ \hline \\ \hline \end{array}$$

(i)



(ii)



<u>Step 2:</u>

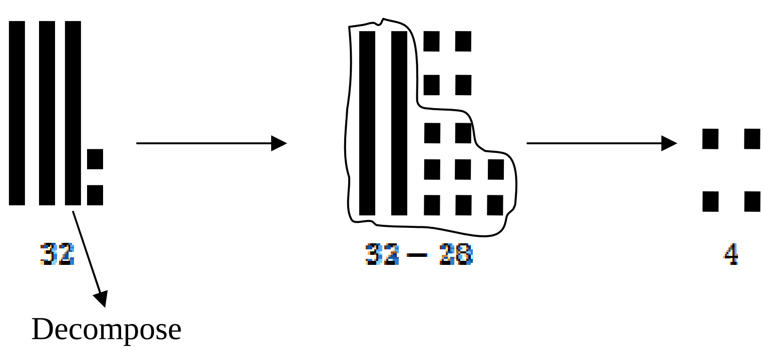
Teacher gives the following exercises to pupils to do in their various groups.

.

(i)
$$\begin{array}{r} 32 \\ -\ 28 \\ \hline \\ \hline \end{array}$$

(ii)
$$\begin{array}{r} 60 \\ -\ 27 \\ \hline \\ \hline \end{array}$$

(i)



Pupils do the above exercises in their various groups.

Closure:

Teacher assigns the following exercises to pupils to practice.

.   (i) $\begin{array}{r} 32 \\ -\ 28 \\ \hline \\ \hline \end{array}$ (ii) $\begin{array}{r} 60 \\ -\ 27 \\ \hline \\ \hline \end{array}$

DAY 6

Topic: Subtraction of whole numbers.

Sub-Topic: Doing compound subtraction at the iconic stage.

Instructional Materials: Diagrams and pictures.

Content Outline:

i. Using semi-concrete materials (diagrams) to solve compound subtraction problems.

Instructional Objectives:

i. Pupil will be able to represent compound subtraction problems involving three-digit numbers with diagrams.

ii. Pupil will be able to decompose and regroup minuend using with diagrams.

Relevant Previous Knowledge:

ii. Pupils can represent subtraction problem involving two two-digit numbers with diagrams.

Introduction

Teacher revises pupils' relevant previous knowledge.

Example: Pupils do the following exercises:

(i)
$$\begin{array}{r} 51 \\ -\ 28 \\ \hline 23 \\ \hline \end{array}$$

(ii)
$$\begin{array}{r} 70 \\ -\ 44 \\ \hline 26 \\ \hline \end{array}$$

Development:

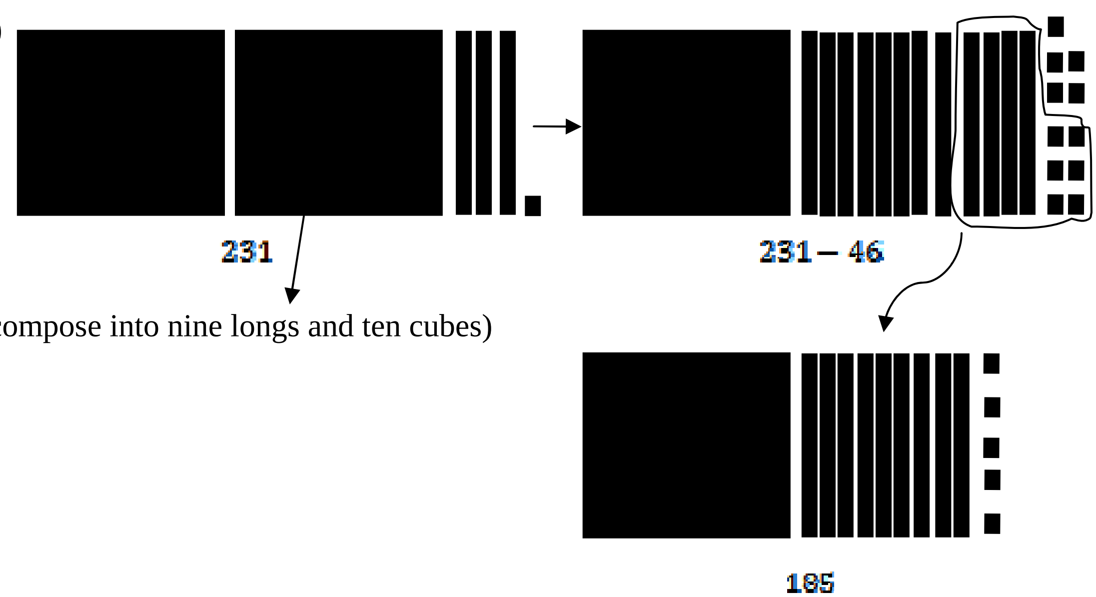
Step 1:

Teacher discusses the representation of compound subtraction involving three-digit numbers with the class.

(i)
$$\begin{array}{r} 231 \\ -\ 46 \\ \hline \\ \hline \end{array}$$

(ii)
$$\begin{array}{r} 350 \\ -\ 262 \\ \hline \\ \hline \end{array}$$

(i)



Step 2:

Teacher gives the following exercises to pupils to do in their various groups.

. (i) $\begin{array}{r} 372 \\ -\ \ 63 \\ \hline \\ \hline \end{array}$ (ii) $\begin{array}{r} 180 \\ -\ 127 \\ \hline \\ \hline \end{array}$

Pupils do the above exercises in their various groups.

Closure:

Teacher assigns the following exercises to pupils to practice.

. (i) $\begin{array}{r} 232 \\ -\ 168 \\ \hline \\ \hline \end{array}$ (ii) $\begin{array}{r} 360 \\ -\ 227 \\ \hline \\ \hline \end{array}$

DAY 7:

Topic: Subtraction of whole numbers Decomposition.

Sub-Topic: Algorithm for

Instructional Materials: Diagrams.

Content Outline: Algorithm for Decomposition.

Instructional Objectives:

Pupil will be able to use algorithm for Decomposition to solve compound subtraction problem.

Relevant Previous Knowledge:

Pupils can use diagrams to solve compound subtraction problem.

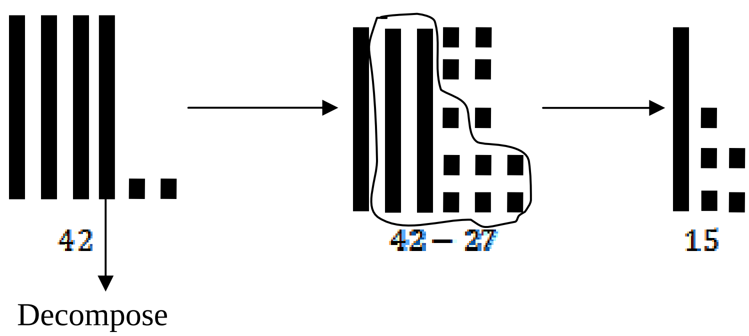
Introduction:

Teacher revises pupils' relevant previous knowledge by asking individuals to use the above mentioned concrete materials to solve compound subtraction problem. Pupils go to the chalkboard to solve compound subtraction problem as they are invited to do so.

Development:

Step 1:

Teacher demonstrates the algorithm for Decomposition using diagrams. For instance, $42 - 27$ can be solved with the aid of diagrams as follows;



The above problem can be solved using crutch as shown below:

$$\begin{array}{cc} H & O \\ 4 & 2 \\ -2 & 7 \\ \hline \\ \hline \end{array} \qquad\qquad \begin{array}{ccc} & H & O \\ & (3) & (1\ 2) \\ & 4 & 2 \\ - & 2 & 7 \\ \hline & 1 & 5 \\ \hline \end{array}$$

Step 2:

Teacher guides pupils to forms small groups of five and gives the following exercises to them to do in the various groups.

$$(i)\quad \begin{array}{r} 9\;1 \\ -\;5\;5 \\ \hline \\ \hline \end{array} \qquad\qquad (ii)\quad \begin{array}{r} 7\;0 \\ -\;4\;9 \\ \hline \\ \hline \end{array}$$

Step 3:

Teacher discusses compound subtraction problems involving three digit- number with the class.

e.g. $$(i)\quad \begin{array}{r} 3\;7\;1 \\ -\;1\;8\;5 \\ \hline \\ \hline \end{array} \qquad\qquad (ii)\quad \begin{array}{r} 2\;7\;0 \\ -\;1\;9\;2 \\ \hline \\ \hline \end{array}$$

$$(i)\quad \begin{array}{rccc} & H & T & O \\ & 3 & 7 & 1 \\ - & 1 & 8 & 5 \\ \hline & & & \\ \hline \end{array} \qquad \begin{array}{ccc} H & T & O \\ & (6) & (11) \\ 3 & 7 & 1 \\ 1 & 8 & 5 \\ \hline & & \\ \hline \end{array} \qquad \begin{array}{ccc} H & T & O \\ (2) & (16) & (11) \\ 3 & 7 & 1 \\ 1 & 8 & 5 \\ \hline 1 & 8 & 6 \\ \hline \end{array}$$

$$(ii)\quad \begin{array}{rccc} & 2 & 7 & 0 \\ - & 1 & 9 & 2 \\ \hline & & & \\ \hline \end{array} \qquad \begin{array}{rccc} & & (6) & (10) \\ & 2 & 7 & 0 \\ - & 1 & 9 & 2 \\ \hline & & & \\ \hline \end{array} \qquad \begin{array}{rccc} & (1) & (16) & (10) \\ & 2 & 7 & 0 \\ - & 1 & 9 & 2 \\ \hline & & 7 & 8 \\ \hline \end{array}$$

Closure:

Teacher assigns the following exercises to pupils to practice.

$$(i)\quad \begin{array}{r} 432 \\ -\ 178 \\ \hline \\ \hline \end{array} \qquad\qquad (ii)\quad \begin{array}{r} 460 \\ -\ 397 \\ \hline \\ \hline \end{array}$$

DAY 8:

Topic: Subtraction of whole numbers.

Sub-Topic: Compound Subtraction at the Abstract Stage

Content Outline: Doing compound subtraction at the abstract stage

Instructional Objectives:

Pupil will be able to solve compound subtraction problem without using concrete or semi-concrete materials.

Relevant Previous Knowledge:

Pupils can find the complement of 1, 4, 7, 54, 89, etc

Introduction:

Teacher revises pupils' relevant previous knowledge by asking individuals to give 1, 4, 7, 9, etc.

Pupils give the complement of the above numbers as 9, 6, 3, 1, etc.

Development:

Step 1:

Teacher discusses problems with the class but this time round verbalizing the algorithms instead of illustrating them.

$$\begin{array}{rr} & 4\ 11 \\ - & 235 \\ \hline & 176 \\ \hline \end{array} \qquad (ii) \quad \begin{array}{rr} & 5\,2\,0 \\ - & 4\,1\,9 \\ \hline & 1\,0\,1 \\ \hline \end{array}$$

Pupils contribute to the discussion and ask questions.

Step 2:

Teacher discusses compound subtraction problems involving subtrahend with successive nines with the class.

$$(i) \quad \begin{array}{rr} & 4\ 31 \\ - & 2\,9\,9 \\ \hline & \\ \hline \end{array} \qquad (ii) \quad \begin{array}{rr} & 7\,2\,0 \\ - & 4\,9\,9 \\ \hline & \\ \hline \end{array}$$

(2)

$$(i) \quad \begin{array}{rr} & 4\ 31 \\ - & 2\,9\,9 \\ \hline & \\ \hline \end{array} \qquad \begin{array}{rccc} & 5 & 3 & 1 \\ & (3) & (n) & (n) \\ - & 2 & 9 & 9 \\ \hline & 3 & 3 & 2 \\ \hline \end{array}$$

Closure:

Teacher gives the exercises below to pupils to do.

DAY 9:

Topic: General revision of subtraction of whole numbers.

Instructional Materials: Dienes'Multibase Arithmetic Blocks.

Content Outline: Discussion on compound subtraction.

Relevant Previous Knowledge:

Pupils have been introduced to compound subtraction.

Introduction:

Teacher revises pupils' relevant previous knowledge by asking them to give base complement of some numbers.

Pupils give the base complement of numbers and go to the chalkboard to solve compound subtraction problem as teacher invites the in turns to do so.

Development:

Step 1:

Teacher revises compound subtraction (two-digit minuend and one- digit subtrahend, two two-digit and three-digits) with the class using concrete and semi-concrete materials.

Pupils go to the chalkboard in turns to solve problems as teacher invites them to do so

Step 2:

Teacher discusses word problems on compound subtraction with the class.

Pupils contribute to the discussion and ask questions.

Closure:

Teacher gives pupils exercises to try at home.

APPENDIX H

COMPUTATION OF MEAN AND STANDARD DEVIATION OF PRETEST SCORES

DEC					BCA				
x/c_1	f	fx	$(x-\bar{x})^2$	$f(x-\bar{x})^2$	x/c_2	f	fx	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
1					1	1	1	225	225
2					2				
3					3				
4					4				
5					5				
6					6				
7					7	1	7	81	81
8	1	8	65.12	65.12	8				
9					9				
10					10				
11	2	22	25.70	51.40	11				
12					12	2	24	16	32
13					13	1	13	9	9
14	3	42	4.28	12.84	14	2	28	4	8
15	3	45	1.14	3. 42	15	5	75	1	5
16	6	96	0.004	0	16	2	32	0	0
17	7	119	0.86	6.02	17	3	51	1	3
18	5	90	3.72	18.60	18	4	72	4	16
19					19	3	57	9	27
20	3	60	15.44	46.32	20	6	100	16	96
	$\Sigma f = 3($	$\Sigma fx = 4$		$\Sigma f(x-\bar{x})^2$		$\Sigma f = 3$	$\Sigma fx = 4$		$\Sigma f(-x)^2 =$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{482}{30} = 16.07$$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{802}{30} = 16$$

$$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{203.72}{30}} = 2.61$$

$$\sigma - \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{302.72}{30}} = 2.61$$

APPENDIX I

COMPUTATION OF MEAN AND STANDARD DEVIATION OF

POSTTESSCORES

DEC					BCA				
x/c_1	f	fx	$(x-\bar{x})^2$	$f(x-\bar{x})^2$	x/c_2	f	fx	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
1					1				
2					2	2	4	82.26	164.52
3	1	3	7	7	3				
4					4				
5					5				
6	2	12	16	32	6	1	6	25.70	25.70
7	2	14	9	18	7	1	7	16.56	16.56
8	7	56	4	24	8	5	40	9.42	47.10
9	8	72	1	8	9	1	9	4.28	4.28
10	2	20	0	0	10	3	30	1.14	3.42
11	2	22	1	2	11	2	22	0	0
12	1	12	4	4	12	2	24	0.86	1.72
13					13	6	78	3.72	22.32
14					14	1	14	2.92	2.93
15	1	15	25	25	15	2	30	15.44	30.88
16					16	1	16	24.30	24.30
17	1	17	49	49	17	2	34	35.16	70.32
18	1	18	64	64	18	1	18	48.02	48.02
19	1	19	81	81	19				
20	1	20	100	100	20				
	$\Sigma f = 3$	$\Sigma fx = 30$		$\Sigma f(x-\bar{x}$		$\Sigma f = 30$	$\Sigma fx = 3$		$\Sigma f(x-\bar{x}$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{300}{30} = 10$$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{332}{30} = 11.07$$

$$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{404}{30}} = 3.73$$

$$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{439}{30}} = 3.83$$

APPENDIX J

COMPUTATION OF MEAN AND STANDARD DEVIATION OF

UNDERSTANDING TEST SCORES

DEC					BCA				
x/c_1	f	fx	$(x-\bar{x})^2$	$f(x-\bar{x})^2$	x/c_2	f	fx	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
0	2	0	4.00	0	0	1	0	36	0
1	9	9	1.69	1.69	1	2	2	25	50
2	2	4	5.29	11.18	2	3	6	16	48
3	7	21	0.08	0.63	3	3	9	9	27
4	1	4	0.49	0.49	4	2	8	4	8
5					5	5	25	1	5
6	2	12	6.97	13.94	6	1	6	0	0
7	1	7	3.70	3.70	7	1	7	1	1
8					8	3	24	4	12
9	1	9	32.49	32.49	9				
10					10	2	20	16	32
11	2	22	59.29	118.58	11	2	22	25	50
12					12	1	12	36	36
13					13	1	13	49	49
14					14	1	14	64	64
15					15				
16					16				
17					17				
18					18				
19					19				
20					20				
	$\Sigma f = 2$	$\Sigma fx = 88$		$\Sigma f(x-\bar{x})$		$\Sigma f = 28$	$\Sigma fx = 1$		$\Sigma f(x-$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{88}{27} = 3.3$$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{168}{28} = 6.0$$

$$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{182.70}{27}} = 2.60$$

$$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{382}{28}} = 3.69$$

APPENDIX K

COMPUTATION OF MEAN AND STANDARD DEVIATION OF

RETENTION TEST SCORES

DEC					BCA				
x/c_1	f	fx	$(x-\bar{x})^2$	$f(x-\bar{x})^2$	x/c_2	f	fx	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
1	7	7	23.91	167.37	1	1	1	71.91	71.91
2	4	8	15.13	60.52	2	1	2	55.95	111.90
3					3	2	6	41.99	83.98
4	3	12	5.57	16.71	4				
5	2	10	0.97	1.58	5				
6	1	6	0.01	0.01	6	2	12	12.11	24.22
7					7	3	21	6.15	18.45
8	1	8	4.45	4.45	8	2	16	2.19	4.38
9	2	18	9.67	19.34	9	3	27	0.23	0.69
10	1	10	16.89	16.89	10	3	30	0.27	0.81
11	3	33	26.11	78.33	11	3	33	2.31	6.93
12	2	24	37.33	74.66	12	1	12	6.35	6.35
13					13	2	26	12.39	24.78
14	1	14	65.77	65.77	14	3	42	20.43	61.29
15	1	15	82.99	82.99	15	1	15	30.49	30.49
16					16	2	32	13.04	13.04
17					17				
18					18				
19					19				
20					20				
	$\Sigma f = 2$	$\Sigma fx = 16$		$\Sigma f(x-\bar{x}$		$\Sigma f = 30$	$\Sigma fx = 5$		$\Sigma f(x-$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{168}{28} = 5.89 \qquad \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{302}{30} = 9.48$$

$$\sqrt{\sigma = \frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{888.62}{28}} = 4.58 \qquad \sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{457.22}{29}} = 3.98$$

APPENDIX L

COMPUTED SPEED OF SUBJECTS ON PRETEST TEST

SUBJECTS	TIME	SCORE	SPEED	POSITION	REMARK
Y_{15}	9.00	20	2.22	1st	
Y_{24}	10.00	19	1.90	2nd	
Y_{6}	11.00	20	1.82	3rd	
Y_{8}	11.00	20	1.82	4th	
Y_{21}	12.00	19	1.58	5th	
Y_{9}	11.00	17	1.55	6th	
X_{3}	10.00	16	1.50	7th	
X_{18}	10.05	18	1.49	8th	
X_{2}	10.20	17	1.47	9th	
Y_{12}	10.50	15	1.43	10th	
X_{1}	11.25	15	1.33	11th	
Y_{7}	11.09	14	1.26	12th	
X_{4}	13.00	11	1.15	13th	
X_{5}	13.10	18	1.15	14th	
X_{6}	14.00	17	1.07	15th	
X_{7}	14.15	16	1.06	16th	
Y_{2}	17.00	18	1.06	17th	
Y_{3}	17.03	18	1.06	18th	
Y_{11}	14.20	15	1.06	19th	
X_{19}	14.30	16	1.05	20th	
Y_{10}	11.45	12	1.05	21st	
X_{20}	14.45	20	1.04	22nd	
X_{8}	14.58	20	1.03	23rd	
X_{21}	14.55	20	1.03	24th	
X_{9}	15.01	15	1.00	25th	
X_{22}	15.00	14	1.00	26th	
Y_{28}	20.00	20	1.00	27th	
X_{26}	15.20	17	0.99	28th	MFT&S(DEC)
X_{29}	15.13	17	0.99	29th	
X_{30}	15.20	17	0.99	30th	CMFT&S

(CON'T) APPENDIX L

COMPUTED SPEED OF SUBJECTS ON PRETEST TEST

SUBJECTS	TIME	SCORE	SPEED	POSITION	REMARK
X_{10}	15.25	14	0.98	31^{st}	
X_{11}	15.13	18	0.94	32^{nd}	
X_{12}	16.01	17	0.94	33^{rd}	
Y_{29}	17.00	16	0.94	34^{th}	
X_{23}	16.20	15	0.93	35^{th}	
X_{24}	16.30	18	0.92	36^{th}	
Y_{23}	20.08	18	0.90	$\boxed{37^{th}}$	MFT&S(BCA)
X_{25}	18.00	17	0.88	38^{th}	
Y_{1}	23.00	20	0.87	39^{th}	
X_{26}	17.50	16	0.86	40^{th}	
Y_{14}	15.03	13	0.86	41^{st}	
Y_{17}	22.00	19	0.86	42^{nd}	
Y_{25}	21.30	18	0.85	43^{rd}	
Y_{19}	24.00	20	0.83	44^{th}	
Y_{27}	18.00	15	0.83	45^{th}	
Y_{13}	18.20	17	0.82	46^{th}	
Y_{22}	20.00	16	0.80	47^{th}	
X_{14}	19.00	16	0.79	48^{th}	
X_{15}	19.20	16	0.78	49^{th}	
X_{16}	19.40	18	0.77	50^{th}	
Y_{18}	22.00	17	0.77	51^{st}	
X_{27}	20.15	16	0.74	52^{nd}	
Y_{26}	20.40	15	0.74	53^{rd}	
X_{17}	21.48	11	0.70	54^{th}	
Y_{20}	24.22	17	0.70	55^{th}	
Y_{30}	22.00	15	0.68	56^{th}	
Y_{16}	24.00	12	0.50	57^{th}	
Y_{13}	30.00	14	0.47	58^{th}	
Y_{5}	23.40	7	0.30	59^{th}	
Y_{4}	12.00	1	0.08	60th	

LEGEND:

- CMFT&S - Common Median Finishing Time And Speed
- MFT&S - Median Finishing Time And Speed
- X - Subjects of The DEC Group
- Y - Subjects of The BCA Group

APPENDIX M

COMPUTED SPEED OF SUBJECTS ON POSTTEST

SUBJECTS	TIME	SCORE	SPEED	POSITION	REMARK
X_{11}	14.50	20	1.38	1st	
X_{2}	14.40	19	1.32	2nd	
X_{3}	18.00	18	1.00	3rd	
Y_{29}	19.50	17	0.87	4th	
Y_{8}	14.03	12	0.86	5th	
Y_{18}	21.00	18	0.86	5th	
X_{29}	10.20	8	0.78	7th	
Y_{1}	19.20	15	0.78	8th	
Y_{22}	19.30	15	0.78	8th	
X_{14}	22.01	17	0.77	10th	
Y_{14}	18.10	13	0.72	11th	
Y_{19}	18.10	13	0.72	12th	
X_{28}	17.00	11	0.65	13th	
X_{1}	14.00	9	0.64	14th	
Y_{3}	11.01	7	0.64	14th	
Y_{26}	26.50	17	0.64	14th	
V_{15}	22.20	14	0.64	14th	
Y_{21}	26.20	16	0.61	18th	
X_{5}	15.00	9	0.60	19th	
X_{10}	10.00	6	0.60	20th	
X_{20}	16.00	9	0.56	21st	
Y_{17}	23.10	13	0.56	21st	
Y_{4}	18.10	10	0.55	23rd	
Y_{9}	24.05	13	0.54	24th	
X_{30}	15.20	8	0.53	25th	
Y_{5}	11.40	6	0.53	25th	MFT&S(BCA)
X_{22}	17.20	9	0.52	27th	
X_{19}	22.10	11	0.50	28th	
X_{24}	16.05	8	0.50	29th	
X_{7}	21.05	10	0.48	30th	MFT&S (DEC) CMFT&S

(CON'T) APPENDIX M

COMPUTED SPEED OF SUBJECTS ON POSTTEST

SUBJECTS	TIME	SCORE	SPEED	POSITION	REMARK
X_{23}	32.48	15	0.46	31^{st}	
Y_{24}	28.20	13	0.46	31^{st}	
Y_{10}	19.04	8	0.42	33^{rd}	
Y_{13}	28.30	12	0.42	33^{rd}	
X_{28}	22.15	9	0.41	34^{th}	
X_{15}	23.50	9	0.38	35^{th}	
Y_{16}	21.10	8	0.38	35^{th}	
X_{27}	33.00	12	0.36	37^{th}	
X_{12}	23.10	8	0.35	38^{th}	
Y_{23}	33.30	11	0.33	39^{th}	
Y_{25}	24.50	8	0.33	39^{th}	
Y_{27}	30.30	10	0.33	39^{th}	
X_{21}	29.17	9	0.33	39^{th}	
X_{26}	26.06	8	0.31	44^{th}	
Y_{28}	35.20	11	0.31	45^{th}	
X_{6}	31.4	9	0.29	46^{th}	
X_{13}	24.08	7	0.29	46^{th}	
X_{16}	34.00	10	0.29	46^{th}	
Y_{11}	35.00	10	0.29	49^{th}	
Y_{12}	28.05	8	0.29	49^{th}	
Y_{20}	28.10	8	0.28	51^{st}	
X_{17}	29.40	8	0.27	52^{nd}	
X_{4}	27.00	7	0.26	53^{rd}	
X_{25}	31.20	8	0.26	53^{rd}	
X_{8}	12.08	3	0.25	55^{th}	
X_{9}	26.30	6	0.23	56^{th}	
Y_{2}	11.50	2	0.17	57^{th}	
Y_{7}	15.00	2	0.13	58^{th}	
Y_{6}	30.28	3	0.10	59^{th}	
Y_{30}					

APPENDIX N

COMPUTED SPEED OF SUBJECTS ON RETENTION TEST

SUBJECTS	TIME	SCORE	SPEED	POSITION	REMARK
Y_{3}	15.10	13	0.86	1st	
Y_{7}	15.20	13	0.86	1st	
Y_{1}	14.20	12	0.85	3rd	
Y_{27}	17.20	14	0.81	4th	
X_{3}	19.00	15	0.79	5th	
Y_{6}	14.00	11	0.79	5th	
Y_{29}	20.45	16	0.78	7th	
Y_{30}	14.20	10	0.70	8th	
X_{9}	15.20	10	0.66	9th	
Y_{20}	22.10	14	0.63	10th	
Y_{16}	29.20	18	0.62	11th	
Y_{4}	10.20	6	0.59	12th	
Y_{15}	17.30	10	0.58	13th	
Y_{21}	26.20	15	0.57	14th	
X_{24}	21.50	12	0.56	15th	
Y_{19}	20.00	11	0.55	16th	
Y_{22}	17.00	9	0.53	17th	
X_{7}	21.00	11	0.52	18th	
X_{27}	27.00	14	0.52	18th	
Y_{2}	21.00	11	0.52	18th	
X_{1}	22.00	11	0.50	21st	MFT&S(BCA)
Y_{28}	28.00	14	0.50	21st	
Y_{17}	21.00	10	0.48	23rd	
Y_{8}	15.00	7	0.47	24th	
Y_{18}	20.40	9	0.44	25th	
Y_{5}	21.50	9	0.42	26th	
X_{4}	23.00	9	0.39	27th	
X_{28}	29.00	11	0.38	28th	
Y_{10}	21.30	8	0.38	28th	
X_{26}	35.00	12	0.34	30th	
Y_{14}	26.00	8	0.31	31st	CMFT&S

(CON'T) APPENDIX N

COMPUTED SPEED OF SUBJECTS ON RETENTION TEST

SUBJECTS	TIME	SCORE	SPEED	POSITION	REMARK
X_{14}	16.50	5	0.30	32^{nd}	
X_{30}	23.20	7	0.30	32^{nd}	
X_{6}	31.00	9	0.29	34^{th}	
Y_{24}	21.30	6	0.28	35^{th}	
X_{21}	29.30	8	0.27	36^{th}	
X_{11}	16.00	4	0.25	37^{th}	
X_{13}	16.30	4	0.25	37^{th}	
Y_{13}	29.00	6	0.21	39^{th}	MFT&S (DEC)
Y_{11}	39.00	7	0.18	40^{th}	
Y_{12}	39.00	4	0.17	41^{st}	
X_{12}	24.20	4	0.17	41^{st}	
Y_{16}	20.00	3	0.15	43^{rd}	
X_{28}	37.10	5	0.13	44^{th}	
X_{27}	16.00	2	0.13	44^{th}	
Y_{26}	28.10	3	0.11	46^{th}	
X_{18}	20.05	2	0.10	47^{th}	
X_{17}	24.30	2	0.08	48^{th}	
Y_{9}	24.00	2	0.08	48^{th}	
X_{8}	15.00	1	0.07	50^{th}	
X_{10}	15.20	1	0.07	50^{th}	
X_{2}	16.20	1	0.06	52^{nd}	
X_{5}	18.00	1	0.06	52^{nd}	
X_{22}	18.00	1	0.06	52^{nd}	
X_{25}	36.00	2	0.06	52^{nd}	
X_{15}	22.20	1	0.05	56^{th}	
Y_{25}	22.00	1	0.05	57^{th}	
X_{16}					
X_{20}					
Y_{28}					

APPENDIX O

Computation for Pretest, Posttest and Retention Test on the Median Test

$$\chi^2 = \frac{T\left(|b_1 d_2 - b_2 d_1| - \frac{T}{2}\right)^2}{T_1 \times T_2 \times T_b \times T_d}$$

Pretest Time Score

Reference to the 2×2 contingency table at page 73 (table 5)

$b_1 = 13, b_2 = 17, d_1 = 14, d_2 = 16, T_1 = 27, T_2 = 33, T_b = 30, T_d = 30, T = 60$

$$\chi^2 = \frac{60\left(|(13 \times 16) - (17 \times 14)| - \frac{60}{2}\right)^2}{27 \times 33 \times 30 \times 30}$$

$$= 0$$

χ^2 value at 5% significant level = 3.84.

Posttest Time Score

Reference to the 2×2 contingency table at page 73 (table 6)

$b_1 = 15, b_2 = 14, d_1 = 13, d_2 = 17, T_1 = 28, T_2 = 31, T_b = 29, T_d = 30, T = 59$

$$\chi^2 = \frac{59\left(|(15 \times 17) - (14 \times 13)| - \frac{59}{2}\right)^2}{28 \times 31 \times 29 \times 30}$$

$$= 0.17$$

Retention Time Score

Reference to the 2×2 contingency table at page 73 (table 6)

$b_1 = 21, b_2 = 8, d_1 = 10, d_2 = 18, T_1 = 31, T_2 = 26, T_b = 29, T_d = 28, T = 57$

$$\chi^2 = \frac{57\left(|(21 \times 18) - (10 \times 8)| - \frac{57}{2}\right)^2}{31 \times 26 \times 29 \times 28}$$

$$= 6.33$$

APPENDIX P

Statistical Instrument for Data Analysis

The formulae used in the analysis of data included the following

A.

- Mean($\bar{x}$) $= \frac{\Sigma fx}{\Sigma f}$
- Standard deviation(s)

$$= \sqrt{Variance} = \sqrt{s^2} = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{n-1}}$$

where x = the scores

$\bar{x}$ = mean

n = sample size

B. The Median Test

- 2 x 2 Contingency Table

	BCA	DEC	TOTAL
ABOVE	b_1	d_1	T_1
BELOW	b_2	d_2	T_2
TOTA	T_b	T_d	T

- $$\chi^2 = \frac{T\left(|b_1 d_2 - b_2 d_1| - \frac{T}{2}\right)^2}{T_1 \times T_2 \times T_b \times T_d}$$

C. The t-test

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{[(n_1 - 1)s_1^2 (n_2 - 1)s_2^2](n_1 + n_2)}{(n_1 + n_2 - 2)(n_1 n_2)}}}$$

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D. $$r_{total\ test} = \frac{KS^2 - \bar{X}(K - \bar{X})}{S^2(K-1)}$$

Statistical Instruments for Data Analysis

where $x =$ scores

$\bar{x}_1$ = mean score of the BCA group

$\bar{x}_2$= mean score of the DEC group

$\sum f$ = sum of frequency

$\sum fx =$ sum of the product of frequency and score

s_1^2 = variance of BCA group

s_2^2= variance of DEC group

n_1 = sample size of BCA

n_2 = sample size of DEC

b_1 = frequency of BCA group above median finishing time.

d_1 = frequency of DEC group above the median finishing time.

b_2 = frequency of BCA group below the median finishing time.

d_2 = frequency of DEC group below the median finishing time.

T_1 = total frequency of both groups above the median finishing time.

T_2 = total frequency of both groups below the median finishing time.

T_b= total frequency of the BCA group.

T_d= total frequency of the DEC group.

T = grand total.

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χ^2 = chi- square

S = standard deviation

K = number of test items

APPENDIX Q

SPSS Analysis

Group Statistics

	PRE-TEST	N	Mean	Std. Deviation	Std. Error Mean
SCORE	DEC	30	16.4000	2.19089	.40000
	BCA	30	16.1000	3.80879	.69539

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	T	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
SCORE	Equal variances assumed	5.115	.027	.374	58	0.710	.30000	0.80222	-1.30583	1.90583
	Equal variances not assumed			.374	46.297	0.710	.30000	.80222	-1.31451	1.91451

Group Statistics

	POSTTEST(ACCURACY TEST)	N	Mean	Std. Deviation	Std. Error Mean
SCORE	DEC	30	10.0000	3.98272	.72714
	BCA	29	10.8621	3.94357	.73230

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Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
SCORE	Equal variances assumed	.144	.705	-.835	57	.407	-.86207	1.03217	-2.92894	1.20481
	Equal variances not assumed			-.835	56.965	.407	-.86207	1.03199	-2.92862	1.20448

(CON'T) APPENDIX Q

Group Statistics

	RETENTION TEST	N	Mean	Std. Deviation	Std. Error Mean
SCORE	DEC	28	6.5357	5.07340	.95878
	BCA	29	9.1379	3.97064	.73733

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	T	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
SCORE	Equal variances assumed	4.192	.045	-2.161	55	.035	-2.60222	1.20433	-5.01575	-.18868
	Equal variances not assumed			-2.151	51.134	.036	-2.60222	1.20951	-5.03026	-.17417

(CON'T) APPENDIX Q

Group Statistics					
	TEST OF UNDERSTANDING	N	Mean	Std. Deviation	Std. Error Mean
SCORE	DEC	27	3.2593	3.12056	.60055
	BCA	28	6.0000	3.93465	.74358

Independent Samples Test										
		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	T	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
SCORE	Equal variances assumed	3.085	.085	-2.855	53	.006	-2.7407	.95985	-4.66596	-.81552
	Equal variances not assumed			-2.867	51.12	.006	-2.74074	.95581	-4.65950	-.82198