UNIVERSITY OF CAPE COAST

FEASIBILITY OF TEACHING QUADRATIC EQUATIONS IN SENIOR HIGH SCHOOL FORM ONE

FRANK SAM

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BY

FRANK SAM

Thesis submitted to the Department of Science and Mathematics Education of the College of Education Studies, University of Cape Coast, in partial fulfilment of the requirements for award of Master of Philosophy Degree in Mathematics Education.

OCTOBER, 2013

DECLARATION

Candidate's Declaration

I hereby declare that this thesis is the result of my own original research and that no part of it has been presented for another degree in this university or elsewhere.

Candidate's Signature: .... Date: ....

Name: Frank Sam

Supervisors' Declaration

We hereby declare that the preparation and presentation of the thesis were supervised in accordance with the guidelines on supervision of thesis laid down by the University of Cape Coast.

Principal Supervisor's Signature: .... Date: ....

Name: Dr. E. M. Wilmot

Co-Supervisor's Signature: ....Date: ....

Name: Mr. John Gyening

ABSTRACT

The study addressed the curriculum problem of the placement of the teaching and learning of quadratic equation in the Senior High School. Its purpose was to assess the ability of S.H.S 1 students to solve quadratic equation by investigating their performance, using four methods.

Stratified sampling method was used to select four schools in four districts in the Central Region. Eight classes were randomly selected for the study. The sample size was 286 of which 160 were girls.

A pretest-posttest non-equivalent comparison group design was used. Treatment group means scores on posttest were compared with their respective mean scores on the pretest. The posttest scores were also used to compare the effectiveness of the methods. The t-test and analysis of covariance (Ancova) were used in the analysis.

The students performed significantly better in the posttest than the pretest. The quadratic formula proved significantly superior to the other methods. Conjugales, factorization and equivalent simultaneous linear equation followed in that order with respect to solving quadratic equation in S.H.S 1. It was concluded that the solution of quadratic equation can be taught effectively in Form One and that the quadratic formula seems to be the best method.

Based on the findings, it was recommended that present core mathematics syllabus should be revised so that quadratic equations would be taught in Form 1 of the S.H.S 1 and also the novel methods - conjugales and equivalent simultaneous linear equation should be included in the S.H.S mathematics curriculum as alternative methods of solving quadratic equations.

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Finally, I would like to share the success of this work with all those who were of assistance to me in various ways.

DEDICATION

This thesis is dedicated to my family for their prayers, humility, support, encouragement, patience and understanding during the course of my study.

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CHAPTER ONE

INTRODUCTION

This chapter covers the background to the study, the statement of the problem, purpose of the study, research questions and hypotheses that would be tested. It also takes into consideration the significance of the study and outlines the limitations and delimitations. Also include in this chapter is the organization of the study. Solution of quadratic equations is one topic that is widely used in mathematics and other related subjects. The familiar methods in Ghanaian senior high schools are; the method of completing the square, the use of quadratic formula, the graphical method, inspection (in the case of incomplete quadratics) and the factorization method.

Background to the Study

As mathematics still proves to be the foundation of many businesses and professions, its importance cannot be ruled out in today's society. According to Coaker (1985) "among the lot of things future employers would really expect from their employees include numeracy, literacy, ability to communicate, trainability, flexibility, economic literacy, willingness to take responsibility and ability to work with others.

Economic progress is stemmed from industrialization which is based on science and technology. These areas of study cannot be separated from mathematics. Thus, mathematics is the focus of the world. Principles of mathematics are used day in; day out in the normal running of man’s activities.

It is a fact that mathematics plays a vital role in the daily life of man all over the world. Mathematics is applied in many professions such as economics, engineering, nursing, medicine, architecture and banking, among others. For instance, in civil engineering, engineers use mathematics to determine the strength of buildings, bridges and other structures. Nurses keep records of patients of the hospital and children at maternal clinics sometimes in the form of graphs. In buying and selling, mathematics is used to determine the change one will receive if one pays more than the cost of the item(s). Also, mathematics is used to determine the item(s) an amount of money can buy.

The subject is used as an essential means of communication. It can be used to describe, illustrate, interpret, predict and to explain. It can also be used to make powerful, concise, precise and unambiguous statements. Symbols in mathematics are used to convey ideas to the audience. Many social activities call for the use of mathematics. In games like scrabble, soccer and others, mathematics is made use of. For instance, in scrabble, the board is square in shape and the scoring is applied by multiplication and addition.

Many other subjects including Geography, Physics, Chemistry, Economics and many others make use of mathematics. In all these cases identified, models are developed which are used to solve related problems. Mathematical models are usually presented in the form of equations. These equations include differential equations, trigonometric equations and algebraic equations of which the quadratic is one typical example. The quadratic equation is any equation that can be expressed in the form $ax^2 + bx + c = 0$ which $a \neq 0$, b and c are real constants and x is a variable.

The implementation of Ghana's most recent educational reform which begun in 1987, brought forth many problems in the objectives, content, administration and management of education. There have been a number of reviews in terms of its coverage, quality, equitability and economic utility under the latest 1987 educational reforms to reflect the needs of the society. The aim of teaching and learning mathematics is either to produce future mathematicians or people who will sufficiently be literates in that, as the core mathematics syllabus states (September, 2007). It is to aid them in performing daily activities such as buying, selling, measuring, counting and weighing. It is also to enable all Ghanaian young person's to acquire the mathematical skills, insights, attitudes and values they will need to learn Mathematics.

According to Coxford and Payne (1984) economist use the quadratic equation to find ways to describe manufacturing cost, sales revenue and business profit. They represent a function for cost reduction arising from large lots of raw materials as $c\ (x) = -0.001x^2 + 10x + 5$; $x \geq 0$ and x is the unit of raw materials.

In physics, the path followed by a projected body (trajectory or projectiles) has the equation of the form of a quadratic curve. The path of trajectory of a body projected with an initial velocity V_0 at an angle of departure θ^0 is given by the relation $y = x\tan\theta - \frac{gx^2}{2v_o^2 cos^2\theta^o}$Where the quantities $V_{0,}$ $tan\ \theta^{\circ}$, $cos\ \theta^{\circ}$ and g are constants. Hence, the equation can be expressed in the form $y = ax - bx^2$ is a parabola.

The understanding of dynamics is much deepened by the works of Galileo on equations of motion in knowing when and how to stop a moving object (Halliday, Robert & Wiley, 1989). This was used in explaining the relationship between the concept of acceleration and quadratic equations. If an object is moving in a direction, without external force acting on it, it continues to move in that direction with a constant velocity. If a particle starts at the point $x_1 = 0$ and moves in that manner for time, *t*, then its resultant position is given as $x = ut$. Usually, the particle has a force of gravity or friction working on it. Newton discovered the production of a constant acceleration *a*, is the effect of a constant force. If the velocity at the start is *u*, then the velocity, *v* after time, *t*, is given as $v = u + at$. Galileo realized that, one can go from this expression to work at the position of the moving particle after sometime. (Halliday, Robert & Wiley, *1989*). In that, if the particle starts with a velocity *u*, then the position*s* at the time, *t*is given as S$= ut + \frac{1}{2}at^2$ where a is the acceleration due to gravity. From the quadratic equation connecting *t* and s, we can determine how far we travel during a period of time, or how long it will take to travel a distance. Another area of concern is the ability to deduce the stopping distance of a car travelling at a given velocity. If a constant de-acceleration –*a*, is applied to slow a car down from speed *u* to a halt, speed zero, then, using $v^2 = u^2 + 2(-a)s$ gives us the stopping distance *s*, as $s = \frac{u^2}{2a}$ when $u = 0$. This shows that increasing ones speed actually quadruples, not doubles, his stopping distance. This is the reason why we should slow down when driving through a settlement since a small reduction in speed, leads to a much larger reduction in stopping distance. Other physical activities which are based on solutions of quadratic equations include the construction of the Holland tunnel under the Hudson River in New York (Reeves & Kilmister, 1952) Radars dishes, reflectors and spotlights, components of microphones and some cables of suspension bridges are all in the shape of parabolas.

The base fact in quadratic equations is that no matter the value of x, x^2 is always no negative. Thus, the equation $x^2 = -1$ has no real roots. This challenge was overcome by the use of the letter *i*, to represent imaginary numbers as a solution to $x^2 = -1$.

$$x^2 = -1$$

$$x = \pm\sqrt{-1}$$

Where $i = \pm\sqrt{-1}$, so that if $x^2 = -4$

Then $x = \pm\sqrt{4}$

$$x = \pm 2\sqrt{-1}$$

$$x = \pm 2i$$

The quantum theory utilizes this concept in the physical world. This theory deals with phenomena at a microscopic level where quantities (electrons and protons of energy) can behave both like particles and like oscillating waves. This oscillating behaviour can be described using *i*. According to Nachi (*1989*) the Schrondinger's equation is the fundamental equation of quantum theory, which is used to calculate the "*wave number*" of a quantity (the probability of it being in a particular location). This is partially a differential equation involving *i*, which can be written as

$$\frac{i\partial u}{\partial t} + v^2 u + v(x)u = 0.$$

This can be used to deduce the motion of the electrons in semi-conductors in order to design integrated circuits with huge number of components which can perform complex tasks. Such circuits are used in the making of many modern technological appliances including computers, DVD player and mobile phones. A mobile phone works by converting your speech into high frequency radio waves and the behaviour of these waves can be calculated using equations involving ***i***, thus, we can say that without the quadratic equation $x^2 = -1$, the mobile phone would not have been invented.

Cordey (*1945*) proves that the vibration of the simple pendulum is an application of quadratic equation. The vibration frequency of simple pendulum can be represented by $T^2 = \left(\frac{4\pi^2}{g}\right)L$, where T is the time of one vibration, L is the length of the pendulum and g is the acceleration due to gravity. This led to the invention of the chronometer. Due to the indispensable relation of many activities to quadratic equation, Sawyer (1970) and Moushovitz–Hadar (*1993*) noted the importance of its study as part of education as the most fundamental of polynomial functions and central to the study of mathematics. Willoughby (1967) noted that to upgrade the study of Mathematics; a pupil will need a good understanding of quadratic equations, since there are many problems in geometry which lead to such equations. This was also observed by Nickle (1942) in a study conducted, concerning the topic of quadratic equations.

It is thus, essential for teachers to effectively teach this topic since the basic methods of mathematical analysis is applied rather than the detailed results. Dubisch (*1963*) stated that more emphasis should therefore be placed on enhancing the student’s logical thought and critical analyses that can be applied to other fields of study, rather than the product of a mathematical problem. Though mathematics is a core subject in the high school curriculum, the performance of students in their final year exams is very low. The Chief examiner's report of the West African Examination Council (1995) attributes the poor performance to the fact that, teachers were unable to complete the syllabus within the allotted time or rushed through the mathematics syllabus due to its load. Further studies of subsequent reports of chief Executive Examiners reports (WAEC 1986, 1996, 1998) revealed that students were particularly unpopular with questions involving factorizing quadratic trinomials.

Candidates of GCE Level Modern Mathematics (June 1986) were asked to find the truth set of $3x^2 - 3x - 289 = 0$ by factorization. According to a report after evaluating the students answers, a very high proportion of students found it difficult to factorize the left hand side of the equation even though it is indicated in the syllabus that factorization is only restricted to the solution of quadratic equations (WAEC, 1986). In another instance, during the Nov/Dec 1998 SSSCE, candidates could not use the conventional method to factorize $x^2 + 2x - 35 = 0$, to find the value of *x*. most candidates resorted to the use of the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, even though some were unable to substitute properly to answer correctly. Also, during Nov/Dec 2008, candidates were asked to solve the quadratic equation $2x^2 + 7x + 2 = 0$ by the method of completing squares and give their answers correct to 3 decimal places. According to a report after evaluating the students answers, a very high proportion of students found it difficult to find the solutions for the question using the completing the squares method.

The development and growth of mathematics falls into about seven eras. Each era is marked by a corresponding civilisation. These civilisations are; Pre-Historic, Egyptian, Babylonian, Greek, Hindu-Arabic, Early and Medieval Europe, and modern civilisation.

Pre-Historic people did very little of what is now called mathematics. They did collection of objects, grouping and counting, and adding them together.

By 8000 BC Egypt had become a centre of learning. Mathematics existed there in its skeletal form. The rudiments of algebra appeared in the Ahmes Papyrus and among them were the first known reference to quadratic equations. But the solution of quadratic trinomials seems to have exceeded by far the algebraic capabilities of the Egyptians.

The Babylonian civilisation followed the Egyptian civilisation from about 4000 BC onwards. The solution of the quadratic trinomials presented the Babylonians with no serious difficulty.

They handled such equations effectively because they had acquired flexible algebraic operations. They could add equal terms to both sides of an equation to transpose terms, and they multiply both sides by like quantities to remove fractions or to eliminate factors. By the additions of $4ab$ to $(a-b)^2$, they could obtain $(a+b)^2$ because they possessed many simple forms of factorisation. They also transformed the quadratic trinomial $ax^2+bx+c=0$ to the normal form $y^2+by=ac$ through the substitution $y=ax$. Thus they could solve problems equivalent to solving $x^2-x=870$ with steps equivalent (equation) to the formula $x = [((p/2)^2 + q) + p/2]$ for the root of the equation $x^2 - px = q$.

Greek civilisation replaced Babylonian civilization at about 600 BC. Two distinct periods are distinguishable in this civilization; the classical period (600 BC – 300 BC) and the Alexandarian Period (300 BC – 600 AD). In the first of these periods, the Pythagoreans used geometrical algebra to solve quadratic equation of the type $x^2 = a^2 - ax$. The solution required methods of geometrical constructions and comparison of areas. Descartes is known to have written detailed instructions on the solution of quadratic equation by geometrical algebra.

During the second period, Euclid, Apollonius and Archimedes all worked on the conic sections and the parabola before 190 B.C. Indeed Archimedes is said to have written a book named "*The Quadratic of the Parabola*" (Kline, 1982). Also Diophantus, who lived in Alexandra, Egypt, about 275 AD, solved quadratic equation by a method similar to completing the square but admitted only the roots. Positive roots were not accepted. It seems also that little effort was made to find all the possible solution.

Babylonian and Hellenistic cultures possibly influenced the Arabs through trade. So from about 200 AD the influence of the Arabs on mathematics began to be felt. Like the Babylonians, their quadratic equations covered three classical cases of quadratic trinomials. These were (1) squares and roots equal to numbers, (2) Squared and roots equal to roots and (3) roots and numbers equal too squares. Square, roots and numbers, in our modern notation are equivalent to x^2, x and numerical quantities respectively.

Mohammed Ibn Musa al-Khowa-rizmi composed an exposition on quadratic equations and their solutions, typical of the mathematics of the Arabic Hegemony. His work was so systematic and exhaustive to the extent that his readers must have had little difficulty mastering them. It would seem that the method of completing the square was meticulously followed and that negative numbers solution was ignored .Only positive roots were accepted (Boyer, 1968). Abd-al-Hamid also used geometrical figures to prove that if the discriminant is negative, a quadratic equation has no solution.

In the same era, the Hindus, working independently of the Arabs, became the first to obtain the algebraic solution of $ax^2 + bx + c = 0$ giving the roots as $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Europe began to assert herself as a centre of learning from about 1100 AD. This led to the Renaissance in the sixteenth century. In Germany there were many amateur and professional mathematicians and many textbooks were produced. Michael Stifel’s Arithmetica Integra was the most important of all the sixteenth century German algebras. It is significant for its treatment of negative numbers, radicals and powers. Through the use of negative coefficients in equations, Stifel was able to reduce the multiplicity of cases of quadratic equations that then existed into what appeared to be a single form. However, despite his understanding of the properties of negative numbers, he failed to admit negative numbers as roots of an equation. Until modern times there was no thought of solving quadratic equations of the form $x^2 + px + q = 0$, where p and q are positive, for the equation has no positive roots. In that century two Galileo Galilei shared that motion of projectiles was parabolic.

In 1545 Jerome Cardan set up and solved the quadratic problem of dividing 10 into two parts whose product is 40 in his book, Ars Magna (the Great Art). "If x is one part, the equation for **x** is $x(10-x)=40$". Cardan obtained $5+\sqrt{(-15)}$ and $5-\sqrt{(-15)}$ and then says these are "sophistic quantities which though ingenious are useless" (Boyer, 1968, p.116).

Bombelli knew in the sixteenth century that by means of complex numbers which quadratic and cubic equation can be completely solved; in other words, that the most general equations of the second and third degree must possess at least one roots which may be a real or complex number. It was also known that imaginary roots of an algebraic equation with real coefficients must come in pairs, that is, if $a-ib$ was a solution two (Boyer, 1968; Dantzig 1947).

In 1631 Thomas Harriot came out with the ingenious idea of putting any equation in the form of polynomial equations to zero (0) so that if α be a root of an algebraic equation, then $x-\alpha$ is a factor of which corresponding polynomial. This fundamental fact reduced the solution of any equation to a problem in factoring, and showed conclusively that if it could be proved that any equation has a root real or complex, it would be ipso facto established that the equation has many roots as it degree indicate; with the reservation, of course, that every root be counted as many times as the corresponding factor enters in the polynomial (Dantzig, 1947). *Girard* also recognized the equality between the degree and the number of roots of an equation, emphasizing the necessity to recognise complex numbers (Dubbey, 1970).

As a prelude to modern mathematics, Vieta made a step in the right direction. From the time of the Egyptians and Babylonian right up to Vieta's work, mathematicians solved equation of the first four degrees with numerical coefficients only (*Kline, 1982*). Thus, equations such as $3x^2 + 5x + 6 = 0$ and $4x^2 + 7x + 8 = 0$ were regarded as different from each other though it was clear that the same method of solution applied to both. Moreover, to avoid negative numbers, an equation such as $x^2 - 7x + 8 = 0$ was for a long time treated in the form $x^2 + 8 = 7x$. Hence, there were many types of equations of the same degree each of which was treated separately. Vieta's contribution was to introducing literal coefficients. Though a number of mathematicians had used letters for sporadic and incidental purposes, Vieta was the first man to do so purposefully and systematically (Kline, 1982). He used letters as general coefficients and not just to represent an unknown or powers of an unknown. Thus all second degree equations could be treated in one swoop by writing (in our notation) $ax^2 + bx + c = 0$, wherein for any numbers, whereas x stands for an unknown quantity or quantities whose values are to be found. At the beginning of the nineteenth century, the general formulas for the solution of a quadratic equation $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for $ax^2 + bx + c = 0$ were known worldwide.

Statement of the Problem

Many of the recent researches have focused on the cognitive components of mathematics and success (McKnight, Magid, Murphy & Mcknight, 2000). This study, which is one of such, will look at the solution of the quadratic equation, which is a pre-requisite skill needed to understand and be able to solve problems not only in mathematics and physics but also in other areas of the senior high school curriculum.

Quadratic equation have been on the Senior High School Mathematics Syllabus before the inception of the New Educational Reforms in Ghana in 1987, and have since then been an integral part of the Senior High School mathematics syllabus. The importance of the topic is stressed by Willoughby (1967) when he writes “Pupils intending to continue the study of mathematics will need a good understanding of quadratic equation and their solutions since there are many problems in geometry which lead to such equation” (p 401). Teaching quadratic equation as topic that has no lead to further mathematical development therefore does a great disservice to the pupils (Movshovitz–Hadar, 1993). Writing on the aims of teaching mathematics, Dubisch (1963) asserts that:

1. Mathematics should be taught with emphasis on the thinking process to equip the students with the habits of logical thought that can be transferred to other subjects.
2. A routine mechanical-like skill is not sufficient; how we teach is of overwhelmingly greater importance than what we teach in the senses that it is the basic methods of mathematical analysis that are applied rather than the detailed results.
3. The student should receive a broad, thorough training in fundamental mathematics. The foundation of mathematic should be taught by the teacher and learned by these mathematics students.

Each student in pre–tertiary school in Ghana learns mathematics and offers it compulsorily at the respective terminal examination. The aims of mathematics education in pre-tertiary schools in Ghana include not only producing students who are mathematically functional in society, but also those who can pursue mathematics and Mathematics–related courses in higher institutions for the achievement of a scientific society.

As to the methods of teaching mathematics, the syllabus contains objective activities and notes for each topic. There is, in addition, a section on the general methodology that teachers are required to use. The methodology is comprehensive; it is not mechanical.

Now in Core Mathematics the solution of quadratic equation is not treated until in the 2^{nd} year (The Curriculum Research and Development Division (C.R.D.D, 2010, mathematics core syllabus for Senior High School) while in Elective mathematics, it is also taught in the 1^{st} year(The Curriculum Research and Development Division (C.R.D.D), 2010, mathematics elective syllabus for Senior High School). Meanwhile, the solution of quadratic equation is a pre-requisite knowledge for learning some topics in Physics in the 1^{st} year of the Senior High School. For example, the student encounter the solution of quadratic equation right in the first year in equation of motion under uniform accelerated such as: $s = ut + \frac{1}{2}at^2$ where S is the distance transverse by a body, u is the initial velocity, as is the uniform acceleration and t is the time elapsed. (The Curriculum Research and Development Division (C.R.D.D), 2010, Physics elective syllabus for Senior High School). This means that the Physics teachers may be compelled to teach it in an ‘on the spot’ manner. This normally involves the use of quadratic formula which is taught perfunctorily or done hastily, and which may be learnt by instrumentally without any proper understanding. This could lead to a wrong attitude of students to the topic in particular and to mathematics in general. The resultant notion among student is that the solution of quadratic equation is difficult and this becomes a barrier later on when the mathematics teacher has to teach it. This is a problem with the curriculum. The problem is the placement of the teaching of quadratic equation in the curriculum.

Four conventional methods for solving quadratic equation(s) commonly taught in Ghanaian schools are (a) solving, by graphical method, (b) solving by factorization (c) solving by completing the square and (d) solving by the quadratic formula.

Gyening and Wilmot (1990) have speculated that "each of the conventional methods of solving quadratic equations is based on a prerequisite skill which is highly specialized" (p.41) that the preliminary teaching of these prerequisite is time succumbing and unduly delays the main skill. Where the object is to facilitate the student with the main skill for use in learning other subjects at the appropriate time, this delay is undesirable.

Purpose of the Study

The second international Congress on mathematics education held in exeter in 1972 recognized "*the need for research on the relationship between topics and on the sequences of topics with a curriculum*" (Howson, 1973, p. 62). The purpose of the study therefore is to address the curriculum problem of the placement of the teaching and learning of quadratic equations in the senior high school with the same efficiency and effectiveness as the other topics in the Form One Core Mathematics syllabus.

Gyening and Wilmot (1999) have speculated that the method of ESLE makes it possible to teach the solution of quadratic equations in the lower forms of the senior high school for optimum results, and hence addresses the curriculum problem identified above. This speculation will also be verified by investigating the performance of Form One students in using methods, namely, quadratic formula, conjugales, factorization, and ESLE to solve quadratic equations.

Research Questions

1. As a result of the treatment, is it possible for Senior High School Form One students to learn quadratic equations as effectively as the mathematics topics in the second term scheme of work
2. Should the answer to question 1 be positive which method(s) will show a significant difference?

Null Hypothesis

1. For each treatment group, the mean score on the pretest does not differ from that on the posttest.
2. There is no significant difference in the mean score on the posttest of the quadratic formula group and the factorization group.
3. There is no significant difference in the mean scores on the posttest of the ESLE group and the quadratic formula group.
4. There is no significant difference in the mean scores of the posttest of quadratic formula and the conjugales group.
5. There is no significant difference in the mean scores on the posttest of the ESLE group and the factorization group.
6. There is no significant difference in the mean scores on the posttest of the ESLE group and the conjugales group.
7. There is no significant difference in the mean scores on the posttest of the factorization group and Conjugales group.

Significance of the Study

When the study gives support to the possibility of teaching anyone method for solving quadratic equations to students in the first year of the S.H.S, the topic will no longer be unduly delayed in the mathematics curriculum. The situation where physics teachers happen to introduce quadratic equations and their solutions to S.H.S students can be revised. Mathematics teachers will be encouraged to teach the topic in form one as noted by Willoughby (1967), those students who will continue their study of mathematics beyond the S.H.S need a good understanding of quadratic equations and their solutions. The advantages inherent in the most feasible method over the others will help lay the proper foundation for the necessary understanding of the topic to facilitate the study of mathematics and math-related subjects beyond the S.H.S level. Teaching quadratic equations in form one of the S.H.S level will bring the age of introduction of quadratics in mathematics at par with international standards. The age of introduction of quadratics in Belgium, France, Germany, Italy, Japan and England are 15, 14-15, 14, 14-16, 14 and 16 respectively (APU, 1985, p.19) that of Ghana will then become 15.

Delimitation of the Study

The study will be carried out in Form One Core Mathematics classes except Physics and Elective Mathematics students. The methods of ESLE and conjugales and two conventional methods, namely, Factorization and Quadratic Formula will be used in the study. Completing the square and the Graphical methods will not be used in the study.

Limitations of the Study

The study is limited by shortcoming in the design. The research design is not perfect. Teaching experiments normally use small samples and take place over long periods. This study however, used a large sample size and lasted for a few contact periods.

Some of the students had problems evaluating $d = \sqrt{b^2 - 4ac}$ especially when a or c or both are negative numbers.

The findings can only be generalized to form one student in the Cape Coast district, Mfantseman district, Lower Denkyira district and Komenda- Edina Eguafo- Abirem Metropolis of the Central region. The reason is the conditions in other parts of the region may be different. Such conditions include good libraries and other learning materials, furnished classrooms and qualified trained mathematics teachers.

Abbreviations

SSSCE: Senior Secondary School Certificate Examination

CRDD: Curriculum Research and Development Division

WAEC: The West African Examination Council

GCE: General Certificate of Education

ESLE: Equivalent Simultaneous Linear Equation

SHS: Senior High School

Ibid: In the same work or passage

DTIA: Directed Trial and Inspection Analysis

DTA: Directed Trial and Analysis

Conjugales: Conjugate Linear Equations

ANCOVA: Analysis of Covariance

LSD: Least Significance Difference

M: Mean

SD: Standard Deviation

MAG: Mathematics Association of Ghana

GES: Ghana Education Service

NGOs: Non-Governmental Organizations

MOE: Ministry of Education

Conventional Methods: Method prescribed by the SHS mathematics syllabus including graphical method, the method of completing the squares and the Quadratic Formula

Organization of the Rest of Study

The rest of the report has been organized into four chapters; chapter two presents the review of related literature. Chapter three looks at the methodology and describes the research design, population and sampling techniques, the instrument to be used, data collection procedure, and the statistical tool that has been employed. The results and discussions of this study constitute chapter four. Chapter five constitutes summary, conclusion and gives recommendation.

CHAPTER TWO

REVIEW OF RELATED LITERATURE

The literature review has been presented in four principal sections. The first section seeks to explore the evolution of quadratic equations and how it has been developed over the years. The second section seeks to explore the applications to which the quadratic function has been put and hence the need to teach or learn its solution in school. The third section gives theoretical considerations as contained in textbooks and journals. The fourth section treats empirical data obtained from previous research related to the present study. In addition to these is a summary of the major findings of the literature review that sets up the state of the art.

Psychology of Learning and Teaching Mathematical Concepts

Good Mathematics teaching is a blend of the teacher's own knowledge of the subject matter, pedagogy, and psychology. The teacher is therefore an important pillar in the teaching and learning process. According to Skemp (1986), learning Mathematics in general requires both intelligent learning and habitual learning approaches. Habitual learning is a situation where the student depends mostly on the teacher to make available the rules for solving the problem. In intelligent learning however the student searches for the rules thereby developing self–confidence in his own abilities to cope with new learning situations. The intelligent learning approach is therefore recommended at the Senior High School level.

Brownell cited in Patricia (1935), also recommended what he referred to as ‘meaningful learning’ in Mathematics. He argued that effective Mathematics instructions must promote an understanding of the concepts, relationships, and process that define arithmetic. Students who encounter problems that are related to quadratic equations in Mathematics and other fields of study must therefore understand the meaning of the concepts and processes that underpin the problem. This idea is supported by Holt (1970) when he noted that teaching a student rules or formulae could be dangerous if the student does not really understand the bases of the rule or formulae. They must therefore be taught the facts and skills about a subject matter to be able to apply the knowledge whenever and wherever the need arises.

Marjorie & Stein (1997) observed that the main task underlying unsuccessful task implementation is lack of alignment between the task and the prerequisite skills needed for the task. Many times students become frustrated in learning topics that are unrelated to their cognitive experience. A student’s cognitive processing of a task during the task implementation stands a better change of remaining at his level if the task is based on good entry behavior appropriate to the new task (Bennett and Desferges, 1988).

Case (1975) also noted on information processing that a ‘narrow span’ teaching hierarchy is more effective than a ‘broader span’ hierarchy in teaching towards a terminal skill or objective. Case’s thesis states that if a task analysis yields a set of subordinate or related skills that contribute to a parent or super ordinate skill and that can be arranged in alternative teaching sequences or hierarchies, then the hierarchy that places the least information–

processing demands on the learner should be chosen. That is, the hierarchy with the narrowest span is preferable.

On sequencing of learning materials, Robert Gagne’s theory of learning may suggest some ideas about how to begin. In Robert Gagne’s theory in which simpler tasks function as actual components (elements) of more complex task? Resnick & Ford (1981) noted that “*the fact that these complex tasks are made identifiable simpler elements make for transfer from the simpler ones to the complex ones”* (p.5). *Gagne* presented examples of approaches to analyzing skills into learning hierarchies. These learning hierarchies can be used to make vital decisions concerning sequencing of topics in the Mathematics curriculum. Cooney, Davis & Henderson (1975) also observed that pre–requisite skills or concepts are used more frequently than any other factor for determining the sequencing for teaching Mathematics topics. Based on these, it is therefore expedient to study the pre–requisite skills needed to solve quadratic equations in the Mathematics curriculum before the topics itself.

The Time Factor in Teaching

The process of teaching and learning is as old as human beings on earth. It is carried out by both animals and humans to teach their young ones for successful adjustment in the environment (Chauhan 2001). Bennett and Carre (1995) emphasized that teachers have difficulty in translating awareness of time into good management of time. Balogun et al (1984) explained management of time as an art or process of tactfully controlling or bringing various elements to work together for some particular within the given period.

According to Kyriacou (1985), early research concentrated on the amount of time students spent on outcome-related tasks, indicated that greater time spent on task behaviour was associated with greater gains in educational attainment. This was generally true whether it resulted from teacher allocating more curriculum time to task behaviour or form individual teachers maintaining task behaviour during a lesson for longer than their colleagues do.

Moyles (1997) reported that time, as the modern world perceives it is a finite, particularly the school day. The concept of time is incompatible, for it rarely balances adequately, with what we want to achieve. Pernet (cited in Moyles, 1997) suggested that first things we really spent is an important first step "Richardson (cited in Moyles, 1997) indicates that people mostly under estimate the time needed for tasks, it is necessary to add around 20 percent to any estimate. He suggested two golden rules regarding time which are worth remembering. One can always make more effective use of time and the only person to make better use of your time is you.

Carol (cited in Duke, 1982) maintained that students will master instructional objectives to the extent that they are both permitted and willing to invest the time necessary to learn the content. Therefore, the time it takes each student to learn will vary depending on such factors as prior student achievement and attitudes. The important implication of this model, of course is the given enough time, most students can learn most content.

Myers (cited in Moyles, 1997) said, in relating to learning time, it must be acknowledge that there is a difference between allocated and engage time. Moyles (1997) suggested that within the National curriculum framework, teachers and schools will be expected to make decisions, decision depending upon other factors including: time of year, whole school decision, and relative length of each term, any ‘fixed’ aspects, strengths and expertise of other staff members, resources available and government views on curriculum provision.

Obanya (1980), indicated that more periods should be allocated to basic and compulsory subjects (e.g. Language and Mathematics). Contributing to time allocation, Cockroft (1982) stated that, the proportion of teaching time given to mathematics has decreased in most schools due to the introduction of additional subjects. This is supported by Pratt (1980) when he stated that Time is the most valuable asset in implementing a curriculum and it is almost always underestimated. The result of this is that both teachers and students are forced to haphazardly go through the syllabus. The teachers need extra time to prepare lessons and materials, administer test and for marking. If some mathematics topics could be learnt in shorter time period then the load on the teacher and the student would be reduced since the time saved could be shifted to other areas. Based on this, Oliver (1965) suggested that, ideally the amount of time given to a bit of content should depend upon how long the students take to learn it. He further stated that less time can be spent on a subject as a result of better method and more effective materials. Time allotment to topics in mathematics depends on the number of activities the teacher had to guide the students through in the learning process as noted by Tarlor & Rchard (1985). It is therefore clear that both the teacher and the student cannot carry out the work without considering time.

Blege (1986) said that school time should be regulated by means of timetable, which should be organized in two ways; stiffly and loosely regulated. He further explained that, one control factor which runs the method of organization of time is the question of control of the curriculum. To what extent should the teacher determine what is to be taught at a particular time, and who is to determine how long a learning activity should last? (Blege, 1986; p. 145).

How the school day is subdivided for instructional purposes, typically, is a macro-level issue handled by school and district policies, state guidelines and contact. Specifically, how do teachers allocate time on daily basis to particular subjects and particular learning activity including time for exercises and examinations? It is at this level that the quality of instruction of frequently determined. While the school teachers cannot be in every classroom for every minute of the day, it is important for them to try to understand how class time is spent and to encourage teachers to allocate it in ways that best promote realization of school goals.

It is generally assumed that students learning are directly related to exposure to instruction. For this reason, schools that serve large numbers of students who are deficient in basic skills often increase the time they must spend studying reading and mathematics. In Washington D.C junior high school students who are working below acceptable levels in mathematics, for example, must take two period of mathematics everyday as part of a programme called intensive Junior High School Instruction. In Ghana and other West African countries extra classes are organized for student who fell the time allotted for teaching and learning is not enough. This view was shared by Mathew (1989) when he that a student's level of attainment was directly related to the length of time actively spent on learning. This opinion was given a further boost by Kraft (1995) that the amount of time spent on the basics of language and mathematics is a critical factor in the achievement level of studies.

Origins and Earliest Solution

There are many kinds of mathematical equations of which the algebraic equation is one. Among the algebraic equations are polynomials from which we get the quadratic equations. Sawyer (1970) observed that of all the polynomials, the quadratic equation seems to be the most important.

The method of solution of this important equation took many forms over the years. According to Smith (1958) the first known solution of a quadratic is the one given in the Berlin Papyrus. Smith (ibid) gives this type of solution and others by Mahavira (c.850), Brahmagupta (c.628). Dubbey (1970) mentions that Galileo and contemporaries applied the distributive law to solve the equation $y = ax - bx^2$. Dubbey (ibid) reports that Al-Khowarismi carried the solution as follows:

$$p\chi - \chi^2 = \chi(p - \chi) = \left(\frac{1}{2}p\right)^2 - \left(\frac{1}{2}p - x\right)^2 = 2$$

He had unfortunately not developed the notations and all operations were written in words.

According to Kramer (1970) algebraists, starting with the Babylonians developed a formula for solving quadratic equations. The cuneiform text book tablet gave verbal instructions that are equivalent to quadratic formula of today’s algebra. It is generally believed that the Babylonians derived this formula by the method of completing the square which was used by Greek and Arab Mathematicians.

The Hindu admitted negative and irrational numbers and recognized that a quadratic having real answers has two formula roots. They unified the algebraic solution of quadratic equations by familiar methods of completing the square. This method is today referred to as the Hindu method, Eves (1964). Among the earliest solutions of quadratic equations is the Harriot method which is also known as the solution of the quadratic equation by method of factorization. This is one and probably the most popular of the four conventional methods of solution currently taught in schools. The other three are solution by graphical methods, solution by the use of formula and solution by completing squares.

According to Dantzig (1947) the Harriot's principle is concerned with a procedure which consists of transposing all terms of an equation to one side of the equality sign and writing it in the form $p(\chi) = 0$ where $p(\chi)$ is a polynomial.

Smith (ibid) also said that the first important treatment of the solution of quadratic and other equation by factorizing is found in Harriot's Artis analyticae Praix (1631), which was published ten years after his death. The principle further demands that the non- zero side of equality sign be factorized into two factors and the null- factor law used. The null-factor law which Harriot applied but did not mention states that a product cannot be equal to zero unless at least one of its factors is zero. Symbolically $mn = 0$, either $m = 0$ or $n = 0$.

Kramer (1970) observed that Harriot used the reasoning that product of two numbers is zero only if at least one of the numbers is zero. Since this is well known method of solution treated in almost every textbook on basic algebra, it is rather not necessary to make any attempt to describe in detail.(see Durell and Robson, 1964; Hall and Knight, 1962; Miller and Green,1962;

Smith and Murray, 1954; Hyatt and Hardesty, 1975). Among those who have recently worked on the solution of quadratic equation namely, Directed Trial and Inspection Analysis (DTIA), D'AMEN and the Equivalent Simultaneous Linear Equation (ESLE). Opoku and Fefoame (1996) and Eshun (1997) ESLE have said in their research in DTIA and ESLE respectively that these methods mentioned are simpler alternative to the conventional methods of solving quadratic equations.

Why Study Solution of the Quadratic Equation

The quadratic equation is one of the most important of all the polynomials to man such that *this topic appears in every mathematic textbook on basic algebra.* For instance Morris (1960) states that Galileo's investigation which produced Newton's law of motion was based on quadratic equation. These laws operate when dealing with moving bodies. Morris (ibid) again mentions the fact that Galileo's work on vertical motion was based on the quadratic and from which he found that the force of gravity acting on all bodies is 32ft/sec^2, referred to as "weight" on the surface of the earth.

On a production line, objects are passed from one operation to another at a lower level by allowing it to slide down inclined planes, an investigation by Galileo based on quadratic equation. Other writers on equation of motion based on the quadratic equations are:

1. Alexsandrov and Kolmogorov (1956) on its application in analytic geometry and mechanics.
2. Brixley and Andree (1966) on the importance of vertical motion.
3. Carslaw and Jaeger (1963) who concerned themselves with dynamical problems.

According to Boot (1964) investors on the stock market use quadratic programming to guide them in their operations. They consider the problem of maximizing a quadratic function subject to linear inequality constraints.

Coxford and Payne (1984) support the above by saying that economists seek efficient ways to describe manufacturing costs, sales, revenue and business profit, assuming that cost, revenue and profits are functions of the number of items produced.

Historical Review

We view the history of mathematics and for that matter the development of quadratic equation from our own perspective and sophistication. There can be no other way but nevertheless we have to try to appreciate the difference between our viewpoint and that of mathematician's centuries ago. Often the way mathematics is taught today makes it harder to understand the difficulties of the past but in those days brilliant major mathematical discoveries often appear as isolated flashes of brilliant thoughts.

Most times these discoveries are a culmination of works by many, often less able, mathematicians over a long period of time. As noted by Burton (1999) no branch of mathematics sprang up, fully grown, through the work of only one person. For example, the controversy over whether Newton or Leibniz discovered calculus can easily be answered. Neither did, since Newton certainly learnt the calculus from his teacher Barrow (Burton, 1999). Of course, I am not suggesting that Barrow should receive the credit for discovering calculus, I am merely pointing out that calculus came out a long period of progress starting with the Greek mathematicians.

Aleksandov, Kolmogorov & Lavrentiev (1956), noted that the second degree equation was solved in early antiquity. The Egyptian, Chinese and Babylonians are reported to be among the first to have delved into problems relating to quadratic equations (Boyer, 1968; Eves, 1964).

The first aspect that finally led to the quadratic equation was the recognition by the Egyptians (1500BC) that it is connected to a very pragmatic problem, which in its turn demanded a ‘quick and dirty’ solution (Smith, 1951). It must be noted that Egyptian mathematics did not know equations and numbers like we do nowadays; it is instead descriptive, rhetorical and sometimes very hard to follow (Smith, 1951). It is known that the Egyptian wise men (engineers, scribes and priests) were aware of this shortcoming- but they came up with a way to circumvent this problem: instead of learning an operation, or a formula that could use to calculate the sides from the area, they calculated the area for all possible sides and shapes of squares and rectangles and made a look-up table (Smith, 1951).

This method works much like the multiplication tables we learnt by heart in school instead of doing the operations properly. Therefore if someone wanted a peace of plot to farm, the engineer would go to his table and find the most fitting design. The engineers did not have time to calculate all shapes and sides to make their own table. Instead, the table they used was a reproduction of a master look-up table. The users did not know if the stuff they were using made sense or not as they didn’t know anything about mathematics (Smith 1951).

The Egyptian method worked fine and gives a more general solution- without the need for tables-seemed desirable. That’s where the Babylonian came into play. Babylonian mathematics had a big advantage over the one used in Egypt, namely the used a number-system that is much like that one we use today (Eves, 1964). Addition and multiplication were a lot easier to perform with this system, so the engineers around 1000BC could always double-check the values in their tables (Eves, 1964). By 400BC they found a more general method called ‘completing the square to solve generic problems involving areas. (Eves, 1964).

The old Babylonians were believed to have solved problems equivalent to solving for the roots of the equation (Boyer, 1968; Eves, 1964). There are scores of clay tablet that suggest that these Babylonians of 2000 BC were familiar with our formula for solving quadratic equations (Burton, 1999). The steps they used were similar to the quadratic formula, although they had to notion of equations. Their effort however gave rise to the concept of quadratic equations, which is now an important tool for revolutionizing the world.

They also had traces of the method of completing the square in their approach to solving problems relating to quadratic equations (Boyer, 1968). However all the problems of the Babylonians had answers that were positive quantities since they were usually dealing with lengths. Research has shown that the Chinese worked on polynomial equations as early as 100BC (Boyer, 1968). However the Chinese, like the Egyptians, also did not use a numeric system, but a double checking of simple mathematical operations was made astonishingly easy by the widespread use of the abacus (Boyer, 1968).

History also has it that the Indians, as early as 500BC, have delved into the mathematics of quadratic equations and so did the Greeks of Euclid era. (Burton, 1999). In fact the first attempts to find a more general formula to solve quadratic equations can be tracked back to geometry and trigonometry.

(Smith, 1951). Bananas Pythagoras (500BC in Croton, Italy) and Euclid (300 BC in Alexandria, Egypt), both used a strictly geometric approach, and found a general procedure to solve the quadratic equation (Smith, 1951). Pythagoras noted that the ratios between the area of a square and the respective length of the side-the square root-were not always integer, but he refused to allow for proportions other than rational (Smith, 1951).

During the 15th century, mathematicians were confronted with problems whose solutions require finding the roots of polynomials. This led the early mathematician such as Ludovico Ferrari (1522-1556), Cardan A Magna (1430-1540), Neil Henrik Abel (1892-1829) to spend sleepless nights trying to find solutions to such problems (Burton, 1999).

At the beginning of the nineteenth century, the theory of the solution of algebraic equation gradually began to occupy the minds of most mathematics (Aleksandov et al., 1956), but the difficulty involved was how to find the solution of an nth-degree equation in one variable. This was probably triggered by the pressure of the demands from many other branches of both mathematics and other applied science. Polynomials of the form;

$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots\ldots\ldots\ldots\ldots\ldots + a_1 x + a_0 x^0 = 0$, where a_n, a_{n-1}, a_{n-2}, a_1, $a_0 \in R$ are the coefficients of x^n, x^{n-1}, x^{n-2} …… x, x^0 respectively were then identified (Mathew, 1973).

Thomas Harriot, in 1621 came out with the idea of putting any polynomial into the form of an equation with the right hand side equated to zero (Dantzig, 1947). This ingenious idea led Harriot to the factor theorm which states that, if α is a root of an algebraic equation in x, then $x - \alpha$ is a factor of the corresponding polynomial (Dantzig, 1947). Thomas Harriot

(1560 – 1631) used the method of factorization to obtain the solution of quadratic equations in his great works on algebra (Dantzig, 1947; Sastry, 1988). Harriot's method of solving quadratic equations is one of the conventional methods currently used in schools. Descartes was also reported to have written detailed instructions on the solution of quadratic equations by geometrical algebra (Sastry, 1988).

In mathematics, a quadratic function is a polynomial function of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$. It takes its name from the Latin word 'quadratus', which means a square. This is because quadratic functions arise in the calculation of areas of squares.

Hag &Weisstern (1959); Budnick (1985) also noted that an *n*th–degree equation in x is called a polynomial. The degree of the polynomial is the largest exponent of x present when the polynomial is cleared of grouping symbols. A polynomial of second–degree is called a quadratic. Briton & Bello (1979), defined a quadratic equation more precisely as, "a second–degree sentence whose standard form is $ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$" (p. 303). Thus, a quadratic equation is a second–order polynomial equation. According to Evans (1959),

> if a conditional equation in one variable was such that its sides were polynomials of degree of two or less, so that by appropriate use of the laws of equality, it could be put in the form $ax^2 + bx + c = 0$, where $a \neq 0$, and $a, b,$ and $c \in R$, then, the equation is said to be an equation of second degree or a quadratic

equation in the variable which occurred in it (p. 256).

To solve a quadratic equation in x therefore implies finding all the values of x that satisfy the equation. That is, the truth set of the equation gives the solution set or roots of the equation (Haag & Weisstern, 1959, p. 105; Miller 1957. p. 175). The word root is synonymous to the word solution.

The first known solution of a quadratic equation is the one given in the Berlin papyrus from the Middle Kingdom (ca. 2160-1700BC) in Egypt. This problem reduces to solving.

$$x^2 + y^2 = 100$$

$$y = \frac{3}{4}x$$

(Smith 1953, p. 443).

The Hindus also found a solution to the quadratic $ax^2 + bx = c$ (Smith, 1953). The method they used was essentially our present day method of "completing the square". According to Smith (1953),the Hindus realized that a quadratic with real roots has two roots, but they could not always find both. The Greeks were also reported to have solved the quadratic equation b) geometric methods. And Euclid's (ca. 325-270 I~C) work contains three problems involving quadratic equations (Smith, 1951). In his work, Arithmatica, the Greek mathematician Diophantus (ca. 210-2(0) solved the quadratic equation, but gave only one root as the Hindus did, even v\ hen both roots were positive (Smith 1951, p. 134). Viéte was also among the first to replace geometric methods of solution with analytic ones, although he apparently did not grasp the idea of a general quadratic equation (Smith 1953. pp. 449-450).

A number of Indian mathematicians gave rules equivalent to the quadratic formula. It was possible that certain altar constructions dating from ca. 500 BC represent solutions of the equation, but there were no traces of records of the method of solution (Smith 1953, p. 444).

The Hindu mathematician $\bar{A}$ryabhata (475 or 476-550) gave a rule for the sum of geometric series that shows knowledge or the quadratic equations with both solutions (Smith 1951. p. 159: Smith 1953, p. 444), while Brahmagupta (ca. 628) appears to have considered only one or them (Smith 1951, p. 159~ Smith 1953, pp.444-445). Similarly, Mahavira (ca. 850) had substantially the modern rule for the positive root of a quadratic. 'Sr$\bar{\iota}$dhara (ca 1025) gave the positive root of the quadratic formula. As stated by Bhaskara (ca 1150; smith 1953. pp 445-446). The Persian mathematicians al-Khowarizmi (ca. 825) and Omar Khayyam (ca. 1100) also gave rules for finding the positive root.

The Hindu method is used in schools in recent times. This method dates back to 1025 AD. It is known as the quadratic formula. It is also 'well known among students as the 'almighty' formula. In this formula, the quadratic equation: $ax^2 + bx^2 + c = 0$, has solutions: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

The Hindus unified the algebraic solution of quadratic equations by the method of completing the square (Eves, 1964). Admitting that a quadratic equation with real roots has two formal roots (x_1 and x_2.),

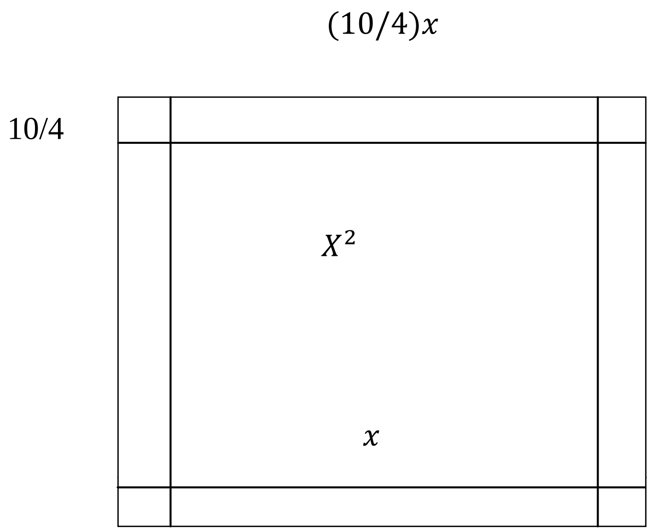
where $x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}, x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

The approach of the Arabic mathematician, Mohammed Ibn Al-Khowarizmi is much different from that of the Hindus (Smith, 1953). In dealing with quadratic equations, Al-Khowarizmi divided them into three fundamental types:

1. $x^2 + ax = b$
2. $x^2 + b = ax$
3. $x^2 = ax + b$

with only positive coefficients admitted (negative quantities standing alone were still not accepted by the Arabic mathematicians). All problems were reduced to these standard types and solved according to a few general rules.

He tackled the solutions of the problem $x^2 + 10x = 39$ with two methods. In his first method he constructed a square with each side of length x to represent x^2. He then added $10x$ to x^2. This is accomplished by dividing $1Ox$ by 4, each part representing the area $\frac{10x}{4}$ as a rectangle (Burton, 1999), and then applying these 4 rectangles to the 4 sides of the square constructed.



This resulted in a figure represented by the expression

$x^2 + 10x = x^2 + 4\left(\frac{10x}{4}\right)$. To make the figure a large square of side $x + \frac{10}{2}$, Al-Khowarizmi added four small squares at the corners, each of which has an area equal to $\left(\frac{10}{4}\right)^2$. Hence, to complete the square he added $4\left(\frac{10}{4}\right)^2 = \left(\frac{10}{2}\right)^2$ which resulted $\left(x + \frac{10}{2}\right)^2 = (x^2 + 10x) + 4\left(\frac{10}{4}\right)^2 = 39 + \left(\frac{10}{2}\right)^2 = 39 + 25 = 64$

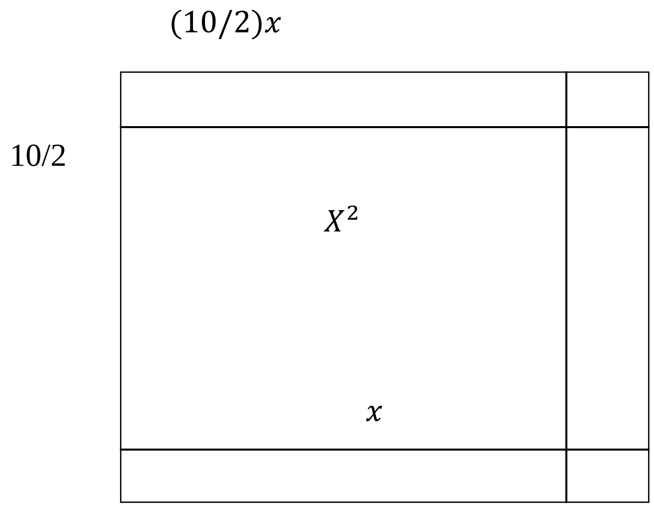
The side of the square must be $x + \frac{10}{2} = 8 \Rightarrow x = 3$

The general form of this type of quadratic, $x^2 + px = q$ is solved by the method of completing the square by adding four squares, each or area $\left(\frac{p}{2}\right)^2$, to the figure representing $x^2 + p$, to get.

$$\left(x + \frac{p}{2}\right)^2 = x^2 + px + 4\left(\frac{p}{4}\right)^2 = q + \left(\frac{p}{2}\right)^2$$

$$\Rightarrow x = \sqrt{\left(\frac{p}{2}\right)^2 + q} - \frac{p}{2}$$

The second method used by Al-Khowarizmi for solving this problem starts with a square of side x and area X2 and two rectangles, with dimensions x and $\frac{10}{2}$, resulting in the area of the entire figure to be $x^2 + 2\left(\frac{10}{2}\right)x$. To complete the square, he added a smaller square of area $\left(\frac{10}{2}\right)^2$. The completed square therefore has an area of $\left(x + \frac{10}{2}\right)^2$. But $\left(x + \frac{10}{2}\right)^2 = x^2 + 2\left(\frac{10}{2}\right)x + \left(\frac{10}{2}\right)^2 = 64$. The side of the square must be $x + \frac{10}{2} = 8 \Rightarrow x = 3$



Again the general form of this type of quadratic, $x^2 + px = q$ is solved by adding a square of side $\left(\frac{p}{2}\right)$, to the figure representing $x^2 + 2\left(\frac{p}{2}\right)^2 x$ to get $\left(x^2 + \frac{p}{2}\right)^2 = x^2 + \left(\frac{p}{2}\right)^2 + 2\left(\frac{p}{2}\right)^2 = q + \left(\frac{p}{2}\right)^2 \Rightarrow x = \sqrt{\left(\frac{p}{2}\right)^2 + q - \frac{p}{2}}$

The Arabic mathematicians made a lasting contribution of their own to the development of the solution of the quadratic equation. For example, the Arabs accepted irrational roots of the quadratic equation, although these were disregarded by the Greeks. They also listed only the positive roots of the quadratic equation when they recognized that it has two roots (Burton. 1999). They did not perceive the reality of the negative solution of to be quadratic equation. The Hindus mathematician, Bhaskara was the first to affirm the existence and validity of negative as well as positive roots (Burton, 1999).

A look at the various attempts revealed that the solution of the quadratic equation was undisputedly a mistress in Euclid' Elements. The algebraic content was always clothed in geometric language.

As identified by Butler, Wren & Bank, (1970) the following five methods are for solving quadratic equations are commonly taught in schools. These are:

1) Solution by graphical methods,

2) Solution by inspection (in case of incomplete quadratics),

3) Solution by the factorization method,

4) Solution by completing the square and

5) Solution by the quadratic formula.

They however assert that the solution by graphical method is not strictly an algebraic method and can only give approximate solutions. Also they claim it is slow and tedious. This point was earlier made by Miller (1957) when he also asserted that the graphical method can sometimes not give the exact roots of a quadratic equation.

Factorization and Difficulty Level of Quadratic Polynomials

Factorization is an important step in the process of finding the roots of a quadratic equation. So in the high school, factorization as a topic is handled with all the seriousness that it desires. Some authors (Amissah, Abbiw, Adjei, Awuah, Dogbe & Eshun 1991; Smith 1987) defined factorization of quadratic polynomials as the reverse of an algebraic expansion of two binomials. Algebraic expansion involves the process of multiplying two or more linear expressions in x (Amissah et al, 1991; Amissah et al 1993; Mitchelmore & Raynor 1988). On this, Amissah et al (1991) indicated that the algebraic expansion of two binomials such as $2x + 3$ and $5x + 9$ is illustrated as

$$(2x + 3)(5x + 9)$$
$$= 10x^2 + 18x + 15x + 27$$
$$= 10x^2 + 33x + 27$$

The result is a quadratic function whose roots can be found. For these authors, the process of using the final results on the right-hand-side of the equation to obtain the two linear factors on the left-hand-side is factorization. In defining factorization of quadratic polynomials (Barnett & Kearns, 1994; Ferris & Busbridge, 1973; Roberts & Stockton 1957) shared similar views with Amissah et al 1991. For instance, Roberts & Stockton (1957) defined factorization as "*the process of separating a quadratic trinomial into linear factors*" (p. 207).

At any rate, the definitions of the various authors (Amissah et al 1991; Barnett & Stockton 1957) suggest that in factorizing trinomials, the product is given and the objective is to find the factors. On the other hand, in binomial expansion, the linear factors are given and the objective is to find the product. Consequently, if teachers of Mathematics do not teach binomial expansions as a pre–requisite skill to factorizing quadratic polynomials, Senior High School students would not be aware of the relation between multiplying linear factors and factorizing quadratic polynomials.

In accordance with the relationship between multiplying and factorizing quadratics, the factorization of the difference of two squares will result into two binomials; whose first terms are positive and second terms differ in sign. For instance, according to Robert & Stockton (1957), to factor $a^2 - b^2$, first find the square root of a^2 and then b^2. This results into a and b respectively. Then add two square roots to form a factor. Similarly, subtract the square roots to form the second factor. Thus $a^2 - b^2 = (a + b)(a - b)$. Some authors (Barnett & Kearns 1994; Ferris & Bushbridge, 1973; Mitchelmore & Raynor, 1988) only introduce the identity $a^2 - b^2 = (a + b)\ (a - b)$ for factorizing the difference of two squares. For instance Mitchelmore & Raynor (1988) have indicated that, "the above identity enables us to factorize any expression which is the difference of two squares" (p. 139). On this, where a

Mathematics teacher, uses such a textbook to give only exercises in order for students to remember the rule and to test their understanding, a gap will be created for the students. This gap will often lead students into the difficulty of factorizing the difference of two squares. Backhouse (1978) contended that "the difficulty found in the last two examples is that $4x^2$ and 1 are not always recognized as being in the form a^2" (p. 40). Thus, understanding this form becomes a hindrance for SHS students.

Incomplete quadratics are of the form $ax^2 + bx$ where a and $b \neq 0$. In factorizing incomplete quadratics, Barnett & Kearns (1994) advocate the process of factorizing out common monomial factors. For instance, to factorize $6x^2 + 15x$, the common factors are found first. The product of the common factor 3 and x is then found to be $3x$ and the special quadratic polynomial factorized into $3x\ (2x + 5)$. Several authors (Amissah et al, 1991; Butler, Wren & Bank, 1970; Levi, 1961; Mitchelmore & Raynor, 1988) share similar views with Barnett & Kearns in the concept of the expression. Gyening (1988) refer to this process as direct application of the distributive law. It is the underlying principle of "taking out the common factor" (p. 7).

Factoring Quadratic Trinomials

Factorizing quadratic trinomial among Senior High School students has long been considered a problem area in mathematics pedagogy all over the world (Autrey & Austin 1979: Usiskin 1980; Erisman 1986; Gendler 1988: Skemp 1966; Jaji 1994). This is because mathematical facts are learned in a disconnected manner and this poses an unnecessary burden on the student's memory. On this, Gyening (1988) observed:

> Due to their mode of presentation, the conventional Methods appear to be only a collection of isolated techniques of solving routine problems and not special cases of well-founded coherent general methods. But as we all know, mathematics is not a mere collection of isolated techniques of solving routine problems. Mathematics is made up of systems with patterns and structures that tie the various facets to the system.

Thus in the conventional methods of factorization we see an incorrect representation of mathematics (p.1).

Definitely, this approach of incorrect representation of mathematical facts, seen in the conventional methods, cannot encourage any proper understanding and application of these ideas and facts to be factorization of trinomials hence in findings the root of a quadratics equation Ferris & Bushbridge (1973) and Mitchelmore & Raynor (1988) have expressed similar viewpoint with respect to representing the methods in secondary schools. He stated that it is even more tedious when the coefficient of x^2 or the constant term has many factors. For instance Ferris & Busbridge (1973) attested, "factorization is more difficult because of the many ways of pairing the numerical factors" (p. 81). These defects in the learning of factorization of quadratic trinomials are reflections of the approaches adopted in the pedagogy of the topic.

Jaji (1994) also reported in his study that 60% of the teachers responded that solving quadratic inequality was considered challenging for the average sixth form student pursuing mathematics. This challenge arises because the method of factorization, the most convenient (Cundy 1968) is a mathematical chore to be endured (Erisman 1986; Gendler, 1987; savage, 1989) and so must be applied on the quadratic inequality or equality for its solution. In a subsequent report, if mathematics teacher do not teach the binomial expansion as a pre-requisites skills to factorization, the students would not be aware of the relations between multiplying two linear factors and factorizing quadratic trinomials (Robert & Stockton, 1956). Consequently, the end result is the difficulty students' fact in sixth form mathematics class. Also, the report was not a triangulated one so as to expose views of students on the topic.

For this reason, the approach has been referred to by some researchers (Autrey & Austin, 1979, Erisman, 1986: Gyening, 1988; Gendler, 1987, Sawyer, 1964; Steinmetz & Cunningham, 1983; Usiskin, 1980) as boring, time consuming, lengthy for beginning students, a mathematical chore to be endured and primitive Sawyer (1964) for instance, criticized that "the trial and error is a rather primitive method, and into his case it may prevent your seeing a very simple feature of the problem (p. 187).

These defects embedded in the learning of factorizing trinomials have resulted in some secondary school students making errors when applying the method of factorization to solve quadratic equation. Such errors, reported by Roberts & Stocken (1956) are in the solution of the quadratic equation give:

$2x^2 - 20x = -42$

$2x(x - 10) = -42$

Either $2x = -42$ or $x - 10 = -42$

Giving $x = -21$ or $x = -32$

Robert & Stockton (1956) reported. “Both results are wrong” (p. 254). The problem with students in the preceding problem is an issue of the principle of divisor of zero known as the zero principle. The principle is, if $pq = 0$ it implies that $p = 0$ or $q = 0$. Students in this problem have associated zero to be equal to -42. Consequently, there arrive at the wrong answers. Similarly, Cornelius & Gott (1988) in their study reported that all the wrong responses to solving of quadratic equations either by factorization or the use of the general quadratic formula came from science students. A wrong response on the trinomial $2x^2 - 3t - 5 = 0$ was reported as:

$$2t - 3 = 5/t$$

$$t(2t - 3) = 5$$

$t = 3t - \sqrt{51}$ (after a page of working) (p. 863).

The wrong responses from science students, is a question of method. Students were either confused to use the method of factorization or the general quadratic formula to solve the problem.

Eventually the inability to use either of the methods was evident in their approach leading to a wrong response to the question.

According to Cornelius & Gott (1988), a gap exists for students between the mathematics in the ‘mathematics” lessons and the mathematics in the ‘science’ lessons. To them, a gap exists because a combination of different languages, methods and sometimes answers leave student in a state of confusion. This confusion will exist if science teachers who claim to have knowledge in mathematics are allowed to teach mathematics instead of allowing mathematics teachers to teach the subject. For this, Johnson (1988) observed, “students in disciplines outside mathematics, but which require mathematics as a tool, are not always highly motivated to study the requisite of mathematics" (p. 895). This leaves a shallow foundation in mathematics for these students to build on.

Conventional Methods for Solving Quadratic Equations

Butler, Wren and Banks (1970) identified five methods for solving quadratic equations commonly taught in schools. These are:

1. Solving by graphical method
2. Solving by inspection (in the case of incomplete quadratics)
3. Solving by factoring
4. Solving by completing the square, and
5. Solving by the quadratic formula

Graphical Method

Butler et al (1970) assert that the graphical method is not strictly an algebraic method and can give only approximate solutions. In addition, they claim that the graphical method is slow and tedious. Miller (1957) also shares the view that the real roots of a quadratic equation sometimes cannot be obtained exactly by graphical methods.

Factorization Method or Solving by Factoring

Factorization is an important step in the process of finding the roots of a quadratic equation. So in the high school, factorization as a topic is handled with all the seriousness that it desires. Some authors (Amissah, Abbiw, Adjei, Awuah, Dogbe & Eshun 1991; Smith 1987) defined factorization of quadratic polynomials as the reverse of an algebraic expansion of two binomials. Algebraic expansion involves the process of multiplying two or more linear expressions in x (Amissah et al, 1991; Amissah et al 1993; Mitchelmore &

Raynor 1988). On this, Amissah et al (1991) indicated that the algebraic expansion of two binomials such as $2x + 3$ and $5x + 9$ is illustrated as $(2x + 3)(5x + 9) = 10x^2 + 18x + 15x + 27 = 10x^2 + 33x + 27$. The result is a quadratic function whose roots can be found. For these authors, the process of using the final results on the right-hand-side of the equation to obtain the two linear factors on the left-hand-side is factorization. In defining factorization of quadratic polynomials (Barnett & Kearns, 1994; Ferris & Busbridge, 1973; Roberts & Stockton 1957) shared similar views with Amissah et al 1991. For instance, Roberts & Stockton (1957) defined factorization as "*the process of separating a quadratic trinomial into linear factors*" (p. 207).

The factoring method consists of expressing the given quadratic equation as the product of two linear factors each of which is set to zero and solved. For example:

$x^2 + 5x + 6 = 0$

$x^2 + 3x + 2x + 6 = 0$

$x(x + 3) + 2(x + 3) = 0$

$(x + 2)(x+3) = 0$

$x + 2 = 0$ *or* $x + 3 = 0$

$x = -2$ *or* $x = -3$

Therefore the solutions are -2 and -3

Commenting on solving by factoring, Butler et al (1970) said that when an equation such as $x^2 + 11x + 18 = 0$ is given in factored form as $(x + 2)(x + 9) = 0$, it is not always clear to students why one has the right to set the factors separately equal to zero and thus gets two linear equations. The justification for this, they claim, should be made clear and that the practice lacks generality in terms of real numbers. This is supported by Richardson (1966) when he claimed that $x^2 - 5x + 6 = 0$ is equivalent to $(x - 2)(x - 3) = 0$, $x - 2 = 0$ or $x - 3 = 0$ which means that x can only be either 2 or 3.

But in the following presentation, Richardson (ibid.) asserts that the third line is erroneous because we cannot make for the number 12 as a statement as "*the product of two quantities can be 0 only when one, or the other (or both) of the quantities is itself zero*" (p. 143).

$x^2 - 5x + 6 = 12$

$(x - 2)(x - 3) = 12$

$x - 2 = 12$ or $x - 3 = 12$

$x = 14$ or $x = 15$

Zero is the only number with this property.

He continued to say that we factor essentially by remembering our experiences in multiplying. Hoffman (1976, p.55) supports Richardson's assertion in the following quote, "*Students who do not have reasonably strong skills in multiplication should not be expected to develop strong skill in factorization*".

Reeves (1952), Miller (1957), Western and Haag (1959), Richardson (1966) and Budnick (1985) all claim many quadratic equations involve trinomials that cannot be factored or are factored by trial and error. In fact Reeves (ibid.) says "*it is impossible to factor a quadratic equation in a real life situation that can be solved by factoring*" (p. 500). So to him, solving quadratic equations by factoring is artificial. For example, the equation $x^2 + 7 = 0$ cannot be solved by factoring. It is however, claimed by Miller (ibid.) that solution by factoring is convenient and simple when it can be applied.

Illustrating solving by factoring, Britton and Bello (1979) say that where the expression $ax^2 + bx + c$ can easily be rewritten as a product of two first-degree expressions, we suppose we have the product $(x + p)(x + q)$. Then by the use of usual properties of addition and multiplication, we may write

$$(x + p)(x + q). =(x + p)x + (x + p)q$$

$$= x^2 + px + qx + pq$$

$$= x^2 + (p + q)x + pq$$

This result can sometimes be used to write a quadratic expression in the product (factored) form. For example, in order to write $x^2 + 5x + 6$ in the factor form, we try to find p and q so that $pq = 6$ and $p + q = 5$. By inspection, we see that $p = 2$ and $q = 3$ are feasible.

Therefore, $x^2 + 5x + 6 = (x + 2)(x + 3)$.

Biwell (1972), however, asserts that solving quadratic equations this way can lead students to pedantic work.

Hoffman (1976) recognizes several different methods available for factoring quadratic expressions such as $3m^2 + 7m + 2$. Among them is the method of decomposition of the linear term that he discussed.

i) Multiply $3m^2$and 2 to get $6m^2$

ii) Decompose $7m$ into the sum of two terms whose product is $6m^2$

This gives $7m = 6m + m$

iii) Factorize $3m^2 + (6m + m) + 2$ by grouping:

$$3m^2 + (6m + m) + 2 = (3m^2 + 6m) + (m + 2)$$

$$= 3m(m + 2) + (m + 2)$$

$$= (m + 2)(3m + 1)$$

This method is based on the observation that

$(ax + b)(cx + d) = acx^2 + adx + bcx + bd$

$= acx^2 + (ad + bc)x + bd$

And that $(acx^2)(bd) = abcdx^2 = (adx)(bcx)$

(Hoffman, 1976, p. 54)

Commenting on the decomposition method for factoring quadratic expressions, and hence solving quadratic equations, Budnick (1985) says that identifying the four constants which satisfy these conditions can be difficult.

Completing the Square

A number of school textbooks treat the method of solving quadratic equations by completing the square as a procedure of putting a quadratic expression as a sum or differences of a perfect square and a number, and then solving for the variable that occurs in it by taking square roots and simplifying. For example;

$$x^2 + 10x + 6 = 0$$

$$x^2 + 10x = -6$$

$$x^2 + 10x + (5)^2 = -6 + (5)^2$$

$$(x + 5)^2 = -6 + 25$$

$$(x + 5)^2 = 19$$

$$x + 5 = \pm\sqrt{19}$$

$$x = -5 \pm \sqrt{19}$$

Therefore, the roots are $-5 - \sqrt{19}$ and $-5 + \sqrt{19}$

$$2x^2 + 4x - 6$$

$$\frac{2x^2}{2} + \frac{4x}{2} - \frac{6}{2} = \frac{0}{2}$$

$$x^2 + 2x - 3 = 0$$

$$x^2 + 2x = 3$$

$$x^2 + 2x + (1)^2 = 3 + (1)^2$$

$$(x + 1)^2 = 3 + 1$$

$$(x + 1)^2 = 4$$

$$(x + 1) = \pm\sqrt{4}$$

$$x + 1 = \pm 2$$

$$x = -1 \pm 2$$

$$x = -1 + 2 \text{ and } x = -1 - 2$$

$$x = 1 \text{ and } x = -3$$

Therefore, the roots are *1* and -3

In completing the square of a quadratic expression, the coefficient of x^2 should always be 1.

Discussing the methods of completing the square, Butler et al (ibid.) said that, the principal function for which this method is taught is to provide a means for developing the general quadratic formula, and hence it is not an end in itself as a method. Hag &Weisstern (1959), in an attempt to explain the method, showed that $(x + a)^2 = x^2 + 2ax + a^2$ and illustrated with $x^2 + 10x + a^2$ concluding that, in general, a is one half the coefficient of x. Thus $x^2 + 10x + a^2 = x^2 + 10x + 5^2 = (x + 5)^2$

Considering the special case of a quadratic with no first-degree term present, to solve $y^2 - k = 0$ to obtain $y = -k$ or $y = k$ and, for purposes of classification, note that;

If $k > 0$, the roots are real,

If $k < 0$, the roots are imaginary,

If $k = 0$, the roots are equal, both equal to zero.

Stover (1978) thinks that the method of completing the square is a basic skill assumed in both pure and applied mathematics courses at college level and should therefore be taught all over again when each new need arises. However, he suggested that the teaching of the technique be moved to such a time when the student has more experience and can better sustain the rather lengthy sequence of steps.

Jones (1985) makes the following assertion about the method of completing the square, "*In my own experience, completing the square was a difficult concept for sixth formers, let alone third years*" (p. 7). Smith (1987) has also said that "*the process of completing the square is often cumbersome*" (p. 79).

Solving by the Quadratic Formula

Most school textbooks apply the method of completing the square to the general quadratic equation, $ax^2 + bx + c = 0$, to obtain the quadratic formula,

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ as follows:

Given $ax^2 + bx + c = 0$

Divide through by a to obtain

$$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$$

Adding $\left(b/_{2a}\right)^2$ to both sides, we get

$$x^2 + \frac{bx}{a} + \left(\frac{bx}{a}\right)^2 + \frac{c}{a} = \left(\frac{bx}{a}\right)^2$$

Re–arranging terms gives

$$x^2 + \frac{bx}{a} + \left(\frac{bx}{a}\right)^2 = \left(\frac{bx}{a}\right)^2 - \frac{c}{a}$$

Hence,

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Taking square root on both sides gives

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

Therefore

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In using the formula to solve, for instance, $4x^2 + 3x - 5 = 0$ the student just substitute $a = 4$, $b = 3$ and $c = -5$ into the formula and simplifies for x. That is:

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 4 \times (-5)}}{2(4)}$$

$$= \frac{-3 \pm \sqrt{(9+80)}}{8}$$

$$= \frac{-3 \pm \sqrt{89}}{8}$$

$$= \frac{-3 \pm 9.433981}{8}$$

$$x = \frac{-12.433981}{8}$$

or

$$\frac{6.433981}{8}$$

x = –1.554 or 0.804 (3 decimal places onl6)

Therefore the roots are –1.554 and 0.804

Western & Haag (1959) theorized that:

The roots of the equation $ax^2 + bx + c = 0$, $a \neq 0$, are given by the quadratic formula,

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

An advantage of the quadratic formula, they observed, is that the numbers produced by the formula have been shown to be the roots, and the only roots, of the quadratic equation. Hence there is no logical necessity for checking the values obtained by the application of the formula except:

1. to catch mistakes in arithmetic,
2. to catch an original equation which was transformed to fit the quadratic equation

Butler, et al (ibid.) claimed that the general quadratic formula is of such importance and usefulness and that it should be thoroughly mastered by every student. Its development requires the use of the method of completing the square and provides an excellent review of operations with literal symbols. The formula itself is indispensable and every student should memorize it and use it until he is perfectly familiar with its form and meaning. It is possible, in a quadratic equation under proper hypothesis of reality or rationality of the coefficients, to determine the nature of the roots from a study of the discriminant alone without solving the equation. Roberts and Stockton (1957) and Brixey and Andree (1966) also agree that the quadratic formula should be memorized.

Britton and Bello (1979) asserted that examples like $3x^2 + x - 5 = 0$, which cannot be factored, display the tremendous advantage of the quadratic formula over other methods of solving quadratic equations. Freund (1956) supported this assertion by writing that the significance of the quadratic formula lies in the fact that proving its result we have actually solved every single quadratic equation. He also said that the quadratic formula reduces the solution of a quadratic equation to the routine of substituting numerical values for *a*, *b*, and *c*. Bigelow (1928) supports these ideas further when he said that a formula has applicability to anyone of a definite class of problems; has properties of conciseness and ready comprehensibility; and that its use affords the student "*economy of thought*" (p. 452).

Bigelow (ibid.) suggests, however, that students must not use the formula until they clearly understand the processes that its prescribes. He suggested further that students should master the technique of the method of completing the square and should develop the formula first.

Stover (1978), on the other hand, said that the quadratic formula lacks the power of "*the quadratic theorem*" (p. 14) he proposed because it does not apply to inequalities and contains the troublesome ± symbolism. Even he said that the quadratic formula is far from being sacrosanct since there is an alternative version:

$$x = \frac{-b_1 \pm \sqrt{({b_1}^2 - ac)}}{a} \text{ Where } b_1 = \frac{b}{2}$$

Non–Conventional Methods for Solving Quadratic Equation

Because of the inherent difficulties associated with the conventional method of solving quadratic equations, Fehr (1951), Stover (1978), Ketcham (1978), Olson (1976), Staib (1977) and Gyening & Wilmot (1999) have proposed various other approaches for solving quadratic equations with certain advantages.

Solving by the ESLE

The method of Equivalent Simultaneous Linear Equations as the name implies, involves an extension application of simultaneous linear equation involving two variables. The main characters here are the arbitrary constants. It is therefore imperative that the quadratic equation is written or transformed into the canonical form to ease the identification of such constants.

If x_1 and $x_2$are the roots of the quadratic equation $ax^2 + bx + c = 0$; then

$$ax_1^2 + bx_1 + c = 0 \quad \text{………………………..} \ (1)$$

$$ax_2^2 + bx_2 + c = 0 \quad \text{………………………..} \ (2)$$

Subtracting equation (2) from (1) we get;

$$ax_1^2 - ax_2^2 + bx_1 - bx_2 = 0$$

$$a(x_1^2 - x_2^2) + b(x_1 - x_2) = 0$$

$$a(x_1 + x_2)(x_1 - x_2) = -b(x_1 - x_2)$$

$$a(x_1 + x_2) = -b \quad \text{………………………..} \ (3)$$

From equation (3), $b = -a(x_1 + x_2)$. And so putting it in equation (1) we have,

$$ax_1^2 - a(x_1 + x_2)x_1 + c = 0$$

$$ax_1^2 - ax_1^2 - ax_1x_2 + c = 0$$

$$-ax_1x_2 + c = 0$$

$$ax_1x_2 = c \quad \text{……………………..} \ (4)$$

Squaring both sides of the equation (3) we get,

$$a^2(x_1 + x_2)^2 = (-b)^2$$

$$a^2(x_1 + x_2)^2 = b^2 \ \text{………………….} \ (5)$$

Multiplying both sides of equation (4) by a we get,

$$a^2x_1x_2 = ac \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(6)}$$

Also it is a property for real numbers that,

$$(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2$$

Multiplying both sides by a^2 gives,

$$a^2(x_1 - x_2)^2 = a^2(x_1 + x_2)^2 - 4a^2x_1x_2$$

$$a^2(x_1 - x_2)^2 = b^2 - 4ac$$

$$a(x_1 - x_2) = \sqrt{b^2 - 4ac} = \text{d where } b^2 - 4ac > 0$$

$$ax_1 - ax_2 = \text{d} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(7)}$$

From equation (3);

$$a(x_1 + x_2) = -b$$

$$ax_1 + ax_2 = -b$$

Thus the simultaneous linear equations;

$$ax_1 + ax_2 = -b$$

$$ax_1 - ax_2 = d$$

Can be solved easily for x_1 and x_2.

Example: Solve the equation $x^2 + 2x - 15 = 0$.

Compared to $ax^2 + bx + c = 0$, $a = 1$, $b = 2$, $c = -15$

$$\text{d} = \sqrt{b^2 - 4ac}$$

$$= \sqrt{2^2 - 4(1)(-15)}$$

$$= \sqrt{4 + 60}$$

$$= \sqrt{64}$$

$$= 8$$

Now let x_1 and x_2 represent the roots of the equation, then,

$$x_1 + x_2 = -2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

$$x_1 - x_2 = 8 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Adding (1) and (2) we get,

$$2x_1 = 6$$

$$x_1 = 3$$

Putting the value of x_1 in equation (1) we get,

$$3 + x_2 = -2$$

$$x_2 = -2 - 3$$

$$x_2 = -5$$

Therefore, the solution is $x = -5$ or 3

Also $x^2 + 5x + 6 = 0$

Compared to $ax^2 + bx + c = 0$

$$a = 1, b = 5, c = 6$$

$$d = \sqrt{b^2 - 4ac}$$

$$d = \sqrt{5^2 - 4(1)(6)}$$

$$d = \sqrt{5^2 - 4(1)(-6)}$$

$$d = \sqrt{25 - 24}$$

$$d = \sqrt{1}$$

$$d = 1$$

Now let x_1 and x_2 represent the roots of the equation, then

$$x_1 + x_2 = -5 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$x_1 - x_2 = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Adding (1) and (2) we get

$$2x_1 = -4$$

$$\frac{2x_1}{2} = -\frac{4}{2}$$

$$x_1 = -2$$

Putting the value of x_1 in the equation (2) we get,

$$-2 - x_2 = 1$$

$$x_2 = -2 - 1$$

$$x_2 = -3$$

Therefore, the solution is -2 or -3

Advantages of ESLE

According to Gyening (1988), a very important educational value of this method is that the student's increase awareness of the mathematical method of replacing a problem by a simpler equation to facilitate solution. Skemp, (1986) refers to this as the essence of the application of the principle of interchangeability which is implicit in the idea of equivalence class.

The method of ESLE is free from all the inherent limitation of its conventional counterparts. Like the method of completing the square or the quadratic formula, this method has got a broad spectrum of applicability. This implies that the high degree of complexity of the method of completing the square could be avoided by making the use of this approach.

The ESLE can easily be taught to all average ability students as well as high achievers because it does not involve any difficult concepts, rigorous manipulations and is simpler than the method of completing the square. In this regard, its stands the chance of proving to be superior to the quadratic formula, whose formidable structure, including the weird–looking double sign ±, has put many a student to flight on first encounter.

The ESLE is the easiest type of simultaneous equations because they have the same coefficient and the signs are negative and positive so there is no need to spend more time to teach any specialized prerequisite skills prior to teaching the method. Time is saved which could be used more gainfully to promote learning for the pupils to achieve mastery.

The Conjugales

This method also requires the solution of two simple linear equations to obtain desired solution. It is however important to write the given equation in the standard or canonical form for easy identification of the constants. For example, given the equation $ax^2 + bx + c = 0, a \neq 0$, it can be restated as;

$$ax^2 + bx = -c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Multiply equation (1) by $4a$

$$4a^2x^2 + 4abx = -4ac \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Add b^2 to both sides of equation (2)

$$4a^2x^2 + 4abx + b^2 = b^2 - 4ac \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

The left hand side of equation (3) is now a perfect square.

$\Rightarrow (2ax + b)^2 = b^2 - 4ac$

$\Rightarrow 2ax + b = \pm\sqrt{b^2 - 4ac}$

Let d be any number equivalent to $\pm 2ax + b$.

Implying that $d = \sqrt{b^2 - 4ac}$

Hence $2ax + b = -d$

or

$$2ax + b = d \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

The solution of these two simple linear equation is the solution set of the quadratic equation.

Example:

The truth set of the equation $x^2 - 5x + 6 = 0$

Compare $x^2 - 5x + 6 = 0$ with the general form of a quadratic equation, $ax^2 + bx + c = 0$ $a = 1, b = -5$ and $c = 6$.

Putting these values into

$$d = \sqrt{b^2 - 4ac},$$

$$d = \sqrt{(-5)^2 - 4(1)(6)},$$

$$d = \sqrt{25 - 24}$$

$$d = \sqrt{1}$$

$$d = 1$$

The two linear equations required can therefore be formed as,

$2x - 5 = -1$ ……………………… (1)

$2x - 5 = 1$ ……………………… (2)

From equation (1) and (2), $x = 2$ and $x = 3$ respectively. Hence the truth set of the equation is $\{x : x = 2, 3\}$.

The truth set of the equation $x^2 - 15x - 100 = 0$

Compare $x^2 - 15x - 100 = 0$ with the general form of a quadratic equation, $ax^2 + bx + c = 0$

$a = 1, b = -15$ and $c = -100$

Putting these values into

$$d = \sqrt{b^2 - 4ac}$$

$$d = \sqrt{(-15)^2 - 4(1)(-100)}$$

$$d = \sqrt{225 + 400}$$

$$d = \sqrt{625}$$

$$d = 25$$

The two linear equations required can be therefore be formed as,

$$2x - 15 = -25 \text{ ……………………….(1)}$$

$$2x - 15 = 25 \text{ ………………………… (2)}$$

From equation (1)

$$2x - 15 = -25$$

$$2x = -25 + 15$$

$$2x = -10$$

$$\frac{2x}{2} = \frac{-10}{2}$$

$$x = -5$$

Also from equation (2)

$$2x - 15 = 25$$

$$2x = 25 + 15$$

$$2x = 40$$

$$\frac{2x}{2} = \frac{40}{2}$$

$$x = 20$$

Hence the truth set of the equation is $\{x: x = -5, 20\}$

We notice that the student is again made aware of the need to replace a complex problem with a much simpler but equivalent one to enable the student to solve it. At the J.S.S level, students learnt to solve linear equations, so it would not be difficult for a student to recall from his/her cognitive background the skills for solving linear equations.

Importance of Quadratic Equation

Apart from giving definitions, methods for solving quadratic equations, proofs for quadratic theorems and propositions for teaching quadratic equations, the literature also cites some practical relevance in various fields for quadratic equations. Reeves (1952) said that the construction of the Holland Tunnel under the Hudson River in New York was based on a quadratic equation and its solution. According to Wheeler and Peeples (1986), radar dishes, reflectors or spotlights, components of microphones, and some cables of suspension bridges are all in the shape of parabolas. Likewise profit and cost functions in business, equilibrium point and Laffer curve in economics, blood velocity and pollution in life sciences, and population growth in the social sciences are all models of quadratic functions. All this provide justification for teaching quadratic equations and teaching the topic well for as Dubisch (1963) said it is the basic methods of mathematical analysis that are applied rather than the detailed results. Quadratic equations should therefore be taught with emphasis on the thinking process to equip the students with the habits of logical thought that can be transferred to other subjects.

Empirical Research

Bornaa (2007) did a study on teaching quadratic equation by algebraic methods, relative variability of prescribed methods of Solutions; Conjugales, Directed Trial Analysis and Equivalent Simultaneous Linear Equation. He found out that; the novel methods group (DTA, Conjugales and ESLE) was significantly higher than the performance of the factorization group on the posttest. In his study, there was no evidence for the assertion that differences exist on measures of accuracy among pairs of the ‘novel methods group’ and therefore seems to suggest by the results of his study, that all the ‘novel methods’ are relatively superior and viable for use in solving questions and problems involving quadratic equations. He recommended educational researchers to conduct a nationwide study similar to it, to assess the efficacy of these methods and to give a broader picture of the findings for generalization.

Kissi–Twum (2006) also undertook a study on the feasibility of teaching quadratic equations in S.H.S (form one). He reported that completing the square is more difficult than ESLE, factorization and the quadratic formula. In his study, it seems more likely that first year students in the S.H.S would cope with solving quadratic equations by the quadratic formula better than the ESLE. He therefore concluded that it is the most feasible method for solving quadratic equations among form one students in the S.H.S. He also suggested that in the near future his work should be duplicated to give more widespread and generally true conclusions that can then be reached on the matter of teaching quadratic equations in the first year and by which method. In his study, lesson took place after siesta in the afternoon, the willingness and the commitment on the part of the students to study the lessons could affect the outcome of the researchers posttest, and hence the correlation coefficient and the outcome of the statistical test.

The students knew the results were not going to be part of their school assessment and therefore they probably did not give it the seriousness it deserved. The Science, Agric and Elective Mathematics students were included.

Attah (1999) undertook a study that compared the method of completing the square and the ESLE among Senior High School students in their first year. He reported that the ESLE group performed better on the achievement posttest than the group that used the method of completing the square. Their respective mean scores were 26.39 and 18.56. He reported also that the ESLE group made 24 wrong ESLE equations, 15 wrong computations of the discriminant and 13 wrong solutions to the equations. For the group that used completing the square, there were 38 wrong completions of the square and 33 wrong solutions of equations.

From the second–half of the 1990s a number of people have reported on the feasibility of teaching the solution of quadratic equations by the ESLE in the first year of the senior high school. Among them are Adams (1997), Anomah (1998), Essah (1999), and Sackey and Adjei (2003). The studies were undertaken separately in places such as Cape Coast, Koforidua and Teshie in Accra. They involve both single–sex and mixed senior high schools. Some of the boys' schools are St. Augustine's High School, Adisadel College and Koforidua High Technical School, Holy Child High School, New Juaben High School/Commercial and Teshie Presbyterian High School are the mixed schools which have taken part in the studies.

The sample sizes in the studies range between 30 and 55. The samples come separately from all the programme areas in the senior high school. The populations from which the samples were randomly selected also come from a cross–section of the socio–economic and geographical backgrounds in Ghana.

The studies used teaching experiments that employed pretest–posttest techniques. The purpose of each study was to investigate the feasibility of form one students in the senior high school using the method of ESLE to solve the quadratic equation.

All the studies agree in their findings and conclusions. Their conclusions may be summarized as: –

1. There were statistically significant differences ($\alpha = 0.05$) between the scores on the pretest (usually second term exams in schools) and the posttest (on quadratic equations).
2. ESLE can be taught effectively as a method for solving quadratic equations to students in the first year of senior high schools.
3. Further research into the feasibility of teaching quadratic equations in the first year of senior high school is however recommended.

The studies showed that some students found it difficult to quote the ESLE equations. Another problem was the identification of the coefficients of the quadratic equations for substitution in the formula for the discriminant. Again substitution of *a* and *b* in the ESLE equations was a problem, especially when *a* and *b* were negative.

Summary of the Literature Review/State of the Art

The literature explains quadratic equations as any equation that can be put in the form of a polynomial of degree two, precisely $ax^2+bx + c = 0$, *a, b, c* are real numbers and are not equal to zero. Conventional and ingenious non–conventional methods are available for solving quadratic equations.

Conventional Methods

The conventional methods used in solving quadratic equations are solving (a)by graphical methods, (b) by inspection, (c) by factoring, (d) by completing the square and by the quadratic formula.

Graphical Method

The graphical methods are slow, tedious and yield only approximate solutions.

Inspection

This method is not seriously treated.

Factoring

1. Setting the factors of an equation such as $(x + 7)(x + 2) = 0$ separately equal to zero lacks generality in terms of real numbers and generally many students do not understand it.
2. Factoring quadratic equations is dependent on students' strong skills in multiplication and is too mechanical.
3. Solution of quadratic equations by factoring is simple and convenient where it can be applied.
4. There are various methods of factoring equations such as decomposition of the linear term and others.

Completing the Square

1. This method is mostly taught to provide a means for developing the general quadratic formula.
2. The method is a basic skill assumed in both pure and applied mathematics courses at college level.
3. The method is difficult and should be delayed until the student has more experience and can better sustain the lengthy sequence of steps.
4. It should be taught all over again when each new need for the method arises.

The Quadratic Formula

1. It is so important, useful and indispensable that it should be mastered by every student.
2. The numbers produced by the formula have been known to be the only roots of the quadratic equation.
3. That the significance of the formula lies in the fact that by proving its result we have actually solved every single quadratic equation.
4. The formula is not sacrosanct. There are alternative versions.
5. The $\pm$ and $\sqrt{\quad}$ symbolism are too overwhelming for most students.

Ingenious Non–Conventional Methods

A number of mathematicians have proposed alternative methods for solving quadratic equations to remedy the difficulties students face with the conventional methods. One of the alternative methods is ESLE.

The ESLE Method

1. Very little effort is required both for the teaching and learning of the ESLE as no specialized pre–requisite skills are needed.
2. ESLE as a method for solving quadratic equations is much easier, simpler and more conducive to real understanding than the method of completing the square.
3. ESLE can be taught effectively to form one student in the senior high school.

CHAPTER THREE

METHODOLOGY

This chapter presents the research design, population, sample and sampling technique, instrument development and pilot study, data collection and data analysis techniques.

Research Design

The study used a pretest – posttest non–equivalent, comparison group design. The purpose of a teaching experiment could be one of curriculum testing and development or one of theory construction. This teaching experiment was designed to assess the potentials of Form One students in Senior High Schools to learn to solve quadratic equations.

In this teaching experiment, lessons were planned and taught to the students. It covered six contact periods of a total duration of four hours. The lessons inuduel four approaches in solving quadratic equations. These were *Factorization*, *Quadratic Formula*, *Conjugales* and *Equivalent Simultaneous Linear Equations* (*ESLE*). One method was taught to each school but the same achievement test was administered to all the subjects. The achievement test's mean scores were then compared with subjects mean scores on their pretest.

Moore (1999) used a similar design but limited it to only the method of *ESLE*.

Population

According to Osuala (1993), difficult problems arise when the universe to be sampled is not precisely defined. Koul (2002) defined a population as the entire aggregation of cases that meet a designated set of criteria. The target population consist of all students in form one in the Central Region. The accessible population was students in form one from Senior High Schools in Cape Coast Metropolis, Mfantseman District, Lower Denkyira District and Komenda Edina EguafoAbirem Municipality. The sample size was 286 of which 160 were girls.

Sample and Sampling

Sarantakos, (1998) stated that sampling in research may be guided by a number of factors and he identified the following as the most common;

1. In many cases a complete coverage of the population is not possible.
2. Complete coverage may not offer substantial advantage over a sample survey. Sampling provides a better option since it addresses the survey population in a short period of time and produces comparable and equally valid results.
3. Studies based on samples require less time and produce quick answers.
4. Sampling is less demanding in terms of labour requirement, since it requires a small portion of the target population.
5. It is also thought to be more economical since it contains fewer elements of the population and requires less printed materials and a reduced general cost.
6. Samples are thought to offer more detailed information and a high degree of accuracy because they deal with relatively small numbers of units.

Table of random numbers was used to select four (4) districts, out of 21 districts in the Central Region of Ghana and based on these factors, a sample of four (4) schools were randomly selected from stratified groups made by location differentiation of the schools; a school from each location group. Stratified sampling involves dividing the population into a number of homogenous groups or strata. Each group contains subject with similar characteristics. A sample is then drawn from each group. The sub samples make up the final sample for the study. The division of the population into homogenous group was based on location. The four (4) selected schools had a total Form One population of 1480 of which 286 students consisting of 160 girls and 126 boys took part in the teaching experiment. The mean age of the entire sample was 15.75 with a standard deviation of 0.68.

The schools were as follows:

Mfantsiman District

1. Methodist Senior High School

Cape Coast Metropolis

2. Academy of Christ the King

Lower Denkyira District

3. Jukwa Senior High School

Komenda Edina Eguafo Abirem Municipality

4. Edinaman Senior High School

In each of these schools, the Assistant Headmaster or Assistant Headmistress (Academic) in consultation with the Mathematics teachers randomly selected two (2) form one classes deficiency of Physics and Elective Mathematics for the teaching experiment. In each school, the two selected classes studied the same method for solving quadratic equations. A round of ballot determined a method for each school. Four (4) Junior High School Students, one of whom represented each school did the balloting. These eight

(8) classes selected were 1business 1, 1 visual art 1, 1 home economics A, 1general art C, 1general art A, 1 business A, 1 visual art A, and 1 home economics 1

Instrument Development and Pilot Study

The pilot study was undertaken at University Practice Senior High School, Cape Coast during the first term precisely during the fifth week of the academic term.

With the permission of the school authority, the Head of Mathematics Department assigned four Form One classes deficiency of Physics and Elective Mathematics, all because the Physics students by then had done Motion which involves quadratic equation and the Elective Mathematics students too by then has also done quadratic functions which also involves quadratic equation and may have effect on the study. These classes were Home Economics, Visual Art, Business A and General Art B.

There were four (4) instructional periods of forty (40) minutes duration each and a further fifty (50) minutes for an achievement test based on the lessons. All the lessons followed the school's normal timetable. Exercises for class discussions and practice were selected from the Form Three students' textbook namely Exercise 3.4B on pages 45 and 46 of the Ghana Senior Secondary School Mathematics Book 3. However, care was taken to exclude the questions previously selected from the textbook for the achievement test from the discussions. The number of students that took the test are in *Table 1*.

Table 1: Pilot Test

Number of students by classes, sex and methods that took the Test

Class	Method	Boys	Girls	Total
1 Bus A	Conjugales	17	23	40
1 H/E B	Factorization	0	35	35
1 G/A B	ESLE	10	21	31
1 V/A	Quadratic Form	18	6	24
Total		**45**	**85**	**130**

In all there were 319 Form One students in the school. The test items followed the same order as they appear in the textbook. Student answered the questions on the question paper because space was created on it. The test was by nature one for achievement that was crafted to elicit both knowledge (concept, recognition, recall) and comprehension of the subject matter of quadratic equations. This was made amply clear to the students throughout the teaching experiment.

Pretest

Twenty multiple–choice test items were constructed for the Mathematics topics in the Second Term Scheme of Work common to all the four randomly selected schools. All the common topics among the four schools constitute pre–requisite skills for the solutions of quadratic equations.

These items were drawn from areas such as factorization of algebraic expressions, solution of linear equations, real number system and numeration system.

The pretest gave the researcher a fair idea of the state of learning readiness for the solution set of quadratic equation by Form One students. These are samples of the pretest items:

Question 4 is stated like this

Solve the equation $x + 3 = -10$

A. 7

B. –30

C. $^{10}/_{3}$

D. –13

E. 3

Question 16

Find the value of $a^2 - 3ab + b^3$ when $a = 2$ and $b = -3$

A. 49

B. 13

C. -5

D. -19

E. 50

Question 10

Given that $y = \frac{2x-5}{3x+2}$, find y when $x = -4$

A. 0.2

B. 0.3

C. 0.5

D. 1.3

E. 2.4

Question 12

Simplify $\frac{a-b}{2}+\frac{2a}{3}$

A. $a - b$

B. $\frac{a-3b}{6}$

C. $\frac{7a+3b}{6}$

D. $7a + 3b$

E. $\frac{7a-3b}{6}$

Question 11

Make a the subject in the relation $v = u + at$

A. $\frac{v+u}{t}$

B. $\frac{v-u}{t}$

C. $\frac{u-v}{t}$

D. $\frac{vu}{t}$

E. $\frac{v}{t} - u$

Question 18

Solve the equation: $\frac{x-2}{3}+\frac{3}{2}=\frac{x}{2}$

A. 5

B. 4

C. 2

D. -1

E. -2

Question 19

Factorize $6am + 15am + 2bm + 5bn$

A. $(2m - 5n)(3a + b)$

B. $(2m + 5n)(3a + b)$

C. $(3m - 5n)\ (3a - b)$

D. $(2m - 5n)(3a - b)$

E. $(3a - b)(3m + 5m)$

Question 20

Find the values of x and y if $x - y = 2$ and $2x + y = 4$

A. $x = 4, y = 2$

B. $x = 2, y = 0$

C. $x = 0, y = 2$

D. $x = 2, y = 4$

E. $x = 2, y = 2$

Posttest

The posttest constituted five (5) essay type items based on quadratic equations. The items required students to find the value(s) of x in the following quadratic equations.

$2x^2 + 3x - 20 = 0$

$4x^2 + 7x + 3 = 0$

Find the solution set of the equations

$18x^2 + 9x - 2 = 0$

$8 - 2x - 3x^2 = 0$

Solve for x in the equation

$$x^2 - 2x - 35 = 0$$

Scoring

The pretest was scored dichotomously. For the posttest, a marking scheme was drawn for each test, such that there was no differential on the basis of the treatment so that no one treatment gained an undue advantage over the other. Each item of the posttest was awarded 10 marks. Every logical Mathematical step was awarded a mark including the final answer.

Validity

According to the American Educational Research Association, American Psychological Association and National Council on Measurement in Education (citd in Amedahe & Gyimah, 2004) validity refers to the degree to which evidence and theory support the interpretation of test scores entailed by proposed uses of tests. In other words, validity refers to the soundness or appropriateness of your interpretations and uses of students' assessment results.

The content validity of the study is related to how adequately the content of the items of the instruments (test) and the responses to the test sampled the domain about which inferences are to be made. Content validity was built into the test from the outset. Thus, at the planning stage a test specification table was drawn on the entire area of content. Also the test items were constructed within the prescribed Mathematics syllabus and similar questions taken from SSSCE past questions. The items of each test were therefore given to some experienced Mathematics teachers for scrutiny. After which the Supervisor went through it for the final scrutiny. These items were then validated following the comments from the experienced teachers and supervisor.

Reliability

The reliability coefficients of the pretest was calculated using Kuder–Richardson's reliability (KR20) and that of the posttest was calculated using the Cronbach Alpha.

The pretest and posttest recorded 0.86 and 0.80 respectively. This is a measure of the internal consistency reliability (McKnight, et al 2000). According to McDaniel (1994) a reliability coefficient that falls within the ranges of 0.80 – 0.84 and 0.85 – 0.89 is interpreted as being useful and good respectively (p. 166). For conceptual purposes, the standardized Cronbach Alpha Coefficient formula is:

$$r = \frac{n({\sigma_t}^2 - \Sigma{\sigma_i}^2)}{(n-1)({\sigma_t}^2)}$$

Where *r* is the reliability coefficient

n is the number of test items

$\sigma_i{}^2$ is the variance of the test scores of the *i*–th term

σ_t^2 is the variance of the scores of the entire test

Data Collection Procedure

A letter introducing the researcher, from the Department of Science and Mathematics Education was presented to the Heads of each of the selected schools two weeks before the start of the data collection exercise. I undertook all the teaching experiments. The time table for the teaching experiment is in table 2.

Table 2: Time Table for Teaching Experiment

School	Classes	Start	End
Edinaman	Business	4th February	11th February
	General Art	4th February	11th February
Methodist High	1 General Art C	12th February	21th February
	1 Business A	14th February	21st February
Jukwa	1Home Economics	22nd February	5st March
	1 Visual Art	27th February	5th March
Academy	1 General Art A	8th March	18th March
	1 Home Economics A	12th March	18th March

Data Analysis

Marks were obtained from two broad sources; Pretest and Protest. Analysis of variance (ANOVA) was used to analyze the pretest scores for mathematical abilities between schools prior to the teaching experiment, and for the effectiveness between the two methods for solving quadratic equations used in each school. This allows to test many means simultaneously to see, if there is any difference among them. The analysis of variance procedure is design to measure the spread in a given data and a portion, the total amount of variance present for all the sample data into two components, one corresponding what happens between the different sets of data and the other corresponding what happens within each sets of data. If there is a little variances within each sample group and there is relatively little difference between the sample unit, then we cannot conclude that the population means are different. Also the one- way analysis of variance is used when the variable testing is the same. The t-test for dependent (paired) samples was used to analyze the mean scores on pretest (topics in the second term scheme of work) and posttest scores in the four methods for solving quadratic equations. The t-test for dependent (paired) samples was also used to analyze the pretest scores and the posttest scores in each class. The t-test for independent samples was used to analyze the posttest between methods by method.

Analysis of covariance (ANCOVA) was also used to analyze the posttest scores in all the schools. This tool assumes a random sampling, homogeneity of variance, and normal distribution. In addition it takes into consideration that the homogeneity of the slopes of the covariance and the dependent variable must be equal. That is the regression coefficient for the regression lines must be equal. The tool was considered valuable in this type of research since the subjects were not randomly assigned to the treatment groups. Also the tool was used to increase the statistical power by controlling variability due to the effects of extraneous variable and reduce bias in statistical analysis as noted by Fisher (1932). League table was also used to come out with the more feasible method (s).

CHAPTER FOUR

RESULTS AND DISCUSSIONS

This chapter presents the general statistics and the test of hypothesis postulated from the research. The results and findings are presented on each hypothesis stated.

The analysis of variance was used to analyze the pretest scores for mathematical abilities between schools prior to the teaching experiment, and for the effectiveness between the two methods for solving quadratic equations used in each school. The t-test for dependent (paired) samples was also used to analyze the pretest scores and the posttest scores in each class. Analysis of covariance (ANCOVA) was used to analyze the posttest scores of the treatment groups. The entire tests were at a 0.05 level of significance.

The research findings are presented hypothesis after hypothesis supported with statistical tables and its discussions. At the end of the chapter there is a summary of the findings related to both the hypothesis and the purpose of the study.

Data on the Pretest and the Posttest compared

Table 3: Descriptive statistics of Posttest and Pretest Scores of Treatment Groups

		Posttest		Pretest	
Method	N	Mean	Standard Deviation	Mean	Standard deviation
Quadratic formula	64	79.72	15.01	72.19	15.38
Factorization	71	75.24	16.17	66.79	16.9
ESLE	70	68.40	17.34	65.79	17.44
Conjugales	81	79.41	16.63	70.93	16.59
Total	286	75.69	16.29	68.92	16.66

Hypothesis 1: For each treatment group, the mean score on the pretest does not differ from that on the posttest.

Table 4: Dependent (Paired) t-test Showing Differences in the Mean Score on Posttest and Pretest of Respondents Scores in the Four Methods of Solving Quadratic Equations

Method	N	Mean	Standard dev.	Mean diff.	t	Sig
Posttest	286	75.75	16.81			
Pretest	286	68.92	16.66	6.83	10.70	.000**

n= 592, source: field data (2013) *p< 0.05(2-tailed),   **p< 0.01(2-tailed)

Dependent t-test was conducted to determine whether significant differences exist between the mean scores of respondents pretest and posttest in solving quadratic equation in Table 4. It was found out that there was significant difference between the mean scores of the posttest (M=75.75, SD= 16.66) and the pretest (M=68.92, SD= 16.66) with respect to topics in second term scheme of work and the use of the four methods in solving quadratic equation and that the null hypothesis is rejected at a significance level of 0.05. The implication is that form one students can learn quadratic equation as effectively as the topics in the second term scheme of work. From Table 4 it was shown that students even did better than on posttest than pretest.

Table 5: Dependent (Paired) t-test showing comparison of performances on the Pretest and the Posttest of Factorization group/1 General Arts A

Test	n	Mean	Standard dev.	Mean diff.	t	Sig
Posttest	37	77.62	17.02			
Pretest	37	66.35	17.98	4.48	7.16	.000**

n= 74, source: field data (2013) *p< 0.05(2-tailed), **p< 0.01(2-tailed)

Dependent t-test was conducted to determine whether significant difference exist between the mean scores of respondents pretest and posttest in solving quadratic equation in Table 5. It was found out that there was significant difference between the mean scores of the posttest (M=77.62, SD=17.02) and the pretest (M=66.35, SD=17.98) and that the null hypothesis is rejected at a significance level of 0.05.

The implication is that Form One student can learn quadratic equation as effectively as the topics in the second term scheme of work. From Table 5 it was also revealed that students even did better than on the posttest than the pretest.

Table 6: Dependent (Paired) t-test showing comparison of performances on the Pretest and the Posttest of Factorization group/ 1 Home Economics A

Test	n	Mean	Standard dev.	Mean diff.	t	Sig
Posttest	34	72.65	15.02			
Pretest	34	67.21	15.19	5.44	3.17	.003*

n= 84, source: field data (2013) *p< 0.05(2-tailed), **p< 0.01(2-tailed)

Dependent t-test was conducted to determine whether significant difference exist between the mean scores of respondents pretest and posttest in solving quadratic equation in Table 6. It was found out that there was significant difference between the mean scores of the posttest (M=72.65, SD=15.02) and the pretest (M=67.21, SD=15.19) and that the null hypothesis is rejected at a significance level of 0.05.

The implication is that Form One student can learn quadratic equation as effectively as the topics in the second term scheme of work. From Table 6 it was also revealed that students even did better on the posttest than the pretest.

Table 7: Dependent (Paired) t-test showing comparison of performances on the Pretest and the Posttest of ESLE group/ 1 Home Economics

Test	n	Mean	Standard dev.	Mean diff.	t	Sig
Posttest	28	71.86	16.39			
				6.14	3.46	.002*
Pretest	28	65.71	19.04			

n= 56, source: field data (2013) *$p< 0.05$(2-tailed), **$p< 0.01$(2-tailed)

Dependent t-test was conducted to determine whether significant difference exist between the mean scores of respondents pretest and posttest in solving quadratic equation in Table 7. It was found out that there was significant difference between the mean scores of the posttest (M=71.86, SD=16.39) and the pretest (M=65.71, SD=19.04) and that the null hypothesis is rejected at a significance level of 0.05.

The implication is that Form One students can learn quadratic equation as effectively as the topics in the second term scheme of work. From Table 7 it was also revealed that students even did better with the ESLE method than the pretest.

Table 8: Dependent (Paired) t-test showing comparison of performances on the Pretest and the Posttest of ESLE group/ 1 Visual Art

Test	n	Mean	Standard dev.	Mean diff.	t	Sig
Posttest	42	66.10	17.80			
Pretest	42	65.83	16.53	0.26	0.12	.902

n= 84, source: field data (2013) *$p< 0.05$(2-tailed), **$p< 0.01$(2-tailed)

Dependent t-test was conducted to determine whether significant difference exist between the mean scores of respondents pretest and posttest in solving quadratic equation in Table 8. It was found out that there was no significant difference between the mean scores of ESLE method (M=66.10, SD=17.80) and pretest (M=65.83, SD=16.53) at $p<0.05$ alpha level and that the null hypothesis is accepted. Even though the mean score of posttest (ESLE method) was higher than the mean score of pretest, it can be concluded that respondent understanding of the use of this method in solving quadratic equations and the pretest.

Table 9: Dependent (Paired) t-test showing comparison of performances on the Pretest and the Posttest of Quadratic Formula group/ 1 Business 1

Test	n	Mean	Standard dev.	Mean diff.	t	Sig
Posttest	31	80.65	14.86	7.90	5.11	.000**
Pretest	31	72.74	15.70			

n= 62, source: field data (2013) *p< 0.05(2-tailed), **p< 0.01(2-tailed)

Dependent t-test was conducted to determine whether significant difference exist between the mean scores of respondents pretest and posttest in solving quadratic equation in Table 9. It was found out that there was significant difference between the mean scores of the posttest (M=80.65, SD=14.86) and the pretest (M=72.74, SD=15.70) and that the null hypothesis is rejected at a significance level of 0.05.

The implication is that Form One student can learn quadratic equation as effectively as the topics in the second term scheme of work. From Table 9 it was also revealed that students even did better with the quadratic formula method than the pretest.

Table 10: Dependent (Paired) t-test showing comparison of performances on the Pretest and the Posttest of Quadratic Formula group/ 1 General Art 1

Test	n	Mean	Standard dev.	Mean diff.	t	Sig
Posttest	33	78.85	15.33	7.18	3.79	.001**
Pretest	33	71.67	15.29			

n= 78, source: field data (2013) *p< 0.05(2-tailed), **p< 0.01(2-tailed)

Dependent t-test was conducted to determine whether significant difference exist between the mean scores of respondents pretest and posttest in solving quadratic equation in Table 10. It was found out that there was significant difference between the mean scores of the posttest (M=78.85, SD=15.33) and the pretest (M=71.67, SD=15.29) and that the null hypothesis is rejected at a significance level of 0.05.

The implication is that Form One student can learn quadratic equation as effectively as the topics in the second term scheme of work. From Table 10 it was also revealed that students even did better with the quadratic formula method than the pretest.

Table 11: Dependent (Paired) t-test showing comparison of performances on the Pretest and the Posttest of Conjugales/ 1 Business A

Test	n	Mean	Standard dev.	Mean diff.	t	Sig
Posttest	42	81.81	11.53	7.05	5.58	.000**
Pretest	42	74.76	14.01			

n= 84, source: field data (2013) *p< 0.05(2-tailed), **p< 0.01(2-tailed)

Dependent t-test was conducted to determine whether significant difference exist between the mean scores of respondents pretest and posttest in solving quadratic equation in Table 11. It was found out that there was significant difference between the mean scores of the posttest (M=81.81, SD=11.53) and the pretest (M=74.76, SD=14.01) and that the null hypothesis is rejected at a significance level of 0.05.

The implication is that Form One student can learn quadratic equation as effectively as the topics in the second term scheme of work. From Table 11 it was also revealed that students even did better with the conjugales method than the pretest.

Table 12: Dependent (Paired) t-test showing comparison of performances on the Pretest and the Posttest of Conjugales/ 1 General Art C

Test	n	Mean	Standard dev.	Mean diff.	t	Sig
Posttest	39	76.82	20.17	10.03	5.59	.000**
Pretest	39	66.79	18.23			

n= 78, source: field data (2013) *p< 0.05(2-tailed), **p< 0.01(2-tailed)

Dependent t-test was conducted to determine whether significant difference exist between the mean scores of respondents pretest and posttest in solving quadratic equation in Table 12. It was found out that there was significant difference between the mean scores of the posttest (M=76.82, SD=20.17) and the pretest (M=66.79, SD=18.23) and that the null hypothesis is rejected at a significance level of 0.05.

The implication is that Form One student can learn quadratic equation as effectively as the topics in the second term scheme of work. From Table 12 it was also revealed that students even did better with the conjugales method than the pretest.

Data on the Various Methods Compared on Treatment Basis

Table 13: ANCOVA output of the adjusted mean scores for the treatment group on the posttest scores

Source of variation	Sum of squares	df	Ms	F	sig
Between groups effect	5891.65	3	1963.88		
				7.42	.000
Within groups	74682.22	282	264.83		
Total	80573.87	285			

Source field data (2013) *p< 0.05(2-tailed), **p< 0.01(2-tailed)

Table 14: Levene's Test of homogeneity of variance

Levene's statistic	Sig
.32	.805

p< 0.05(2-tailed), **p< 0.01(2-tailed) Equal variance assumed

This section compares performances on the posttest method by method.

Quadratic Formula and Factorization Compared

Hypothesis 2: There is no significant difference in the mean scores on the posttest of the quadratic formula group and the factorization group.

Table 15 presents the data on this hypothesis. The two schools involved here are Edinaman Senior High and Academy of Christ the King whose mathematical abilities before the teaching experiment were not significantly different (Appendix F). During the teaching experiment Edinaman Senior High learnt to solve quadratic equations by the method of quadratic formula whiles Academy of Christ the King learnt the method of factorization.

Table 15: Independent t-test of the Adjusted Posttest Scores Showing Differences in the use of Quadratic Formula method and Factorization method in Solving Quadratic Equation

Method	n	Mean	Standard dev.	Mean diff.	t	Sig
Quadratic Formula	64	79.72	15.01			
				4.48	1.66	.099
Factorization	71	75.24	16.17			

n= 135, source: field data (2013) *$p< 0.05$(2-tailed), **$p< 0.01$(2-tailed)

Independent t-test was conducted to determine whether significant differences exist between the mean scores of respondents with the use quadratic formula and factorization in solving quadratic equation in Table 15. It was revealed that there were no statistically significant difference between the mean scores of quadratic formula *(M= 79.72, SD= 15.01)* and factorization *(M=75.24, SD= 16.17)* at $p< 0.05$ alpha level in solving quadratic equation and that the null hypothesis is accepted. Even though the mean score of quadratic formula was higher than factorization it can be concluded that respondents understanding of the use of these methods in solving quadratic equations are not different.

Quadratic Formula and ESLE Compared

Hypothesis 3: There is no significant difference in the mean scores on the posttest of the quadratic formula group and the ESLE group.

Table 16 presents the data on this hypothesis. The two schools involved here are Edinaman Senior High and Jukwa Senior High whose mathematical abilities before the teaching experiment were not significantly different (Appendix F).

During the teaching experiment Edinaman Senior High learnt to solve quadratic equations by the method of quadratic formula whiles Jukwa Senior High learnt the method of ESLE.

Table 16: Independent t-test of the Adjusted Posttest Scores Showing Differences in the use of Quadratic Formula method and ESLE Method in Solving Quadratic Equation

Method	n	Mean	Standard dev.	Mean diff.	t	Sig
Quadratic formula	64	79.72	15.01	11.32	4.02	.000**
ESLE	70	68.40	17.34			

n= 134, source: field data (2013) *p< 0.05(2-tailed), **p< 0.01(2-tailed)

Independent t-test was computed to determine whether significant differences exist between the mean scores of respondents with the use quadratic formula and ESLE in solving quadratic equation in Table 16. It was revealed that there were statistically significant difference between the mean scores of quadratic formula *(M= 79.72, SD= 15.01)* and ESLE *(M=68.40, SD= 17.34)* at p< 0.05 alpha level in solving quadratic equation and that the null hypothesis is rejected. The implication is that respondents understood quadratic formula method better in solving quadratic equation than the use of ESLE method.

Quadratic Formula and Conjugales Compared

Hypothesis 4: There is no significant difference in the mean scores on the posttest of the quadratic formula group and the conjugales group.

Table 17 presents the data on this hypothesis. The two schools involved here are Edinaman Senior High and Methodist Senior High whose mathematical abilities before the teaching experiment were not significantly different (Appendix F).

During the teaching experiment Edinaman Senior High learnt to solve quadratic equations by the method of quadratic formula whiles Methodist Senior High learnt the method of conjugales.

Table 17: Independent t-test of the Adjusted Posttest Scores Showing Differences in Quadratic Formula method and Conjugales method in Solving Quadratic Equation

Method	n	Mean	Standard dev.	Mean diff.	t	Sig
Quadratic formula	64	79.72	15.01	0.311	0.12	.906
Conjugales	81	79.41	16.63			

n= 145, source: field data (2013) *p< 0.05(2-tailed), **p< 0.01(2-tailed)

Independent t-test was conducted to determine whether significant differences exist between the mean scores of respondents with the use quadratic formula method and Conjugales method in solving quadratic equation in Table 17. It was found out that there were no significant difference between the mean scores of quadratic formula method *(M= 79.72, SD= 15.01)* and Conjugales method *(M=79.41, SD= 16.63)* at $p< 0.05$ alpha level in solving quadratic equation and that the null hypothesis is accepted. Even though the mean score of quadratic formula method was higher than Conjugales method, it can be concluded that respondents understanding of the use of these methods in solving quadratic equations are not different.

ESLE and Factorization Compared

Hypothesis 5: There is no significant difference in the mean scores on the posttest of the ESLE group and the factorization group.

Table 18 presents the data on this hypothesis. The two schools involved here are Jukwa Senior High and Academy of Christ the King whose mathematical abilities before the teaching experiment were not significantly different (Appendix F). During the teaching experiment Jukwa Senior High learnt to solve quadratic equations by the method of ESLE whiles Academy of Christ the King learnt the method of factorization.

Table18: Independent t-test of the Adjusted Posttest Scores Showing Differences in the use of Factorization method and ESLE method in Solving Quadratic Equation.

Method	n	Mean	Standard dev.	Mean diff.	t	Sig
Factorization	71	75.24	16.17	6.84	2.42	0.017*
ESLE	70	68.40	17.34			

n= 145, source: field data (2013) *p< 0.05(2-tailed), **p< 0.01(2-tailed)

Independent t-test was conducted to determine whether significant differences exist between the mean scores of respondents with the use factorization method and ESLE method in solving quadratic equation in Table 18. It was found out that there were statistically significant difference between the mean scores of factorization method *(M= 75.24, SD= 16.17)* and ESLE method *(M=68.40, SD= 17.34)* at $p< 0.05$ alpha level in solving quadratic equation and that the null hypothesis is rejected. The implication is that respondents understood factorization method better in solving quadratic equation than the use of ESLE method.

ESLE and Conjugales Compared

Hypothesis 6: There is no significant difference in the mean scores on the posttest of the ESLE group and the conjugales group.

Table 19 presents the data on this hypothesis. The two schools involved here are Jukwa Senior High and Methodist Senior High whose mathematical abilities before the teaching experiment were not significantly different (Appendix F). During the teaching experiment Jukwa Senior High learnt to solve quadratic equations by the method of quadratic formula whiles Methodist Senior High learnt the method of factorization.

Table 19: Independent t-test of the Adjusted Posttest Scores Showing Differences in the use of ESLE method and Conjugales method in Solving Quadratic Equation

Method	n	Mean	Standard dev.	Mean diff.	t	Sig
ESLE	70	68.64	17.36			
				-10.77	-3.89	.000**
Conjugales	81	79.41	16.36			

n= 150, source: field data (2013) *p< 0.05(2-tailed), **p< 0.01(2-tailed)

Independent t-test was conducted to determine whether significant differences exist between the mean scores of respondents with the use of ESLE method and Conjugales method in solving quadratic equation in Table 19. It was found out that there were statistically significant difference between the mean scores of ESLE method *(M= 68.64, SD= 17.36)* and Conjugales method *(M=79.41, SD= 16.36)* at p< 0.05 alpha level in solving quadratic equation and that the null hypothesis is rejected. The implication is that respondents understood Conjugales method better in solving quadratic equation than the use of ESLE method.

Conjugales and Factorization Compared

Hypothesis 7: There is no significant difference in the mean scores on the posttest of the conjugales group and the factorization group.

Table 20 presents the data on this hypothesis. The two schools involved here are Edinaman Senior High and Academy of Christ the King whose mathematical abilities before the teaching experiment were not significantly different (Appendix F). During the teaching experiment Edinaman Senior High learnt to solve quadratic equations by the method of quadratic formula whiles Academy of Christ the King learnt the method of factorization.

Table1 20: Independent t-test of the Adjusted Posttest Scores Showing Differences in the use of Factorization Method and Conjugales Method in Solving Quadratic Equation

Method	n	Mean	Standard dev.	Mean diff.	t	Sig
Factorization	71	75.24	16.17			
				-4.17	-1.58	.117
Conjugales	81	79.41	16.36			

n= 152, source: field data (2013) *$p< 0.05$(2-tailed), **$p< 0.01$(2-tailed)

Independent t-test was conducted to determine whether significant differences exist between the mean scores of respondents with the use of factorization method and Conjugales method in solving quadratic equation in Table 20. It was found out that there were statistically no significant difference between the mean scores of factorization method *(M= 75.24, SD= 16.17)* and Conjugales method *(M=79.41, SD= 16.36)* at $p< 0.05$ alpha level in solving quadratic equation and that the null hypothesis is accepted. Even though the mean score of Conjugales method was higher than factorization method, it can be concluded that respondents understanding of the use of these methods in solving quadratic equations are not different.

All Four Methods Compared

Table 21: Comparison of four Methods of Solving Quadratic Equations

Method (I)	Method (J)	Mean Difference (I-J)	Std. Error	Sig.
Quadratic Formula	Factorization	4.48	2.81	.111
ESLE Factorization		6.84*	2.74	.013
Conjugales	Factorization	4.17	2.65	.116
Quadratic Formula	ESLE	11.32*	2.81	.000
Quadratic Formula	Conjugales	.31	2.72	0.909
Conjugales	ESLE	11.01*	2.66	.000

Source field data (2013)*p< 0.05(2-tailed), **p< 0.01(2-tailed)* significant

I (treatment with greater mean) J (treatment with lesser mean)

Table22: Points gained by treatments in the pairwise comparisons

Treatment groups	ELSE	Quadratic formula	Conjugales	Factorization	Total
Quadratic formula	4		2	2	8
Factorization	3	2	2		7
ESLE		0	0	1	1
Conjugales	4	2		2	8

Analysis of covariance (ANCOVA) was computed to test whether statistically significant difference exit among the mean scores of treatment groups with respect to the use of the four methods in solving quadratic equation. Levene's test of the homogeneity of variance was also computed to determine the appropriate post hoc multiple comparison to be used to determine where significant differences actually existed among the four methods in view of the fact that the F–test showed significant difference. The result showed that the variances that existed among the means of the four methods were statistically not significant at p< .05 alpha level. This implies that equal variance assumed among the four methods. Since equal variance was assumed, Least Significant Difference (LSD) was chosen as the appropriate post hoc multiple comparison technique for the comparison of the mean differences among the four methods. It was found out that quadratic formula method (M= 79.72, SD= 15.01) was significantly different from ESLE method (M= 68.40, SD= 17.34) however, it was not significantly different from factorization method (M= 75.24, SD= 16.17) and conjugales method (M= 79.41, SD= 16.36). Factorization method (M= 75.24, SD= 16.17) was also significantly different from ESLE method (M= 68.40, SD= 17.34) but was not significantly different from quadratic formula (M= 79.72, SD= 15.01) and conjugale (M= 79.41, SD= 16.36). Conjugales method (M= 79.41, SD= 16.36) was significantly different from ESLE method (M= 68.40, SD= 17.34). Also by awarding four points to a method when it is significantly higher at $p < 0.01$ and zero point when it is significantly lower. Three points to a method when it is significantly higher at $p < 0.05$ and one point when it is significantly lower. Two points has been awarded to each method when there is no significantly difference between them Table 22 displays the results.

From table 22 the quadratic formula and conjugales had eight points each. There are totals of eight, seven and one points respectively for quadratic formula and conjugales, factorization and ESLE. These results show that the students can learn best to solve quadratic equations by quadratic formula and conjugales, factorization and ESLE in that order.

Chapter Summary of Discussion and Interpretation of the Findings

The data have been statistically analyzed with t-tests and ANCOVA at significance levels of 0.05. When the pretest scores were compared with the posttest scores, there was significant difference between the mean scores and it was in favour of the posttest. The t-test for dependent (paired) samples was also used to analyze the pretest scores and the posttest scores in each class, seven out of the eight samples showed significantly differences between the mean scores. All the seven samples showed statistically higher performances on the posttest on quadratic equations than pretest. The other class, 1Home Economics in Jukwa, it was found out that there was statistically no significant difference between the mean scores of ESLE method (M=66.10, SD=17.80) and pre-test (M=65.83, SD=16.53) at $p<0.05$ alpha level and that the null hypothesis was accepted. Even though the mean score of posttest (ESLE method) was higher than the mean score of pretest, the respondent understanding of the use of this method in solving quadratic equations and the pretest were not different. It has therefore been concluded that it is possible for the entire sample to learn solution of quadratic equations as effectively as the other mathematics topics in the second term scheme of work.

Anomah (1998), Moore (1999), Essah (1999) and Sackey and Annor–Adjei (2003) have concluded and confirmed each other's finding that quadratic equations can be taught effectively in the first year of the Senior High School by the method of ESLE. This is confirmed and affirmed by the current study. But the current study also reveals that quadratic equations can be taught to the first year students by the methods of factorization, conjugales, ESLE and quadratic formula. In fact, the current study shows that students can best learn to solve quadratic equations by the quadratic formula. In the studies of Agyapong (2000) and Bornaa (2007) to compare the conjugales with factorization method on measures of retention, they concluded in favour of conjugales. The results of this study seem to confirm this assertion.

The students' performance by ESLE is significantly lower than that of the quadratic formula and that of conjugales is higher than the factorization method but in all the quadratic formula or conjugales seems to be the best in this study.

There were significantly no differences between the mean scores on the pretest in all the schools which gave room to compare method by method and the following findings were made;

1 It is possible to teach quadratic equations to Form One students in the Senior High School with the same effectiveness and efficiency as the topics in the second term scheme of work.

2 Students can learn best to solve quadratic equations using the quadratic formula and conjugales.

3 Conjugales appears to be easier than factorization as a method for solving quadratic equations.

4 Factorization appears to be easier than ESLE as a method for solving quadratic equations.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

This chapter gives an overview of the research problem and methodology, a summary of the findings of the thesis. It draws conclusions based on the findings. The chapter concludes with some recommendations for mathematics educators, curriculum developers and suggestions for future research.

Summary

Quadratic equations have wide and varied application in both mathematics and other related disciplines. Therefore, it is very important for further study in mathematics and must be taught well (Movshovitz-Hadar, 1993: Willoughby: 1967). In the Senior High School, the topic is taught in the first year in Elective Mathematics and in the second year in core mathematics. However, the ability to solve quadratic equations is pre-requisite knowledge for learning uniformly accelerated motion as a topic in physics in the first year (CRDD, 2010). The placement of the topic in the curriculum is therefore the problem. This study sought to ascertain whether quadratic equations could be taught in Senior High School Form One, and to investigate the students in using four methods, namely Quadratic Formula, Factorization, ESLE and Conjugale for solving quadratic equations. The research was a teaching experiment (Skemp, 1987) using pretest–posttest non–equivalent comparison group design. It was designed to assess the potentials of Form One students in the Senior High School to learn to solve quadratic equations. The four methods for solving quadratic equations identified above were taught to 286 students from eight classes in four Senior High Schools. The schools were selected after a stratified random sampling. However these classes were randomly assigned to the treatment (method of factorization, ESLE, quadratic formula and conjugales).

The pretest constituted twenty (20) multiple choice items. The posttest was made up of five (5) essay type items. The reliability coefficient for the pretest and the posttest were 0.86 and 0.80 respectively. Before the instructions commenced a pretest was administered. Then the posttest came after the teaching instructions. The pretest was scored dichotomously and the posttest was scored using a marking scheme which was drawn for the test.

Data obtained on the pretest and the posttest were analyzed with t-test and ANCOVA at $\alpha = 0.05$. Though a quantitative research approach was used in the study, the findings cannot be used to be representative of other situations in other localities in the country. This calls for replication of the study in other parts of the country by including urban and rural areas to make inter–regional comparisons in order to provide a basis for general conclusion.

Major Findings

The following were the major findings of the study:

1. It is possible to teach quadratic equations to Form One students in the Senior High Schools with the same effectiveness and efficiency as the other topics in the Form One Core Mathematics Syllabus.
2. The students can best learn to solve quadratic equations using the quadratic formula.
3. ESLE is the most difficult method for the students.

4. Factorization appears to be easier than ESLE as a method for solving quadratic equations.
5. There was also significant difference on the mean scores of pretest (topics in the second term scheme of work) and posttest (various methods used in solving quadratic equations).
6. There were significant differences existed among the methods except quadratic formula and factorization method, quadratic formula and conjugales.
7. These occurred between the following pairs of treatment groups:
   i) The quadratic formula and ESLE groups in favour of the quadratic formula group.
   ii) The factorization and ESLE groups in favour of the factorization group.
   iii) The conjugales and ESLE groups in favour of the conjugales group.

Conclusion

That the current study shows that quadratic equations can be taught to first year students in the Senior High School is proof that those students possess the necessary schemas for learning the topic. The kind of questions that were used for the researcher's posttest and the process required to solve them were not such as could be memorized as a rule. None of the four methods of solving quadratic equations used in the study lends itself to memorization without proper understanding. What is required therefore is for teachers to build up the requisite knowledge for solving quadratic equations, piece–by–piece, out of the schemas currently available to the students (Skemp, 1987).

The syllabus of WASSCE placed emphasis on the use of factorization, method of completing the square and the quadratic formula. Among these methods, the method of factorization is widely used in solving quadratic equation (Cornelius and Gott, 1988). The Senior High School Core Mathematics Teaching Syllabus also prescribed only the method of factorization for solving quadratic equations. By the results of this study the "*novel methods*" (Conjugales and ESLE) have proved to be equally effective as the conventional methods and therefore can run side–by–side with the factorization and quadratic formula.

Previous research and the theoretical framework are more favourably disposed to the quadratic formula and ESLE. Factorization and completing the square are less favoured. But on the basis of the current study, it seems more likely that first year students in the Senior High Schools would cope with solving quadratic equations by the quadratic formula better than the ESLE. Also conjugales method was more favoured than the ESLE. It is therefore concluded that the most feasible method for solving quadratic equations among Form One students in the Senior High Schools is the quadratic formula.

Recommendations

The following suggestions are made to the Ghana Education Service and other stakeholders for the application of the consideration.

1. The present Core Mathematics Syllabus should be revised so that quadratic equations would be taught in Form One of the Senior High Schools.
2. The quadratic formula should be taught to Form One students for solving quadratic equations.
3. The quadratic formula should be included in the Form One Mathematics textbooks at the next revision.
4. The Mathematics Association of Ghana (MAG), GES, NGOs and other interested parties should include workshops or in–service training in their

programmes to expose the Conjugales and ESLE methods of solving Quadratic Equations to teachers.

5. Both examining and professional bodies that set the final Senior High School assessment questions should be exposed and encouraged to use Conjugales and ESLE in solving Quadratic Equations.
6. The introduction of the Conjugales and ESLE in the curriculum of the teacher training institutions and universities would also not be out of place since the newly trained teachers would eventually replace those already serving.

Suggestions for Future Research

This study is a pointer to the urgent need for GES, MOE, NGOs and Educational researchers to conduct a nationwide study, similar to it, to assess the feasibility of teaching quadratic equations in Senior High School Form One with retention test and to give a broader picture of the findings for generalization.

The pedagogy of teaching “*novel methods*” (Conjugales and ESLE) is a virgin and ripe area for educational research to delve in at the College of Education and University levels.

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APPENDICES

APPENDIX A

PRETEST QUESTIONS **TIME: 50 Minutes**

SCHOOL: ____

DATE:________________

IDENTIFICATION NUMBER: ______________________________

AGE:_________ SEX: ________

Note: Circle only the correct answer

1. If b =5 a = 2 and c = 3, find the value of $\sqrt{b^2 - 4ac}$

   A. 2 B. 5 C. 1 D. 6

   E. 7

2. Simplify $\dfrac{3\frac{1}{4} - 1\frac{3}{5}}{4\frac{2}{5}}$

   A. 0.125 B. 0.375 C. 0.5 D. 0.625

   E. 0.875

3. Find the common factor of $x(y+z)$ and $-k(y+z)$

   A. $x - k$ B. y C. x D. $y + z$

   E. $z - y$

4. Solve the equation $x + 3 = -10$.

   A. 7 B. –30 C. $^{-10}/_{3}$ D. –13

   E. 3

5. Evaluate $\sqrt{90^2 - 4 \times 18 \times 52}$

   A. 54 B. 56 C. 66 D. 64 E. 90

6. Evaluate $86.5^2 - 13.5^2$.

   A. 5,329 B. 7,300 C. 7,482.25 D. 7,664.5 E. 10,000

7. Find the values of x and y in the following pair of questions

$$x + 5y = 6$$

$$4x + 5y = 9$$

A. 1,1 B. 2,2 C. 3,3 D. 4,4

E. 5,5

8. Multiply ($2a - b$) by ($2a + b$)

A. $4a^2 - 4ab - b^2$ B. $4a^2 + ab + b^2$ C. $4a^2 - ab - b^2$

D. $4a^2 + b^2$ E. $4a^2 - b^2$

9. Find the set of value(s) of x for which the expression $\frac{2+x}{x-3}$is not defined.

A. {–3} B. {–2} C. {–3, 2} D. {–2, 3}

E. {3}

10. Given that $y = \frac{2x-5}{3x+2}$, find y when $x = -4$.

A. 0.2 B. 0.3 C. 0.5 D. 1.3 E. 2.4

11. Make a the subject in the relation $v = u + at$

A. $\frac{v+u}{t}$ B. $\frac{v-u}{t}$ C. $\frac{u-v}{t}$ D. $\frac{vu}{t}$

E. $\frac{v}{t} - u$

12. Simplify $\frac{a-b}{2} + \frac{2a}{3}$.

A. $a - b$ B. $\frac{a-3b}{6}$ C. $\frac{7a-3b}{6}$ D. $7a + 3b$

E. $\frac{7a+3b}{6}$

13. Express $7\frac{3}{35}$ to 3 decimal places.

A. 0.7003 B. 7.003 C. 7.03 D. 7.12

E. 7.086

14. Find the truth set of the equation $\frac{x}{3} - \frac{x}{5} = 4$.

A. 2 B. 30 C. 15 D. . $7\frac{1}{2}$

E. $5\frac{1}{2}$

15. If 3*y*:32 = 9:16, find the value of *y*.

A. 6 B. $^{17}/_{15}$ C. 3 D. $2\,^{1}/_{4}$

E. $^{3}/_{2}$

16. Find the value of $a^2 - 3ab + b^3$ when $a = 2$ and $b = -3$.

A. 49 B. 13 C. –5 D. –19E. 50

17. If p and q are the pair of factors of 96 and $p - q = 4$, find the values of p and q

A. $-24, 4$ B. $24, 4$C. $12, -8$ D. $-12, 8$

E. 12, 8

18. Solve the equation: $\frac{x-2}{3} + \frac{3}{2} = \frac{x}{2}$

A. 5 B. 4 C. 2 D. –1 E. –2

19. Factorize $6am + 15am + 2bm + 5bn$

A. $(2m - 5n)(3a + b)$ B. $(2m + 5n)(3a + b)$

C.$(3m - 5n)(3a - b)$ D. $(2m - 5n)(3a - b)$

E. $(3a - b)(3m + 5m)$

20. Find the values of x and y if $x - y = 2$ and $2x + y = 4$

A. $x = 4, y = 2$ B. $x = 2, y = 0$ C. $x = 0, y = 2$

D. $x = 2, y = 4$ E. $x = 2, y = 2$

APPENDIX B

POSTTEST QUESTIONS

Time: 50minuttes

SCHOOL: ____

DATE:________________

IDENTIFICATION NUMBER: _______________________________

AGE:__________ SEX: ________

Answer all questions on this sheet

Show all necessary working

1. Solve for x in the equation: $x^2 - 2x - 35 = 0$.

2. Find the solution set of the equation: $8 - 2x - 3x^2 = 0$

3. Solve for the value(s) of $\boldsymbol{x}$ in the equation: $2x^2 + 3x - 20 = 0$

4. Find the solution set of the equation: $18x^2 + 9x - 2 = 0$

5. If $4x^2 + 7x + 3 = 0$, find the value(s) of x.

APPENDIX C

ANSWERS TO PRETEST QUESTIONS

1. D
2. B
3. D
4. D
5. C
6. B
7. A
8. E
9. E
10. D
11. B
12. C
13. C
14. D
15. A
16. C
17. E
18. A
19. B
20. B

APPENDIX D

MARKING SCHEME 1

POST TEST **CONJUGALES**

QUESTION DETAILS	MARK
1. Comparing $x^2 – 2x – 35 = 0$ with $ax^2 + bx + c = 0$,	
$a = 1, b = –2$ and $c = –35$	M1 (for a, b and c all correct)
$2ax + b = \pm d$ where $d = \sqrt{(b^2 - 4ac)}$	
$d = \sqrt{((-2)^2 - 4 \times 1 \times -35)}$	M1 for substitution
$d = \sqrt{(4 + 140)} = \sqrt{144}$	M1 for simplification
$d = \pm 12$	A1 for d
$2(1)x – 2 = –12$	M1 for conjugales 1
$2x – 2 = –12$	
$x = \frac{-12+2}{2}$	M1 for simplification
$x = –5$	
$2x – 2 = 12$	M1 for conjugales 2
$x = \frac{12+2}{2}$	M1 for simplification
$x = 7$	
Hence the values of x are –5 and 7	[A1 for –5, A1 for 7]
2. Comparing $8 – 2x – 3x^2 = 0$ with $ax^2 + bx + c = 0$,	
$a = –3, b = –2$ and $c = 8$	M1 (for a, b and c all correct)
$2ax + b = \pm d$ where $d = \sqrt{(b^2 - 4ac)}$	
$d = \sqrt{((-2)^2 - 4 \times -3 \times 8)}$	M1 for substitution
$d = \sqrt{(4 + 96)} = \sqrt{100}$	M1 for simplification
$d = \pm 10$	A1 for d
$2(–3)x – 2 = –10$	M1 for conjugale 1
$–6x – 2 = –10$	
$x = \frac{-10+2}{-6}$	M1 for simplification
$x = {}^{-8}/_{-6} = {}^{4}/_{3}$	M1 for conjugale 2

	$-6x - 2 = 10$	M1 for simplification
	$x = \frac{10+2}{-6}$	
	$x = -2$	[A1 for $^4/_3$, A1 for –2]
	Hence T.S = $\{x : x = -2, \frac{4}{3}\}$	
		M1 (for a, b and c all correct)
3.	Comparing $2x^2 + 3x - 20 = 0$ with $ax^2 + bx + c = 0$,	
	$a = 2$, $b = 3$ and $c = -20$	
	$2ax + b = \pm d$ where $d = \sqrt{(b^2 - 4ac)}$	M1 for substitution
	$d = \sqrt{(3^2 - 4(2)(-20))}$	M1 for simplification
	$d = \sqrt{9 + 160} = \sqrt{169}$	A1 for d
	$= \pm 13$	M1 for conjugales 1
	$2(2)x + 3 = -13$	
	$4x + 3 = -13$	M1 for simplification
	$x = \frac{-13-3}{4}$	
	$x = -4$	M1 for conjugales 2
	$4x + 3 = 13$	M1 for simplification
	$x = \frac{13-3}{4}$	
	$x = ^{10}/_{5} = ^{5}/_{2}$	[A1 for –4, A1 for $\frac{4}{2}$]
	Hence T.S = $\{x : x = -4, ^5/_2\}$	
4.	Comparing $18x^2 + 9x - 2 = 0$ with $ax^2 + bx + c = 0$,	M1 (for a, b and c all correct)
	$a = 18$, $b = 9$ and $c = -2$	
	$2ax + b = \pm d$ where $d = \sqrt{(b^2 - 4ac)}$	M1 for substitution
	$d = \sqrt{9^2 - 4(18)(-2)}$	M1 for simplification
	$= \sqrt{81 + 144} = \sqrt{225}$	A1 for d
	$= \pm 15$	M1 for conjugales 1
	$2(18)x + 9 = -15$	
	$36x + 9 = -15$	M1 for simplification
	$x = \frac{-15-9}{36}$	M1 for conjugales 2
	$x = ^2/_3$	M1 for simplification

	$36x + 9 = 15$	[A1 for $-{}^{2}/_{3}$, A1 for ${}^{1}/_{6}$]
	$x = \frac{15-9}{36} = \frac{6}{36}$	
	$x = {}^{1}/_{6}$	
	Hence T.S = $\{x : x = {}^{1}/_{6}, {}^{-2}/_{3}\}$	M1 (for *a*, *b* and *c* all correct)
5.	Comparing $4x^2 + 7x + 3 = 0$ with $ax^2 + bx + c = 0,$	M1 for substitution
	$a = 4, b = 7$ and $c = 3$	M1 for simplification
	$2ax + b = \pm d$ where $d = \sqrt{(b^2 - 4ac)}$	A1 for d
	$d = \sqrt{7^2 - 4(4)(3)}$	M1 for conjugales 1
	$d = \sqrt{49 - 48} = \sqrt{1}$	
	$d = \pm 1$	M1 for simplification
	$2(4)x + 7 = -1$	
	$8x + 7 = -1$	M1 for conjugales 2
	$x = \frac{-1-7}{8}$	M1 for simplification
	$x = -1$	
	$8x + 7 = 1$	
	$x = \frac{1-7}{8}$	[A1 for ${}^{-3}/_{4}$, A1 for –1]
	$a = {}^{-6}/_{8} = {}^{-3}/_{4}$	
	Hence T.S = $\{x : x = {}^{-3}/_{4}$, A1 for –1$\}$	

APPENDIX D (Continued)

MARKING SCHEME 2

POSTTEST **QUADRATIC FORMULA**

QUESTION DETAILS	MARK
1. Comparing $x^2 – 2x – 35 = 0$ with $ax^2 + bx + c = 0$,	
$a = 1, b = –2$ and $c = –35$	M1 (for a, b and c all correct)
$x = \frac{-b \pm\sqrt{b^2 - 4ac}}{2a}$	M1 for correct formula
$x = \frac{--2 \pm\sqrt{(-2)^2 - 4(1)(-35)}}{2(1)}$	M1 for substitution
$x = \frac{2 \pm\sqrt{4+140}}{2}$	M1 for simplification
$x = \frac{2 \pm\sqrt{144}}{2}$	M1 for simplification
$x = \frac{2 \pm 12}{2}$	
$x = \frac{2-12}{2}$	M1 for simplifying radical
$x = –5$	A1 for $x = –5$
$x = \frac{2+12}{2}$	A1 for $x = 7$
$x = 7$	
Hence T.S = $\{x{:}x = –5, 7\}$	[A1 for –5, A1 for 7]
2. Comparing $8 – 2x – 3x^2 = 0$ with $ax^2 + bx + c = 0$,	
$a = –3, b = –2$ and $c = 8$	M1 (for a, b and c all correct)
$x = \frac{-b \pm\sqrt{b^2 - 4ac}}{2a}$	M1 for correct formula
$x = \frac{--2 \pm\sqrt{(-2)^2 - 4(-3)(8)}}{2(-3)}$	M1 for substitution
$x = \frac{2 \pm\sqrt{4+96}}{-6}$	M1 for simplification
$x = \frac{2\pm\sqrt{100}}{2}$	M1 for simplification
$x = \frac{2\pm10}{-6}$	M1 for simplifying radical
$x = \frac{2+10}{-6}$	
$x = –2$	A1 for $x = –2$

$x = \frac{2-10}{-6}$

$x = {}^{-8}/_{-6}$

$x = {}^{4}/_{3}$ — A1 for $x = {}^{4}/_{3}$

Hence T.S = $\{x : x = -2, {}^{4}/_{3}\}$ — [A1 for –2, A1 for ${}^{4}/_{3}$]

3. Comparing $2x^2 + 3x - 20 = 0$ with $ax^2 + bx + c = 0,$

$a = 2, b = 3$ and $c = -20$	M1 (for a, b and c all correct)
$x = \frac{-b \pm\sqrt{b^2 - 4ac}}{2a}$	M1 for correct formula
$x = \frac{-3 \pm\sqrt{3^2 - 4(2)(-20)}}{2(2)}$	
	M1 for substitution
$x = \frac{-3 \pm\sqrt{9 + 160}}{4}$	
$x = \frac{-3 \pm\sqrt{169}}{4}$	M1 for simplification
$x = \frac{-3 \pm 13}{4}$	M1 for simplification
$x = \frac{-3 + 13}{4}$	M1 for simplifying radical
$x = {}^{10}/_{4} = {}^{5}/_{2}$	A1 for $x = {}^{5}/_{2}$
$x = \frac{-3-13}{4}$	
$x = -4$	A1 for $x = -4$
Hence T.S = $\{t : t = {}^{5}/_{2}, -4\}$	[A1 for ${}^{5}/_{2}$, A1 for –4]

4. Comparing $18x^2 + 9x - 2 = 0$ with $ax^2 + bx + c = 0,$

$a = 18, b = 9$ *and* $c = -2$	M1 (for a, b and c all correct)
$x = \frac{-b \pm\sqrt{b^2 - 4ac}}{2a}$	M1 for correct formula
$x = \frac{-9 \pm\sqrt{9^2 - 4(18)(-2)}}{2(18)}$	
	M1 for substitution
$x = \frac{-9 \pm\sqrt{81 + 144}}{36}$	
$x = \frac{-9 \pm\sqrt{225}}{36}$	M1 for simplification
$x = \frac{-9 \pm 15}{36}$	M1 for simplification
$x = \frac{-9+15}{36}$	M1 for simplifying radical

$x = \frac{6}{36}$

$x = {}^{1}/_{6}$ — A1 for $x = {}^{1}/_{6}$

$x = \frac{-9-15}{36}$

$x = -{}^{2}/_{3}$ — A1 for $x = {}^{-2}/_{3}$

Hence T.S = $\{r : r = {}^{1}/_{6}, {}^{-2}/_{3}\}$ — [A1 for ${}^{1}/_{6}$, A1 for ${}^{-2}/_{3}$]

5. Comparing $4x^2 + 7x + 3 = 0$ with $ax^2 + bx + c = 0,$

$a = 4, b = 7$ and $c = 3$ — M1 (for a, b and c all correct)

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — M1 for correct formula

$x = \frac{-7 \pm \sqrt{7^2 - 4(4)(3)}}{2(4)}$ — M1 for substitution

$x = \frac{-7 \pm \sqrt{49-48}}{8}$ — M1 for simplification

$x = \frac{-7 \pm \sqrt{1}}{8}$ — M1 for simplification

$x = \frac{-7 \pm 1}{8}$ — M1 for simplifying radical

$x = \frac{-7-1}{8}$

$x = -1$ — A1 for $x = -1$

$x = \frac{-7+1}{8}$

$x = {}^{-6}/_{8} = {}^{-3}/_{4}$ — A1 for $x = {}^{-6}/_{8}$

Hence T.S = $\{x : x = -1$, A1 for ${}^{-3}/_{4}\}$ — [A1 for -1, A1 for ${}^{-6}/_{8}$]

APPENDIX D (Continued)

MARKING SCHEME 3

POSTTEST **FACTORISATION**

QUESTION DETAILS	MARK
1. $x^2 - 2x - 35 = 0$	
$x^2 - 7x + 5x - 35 = 0$	M1 either $7x$ or $-5x$ correct
$(x^2 - 7x) + (5x - 35) = 0$	M1 for grouping
$x(x - 7) + 5(x - 7) = 0$	M1 for 5, M1 for -7
$(x + 5)(x - 7)$	A1 for $x + 5$, A1 for $x - 7$
$x + 5 = 0$ *and* $x - 7 = 0$	M1 for $x + 5 = 0$, M1 for $x - 7 = 0$
Hence T.S = $\{x : x = -5, 7\}$	[A1 for –5, A1 for 7]
2. $8 - 2x - 3x^2 = 0$	
$8 - 6x + 4x - 3x^2 = 0$	
$(8 - 6x) + (4x - 3x^2) = 0$	M1 either $-6x$ or $4x$ correct
$2(4 - 3x) + x(4 - 3x) = 0$	M1 for grouping
$(2 + x)(4 - 3x) = 0$	M1 for 2, M1 for x
$2 + x = 0$ and $4 - 3x = 0$	A1 for $2 + x$, A1 for $4 - 3x$
Hence T.S = $\{x : x = -2, {}^4/_3\}$	M1 for $2 + x = 0$, M1 for $4 - 3x = 0$
	[A1 for –2, A1 for $^4/_3$]
3. $2x^2 + 3x - 20 = 0$	
$2x^2 + 8x - 5x - 20 = 0$	
$(2x^2 + 8x) - (5x + 20) = 0$	
$2x(x + 4) - 5(x + 4) = 0$	M1 either $8x$ or $-5x$ correct
$(2x - 5)(x + 4) = 0$	M1 for grouping
$2x - 5 = 0$ and $x + 4 = 0$	M1 for $2x$, M1 for -5
Hence T.S = $\{x : x = {}^5/_2, -4\}$	A1 for $2x - 5$, A1 for $x + 4$
	M1 for $2x - 5 = 0$, M1 for $x + 4 = 0$
4. $18x^2 + 9x + 2 = 0$	[A1 for $^5/_2$, A1 for –4]
$18x^2 + 12x - 3x + 2 = 0$	
$(18x^2 + 12x) - (3x + 2) = 0$	
$6x(3x + 2) - (3x + 2) = 0$	

APPENDIX D (Continued)

	$(6x - 1)(3x + 2) = 0$	M1 either $12x$ or $-3x$ correct
	$6x - 1 = 0$ and $3x + 2 = 0$	M1 for grouping
	Hence T.S = $\{x : x = {}^{1}/_{6}, {}^{-2}/_{3}\}$	M1 for $6x$, M1 for -1
		A1 for $6x - 1$, A1 for $3x + 2$
5.	$4x^2 + 7x + 3 = 0$	M1 for $6x - 1 = 0$, M1 for $3x + 2 = 0$
	$4x^2 + 4x + 3x + 3 = 0$	
	$(4x^2 + 4x) + (3x + 3) = 0$	[A1 for ${}^{1}/_{6}$, A1 for ${}^{-2}/_{3}$]
	$4x(x + 1) + 3(x + 1) = 0$	
	$4x + 3 = 0$ and $x + 1 = 0$	
	$x = {}^{-3}/_{4}$ and $x = -1$	M1 either $4x$ or $3x$ correct
	Hence T.S = $\{x : x = {}^{-3}/_{4}, -1\}$	M1 for grouping
		M1 for $4x$, M1 for 3
		A1 for $4x + 3$, A1 for $x + 1$
		M1 for $4x + 3 = 0$, M1 for $x + 1 = 0$
		[A1 for ${}^{-3}/_{4}$, A1 for -1]

APPENDIX D (Continued)

MARKING SCHEME 4

POSTTEST **ESLE**

QUESTION DETAILS	MARK
1. Comparing $x^2 - 2x - 35 = 0$ with $ax^2 + bx + c = 0$	
$a = 1, b = -2$ and $c = -352$	M1 (for a, b, and c all correct)
$ax_1 + ax_2 = -b$	
$ax_1 - ax_2 = d$	
$d = \sqrt{b^2 - 4ac}$	
$d = \sqrt{(-2)^2 - 4(1)(-35)}$	
$d = \sqrt{4 + 140} = \sqrt{144}$	M1 for substitution
$d = \pm 12$	M1 for simplification
$x_1 + x_2 = - -2$	A1 for d
$x_1 - x_2 = 12$	M1 for *ESLE* 1
$2x_1 = 14$ and $x_1 = 7$	M1 for *ESLE* 1
$7 + x_2 = 2$ and $x_2 = -5$	M1 for addition
Hence the values of x are -5 and 7	M1 for substitution
	[A1 for–5, A1 for 7]
2. Comparing $8 - 2x - 3x^2 = 0$ with $ax^2 + bx + c = 0$	
$a = -3, b = -2$ and $c = 8$	M1 (for a, b, and c all correct)
$ax_1 + ax_2 = -b$	
$ax_1 - ax_2 = d$	
$d = \sqrt{b^2 - 4ac}$	

$d = \sqrt{(-2)^2 - 4(-3)(8)}$	M1 for substitution
$d = \sqrt{4 + 96} = \sqrt{100}$	M1 for simplification
$d = \pm 10$	A1 for d
$-3x_1 - 3x_2 = --2$	M1 for *ESLE* 1
$-3x_1 + 3x_2 = 10$	M1 for *ESLE* 1
$-6x_1 = 12$, and $x_1 = -2$	M1 for addition
$6 - 3x_2 = 2$ and $x_2 = {}^4\!/_3$	M1 for substitution
Hence the values of x are 2 and ${}^4\!/_3$	[A1 for 2, A1 for ${}^4\!/_3$]
3. *Comparing* $2x^2 + 3x - 20 = 0$ with $ax^2 + bx + c = 0$	
$a = 2, b = 3$ and $c = -20$	M1 (for a, b and c all correct)
$ax_1 + ax_2 = -b$	
$at_1 - at_2 = d$	
$d = \sqrt{(b^2 - 4ac)}$	
$d = \sqrt{3^2 - 4(2)(-20)}$	M1 for substitution
$d = \sqrt{9 + 160} = \sqrt{169}$	M1 for simplification
$d = \pm 13$	
$2x_1 + 2x_2 = -3$	A1 for d
$2x_1 - 2x_2 = 13$	M1 for *ESLE* 1
$4x_1 = 10$ and $x_1 = {}^5\!/_2$	M1 for *ESLE* 1
$2\left({}^5\!/_2\right) + 2x_2 = -3$	M1 for addition
$5 + 2x_2 = -3$ and $x_2 = -4$	M1 for substitution
Hence the values of x are ${}^5\!/_2$ and -4	[A1 for 2, A1 for ${}^4\!/_3$]

4. Comparing $18x^2 + 9x – 2 = 0$ with $ax^2 + bx + c = 0$	
$a = 18$, $b = 9$ and $c = –2$	M1 (for a, b, and c all correct)
$ax_1 + ax_2 = –b$	
$ax_1 – ax_2 = d$	
$d = \sqrt{(b^2 - 4ac)}$	
$d = \sqrt{9^2 - 4(18)(-2)}$	M1 for substitution
$d = \sqrt{81 - 144} = \sqrt{225}$	M1 for simplification
$d = \pm 15$	A1 for d
$18x_1 + 18x_2 = –9$	M1 for *ESLE* 1
$18x_1 – 18x_2 = 15$	M1 for *ESLE* 2
$36x_1 = 6$ and $x_1 = {}^{1}/_{6}$	M1 for addition
$3 + 18x_2 = –9$ and $x_2 = {}^{-2}/_{3}$	M1 for substitution
Hence the values of x are ${}^{1}/_{6}$ and ${}^{-2}/_{3}$	[A1 for 20, A1 for –5]
5. Comparing $4x^2 + 7x + 3 = 0$ with $ax^2 + bx + c = 0$	
$a = 4$, $b = 7$ and $c = 3$	M1 (for a, b and c all correct)
$ax_1 + ax_2 = –b$	
$ax_1 – ax_2 = d$	
$d = \sqrt{(b^2 - 4ac)}$	
$d = \sqrt{7^2 - 4(4)(3)}$	M1 for substitution
$d = \sqrt{49 - 48} = \sqrt{1}$	M1 for simplification
$d = \pm 1$	A1 for d
$4x_1 + 4x_2 = –7$	M1 for *ESLE* 1

APPENDIX D (Continued)

$4x_1 - 4x_2 = 1$	M1 for *ESLE* 2
$8x_1 = -6$ and $x_1 = {}^{-3}/_{4}$	M1 for addition
$-3 + 4x_2 = -7$ and $x_2 = -1$	M1 for substitution
Hence the values of x are ${}^{-3}/_{4}$ and -1}	[A1 for ${}^{-3}/_{4}$, A1 for -1]

APPENDIX E

Determination of the Test Reliability Coefficient of Posttest

$$r = \frac{n({\sigma_t}^2 - \Sigma{\sigma_i}^2)}{(n-1)({\sigma_t}^2)}$$

$n = 5$

${\sigma_1}^2 = 8.34551077$

${\sigma_2}^2 = 6.00921660$

${\sigma_3}^2 = 5.98100433$

${\sigma_4}^2 = 5.59506519$

${\sigma_5}^2 = 6.21355981$

$\Sigma{\sigma_i}^2 = 32.14435677$

${\sigma_t}^2 = 88.979098641$

$\therefore r = \frac{5(88.979098641 - 32.14435677)}{(5-1)(88.979098641)}$

$r = \frac{4(56.3474187)}{4(88.979098641)}$

$r = 0.7984282649$

$r \approx 0.80$

APPENDIX E (Continued)

Determination of the Test Reliability Coefficient of Pretest

$$KR20 = \left[\frac{n}{n-1}\right]\left[1 - \frac{\sum SD_i^2}{SD_x^2}\right]$$

$n = 20$

$\sum SD_i^2 = 13.16713322$

$SD_x^2 = 71.77720355$

$\therefore KR\ 20 = \left[\frac{20}{20-1}\right]\left[1 - \frac{13.16713322}{71.77720355}\right]$

$KR20 = \left[\frac{20}{19}\right][1 - 0.1834445]$

$KR20 = (1.052631579)(0.8165555)$

$KR20 = 0.8595192127$

KR20 ≈ 0.86

APPENDIX F

LESSON NOTES BY CONJUGALES METHOD

DAY/DATE:

SCHOOL: Methodist High School

PROPOSED TIME: (80 minutes)

TOPIC: Solution of quadratic equations by conjugales.

REFERENCE: Abbiw, M.K et al, 2002, Mathematics 300.2, Accra; Ministry of Education,

Asiedu, P. (2005), Aki Ola Series Core Mathematics, Accra; Aki Ola Publication

Ghana S.S.S Mathematics Book 2 and Book 3 (Amissah S.E. et al, 1991)

INSTRUCTIONAL OBJECTIVES: Students should be able to;

1. Identify a quadratic equation.
2. Explain the general form of a quadratic equation.
3. Define a quadratic equation.
4. Solve quadratic equations by conjugales.

RELEVANT PROVISIONS KNOWLEDGE ASSUMED

1. Students can identify the constants, variables and coefficients of the variable of an equation.
2. Students can substitute values into a formula and simplify.
3. Students can find the positive square root of numbers.
4. Students can solve linear equations.

Teaching and Learning Materials

Various posters of equations and step by step instructions for solving quadratic equations.

Subject Matter/Time	Teachers Activity	Students' Activity	Main Ideas
Introduction	Teacher writes the topic on the board. Paste on the board a card with the following equations: 1. $ax^2 + bx + c = 0$ 2. $ax^2 + bx = 0$ 3. $ax + b = 0$ 4. $-ax^2 = 0$ 5. $x^3 + bx + cx + d = 0$ Asks students to identify and categorize the above equations under linear, quadratic and cubic.	Students identify the equations with common highest exponent and put them into the following categories: 1. cubic equations 2. quadratic equations 3. linear equation	Equation (5) is a cubic equation. Equation (1), (2) and (4) are quadratic equations. Equation (3) is a linear equation.

Development			The general form of a quadratic equation.
1. Explain the general form of the quadratic equation.	Teacher explains the general quadratic equation. Any equation of the form $ax^2 + bx + c = 0$ where $a \neq 0$ is a quadratic equation. Why is $a \neq 0$ both necessary and sufficient for an equation to be a quadratic equation? What if $b = 0$? What if $c = 0$?	If $a = 0$, the equation becomes $bx + c = 0$, which is a linear equation. The equation becomes $ax^2 + c =$	$a \neq 0$, is a necessary and sufficient condition.

		0 if $b = 0$.	
	What if b = 0 and c = 0?	The equation becomes $ax^2 + bx = 0$ if $c = 0$ The equation becomes $ax^2 = 0$ if b and c are both equal to zero. This is a quadratic equation	
2. Examples and non examples of quadratic equations	Teacher asks students to each write down two examples and two non examples of quadratic equations.	Examples: $6t^2 + 3t – 2 = 0$ $16x^2 + 3 = 0$	Examples and non examples of quadratic equations.
3. Definition of a Quadratic Equation	Ask students to give their own definition of a quadratic equation.	Non examples: $17x – 5 – 0$	Definition of a quadratic equation

		$6g^3 + 18g = 0$ A second degree polynomial whose standard form is $ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$ is a quadratic equation.	
4. Square root of discriminant d = $\sqrt{b^2 - 4ac}$	Teacher explains that there is a relationship between the roots of any quadratic equation and its square root of discriminant $d = \sqrt{b^2 - 4ac}$. When $d = 0$, the roots are real and equal, When $d > 0$, the roots are real and unequal. When $d < 0$, the roots are imaginary or complex. This is why it is called the	Students listen and take notes as teacher explains the discriminant.	The square root of discriminant is $d = \sqrt{b^2 - 4ac}$ $d = 0$, implies real and equal roots. $d > 0$, implies real and unequal roots.

	discriminant. For our purpose we shall limit ourselves to real roots only. We shall limit ourselves to positive square roots only illustrations. Find d when $3x^2 + 7x + 2 = 0$. Solution What are the values of a, b and c. Substitute these values into $d = \sqrt{b^2 - 4ac}$ and simplify. Teacher asks students to find the square	$a = 3$, $b = 7$ and $c = 2$. $d = \sqrt{7^2 - 4 \times 3 \times 2} = \sqrt{49 - 24}$	$d < 0$, implies imaginary or complex roots. For our purpose $d > 0$.

	root of the discriminants of the following equations: 1. $-9x^2 - 3x + 6 = 0$ 2. $p^2 + 2p - 35 = 0$	$d = \sqrt{25} = 5$ Students work out the square root of discriminants as follows: 1. $d = \sqrt{(-3)^2 - 4 \times 6 \times -9} = 15$ 2. $d = \sqrt{2^2 - 4 \times 1 \times -35} = 12$
5. Forming the conjugales	Teacher explains to students that in order to solve the quadratic equations by this method we make use of two conjugales.	

(CONJUGALES). These equations take the form

$2ax + b = -d$

or

$2ax + b = d$

In the general quadratic equation what do a and b stand for, and what does d stand for?

Form the CONJUGALES of the above equations.

a is the coefficient of x^2, b is the coefficient of x and d is the square root of the discriminant.

1. $a = 9$, $b = -29$, $c = 6$, $d = 25$

   $18t - 29 = 25$

   or

   $18t - 29 = -25$

2. $a = 1$, $b = -2$, $c = -3$, $d = 4$

   $12t - 4 = 4$

   or

   $12t - 4 = -4$

6. Solving quadratic equations by the use of CONJUGALES. i. Guidelines ii. Examples: Applying The Guidelines	Paste a cardboard containing the steps in solving quadratic equations on the board. 1. Write the equation in its canonical form. 2. Identify *a*, *b* and *c*. 3. Calculate the value of *d*. 4. Write down the CONJUGALES 5. Solve for the *x* in the CONJUGALES. Let us solve these equations using the guidelines. 1. $9t^2 - 29t + 6 = 0$ 2. $8p^2 - 2p - 3 = 0$	Students read through guidelines and ask for clarification.	Step–by–step procedure for solving the quadratic equation by CONJUGALES. Form the CONJUGALES

Guiding questions		Solving ESLE
1. What are the values of *a*, *b*, *c* and *d*?	1.$a = 9$, $b = –29$, $c = 6$ and $d = 25$	
2. Write down the CONJUGALES	$18t – 29 = 25$ ……….. (1) $18t – 29 = –25$ ……. (2)	
3. Solve for the values of *t* in the equation.	From (1), $18t = 54$, $t = 3$ From (2), $18t = 4$, $t = {}^{2}/_{9}$	
4. Check to find out whether the values satisfy the above equation.	$9({}^{2}/_{9})^{2} – 29({}^{2}/_{9}) + 6 = 0$ and $9(3)^{2} – 29(3) + 6 = 0$	Exercise
5. What does this mean? Guide students through questions.	This means 3 and ${}^{2}/_{9}$ are the roots of the equation $9x^{2} – 29x$ +	

		$6 = 0$.	
7. Class Exercise	Solve 1. $8v^2 + 10v – 8 = 0$ 2. $12x^2 – 10x + 12 = 0$	Students independently solve the equations.	
8. Conclusion Homework	Solve the following equations 1. $x^2 + 4x + 3 = 0$ 2. $2s^2 + 6s + 8 = 0$ 3. $4p^2 – 6p + 16 = 0$ 4. $5m^2 + 20m + 96 = 0$ 5. $3g^2 + 7g – 15 = 0$		Homework

SECOND LESSON NOTES

Subject Matter/Time	Teachers Activity	Students' Activity	Main Ideas
Introduction	Write the topic on the board.		
	Review the guidelines for solving the general quadratic trinomials.	Students mention steps.	Procedure for solving quadratic equations.
Development			
1. Discussion of home work with students	Call students to reproduce their work on the board for discussion.	Five students reproduce a question each on the board for discussion.	Discussion of homework
More class exercise	Solve the following equations. 1. $6x^2 + 9x + 3 = 0$ 2. $5y + 3 = -2y^2$	Students individually solve the equations.	Exercise

	3. $^{1}/_{2}x^{2} = 3x + 1 = 0$
	Teacher goes round to render assistance to individual students who are in difficulty.
2. Conclusion	Discuss with students the schedule for the posttest.

APPENDIX F (Continued)

SOLVING QUADRATIC EQUATIONS BY ESLE METHOD

Day/Date:

School: Jukwa Senior High School

Proposed Time: (80 mins)

Topic: Solutions of Quadratic Equations by ESLE

Reference: AbbiwM.K et al, 2002, Mathematics 300.2, Accra; Ministry of Education

Asiedu, P (2005), Aki Ola Series Core Mathematics, Accra, Aki Ola Publication

Ghana SSS Mathematics Book 2 and Book 3 (Amissah S. E et al 1991).

Instructional Objectives: Students should be able to

1. Identify a quadratic equation.
2. Explain the general form of a quadratic equation.
3. Define a quadratic equation.
4. Solve quadratic equations by ESLE.

Relevant Previous Knowledge Assured:

1. Students can identify the constants, variable and coefficients of the variables of an equation.
2. Students can substitute values into a formula and simplify.
3. Students can find the positive square root of numbers.
4. Students can solve simultaneous linear equations.

Teaching and Learning Materials:

Various posters of equations and step–by–step instructions for solving quadratic equations.

Subject Matter/Time	Teachers Activity	Students' Activity	Main Ideas
Introduction	Teachers write the topic on the board. Paste on the board a card with the following equations: 1. $ax^2 + bx + c = 0$ 2. $ax^2 + bx = 0$ 3. $ax + b = 0$ 4. $-ax^2 = 0$ 5. $x^3 + bx^2 + cx + d = 0$		
	Ask students to identify and categorize the above equations under linear, quadratic and cubic.	Students identify the equations with common highest exponent and put them into the following categories. a. Cubic equations b. Quadratic equations	Equation (5) is a cubic equation. Equation (1), (2) and (4) are quadratic equations.

		c. Linear equations	Equation (3) is a linear equation.
Development: 1. Explain the general form of the quadratic equation.	Teacher explains the general quadratic equation. Any equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$ is a quadratic equation.		The general form of a quadratic equation.
	Why is $a \neq 0$ both necessary and sufficient for an equation to be a quadratic equation?	Ans If $a = 0$, the equation becomes $bx + c = 0$, which is a linear equation.	$a \neq 0$, a necessary and sufficient condition.
	What if $b = 0$?	If $b = 0$, the equation becomes $ax^2 + c = 0$.	
	What if $c = 0$?	If $c = 0$, the equation becomes $ax^2 + bx = 0$.	

	What if $b = 0$ and $c = 0$?	The equation becomes $ax^2 = 0$ if b and c are both equal to zero. This is a quadratic equation.	
2. Examples and non-examples of quadratic equations.	Teacher asks students to each write down two examples and two non-examples of quadratic equations.	Examples $8q^2 + q + 2 = 0$ $x^2 - x = 0$ Non-examples $y + 3 = 0$ $x^3 + x^2 + x = 7$	Examples and non-examples of quadratic equations.

3. Definition of a quadratic equation	Ask students to give their own definition of a quadratic equation.	A second–degree polynomial whose standard form is $ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$ is a quadratic equation.	Definition of a quadratic equation.
4. The square root of the discriminant $d = \sqrt{b^2 - 4ac}$	Teacher explains that there is a relationship between the roots of any quadratic equation and its square root of the discriminant $d = \sqrt{b^2 - 4ac}$. When $d = 0$, the roots are real and equal. When $d>0$, the roots are real and unequal. When $d < 0$, the roots are imaginary or complex. This is why it is called the discriminant.	Students listen and take notes as teacher explains the square root of the discriminant.	The square root of the discriminant is $d = \sqrt{b^2 - 4ac}$ $d = 0$, implies real and equal roots. $d> 0$, implies real and unequal roots. $d< 0$, implies imaginary or

For our purpose we shall limit ourselves to real roots only. We shall also limit ourselves to positive square roots only.

complex roots.

Illustration

Find d when $2x^2 + 5x + 2 = 0$

Solution

What are the values of a, b and c.

Substitute these values into

$d = \sqrt{b^2 - 4ac}$ and simplify.

For our purpose $d > 0$.

$a = 2, b = 5, c = 2$

$d = \sqrt{5^2 - 4 \times 2 \times 2} = \sqrt{25 - 16}$

$\sqrt{9} = 3$

	Teacher asks students to find the square root of the discriminants of the following equations. 1. $-9x^2 - 3x + 6 = 0$ 2. $p^2 - 2p + 35 = 0$	1. $d = \sqrt{(-3)^2 - 4 \times 6 \times -9} = 15$ 2. $d = \sqrt{2^2 - 4 \times 1 \times -35} = 12$	
5. Forming the ESLE	Teacher explains to students that in order to solve the quadratic equations by this method we make use of two Equivalent Simultaneous Linear Equations (ESLE). These equations take the form. $ax_1 + ax_2 = -b$ $ax_1 - ax_2 = d$		The ESLE are $ax_1 + ax_2 = -b$ $ax_1 - ax_2 = d$ where x_1 and x_2 are the roots of the equation $ax^2 + bx + c = 0$, where $a \neq 0$ and

In the general quadratic equation, what do *a* and*b* stand for, and what do *d* stand for?	*a* is the coefficient x^2, *b* is the coefficient of *x* and *d* is the square root of the discriminant.	$d = \sqrt{b^2 - 4ac}$
Form the ESLE of the following equations: 1. $9x^2 – 29x + 6 = 0$ 2. $p^2 – 2p + 1 = 0$	1. $a = 9, b = 29, c = 6, d = 25$ $9x_1 + 9x_2 = 29$ $9x_1 – 9x_2 = 25$ 2. $a = 1, b = –2, c = 1, d = 0$ $x_1 + x_2 = 2$ $x_1 – x_2 = 0$	

6.	Solving quadratic equations by the use of ESLE.	Paste a cardboard containing the steps in solving quadratic equations by ESLE on the board.		Step–by–step procedure for solving quadratic equations by ESLE.
	i) Guidelines	1. Write the equation in its canonical form. 2. Identify *a*, *b*, and *c*. 3. Calculate the value of *d*. 4. Write down the *ESLE*. 5. Solve for the x_1 and x_2.	Students read through guidelines and ask for clarification.	
	ii) Examples: Applying the guidelines	Let us solve these equations using the guidelines. 1. $9t^2 - 29t + 6 = 0$ 2. $p^2 - 2p + 1 = 0$		

Guiding questions

a. What are the values of *a*, *b*, *c* and *d*?	1. $a = 9, b = -29, c = 6$ *and* $d = 25$	
b. Write down the *ESLE*.	$9t_1 + 9t_2 = 29$ _______________ *(1)* $9t_1 - 9t_2 = 25$ _______________ *(2)*	Form the *ESLE.*
c. Solve for the values of x_1 *and* x_2.	Adding (1) and (2) $18t_1 = 54, t_1 = 3$ Put $t_1 = 3$ into (1) $27 + 9t_2 = 29, t_2 = {}^2/_9$	Solving the *ESLE.*
d. Check to find out whether these values satisfy the above equation.	$9({}^2/_9)^2 - 29({}^2/_9) + 6 = 0$ and $9(3)^2 - 29(3) + 6 = 0$	
e. What does this mean?	This means 3 and ${}^2/_9$ are the roots of the equation$9x^2 - 29x + 6 = 0$.	

7. Class Exercise	Teacher guide students through question 2. Solve 1. $3x^2 - 10x - 8 = 0$ 2. $2x^2 - 10x + 12 = 0$	Students independently solve the equation.	Exercise

8. Conclusion:	Solve the following equation.	
Homework	1. $2x^2 + 8x + 3 = 0$ 2. $s^2 + 3s + 10 = 0$ 3. $2p^2 - 10p + 5 = 0$ 4. $5m^2 + 20m + 80 = 0$ 5. $3g^2 + 7g - 15 = 0$	Homework

SECOND LESSON NOTES

Subject Matter/Time	Teachers Activity	Students' Activity	Main Ideas
Introduction	Write the topic on the board		
	Review the guidelines for solving the general quadratic trinomial.	Students mention steps.	Review of procedure for solving quadratic by *ESLE*.
Development 1. Discussion of homework with students	Call students to reproduce their work on the board for discussion.	Five students reproduce a question each on the board for discussion.	Discussion of homework
	Solve the following equations.	Students individually solve the equations.	Exercise
2. More class exercise	1. $3x^2 + 6x + 9 = 0$ 2. $10y + 3 = -4y^2$ 3. $5x^2 + 10x + 1 = 0$		

	Teacher goes round to render assistance to individual students who are in difficulty.
3. Conclusion	Discuss with students the schedules for the posttest.

APPENDIX E (Continued)

SOLVING QUADRATIC EQUATIONS BY THE FACTORIZATION METHOD

DAY/DATE:

PROPOSED TIME: 7:00 – 8:20 (80 MINS)

TOPIC: Solutions of Quadratic Equations by Factorizing

REFERENCE: AbbiwM.K. et al, 2002, Mathematics 300.2, Accra; Ministry of Education.

Asiedu, P (2005), Aki Ola Series Core Mathematics, Accra, Aki Ola Publication

Ghana SSS Mathematics Book 2 and Book 3 (Amissah S. E et al 1991).

INSTRUCTION OBJECTIVES: Students should be able to:

i. Identify a quadratic equation.

ii. Explain the general form of a quadratic equation.

iii. Define a quadratic equation.

iv. Solve quadratic equations by factorization method.

RELEVANT PREVIOUS KNOWLEDGE

i. Students can group terms with common factor.

ii. Students can find factors of real numbers and simple mathematical expressions involving variables.

iii. Students can use the zero property of multiplication to solve simple linear equations

Teaching and Learning Materials

Various posters of equations and step-by-step instructions for solving quadratic equations.

Subject Matter	Teacher's Activity	Student's Activity	Main Ideas or Core Points
Introduction Proposed time: 60 mins	Teacher writes the topic on the board. Paste on the board, a card with the following equations: 1. $ax^2 + bx + c = 0$ 2. $ax^2 + bx = 0$ 3. $x^3 + bx^2 + cx + d = 0$ 4. $ax + b = 0$ Teacher asks students to identify and categorize the above equations under linear, quadratic and cubic.	Students identify the equation with common highest exponent and put them into the following categories: A. Cubic equations B. Quadratic equations C. Linear equations	An equation (3) is a cubic equation. Equation (1) and (2) are quadratic equations. Equation (4) is a linear equation. The general form of a quadratic equation

Development:			
Explain the general form of the quadratic equations.	Teacher explains the general quadratic equation. Any equation of the form $ax^2 + bx + c = 0$ where $a \neq 0$ is a quadratic equation.		
	Why is $a \neq 0$ both necessary and sufficient for an equation to be a quadratic equation?	Ans: if $a = 0$, the equation becomes $bx + c = 0$, which is a linear equation.	$a \neq 0$, a necessary and sufficient condition.
	What if b=0?	Ans: the equation becomes $ax^2 + c = 0$ if $b = 0$	
	What if c=0?	Ans: the equation becomes $ax^2 + bx = 0$ if $c = 0$	

	What if b=0 and c=0?	Ans: the equation becomes $ax^2 = 0$. This is a quadratic.	
Examples and non-examples of quadratic equations	Teacher asks students to write down two examples and two non-examples of quadratic equations.	Examples: $3p^2 + 4p + 6=0$ $10x^2 + 5=0$ Non–examples: $x + 9 = 0$	Examples and non-examples of quadratic equations.

		$5t^3 + 11t = 0$	
Definition of a quadratic equation	Ask students to give their own definition of a quadratic equation.	A second-degree polynomial whose standard form is $ax^2+bx+c=0$, where a, b and c are real numbers and $a \neq 0$ is a quadratic equation.	Definition of a quadratic equation.
Solving the general trinomial quadratic equation:$ax^2+bx+c=0$	Paste a card board containing the steps in solving quadratic equations on the board	Students read through the guidelines and ask for clarifications.	
	Step instructions:		
i. Guidelines	i. If there is a common factor of the coefficients and the constants, divide the equation through by the common factor.		Step by step procedure for solving quadratic equations.

ii. Obtain *ac* and find two of its factors such that their sum will be equal to b.

iii. Multiply each of the identified factors by the variable and replace *bx* in the equation with these two terms.

iv. Factorize by the method of grouping. Apply the zero principle(i.e. equate each factor to zero)

v. Solve each linear equation for the variable.

ii. Examples:

Applying the guidelines

Let us solve these equations using these guidelines:

1. $3x^2 - 8x + 5 = 0$
2. $r^2 + 4r - 5 = 0$

Solution:

What are the values of a, b and c in equation 1?

$a = 3$, $b = –8$, $c = 5$

What is the product of a and c?

$ac = 15$

Which is the pair of factors of 15 whose sum is equal to –8?

–3 and –5

Multiply through by each of the factors.

Replace $–8x$ by $–3x – 5x$ in equation 1.

Group terms with common factors.	$-3x$ and $-5x$
Factorize the equation.	$3x^2 - 3x - 5x + 5 = 0$
Apply the zero principle.	$(3x^2 - 3x) - (5x - 5) = 0$
Solve the linear equations 1 and 2	$3x(x - 1) - 5(x - 1) = 0$
	$(3x - 5)(x - 1) = 0$
	Either $3x - 5 = 0$
	or $x - 1 = 0$
	$3x = 5$or $x = 1$
	Therefore, $x = {}^{5}/_{3}$ or 1
	$3x^2 - 8x + 5 = 0$
Check your answers	$3(1)^2 - 8(1) + 5 = 0$

		$3\left({}^{5}/_{3}\right)^{2} - 8\left({}^{5}/_{3}\right) + 5 = 0$	
5. Class Exercise	Teacher guide students through the question 2 Solve: 3. $v^2 + 2v - 35 = 0$ 4. $x^2 - 2x - 3 = 0$	Students independently solve equation.	Exercise
6. Conclusion Homework	Solve the following equations: 1. $m^2 + 5m + 6 = 0$ 2. $3t^2 + 5t + 2 = 0$ 3. $q^2 + q - 72 = 0$ 4. $p^2 + 4p - 5 = 0$		Homework

5. $x^2 - 5x - 14 = 0$

6. $3r^2 + 7r - 15 = 0$

APPENDIX F (Continued)

SOLVING QUADRATIC EQUATIONS BY THE QUADRATIC FORMULA METHOD

DAY/DATE:

PROPOSED TIME: 7:00 – 8:20 (80 MINS)

TOPIC: Solutions of Quadratic Equations by Quadratic Formula

REFERENCE: AbbiwM.K. et al, 2002, Mathematics 300.2, Accra; Ministry of Education

Asiedu, P (2005), Aki Ola Series Core Mathematics, Accra, Aki Ola Publication

Ghana SSS Mathematics Book 2 and Book 3 (Amissah S. E et al 1991).

INSTRUCTION OBJECTIVES: Students should be able to:

v. Identify a quadratic equation.

vi. Explain the general form of a quadratic equation.

vii. Define a quadratic equation.

viii. Solve quadratic equations by quadratic formula method.

RELEVANT PREVIOUS KNOWLEDGE

iv. Students know constants and variables and coefficients.

v. Students can explain the principle of square roots.

vi. Students can substitute variables into formulas and simplify them

vii. Students can find the positive squares of numbers

Teaching and Learning Materials

Various posters of equations and step-by-step instructions for solving quadratic equations.

Subject Matter/Time	Teachers Activity	Students' Activity	Main Ideas
Introduction	Write the topic on the board. Also write the following equations on the board. 1. $ax + b = 0$ 2. $ax^2 + bx + c = 0$ 3. $ax^3 + bx^2 + cx + d =$ Then ask the students to distinguish among (1), (2) and (3) above.	Students explain that 1. is a linear equation 2. is a quadratic equation; and 3. is a cubic equation The highest index of (1) is one, that of (2) is two and that of (3) is three	$ax + b = 0$ is a linear equation $ax^2 + bx + c = 0$ is a quadratic equation. $ax^3 + bx^2 + cx + d = 0$ is a cubic equation.

Development 1. Explain the general form of a quadratic equation.	Teacher explains the general quadratic equation. Any equation of the form $ax^2 + bx + c = 0$, $(a \neq 0)$ is a quadratic equation.		The general quadratic equation is $ax^2 + bx + c = 0$, $a \neq 0$.
	Why is the condition $a \neq 0$ necessary?	If $a = 0$, the equation becomes $bx + c = 0$ which is linear.	$a \neq 0$ is a necessary condition
	What if $b = 0$?	The equation becomes $ax^2 + c = 0$	
	What if $c = 0$?	Then $ax^2 + bx = 0$ $ax^2 = 0$ is the new equation.	
		Yes. Because, ax^2, $a \neq 0$ is the	ax^2, $a \neq 0$ is a

	What if both *b* and *c* are all zero? Are all these quadratic equations? Why?Everybody should write two examples of quadratic equations.	sufficient term in the equation for it to be quadratic.	sufficient condition for a quadratic equation.
2. Examples and non–examples of quadratic equations.	Write two non–examples of quadratic equations.	Examples $3x^2 + 5x - 2 = 0$ $9x^2 - 3 = 0$ Non- example $4x + 5 = 0$ $10x^3 + 4x + 2 = 0$	
3. Relating particular quadratic equations to the general			

quadratic trinomial $ax^2 + bx + cx = 0$.	Let us compare our examples with the general quadratic equation $ax^2 + bx + c = 0$.	
	For $4x^2 + 8x + 2 = 0$, give the values of a, b and c.	$a = 4$, $b = 8$, $c = 2$
	And for $8x^2 – 4 = 0$?	$a = 8$, $b = –4$, $c = 0$
	Now for the following equations find a, b, c. (Write the equations on the board.) 1. $2x^2 + 10x+10 = 0$ 2. $9x^2 + 3x – 1 = 0$ 3. $18 + 8m – 10m^2 = 0$ 4. $p^2 + 15p + 30 = 0$	1. $a = 2$, $b = 10$, $c = 10$ 2. $a = 9$, $b = 3$, $c = –1$ 3. $a = –10$, $b= 8$, $c = 18$ 4. $a = 1$, $b = 15$, $c = 30$ Students give the answer for both

4. The principle of square root	What is $6^2 = ?$ What is $(-6)^2 = ?$ Now what is $\sqrt{36}$? So for our purpose $\sqrt{m}$ is $\pm\, n$ where $m = n^2$. Find: 1. $\sqrt{81}$ 2. $\sqrt{144}$	questions. $\sqrt{36} = 6$ or -6 Students find the square roots as $\sqrt{81} = 9$ or -9 $\sqrt{144} = 12$ or -12	$\sqrt{m} = \pm\, n$, where $m = n^2$.
5. Deriving the Quadratic Formula			As under Teacher’s Activity

Students follow the derivation,

ask questions, make contributions

and copy the derivation.

Teacher guides the students to solve the general quadratic trinomial $ax^2 + bx + c = 0$ *as follows;*

Step 1: Subtract *c* from both sides of the equation.

$ax^2 + bx = -c$

Step 2: Divide by *a*

$x^2 + \frac{bx}{a} = \frac{-c}{a}$

Step 3: Take ½ the coefficient of x, square and add to both sides. $\frac{1}{2}(^{b}/_{a}) = \frac{b}{2a}$ $\left[\frac{1}{2}\left(\frac{b}{a}\right)\right]^2 = \frac{b^2}{4a^2}$ $x^2 + \frac{bx}{a} + \left(\frac{b}{2a}\right)^2 =$ $= \frac{-c}{a} + \left(\frac{b}{2a}\right)^2$ Step 4: Write as a square of a binomial on the left and simplify on the right. $$\left(\frac{x+b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$ $$\left(\frac{x+b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$	The two roots are $x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$	Same as Teacher's Activity

Step 5: Solve the equation for

$$\left(x + \frac{b}{2a}\right) = \pm\sqrt{\frac{b^2 - 4ac}{2a}}$$

Step 6: Solve for *x*:

$$x = \frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula should be memorised. Now ask the students to state the two roots implied in the formula.

The two roots are

$x_1 =$

$\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and

$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

6. Advantages of the formula

The formula can be used to solve all quadratic equations. The

7. Applying the formula to solve quadratic equations	Explain to students that the formula, unlike the factorization can be used to solve all quadratic equations. Also unlike the graphical methods, the formula gives accurate roots, not approximate solutions. Let us use the formula to solve the following equation $4x^2 + 3x - 1 = 0$. What are the values of *a*, *b* and *c* in the	Students answer as follows $a = 4$, $b = 3$, $c = -1$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-3 \pm \sqrt{3^2 - 4(4)(-1)}}{2(4)}$ $= \frac{-3 \pm \sqrt{9 + 16}}{8}$ $= \frac{-3 \pm \sqrt{25}}{8}$ $= \frac{-3 \pm 5}{8}$ $= \frac{-3 + 5}{8}$ *or* $\frac{-3-5}{8}$	answers can be obtained with a fairly high degree of accuracy.

	equation? Substitute for *a, b, c,* in the formula. What do you get? Now simplify.	$= \frac{2}{8}$ *or* $\frac{-8}{8}$ $= \frac{1}{4}$ *or* -1	$x = 1$ *or* 3
	You may put these values of x into equation for a check.	Students solve the equation on their own.	
8. Class Exercise			
Conclusion:			
Homework	Solve $x^2 - 4x + 3 = 0$		

Homework

Solve the following equation.

1. $x^2 + 4x + 3 = 0$
2. $s^2 + 6s + 8 = 0$
3. $p^2 - 5p + 6 = 0$
4. $m^2 + 20m + 96 = 0$

Step 5: Solve the equation for

$$\left(x + \frac{b}{21}\right) = \pm\sqrt{\frac{b^2 - 4ac}{2a}}$$

Step 6: solve for x:

$$x = \frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{2a}}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula should be memorized.

Now ask the students to state the two roots implied in the formula

The two roots are

$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and

$$x_2 = \frac{-b \ + \ \sqrt{b^2 - 4ac}}{2a}$$

The two roots are

$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

6. Advantages of the formula	Explain to students that the formula, unlike the factorization can be used to solve all quadratic equations. Also unlike the graphical methods, the formula gives accurate roots, not approximate solutions.	Students answer as follows *a = 4, b = 3, c = –1* $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-3 \pm \sqrt{3^2 - 4(4)(-1)}}{2(4)}$	The formula can be used to solve all quadratic equations. The answers can be obtained with a fairly high degree of accuracy.
7. Applying the formula to solve quadratic equations	Let us use the formula to solve the following equation $4x^2 + 3x - 1 = 0$. What are the values of *a*, *b* and *c* in the equation? Substitute for *a*, *b*, *c*, in the formula. What do you get? Now simplify.		

	You may put these values of x into equation for a check.	$= \frac{-3 \pm \sqrt{9+16}}{8}$ $= \frac{-3 \pm \sqrt{25}}{8}$ $= \frac{-3 \pm 5}{8}$ $= \frac{-3+5}{8}$ *or* $\frac{-3-5}{8}$ $= \frac{2}{8}$ *or* $\frac{-8}{8}$ $= \frac{1}{4}$ *or* -1	$x = 1$ *or* 3
8. Class Exercise	Solve $x^2 - 4x + 3 = 0$	Students solve the equation on their own.	

Conclusion:	Solve the following equation.	Homework
Homework	9. $x^2 + 4x + 3 = 0$ 10. $s^2 + 6s + 8 = 0$ 11. $p^2 - 5p + 6 = 0$ $m^2 + 20m + 96 = 0$	

SECOND LESSON NOTES: SOLVING QUADRATIC EQUATION BY QUADRATIC FORMULA

Subject Matter/Time	Teachers Activity	Students' Activity	Main Ideas
Introduction	Write the topic on the board.		
	Review the quadratic formula. Discourage students from misquoting the formula.	Students quote and write the formula:	
	e.g. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Development			
1. Inspection and discussion of students' homework	Call students to the board to reproduce their work for discussion.	A number of students reproduce a question each on the board for discussions and corrections.	If the equation has fractional coefficients, it should be cleared off the fractions first.

2. More Class Exercise	Copy more questions including $3q + \frac{6}{q} = 11$ for class exercises. Go round for inspection.	Students work the problems themselves.	$3q + \frac{6}{q} = 11$ is not a quadratic equation. It only requires knowledge of solving quadratic equation.
3. Discussion and scoring of exercises	Discuss the exercises with students by working them on the board.	Students contribute to discussions. They exchange books and mark them.	
4. Importance of Quadratic functions and equations	Explain to students that a good grasp of the subject matter of quadratics is necessary for further studies in Mathematics, Economics, Physics, Engineering, Population Studies and the Life Sciences.	Students take down notes.	Quadratic equations can be applied in other subject areas. They also have practical relevance.

Conclusion: Homework	Solve the following equations: 1. $x^2 – 5x – 6 = 0$ 2. $x^2 – 11x – 30 = 0$ 3. $3r^2 + 18r + 27 = 0$ 4. $q^2 – 81 = 0$	Students copy exercises for homework.

APPENDIX G

Descriptive on the pretest scores

Method	N	Mean	Standard deviation
Quadratic formula	64	72.1875	15.37559
Factorization	71	66.7606	16.58525
ESLE	70	65.7857	17.43800
Conjugales	81	70.92591	16.56636
Total	286	68.9161	16.66112

Test of Homogeneity of Variances

Pretest

Levene statistic	df1	df2	Sig
0.279	3	282	0.841

ANOVA

	Sun of square	Df	Mean square	F	Sig
Between groups	2027.965	3	675.988	02473	0.063
Within groups	77086.021	282	273.355		
Total	79113.986	285			

APPENDIX H

Test Result-Quadratic Formula

Identification number	Age	Sex	Pretest	Posttest	Identification number	Age	Sex	Pretest	Posttest
1	15	M	80	96	16	16	F	80	90
2	16	F	90	94	17	15	F	70	94
3	16	F	80	82	18	16	F	50	68
4	16	F	85	86	19	16	F	85	96
5	16	F	75	94	20	18	F	90	92
6	15	F	50	68	21	15	F	85	92
7	16	F	80	90	22	14	F	80	88
8	15	F	90	98	23	16	F	95	98
9	15	F	55	66	24	15	F	65	74
10	15	F	100	98	25	15	F	50	35
11	17	F	65	74	26	15	F	75	82
12	15	F	50	66	27	15	F	65	78
13	15	F	45	56	28	15	F	80	88
14	15	F	50	48	29	16	F	70	54
15	15	F	55	62	30	15	F	90	94

APPENDIX H (Continued)

Quadratic Formula Result

31	15	F	75	44	49	17	M	90	88
32	18	M	45	52	50	19	M	75	80
33	15	M	80	94	51	17	M	45	70
34	15	M	75	90	52	17	M	90	100
35	14	F	85	80	53	18	M	55	58
36	15	M	70	94	54	20	M	85	92
37	21	M	50	52	55	16	M	75	80
38	16	M	80	78	56	17	M	50	64
39	15	F	80	90	57	15	M	60	78
40	18	M	85	94	58	15	M	70	54
41	16	M	70	98	59	17	F	90	96
42	17	M	55	44	60	15	M	75	82
43	16	M	70	76	61	16	M	100	100
44	15	M	90	74	62	18	M	65	68
45	16	M	80	88	63	16	F	75	41
46	15	F	80	94	64	20	M	65	38
47	16	M	75	72					
48	15	F	40	64					

APPENDIX H (Continued)

Test Result- Factorization

Identification number	Age	Sex	Pretest	Posttest	Identification number	Age	Sex	Pretest	Posttest
1	15	M	75	68	17	14	F	90	98
2	16	M	75	78	18	16	M	65	68
3	14	M	80	96	19	16	F	80	96
4	16	M	65	88	20	15	F	70	76
5	16	M	80	92	21	16	F	75	94
6	17	M	65	72	22	15	M	65	58
7	15	F	100	100	23	15	M	90	90
8	15	F	755	80	24	14	M	90	94
9	15	F	60	78	25	15	M	70	86
10	16	F	50	74	26	16	M	50	76
11	15	F	40	58	27	18	M	650	90
12	16	M	85	98	28	15	F	30	54
13	15	F	75	96	29	16	F	50	52
14	15	F	55	78	30	15	F	50	56
15	15	F	50	68	31	15	F	80	84
16	15	F	55	52	32	15	M	25	32

APPENDIX H (Continued)

Test Result- Factorization

33	18	M	90	100	49	16	F	60	76
34	17	M	75	82	50	16	F	70	76
35	15	F	50	66	51	15	M	95	100
36	15	M	54	74	52	17	M	65	74
37	15	M	85	80	53	16	M	80	82
38	15	F	70	76	54	15	F	35	20
39	15	M	55	50	55	16	M	65	60
40	15	F	65	68	56	18	F	70	80
41	16	F	75	80	57	15	F	80	76
42	16	F	50	58	58	17	F	85	70
43	16	M	50	62	59	16	F	60	64
44	18	F	40	54	60	17	F	50	60
45	15	F	55	74	61	17	F	75	88
46	15	F	50	60	62	17	M	95	80
47	15	F	80	68	63	15	F	60	64
48	16	M	75	78	64	16	F	65	80
47	16	M	80	79	65	15	M	50	78
48	16	M	60	74	66	15	M	80	92

APPENDIX H (Continued)

Test Result- Factorization

67	15	F	85	84
68	16	M	85	88
67	15	M	80	98

Test Results- ESLE

Identification number	Age	Sex	Pretest	Posttest	Identification number	Age	Sex	Pretest	Posttest
1	16	M	65	52	10	15	M	65	70
2	14	F	75	78	11	15	M	65	70
3	16	F	60	54	12	16	M	80	84
4	16	F	80	90	13	16	M	50	68
5	16	M	90	98	14	16	M	75	66
6	15	M	30	54	15	16	M	80	92
7	16	M	20	36	16	16	M	100	100
8	16	M	45	66	17	15	M	55	68
9	17	M	50	58	18	19	M	65	52

APPENDIX H (Continued)

Test Results- ESLE

19	15	M	50	62	36	15	F	50	54
20	16	F	75	68	37	17	F	55	68
21	17	M	65	64	38	15	F	80	62
22	17	M	90	98	39	15	F	75	76
23	15	M	70	92	40	15	F	65	50
24	15	M	80	82	41	15	F	65	60
25	17	M	80	82	42	15	F	70	72
26	16	M	35	46	43	15	F	85	80
27	15	M	65	74	44	16	F	80	86
28	20	M	80	80	45	15	F	65	58
29	15	M	50	36	46	15	F	90	76
30	15	F	60	60	47	16	F	65	88
31	16	F	40	46	48	17	F	55	64
32	15	F	60	68	49	16	F	90	82
33	15	F	70	80	50	15	F	70	74
34	15	F	70	72	51	18	F	65	74
35	15	F	45	30	52	15	F	50	54

APPENDIX H (Continued)

Test Results- ELSE

53	17	M	75	78	68	15	M	95	96
54	15	F	80	86	69	15	F	50	30
55	15	F	50	56					
56	15	F	40	32					
57	15	F	100	98					
58	16	F	70	66					
59	15	F	75	84					
60	15	F	80	62					
61	18	M	65	38					
62	16	F	50	48					
63	15	F	45	74					
64	17	F	30	86					
65	15	F	45	48					
66	15	F	70	66					
67	16	F	85	88					

APPENDIX H (Continued)

Test Result-Conjugales

Identification number	Age	Sex	Pretest	Posttest	Identification number	Age	Sex	Pretest	Posttest
1	16	F	90	96	17	16	F	50	56
2	15	F	80	82	18	15	F	30	18
3	17	F	80	88	19	15	F	50	68
4	16	F	75	92	20	15	F	45	42
5	16	M	95	90	21	17	F	50	45
6	17	M	60	62	22	16	M	55	80
7	14	F	70	82	23	15	F	40	46
8	15	M	65	76	24	16	F	50	76
9	15	F	75	98	25	14	F	65	90
10	16	F	74	84	26	15	F	90	96
11	15	F	100	100	27	15	F	80	92
12	15	M	45	44	28	15	M	90	98
13	15	F	50	58	29	17	F	70	72
14	15	F	85	96	30	15	F	70	98
15	15	M	80	92	31	16	F	90	92
16	15	M	70	66	32	19	F	60	94

APPENDIX H (Continued)

Test Result-Conjugales

33	14	F	100	100	52	16	F	85	88
34	15	M	65	76	53	15	M	85	92
35	15	M	45	64	54	16	F	80	86
36	15	M	40	72	55	15	M	55	72
37	16	M	65	82	56	15	F	45	64
38	15	M	50	56	57	17	F	95	100
39	15	M	60	88	58	15	F	55	76
40	15	F	80	76	59	16	F	80	74
41	17	F	80	98	60	16	F	80	88
42	15	F	75	78	61	15	M	85	88
43	15	M	95	98	62	17	M	50	74
44	15	F	80	92	63	17	F	80	90
45	16	M	65	72	64	19	M	90	92
46	16	F	70	70	65	17	M	55	62
47	17	M	65	64	66	15	M	80	88
48	17	F	80	90	67	15	M	90	92

49	17	M	60	66	68	16	M	65	72
50	16	M	60	62	69	15	F	50	66
51	16	M	100	100	70	17	F	80	74

APPENDIX H (Continued)

Test Result-Conjugales

71	15	M	75	86
72	15	M	95	88
73	15	M	80	86
74	15	M	80	88
75	15	F	70	64
76	16	M	60	80
77	14	F	70	74
78	16	F	70	70
79	15	F	80	90
80	15	M	100	100
81	15	M	65	96