UNIVERSITY OF CAPE COAST

INCORPORATING GEOGEBRA SOFTWARE IN THE TEACHING OF CIRCLE THEOREM AND ITS EFFECT ON THE PERFORMANCE OF STUDENTS

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BY

ANTHONY KWADWO BADU-DOMFEH

Thesis submitted to the Department of Mathematics and Information Communication Technology Education of the Faculty of Science and Technology Education, College of Education Studies, University of Cape Coast, in partial fulfilment of the requirements for the award of Master of Philosophy Degree in Mathematics Education

JUNE 2020

DECLARATION

Candidate's Declaration

I hereby declare that this thesis is the result of my own original research and that no part of it has been presented for another degree in this university or elsewhere.

Candidate's Signature: .... Date: ....

Name: Anthony Kwadwo Badu-Domfeh

Supervisors' Declaration

We hereby declare that the preparation and presentation of the thesis were supervised in accordance with the guidelines on supervision of thesis laid down by the University of Cape Coast.

Principal Supervisor's Signature: .... Date: ....

Name: Dr Kofi Ayebi-Arthur

Co-Supervisor's Signature: ....Date: ....

Name: Dr Rosemary Twum

ABSTRACT

The purpose of the study was to investigate the effect of *GeoGebra* software on senior high school students' academic performance in Circle Theorems. The study also assessed the students' attitudes towards the use of *GeoGebra* in teaching and learning of Circle Theorems. The study made use of simple random sampling (lottery method) and purposive sampling techniques select two Form 2 classes from two different schools within the Bono Region. The sample size for the study was 78 students. The study used a pre-test post-test non-equivalent research design. The pre-test scores were used to establish that students in the two schools had similar or equivalent abilities in geometry before the intervention. The post-test scores were used to determine the differences in academic performance between students who were taught Circle Theorems with the use of the *GeoGebra* software and students who were taught using the conventional method of teaching to learn Circle Theorem. The students who used in the *GeoGebra* software in the teaching and learning of geometry performed better than their peers who did not use the software. The study also found that the use of *GeoGebra* in teaching and learning increased the interest of students to learn geometry. Based on the findings, it was recommended that integrating the use of computers and interactive educational software into the teaching and learning of mathematics will help improve students understanding in mathematics and find the geometry interesting to learn.

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DEDICATION

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CHAPTER ONE

INTRODUCTION

Geometry, as an aspect in mathematics, plays an essential role in helping how mathematicians, as well as learners of mathematics, appreciate and understand the space, shape and orientation of various bodies and objects in this world. One aspect of geometry that enables mathematicians and learners of mathematics appreciate circular space, shape and orientation in this world is Circle Theorems. In that regard, learning geometry, especially at the senior high level, requires more innovative approaches that will enhance learners' understanding and skills in geometry. Some of these innovative approaches that are known to help learners' understanding of geometry are the use of technological devices, particularly integrating the use of mathematics software into the teaching and learning of geometry. This study gravitates towards the use of interactive software in the teaching and learning of geometry.

The chapter comprises of background to the study, statement of the problem, the purpose of the study, objectives of the study, research questions and hypothesis, significance of the study, limitation and delimitation of the study, and organization of the entire study.

Background to the Study

Several computer programs have been developed for commercial, educational, social and administrative purposes since the 1980s. Computer programs are created to perform specific tasks or to solve a class of problems. These computer programs are designed to make work easier and efficient. Fogg

(2002) asserted that computing has changed social and cultural dynamics in so many ways. The use of computer programs has had numerous applications and benefits. In the USA for instance, technological tools like computers and computer software have been integrated into every aspect of their education system (Eady & Lockyer, 2013). Computer programs are being used administratively and in classroom teaching and learning. New concepts such as the inverted classroom and Computer-Assisted Instruction (CAI) are being used in teaching (Trelease, 2016).

The use of computers in teaching and learning has become a catalyst for positive change in the approach to teaching and learning, especially in mathematics (Gunga & Ricketts, 2007). As such, it is within the domain of mathematics teachers to create instructional techniques that will help students in learning mathematics. From the mid-2000s, emphasis has been placed on improving student performance with the use of technology as a tool in teaching (Ross, Morrison, & Lowther, 2010). Policymakers and educators, therefore, have been reviewing programmes and instructional practices that have an effective impact on instruction and on student performance (Shieh, 2012).

The Mathematics discipline has also been greatly influenced by the use of computer programs. Computer programs such as MATLAB, Maxima, GraphCalc, Scilab, Microsoft Mathematics, GeoGebra, etc., have all been developed for various uses in Algebra, Calculus, Geometry, Trigonometry, Statistic and Probability. Some studies in the 1990s and 2000s have provided evidence of the positive effects of the use of interactive software on students' learning (Christmann, Badgett & Lucking, 1997; Cavanaugh, 2001; Page, 2002; Fried, 2008). These studies show that technology, that is, the use of interactive software helps to improve students' learning and achievement. Agyei and Benning (2015) assert that the use of computer (interactive) software in teaching has helped improve pedagogical, content knowledge, and technological skills of mathematics teachers. Using technology to teach and learn any subject matter is essentially employing computers and other technical tools such as calculators, projectors, software and the internet, inside and outside the classroom context (Chai, Koh & Tsai, 2013).

Mathematics is a very important subject across all levels of education in Ghana. At the second cycle level, it is a subject that must be learned by all senior high school students. A pass grade is a requirement for tertiary education in the country. A pass in mathematics forms part of the requirement for recruitment to state agencies such as the police service, military and immigration services.

The Senior High School Mathematics curriculum describes seven major areas that cover the content of mathematics syllabus (CRDD, 2010). They are; Numbers and Numeration, Plane Geometry, Mensuration, Algebra, Statistics and Probability, Trigonometry and Vectors and Transformation in a Plane. Geometry covers these four major areas: Plane Geometry, Mensuration, Trigonometry and Vectors and Transformation in a Plane.

Geometry is a very essential aspect in the field of mathematics. It is an area of mathematics that deals with relationships and properties of points, lines, plane and solid figures. Geometry gives an appreciable understanding of the world (Schopenhauer, 2016). It is an integral part of the modern world with respect to architecture, machines, and practically everything that is created with mathematics. In this regard, teaching and learning of Geometry is encouraged and recommended for all basic and high school education in Ghana. Teaching geometry involves helping students to be able to visualise clearly figures and shapes that may not be easily seen and understood by students (Noss, Healy, & Hoyles, 1997). This, however, is what makes Geometry a complicated area of learning.

There have been a number of resources that have been developed to aid the teaching and learning of Geometry (Schopenhauer, 2016). These include construction instruments such as the compass, dividers, protractors and set squares. Geometry charts are also used by teachers to help improve conceptual understanding of the discipline (Hooper & Rieber, 1995). Similarly, computer programs have become advantageous resources that have aided understanding of geometric concepts (Li & Ma, 2010)

There have been several computer programs that have enhanced students' achievement in mathematics (Oldknow, Taylor, & Tetlow, 2010). One such programme is the *GeoGebra* software which was created by Marcus Hohenwarter and Yves Kreis in 2001. It was created for teaching and learning of mathematics at pre-tertiary and tertiary levels. The *GeoGebra* software is a Computer Algebra System. It is at the same time a Dynamic Geometry System. In other words, the *GeoGebra* software consolidates elements of a Dynamic Geometry System and the elements of Computer of a Computer Algebra Systems. It connects mathematics with algebra, geometry and analytics (Hohenwarter, Hohenwarter, Kreis, & Lavicza, 2008). The Computer Algebra System is a mathematics programme that operates with mathematical expressions in a similar way as the traditional manual computations. It provides an advantage to users working in a mathematics field that requires manipulation of mathematical expressions. The DGS is an interactive software that allows the construction and manipulation of geometric figures in plane geometry.

Geogebra was created both for teachers and students to make mathematics education easy and meaningful. *GeoGebra* can be characterised as a compelling and imperative tool in building up connections between geometry and algebra concepts (Pierce & Stacey, 2011). The product can, therefore, be used by students going from primary school level to those of senior high school level. The software can also be used for simple constructions to solving functions. Velichova (2011) has suggested that students can use *GeoGebra* to explore mathematics alone or with the teacher, who acts as a guide. Finally, the software enhances students' ability to visualise geometrical concepts (Velichova, 2011). The present study explored the use of *GeoGebra* in the teaching of Geometry in Senior High Schools.

Statement of the Problem

The teaching and learning of geometry have been of special interest to educational stakeholders, mathematicians and scientist across the world (Mesa, Gómez, & Cheah, 2012). The skills and knowledge learned from Geometry help nurture students' ability to think critically and solve problems (Clements, 2004). At the senior high school level, the goal of teaching Geometry has been to ensure students obtain mathematical skills, insights, attitudes applicable to solve mathematics problems. Therefore, there is the need to find innovative ways that can help students to improve their ability to understand theorems and concepts in geometry.

GeoGebra is a computer programme that has had extensive use in mathematics especially geometry (Diković, 2009; Hohenwarter, & Fuchs, 2004). *GeoGebra* has been used as a teaching aid to improve students understanding geometry. *GeoGebra* builds students understanding by creating geometric visualizations in two dimensional and three-dimensional forms. Furthermore, in the area of geometry, the use of interactive software as a tool in teaching has been shown to actually enhance students' performance (Velichova, 2011).

Performance in mathematics is of paramount interest to all stakeholders in education. Students performance in the West African Senior School Certificate Examination (WASSCE) provides a standard yardstick for determining whether students are able to perform well in mathematics or not. From the early 2010s, there has been a decline in students achievement in Core – Mathematics in the WASSCE (Akaboha & Kwofie, 2016). One area of Core – Mathematics most students do not perform particularly well is Plane geometry II (Circle Theorem) (WAEC, 2011; 2014; 2015). Students have failed to demonstrate the required knowledge and understanding in this area, especially when applying two or more circle theorems in solving a given problem (WAEC, 2011; 2014; 2015). The present study therefore investigated the use of *GeoGebra* software in an interactive geometry classroom to improve the performance of senior high school students in Geometry (Circle Theorems).

Purpose of the Study

The purpose of this study was to investigate the effect of using *GeoGebra* on the performance of senior high school students in Geometry (Circle Theorem) in the Bono Region of Ghana.

Objectives of the Study

The study was guided by the following objectives:

1. To find the difference in academic performance between students who are taught using *GeoGebra* and students who are not taught using *GeoGebra* in the teaching and learning of geometry.
2. To assess the attitudes of students toward the use of *GeoGebra* in the teaching and learning of geometry.

Research Questions and Hypothesis

Research questions

The research questions that underpinned the study were the following:

1. What is the difference in academic performance between students who are taught using *GeoGebra* and students who are not taught with *GeoGebra*?
2. What are the attitudes of students toward the use of *GeoGebra* in the teaching and learning of geometry?

Hypothesis

The following null hypothesis was tested in the study:

H_o: There is no significant difference in the achievement scores between students learning mathematics using the *GeoGebra* software and students not using the *GeoGebra* software to learn mathematics.

Significance of the Study

The findings of the study have a number of benefits. Firstly, it is hoped that the software will be a useful tool in the teaching of geometry in Senior High Schools. Students will find it easy to solve geometry problems using the software. Lastly, the findings will add to the literature on teaching and learning of geometry.

Limitation

The study was based on a sample of senior high school students from the Bono Region of Ghana. The population was from two public senior high schools that followed the structured academic calendar as prescribed by the Ghana Education Service. As such there was a limitation since the sampling procedure was based on purposively selecting a sampling unit. The sampling procedure therefore limits the extent of application of the results.

Delimitation

The scope of this study was limited to two senior high schools in the Bono region of Ghana. The research focused on teaching and learning of only geometry. Therefore, the applicability of the software was not extended to other subjects in other education levels. Although there are many computer programs that can be used as tools in teaching geometry, this study focused only on the use of *GeoGebra.*

Organisation of the Study

The thesis consists of five chapters. Chapter One presents the background to the study, the statement of the problem, the purpose of the study, research questions, and the significance of the study, delimitation and limitations of the study and the organisation of the study. Chapter Two reviews the related literature. Chapter Three discusses the methodology while Chapter Four presents the analyses of the data collected. The thesis ends with Chapter Five, where the summary, conclusions and recommendations are presented.

Chapter Summary

This chapter gave an overall background which provided focus and direction for the study. The chapter first presented interactive software as innovative tool in the teaching and learning of geometry. The problem of the study was then presented which underscored the need for conducting the study. The chapter went further to state the purpose of the study and the specific objectives of the study. This was followed by the research questions and hypothesis. The significance of the study was then discussed. The limitation encountered during the course of conducting the study was discussed and the delimitation was also presented. The last section, which is the organisation of the study was then presented before the summary of the chapter.

CHAPTER TWO

LITERATURE REVIEW

Introduction

This chapter explores various literature under the following themes: technology in education, the use of technology in education in Ghana, the use of technology in Mathematics education, the teaching and learning of Geometry, Dynamics Geometry Software in Mathematics Education and the use of *GeoGebra* in mathematics education. The last sections provide the theoretical framework of the study and a summary of the chapter respectively.

Technology in Education

The use of technology in education began with the first Computer Assisted Instruction (CAI) developed by researchers at the IBM in the 1950s (Reiser, 2001). In 1959, Donald Bitier of the University of Illinois employed the first large scale use of computers in education (Choi, Dailey-Hebert, & Estes, 2016). During the 1980s, educators and researchers' interest in the use of technology in education initiated the use of certain applications in some subject areas (Reiser, 2001). One technological tool that that was of particular interest was the microcomputer (that is, personal computer) since it was inexpensive and could perform several functions that other large computers performed. By the 1990s, technological advancements started to have a significant impact on the nature and content of instructional practices (Spector & You-qun, 2016). Technological tools helped instructors to direct learning outcomes to include the development of problem-solving skills and creating the "independent learner". Juniu (2011) observed that the introduction of word processors, databases, spreadsheets and other tools helped both teachers and learners to adopt easier way of teaching and learning without relying on the use mental capacity on trivial tasks.

Technological tools are now integral parts of modern educational systems. Krueger and Kumar (2004) have observed that in developed countries new technologies have integrated into the structure, concepts and methods of how teaching and learning are done. Pilli and Aksu (2013) identified two broad categories of computers application in education. Pilli and Aksu (2013) asserted that a computer can be an object of instructional lesson or process, or can be used as an instructional device. In the first case, computer literacy courses or programmes are essentially concerned with how the computer or other related technologies become objective of instructional lesson. In the second case, the computer as an instructional device is concerned with how the computer or other related technologies can be used as aid or tool to help achieve instructional goals. The idea of using a computer as an instructional device is termed as Computer Assisted Instruction (CAI) or blended instruction (Tayebinik & Puteh, 2013).

Computer-Assisted Instruction or blended instruction is essentially the use of computers or other related technologies to express any subject matter. Kaur (2013) defined blended instruction as the approach to teaching and learning that mix the conventional method of teaching with the use of technological tools. A blended instruction or CAI is used as a medium to deliver teaching and learning content and activities in a classroom setting. With new advancements in technology, the structure and nature of CAI have become a less rigid system of classroom teaching. However, certain core components are necessary for its implementation. Cotton (2008) suggests that three components of a CAI or blended instruction should include a competent teacher or instructor, the learners or students and the computer interactive environment. Cotton (2008) asserts that the components of the blended instruction or the CAI is a union between the elements of the conventional method of instruction and the computer interactive environment.

The use of technology in education has had numerous benefits in teaching and learning. One main benefit of using technology in teaching and learning is the speedy processing of information. The capacity of a computer used in learning makes it easier for the learner to receive instructional information in either textual, diagrammatic or animated form (Berney & Bétrancourt, 2016). Roehl, Reddy and Shannon (2013) assert that technology in a learning environment allows for interaction between learners and teachers, connectivity of ideas and concepts during the instructional process; it also enables the teacher to effectively control the learning environment. Another advantage of technology in education is that it improves students' interest in learning. Several studies have shown that technology used in teaching and learning increases learners' enthusiasm which results in improved student achievement (Martin & Ertzberger, 2013; Shieh, 2012; Cifuentes, Maxwell, & Bulu, 2011). The use of technology in education ensures that student achievement can be measured in a number of ways and provide instant feedback (Pellegrino & Quellmalz, 2010).

Although the use of technology in blended instructions has been very beneficial to the teaching and learning process of a number of subject areas, there have been some limitations that have hindered the effective implementation of some of these technologies. Arkorful and Abaidoo (2015) for example, have pointed out certain causes as the limitations to the use of technology in education. They include the lack of funding for acquiring technological tools, teacher or instructor incompetency in the use of the technology, and the absence of a curriculum with technology integrated into its instructional objectives and processes. Arkorful and Abaidoo (2015) have also pointed out that one of the major hindrances to implementing technology in education, especially in developing countries, is how to fund it. Most poor schools cannot afford to buy technological tools because they are expensive; meanwhile facilities such as expensive building blocks for well-endowed schools are put up. Arkorful and Abaidoo (2015) have therefore opined that with the provision of computers, many learners could have more interactive periods of learning.

Another major concern is teacher incapability or incompetence in the usage of technology in classrooms. Even with the presence of the computers and other technological tools in the educational settings, the absence of skilled teachers remains a significant restrictive factor in using technology in teaching and learning (De Grove, Bourgonjon & Van Looy, 2012). This implies that the teacher factor remains a very important aspect of a blended instruction or CAI.

The Use of Technology in Education in Ghana

The speed of improvement in technology has brought about a lot of changes in the 1990s to the 2010s. This rapid development of technological resources has influenced the needs of modern society (Kozma & Voogt, 2003). With the impact of new technological advances on the working environment, educational authorities and various stakeholders endeavour to rebuild their educational programmes as well as and classroom infrastructure, to ensure effective teaching and learning (Butcher, 2015). This rebuilding process has required successful dissemination of ICTs into existing programmes with the focus of providing students' the opportunity of meaningful learning and enhancing professional integrity (Wang, 2008). The integration of technology in Ghanaian schools as well as in Africa is largely expanding and developing progressively as a result of various initiatives put in place to promote technological usage in schools (Buabeng-Andoh, 2012). Technology is viewed as a vital tool to improve learner participation in the learning environment. Technology along these lines give a window of opportunity to educational establishments to use innovations to supplement and bolster teaching and learning processes.

Although a lot of research on factors determining the use of ICT in education are from developed nations, recent studies show that third world countries are discovering ways to address challenges with respect to ICT in training education (Agyei & Voogt, 2011). A typical example is when in 2006 education ministers in West Africa adopted a ten-year plan in which education in science and technology was to undergo reform at all educational levels (Kuyini, 2013). The respective education divisions were to find ways of creating the right conditions within the education system to effectively utilize technology, and thus support teaching and learning.

The Government of Ghana in its endeavour to make use of technology in teaching, began in 2007 (Kuyini, 2013). Incorporating technology into Ghana's educational framework started in September 2007 as a component of the administrative drive to enhance the nature of teaching and learning in the country's educational system. The combination of ICT into Ghana's educational framework was formally presented as a major aspect of educational changes that started in September 2007 as a component of the administrative drive to improve teaching strategies in the country's educational system. A noteworthy requirement of the 2007 educational reform was to ensure that all students in senior high schools in Ghana should obtain essential ICT proficiency skills and apply these in their studies. The capacity to completely incorporate ICT into the education system and embarking on situational investigations of the diverse settings and difficulties that exist in Ghana’s educational structure with respect to technological use will require the essential step to investigate.

The Use of Technology in Mathematics Education

Technological advances and innovations have brought about new ways to the procedure of learning and teaching. These advances required changes in the capabilities of the individual and society. Butcher (2015) asserts that the essence of new technologies in education is to influence how we learn and practice mathematics. The use of ICTs in the learning environment helps teachers to keep up with the goals of the instructional period, and it also gives learners opportunities for developing adequately.

Various studies have been conducted to study the implications of technology use in the classroom setting. In one of these studies, Ross and Bruce (2009) researched the use of the interactive programme “Critical Learning Instructional Paths Support” (CLIPS) in a few Grades 11 and 12 classrooms to enhance understanding of fractions. This programme was intended to give the students independent lessons, including video presentations and assigned readings. Quizzes and other activities followed. The results showed that students benefitted the most when learning tool was presented in class. The implication is that the use of technology during an instructional lesson provides students with the assistance needed for learning.

Timmerman and Kruepke (2006) performed a meta-analysis on the influence of CAI on college student performance. They found a general positive effect of students being presented to a computer-assisted instruction. The analysis revealed that CAI packages that were planned particularly for one course largely affected students' performance than that of general CAI packages. They likewise discovered that gains were higher when the technology was utilised reliably throughout the course, not similarly as a one-time example. The authors also compared CAI packages that were able to provide feedback to students with ones that did not. The results showed no evidence to support the hypothesis that CAIs that provided feedback to students produced a more prominent change in students' performance. The study noticed that students just briefly viewed the feedback on the screen before clicking to proceed onward. One of the difficulties of the CAI package was encouraging students to use the package as the instructor intends. Timmerman and Kruepke (2006) study focused on various technological packages (that is, programmes or software) while Bruce and Ross (2010) study focused on only just one technological package (programme). This study focused on only one technological package, that is, *GeoGebra*.

Gunga and Ricketts (2011) have observed that ICT devices give capable scope of visual portrayals which help teachers to draw students' attention to mathematical concepts. That is why ICT devices like computers, web-based applications, graphic calculators, dynamic mathematics/geometry software, are being used in senior high school classrooms in advanced countries. Hence many studies have been done in those countries to evaluate the viability of technology in mathematics education (Skryabin, Zhang, Liu & Zhang 2015). A study by Amarin and Ghishan (2013) on the impact of educational technology on the learner interactions, for example, has shown that when educational technology is incorporated into conventional teaching practice, students' interest and motivation towards learning are increased. Amarin and Ghishan (2013) pointed out that integrating technology into lesson presentations makes the lesson to be constructivist in nature.

The need to improve students' achievement in mathematics is generally determined by the method of how instructional content is delivered to students (Pierce & Stacey, 2011). Chang and Lee (2010) pointed out that when is instructional lessons are well organised, learners are able to have a broad understanding of the concepts and theories that are applicable in mathematical solutions. Several researchers have suggested that one effective method of improving students' performance in mathematics is by using technological tools to teach in a classroom (Jang, 2010; Cheung & Slavin, 2013; Eyyam & Yaratan, 2014). These researchers have advocated educational institutions as well as mathematics instructors blend the use of technological resources in a conventional classroom setting.

From the prior discussion, technology innovation plays an essential part in mathematics instruction; in addition, it encourages mathematics teachers to better stimulate the attention of students to understand mathematics concepts (Khouyibaba, 2010). However, the coordination of technology in the learning and teaching of mathematics requires unique consideration in pedagogy and content of a mathematics lesson. Technological situations enable educators to adjust their methods and strategies to adequately address their students' needs (NCTM, 2008). By incorporating educational tools into teaching practices, educators can give innovative opportunities for supporting students' learning.

Teaching and Learning of Geometry

Teaching Geometry can be done using different activities. Choosing a particular activity depends on the objectives of the lesson. Such activities usually have an impact on how students learn geometry. A typical way of learning geometry is by the use of diagrams. The use of diagrams helps to bring out concepts and it communicates a kind of common relationship. Schwartz and Heiser (2006) opined that the use of diagrams helps students demonstrate spatial reasoning. In that regard, the use of diagrams in teaching and learning geometry involves the teacher being able to represent visually geometric drawings and help students recognise graphical or geometric relationships from the diagrams. Several studies have investigated the impact of geometric teaching and learning activities on students learning. Yerushalmy, Chazan and Gorgon (1990) conducted a study on the effect of high school students' use of diagrams in geometry. An inquiry approach was used to teach high school geometry courses with the use of the

"Geometric Supposers" from 1984 to 1988. The authors used "Geometric Supposers" when they identified three factors that hinder students when examining and interpreting diagrams. The three factors were; geometric diagrams that are specific, frequently using diagrams generates confusion with standard diagrams and lastly, one diagram can be visualised in different ways. Information collected on students that learned with "The Supposer" and students that did not use "the Supposer" from different senior high schools. The findings showed that students made use of the diagrams in their work after the introduction of "the Supposer". The researchers concluded that students that used "The Supposer" had a better understanding of geometric diagrams than the students that did not use it.

Another study by Noss, Healy, and Hoyles (1997) was done to demonstrate the significance of spatial reasoning in learning geometry. The study was to explore the relationship between learners' actions and visualisations and the means by which these were articulated. The researchers made use of Microworld *Mathsticks,* which is designed to help students develop mathematical meanings by building relationships between their actions and the symbolic representations they develop. They used a case study of two students. The results revealed that visualisations help students to make relationships from spatial and practical scenarios. The study also revealed that visualisation helps students to make algebraic relationships from diagrammatic representations.

An important goal of teaching geometry is to develop deductive reasoning in students. In that regard, Jones, Fujita, and Ding (2006) conducted a study on the teaching and learning approaches in geometry. They considered teaching approaches in China and Japan. They suggested that learning geometry should incorporate using modelling to apply geometric concepts, deductive reasoning, and the use of a problem solving approach in a variety of contexts. They concluded that an important way of improving the teaching of geometry is through the development of good pedagogical models with the support of well-designed learning tasks and tools.

Using games is also one way of creating geometric activities which aid teaching and learning in Geometry. Herbst, Gonzalez, and Macke (2005) demonstrated this with a study done to investigate how a teacher may prepare the ground for students to meaningfully define a figure. The authors used 53 students from two high school geometry classes. The students participated in a game called "Guess My Quadrilateral". The goal was to ascertain students' previous knowledge of quadrilaterals. A questionnaire was given to students before the instruction. The researchers planned the instruction and implemented it in the course of three weeks. The game involved a context of a neighbourhood of special quadrilaterals. The game was to make students concentrate on each quadrilateral and differentiate it from its neighbour. The findings of students' responses revealed that as students talked about the properties of figures, they were able to draw these figures instead of describing them. The authors concluded that students were able to use the information from the game to assess the properties of the quadrilaterals they discussed.

Another study by Foster and Shah (2015) was done to find out how games could aid learning in an educational setting. The researchers used the Play, Curricular Activity, Reflection, Discussion (PCaRD) model as the teaching strategy. The study was conducted at a senior high school using a mixed-methods approach with experimental and control groups. Three games were implemented with the use of PCaRd model for a time duration of one year. Foster and Shah (2015) used pre-and post-tests to determine the achievement gains. They reported that the PCaRD model helped students learning of geometry. Foster and Shah (2015) asserted that the PCaRD model also helped teachers to adapt games into their teaching strategies.

Students' thought processes are also very important to how students demonstrate understanding when learning geometry. Moscucci, Piccione, Rinaldi, Simoni, and Marchini (2005) conducted a study that looked into the strategies students use to understand geometrical concepts. The purpose of the study was to investigate students' thinking about isosceles triangles. A sample of 105 students from six third grade classrooms in Italy was used in the study. The researchers studied how the "orientation' of a particular drawing influenced the way students perceived isosceles triangles. The researchers observed that students' strategies to solutions showed different naïve methods of measurement in geometry. The researchers reported that students learned better with short activities. Marchett, Medici, Vighi, and Zaccomer (2005) also investigated students' thinking processes in relation to the area and perimeter of geometric figures. The study involved 130 fourth and fifth-year primary school students in Italy. The study looked at conflicting ideas of perimeter and area. The researchers used two worksheets to discuss pupil's reasoning skills. They found that the pupils were able to make comparisons between areas than perimeters. In this regard, the pupils were able to attempt problems involving areas with more appropriate strategies. It was also found that students were tricked by how they visually perceived the geometric figures. The researchers also realised that students like working on single geometric shapes rather comparing them.

Teaching geometry is done at all levels of education. At the university level, for instance, the use of geometric visualisations and activities are also applicable to non- Euclidean geometry. One study by Kaisari and Patronis (2010) investigated how university students construct a model of elliptic geometry. The aim of the study was to find the ways geometrical meanings can be developed through context and practices. The researchers assumed that reformulation of Euclid's axioms and development of models for elliptic geometry reveal relevant relationships between basic and higher geometry. Students were tasked to work one of them throughout a semester and discuss their work as a team. The students were given the opportunity to express their own views and have interactions on various tasks. The study revealed that irrespective of the ways students use geometrical concepts, they are able to interact and influence the understanding of their peers.

One aspect of geometry that plays an important role in how students appreciate and apply geometric concepts is circle geometry. There have been very few studies on circle geometry in literature. Some of these studies on circle geometry have focused on the construction of geometrical shapes and objects and the use of geometrical problems to develop reasoning in learners. An example of such a study by Canada and Blair (2006) was conducted to investigate the intersections of a circle and square. Canada and Blair (2006) sought to find out how many points of intersection exist between a circle and square. The participants of the study included students and pre-service teachers. The authors initiated tasks in the study that was to help students develop mathematical arguments, conjectures and make connections between mathematical ideas. The task in the study was for the participants to construct a six-point intersection and find out other possible points of intersection. Canada and Blair noted students were able to determine possible 0, 1, 2 and 3 points of intersection even with the imprecisely drawn circles and squares. Canada and Blair (2006) also noted that students' drawings of 5, 6 and 7 point intersections resulted in different conclusions depending on how accurate the drawings were. The researchers concluded that students were able to make accurate discoveries and conclusions of geometric concepts when diagrams were precisely drawn.

Neel-Romine, Paul and Shafer (2012) conducted a study that tasked students to write and critique the definition of a circle. The study comprised of Sixth-grade students from one school. The researchers used an activity to help students develop the definition of a circle. Students were put in groups and tasked to write the definition of a circle. Counter examples were used to test students' definitions and also guide students in exploring how to create a circle. Neel-Romine, Paul and Shafer (2012) noted that students were able to write down definitions such as "round shape" and "looks like an orange or a coin". The authors also pointed out that students were able to write true definitions of a circle such as "a circle contains a diameter and a radius" and "all diameters are the same length". The study also revealed that even with the use of pencils and paper clips, students struggled to define the radius as equidistant. The authors concluded that after a series of activities students were able to properly define a circle and write out its properties.

Another study that focused on circle geometry was conducted by González and DeJarnette (2013) who sought to use problem – based instruction to develop reasoning skills. The study comprised of 22 students who were made to participate in a number of activities. The students were given a circle problem in a problem – based lesson. The objective of the lesson was to help students view the problem as one that required more than numerical calculation. The problem was given at the time when students had covered all the prerequisite mathematical concepts needed to solve the problem. The problem also allowed students to develop their strategies, hence students were not required to limit themselves in applying specific theorems and concepts in solving the problem. The study revealed that almost all students used key features of the problem (that is, diagram) to apply multiple strategies to solve the problem. Some of these strategies included marking out right angles, using helping lines (that is, dotted lines) and applying Pythagoras theorem. González and DeJarnette (2013) pointed out that the task of solving the problem enabled learners to analyse the problem, develop a strategy and implement it, make connections between different concepts and theorems in mathematics and finally reflect on the solution.

In summary, focus on helping learners to visualise geometric concepts and analyse the relationships between these concepts helps improve understanding of geometry. This study makes use of visualisations as well as diagrams with the aid of interactive software to help improve senior high students' achievement in mathematics.

Dynamics Geometry Software in Mathematics Education

Dynamic Geometry Software (DGS) are computer programs which provide the interactive environment for users to draw and measure geometric figures. It also allows users to measure and calculate variables of geometric figures and allows users to develop relationships between geometric figures (Hollebrands, 2007). DGS enable users to alter the orientation of the geometric figures and change its appearance while preserving the mathematical relationships on the figure (Hollebrands, 2007). These computer programs make the visual figures to be enriched with dynamic movements that help students to understand mathematical concepts and develop strategies for solving problems. Visualisation is an important aspect of geometric reasoning (Presmeg, 2006). With the use of DGS, students are able to drag and move points of a geometric figure to observe changes in the relationships on the figures. There several DGS environments such as *GeoGebra*, *Cabri 3D*, and *Geometer's Sketchpad*.

The Dynamic Geometry Software, *Cabri 3D*, is used in exploring three-dimensional geometric figures. It was developed in 2004 by a French company called Cabrilog. The *Cabri Geometry* is a commercial product and was designed to be used in the areas of geometry and trigonometry. The software allows animations of geometric figures which is a significant advantage over geometric figures drawn on board. The software also allows the user to make relationships between points on a geometric figure. The software also provides good graphing and display functions. These functions allow the user to make connections between geometry and algebra. The programme can be run on Windows or the Mac OS.

Another software for teaching geometry is the *Geometer's Sketchpad*. It is a commercial interactive software that is used to explore various aspects of geometry and other areas of mathematics. *Geometer's Sketchpad* incorporates the traditional Euclidean tools for geometric constructions. Figures such as the pentagon or decagon with the software just as it can be constructed with a compass and ruler. The software allows the user to make transformations that may otherwise be difficult to construct with a compass and ruler. It also allows the animation of objects. The software allows the user to create a number of objects that can be used to solve very difficult math problems. The program allows the determination of the midpoints and mid segments of objects. One main disadvantage of the software is that it is a paid software. Therefore, it cannot be easily accessible to students and teachers.

GeoGebra is an open-source mathematics learning software. *GeoGebra* that consolidates elements of different dynamic representations and provides a rich variety of computational tools that can be used for modelling and simulations. *GeoGebra* has a user-friendly interface and has a web platform that makes it convenient for users across the world to access a number of mathematical resources. Figure 1 shows the interface of the *GeoGebra* software.

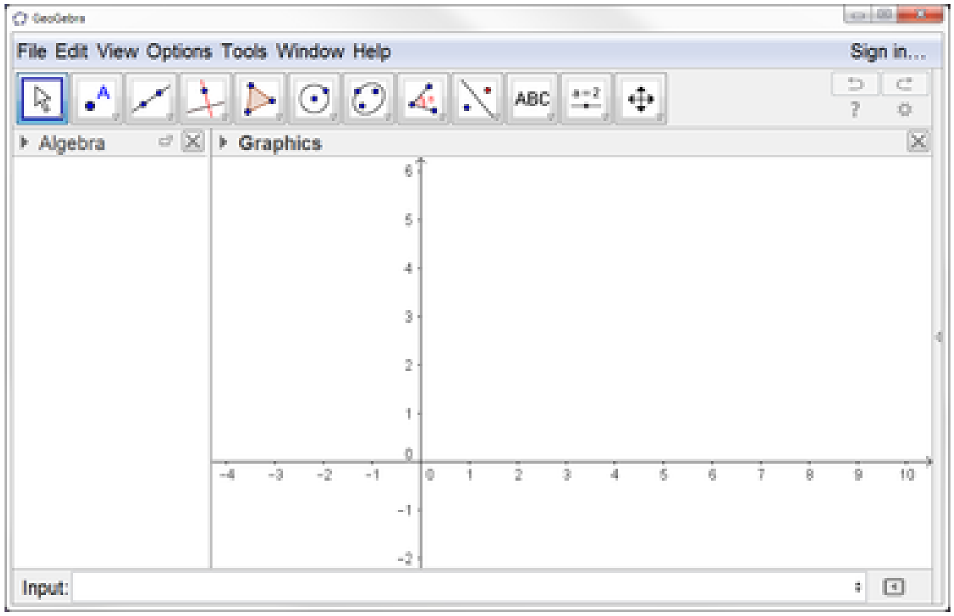

Figure 1. *GeoGebra* interface

Source: wiki.geogebra.org

In the mathematics classroom, *GeoGebra* can be used to teach a geometric concept. The process of using *GeoGebra* in an interactive geometric classroom starts by introducing the concept to be taught. The teacher or instructor then uses the tools available in the *GeoGebra* software to draw or animate the concept for

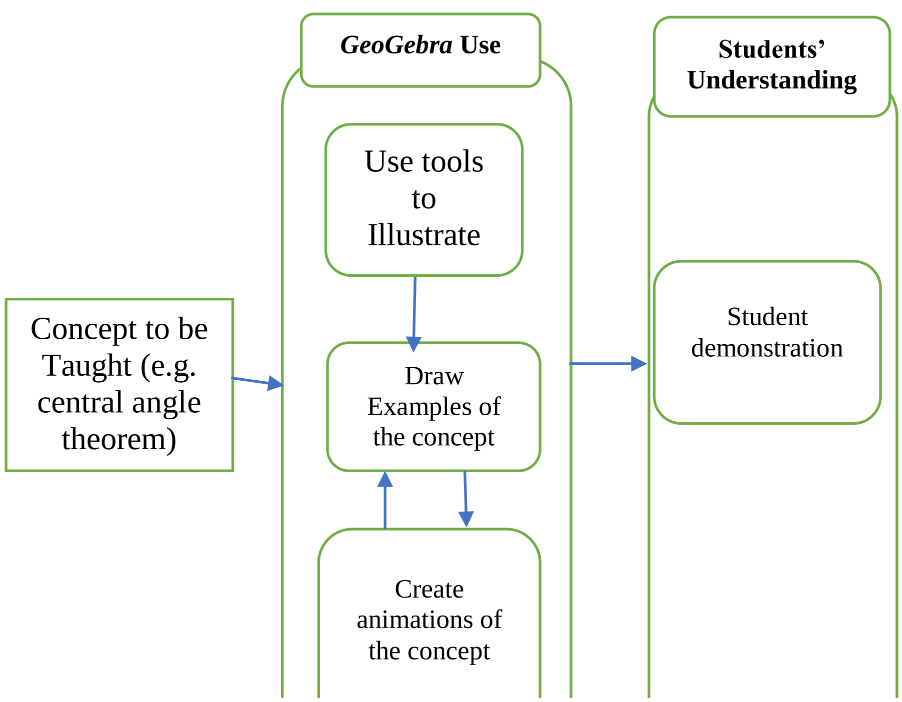

Figure 2. Process of using *GeoGebra* in the classroom.

clear visualization. The *GeoGebra* software displays a diagram which gives a clear visualisation (possible orientations) of the mathematical concept that the teacher wants to teach. Some the tools available in the software include the point with circle tool, ruler, arc tool, angle tool, angle with given size tool, line tool, line segment tool, line segment with given size tool, line vector tool, bisector tool, perpendicular bisector tool, etc. See Figure 2: Process of Using *GeoGebra* in the classroom

GeoGebra offers other tools to make the drawn concept to be manipulated into different examples. These examples are then to be stored as *Geogebra* file and practised by students. Students understanding of the concept is demonstrated when they are able to draw different examples of the same concept. The teacher knowledge in the use of *GeoGebra* is imperative to ensure proper demonstrations with the software.

GeoGebra has expanded internationally as a result of online *GeoGebra* Wiki, global and local professional conferences. *GeoGebra* provides a number of digital tools that enable users to develop mathematical relationships of practical situations, synthesise and personally manipulate diagrams using a number of representations and modelling tools. This allows learners to be to formulate very abstract mathematical ideas. With these advantages, the present study integrated the *GeoGebra* software in classroom instruction of geometry.

There have been a number of studies that have focused on integrating DGS in mathematics teaching and learning. Strausova and Hasek (2013) conducted a study to investigate visual proofs with the use of DGS. The researchers suggested that diagrams play a vital role in helping learners conceptualised various mathematical properties. They also suggested that a geometric property or theorem can be proven using a diagram. Strausova and Hasek (2013) asserted students appreciate geometric proofs rather than the use of only words. Karaibryamov, Tsareva and Zlanatov (2012) also investigated courses optimisation in geometry with the use of the DGS. The researchers used a new method with the aid of DGS to teach geometry in at the tertiary level. The effective ways in which courses in geometry can be taught. Karaibryamov, Tsareva and Zlanatov (2012) reported that the use DGS helped optimised the teaching process by ensuring that there was enough time for drawing, generalise a large of problems and help form creative way of reasoning.

Ertekin (2014) also investigated the effect of Cabri 3D on students' geometric ability in Geometry. The purpose of the study was to determine if the students could properly write out the equation of a given plane and draw a graph of the plane. Another objective of the study was to determine if students could find the normal vector of a given plane. The study comprised of 78 students grouped into two groups. The two groups were the experimental group and the control group. Students in the experimental group were taught with the Cabri 3D while students in the control group were taught without the Cabri 3D. The study found that students that were taught with the Cabri 3D were more successful in identifying identify special plane and the corresponding normal vector. The students taught with Cabr 3D were also more successful in drawing out corresponding diagrams for the special planes.

Another study was conducted by Donevska-Todorova (2015) to focused on students' understanding of the scalar product. The study sought to use DGS in a dynamic geometry environment to aid students understanding of dot product. The sample used in the study comprised of 12th-grade students learning in a dynamic geometry environment (DGE). The findings of the study showed that that DGS helped students to acquire a deeper understanding of dot product. Donevska-Todorova (2015) argued that the use of a number of diagrammatic representations helped students to gain more insight about a particular mathematical concept.

From the prior discussion of the DGS and the literature available, the appropriate use of DGS holds a significant advantage in mathematics education. The succeeding section gives an empirical review of the use of *GeoGebra* in mathematics education.

***GeoGebra* Use in Mathematics Education**

Several research studies have revealed that *GeoGebra* can be used to improve students' performance. One of such studies was by Saha, Ayub and Tarmizi (2010). The research was to find out the effect of using the *GeoGebra* software to teach Coordinate Geometry on students' performance. The study consisted of 53 secondary school students who were grouped into control group, and experimental group. The experimental group were taught using the *GeoGebra* software, while the control group did not use the software. The result was that the experimental group performed significantly better than the control group. Thus, the implication was that a blended instruction using the *GeoGebra* software was a strong complement to conventional method of teaching.

Selçik and Bilgici (2011) also conducted a study with 32, seventh-grade students from two schools to explore how effective *GeoGebra* is in the teaching and learning of the polygons. The study used an achievement test as the research instrument. The sample was grouped into the experimental group (17 students), who were taught using the software, while the control group (15 students) were instructed without using the software for 11 hours in an elementary school. The findings of the study indicated that there was a significant difference between the experimental group and the control group. This outcome demonstrates that using *GeoGebra* upgraded students' achievement scores more than conventional instruction does. It was additionally discovered that the experimental group learnt more effectively using *GeoGebra* and retained what they learnt more than the students that learnt in a computer-free environment.

The results of Selçik and Bilgici (2011) was similar to those of Zengin, Furkan and Kutlucaa (2011) study. Zengin, Furkan and Kutlucaa (2011) study was to determine the effect of *GeoGebra* on students' achievement in trigonometry. The study comprised of 51 students grouped into two groups; a control group and experimental group. The results of Zengina, Furkanb and Kutlucaa (2011) revealed that the *GeoGebra* software was able to improve students' achievement in trigonometry better than the traditional method of teaching and learning. Zengina, Furkanb and Kutlucaa (2011) pointed out the use of *GeoGebra* in teaching and learning was an effective strategy of a constructivist form of instruction.

Another study by Zakaria (2012) was conducted to find out the impact of using *GeoGebra* on students' performance in mathematics. The participants of the study comprised of 284 secondary school students. The students were put into an experimental group, who were taught using *GeoGebra* and a control group, who were taught using the conventional method of teaching. The findings of the study showed that the students in the experimental group performed better in the post-examination than their peers in the control group. The findings also revealed that there was no significant difference in performance between the boys and the girls. Zakaria (2012) concluded that integrating *GeoGebra* into teaching and learning aids conceptual and procedural learning of learners.

Shadaan and Leong (2013) also conducted a study to find the effect of *GeoGebra* on students understanding in the teaching and learning of circles. The study involved 53 Year 9 students grouped into two groups; the treatment group and control group. The study made use of a pre-test and post-test to determine the effect of *GeoGebra* on the students' achievement in learning circles. The findings from the study showed that students that used the *GeoGebra* software to learn circles performed better than peers that did not use the software. Shadaan and Leong (2013) also revealed that students had an overall positive perception of the use of *GeoGebra.*

Mukiri (2016) sought to find out how *GeoGebra* could be applied in the teaching and learning of mathematics in secondary schools in Kenya. Mukiri (2016) also sought to find the effect of using *GeoGebra* to teach among boys and girls. The study employed a mixed-methods design of qualitative and quantitative approaches. The study comprised of 33 teachers and 270 students. The results of the study revealed that mathematics teachers' uptake of technology was slow. The study also found students that used the *GeoGebra* software to perform better than students that did not use the software. Mukiri (2016) established that gender difference did not affect students' performance after the application of the GeoGebra in the classroom.

Chimuka (2017) also conducted a study on the effect of integrating *GeoGebra* on students' performance in mathematics. The study used *GeoGebra* in teaching circle geometry to senior high school students. The study made use of quasi-experimental non-equivalent control group design to compare the achievement of students that used the *GeoGebra* software and students that were taught with the conventional method of teaching. The study comprised of 47 students grouped into control and experimental groups.

The researcher found the average performance of students' that used the *GeoGebra* software higher than the performance of students that did not use the *GeoGebra* software. Chimuka (2017) also focused on the study on Van Hiele's theory of Geometric thinking. The results also indicated students taught with the *GeoGebra* perform significantly than their peers that did not use the software at Van Hiele's Levels 1 and 2. However, there was no significant difference in achievement the two groups at Van Heile's Levels 3, 4 and 5.

Another study that focused on the effect of GeoGebra, as an IPad application, on student achievement was conducted by Martinez (2017). The study employed an experimental non-equivalent pre-test and post-test design with the use of a treatment and control group. The study comprised of 56 students. Martinez (2017) found that students' scores improved when *GeoGebra* was incorporated into teaching.

A study that focused on circles was conducted by Tay and Wonkyi (2018) on the effect of *GeoGebra* on senior high school performance in *GeoGebra*. The study also employed quasi-experimental non-equivalent control group in which 49 students were used for the study. The study used purposive sampling to grouped the students into two groups from two schools in which one group was taught with the use of *GeoGebra* while the other group was taught with the traditional method. Tay and Wonkyi found that there was a positive effect for students that used the *GeoGebra* software to learn geometry. Tay and Wonkyi (2018) also found that teaching with *GeoGebra* made classroom lessons more interesting and practical to students.

Aizikovitsh-Udi and Radakovic (2012) conducted a study to demonstrate the impact of GeoGebra to enhance students understanding of probability and Bayes Theorem. The purpose of the study was to determine the influence of *GeoGebra* in helping students to understand mathematical concepts that are abstract. The research instrument used in the study was an achievement test. The findings of the study showed that integrating *GeoGebra* in classroom instruction improved students critical thinking skills in solving probability questions. The researchers concluded that visualisations from the use of *GeoGebra* helped students have a better understanding of mathematical concepts. While all the previously discussed studies focused on determining the effect of integrating *GeoGebra* into classroom teaching and learning on learners' performance, Aizikovitsh-Udi and Radakovic (2012) used the *GeoGebra* software to help foster critical thinking skills of learners.

In consideration of all the prior discussion of the literature on the use of *GeoGebra*, the use holds a significant advantage in mathematics education. It can, therefore, be concluded that the use of DGS in an educational setting can be a useful technological tool that makes the teaching and learning more effective and interesting.

Theoretical Framework for the Study

This study adopted Rogers' (2010) Diffusion Innovation Model. Rogers asserted diffusion as "the process in which an innovation is communicated through certain channels over time among the members of a social system" (p, 37). According to Rogers, the attitude of people towards the introduction of a new technology is a significant factor to process of its diffusion. Applicable in this study, is the use of *GeoGebra*, which is the new technology (innovation) and its effects on students learning. Since Rogers uses the terms innovation and technology interchangeably, the diffusion of innovation framework seems particularly suited for the study of the diffusion of ICT.

Rogers proposed features of innovations that helped in the innovation-diffusion process. These features include; relative advantage, compatibility, complexity, trialability, and observability. These attributes were applicable to this study in its effort to find out the implications of using the *GeoGebra* software to teach Circle Theorem.

Chapter Summary

The present study reviewed literature that shows various studies that have provided many insights into the use of technology (particularly interactive software) in the teaching and learning of geometry. While there has been much attention on the teaching and learning of geometry in general, there has been little focus on the use of interactive software to learn Circle Theorem in particular.

Studies involving the use of computer programs have focused on students' ability to identify geometric objects and improving students' spatial and deductive reasoning skills when solving a problem in geometry. Therefore, there is no explicit focus given to students' knowledge and skills in circle geometry concepts or using interactive software to improve the teaching and learning of these concepts. This makes the present study is unique because of its attention on senior high school students in Ghana and on students' achievement in Circle Theorems.

CHAPTER THREE

RESEARCH METHODS

Introduction

This chapter describes the various methods that were employed in generating and analysing the data used in the study. The description covers the research design, study area, population of the study, sample and sampling procedure, research instrument, administration of instrument, data collection and data analysis.

Research Design

The study employed a quasi-experimental design. A quasi-experiment is an empirical study used to appraise the causal effect of an intervention (treatment) on a target population form which participants are not randomly assigned in groups (Creswell, 2014). The research design involved non-random assignment of participants into two groups namely treatment (experimental) and control groups. This design enabled the researcher to study the cause and effect integrating the *GeoGebra* software into teaching and learning to improve students' performances (Creswell, 2014).

The study sought to find the implications of using the *GeoGebra* software to teach Circle Theorem to senior high school students. In that regard, teaching lessons were planned and taught to the students. The performance of students (experimental group) taught using *GeoGebra* was compared with those taught without *GeoGebra* (control group). The control group was taught using conventional method of teaching as such the use of a marker-board, maker and cardboard presentations.

Study Area

The study area consisted of two schools located in two districts in the Bono Region of Ghana. The Bono Region is located in Mid-West part of Ghana, bordered to the north by the Savannah Region and to the east by the Bono East Region and to the south by Ahafo, Ashanti and Western regions and to the West, Cote D'Ivoire.

Population

The population for the study comprised of all senior high school students in the Bono Region in Ghana. There are 28 senior high schools in the region consisting of both public and private senior high schools. The target population consisted of all Form Two senior high school students in the Bono Region. The two schools used for the study were School A and School B. School A was the site for the experimental group while School B was the site for the control group.

Sampling Procedure

The sample was derived from the target population. There are 12 districts in the Bono Region of Ghana. Two districts were randomly selected from the region using the lottery method. The selected districts were Sunyani West District and Wenchi Municipality. In the Sunyani West District, purposive sampling was used to select one senior high school (School A) that had adequate ICT facilities as the site for the experimental group. Purposive sampling technique was used because it enabled the researcher to pick samples that were "information-rich" schools (Creswell, 2014). Another senior high school (School B) was selected in the Wenchi municipality using simple random sampling. In the two respective schools, Form Two classes were purposively selected, because the topic, Circle Theorem, which was chosen for the study is taught in Form Two as stated by the Senior High School Mathematics Curriculum. In school A, one Form Two class was randomly selected as the experimental group while in school B, one Form Two class was also randomly selected as the control group.

Data Collection Instrument(s)

To determine the influence of the independent variable (instruction with *GeoGebra* and conventional method of teaching) on the dependent variable (achievement test), three instruments, namely pre-test, post-test and the questionnaire were used. The pre-test was a test administered to the two groups in order to determine whether students had equivalent knowledge in geometry before the interventions were administered. The post-test was a test administered to students in both groups to determine the effect of the interventions. All the items in the pre-test were based on the Plane Geometry topic in the Senior High School Mathematics curriculum while the items in the post-test were based on the treatment topic, Circle Theorems.

Pre-test and post-test instruments

The pre-test and post-test each comprised 10 theory questions in geometry from which learners were tasked to answer all. The items in the pre-test (Appendix A) were based on the geometric concepts of the Plane Geometry I topic of the senior high school curriculum. The study used the pre-test to determine if the students in both groups had equivalent level of knowledge and understanding required to learn the Circle Theorem topic.

The post-test (Appendix B) was designed to assess students' mastery of concepts and relationships in Circle Theorem after the intervention (treatment). The post-test for both groups contained the same number of items and content.

Questionnaire

The research questionnaire (Appendix C) was designed to solicit information on students' attitudes and motivations toward mathematics, and attitudes toward the use of the software. The questionnaire was mainly administered to students in the experimental group. The questionnaire used a four-point Likert scale. The question items 1 – 4 sought information on students understanding of the Circle Theorems (Conceptual Understanding) with the use of *GeoGebra*. Question items 5 – 7 sought information on how students' ability in solving problems. Question 8 – 10 sought information on student appreciation of the *GeoGebra*-based classroom.

Reliability and validity of research instruments

The reliability and validity of the instruments were ensured by pre-testing them on 10 Form Two students of St. James Senior High School in Sunyani, to ensure that the test items do not contain inexcusable errors and that they were relevant to the purpose of the study. To ensure reliability, the pre-test and post-test was given to two experienced mathematics teachers to score the students. There was little discrepancy between the scores of the two teachers. The reliability of the questionnaire was evaluated using Cronbach's alpha. High alpha coefficients (above 0.7) are generally considered to indicate high internal consistency of the scores (Bryman & Cramer, 2005). Alpha value 0.735 was obtained for the research questionnaire. Thus, the questionnaire was judged to be reliable.

For the present study, content validity was assured for the pre-test and post-test. Polit and Beck (2006) assert that content validity is a method of measuring or determining a consensus among experts with regards to the quality of a specific test item. Polit and Beck (2006) pointed out expert judgement is main approach to check whether a test has content validity. In that regard, two mathematics teachers from the two schools and three experience mathematics teachers evaluated the pre-test and the post-test and made recommendations to the instruments.

Data Collection Procedures

Prior to the start of the study, the researcher established a good rapport with authorities in the schools that were to be used for the research. The researcher also sought approval from the Institutional Review Board (IRB) of the University of Cape Coast to implement the research. After consent was given from IRB (see Appendix I), a letter introducing the researcher from the Department of Mathematics and ICT Education was presented to the Heads of the selected schools two weeks prior to data collection exercise (see Appendix J).

Administration of pre-test instrument

Three days before the start of the intervention, the researcher conducted the pre-test for students in the experimental group. The pre-test was to ensure the students had the relevant previous knowledge needed to learn Circle Theorems. In the control group, the researcher also administered the pre-test two days before the intervention, that is, the teaching of Circle Theorem using the conventional method of teaching. The pre-test papers were marked and the data stored on a computer.

Instruction on circle theorems

The intervention (treatment) took a period of seven days to teach Circle Theorems and apply Circle theorems in solving circle problems. Circle Theorems were taught in a *GeoGebra* supported environment to the experimental group. The researcher was the teacher for the experimental group. Lesson plans and materials were prepared before the intervention period, by the researcher. Each lesson and activity was reviewed by both the researcher before delivery of the instruction.

The control group was taught using the conventional method of teaching for a duration of seven days. instructional lessons were also prepared by the researcher and the teacher from School B. Conventional method of teaching for this research involved of the teacher using the marker-board with the marker and working each circle theorem with the students in school B. The researcher was the teacher for the two groups.

The intervention involved teaching the following eight Circle theorems:

1. Circle Theorem 1 - Angle at the Centre (Central Angle Theorem)
2. Circle Theorem 2 - Angles in a Semicircle
3. Circle Theorem 3 - Angles in the Same Segment
4. Circle Theorem 4 - Cyclic Quadrilateral
5. Circle Theorem 5 - Tangents from a Point to a Circle

6. Circle Theorem 6 - Tangents from a Point to a Circle II
7. Circle Theorem 7 - Alternate Segment Theorem
8. Circle Theorem 8 – Perpendicular Bisector of a Chord

The next section provides description of each lesoon for both groups.

Treatment group

First and second lessons

The researcher began the first lesson by introducing the concept of circle properties and the relationships that can be drawn from these properties. After that the basic tools of the *GeoGebra* software were introduced to students. The researcher used about twenty minutes for this introduction. The students said that the software was user-friendly and they could use it with ease. Throughout the lesson, the researcher observed that students were adapting well with the software.

After the introduction, the first theorem was explained to the students. Figure 3 is a diagram explaining the first theorem which illustrates the central angle theorem. The central angle theorem states that the angle at the centre of the circle (angle β) created by two radii is twice the angle at the circumference of the circle (angle α). The *GeoGebra* software was used to help students easily visualise the relationship between the two angles of the central angle theorem. Figure 4 illustrates how the *GeoGebra* software was used to turn or alter the diagram in Figure 3 into another diagram representing the same central angle theorem. In this way, students were able to recognise the circle diagram in Figure 4 as the same theorem in Figure 3. Throughout the lesson and subsequent lessons, the software was used in this manner to help students in recognising the circle theorem in a variety of circle diagrams.

The *GeoGebra* software gave students the advantage of easily computing the values the angles in the diagrams. Students were expected to draw their own relationship from the diagram and find the value of the two designated angles in the diagram. Some students were able to use the software to estimate the values of the angles. The researcher asked students to make different similarities of the central angle theorem and estimate the angles in the diagrams that they have drawn on their own. The first lesson ended with all of the students been able to draw at least one diagram of the central angle theorem.

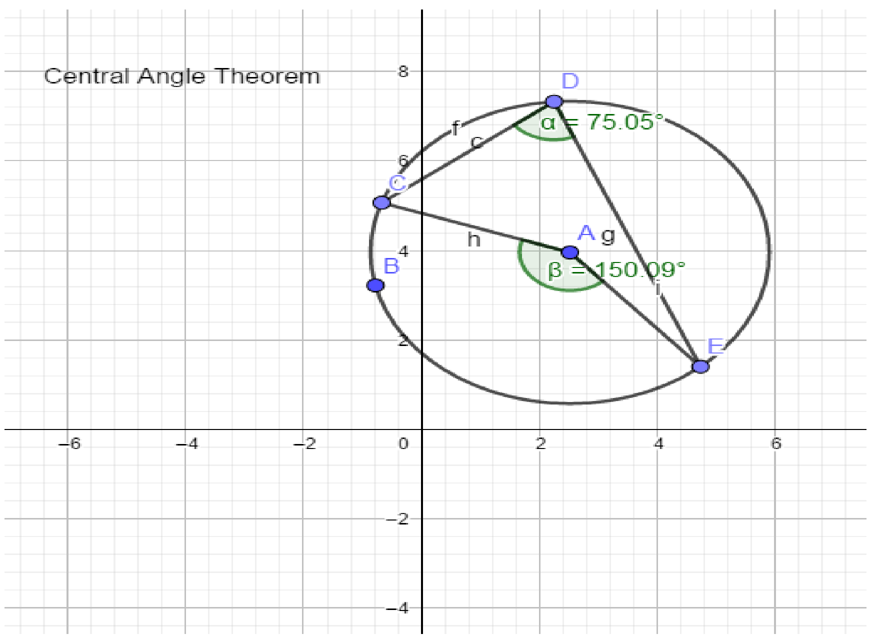

Figure 3. Circle theorem 1 (central angle theorem)

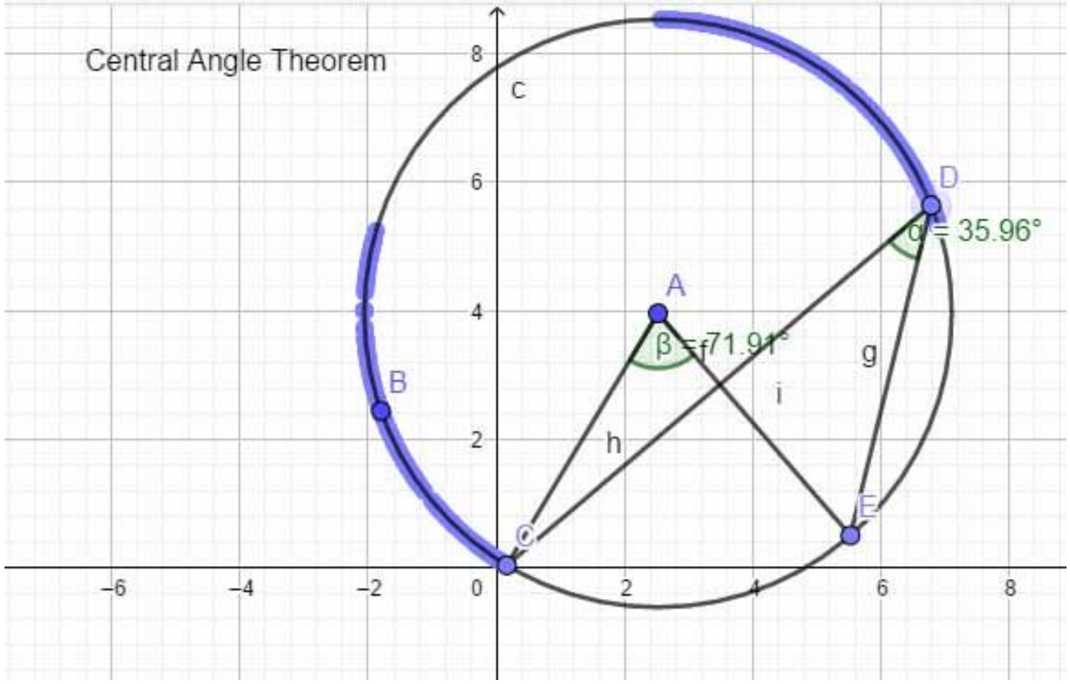

Figure 4. Altered diagram of circle theorem 1

The second lesson began by revising Circle Theorem 1. Most of the students were able to verbally explain the first theorem and draw it freely with the *GeoGebra* as well as on their own. The students were able to draw their versions of the first theorem. About thirty minutes into the second lesson, the researcher then used the first theorem to introduce the second theorem (Circle Theorem 2). The second theorem describes an inscribed angle that is across a diameter will always be equal to 90°. Students were expected to relate the second theorem as an extension of the first theorem (see Figure 5). Most students were able to discover the reasoning of the second theorem by stating that the inscribed angle at the circumference of the circle is half the central angle (180°). As the final task of the second lesson, the researcher requested the students to find angles in four diagrams that they drew on their own. The second lesson ended with students using the *GeoGebra* software to create diagrams that transformed from the first theorem to the second theorem.

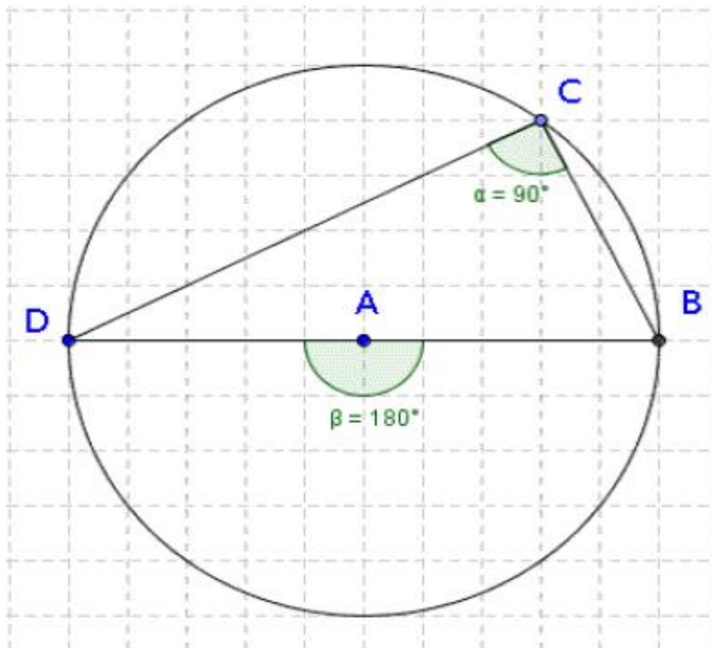

Figure 5. Circle theorem 5: angles in a semi circle

Third lesson

In the third lesson, the researcher introduced the third theorem (Circle Theorem 3) and forth theorem (Circle Theorem 4). Circle Theorem 3 was introduced as another type of inscribed angle in a circle (see Figure 6). The researcher used an already drawn image of the Circle Theorem 3 to explain the concept of the third theorem. The researcher then asked students to the tools in the *GeoGebra* software to draw other diagrams of the third theorem but not exactly the same as the one the researcher introduced to the students. The goal of this task was to let students discover the right mathematical relationship in whatever diagram they drew and to determine if students were developing competency with the use of the *GeoGebra* software.

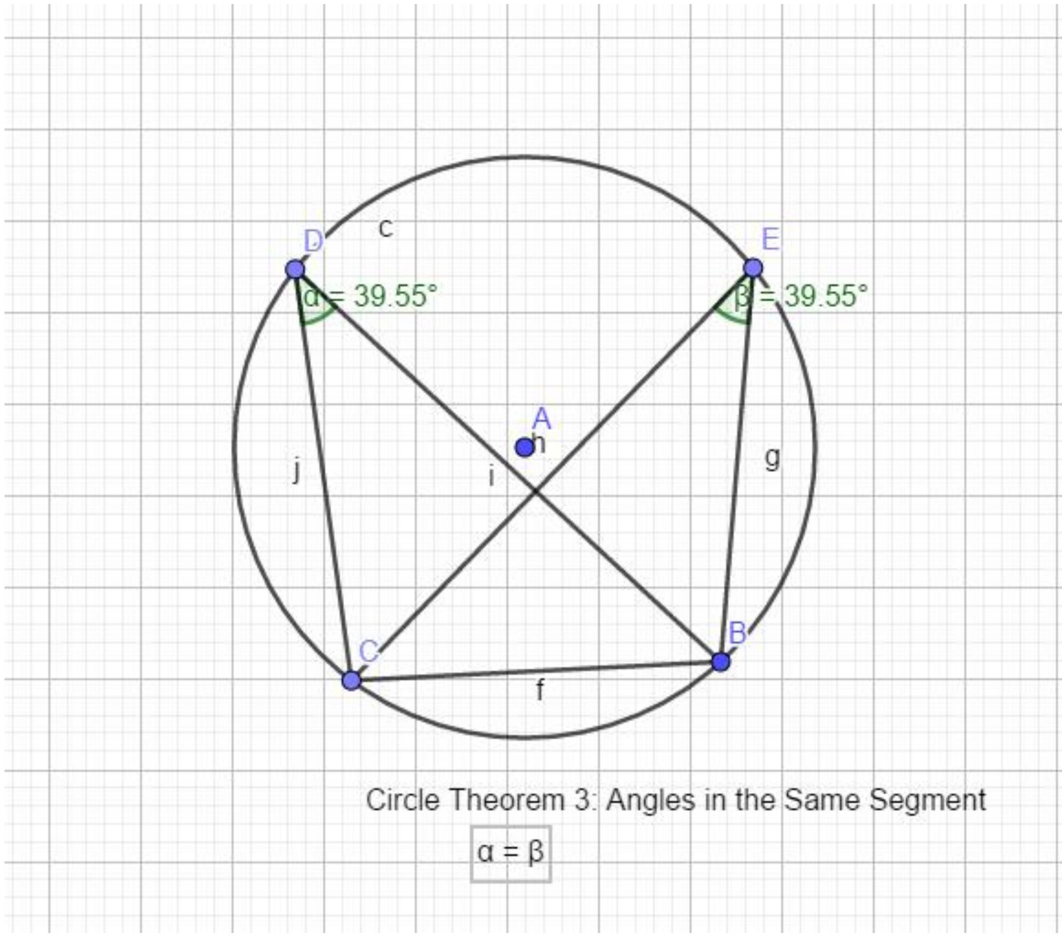

Figure 6. Circle theorem 3 (angles in the same segment)

When students were able to establish the mathematical relationship in the third theorem, the researcher used another geometric diagram the concept of Circle Theorem 4. Figure 7 shows a geometric diagram of Circle Theorem 4. Circle Theorem 4 (cyclic quadrilateral) illustrates a quadrilateral inscribed in a circle. The students were again tasked to draw different variations of the Circle Theorem 4 and use the drawn diagram to deduce the right mathematical relationship. The students were expected to state the right mathematical relationship of the cyclic quadrilateral which showed that the sum of the diagonally opposite angles is equal to 180°. After the students were to able draw the diagrams of the Circle Theorem 4 the researcher decided to introduce Circle Theorem 5 in the last few minutes of the lesson. The researcher concluded the lesson by asking students to manually draw diagrams of the third, fourth and fifth theorem as a kind of homework.

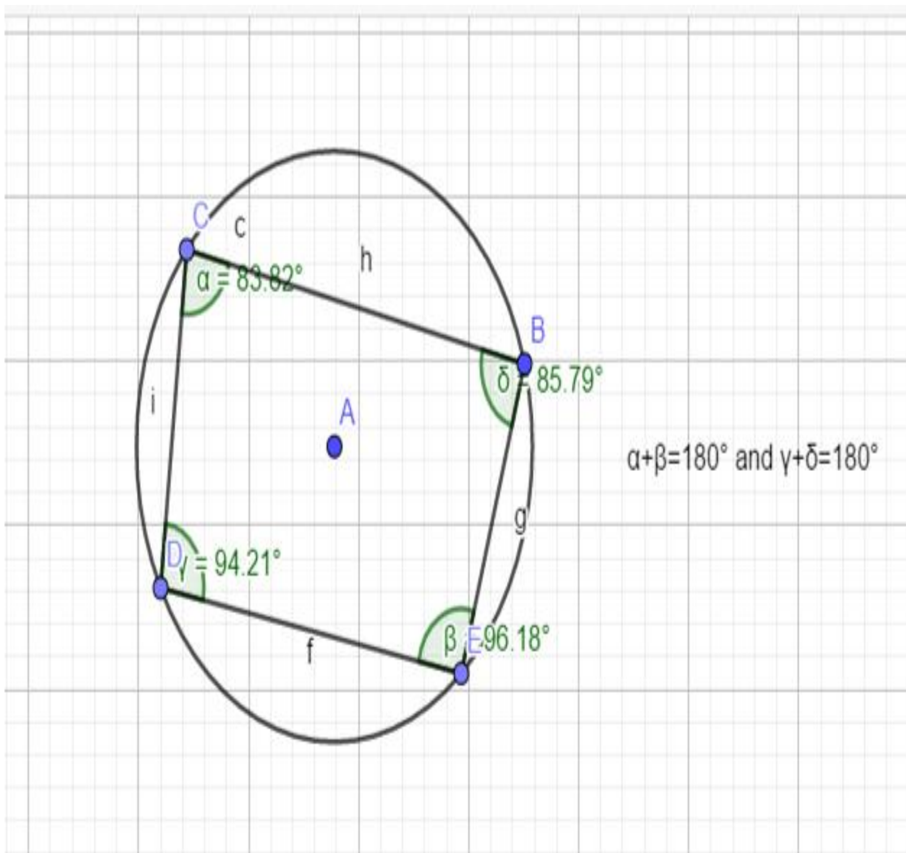

Figure 7. Circle theorem 4 (cyclic quadrilateral)

Fourth lesson

The fourth lesson focused on the last four theorems. The researcher began the lesson by reviewing the previous lesson and the home work that was given to students. Most of the students did the homework. The students were able to properly draw diagrams of the Circle Theorem 3 and Circle Theorem 4 but not all of them could not properly establish the mathematical relationship of the Circle Theorem 5. Students inability to establish the mathematical relationship in the fifth theorem was understandable since Circle Theorem 5 emphasised the concept of converging tangents lines. Circle Theorem 5 showed that lengths of two tangents lines from the circle that converge or meet a point are equal. Figure 8 displays the fifth theorem.

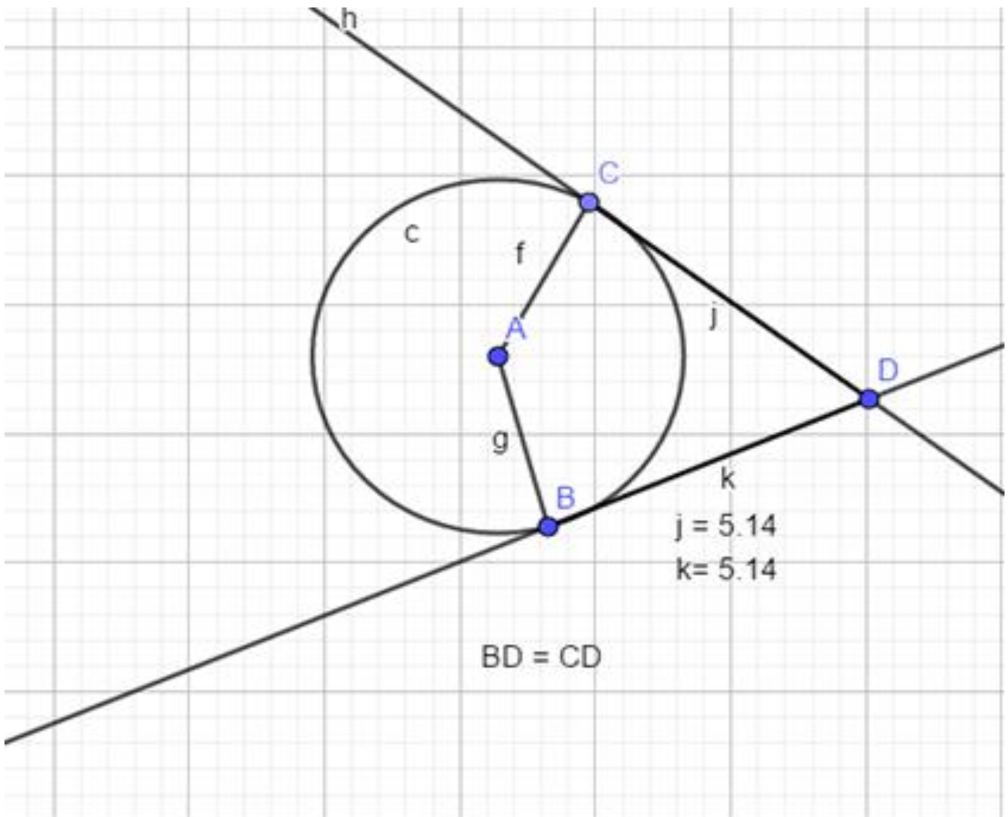

Figure 8: Circle theorem 5 (tangents from a point to circle)

The researcher then explained the concept of the tangents lines to the students and used the *GeoGebra* software to prove the equality of the lengths of the tangents. The researcher then used the diagram of Circle Theorem 5 to introduce the Circle Theorem 6. Circle Theorem 6 was described as the angle the tangents makes with the radii of the circle is equal to 90°. Figure 9 shows the Circle Theorem 6.

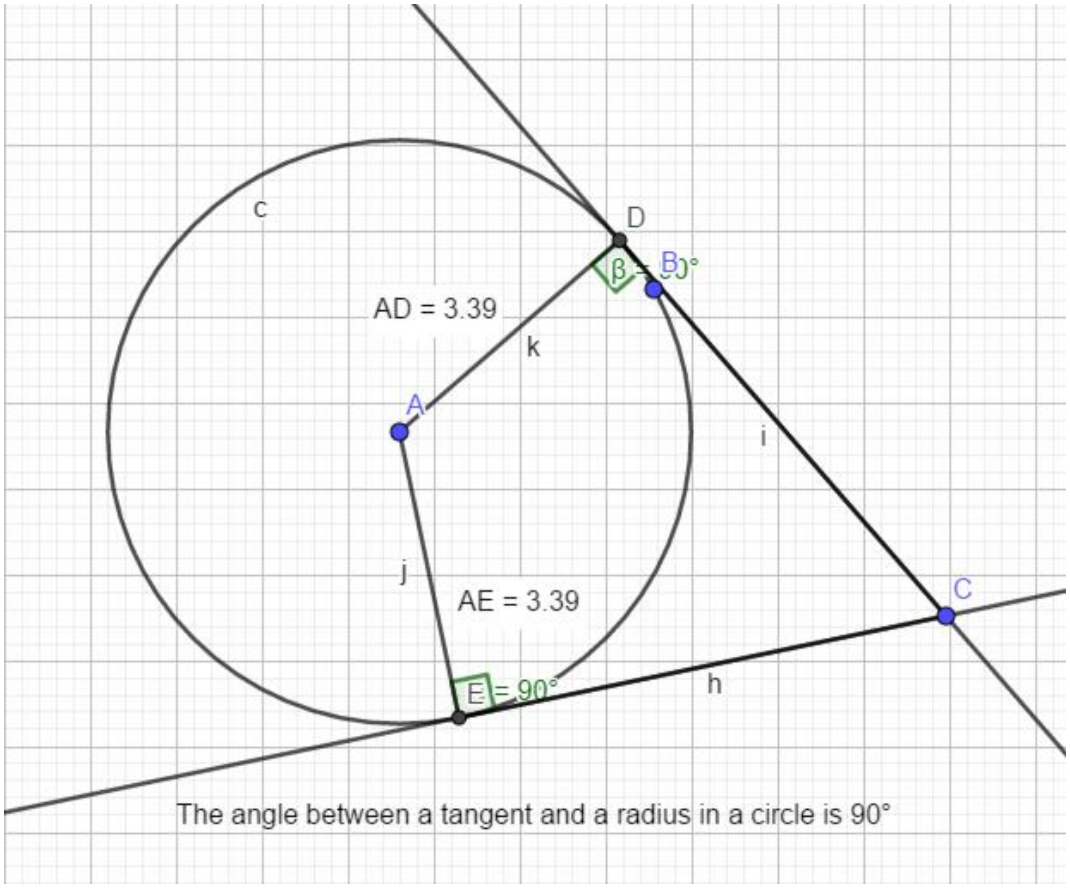

Figure 9. Circle theorem 6 (tangents from point to circle II)

Linking the Circle Theorem 5 and Circle Theorem 6 was to help students understand the concept of tangents on circles. The students were asked to draw diagrams of kind expressing right angles between the radii of the circle and tangents, which illustrates the sixth theorem. Almost all the students were able to deduce the right mathematical relationships of the Circle Theorem 6. The concept of the Circle Theorem 5 and Circle Theorem 6 enabled the researcher to introduce the seventh theorem (Circle Theorem 7) to the students. Circle Theorem 7 was illustrated as the angle between the tangent and the chord at a point on the circumference of a circle is equal to the angle in the alternate segment (see Figure 9). In explaining Circle Theorem 7 to the students, the *GeoGebra* was used to show a clear indication of the angle between the tangent and the chord and the angle at the alternate segment. At this point, students were able to use *GeoGebra* to draw a diagram showing all the three theorems taught in the lesson.

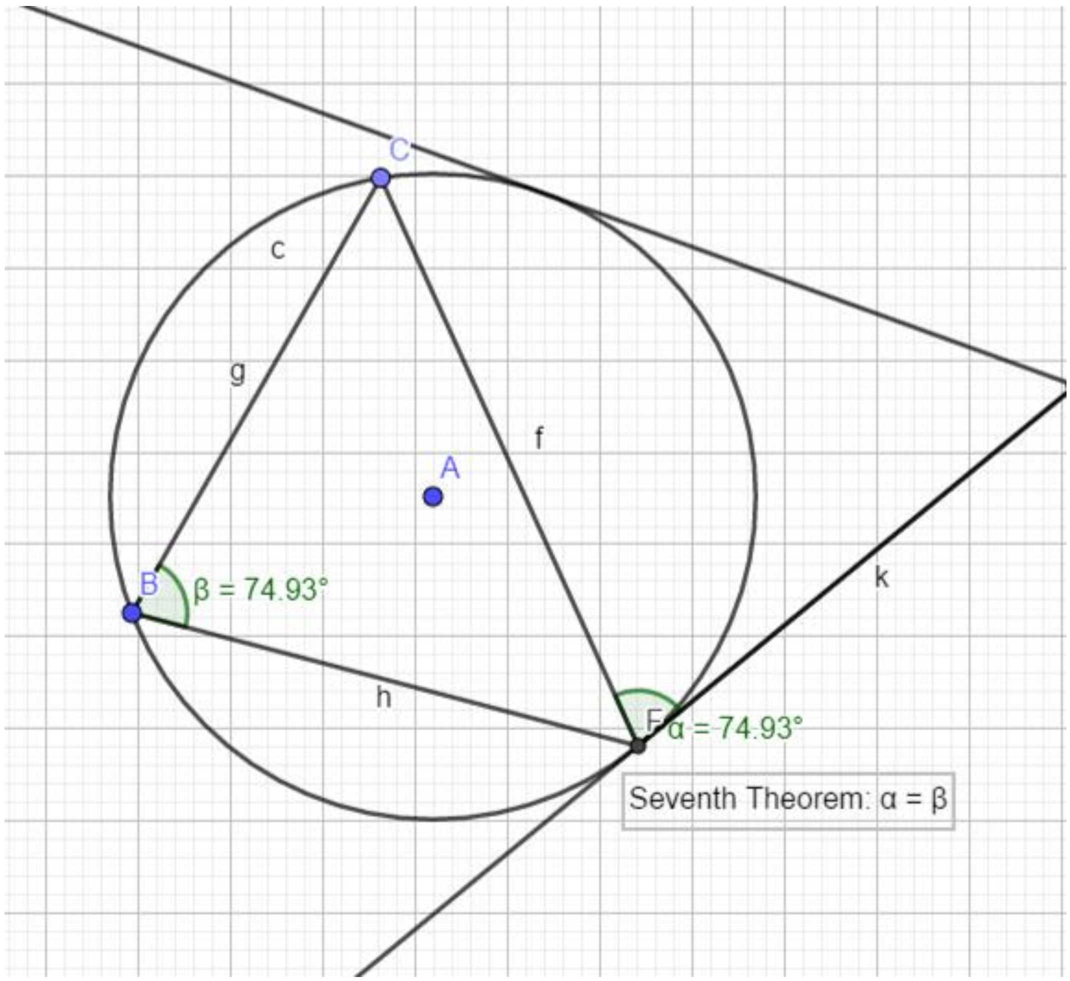

Figure 10. Circle theorem 7 (alternate segment theorem)

Circle Theorem 8 was then explained to students. A diagram was used to explain Circle Theorem 8 as the perpendicular bisector that passes through the centre of a circle makes a right angle with a chord in that circle (See Figure 11). Some of the students easily understood that the Circle Theorem 8 from their knowledge of perpendicular bisector in construction in basic school. It was observed that most of the students could easily replicate this theorem in a number of ways with *GeoGebra*. The lesson was concluded by reviewing all the four theorems taught in the lesson.

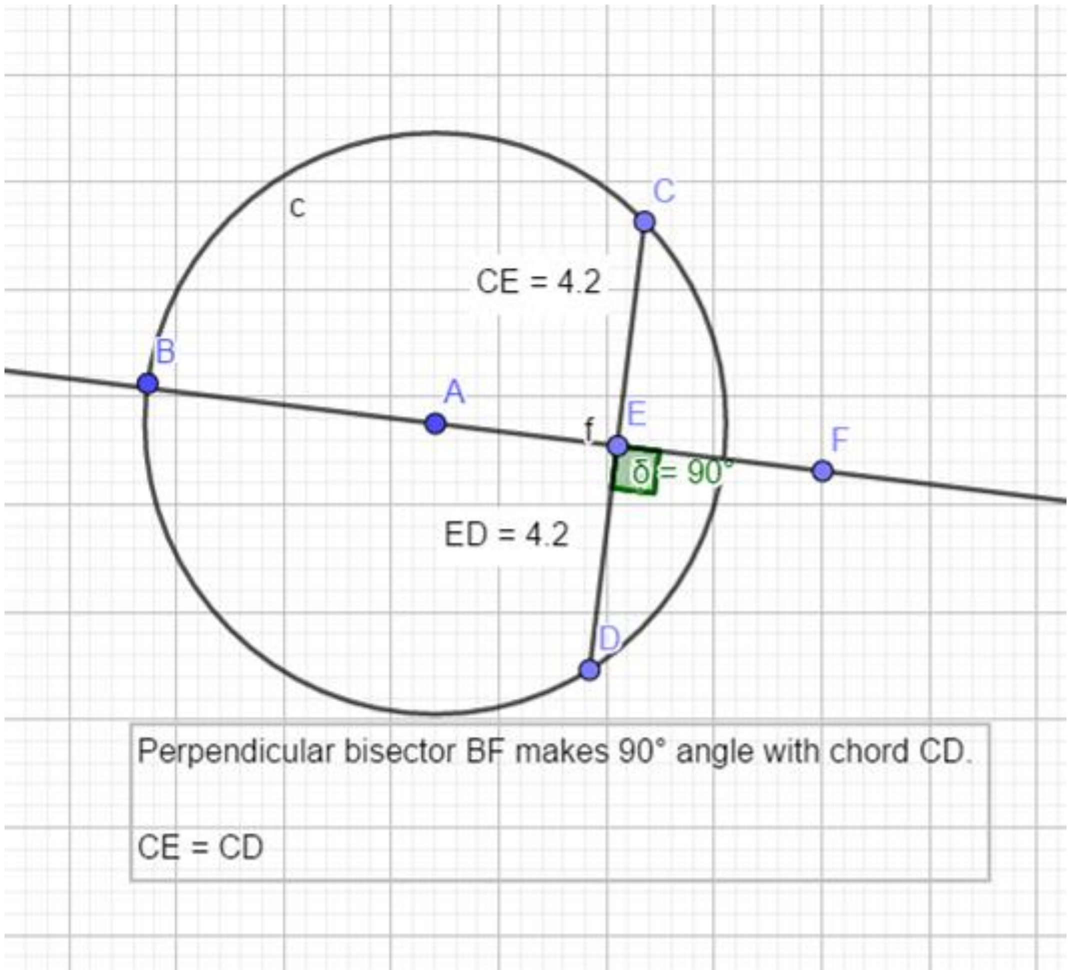

Figure 11. Circle theorem 8

Fifth Lesson

The fifth lesson involved two activities designed to review and improve students understanding of the Circle Theorems. In the first activity, students were asked to use the *GeoGebra* software to explore and come out with their own diagrams illustrating any of the theorems. This exercise required students to work in groups. Here, it was observed some students (15 students) finished this task in less than 30 minutes. In the second activity, students were shown a diagram of a circle problem in which the students were required to reveal the mathematical relationships that illustrated any of the circle theorems (see Figure 12). Students were put in eight groups for this activity. Four groups were able to come with the right theorems found in the diagram.

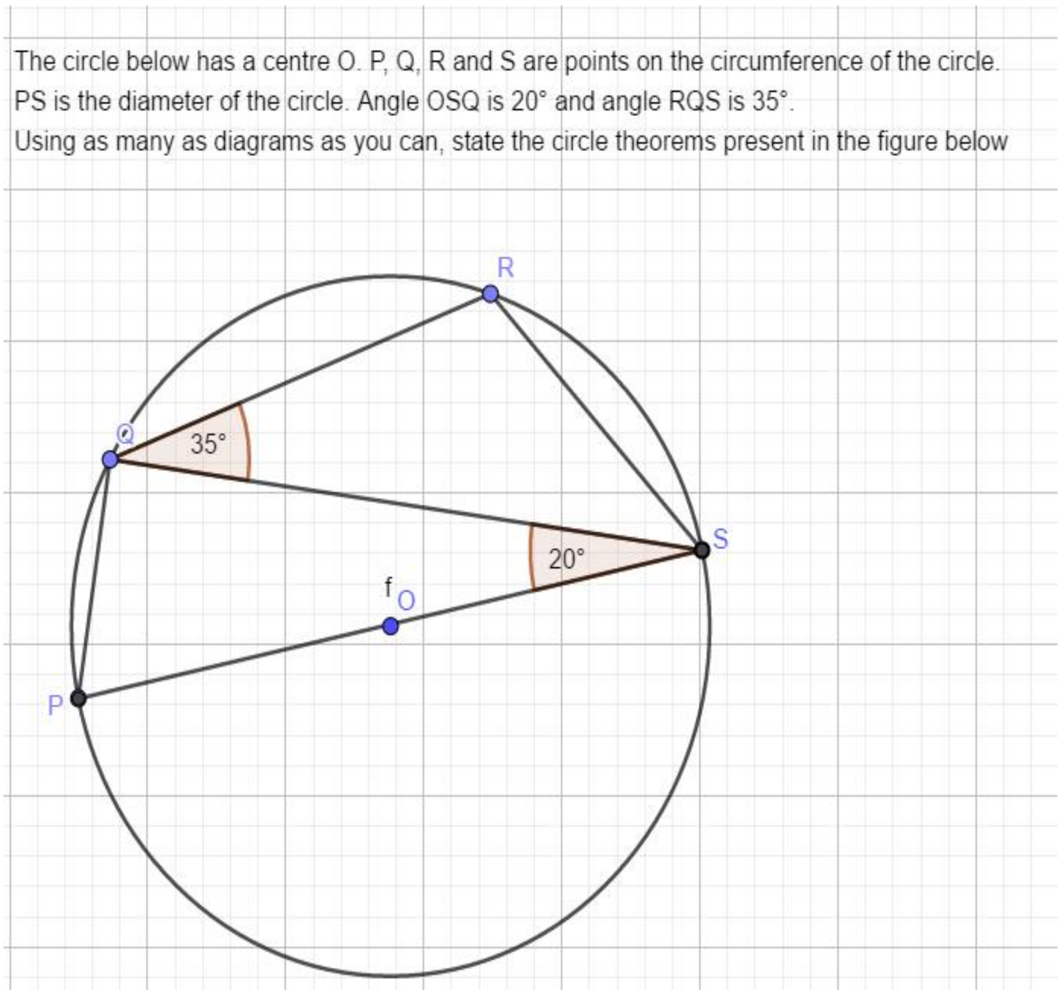

Figure 12. Second activity, lesson 5 (experimental group).

The researcher then showed two different diagrams of the same circle problem illustrating Circle Theorem 2 and Circle Theorem 4 present in the circle problem (See Figures 13 & 14). After showing these diagrams to students, they were able to state the right mathematical expression for each theorem. The lesson ended with the researcher pointing certain issues in trying to recognise a circle theorem in a given diagram.

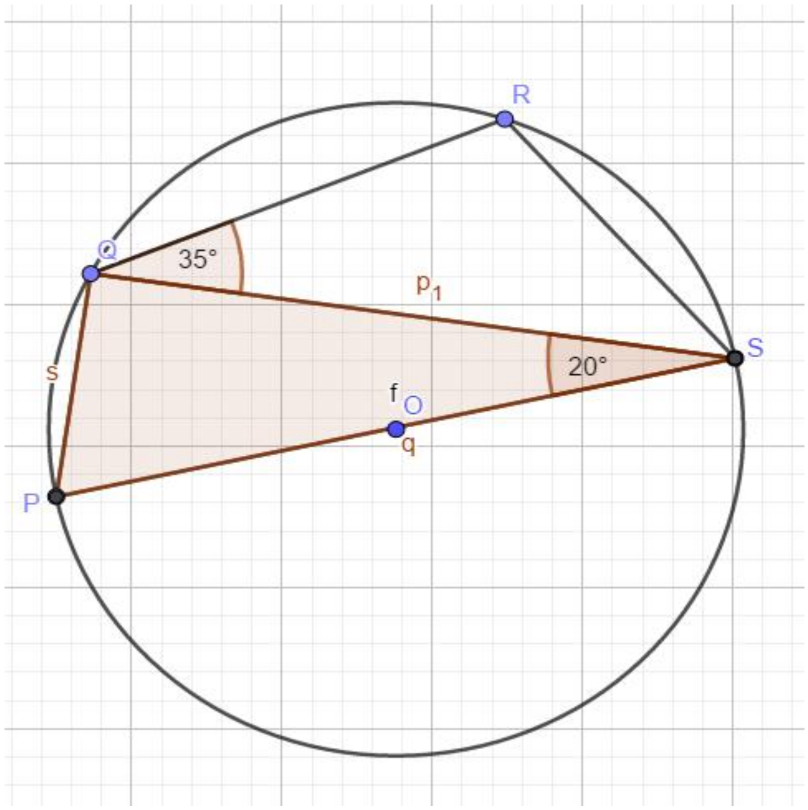

Figure 13. Diagram of circle problem illustrating circle theorem 2.

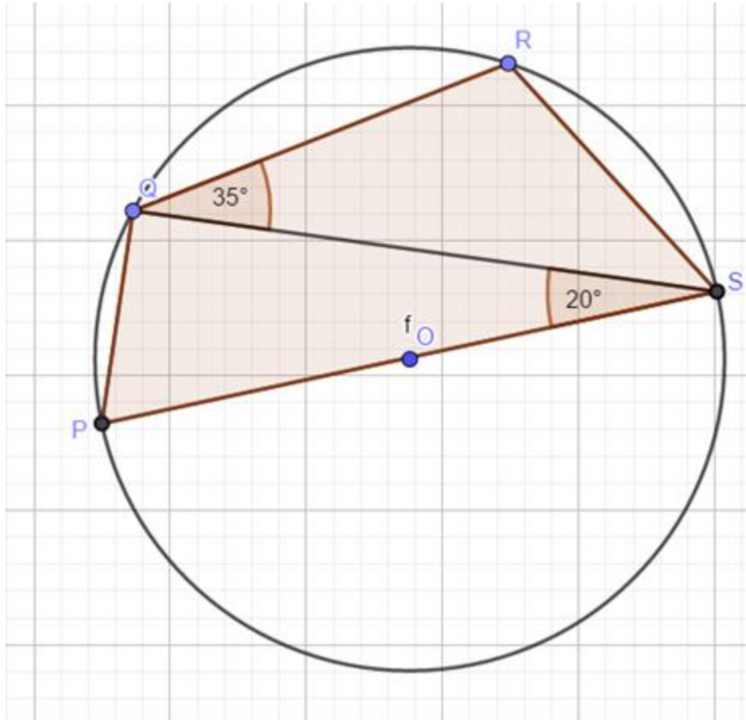

Figure 14

Sixth Lesson and Seventh lesson

In the sixth lesson, students were given an activity that required students to write mathematical relationships in a worksheet (Appendix F) provided to them. The goal of the first activity was to enable to recognise circle theorems in circle diagrams. After the first activity, a second activity was also given to students find certain angles in a worksheet (Appendix G). The goal of the second activity was to help students use their knowledge of circle theorem to find the angle marked α in the circle diagrams. The sixth lesson was concluded by going over students' responses and ensuring that all corrections to any error were made.

The seventh lesson was used to solve questions involving circle theorems without the use of the *GeoGebra* software. The students were required to apply the knowledge and understanding of the eight theorems to solve the sample questions.

This exercise lasted for 30 minutes. The rest of the lesson period was used to allow students to correct any errors they made in answering the questions.

Control group

First and Second Lesson

The first lesson began with the researcher reviewing relevant previous knowledge on Plane Geometry. Students were asked to use of the instruments in the mathematical sets provided for them in drawing diagrams of an acute angle, obtuse angle and a reflex angle. This was done to help students familiarise themselves with the use of the mathematical instruments in drawing geometric diagrams. The researcher then introduced the Circle Theorem to the students. Circle Theorem 1 was explained to the students on the marker board. The researcher used a variety of diagrams to explain the different ways the central angle can be illustrated.

Students were asked to demonstrate these diagrams on their own with the use of the mathematical instruments. The goal was to ensure students are able to make accurate measurements of the angles in the drawn diagrams. It was observed that on the first try, few students were able to successfully draw a diagram to illustrate the theorem and accurately measure the angles in the diagram. Most students, on the other hand, found the use of the mathematical instruments to draw to be frustrating and hence were not able to properly draw the diagram. However, after trying a number of times, almost all the students were able to a diagram, with accurately measured angles, to illustrate the central angle theorem. Most of the students also wrote out the right mathematical relationship of the theorem of their respective diagrams. The researcher then introduced the second theorem as an extension of the first theorem. Examples of the second theorem were drawn on the marker board to show the variety of ways the second theorem can be represented. Students were able to draw the second theorem successfully and had no problem measuring the right angle and establishing the mathematical relationship of the theorem. The lesson concluded with students drawing so many diagrams of the first and second diagram on their own.

The second lesson focused on the third and fourth theorem. The theorem was introduced as another type of inscribed angle theorem. In this lesson, the researcher used drawn diagrams of the third theorem on card board to represent the theorem. Another set of diagrams with designated angles were representing the third theorem were shown to students. The students were put in groups and tasked to write out the mathematical relationships of the third theorem as shown in the diagrams. The goal of this exercise was to help students use their imagination to come out with the right theorem without drawing it first. The exercise lasted for about 15 minutes. All the groups were able to write out the mathematical relationship presented in the diagrams.

The fourth theorem was introduced with an explanation of the properties of quadrilaterals. The researcher explained that the sum of interior angles in a quadrilateral is 360°. The teacher explained that a quadrilateral inscribed in a circle such that all the points of the quadrilateral are on the circumference of the circle is termed a cyclic quadrilateral. The properties or mathematical features of the cyclic quadrilateral were explained to the students. The researcher then used a diagram drawn on the board to illustrate the various ways cyclic quadrilateral can be represented. The lesson concluded with students drawing examples of the cyclic quadrilateral on their worksheets.

Third and fourth lesson

The third lesson was used to teach Circle Theorem 5 and Circle Theorem 6. Using a pair of compasses and protractor, the teacher used drew a diagram illustrating the Circle Theorem 5 on the board. The diagram was used to explain the Circle Theorem 5 by stating that the lengths of the tangents to the point of circle are equal. The researcher drew another diagram of the Circle Theorem 5 on board. The students were encouraged to draw an example of the Circle Theorem 5 on their worksheet. Each student was able to draw at least two diagrams illustrating the Circle Theorem 5. The teacher used the already drawn diagrams to point out the Circle Theorem 6. Here, students were encouraged to discover the Circle Theorem 6 by measuring accurately the angle of the radius to the tangent line on the circle. It was observed that some of the students were able to establish the Circle Theorem 6 in the respective diagrams. The students that were unable to do this were given the opportunity try again. The teacher concluded the lesson by going over the two theorems and pointing out certain mathematical relationships found the diagrams students drew.

The fourth lesson focused on Circle Theorem 7 and Circle Theorem 8. Circle Theorem 7 was introduced to the students using a diagram of Circle Theorem 5. Students were asked to draw an inscribed triangle in the circle such that one point of the triangle intersected with the point of the tangent on the circle. The students were guided to find two angles that were and formed the basis of the

Alternate Segment Theorem. On the first try, few students were observed to have drawn the diagram and written the right mathematical relationships of the Alternate Segment Theorem. Students were encouraged to try drawing two other diagrams in different ways. By the end of the lesson, all the students had drawn at least one diagram of the Alternate Segment Theorem.

Fifth and Sixth lesson

The fifth and sixth lessons were a number of activities intended to enhance the understanding of the Circle Theorems. The first activity in the fifth lesson required each student to draw two circle diagrams in which each diagram illustrated two circle theorems. In this activity, students were not required to use mathematical sets to draw these diagrams. The objective of the activity was to enable students to recognise two or more circle theorems in a single circle diagram.

In the sixth lesson, the first activity (Appendix F) required students to identify the mathematical relationships of circle theorems that were present in the circle diagrams. The purpose of the first activity was to enable the students to recognise circle theorems that were present in the circle diagrams. After the first activity, a second activity was also given to students find certain angles of circle diagrams in a worksheet (Appendix G). In the second activity, students were required to solve for the angle marked α for each circle diagram on the worksheet. The second activity was to help students use their knowledge of Circle Theorems to find the angle marked α in the circle diagrams. The second activity lasted for 20 minutes. The researcher concluded the sixth lesson by going over students' responses and ensuring that students made all corrections to any error that was error in the second activity.

Seventh Lesson

The seventh lesson was used to answer questions (Appendix H) involving circle theorems. The students were required to solve the questions individually. The students were required to apply their knowledge and understanding of the eight theorems to solve the sample questions. This exercise lasted for 30 minutes. The rest of the lesson period was used to allow students to make corrections to certain errors they made in answering the questions. The lesson concluded with the researcher going over the entire topic.

Administering post-test and research questionnaire

At the end of the entire intervention period for the experimental and control groups, all the participants were administered the Post-Test to determine the effect of the intervention. Both groups were examined on the same set of questions in the post-test. The papers were marked and the marks recorded into the computer as quantitative data. After the post-test, a research questionnaire was administered to the experimental to evaluate their perception on the use of the *GeoGebra* software in teaching geometry. All data were prepared for analysis and interpretation of results.

Data Processing and Data Analysis

Multiple techniques were required for data analysis. Data analysis involved both descriptive and inferential statistics.

Descriptive statistics

Descriptive statistics were used to determine average scores for both the pre- and post-test of both experimental and control groups. Other descriptive statistical techniques such as standard deviation and range were used to determine how scattered or how clustered the scores derived from the pre- and post-tests. Histograms were also used to determine the nature of spread of the post-test marks for both the control and experimental group.

Inferential statistics

Analysing pre-test and post-test

The inferential statistics was used to analyse the data from the post-test. Inferential statistics are statistical techniques used to determine if the results from the analysis of data taken from a sample would give identical results for the whole population. (Osborne & Costello, 2009). The t-test statistical technique was used to determine whether there was a significant difference between the mean marks of the post-test of the experimental group and the control group.

Chapter Summary

This study made use of the quasi-experimental design with a non-equivalent group assignment. The study was conducted in the Bono Region of Ghana in two different schools. In selecting the sample, simple random sampling (lottery method) and purposive sampling techniques were used to select two Form 2 classes from two different schools within the Bono Region. Pre-test, post-test and the questionnaire were used to collect the data needed for the study. The study used both descriptive and inferential statistical techniques to analyse the data.

CHAPTER FOUR

RESULTS AND DISCUSSION

Introduction

In this chapter, the findings from the study are presented. In addition, the findings are discussed in relation to the two research questions and the null hypothesis that were formulated to guide the study. The findings are organised and presented using tables, figures, descriptive and inferential statistics. The chapter finally concludes with a summary of the discussion.

Preliminary Analysis

The sample size for the study was 78 students comprising of 31 female and 47 male students. The control group comprised of 40 students while the experimental group comprised of 38 students. All the students were present for the pre-test and post-test. The questionnaire was administered to all the students in the experimental group. The sex distribution of the students in both groups is shown in Table 1.

Table 1: Sex distribution

Sex	Control	Experimental	Total
Male	24	23	47
Female	14	17	31
Total	40	38	78

Source: Field Work, 2018.

The average age for the control group was 17.5 years while the average age for the experimental group was 17.9 years. The age distribution for both groups shows that the students were of similar age group.

Results of Pre-Test

The pre-test was conducted before the intervention for both groups. The purpose of the pre-test was to determine if students in both had equivalent competencies in Plane Geometry I before the introduction of the interventions. The pre-test was marked with the test having a total mark of 25 (see Appendix D).

The findings of the pre-test are shown in Table 2. The average mark for the experimental group (M = 20.06; SD = 3.371) was almost the same as the average mark for the control group (M = 19.65; SD = 3.662).

Table 2: Descriptive statistic for pre-test

Statistic	Control	Experimental
N	40	38
Mean	19.65	20.06
Standard Deviation	3.662	3.371
Minimum	13	13
Maximum	25	24
Range	12	11

Source: Field Work, 2018.

The findings from the pre-test shows that the two groups have almost equal average marks. A student in the control group had a perfect score in the pre-test. The range statistic for both the control and experimental groups was very close. The ranges were 12 for experimental group, and 11 for control group. Both groups also had equivalent standard deviations that were 3.66 and 3.37 respectively. These results show that the spread of marks from the mean marks of both groups were not much different.

In order to determine if statistically there was no difference in pre-test marks, an independent sample t-test was conducted. The results of the t-test showed that the performance (pre-test marks) of the experimental group (M = 20.06; SD = 3.66) was not statistically and significantly greater than the performance (pre-test marks) of control group (M = 19.65; SD = 3.37); $t(76) = 1.263$; $p = 0.211$.

The implication of the findings of the pre-test is that students in both groups had identical level of geometric ability before the intervention; therefore, any differences in the performance of students in Circle Theorem after the intervention is as a result of the intervention.

Difference in Academic Performance Between Students Who Are Taught Using *Geogebra* and Students Who Taught Using Conventional Method of Teaching in The Teaching and Learning of Geometry (Circle Theorems)

The findings of the post-test were used to answer the first objective of the study. The analysis of the post-test is presented in three parts; the first part presents the descriptive information for the post-test. The second part presents the analysis in which the post-test results was to test the null hypothesis set in the study. The third part presents the percentage and grade comparison of the post-test results.

The post-test was conducted to the two groups after the intervention. The purpose of the post-test was to determine the influence of the *GeoGebra* software in teaching Circle Theorems. The post-test comprised of 10 theory questions and the total marks for the test was 65 marks. All the students in both groups were present for the post-test.

Descriptive information for post-test

The findings of the post-intervention test were that the average mark ($M = 51.55$; $SD = 6.328$) for the experimental group was higher than the control group average mark ($M = 40.30$; $SD = 7.643$). The difference between the mean scores of the two groups is 11.25. This implies that the mean mark of the experimental group was better than the mean mark of the control group by 11.25 marks. The highest mark for the experimental group was 61 while the maximum mark was 55. The minimum mark for the experimental group was higher than the minimum mark of the control group. The control group obtained a minimum mark of 30 while the experimental obtained a minimum mark of 38. This shows that all the students that used the software to learn the Circle Theorems was able to get more than half of total mark.

The range of marks in the experimental group was comparatively better than the range of marks in the control group. In the experimental group, the range between the highest mark and the lowest mark was 23 while for the control group the range between the highest mark and lowest mark was 25. This shows that there was a wider difference between the maximum mark and minimum mark in the control group. The experimental group had a standard deviation of 6.328 while the standard deviation for the control group is 7.643. From the measure of the standard deviation, it can be implied that the spread of marks from the mean mark of the control group was slightly greater than the spread of marks from the mean of the experimental group. Table 3 displays the descriptive statistics of the post-test marks of both groups.

Table 3: Descriptive statistic for post-test

Statistic	Control	Experimental
N	40	38
Mean	40.30	55.15
Standard Deviation	7.643	6.328
Minimum	30	38
Maximum	55	61
Range	25	23
Skewness	0.288	- 0.423

Source: Field Work, 2018.

The measure of the skewness also gives the description of the spread of the marks. The skewness shows how symmetric the marks of post-test for both groups are. The marks for the control group had a measure of skewness to be 0.288 while The experimental group had a skewness of -0.423. This means that the distribution for the distribution of marks for the control group was slightly skewed while the marks for the experimental group is negatively skewed. Figures 15 and 16 the histograms and normal (bell-shaped) curves for the experimental group and control group respectively.

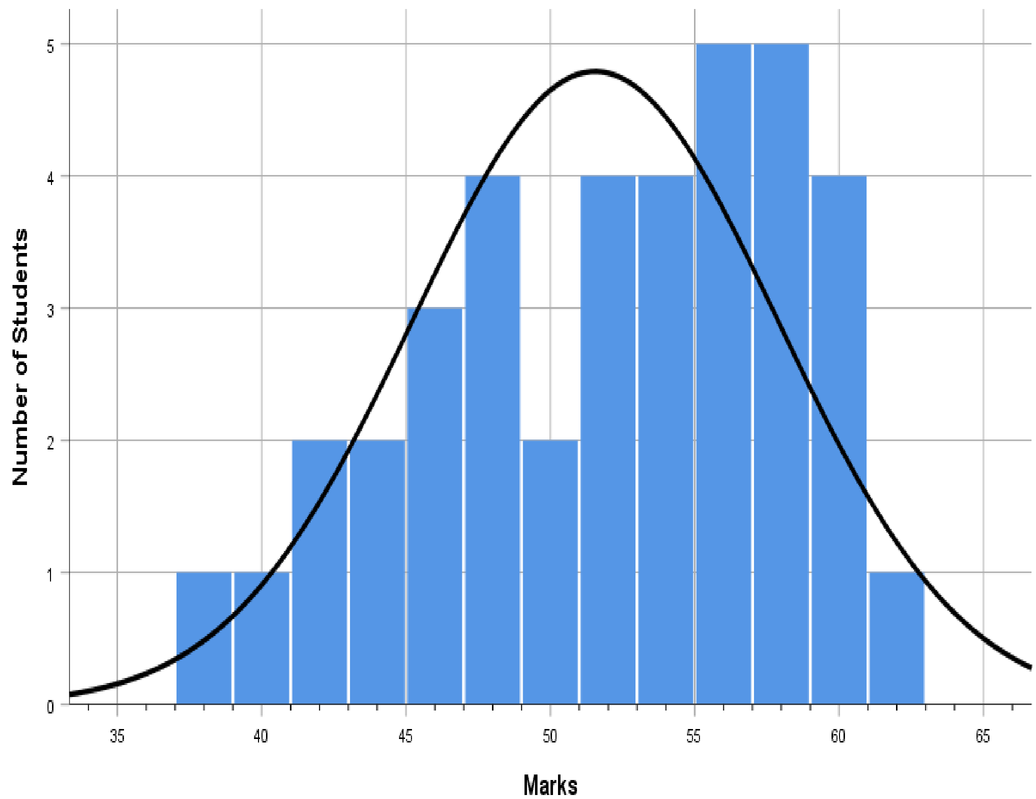

Figure 15. Histogram and normal curve of post-test marks of experimental group

Figure 15 shows that the spread of marks (that is, bell shaped curve) for students that used the software to learn Circle Theorems is not symmetrical. The distribution, which is slightly skewed to the left, indicates the most of the marks obtained in the experimental group is greater than the mean mark (that is 51.55) of that group. Viewed in this way, it can be implied that the number of students that scored marks above the mean mark is greater than to the number of students that scored below the mean mark. This accounts for the negative value of the skewness.

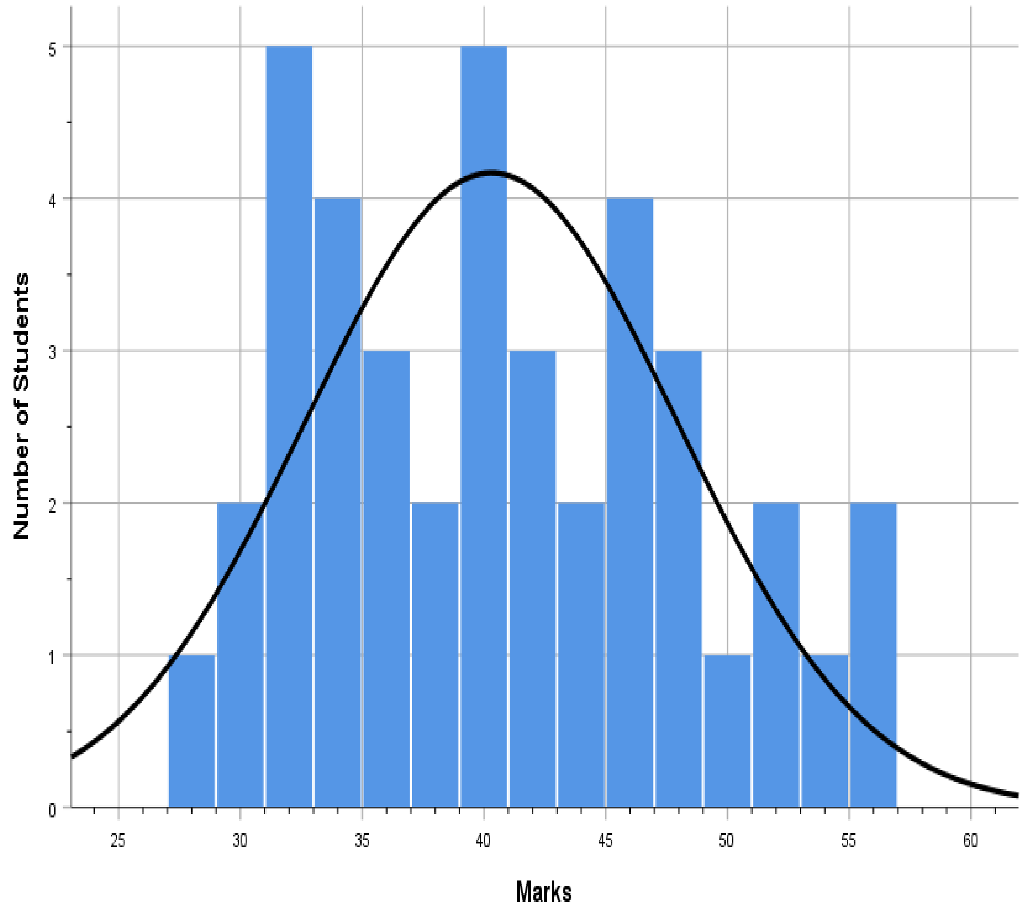

Figure 16. Histogram and normal curve of post-test marks of control group

The distribution in Figure 16 shows that the number of students in the control group who obtained marks between minimum mark (30) and the mean mark (40.30) is almost equal to the number of students that obtained marks between the mean mark and maximum mark. This accounts for the fact the bell-shaped curve is slightly skewed right, that is, the right tail of the curve is longer. This situation also accounts for the positive value of the skewness for the control group.

From the findings in Table 3, the students that used the software to learn Circle Theorem performed better than their counterparts that did not use the software to learn Circle Theorem. These findings reflect those of Arbain and Shukor (2015) who indicated that learners that used *GeoGebra* as a tool in the learning process performed better than their peers who did not use the interactive software. In their study, they pointed out that the average performance of the learners that used *GeoGebra* was better than the learners that did not use the software.

The results of the present study also reflect those of Kaya and Öçal (2018) who conducted a study on the meta-analysis of various studies on the impact of *GeoGebra* on learners' performance in mathematics. Kaya and Öçal (2018) found that in each study included in their study, the software positively influenced leaners' academic performance in mathematics. Chan and Leung (2014) also conducted a study on the meta-analysis of a number of studies on the effect of Dynamic Geometry Software (DGS) in enhancing leaners' performance in mathematics. Chan and Leung (2014) found the influence of DGS in teaching on leaners mathematics performance was comparatively better than conventional instruction. They also stated that short-term instruction with the use of DGS brings significant improvement in mathematics achievement among students. From the findings in the present study, it is evident students that used the *GeoGebra* software found it beneficial with regards to their average performance.

Determining significant difference in post-test marks

In order to determine if the average performance between the control group and the experimental group was significant, independent sample t-test was conducted for the post-test results. Preliminary analysis showed there was no violation of the assumptions of the t-test. This means that the scale of measurement was interval scale, the marks of the two groups were independent of each other, the post-test marks assume a normal distribution and there was homogeneity of variance. The t-test was conducted to determine if the performance between the two groups was significant. The following null hypothesis was tested at 95% confidence interval:

Null hypothesis (H_0): There is no significant difference in the achievement scores between students learning mathematics using the *GeoGebra* software and students not using the *GeoGebra* software to learn mathematics.

Table 4: Post-test group statistics

Group	N	Mean	Standard Deviation	T	Df	P
Control	40	40.30	7.643	7.062	76	0.000
Experimental	38	55.15	6.328			

Source: Field Work, 2018.

From Table 4, the t-test showed that the performance (post-test marks) of the experimental group ($M = 55.15$; $SD = 6.328$) was statistically and significantly greater than the performance (post-test marks) of control group ($M = 40.62$; $SD = 7.643$); $t\,(76) = 7.062$; $p = 0.000$. The magnitude of difference (mean difference = 11.253, CI = 99%: 7.043 to 15.462) in the means was very large (eta squared = 0.396).

Form the results of the t-test, the null hypothesis was rejected, because, there was a significant difference in the post-test marks between students who used the *GeoGebra* software to learn mathematics and students who did not use the *GeoGebra* software to learn mathematics.

The analysis of the t-test on the performance of students taught with the use of *GeoGebra* and students taught without the use of *GeoGebra* showed that the average (mean) performance of students taught with the use *GeoGebra* was significantly greater than the average (mean) performance students taught without the use of *GeoGebra*. This finding implies that *GeoGebra* helped students in the treatment group to have a good conceptual understanding of the diagrams in the post-test questions thus improved the methods of solutions and accuracy of their work. The ability to properly visualise geometric concepts helps learners improve in achievement tests (Shadaan & Leong, 2013; Pittalis & Christou, 2010).

For students that were taught using the conventional method, few examples were used in teaching the teaching, because explaining the theorems within the limited time frame of the lesson would require the teacher to limit the number of diagrams to illustrate each circle theorem. Limiting the number of diagrams for each theorem does not help students grasp the concepts very well. One reason students in the control group did not perform comparatively better in post-test was because students were unable to properly recognize or visualize the concept or theorem present in the question. For instance, one question most students in the control group could not answer fully was Question 3.

In the diagram above, A, B, C and D are points on a circle. $|AB| = |AC|$, $\angle 50°$ and $\angle ABD = 30°$. Calculate the value of $\angle CAD$.

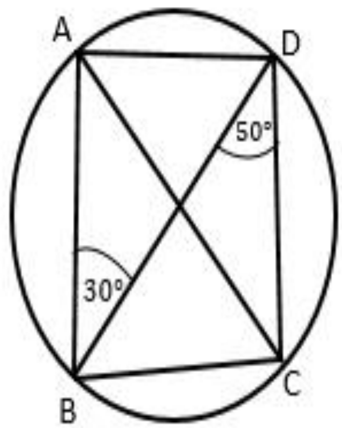

Figure 17. Question 3 of post-test

The students were expected to apply their previous knowledge of isosceles triangles to recognize the $\angle ABD$ + $\angle DBC$ as one angle of the isosceles triangle ΔABC inscribed in the circle in Figure 13. As a result, 34 out of 40 students, representing 85%, in the control group were not able to fully answer the question. However, 23 out the 40 students were able to recognise and fully stated the theorem $\angle BDC = \angle BAC$ and hence found $\therefore\ \angle BAC = 50°$.

The *GeoGebra* software allows learners to explore the different properties of circles, which improves their relational understanding of geometric concepts (Muriki, 2016). The interactive nature of *GeoGebra* allows students to examine the several geometric diagrams as well as concepts. Learning with *GeoGebra* improves students' digital exposition, which is positively associated with cognitive performance (Di Giacomo, Ranieri & Lacasa, 2017). Therefore, it becomes easier for students to assimilate geometric concepts as well as mathematical concepts.

The findings of the study reflect those of Saha, Ayub and Tarmizi (2010), Silcik and Bilgici (2011) and Zakaria (2012) which showed that a blended instruction for example, with the use of *GeoGebra* as a complement to the conventional instruction is better than the conventional instruction alone. The finding of the present study also supports Bu and Schoen (2011) research that the use of *GeoGebra* enabled students to illustrate visually various forms of a geometric concept. In the same manner, the finding of this study supports that of Zengin, Furkan and Kutluca (2012) who found a meaningful difference in mathematics achievement between students that had lessons with *GeoGebra* and the students that did not use the software. The findings are further validated by Shadaan and Leong (2013) who found out that the mathematics achievement of students taught with *GeoGebra* was statistically significant than their peers taught using the conventional approach. The results of this study and those of other studies show the potential of integrating technology into Senior High School education. The benefits of integrating technology also help students to be skilful in ICT and makes the learning environment more enjoyable.

The *GeoGebra* software can support regular teaching and learning of geometry in Senior High Schools (Tay & Wonkyi, 2018). From the present study, it was observed that with the use of *GeoGebra*, students visualising and reasoning skills improved. The software enabled students to present the geometric concepts in a number of ways that helps to enhance mathematics learning (Baki, Kosa & Guven, 2011). In the post-test, more students in the experimental group were able to demonstrate good reasoning especially in Question 7 than in the students in the control group.

In the figure, PQRS is a cyclic quadrilateral, $|QR| = |SR|$, $\angle SPT = x°$ and $\angle QSR = 37°$. Find the value of x.

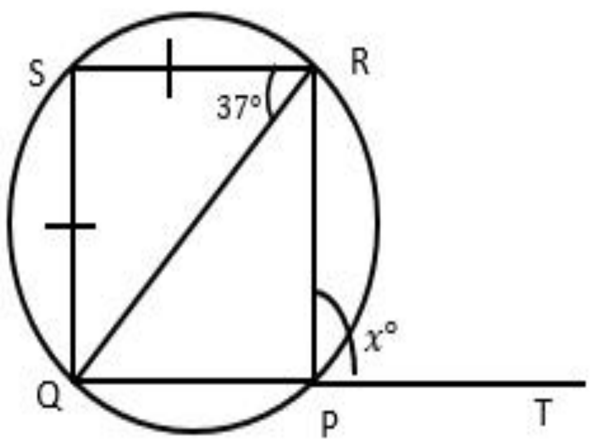

Figure 18. Question 7 of post-test

In question 7 of the post-test, students were required to demonstrate that ΔSRQ is isosceles and establish that $\angle SRQ$ and $\angle SPQ$ were opposite angles of the cyclic quadrilateral. At this point, 32 out of 38 were able to do this in the experimental group. The ability of connect these geometric concepts ultimately led them to find the final answer to the question.

Another observation was how the students demonstrated their ability to recall the circle theorems in solving the questions. For instance, most of the students that used the software, that is 36, recognised and recalled the theorem $2 \times \angle YXZ = minor\ angle\ \angle YOZ$ illustrated in the question 10.

In the diagram, X, Y and Z are points on the circle with centre O. Of $\angle YXZ = 46°$, find the reflex $\angle YOZ$.

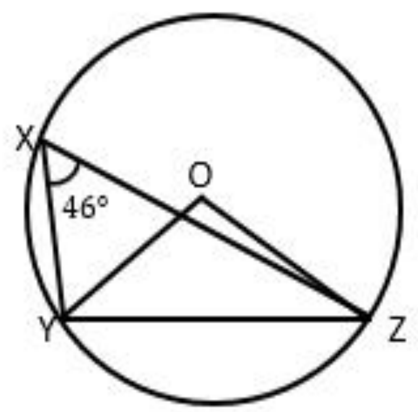

Figure 19. Question 10 of post-test

This observation reflects the findings of Saha et al (2010) who pointed out that the interactive of DGS enhanced learners' ability to recall facts and concepts. Saha et al (2010) suggested that understanding and recall of concepts is a prerequisite for a learner to be able to solve mathematical questions. Therefore, it can be inferred that the use of *GeoGebra* helped students to properly understand and recall mathematical concepts in solving mathematics problems.

As pointed out by Chimuka (2017), a blended instruction with the use of *GeoGebra* forms part of CAI packages in mathematics. The blended instruction allows students to learn through a number of activities that are applicable to different learning styles (Verhoef, Coenders, Pieters, van Smaalen & Tall, 2015). Wang, Shen, Novak and Pan (2009) also pointed out that the blended instruction with technology integration increases student interest and keep students focused in the classroom for longer periods.

Integrating *GeoGebra* in the teaching of Circle Theorems in this study demonstrated that a blended instruction with technology helps improve students' achievement in mathematics. The blended instruction with the use of technology allows teachers to differentiate and personalise teaching and learning styles. The finding of the study is further supported by Bhagat and Chang (2015) who found out that the performance of students taught with *GeoGebra* as part of a blended instruction in mathematics is better than students who were taugh with the conventional method of teaching.

Percentage comparison of post-test marks

The findings of the post-test results were also analysed by finding the percentage equivalent of the post-test results. From the findings, the percentage value of the mean mark of 79.31% for the experimental group was greater than the percentage value, 62.30% of the mean mark for the control group. Table 5 shows the findings of the percentage comparison of the post-test marks.

Table 5: Percentage comparison of post-test marks

Group	Mean Mark	Percentage
Control	40.30	62.30
Experimental	51.55	79.31

Source: Field Work, 2018.

From Table 5, it can be deduced that students that use the software perform better than their peers that did not use the software. This finding reflects the findings of Tay and Wonkyi (2018) who suggested that *GeoGebra* helps to improve mathematics achievement in Ghanaian Senior High Schools. The finding also supports Eyyam and Yaratan (2014) finding that indicated that technology helps students to perform better academically. From the findings of the study, it can be inferred that *GeoGebra* helps students perform better than they would have performed with the conventional teaching and learning approach.

Students' Attitudes toward the use of *GeoGebra* in the Teaching and Learning of Geometry

This study also assessed the attitudes of students towards the use of *GeoGebra* in the teaching and learning of Geometry (Circle Theorems). A questionnaire of 10 items was used to solicit information regarding students' attitudes toward the use of *GeoGebra* in the teaching and learning of geometry. Students responded to the statements using a 4-point forced Likert scale: *Strongly Agree*, *Agree*, *Disagree* and *Strongly Disagree*. The results of the survey are presented in order of the following themes; students understanding of the Circle Theorems, students' ability in solving problems and student interest/acceptance of the *GeoGebra*-based classroom.

Students' understanding of circle theorems

The study assessed students' perception of how they understood circle theorems with the use of the *GeoGebra* software. Question items on the students' questionnaire particularly sought information on whether *GeoGebra* helped students understood inscribed angle theorems, positions of an angle in a semicircle and segment, positions of a cyclic quadrilateral and relationship of angles in the circle and the tangent. Table 6 displays the findings of students' responses on conceptual understanding of the theorems.

Table 6: Students understanding of circle theorems

Statement	Response							
	Strongly Agree		Agree		Disagree		Strongly Disagree	
	N	%	N	%	N	%	N	%
GeoGebra helped me to understand the inscribed angle theorems.	28	73.7	10	26.3	0	0	0	0
GeoGebra helped me to visualise various positions of an angle in a semicircle and segment.	16	42.1	22	57.9	0	0	0	0
With the use of *GeoGebra*, I was able to draw various positions of a cyclic quadrilateral.	17	44.7	23	60.5	0	0	0	0
The use of *GeoGebra* helped me to understand the relationship of angles in the circle and the tangent.	21	55.3	16	42.1	1	2.6	0	0

Source: Field Work, 2018.

The findings of students' understanding of circle theorems showed that all the students that used the software to learn circle theorems agreed that *GeoGebra* enhanced their understanding of inscribed angle theorems. The students confirmed that the software helped them visualise various orientations of diagrams showing an angle in a semi-circle and segment. All the students also acknowledged that the software helped them to draw various orientations of diagrams illustrating a cyclic quadrilateral. With regard to whether the software helped students to understand the relationship of angles made between circle and tangents, all but one was in agreement.

From the findings, students perceived that learning with *GeoGebra* enhanced their understanding of the concepts in circle theorems. This reflects Barmby, Harries, Higgins and Suggate (2007) suggestion that a mathematical concept is understood when learners build mental connections of mathematical ideas. The students' perceptions implied that *GeoGebra* enabled them build the connection of two or more circle diagrams that illustrates a circle theorem. These connections enabled the students to illustrate the circle theorems with diagrams on their own. The students' ability to establish connection or relationships between geometric figures reflects Hollerbrands' (2007) study that pointed out that DGS such as *GeoGebra*, allows learners to develop relationships between geometric figures. Hiebert (2000) asserted that the strength of understanding of learners is determined by their level of understanding. It is expected that the use of *GeoGebra* can enhance students' relational understanding of geometry, which would result in improved performance in Geometry.

Students ability in solving problems

The study also sought students' opinion on how the *GeoGebra* software aided them in solving problems in Geometry. Three items in the questionnaire were formulated to find out, firstly, whether visualisations from *GeoGebra* helped students come out with the right theorems on their own. In addition, it sought to find out if by using *GeoGebra* students had fewer problems applying the theorems and lastly, whether *GeoGebra* helped the students solve problems by using the right approach. Table 7 shows the results of students' ability in solving problems.

Table 7: Students ability in solving problems

Statement	Strongly Agree		Agree		Disagree		Strongly Disagree	
	N	%	N	%	N	%	N	%
The visualisations from *GeoGebra* helped me to come out the right theorems on my own	13	34.2	22	57.9	1	2.6	2	5.3
By using *GeoGebra* I had fewer problems understanding the theorems	19	50	12	32.6	7	18.4	0	0
After using *GeoGebra*, I am able to solve problems by using the right approach.	21	52.5	13	34.2	4	10.5	0	0

Source: Field Work, 2018.

Table 7 shows the results of students' ability in solving problems. From the table, 35 students agreed that the visualisations from *GeoGebra* helped them to come with the right theorems on their own. Three students, however, did not agree that *GeoGebra* helped in developing the right theorems. Thirty – one students also indicated that they had fewer problems by using the *GeoGebra* software. Seven students, however, did not agree that by using *GeoGebra*, they had fewer problems understanding the theorems. Twenty – one students also indicated that they were able to use the right approach in solving problems after learning with *GeoGebra*, while four students disagreed that they were able to use the right approach after using *GeoGebra*.

From the findings of the study, it can be inferred that students find geometry easier to learn when the diagrams that illustrate the geometric theorems are presented using a number of diagrams of different visualisations. Diković (2009) suggested visualisations from DGS Software helps students to grasp the different concepts that are otherwise difficult to understand. Strausova and Hasek (2013) also suggested that pictures and diagrams are essential in helping learners understand various mathematical concepts and the use of a proper diagram can help students establish the proof of a mathematical property or theorem. They pointed out that mathematical proof that is established or presented by diagrams and physical demonstrations are more appealing to learners than proofs introduced to them in written form. The argument by Strausova and Hasek (2013) is reflected in the responses from the students in the experimental group' where most of the students indicated that the visualisations from the *GeoGebra* software made them deduce the right circle theorems.

Certain learning obstacles are eliminated or reduced during the learning of mathematical concepts when using *GeoGebra*, especially those that are abstract to understand with the use of board and maker (Gono, 2016; Mukiri, 2016). Some of these learning obstacles include uncertainty of learning task difficulty in reaching the learning goal and poor connectivity of learning task to concrete objects. Duval (2006) asserted that when learning obstacles are reduced by the use of either a teaching strategy or teaching aid, students encounter fewer or no problems in understanding mathematical concepts. Dixon and Brown (2012) also pointed out that when students encounter little or no problem in learning geometric concepts, their ability to solve problems is improved. This assessment by Dixon and Brown (2012) is reflected in the fact that most of the students agreed that they were able to use the right approach in solving problems.

Students interest in the lesson

Data on how students interest or appreciation of the use of the *GeoGebra* software in teaching and learning was sought with the use of the research questionnaire. Three question items in the questionnaire sought information on how students' interest of the *GeoGebra* lesson. The three items were formulated to find out, firstly, whether the teacher interacted more with the students during the class lessons and if the students enjoyed the class lesson with the use of the *GeoGebra* software. The items were further used to find whether the students felt confident in their ability to solve questions in circle theorems in the future. Table 8 displays the findings of students' interest of the lesson.

Table 8: Students interest/acceptance of the lesson

Statement	Strongly Agree		Agree		Disagree		Strongly Disagree	
	N	%	N	%	N	%	N	%
The teacher interacted more with me during the class lessons.	22	57.9	13	34.2	3	7.9	0	0
I enjoyed the class lesson with the use of *GeoGebra*.	19	50	16	42.1	2	5.3	0	0
After the class lessons I felt	18	47.4	20	52.6	0	0	0	0

confident in my ability to solve questions in circle theorems.	

Source: Field Work, 2018.

From Table 8, the findings showed that 35 students agreed that the teacher interacted more with them during the lesson. However, three students indicated that the teacher did not interact more with them. Thirty-five students also indicated that they enjoyed the lesson with the use of *GeoGebra* while two students did not agree. All the students agreed that they were confident in their ability to solve circle theorems when they used the *GeoGebra* software.

The findings of this study showed that a blended instruction which incorporates *GeoGebra* in teaching creates the appropriate environment for teacher to interact with students more. By using interactive software, teaching and learning process becomes an activity – form of learning where students are engaged with other students and with the teacher to explore the geometric theorems (Laborde, Kynigos, Hollebrands, & Strässer, 2006). Teacher – students' interaction serves as an important component of mathematics education (Yeh & Santagata, 2015).

The students' views regarding incorporating *GeoGebra* in teaching highlights the positive impact it has on their attitude towards learning Geometry. With *GeoGebra*, students feel encouraged to learn Geometry and it also makes them feel confident in their ability to learn and work out Geometry problems. Hanus and Fox (2015) have suggested that students who have more interaction with the teacher during lessons tend to enjoy the lessons. This is in line with the most students' response in this study that they enjoyed the lesson with the use of the software.

The students were able to learn more on the circle diagrams in a much more interactive way. This interactive way of learning probably accounts for the students' assessment that they enjoyed the lesson with the use of *GeoGebra* software. The use of *GeoGebra* in teaching and learning adds an important aspect to students' learning; which is students' interest in learning. Since *GeoGebra* is an interactive programme, its usage in the classroom engages students to practise the constructions of a number of circle theorems on their own.

The consistent practice of these circle theorems helps students develop more interest in the mathematics subject as well as enjoy the class lessons (Stigler & Hiebert, 2009). The opportunity and medium *GeoGebra* creates for students to practise circle diagrams and theorems also help them develop confidence in their ability to solve geometric problems. All the students indicated that they felt confident in the ability to solve questions in Circle Theorems after being taught with *GeoGebra*. This finding reflects Arbain, and Shukor (2015) assessment that *GeoGebra* enhances motivates students to learn mathematics and increases their interest in the subject.

Chapter Summary

The study sought to find out the influence of using *GeoGebra* on senior high school students' achievement in geometry. The findings from the study have shown that *GeoGebra* can help students to gain significant achievements in geometry. The findings showed that the students who used the software performed better than the students who did not use the software to learn geometry. The findings also revealed thatthe use of *GeoGebra* increased the interest of students' in learning circle theorems.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Overview

This chapter gives an overview of the purpose of the study, methodology and highlights findings of the study. Conclusions based on the findings of the study are offered. The chapter concludes with recommendations to draw attention to the implications of incorporating *GeoGebra* into the teaching of mathematics and provides suggestions for further research.

Summary of the Study

This study sought to find out the influence of *GeoGebra* on the performance of senior high school students' in Geometry in the Bono Region of Ghana. This was done by answering two research questions and a hypothesis. The two research questions and the hypothesis were stated as:

Research Question 1: What is the difference in academic achievement between students who are taught using *GeoGebra* and students who are not taught using *GeoGebra* in the teaching and learning of geometry?

Research Question 2: What are students' attitudes toward the use of *GeoGebra* in the teaching and learning of geometry?

Hypothesis H_o: There is no significant difference in the achievement scores between students learning mathematics using the *GeoGebra* software and students not using the *GeoGebra* software to learn mathematics.

This study was conducted using quasi-experimental research design. The research design involved non-random assignment of participants into two groups namely treatment (experimental) and control groups. Purposive sampling was used in determining the two study sites and the two classes used for the study. 78 participants from the two study sites were used for the study. The study involved the four stages; the pre-test, intervention (or treatment stage), post-test and the administration of the research questionnaire. Analysis of data gathered from the pre-test and post-test was used to answer the first research question and hypothesis of the study. The second research question was answered from the analysis of the data retrieved from the research instrument administered. The analysis of data from the pre- and post-test was done using frequencies, percentages, means, standard deviations, range, skewness and histograms and t-test. Frequency distribution tables were used to analyse the data from the questionnaire.

Key Findings

The key findings are presented in this section. The study revealed three key findings of the study. The key findings are discussed as follows:

1 The mean performance of students that were taught with the software was better than the mean performance of students that were taught with the conventional method.

2. There was a significant difference in achievement between the treatment group and the control group, with the treatment group performing better the control group.

3. The *GeoGebra* software improved students' understanding of circle theorems and their ability to solve problems in circle theorems. The

findings also revealed the use of *GeoGebra* increased students' interest in learning circle theorems.

Conclusions

Three conclusions were drawn findings of this study. Firstly, it can be concluded that *GeoGebra* as a teaching and learning aid helps improves academic performance in mathematics. This implies that when senior high school students are taught mathematics with the use of *GeoGebra*, the students perform better as compared to being taught using the conventional method of teaching.

Secondly, with the use of *GeoGebra* in teaching and learning of mathematics, students are able to improve their understanding of geometric concepts and ability in solving problems. This implies that significant improvement in performance of students is attributed to students' ability to establish the relationships that exist between concepts and theorems in mathematics.

Thirdly, the application of *GeoGebra* in the teaching and learning of circle theorems motivates students towards learning mathematics in general. This means that using *GeoGebra* to teach Geometry as a blended instruction increases students interest in the subject.

The findings of the study show that innovative methods (such as incorporating *GeoGebra* into teaching and learning) have proven to be more effective than conventional teaching method and as such, it can be integrated into classroom teaching as blended instruction. Previous studies are also favourably disposed to integrating *GeoGebra* into mathematics education. It is therefore concluded that *GeoGebra* helps to improves senior high school students' performance in Geometry, particularly Circle Theorems.

Recommendations

From the findings of this study, the following recommendations are made for Ghana Education Service and other stakeholders for application.

1. Introducing pre- and in-service teachers to *Geogebra* as an innovative way to teach mathematics will be most helpful in raising the performance of senior high school students. This introduction could be done through workshops and seminars organised by Ghana Education Service and/or other stakeholders in education. This will help enhance the teaching skills and strategy of teachers in mathematics.
2. Integrating the use of computers and interactive educational software into the Mathematics Senior High School curriculum by curriculum developers could also help teachers guide students to understand of geometric concepts and improve students' ability in solving mathematics problems. This will help to improve students' performance significantly.
3. Interactive educational software and computers could be incorporated into teaching and learning activities. This will help make learning mathematics more interesting to students. This could be done by providing resource materials such as educational applet devices, computers and mathematical instruments.

Suggestions for Further Studies

The following are suggestions for further research:

1. Further studies with the use of *GeoGebra* could be done in other areas in mathematics where students have demonstrated poor performance such as trigonometry.
2. This study was limited to only two Form Two classes from two senior high schools. Further studies could be replicated for a much larger sample for better generalisation.

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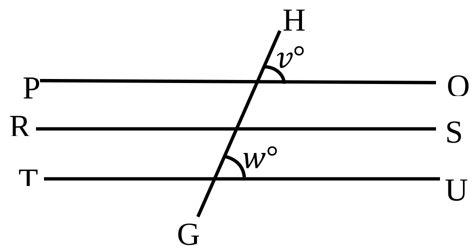
APPENDICES

APPENDIX A

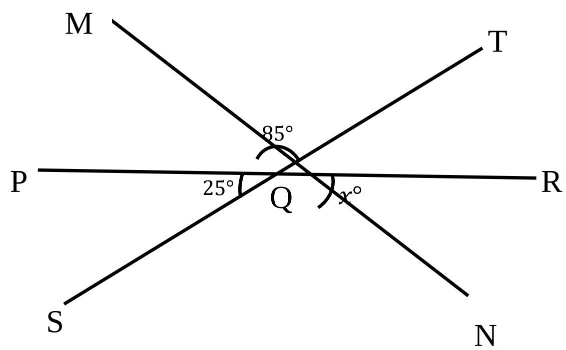
PRE-TEST

1.



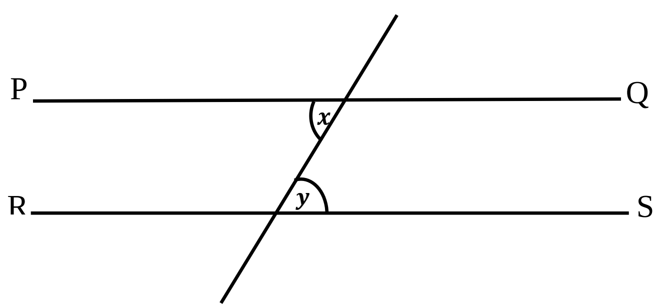
PQ, RS, TU and GH are straight lines, $PQ \parallel RS \parallel TU$. What kind of angles are V and W?

2.



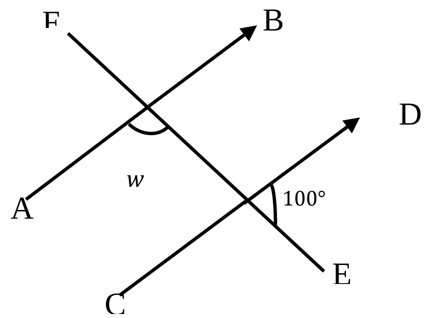
PQR, SQT, MQN are straight lines. $\angle PQS = 25°$ and $\angle MQT = 85°$. Find the value of x.

3.



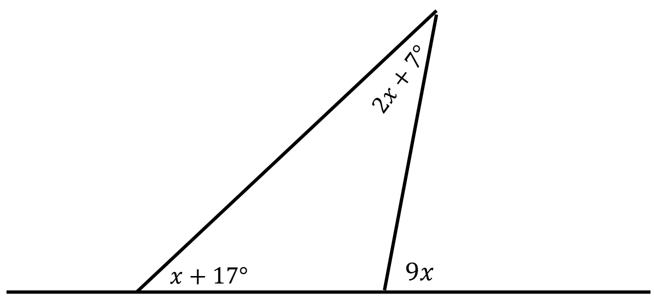
In the diagram $PS \parallel RS$. What are angles x and y?

4.



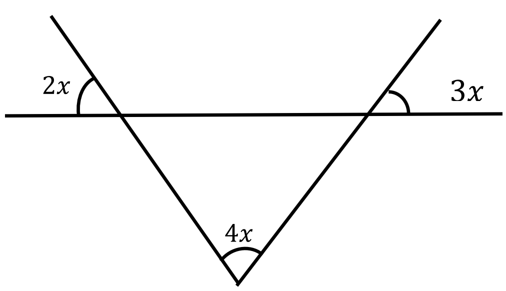
In the diagram AB and CD are parallel lines. What is the value of w?

5.



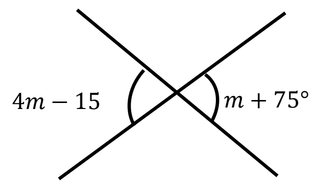
Find the value of $9x$ in the diagram.

6.



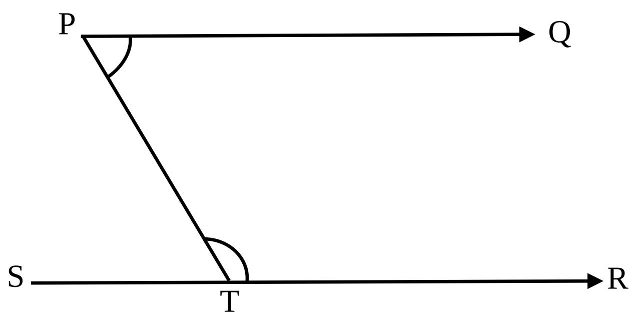
Calculate the value of x in the diagram above.

7.



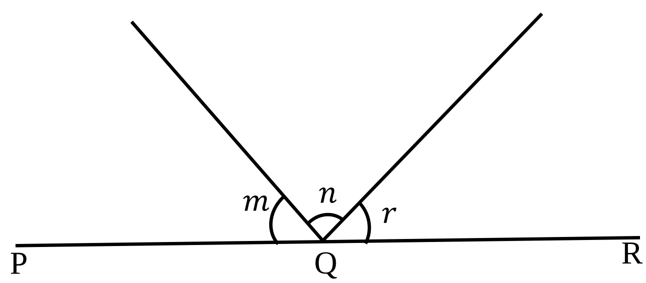
What is the value of m in the diagram above?

8.



In the diagram above, $PQ \parallel SR$ and $\angle PTR = 128°$. What is the size of $\angle QPT$?

9.



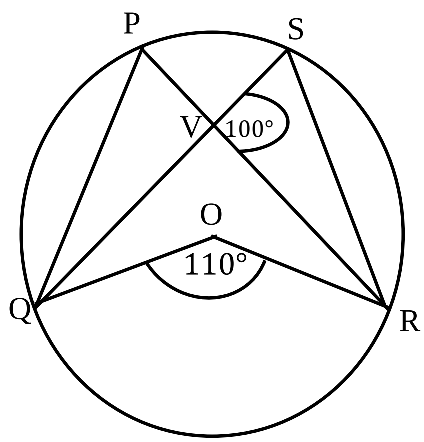
In the diagram, PQR is a straight line, $(m + n) = 120°$ and $(n + r) = 100°$. Find the value of $(m + r)$.

10. What are the interior angles on the same side of a transversal on two parallel lines?

APPENDIX B

POST-TEST

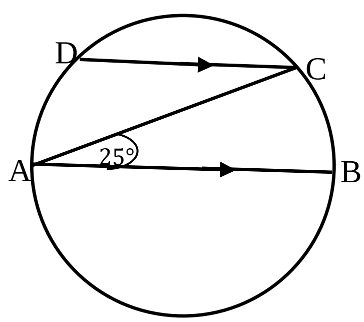
1. In the diagram, PQRS is a circle with centre O. QS and PR interest at V. $\angle POR = 110°$ and $\angle RVS = 100°$. Calculate $\angle PQS$.



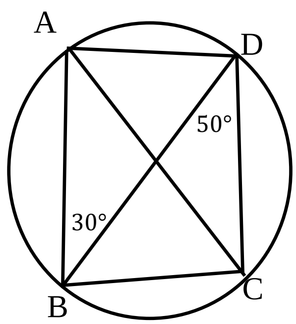
2. In the diagram (not drawn to scale), AB is a diameter of the circle ABCD. DC is parallel to Ab

   and $BAC = 25°$.

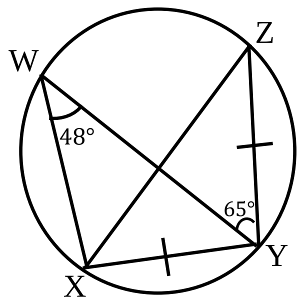
   Calculate $\angle CAD$.



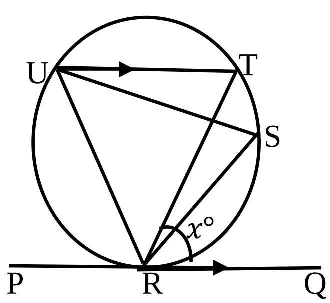
3. In the diagram above, A, B, C and D are points on a Circle. $|AB| = |AC|$, $\angle BDC = 50°$ and $\angle ABD = 30°$. Calculate the value of $\angle CAD$.



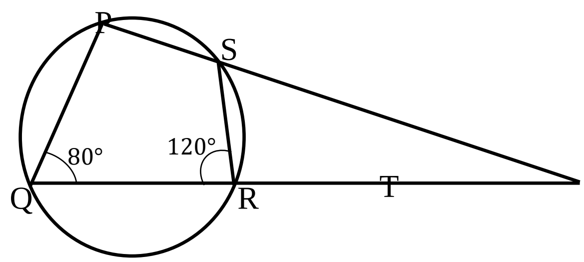
4. In the diagram, |ZY|=|XY|, ∠WYZ=65° and ∠XWY=48°. Find ∠WYX.



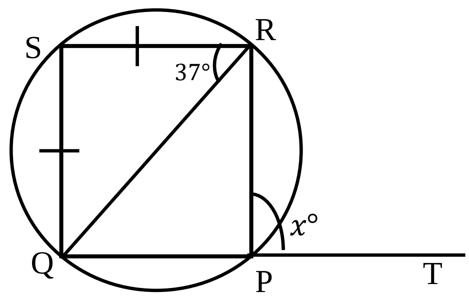
5. In the figure, PQ is a tangent to the circle at R and UT is parallel to PQ. If $\angle TRQ = x°$, find $\angle URT$ in term of *x*.



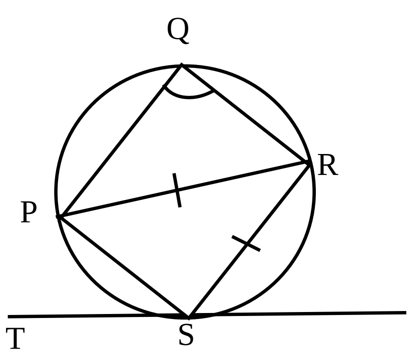
6. In the diagram, PQRS is a cyclic quadrilateral. PS and QR are produced to meet T. If $\angle PQR = 80°$ and $\angle QRS = 120°$, find $\angle RTS$.



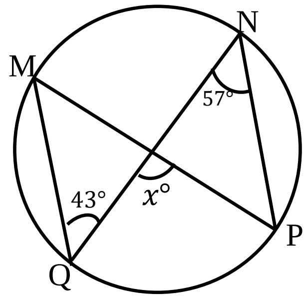
7. In the figure, PQRS is a cyclic quadrilateral, $|QS| = |SR|, \angle RPT = x°$ and $\angle QSR = 37°$. Find the value of x.



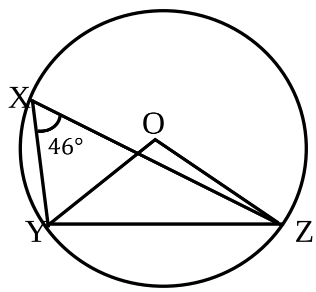
8. In the diagram, TS is a tangent to the circle at S. $|PR|$ = $|RS|$ and $\angle PQR = 117°$. Calculate $\angle PST$.



9. Find the value of x in the diagram above.



10. In the diagram, X, Y and Z are points on the circle with centre O. If $\angle YXZ = 46°$, find the reflex $\angle YOZ$.



APPENDIX C

QUESTIONNAIRE FOR STUDENTS

This questionnaire is supposed to provide data to be used in a research. Please kindly answer the questions as frankly as possible. Your confidentiality is assured.

Tick (√) the appropriate option.

i. Sex: Male[ ] Female[ ]

ii. Age: …………………………………………….

iii. Form: …………………………………………….

Tick (√) one option for each statement to indicate your degree of agreement with each statement.

	Statement	Strongly Agree	Agree	Disagree	Strongly Disagree
1	*GeoGebra* helped me understand the inscribed angle theorems.				
2	*GeoGebra* helped me visualise various positions of an angle in a semicircle.				
3	With the use of *GeoGebra* I was able to draw various positions of a cyclic quadrilateral.				
4	The use of *GeoGebra* helped me understand the relationship of angles in the circle and the tangent				

5	The visualisations from *GeoGebra* helped me come out the right theorems on my own				
6	By using *GeoGebra* I had fewer problems understanding the theorems.				
7	After using *GeoGebra*, I am able to solve problems by using the right approach.				
8	I enjoyed the class lesson with the use of *GeoGebra*.				
9	I am happy if the teacher uses *GeoGebra* in teaching Geometry.				
10	After the class lessons I felt confident in my ability to solve questions in circle theorems.				

APPENDIX D

MARKING SCHEME FOR PRE-TEST

Solution

1. Corresponding Angles B1 1 mark
2. $\angle RQT = \angle PQS$

	$\angle RQT = 25°$ [Vertically Opposite Angles]	BI	
	$85° + 25° + x° = 180°$	M1	
	$x = 70^{\circ}$ [Correct Solving for x]	A1	3 marks
3.	*Alternate Angles*	B1	1 mark
4.	$\angle CGE = 180° - 100°$		
	$\angle CGE = 80°$	M1	
	$w = 80^{\circ}$ [Angle w corresponds with $\angle CGE$]	A1	2 marks
5.	$y = (x + 17) + (2x + 7)$	M1	
	$y = 3x + 24$	M1	
	$y = 36°$	A1	3 marks
6.	$2x + 3x + 4x = 180^{\circ}$	M1	
	$9x = 180°$	M1	
	$x = 20$	A1	3 marks
7.	$4m - 15° = m + 75°$	M1	
	$3m = 90^{\circ}$	M1	
	$m = 30°$	A1	3 marks
8.	$\angle QPT = 180° - 128°$	M1	
	$\angle QPT = 52°$	A1	2 marks
9.	$M + n + r = 180$		
	$M + n = 120$		
	$N + r = 100$	M1	
	Solving simultaneously;	M1M1	

$m = 80, r = 60$	A1	
$m + r = 140$	M1A1	6 marks
10. Supplementary Angles	B1	1 mark
Total		**25 marks**

APPENDIX E

MARKING SCHEME FOR POST-TEST

1.	$\angle QPR = \frac{1}{2} \times 110°$ $\angle QPR = 55°$ $\angle PVQ = 100°$ (Vertically Opposite) $\angle PQS = 180° - (55° + 100°)$ OR $180° = \angle PQS + 55° + 100°$ Correct solving for ∠PQS $\angle PQS = 25°$	M1 A1 M1 M1 M1 A1 **6 Marks**
2.	$\angle ACD = \angle BAC$ $\angle BAC = 25°$ $\angle ADB = 90°$ $\angle BDC = \angle BAC$ $\angle BDC = 25°$ i.e angles formed by the same chord. $\angle ADC = 90° + 25°$ $\angle ADC = 115°$ $180° = \angle CAD + 115° + 25°$ Correct Solving for ∠CAD $\angle CAD = 40°$	B B1 B1 M1 A1 M1 M1 A1 **8 Marks**

3.	$\angle BAC = \angle BDC$ $\angle ABC = \angle ACB$ $\angle ABC = \frac{180° - 50°}{2}$ $\angle ABC = 65°$ $\angle CBD = \angle ABC - \angle ABD$ $\angle CBD = 65° - 30°$ $\angle CBD = 35°$ $\angle CBD = \angle CAD$ $\therefore \angle CAD = 35°$	M1 M1 M1M1 A1 B1 M1 A1 B1 A1 **10 marks**
4.	$\angle XZY = \angle XWY$ $\angle XZY = 48°$ $\angle YXZ = \angle XZY$ $\angle YXZ = 48°$ $\angle WYX = 180° - (48° + 48° + 65°)$ Correct solving of $\angle WYX$ $\angle WYX = 19°$	B1 B1 M1 M1 M1 A1 6 marks
5.	$\angle RUT = \angle TRQ$ $\angle RUT = x$ $\angle PRU = \angle RUT$ $\angle PRU = x$ $\angle URT + x + x = 180°$	B1 B1 B1 B1 M1

	∠URT + 2x = 180° ∠URT = 180° – 2x	M1 A1 7 marks
6.	∠QPS = 180° – 120° ∠QPS = 60° ∠RTS + 80° + 60° = 180° ∠RTS+140 = 180 Correct solving for ∠RTS ∠RTS = 40°	M1 A1 M1 M1 M1 A1 6 marks
7.	∠RQS = ∠QRS ∠RQS = 37° X + 37° +37° = 180° Correct solving for x x = 106°	B1 B1 M1 M1 A1 5 MARKS
8.	∠PQR + ∠PSR = 180 Correct solving for ∠PSR ∠PSR = 63° ∠PSR = ∠RPS ∠RPS = 63° ∠PRS = 180° - (63° + 63°) Correct solving for ∠PRS	B1 M1 A1 B1 M1 M1 A1

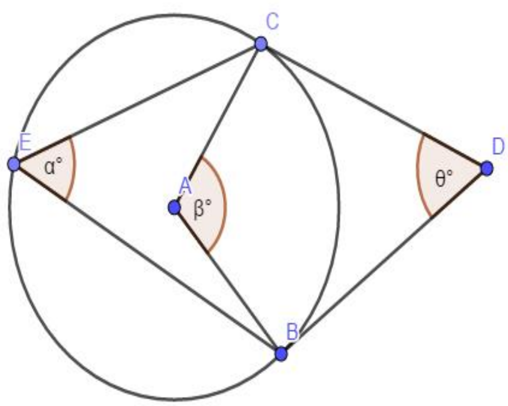
	$\angle PRS = 54°$ $\angle PST = \angle PRS$ $\angle PST = 54°$	B1 A1 9 marks
9.	$\angle PMQ = \angle PNQ$ $\angle PMQ = 57°$ $x = 43° + 57°$ $x = 100°$	B1 B1 M1 A1 4 marks
10.	$\angle YOZ = 2\ (46°)$ $\angle YOZ = 92°$ Reflex $\angle YOZ = 360° - 92°$ Reflex $\angle YOZ = 268°$	M1 A1 M1 A1 4 marks
Total		**65 Marks**

APPENDIX F

ACTIVITY I

WORKSHEET

Please write out all the circle theorems present in the diagrams below.

<table>
<tr><th>Circle Diagram</th><th>Solution</th></tr>
<tr><td>
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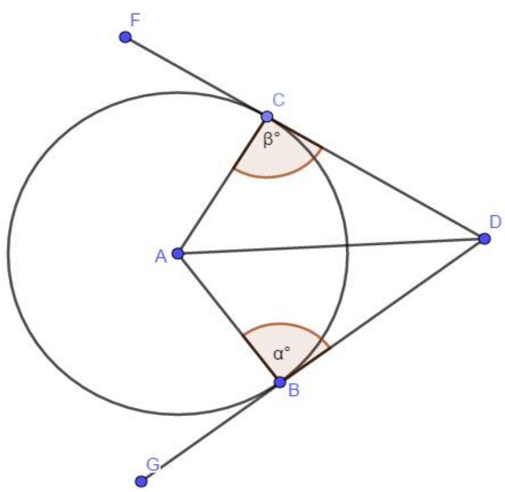
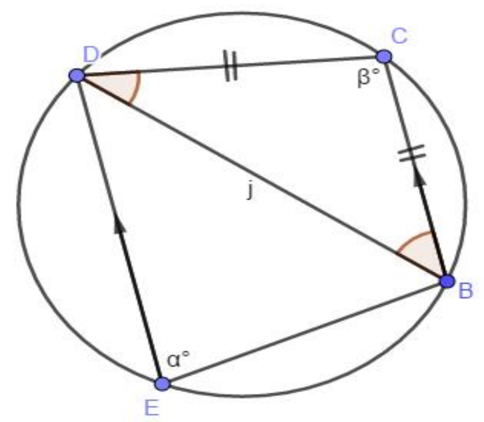
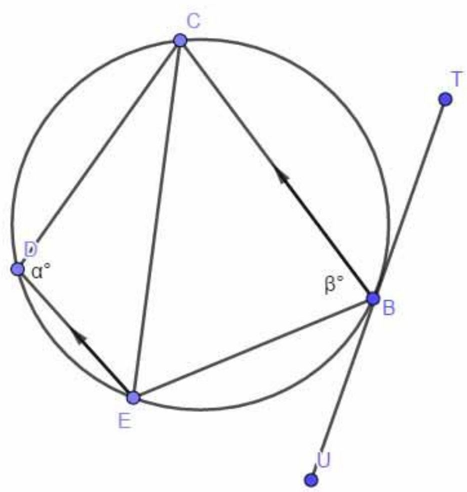
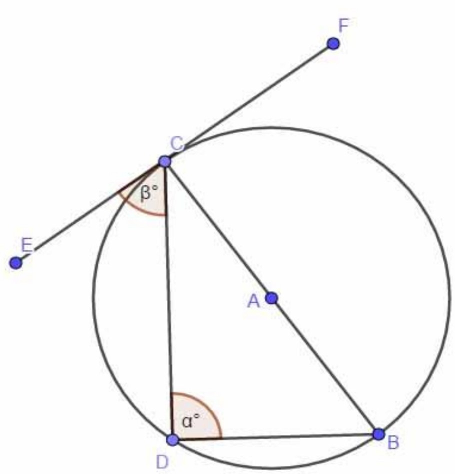
<tr><td>
</td><td></td></tr>
</table>

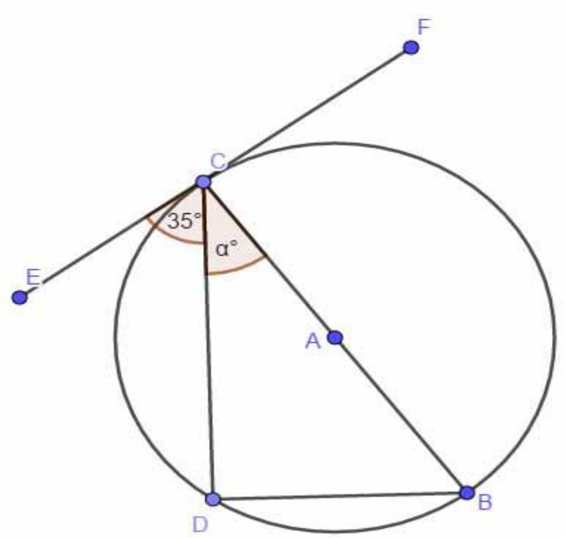
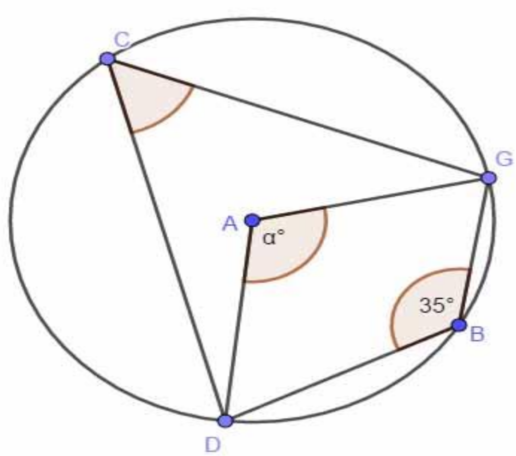
D C β° j B α° E	
C T D α° β° B E U	
F C β° E A α° D B	

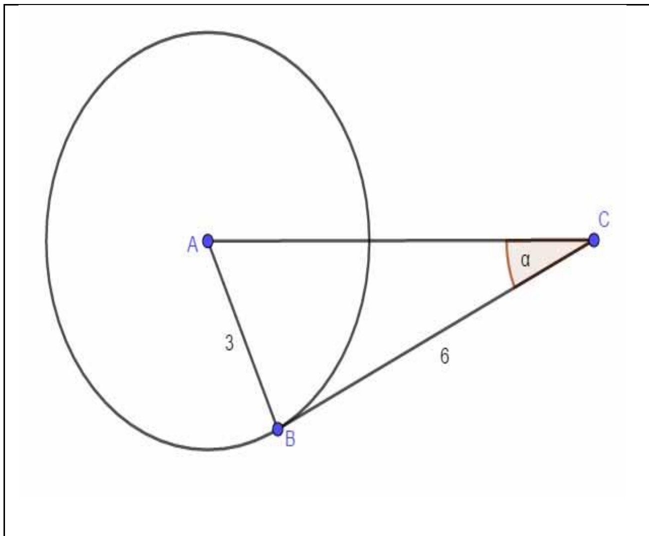
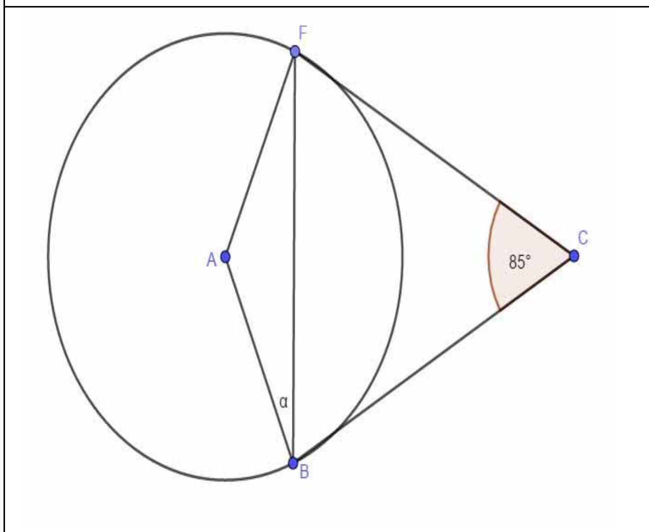
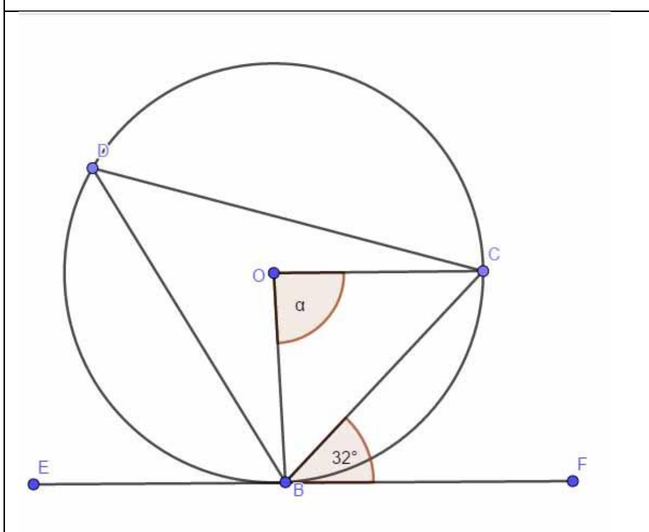
APPENDIX G

ACTIVITY II

WORKSHEET

For each circle diagram find the angle marked α and provide your reason for the answer. All the diagrams are not drawn to scale.

Circle Diagram	Solution
	
	

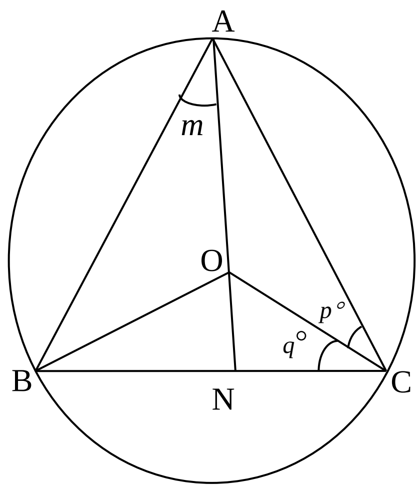
A C α 3 6 B	
F A C 85° α B	
D C O α E 32° F B	

APPENDIX H

PRACTICE EXERCISE

Answer the following questions. Please show working for each problem.

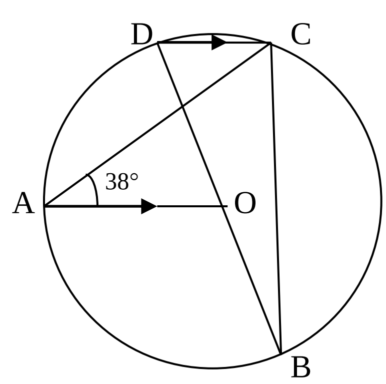
1. In the figure below, A, B and C are three points on a circle with centre O. if $\angle OAB = m°$, $\angle OCA = p°$ and $\angle OCB = q°$, show that $p + q + m = 90°$.



2. In the diagram below, A, B, C, D are points on the circumference of a circle centre O. if $|AO|$ is parallel to $|DC|$ and $\angle OAC = 38°$, find

   i. $\angle ACB$;

   ii. $\angle DBC$



APPENDIX I

ETHICAL CLEARANCE LETTER

UNIVERSITY OF CAPE COAST

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C/O Directorate of Research, Innovation and Consultancy

OUR REF: UCC/IRB/A/2016/258
YOUR REF:
OMB NO: 0990-0279
IORG #: IORG0009096

11TH JUNE, 2018

Anthony Kwadwo Badu-Domfeh
Department of Maths and ICT Education
University of Cape Coast

Dear Mr. Badu-Domfeh,

ETHICAL CLEARANCE –ID: (UCCIRB/CES/2018/04)

The University of Cape Coast Institutional Review Board (UCCIRB) has granted **Provisional Approval** for the implementation of your research protocol titled ***Incorporating GeoGebra software in the teaching of geometry on and its effect on students' performance.*** This approval requires that you submit periodic review of the protocol to the Board and a final full review to the UCCIRB on completion of the research.

The UCCIRB may observe or cause to be observed procedures and records of the research during and after implementation.

Please note that any modification of the project must be submitted to the UCCIRB for review and approval before its implementation.

You are also required to report all serious adverse events related to this study to the UCCIRB within seven days verbally and fourteen days in writing.

Always quote the protocol identification number in all future correspondence with us in relation to this protocol.

Yours faithfully,

Samuel Asiedu Owusu, PhD
UCCIRB Administrator

ADMINISTRATOR
INSTITUTIONAL REVIEW BOARD
UNIVERSITY OF CAPE COAST
Date: 11 – 06 – 18

APPENDIX J

INTRODUCTORY LETTER TO SCHOOL A

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION
DEPARTMENT OF MATHEMATICS AND I.C.T EDUCATION

Telephone: 0332096951
Telex: 2552, UCC, GH
Telegrams & Cables: University, Cape Coast
Email: dmicte@ucc.edu.gh

University Post Office
Cape Coast, Ghana

Your Ref:

Our Ref: DMICTE/P.3/V.1/014

Date: 8th June, 2018

The Headmaster
Odumaseman Senior High School
P. O. Box 938
Sunyani, B/A

Dear Sir,

RESEARCH VISIT

The bearer of this letter, Mr. Anthony Kwadwo Badu-Domfeh, with registration number ED/MDP/15/0009 is an MPhil. (Mathematics Education) student of the Department of Mathematics and ICT Education, College of Education Studies, University of Cape Coast.

As part of the requirements for the award of a master's degree, he is required to undertake a research visit to your school with the purpose of collecting data on the topic "**INCORPORATING GERBERA SOFTWARE INTO THE TEACHING OF GEOMETRY AND ITS EFFECT ON STUDENTS' PERFORMANCE.**"

I would be grateful if you could give him the necessary assistance he may need.

Thanks for your usual support

Yours faithfully,

Dr Kofi Ayebi-Arthur
SUPERVISOR

APPENDIX K

INTRODUCTORY LETTER TO SCHOOL B

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION
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Telephone: 0332096951
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University Post Office
Cape Coast, Ghana

Your Ref:

Our Ref: DMICTE/P.3/V.1/015

Date: 8th June, 2018

The Headmaster
Wenchi Methodist Senior High School
P. O. Box 88
Wenchi, B/A

Dear Sir,

RESEARCH VISIT

The bearer of this letter, Mr. Anthony Kwadwo Badu-Domfeh, with registration number ED/MDP/15/0009 is an MPhil. (Mathematics Education) student of the Department of Mathematics and ICT Education, College of Education Studies, University of Cape Coast.

As part of the requirements for the award of a master's degree, he is required to undertake a research visit to your school with the purpose of collecting data on the topic "**INCORPORATING GEOGERA SOFTWARE INTO THE TEACHING OF GEOMETRY AND ITS EFFECT ON STUDENTS' PERFORMANCE.**"

I would be grateful if you could give him the necessary assistance he may need.

Thanks for your usual support.

Yours faithfully,

For
Dr Kofi Ayebi-Arthur
SUPERVISOR