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A MARKOV REGIME SWITCHING APPROACH TO ESTIMATING THE VOLATILITY OF JOHANNESBURG STOCK EXCHANGE (JSE) RETURNS

Abstract

The study used the Markov regime switching model to investigate the presence of regimes in the volatility dynamics of the returns of JSE All-Share Index (ALSI). Volatility regimes are as a result of sudden changes in the underlying economy generating the market returns. In all, twelve candidate models were fitted to the data. Estimates from the regime switching model were compared to the industry standard non-switching GARCH (1,1) using the Deviance Information Criteria (DIC). The results show that the two-regime switching EGARCH model with skewed Student t innovations describes better the return of the JSE Index. Additionally, we backtest the model results in order to confirm our findings that the two-regime switching EGARCH is the best of the models for the sample period.

Keywords   Bayesian methodology, equity markets, Johannesburg Stock Exchange, Markov chain Monte Carlo simulation, Markov regime switching

JEL Classification   G15, G17

INTRODUCTION

The estimation of volatility of returns to financial assets has been a major preoccupation of both practitioners and academics in finance and its related disciplines ever since financial markets as we know them today existed. The work of Markowitz (1952) gave risk a renewed pre-eminence in finance, tying financial returns to the amount of risk investors assume. Risk plays an important role in today's financial markets as a pricer in derivative trading, a key metric in the regulation of financial institutions, among others. Lately, it has become a traded quantity in its own right (Whaley, 2013; Zhang & Zhu, 2006). Knowing the level of risk exposure serves as a guide to market actors in allocating their wealth among a series of risky investments depending on their preferences.

But volatility remains a somewhat elusive concept. Attempts at estimating it have spawned an industry with various models in current use in finance as each tries to capture an aspect of volatility over the years (Engle & Patton, 2007). The generalized autoregressive regressive conditional heteroscedasticity (GARCH) model of Bollerslev (1986) has served as the fulcrum around which incremental models to capture aspects such as persistence, leverage and asymmetry beyond volatility clustering that the original autoregressive conditional heteroscedasticity (ARCH) of Engel (1982) sought to capture. In the high frequency domain, practitioners and researchers use realized volatility built on

the stochastic definition of quadratic variation due to Karatzas and Shreve (2012). This area has seen a lot of activity and ongoing research with a suite of refinements to the concepts of market microstructure noise to filter out the noise introduced by the discretization schemes of the data (Barndorff-Nielsen & Shephard, 2002; Oomen, 2005; Zhang, Mykland, & Aït-Sahalia, 2005; Corsi, Mittnik, C. Pigorsch, & U. Pigorsch, 2008).

Past crises in financial markets have inspired the search for better and more robust volatility models. These models have always crumbled in the face of stresses in the markets. The assumptions underlying them have been either too brittle or too complicated for the practitioners implementing them in day-to-day trading. Since the global financial crisis of 2007–2009, focus has shifted back to sound, less complicated risk models and prudent risk management in asset markets. Incorporating regimes in volatility models is one way of preventing large-scale shocks to the financial system. Indeed, an investor will like to know how the risk profile of his/her investments is changing over time with developments in the underlying economy. This information is vital and the risk-averse investor will keep calibrating his/her investment strategies taking advantage to the granular nature of volatility to stay or get out of particular markets.

Yalama and Celik (2013) studied volatility extensively in emerging markets and pointed to the presence of long memory characteristics of diverse types and levels of volatility for distinct sampling periods. This would suggest some form of finite mixture of the data generated by the underlying processes manifesting as either regime switching or possible non-stationarity in the returns from financial markets in the long term. Equity returns series contain heterogeneities that map to specific regimes generating that data. In particular, histograms of returns exhibit fat tails and high central peaks, which characterize mixtures of distributions with different statistical descriptors.

In their work on the broader economy in fifty countries over time, Engle and Rangel (2008) identified five macroeconomic conditions driving volatility in these stock markets. These are high inflation, low growth rate of output, high volatility of the short-term interest rate, high volatility of the growth of gross domestic product and high volatility of the inflation rate. These conditions, to varying degrees, have been largely present in the South African economy in recent years as reported by Marx and Struweg (2015). In an earlier study, Seleteng, Bittencourt, and Van Eyden (2013) stated that the South African economy is buffeted now and then by these conditions. Against this backdrop, one will expect to see the effects in the form of regimes on the returns and volatility of returns in South Africa's equity markets. These regimes have different statistics in terms of means, variances and correlations. This is the motivating driver in our use of a regime switching model in estimating the volatility of the returns from the JSE All-Share Index.

This study makes a vital contribution to the finance and volatility literature in the South African context by using the Markov regime switching model to investigate the presence of regimes in the volatility dynamics of the returns of JSE All-Share Index (ALSI). In fact, this study distinguished itself from other recent papers. For example, Muller and Ward (2013), Niyitegeka and Tewar (2013) analyzed volatility using single regime models. There was no attempt in these studies to incorporate regime switching. Moreover, Babikir, Gupta, Mwabutwa, and Owusu-Sekyere (2012) investigated structural changes in GARCH models with the levels of the index of the JSE. Our understanding of the literature on structural changes and regime switching in econometrics, for example, Song (2014) and in social sciences (Valadkhani & O'Mahony, 2018) convinced us that the terms are not synonymous. Indeed, Brooks, Davidson, and Faff (1997) hinted at the effects of political changes on the volatility of the stock market in their study, but fell short of doing the modeling to characterize the resulting regime changes. Our work has picked from there and this is an important contribution to the discipline.

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1. OVERVIEW OF MARKOV REGIME SWITCHING MODEL

Hints of the presence of regime switching in financial models have a long history in econometrics starting with the work of Cosslett and Lee (1985), Goldfeld and Quandt (1973). They gained popularity with the pioneering paper of Hamilton (1989). Yet academics and practitioners did not incorporate regime switching into volatility models because of their path-dependent nature and intractability until the work of Gray (1996) who used Markov chain Monte Carlo approach to analyze these models using interest rates. Since then, there has been an explosion of regime switching models in all areas of the finance and economics disciplines.

Engle and Patton (2007) argued that a volatility model should be able to forecast volatility. Forecasting volatility involves finely picking key stylized facts such as prolonged persistence, asymmetry, leverage and mean reversion. These characteristics vary with the underlying dynamics at play in a given economy. Regime switching conveys investing signals to the financial markets. According to Eichengreen and Tong (2003), such switching signals to economic actors of an impending change in the future economic direction and therefore how they shape their trading decisions.

Lamoureux and Lastrapes (1990) pointed out GARCH models inability to forecast out of sample. Such criticism are not out of place. Several authors have recently reached the same conclusions on the usefulness of GARCH as risk management and trading tools (Nwogugu, 2006; Guo & Cao, 2011; Calvet & Fisher, 2004; Lux, 2008). Taking cognizance of such criticism of GARCH models, Klaassen (2002) earlier recommended improvements in forecasting of GARCH models by incorporating regimes in the modeling of volatility. GARCH models appear to overestimate volatility during high volatility periods.

Liu, Margaritis, and Wang (2012) examined the volatility of equities and found substantial improvements in characterizing the evolving risk using regime switching. The unconditional volatility

reported by single regime models would appear to be a complex weighting of several averages, the result of a mixture of distributions in the given series used in estimation of the GARCH model. Each regime is characterized by its own distribution and hence moments. Behavior of GARCH models out of sample discounts the claims in, for example, Hansen and Lunde (2005) about the ubiquity of the GARCH (1,1). We note that the comparison was among the class of GARCH models without the benefit of switching.

2. PROBLEM STATEMENT

This study is motivated by the changing dynamics underlying the return generation process, which, we argue, should manifest in regime switching of the volatility of returns across time. We propose that the volatility of the JSE returns follows a Markov switching process with multiple clearly defined regimes. The study assesses the predictive accuracy of the models using the Deviance Information Criteria (DIC) for the purpose of model comparison and selection.

3. MODEL SPECIFICATION AND ESTIMATION

Let $\left\{y_t\right\}_{t=1}^{t=T}$ be a vector of continuously compounded returns, which have been demeaned. We take a computational point of view by adopting a Bayesian methodology using largely the notation of Ardia, Bluteau, Boudt, Catania, and Trottier (2016). The return is specified as:

$$y_t \big| \left(s_t = k, I_{t-1}\right) \sim D\left(0, \sigma_{k,t}^2, \gamma_k\right), \qquad (1)$$

with $D\left(0, \sigma_{k,t}^2, \gamma_k\right)$ representing the distribution of the returns y_t specific to regime k with zero mean, variance $\sigma_{k,t}^2$ and shape parameter γ_k. The latent variable s_t takes on values in the vector $\left\{1, 2, \ldots, K\right\}$ representing non-overlapping regimes R_i such that $R_i \bigcap R_j = \varnothing$ for any $i \neq j$. The regimes are thus piecewise models describing the heteroscedasticity dynamics specific to a particular regime R_k.

It is assumed that the latent specifier s_t is a first-order homogeneous Markov process, which

for a two-regime volatility model evolves with a transition matrix

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}, \qquad (2)$$

where the notation is such that $p_{ij} = P\left(s_t = j \mid s_{t-1} = i\right)$ is the probability of a transition from regime $s_{t-1} = i$ to $s_t = j$. Being a probability, we require that $0 < p_{ij} < 1$ for all $i, j \in \{1, 2, \ldots, K\}$ with the additional Markov property that $\sum_{j=1}^{K} p_{ij} = 1$ for all $i \in \{1, 2, \ldots, K\}$.

Following Haas, Mittnik, and Paolella (2004), the conditional variance, $\sigma_{k,t}^2$, of the returns y_t follows a regime switching GARCH conditional on s_k in regime k given by:

$$\sigma_{k,t}^2 = f\left(y_{t-1}^q, \sigma_{k,t-1}^2, \varphi_k\right), \qquad (3)$$

where $f(.)$ is a function for the conditional variance with q as either one or two depending on the GARCH specification and φ_k is the vector of regime specific parameters.

Estimation of the parameters is via a likelihood function. We let $\Phi = \{\Theta_1, \varphi_1, \ldots, \Theta_k \varphi_k, P\}$ represent the vector of model parameters. The likelihood function is given by:

$$L\left(\Phi \mid G_T\right) = \prod_{t=1}^{T} f\left(y_t \mid \Phi, G_{t-1}\right), \qquad (4)$$

where $f\left(y_t \mid \Phi, G_{t-1}\right)$ is the probability of y_t with the filtration G_{t-1} given the parameter vector Φ. For the regime switching GARCH model, the conditional density of the returns y_t is:

$$f\left(y_t \mid \Phi, G_{t-1}\right) = \sum_{i=1}^{K}\sum_{j=1}^{K} p_{ij} w_{i,t-1} f_D\left(y_t \mid s_t = j, \Phi, G_{t-1}\right), \quad (5)$$

where $w_{i,t-1} = P\left(s_{t-1} = i \mid \Phi, G_{t-1}\right)$ is the Hamilton filter of the probability of state i at a time $t-1$.

In the Bayesian methodology used in this paper, we estimate the density function $f\left(y_t \mid \Phi, G_{t-1}\right)$ via MCMC by combining the likelihood with a prior $f(\Phi)$ to get the posterior distribution $f\left(\Phi \mid G_T\right)$. The posterior form of the distribution is unknown, since we did not rely on a conjugate prior; hence, we use the adaptive Metropolis-Hastings random walk sampler first proposed by Haario, Saksman, and Tamminen (1999) and later

expanded to its current form for fast convergence by Vihola (2012) for our simulations. The respective regime switching volatility specifications for GARCH (Bollerslev, 1986), exponential GARCH (EGARCH) (Nelson, 1991) and the GARCH of Glosten, Jagannathan, and Runkle (1993) (GJR-GARCH) are specified as follows.

3.1. Regime switching GARCH

The regime switching GARCH specification is:

$$\sigma_{k,t}^2 = \alpha_{0,k} + \alpha_{1,k} y_{t-1}^2 + \beta_k \sigma_{k,t-1}^2, \qquad (6)$$

for regime $k = 1, 2, \ldots, K$. The parameters to be estimated is the vector space $\{\alpha_{0,k}, \alpha_{1,k}, \beta_k\}$. The parameter are restricted to $\alpha_{0,k} > 0$, $\alpha_{1,k} > 0$ and $\beta_k \geq 0$. To ensure the specific k regime is stationary, we require that $\alpha_{1,k} + \beta_k < 1$.

3.2. Regime switching EGARCH

We specify the regime switching EGARCH as:

$$\log\left(\sigma_{k,t}^2\right) = \alpha_{0,k} + \alpha_{1,k}\left(\left|\vartheta_{k,t-1}\right| - E\left[\left|\vartheta_{k,t-1}\right|\right]\right) + {} \\ + \alpha_{2,k} y_{t-1} + \beta_k \log\left(\sigma_{k,t-1}^2\right), \qquad (7)$$

for the regimes $k = 1, 2, \ldots, K$. The expectation $E[.]$ is taken with respect to the conditional distribution of regime k with the standardized innovations denoted by $\vartheta_{k,t-1}$. The parameters $\varphi_k = \{\alpha_{0,k}, \alpha_{1,k}, \alpha_{2,k}, \beta_k\}$ are to be estimated. The log specification ensures that the variance is positive. Piecewise stationarity requires that $\beta_k < 1$.

3.3. Regime switching GJR-GARCH

The regime switching GJR-GARCH is specified by the equation:

$$\sigma_{k,t}^2 = \alpha_{0,k} + \left(\alpha_{1,k} + \alpha_{2,k} I\{y_{t-1} < 0\}\right) y_{t-1}^2 + {} \\ + \beta_k \sigma_{k,t-1}^2, \qquad (8)$$

for the regime $k = 1, 2, \ldots, K$. The function $I\{.\}$ is an indicator function yielding one if the condition holds and zero otherwise. $\varphi_k = \{\alpha_{0,k}, \alpha_{1,k}, \alpha_{2,k}, \beta_k\}$ is the parameter to be estimated. For the variance to be positive, we re-

strict $\alpha_{0,k} > 0$, $\alpha_{1,k} > 0$, $\alpha_{2,k} \geq 0$, $\beta_k \geq 0$. To ensure that each regime is stationary, we require $\alpha_{1,k} + \alpha_{2,k} E\left[\in_{k,t}^2 I\{\in_{k,t} < 0\} \right] + \beta_k < 1$.

4. DATA AND ANALYSIS

The data sample spanning fifteen years from January 2, 2003 to December 29, 2017 is made up of the closing level of the JSE All-Share Index. The data was calibrated into a two-state regime switching volatility to the returns under consideration. A time series plot shown in Figure 1 displays a rising trend of the index level over time. There was a severe dip in the level during 2008–2009 coinciding with the global financial crisis.

Figure 2 shows the return series obtained through differencing the log of the indices. The graph reveals the characteristics typical of financial returns identified in Cont (2001). There was a slight dip in volatility in 2005. Volatility picked up in 2006,

becoming turbulent during the financial crisis of 2008–2009 with large absolute returns before settling moderately in the ensuring years. Volatility clusters can be seen punctuated by periods of relative tranquillity beyond 2009. Such episodes of elevated volatility pose risk to investment portfolios relying on single regimes. It is also seen that the series taken as a whole is not stationary, but perhaps piecewise stationary over relatively short periods.

Figure 3 shows the histogram and the Q-Q plots of the returns. We see a marked departure from normality at the tails. The distribution of the returns is slightly left skewed with heavy tails displayed in the Q-Q plot on the right.

The departure from normality moderate as confirmed by the statistics in Table 1.

A formal Cramer-von Mises normality test (Csorgo & Faraway, 1996) for the returns yielded a p-value of $7.37 \cdot 10^{-10}$ confirming the dis-

Figure 1. Trend of JSE All-Share Index over the period January 2003 – December 2017
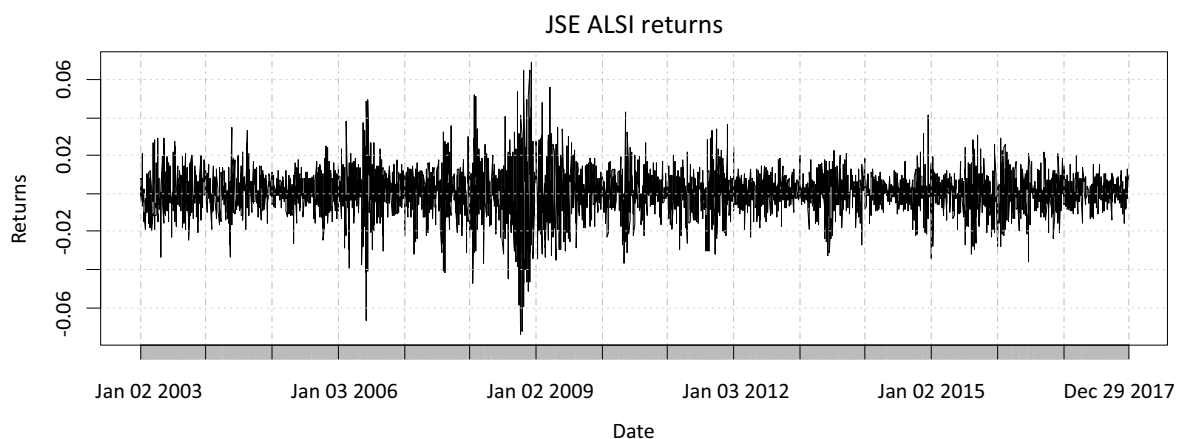
Figure 2. The log returns of JSE All-Share Index over the period January 2003 – December 2017

Table 1. Summary statistics of the returns

Sample size	Mean	Standard deviation	Minimum	Maximum	Skew	Kurtosis
3749	0	0.01	–0.07	0.07	–0.17	3.67

tribution is not normal. We run the Bayesian regime switching versions of GARCH, EGARCH and GJR-GARCH respectively for both single and two-regimes using the MSGARCH package (Catania, Ardia, Bluteau, Boudt, & Trottier, 2018) of the *R* language (R Core Team, 2018). Our choice of a maximum of two-regimes is based on the work of Hardy (2001). Both Student *t* and skewed Student *t* innovations for these set of models generated a total of twelve candidate models as shown in Table 2.

In running the Bayesian algorithm, we first estimated the GARCH coefficients using regime switching maximum likelihood estimation. The coefficients from this first stage served as the starting values for the MCMC sampling. We used 12,500 simulations with a burn-in of 5,000, thinning at each tenth. The selection criterion used to discriminate among the models is the fit statistic,

Deviance Information Criteria (DIC), proposed by Spiegelhalter, Best, Carlin, and Van Der Linde (2002). The two-regime EGARCH versions had the lowest DIC of –23923.8993 and –23877.215 for both skewed and Student *t* innovations, respectively. The single-regime volatility models of all the GARCH versions posted high DIC value. This confirms the presence of regimes in the returns of the JSE Index. The dominance of the EGARCH models signals the preponderance of leverage effects observed by Black (1976) in financial returns. Negative returns dominate the conditional volatility much more than positive returns of the same magnitude.

We run diagnostic tests for the two-regime EGARCH with skewed Student *t* innovations. The trace plots are displayed in Figure 4. The trace plots are largely stationary and the kernel densities normal. This shows converge of the chains.

Table 2. Deviance Information Criteria of the models

Model	Deviance Information Criteria	
	Skewed student *t*	**Student *t***
Single-regime GARCH	–23803.6971	–23767.199
Two-regime GARCH	–23803.5375	–23764.565
Single-regime EGARCH	–23906.3736	–23861.601
Two-regime EGARCH	–23923.8993	–23877.215
Single-regime GJR-GARCH	–23893.1734	–23850.853
Two-regime GJR-GARCH	–23901.5891	–23854.314
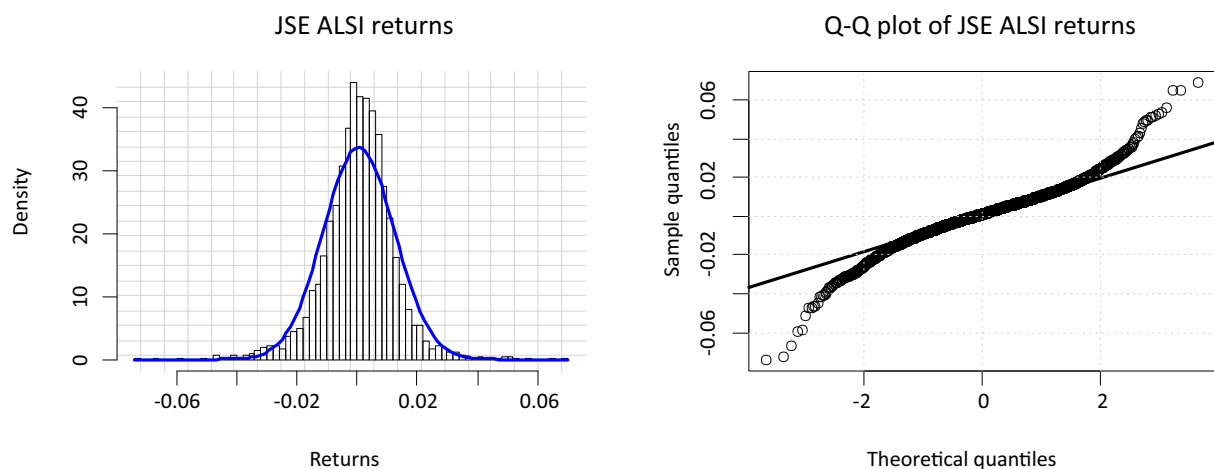
**Figure 3.** Histogram and the Q-Q plots of the returns
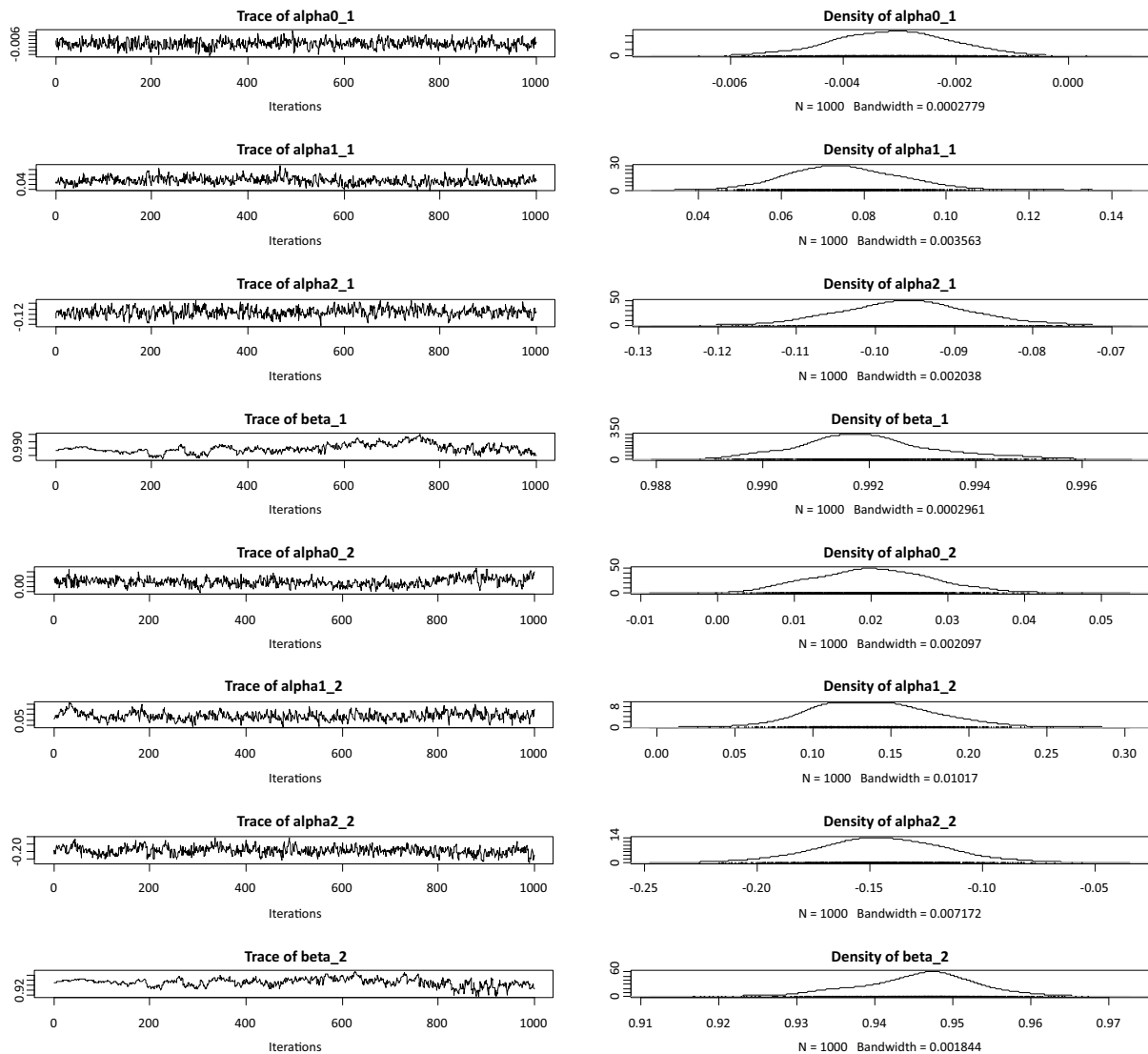
Figure 4. Trace of MCMC samples for the parameters of the two-regime EGARCH model
with skewed Student *t* innovations

5. RESULTS

The results for the two-regime EGARCH model is presented in Table 3. Column 5 of Table 3 shows the relative numerical efficiencies of the estimates. Low values of relative numerical efficiencies shows our samples are independent (Geweke, 1989). The values with the exception of α_{01} are low and therefore we can rely on the estimates for inference.

The results show stable probabilities of 0.7984 and 0.2016 for the low and high regime, respectively. This implies the low regime is dominant compared to the high volatility regime. Different estimates as shown in Table 3 are an indication of heteroge-

neity of the return process of the JSE series. There are different reactions to past negative returns as reported by the respective $\alpha_{21} = 0.0961$ and $\alpha_{22} = 0.1449$. Volatility is more persistent in regime two than in regime one. This is in line with empirical observations of persistence in volatility during market turbulence (Bentes, 2014). The unconditional volatility in the first regime is 15.93% against the higher 20.1% for regime two. The lingering persistent observed in regime two is therefore in line with reported behavior of financial returns in the literature. The smoothed probability of the high volatility regimes together with the filtered conditional volatility for the two-regime EGARCH is shown in Figure 5.
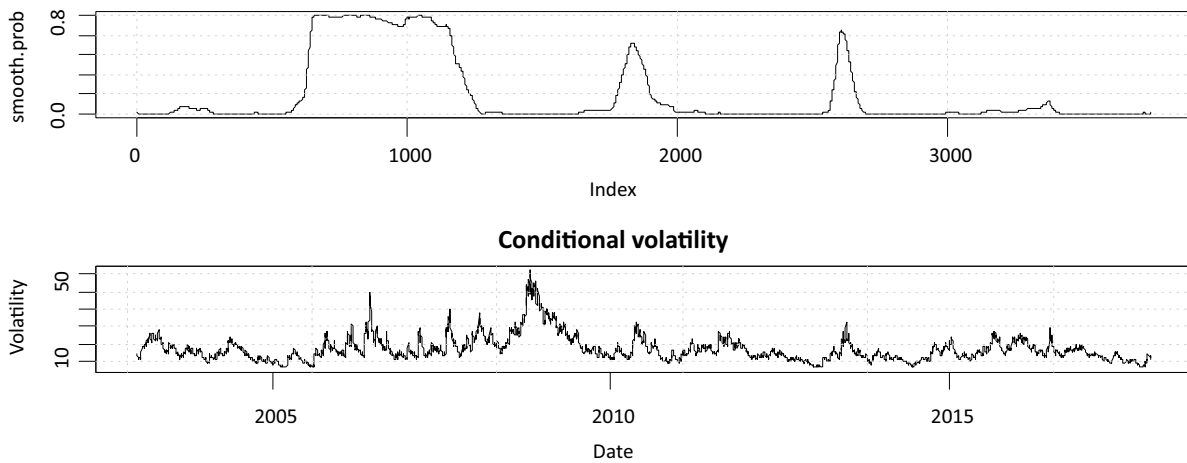
Note: Second regime bottom: filtered conditional volatility.

Figure 5. The smoothed probabilities of the high volatility regime

Table 3. Two-regime EGARCH with skewed Student *t* innovations

Coefficients	Mean	SD	SE	TSSE	RNE
α_{01}	0.0031	0.0011	0	0	0.4737
α_{11}	0.0755	0.0134	0.0004	0.0013	0.1045
α_{21}	0.0961	0.0081	0.0003	0.0004	0.3528
β_1	0.6830	0.0013	0	0.0003	0.0163
v_1	99.6107	0.0525	0.0017	0.0124	0.0178
ξ_1	0.9037	0.0282	0.0009	0.0033	0.0733
α_{02}	0.0204	0.0079	0.0002	0.0009	0.0761
α_{12}	0.1384	0.0382	0.0012	0.0028	0.1862
α_{22}	0.1449	0.0273	0.0009	0.0027	0.1008
β_2	0.7249	0.0078	0.0002	0.0012	0.043
v_2	98.4535	0.1942	0.0061	0.0544	0.0127
ξ_2	0.7164	0.0455	0.0014	0.0114	0.016
ρ_{11}	0.9987	0.0005	0	0.0001	0.021
ρ_{21}	0.0051	0.0007	0	0.0003	0.0066

It can be seen from Figure 5 that the smooth probabilities track closely the evolution of the volatility as described by the conditions volatility on the bottom diagram.

We check the robustness of our results out of sample by backtesting the industry standard GARCH (1,1), the single- and two-regime EGARCH with skewed Student *t* disturbances. Our aim is to assess the model, which is the most accurate in predicting correctly the loss at the 5% quantile. We computed the *p*-values of correct conditional coverage of the value-at-risk using the conditional coverage (CC) test of Christoffersen (1998). The results are displayed in Table 4.

Table 4. Results of the backtest

Model	GARCH (1,1)	One-regime EGARCH sstd	Two-regime EGARCH sstd
p-value	0.0183	0.0273	0.0742

From the result in Table 4, we see that the best performing model is the two-regime EGARCH with the skewed Student *t* innovations.

CONCLUSION

We investigated the switching behavior of volatility regimes in the market returns of the JSE All-Share Index. Using the DIC fit statistic, we confirmed that regimes exist in the returns across time. The results revealed regimes in the volatility of returns of the JSE All-Share Index. We found that the two-regime switching EGARCH fits the data better using the Deviance Information Criteria (DIC) as a model selec-

tion metric. A key finding of this study is the dominance of the EGARCH for both skewed Student *t* and Student *t* specification for the disturbances. Returns on the JSE therefore respond non-symmetrically to leverage effects similarly to other equity markets.

The information content in a changing regime in equity markets is an important signal for both trading and policy alike. Low volatility equity-linked products suffer substantial losses when markets are roiling. We can see the changing behavior of volatility as exhibited by the markedly different regimes. The unconditional variances are clearly distinct indicating a clear change in the market regime from a period of low volatility to one of high volatility in the data sample.

Investors and traders are able to evaluate the risk exposure of an investment conditionally on different regimes of the market. Understanding the risk of crises particularly in bad times underpins today's trading environment. When markets enter the turbulent phase, it is essential that small and retail investors are cautioned to steer clear of risky investments for which they have no risk appetite. Investment firms trading equities on the JSE assuming a single volatility regime are left exposed to the varying dynamics or failing to take advantage of the heightened risk, if they are active investment firms, to enhance the returns.

Switching in volatility enhances robust pricing of equities and makes adequate risk-based capital requirements more transparent. A heightening volatility indicates stresses in the market are coming to the fore. This is itself an indication of flashing red lights in the underlying economy. Policy should be used to address this potential market turbulence with a view to moderating market volatility from the potentially damaging erosion of investor confidence in equities. Some volatility is good for the market as long as it enables orderly trading. The most damaging high volatility regimes can be filtered using policy tools in the hands of the South African monetary and fiscal authorities.

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