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Maize Price Stabilization in Ghana: An Application of a Continuous-Time Delay Differential Equation Model with Buffer Stock

Martin Anokye[1*] and Francis T. Oduro[2]

[1]*Faculty of Applied Science, Kumasi Polytechnic, P.O.Box 854, Kumasi, Ghana.*
[2]*College of Science, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana.*

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Abstract

The paper is intended to use mathematical models for controlling fluctuations in the price of maize by employing a nonlinear continuous time delay differential equation derived from linear demand and nonlinear supply functions of price. The delay parameters reflect the realities prevailing at the local market. These models are formulated from parameters estimated from real economic data of maize price demand and production in the Ashanti Region of Ghana through the use of regression methods. The data is obtained from the Ministry of Food and Agriculture, Statistical Directorate Kumasi-Ghana, pertaining to years from 1994 to 2013.
The results of the study are connected to the assertion that commodity price is dependent on planting time, storage time, relaxation time and total production time. If all these individual time segments are combined as one for supply delay to make up total storage time, which is the delay for the buffer, then price oscillations would be drastically reduced as shown in this paper. The study is, also, an improvement on the work done by earlier workers, whose discrete time models are limiting cases of the delay buffer stock model used in this study. The efficiency of a buffer system is proven to be dependent on delay variation suitably enough to be used by buffer stock operators.
It is noted that, the farther the buffer stock delay and supply delay are reduced, the more stable the price becomes and the effects of buffer stock are felt more by stakeholders. The results of the analysis provide an average stable price of maize as GH¢ 30.49 compared to the actual

*Corresponding author: mafanokye@yahoo.co.uk;

average price of GH¢30.27. The equilibrium price in turn provides the average equilibrium weight of 2931.6 and 8217.6 metric tons for demand and supply respectively. The excess supply is kept in stock and when needed it is released in the next market period. The standard deviation also is reduced to 0.1602 compared to the original 29.48 before the application of buffer stock scheme. Thus, before the application of buffer stock scheme, price oscillated between two price points and could not converge. This affirms the fact that buffer stock acts as a reserve against short-term shortages and dampens excessive fluctuations.

We draw inferences from this study that researchers should rather use continuous time nonlinear delay models as they reflect the realities in most real-life economic problems.

Keywords: Buffer stock, price stabilization, delay-differential equation.

1 Introduction

The prices of agricultural products such as maize, wheat and the like tend to fluctuate more than prices of manufactured products and services. This is largely due to the instability in the supply of agricultural products coupled with the fact that demand and supply are price inelastic [1]. It is believed that in the agricultural industry, price volatility and scarcity is a great threat in the distribution system of seasonal staple foods including maize produced by farmers. There are a lot of disparities in terms of supply during the harvest and planting seasons which in effect could create inconvenience to stakeholders such as producers, consumers, and government [2]. In almost all business environments, extensive studies have been done on demand because demand uncertainty is inherent. However, supply uncertainty has received much less research attention although supply has a significant impact on achieving price stability. Moreover, it is critical for any profit oriented business to move the right quantity of commodities to the right location at the right time in order to turn potential consumer demand into revenue [3].

In agricultural economics, detecting the source of price fluctuations is one of the most important issues. The quest to unravel the reason for the recurring cycles in the production and prices of commodities on regular bases, gave birth, in the thirties, to the cobweb model [4], which describes price fluctuations in a single market for a commodity that takes one unit of time to produce.

In Ghana, as in other developing countries, staple food such as maize normally accounts for quite a significant share of poor households' budgets. The poor have limited possibilities to insure against adverse price shocks. An implementation of food price stabilization policies is an important way by which governments in these countries can express their concern regarding the problem [5].

1.1 Price Stabilization Systems

Many policy interventions have been used by stakeholders to stabilize price, improve quality of maize and increase maize production in Ghana for both local consumption and export to the world market. Price stability depends upon matching supply with demand because both supply and demand uncertainties can present a major obstacle to achieving this goal.

One way to smoothen out fluctuations in prices is to operate price support schemes through the use of buffer stocks. These schemes seek to stabilize the market price of agricultural products by buying up supplies of the product when harvests are plentiful and selling stocks of the product onto the market when supplies are low [1].

1.1.1 Buffer stock system

The empirical results from Myers et al. [6] also paint a relatively favourable picture of the buffer stock scheme. In absence of the scheme, demand shocks are the dominant source of variability, but stockholding did blunt the effects of these shocks.

There is a raging scientific debate regarding the effects of storage, considered by most as a stabilizer on commodity prices and storage is placed at the core of the explanation of the intriguing characteristics of price dynamics. Recently, it is hypothesized that endogenous chaotic behavior of markets is the cause of commodity price fluctuations. A nonlinear cobweb (that shows chaotic price series) model with storage, risk averse agents and adaptive expectations was developed, and like the theory of competitive storage, the nonlinear cobweb model found to reduce price variability [7].

Sutopo [8] observed in previous researches that, buffer stock models have been developed separately based on nonlinear optimization and econometrics methods. While optimization methods had been used to determine the level of availability with schemes consisting of time and quantity of buffer stock, the econometrics methods on the other hand had been used to determine the equilibrium price using the selling-price and the amount of buffer stock. Therefore, they developed a buffer stock model that integrates both methods using decision variables which consist of quantity, time and price to achieve stability.

Athanasiou et al. [9] presented a nonlinear cobweb model with supply increasing according to certain a piecewise linear supply function. Athanasiou et al. [9,10] also proposed naive expectation to test the effect of governmental interventions through the provision of reserve from buffer stocks operation to weakening commodity price fluctuations. Their model proved that if the storage capacity for a particular commodity is sufficiently large then there exists a simple stabilization policy, such that the equilibrium price is a global attractor for the corresponding closed-loop system.

Anokye and Oduro [11] developed a linear cobweb model which studied the phenomenon of commodity price fluctuations with the assumptions that fresh tomatoes have no equal substitutes and also no exogenous shocks needed to generate price fluctuations. The model showed unstable price oscillations around the equilibrium point. However, after incorporating a buffer stock scheme into the model, price stability was achieved in the short run while in the long run buffer system suffered instability until the average supply was reviewed.

Edwards and Hallwood [12] developed linear econometric models (linear supply and demand) to determine the amount of commodities that Government must store, so that the amount of stocks to be released in order to stabilize prices could also be determined.

Price dynamics of a rice market was examined by Brennan [13] using dynamic programming techniques that parameterized the case of Bangladesh which is characterized by high price elasticity (due to income effects) and high storage and interest costs. In this model various storage interventions (both public and private) were explored, and the results show that both interventions have positive impact in ensuring food security and fair prices for the poor.

Nguyen [14] proposed a simple rule for the buffer stock authorities to stabilize both price and earnings in all circumstances, except when market is instable. The model dampens price fluctuations in all periods and not only in the periods when a freely fluctuating price would fall but also outside the chosen limits.

Jha and Srinivasan [15] evaluate the impact of food buffer stocks in stabilizing prices of food commodities when private external trade is liberalized with multi-market equilibrium framework adopted in the model as endogenous factor. The results from the analysis indicated that under liberalized trade, buffer stock scheme is ineffective for stabilizing domestic prices. It is therefore prudent to use trade restrictions when buffer scheme is implemented to ensure price stabilization. Soltes et al. [16] on the other hand, used an inventory (buffer stock) model in which price tends to an equilibrium point not only monotonically but also oscillates around the equilibrium point, and they attributed the cause of the oscillations to the order of the differential equations.

1.1.2 Time varying effect system

The effect of time varying velocity on output responses to policies for reducing and/or stopping inflation was explored through the use of dynamic general equilibrium model in which a time-varying component (effect) was introduced as endogenous parameter for analyzing optimal speed of disinflation. The solution of this nonlinear model revealed that output losses would be much larger when disinflation boom disappears. It was also found from the analysis that gradual disinflation of low inflation is undesirable given its overall impact on the economy [17].

Mackey [18] developed a price adjustment model with state dependent production and storage delays for the study of price dynamics in a single commodity market. Conditions for stable equilibrium price were derived in terms of a variety of economic parameters and it was found that, when stability of the equilibrium price is lost, Hopf bifurcation occurs, giving rise to an oscillatory commodity price with a period between two and four times the equilibrium production-storage delay.

Liz and Rost [19] studied the price dynamics in a commodity market governed by a balance between demand and supply with delay, by employing a delay differential model. They also did a thorough study of the discrete time case and then used the results obtained to get new sufficient conditions for the global convergence of the solution to the positive equilibrium in the continuous time case. When the delay is large and the price system is unstable, they provide bounds to limit amplitude of the oscillations that are quite sharp. Matsumoto [20] used nonlinear delay differential equations and found out that time delay effects has strong stabilizing effect in minimizing cyclic oscillations.

Ruediger et al. [21] assessed how weather and time-varying business volatility affect the price setting of firms and thus the transmission of monetary policy into the real economy. The results of their analysis in twofold suggest that, sharp business volatility increases the probability of a price change, tripling of volatility caused the average quarterly price change to also increased. Secondary, the increase in volatility caused monetary policy output to decline. Anokye and Oduro [22] also assessed the effects of a delay parameter τ on price oscillations and observed that oscillations (price fluctuations) are suppressed for $\tau \leq 0.5$, using the nonlinear delay equation.

All the models reviewed so far (under buffer stock systems) have failed to consider effects of time that elapsed before supply changes in response to market dynamics. They also fail to consider the effects of this time (delay) on buffer stock operation in stabilizing prices of food commodities. Mackey [18], on the other hand, used delay parameter and has proven that, storage delay can rather be a destabilizing factor for price.

However, this paper is intended to use mathematical models for controlling price fluctuations of maize at the market and yet require that the structure and parameters of the model reflect the realities prevailing at the local market while showing that the time varying parameter (delay) is a price stabilizer. We therefore introduce time varying parameter model by employing a nonlinear continuous time delay differential equation derived from linear demand function of price and nonlinear supply function of price with delay. This model would mimic the model used by Soltes [16] to demonstrate the positive impact of the time varying effects on price because according to Papachristodoulou [23], analysis of nonlinear delay differential equations is difficult and so researchers are constrained to the investigation of the properties of their nonlinear un-delayed versions, as was done in Soltes work.

The model would also have its parameters estimated from real economic data of maize price demand and production in the Ashanti Region of Ghana through the use of regression (the same data are used in Anokye and Oduro [22]; Anokye et al. [24]).The model would then be integrated with a buffer stock model that has a supply delay parameter.

It is believed that by the introduction of time varying parameter in the models, stakeholders will appreciate its dynamics on prices and be encouraged to adopt this model to devise mechanisms to ensure food availability and the stabilization of prices.

1.2 Effects of Nonlinear Delay Differential Equations

In fact, a great deal of recent economic modelling has almost entirely ignored the potential role of delays in generating economic fluctuations and a good mathematical setting in which to consider this gap is provided by delay-differential equations [25,19]. While it is assumed that consumers base their buying decisions on the current market price, for most commodities there is actually a finite time τ that elapses before a change in production occurs and this time lag can be affected by several factors [18].

Price dynamics of maize in Ghana was studied based on the price stability conditions of two continuous time delay differential equations using a numerical approach. The results from the analysis suggest that researchers use nonlinear models instead of linear models in solving most real-life economic problems because the nonlinear delay differential models seemed more realistic than those of the linear differential models [22]. For instance, the dynamics of human endocrine level was studied over time using nonlinear time delayed dependent model and it was found that daily variation quantified by time-delayed mutual information system intuitively provided the expected diurnal variation in glucose levels amongst a random population of humans as opposed to the population with no diurnal variation [26]. Also, delay time-dependent stimulation using micro-fluidics technology has proven valuable in eliciting previously unseen cellular responses in physiological and cellular processes within the framework of HIV models, thereby potentially allowing researchers to observe new mechanisms in the pathway [27].

2 Methods

In order to study price dynamics of maize in Ghana, records of price and production of maize from seventeen major market centres in the Ashanti Region were selected. This secondary data pertaining to maize covered the years 1994 to 2013, were collected by the Ministry of Food and Agriculture, Statistical Directorate Kumasi-Ghana and are shown in Table 1. Nonlinear delay differential equations are derived from linear demand and nonlinear supply (with delay) functions whose parameters are formulated from the data. This model is then incorporated with a buffer stock scheme to study the dynamics of price stabilization.

The Table 1 contains the quarterly price and production of maize data at the right and left sections of the table respectively. They are average price and production points across the seventeen (17) market centres. Find beneath statistical characteristics of the data contained in this table.

Table 2 shows descriptive statistics of the data in Table 1 which includes range, mean, skewness over 1 (moderate), kurtosis greater than 0 but within the expected value of 3 and their respective standard errors.

2.1 Continuous Time Non-Linear Model

Considering a simple nonlinear delay differential equation (up to quadratic term) for the supply function of price as:

$$S(p(t)) = b + \beta p(t-\tau) - \delta p^2(t-\tau) \qquad (*1)$$

and linear demand function of price as

$$D(p(t)) = a - \alpha p(t) \qquad (**1)$$

Table 1. Price and production of maize in Ashanti Region of Ghana

Year	Prices of maize (Bag of 100 kg) in GH¢				Production of maize (Metric tons)				Total
	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter	
1994	1.23	1.78	1.49	1.67	10198.13	14830.01	12403.34	13878.19	51309.67
1995	2.69	3.53	1.64	2.22	15947.12	20917.51	9730.17	13171.87	59766.67
1996	2.80	3.41	4.61	4.45	10152.87	12357.34	16710.75	16125.71	55346.67
1997	6.51	8.73	6.74	-	17268.13	23180.01	17886.86	-	58335.00
1998	5.06	5.62	4.43	4.26	19354.45	21514.8	16930.53	16287.52	74087.30
1999	4.64	4.94	5.76	4.60	15016.85	15978.78	18646.42	14875.28	64517.33
2000	9.32	11.05	9.90	10.39	14663.71	17381.97	15579.11	16342.87	63967.66
2001	13.87	18.76	12.98	13.04	13403.83	18127.28	12539.71	12595.85	56666.67
2002	14.83	15.38	10.97	11.36	25347.56	26300.98	18760.61	19417.51	89826.66
2003	14.03	17.06	16.32	14.49	14649.34	17818.76	17039.9	15132	64640.00
2004	17.80	21.67	23.46	23.06	12629.82	15372.77	16646.07	16362.01	61010.67
2005	29.67	45.02	34.25	28.56	11638.64	17660.19	13436.17	11203.67	53938.67
2006	25.91	27.39	20.98	18.87	15226.02	16097.1	12328.32	11090.56	54742.00
2007	25.85	32.34	26.41	26.06	13190.26	16500.28	13474.43	13296.04	56461.01
2008	32.34	56.66	56.72	49.39	10103.38	17700.44	17717.1	15428.41	60949.33
2009	60.79	71.77	55.13	52.21	15780.74	18631.95	14311.43	13552.55	62276.67
2010	53.29	55.51	52.76	46.82	21597.84	22497.63	21385.72	18976.81	84458.00
2011	55.10	75.41	81.34	89.95	10572.89	14470.08	15607.96	17260.74	57911.67
2012	110.69	124.44	90.97	76.00	18169.81	20426.27	14932.32	12475.61	66004.01
2013	77.09	76.24	74.86	81.67	17036.49	16847.17	16542.22	18047.12	68473.00

Table 2. Descriptive statistics

	N	Range	Mean		Std. Deviation	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Std. error	Statistic	Statistic	Std. error	Statistic	Std. error
Price	79	123.21	30.27	3.31709	29.48293	1.160	.271	.584	.535
Production	79	16570.81	16008.72	383.99735	3413.04306	.556	.271	.604	.535
Valid N (listwise)	79								

where *a*, *b*, α, β and τ are positive constants. The supply of maize is assumed to increase with increase in price until supply exceeds demand where the trend changes. The models also perform on the assumptions that, maize has no equal substitutes and there are no exogenous shocks needed to generate price fluctuations. The market price is determined by only the available supply in a single market and the rate of change of the price is proportional to the difference between the supply and demand functions [11,18,28].

We therefore have;

$$p'(t) = \left[(a-b) - \alpha\, p(t) - \beta\, p(t-\tau) + \delta\, p^2(t-\tau) \right] \text{ where, } \tau>0, \text{ on } [0, b], b>0 \qquad (1)$$

This is a simple nonlinear delay differential equation which mimics discrete time nonlinear difference equation modelled by [29] and cannot have an exact solution in practice obtained using an analytical method. Therefore, a numerical method is applied employing MatLab solver dde23 (code in appendix 1).

This equation with a single delay like all DDEs are usually solved in a stepwise fashion with a principle called the method of steps: equation (1) would have an initial function (also known as history function) as p(t) = s(t) defined over the interval [-τ, 0] and then its solution is mapped onto solutions of other functions. Thus, the solution of this equation is going to be a mapping from functions on the interval [t-τ, t] into functions on the interval [t, t+τ], [t+τ, t+2τ], etc., from time points t=0, τ, 2 τ,...

In other words, the solutions of this dynamical system can be considered as sequence of functions $p_0(t)$, $p_1(t)$, $p_2(t)$,.... defined over contiguous time interval of length τ [30].

2.2 Continuous time non-linear Model with Buffer Stock

We shall study how the dynamics of price adjustment will be affected when buffer stock in incorporated into equation (1). It is assumed that buffer has negative effect on price. Buffer stock is mathematically formulated as given below:

$$p'(t) = \left[D(p(t)) - S(p(t)) \right] - \int_{\tau}^{t} \left[S(p(u)) - D(p(u)) \right] du \qquad (2)$$

The second term is the integral of past differences which expresses the accumulated stock (say G) in the buffer. If G >0, then G causes downward adjustment of price because government releases stocks to the market. If G<0 he rather buys from the market to adjust price upward while G=0 denotes no interference from the government [9]. Equation (2) is a price adjustment model obtained from continuous time delay integro-differential equation that mimics the integro-differential model used by [16]. This equation can be transformed into differentiation by simplifying it into the form given by:

$$p''(t) = \left[D'(p(t)) - S'(p(t)) \right] - \left[S(p(t)) - D(p(t)) \right] \qquad (3)$$

The following equation is obtained when the equation (1) fixed into the equation (3);

$$p''(t) = p'(t) - \left[(b-a) + \alpha\, p(t) + \beta\, p(t-\tau) - \delta\, p^2(t-\tau) \right] \qquad (4)$$

Equation (4) is a second order continuous time nonlinear delay differential equation which also mimics the second order linear differential equation used by [16] which can also be solved numerically using MatLab solver dde23. This equation has to be converted into two first order delay differential equations before one can use MatLab dde23 to solve it (code in appendix 2).

The Model Performs with the following Assumptions:

1. No equal substitutes for maize, no exogenous shocks to cause price fluctuations and market price is determined by the available supply in the market
2. The system relies on starting with a good harvest, indeed, without stocks in the system it is not possible to react to a poor harvest. Buffer stocks do not prevent the initial problem from arising.
3. A government or agency can establish a target price, and then guarantee to pay farmers and growers this price, whatever output is produced. If the market price rises above this guaranteed price, the market price will prevail. But if the market price falls below the guaranteed price, then the guaranteed price will prevail.
4. There are no extra costs of storage which the government or the agency has to incur. Government is the only one who operates the inventory and controls the market

3 Results and Discussion

This paper seeks to study whether the delay in supply affects price of maize and this mathematical model used in achieving price stability using real economic data of maize price and production in Ashanti Region, Ghana. Modelling of various functions and their parameter estimates would be done by the use of SPSS and then the numerical solution of this continuous time delay differential equation run using MatLab.

3.1 Preliminary Analysis of Price and Production Data

The data are checked to correct any errors and then SPSS and MatLab used to verify the stationary status for both price and production data sets before formulating the demand and supply functions of price using regression analysis.

The Fig. 1 shows the time series plot (behaviour) of price. It indicates that price data is non-stationary and also exhibits nonlinearity characteristics in the form close to quadratic.

3.2 Parameter Estimates

The following table contains the coefficients of the parameters estimated from the data analyzed. All the parameter values are checked and they are statistically significant.

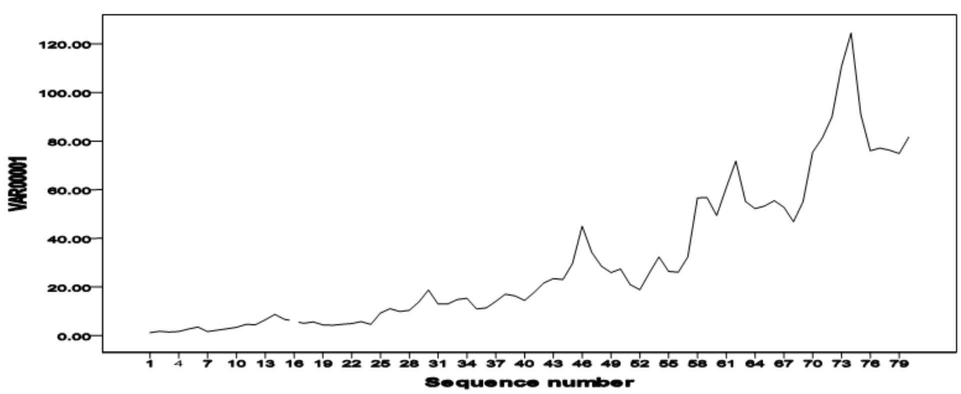

Fig. 1. Time series plot of price

The Table 3 contains the parameter values of demand and supply functions. The regression was carried out through the origin because in both cases the intercepts were not statistically significant. It also contains parameter values of linear demand function of price (1), while (2) is parameter values for quadratic supply function of price.

3.3 Demand Function of Price

The linear demand function of price from the Table 3 is given below with its parameter estimates checked to be statistically significant. This function (in the form of equation **1) was obtained from price data of order two (2) differencing and production data of order one (1) differencing.

$$D(p(t)) = -96.16\,p(t) \text{ where } a = 0 \tag{5}$$

3.4 Supply Function of Price

Similarly nonlinear supply functions with time delay τ are given below. This function in the form of equation (*1) was obtained with no order of differencing from price or the production data.

$$S(p(t)) = 354.28\,p(t-\tau) - 2.78\,p^2\left(t-\tau\right) \text{ where b = 0} \tag{6}$$

The delay τ expresses time that is needed to realise change of supply in dependence on trend of price. Thus current production depends on the past price.

3.4.1 Analysis of nonlinear model

From equations (5) and (6), the rate of change of price is given by the following equation, which is in the form of equation (1) with τ=1 and *a*=*b*=0:

$$\tfrac{1}{96.16}P'(t) = -P(t) - 3.68P(t-1) + 0.03P^2(t-1) \tag{7}$$

Table 3. Coefficients of demand and supply functions

Model	**Unstandardized coefficients**		**Standardized coefficients**			**Demand function**
	B	**Std. error**	**Beta**	**t**	**Sig.**	
1. Price	-96.16	20.26	-0.49	-4.75	0.00	
a. Dependent Variable: Production			b. Linear Regression through the Origin			
2. Price	354.28	36.89	1.72	9.60	0.00	**Supply function**
Price**2	-2.78	0.45	-1.10	-6.14	0.00	
a. Dependent Variable: Production_Lag			b. Linear Regression through the Origin			

Based on same assumption for smoothness of the solution using MatLab solver dde23, equation (7) is divided by 96.162 and the history function set at *P(t)*=1.23, when t≤0, and on the interval [0, 100]. The solution of equation (7) in numerical form is presented graphically as follows (see Fig. 2).

It is clear from the Fig. 2, above that the solution of equation (7) oscillates between two (2) equilibrium (price) points and would never converge to either equilibrium price due perhaps to inflation, food insufficiency and/or producers' sensitivity towards price.

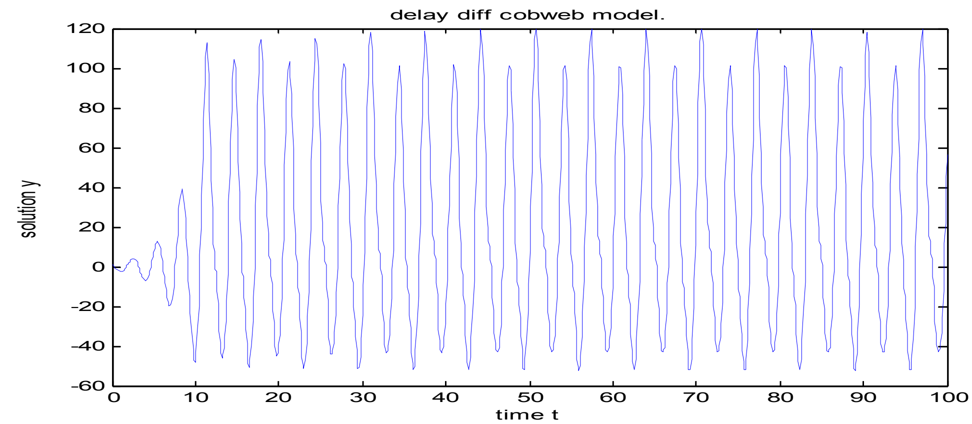

Fig. 2. Oscillations of price around 2 equilibrium points for τ=1

3.5 Time Varying Effects on Price Stability

The varying effects of delay parameter τ on price oscillations (fluctuations) are discussed. The delay parameter can decrease or increase fluctuations of price (Liz and Rost, 2013). This time varying analysis helps one to have fore knowledge of time needed to respond to supply (market) dynamics and keep the system symmetrical about the equilibrium price.

The oscillations with time are suppressed for τ ≤ 0.5, as shown in Fig. 3, below using equation (7).

Fig. 3 indicates that price would be in stable equilibrium when factors affected by time lag in the system are improved.

The price oscillations (fluctuations) start to increase as seen in Fig. 4, and with time become asymmetric (just like Fig. 2) about the equilibrium for τ > 0.5, using the same equation (7). This indicates that price would never be in stable equilibrium until factors affected by time lag, such as that required for increasing supply, transporting commodities to market centres and increasing of storage capacity are considered and improved [18].

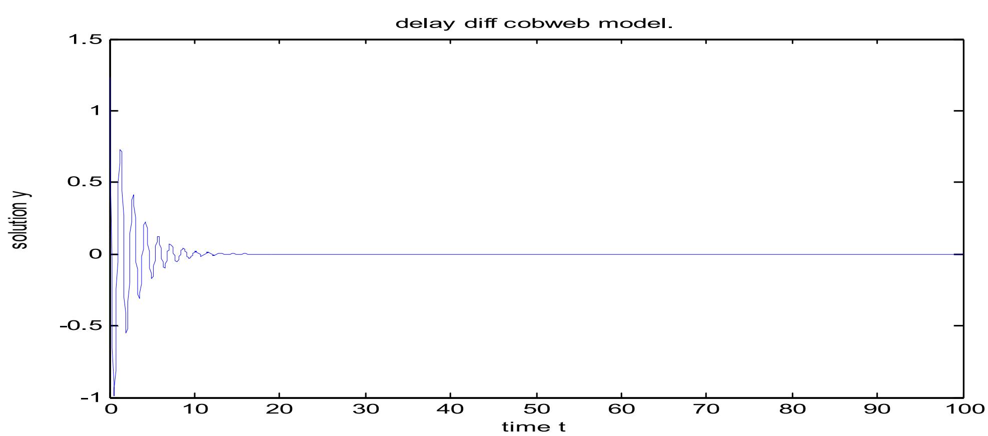

Fig. 3. Oscillation of price about equilibrium with delay τ ≤ 0.5

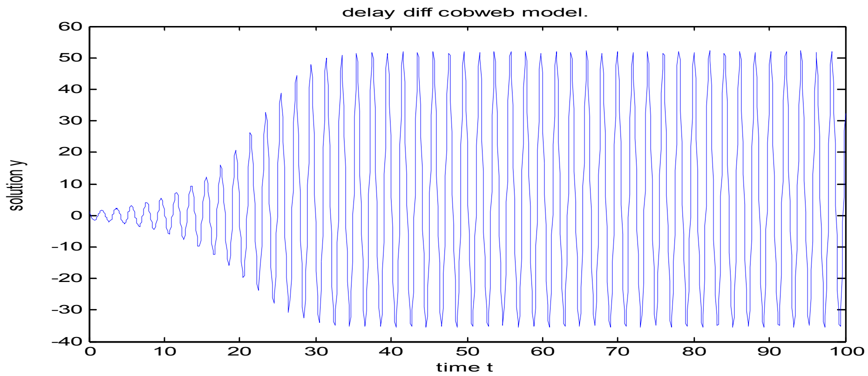

Fig. 4. Oscillation of price about equilibrium with delay τ > 0.5

3. 6 Dynamics of Price Stabilization with Buffer Stock

The buffer stock operator is always committed to price stability in the strict sense of achieving and maintaining a constant price level.

It is clear from Fig. 3, above that if delay in response to supply dynamics is improved then price stability can be achieved. From equation (4), the buffer stock equation is obtained as follows;

$$\tfrac{1}{96.16}\, p''(t) = \left[((-p(t) - 3.68p(t-\tau) + 0.03p^2(t-\tau))'\right] - \left[p(t) + 3.68p(t-\tau) - 0.03p^2(t-\tau)\right] \quad (8)$$

Where *b*=*a*=0. If delay is fixed at 0.45 for the inventory (buffer stock) delay and supply delay system (without buffer scheme), the following price oscillation graph (Fig. 5, below) is obtained:

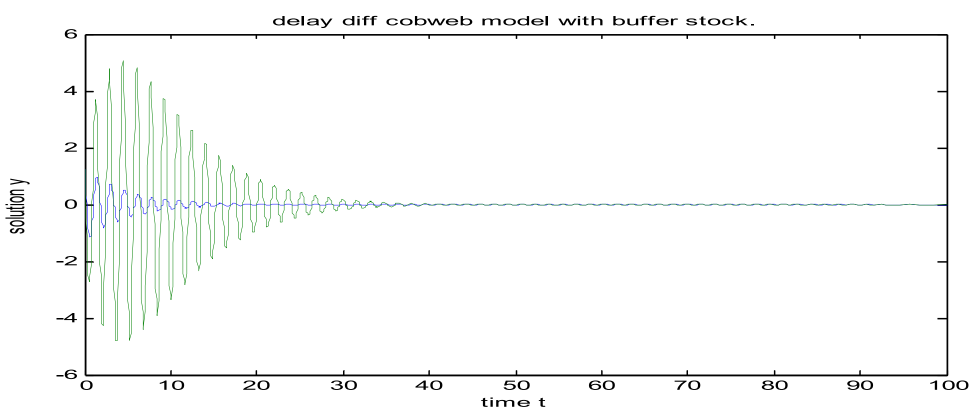

Fig. 5. Oscillations of Price about equilibrium with Buffer Stock, τ=0.45

From Fig. 5, above, the buffer stock delay and the supply delay are of different oscillation length which signifies no effects on price from the buffer operation. For the buffer stock to have greater impact, the two systems (buffer stock and supply) should synchronize.

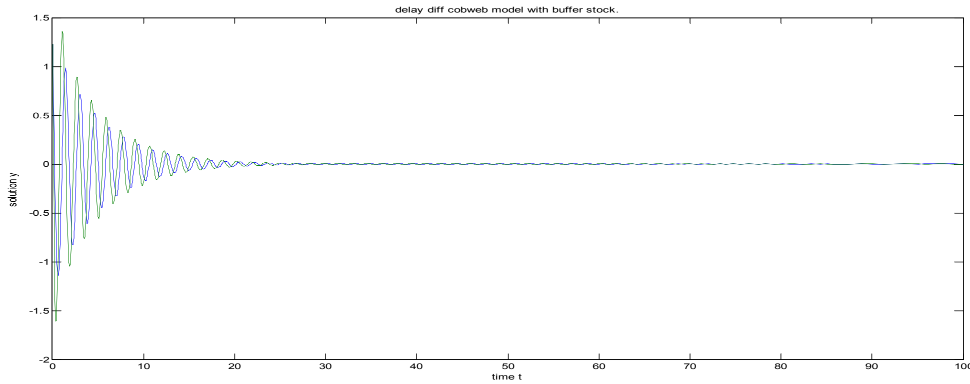

Fig. 6. Oscillations of Price about equilibrium with Buffer Stock, τ=0.22

Now from Fig. 6, when the delay for the buffer stock is reduced to τ =0.22, the two systems are synchronized which shows the impact of the buffer stock scheme on the price oscillations of maize. The oscillations as well as amplitude (wave length) are also reduced.

It is clear from Fig. 7, that the further the buffer stock delay (τ=0.20) and supply delay (τ=0.35) are reduced, more stable the price becomes. Now the effects of buffer stock are felt more by stakeholders.

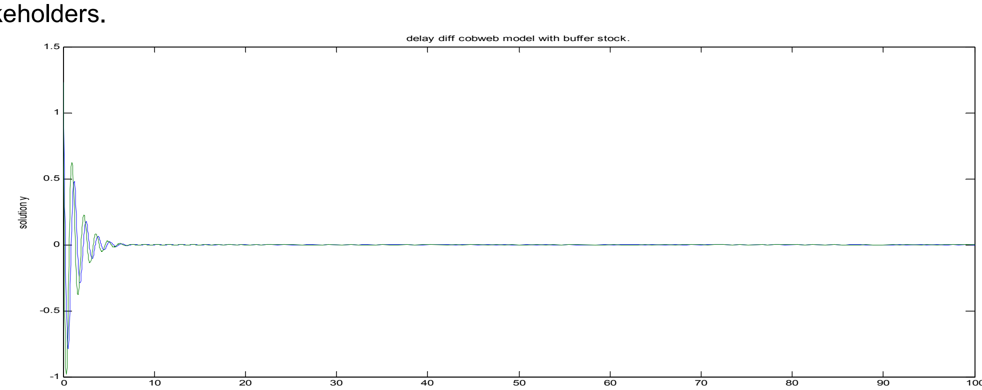

Fig. 7. Oscillations of Price about equilibrium with Buffer Stock Delay, τ=0.20

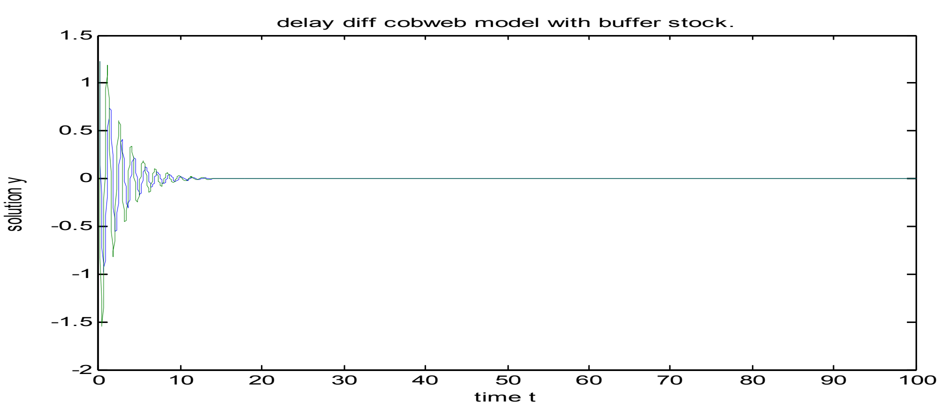

Fig. 8. Oscillations of Price about equilibrium with Buffer Stock Delay, τ=0.21

Table 4. Descriptive statistics of maize price

	N	Range	Minimum	Maximum	Mean		Std. Deviation	Variance	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Statistic	Std. error	Statistic	Statistic	Statistic	Std. error	Statistic	Std. error
Price	7	.49	.05	.54	.3170	.06054	.16016	.026	-.398	.794	.336	1.587
Valid N (listwise)	7											

From Figs. 6 and 7, we obtain best average delay times of τ=0.21 and τ=0.40 for buffer and supply respectively. These delay times give Fig. 8 which provides an average stable price of maize (from the simulated data in Table 3 above) as GH¢ 30.49, that is very close to the mean maize price (from the raw data) of GH¢ 30.27, obtained in Table 2.

Thus the mean price value (GH¢ 0.3170) from the Table 4 above is multiplied by 96.16 to give us the stable equilibrium price of GH¢ 30.49, which in turn provides the equilibrium average demand and supply respectively as 2931. 6 metric tons and 8217.6 metric tons. The excess supply is kept in stock for the next market period. When at another period demand exceeds supply, then the appropriate difference is released from the buffer to the market in order to keep price in equilibrium.

Table 4 shows the statistics of the simulated data (within time points of stability) when the buffer stock was run with Matlab dde23 solver. The values include range, mean, skewness less than 1 (moderate), kurtosis greater than 0 but within the expected value of 3 and their respective standard errors. The standard deviation is reduced to 0.1602 compared to 29.48 in the Table 2, and the same applies to the variance.

These affirm the fact that buffer stock acts as a reserve against short-term shortages and dampens down excessive fluctuations [31], which in turn disputes the assertion by Mackey [18], who based his argument on the fact that commodity price is dependent on planting time, storage time, relaxation time and total production time. However, if all these individual time parameters are combined as one for supply delay and make up storage time, the delay for the buffer, then price oscillations would be drastically reduced as shown in this paper.

The efficiency of the buffer system is dependent on delay variation suitable enough to be used by the buffer stock operator. The models used by Athanasiou et al. [9] and others are discrete-time dependent which is a limiting case of the delay buffer stock model used in this study. Thus, their models lack the time varying parameter. This study also disagrees with Mackey [18], in that, when storage delay is used as a buffer stock delay that is well managed, it could rather be a commodity price stabilizer. This study, moreover, disputes an assertion by Soltes et al [16] and indicates the model to be good for price stabilization.

4 Conclusion

The paper is intended to use mathematical models (which mimic a model developed by Soltes et al. [16] for controlling prices of maize by employing a nonlinear continuous time delay differential equation derived from linear demand and nonlinear supply functions of price with delay parameters reflecting the realities prevailing at the local market. These models are formulated from parameters estimated from real economic data of maize price demand and production in the Ashanti Region of Ghana through the use of regression methods. The data is obtained from the Ministry of Food and Agriculture, Statistical Directorate Kumasi-Ghana, from 1994 to 2013.

The results of the study dispute an assertion by Mackey [18], who based his argument on the fact commodity price is dependent on planting time, storage time, relaxation time and total production time. If all these individual time parameters are combined as one for supply delay and constitute storage time, the delay for the buffer, then price oscillations would be drastically reduced as shown in this paper. The study also improved on the work done by Athanasiou et al. [9], [16] and others, whose models are discrete-time dependent which is a limiting case of the delay buffer stock model used in this study. The efficiency of the buffer system is shown to be dependent on delay variation suitable enough to be used by a buffer stock operator.

It is noted that, the further the buffer stock delay and supply delay are reduced, the more stable the price becomes and the effects of buffer stock are felt more by stakeholders. The results of the analysis provide an average stable price of maize as GH¢ 30.49 compared to the actual average price of GH¢ 30.27. The equilibrium price in turn provides the average equilibrium metric tons of 2931. 6 and 8217.6, for demand and supply respectively. The excess supply is kept in stock and, when needed, is released in the next market period.The standard deviation is also reduced to 0.1602 compared to 29.48 in the Table 2, and the same applies to the variance.

However, before the application of a buffer stock scheme, price oscillated between two price points and could not converge. This affirms the fact that buffer stock acts as a reserve against short-term shortages and dampens down excessive fluctuations.

Our findings need to be interpreted in the light of its limitations, some of which could be addressed in further research. Further analysis is required on the impact of buffer stocks on price of the maize when private storage are allowed to compete with the government's in the context of price stabilization. Also foreign substitutes should be considered in the model formulation to assess their impact.

We draw inferences from this study that researchers should rather use continuous time nonlinear delay models as they reflect the realities in most real-life economic problems.

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Competing Interests

Authors have declared that no competing interests exist.

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APPENDIX

Appendix 1

The code similar to the one in appendix 2 without the buffer stock

Appendix 2

```
function ddebuffer2
%DDEPRICE  Solution for DDE23.
%   This code for delay differential equations with buffer stock
%   ddeprice are solved on [0, 100] with history y_1(t) = 1.23 and y'_1(0) = 0 ,for t <= 0.
%   the buffer used the same history function
%   The lags are specified as a vector [a,b] (they are varied), the delay              differential
%   equations are coded in the subfunction DDEPRICE, and the history is
%   evaluated by the function histprice. Because the history is constant it
%   could be supplied as a vector:
%   sol = dde23(@ddeprice,[a,b],[1.23;0],[0, 100]);
sol = dde23(@ddebuffer,[0.40,0.21],@histbuffer,[0, 100]);
figure;
plot(sol.x,sol.y)
title('delay diff cobweb model with buffer stock.');
xlabel('time t');
ylabel('solution y');
x = linspace(0,20,100);
y = deval(sol,x,1);
% --------------------------------------------------------------------------

function s = histbuffer(t)
% Constant history function for ddeprice.
s = [1.23,1.23];

% --------------------------------------------------------------------------
function dydt = ddebuffer(t,y,Z)
% Delay Differential equations function for DDEPRICE.
ylag1 = Z(:,1);
ylag2 = Z(:,2);
dydt = [ (-1*y(1)-3.68*ylag1(1)+0.029*ylag1(1)^2);
   ((-1*y(1)-3.68*ylag1(1)+0.029*ylag1(1)^2)-(1*y(2)+3.68*ylag2(2)-0.029*ylag2(2)^2))];
```


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