

# Nonlinear Conductivity in Graphene

S. S. Abukari<sup>a,\*</sup>, S. Y. Mensah<sup>a</sup>, R. Musah<sup>c</sup>,  
N. G. Mensah<sup>b</sup>, K. A. Dompok<sup>a</sup>

<sup>a</sup>*Department of Physics, College of Agriculture and Natural Sciences, U.C.C, Ghana.*

<sup>b</sup>*Department of Mathematics, College of Agriculture and Natural Sciences, U.C.C, Ghana*

<sup>c</sup>*Department of Applied Physics, University for Development Studies, Navorongo, Ghana*

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## Abstract

We consider the tight-binding approximation for the description of energy bands of graphene, together with the standard Boltzmann transport equation and constant relaxation time, an expression for the conductivity was obtained. We predicted strong nonlinear effects in graphene which may be useful for high frequency generation.

Keywords: Graphene, Mathematical model, Nonlinear effects, High Frequency Radiation.

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## 1. Introduction

Graphene was discovered by Novoselov et al in 2004 [1]- [3] and has since attracted a great deal of interest due to its unique properties like mechanical, electrical, thermal, etc arising from its highly symmetric two-dimensional honeycomb-lattice structure [4, 5]. This makes graphene potentially applicable in carbon-based nanoelectronics [6, 7]. The nonquadratic energy spec-

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\*Corresponding author

Email: [sulemana70@gmail.com](mailto:sulemana70@gmail.com)

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trum of graphene allows it to exhibit nonlinear phenomena [8] and its non-additivity property provides the mutual dependence of electron or hole motion in orthogonal directions [9]. Several unique physical properties of graphene have been studied both theoretically and experimentally [8]- [12].

Investigation into electronic properties of graphene has revealed that the electron dispersion law is linear in momentum near the Fermi points and thereby causing the quasiparticles to behave like massless Dirac Fermions [12]. However, the tendency of graphene to absorb impurities on its surface and interacts with the impurities electrons results in the formation of nonparabolicity of the energy band which makes the electronic properties of graphene essentially nonlinear under moderate electric fields [13]. This nonlinearity makes graphene to exhibit plethora of transport phenomena [12]- [20]. Under different conditions of an external electric field, an electron in graphene is predicted to reveal a variety of physical effects such as Bloch oscillations, self-induced transparency, absolute negative conductance, etc. Electronic properties and electronic transport in graphene is the subject of many theoretical papers [12]- [20]. Nevertheless, the electrodynamic properties of graphene is worth further studying because it is the basis for developing carbon-based devices. Using the kinetic transport equation, we shall in this work study the effect of high frequency (hf) conductivity in graphene by following the approaches of [21]- [26].

## 2. Theory

Proceeding as in references [21]- [26], we consider the motion of an electron in the presence of high frequency electric field  $E(t)$ . The electric field  $E(t)$

is directed along the graphene axis and the conductivity is derived using the Boltzmann kinetic equations describing electron transport in graphene for the distribution functions in the relaxation time approximation as follows:

$$\frac{\partial f_s}{\partial t} + eE(t)\frac{\partial f_a}{\partial \mathbf{p}} = \frac{F - f_s}{\tau} \quad (1)$$

$$\frac{\partial f_a}{\partial t} + eE(t)\frac{\partial f_s}{\partial \mathbf{p}} = \frac{-f_a}{\tau} \quad (2)$$

where  $e$  is the electron charge,  $\mathbf{p}$  is the electron dynamical momentum,  $F$  is the equilibrium distribution function and  $f_s$  as well as  $f_a$  the symmetric and antisymmetric distribution functions respectively. Solving Eqs.(1) and (2) in a constant electric field yields,

$$\frac{\partial^2 f_a^o}{\partial \xi^2} - \chi_o^2 f_a^o = \frac{\partial F}{(eEa\tau)\partial \xi} \quad (3)$$

where  $\xi_o^2 = \frac{\pi^2}{(eEa\tau)^2}$  and  $\mathbf{p} = \frac{\pi}{a}\xi$ . See [26]

Using  $\frac{\partial F}{\partial \xi} = \frac{\partial F}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial \xi}$ , the solution to Eq.(2) for the necessary boundary conditions  $f_a(-1) = f_a(+1) = 0$  is

$$f_a^o = \varepsilon \frac{\partial F}{\partial \varepsilon} \frac{eaE\tau}{(eaE\tau)^2 + 1} \quad (4)$$

The spectrum of electrons in graphene is given by [24,25].

$$|\varepsilon(p)| = \pm \frac{3\gamma_o b}{2\hbar} |\mathbf{p} - \mathbf{p}_F| \quad (5)$$

where  $\gamma_0 \approx 2.7eV$ ,  $b = 0.142nm$  is the distance between the neighbouring carbon atoms in the graphene. + and - signs are related to the conduction and valence bands respectively. With  $p_F$  as the constant quasimomentum

corresponding to the particular Fermi point. The current density of the mobile electron in the 1<sup>st</sup> Bz for graphene is given as [24, 25]

$$j_x = \frac{2e}{(2\pi\hbar)^2} \int \int v_x f_a^o d^2\mathbf{p} \quad (6)$$

and the quasiclassical velocity  $v_x(p)$  of an electron moving along the graphene axis can be

$$v_x(p) = \frac{\partial\varepsilon}{\partial p} = \frac{a}{\pi} \frac{\partial\varepsilon}{\partial\xi} = \frac{a}{\pi} \text{grad}\varepsilon \quad (7)$$

and writing

$$d^2\mathbf{p} = \frac{\pi}{a} d^2\xi = \frac{\pi}{a} \frac{ds}{|\frac{a}{\pi} \text{grad}\varepsilon|} dE \quad (8)$$

$ds$  is the element of the length of the curve.

Substituting Eqs.(3),(6) and(7) into(5) we get,

$$j_x = -\frac{e^2}{2\pi(\hbar)^2} \frac{E\tau}{(eaE\tau)^2 + 1} \int |\varepsilon| \frac{\partial F}{\partial\varepsilon} d\varepsilon \quad (9)$$

Using (4), we obtain

$$j_x = \frac{8\ln 2}{\pi(\hbar)^2} \frac{\tau e^2 k_B T E}{(\omega_B \tau)^2 + 1} \quad (10)$$

and

$$\sigma_x = \frac{8\ln 2}{\pi(\hbar)^2} \frac{\tau e^2 k_B T}{(\omega_B \tau)^2 + 1} \quad (11)$$

where  $\omega_B \tau = eaE\tau$ . See![26]

By linearizing Eqs. (1) for the perturbation

$$E(t) = (E + E_{\omega,k} e^{-i(\omega t + kx)}), f_s = f_s^o + f_s^1 e^{-i(\omega t + kx)}, f_a = f_a^1 e^{-i(\omega t + kx)} \quad (12)$$

we obtain

$$\frac{\partial^2 f_a^1}{\partial \xi^2} - \chi_\omega^2 f_a^1 = \pi^2 \frac{(i\omega\tau - 1)}{(eaE\tau)^2} f_a^o \frac{E_{\omega k}}{E} - \frac{\partial^2 f_a^o}{\partial \xi^2} \frac{E_{\omega k}}{E} \quad (13)$$

where

$$\chi_\omega^2 = \frac{\{1 - 2i\omega\tau - (\omega\tau)^2\}}{(eaE\tau)^2} \quad (14)$$

Using  $\frac{\partial F}{\partial \xi} = \frac{\partial F}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial \xi}$ , the solution to Eq(11) for the necessary boundary conditions  $f_a(-1) = f_a(+1) = 0$  is

$$f_a^1 = \left[ \varepsilon \frac{\partial F}{\partial \varepsilon} \left( \frac{(eaE\tau)}{(eaE\tau)^2 + 1} \right) \frac{E_{\omega k}}{E} \left\{ \frac{i\omega\tau - 1 - (eaE\tau)^2}{(eaE\tau)^2 - (\omega\tau)^2 + 1 - 2i\omega\tau} \right\} \right] \quad (15)$$

Substituting Eqs.(12),(6)and(7) into(5) and using (4), we obtain we get,

$$j_x = -\frac{e^2}{2\pi(\hbar)^2} \left( \frac{(eaE\tau)}{(eaE\tau)^2 + 1} \right) \left\{ \frac{i\omega\tau - 1 - (eaE\tau)^2}{(eaE\tau)^2 - (\omega\tau)^2 + 1 - 2i\omega\tau} \right\} \times \int |\varepsilon| \frac{\partial F}{\partial \varepsilon} d\varepsilon \quad (16)$$

where  $\int |\varepsilon| \frac{\partial F}{\partial \varepsilon} d\varepsilon = K_\beta T \ln 2$  making Eqn.(16) yield

$$j_x = -\frac{K_\beta T \ln 2 e^2}{2\pi(\hbar)^2} \left( \frac{(eaE\tau)}{(eaE\tau)^2 + 1} \right) \left\{ \frac{i\omega\tau - 1 - (eaE\tau)^2}{(eaE\tau)^2 - (\omega\tau)^2 + 1 - 2i\omega\tau} \right\} \quad (17)$$

$$\sigma_x(\omega) = \sigma_o \left\{ \frac{i\omega\tau - 1 - (eaE\tau)^2}{(\omega_B\tau)^2 - (\omega\tau)^2 + 1 - 2i\omega\tau} \right\} \quad (18)$$

where  $eaE\tau = \omega_B\tau$  and  $\sigma_o = \frac{2\ln 2 \tau e^2 k_B T}{\pi(\hbar)^2} \frac{\omega_B\tau}{(\omega_B\tau)^2 + 1}$ . See [26]

### 3. Results, Discussion and Conclusion

We present the results of a kinetic equation approach of a graphene subject to constant electric field  $E$ . The electric field  $E$  is directed along the

graphene axis. Exact expression for the direct conductivity was obtained in eq.(9). The nonlinearity is analyzed using the dependence of the normalized conductivity  $j_x$  as a function of  $E$ . . Figure 1 illustrates the dependence of  $j_x$  on  $E$ . The figure shows a linear dependence of  $j_x$  on  $E$  at weak values of  $E$ . As  $E$  increases,  $j_x$  increases, and at a some value of  $E$ ,  $j_x$  reaches a maximum value. Further increase in  $E$  results in the decrease of  $j_x$ . The slope of the curve is the dc differential conductivity,  $\sigma_x = dj_x/dE$ . See equation (10). If  $E \approx 1$ , the Bloch frequency is in the terahertz frequency range and if  $E > 1$ ,  $\sigma_x$  is negative and therefore graphene demonstrates negative differential conductivity (NDC). Figure 2 elucidates the graph of conductivity component  $(\sigma_x(\omega))/\sigma_o$  obtained in Eq. (14) on the normalized frequency  $\omega/\omega_B$ . The dimensionless complex conductivity  $(\sigma_x)/\sigma_o$  depends strongly and nonlinearly on the normalized frequency  $\omega/\omega_B$ . We observed that the real part of the complex conductivity will become more negative with increasing frequency, until a resonance minimum occurs just before the Bloch frequency  $\omega_B\tau$ . This negative-conductivity resonance close to the Bloch frequency makes the graphene an active medium for a Bloch oscillator without domain instabilities induced by negative dc conductivity. In summary, using the solution of the Boltzmann's transport equation with constant relaxation time  $\tau$  approximation, we obtained an exact expression for the conductivity of graphene. We noted a strong nonlinear effects which may be useful for the generation of high frequency radiation.

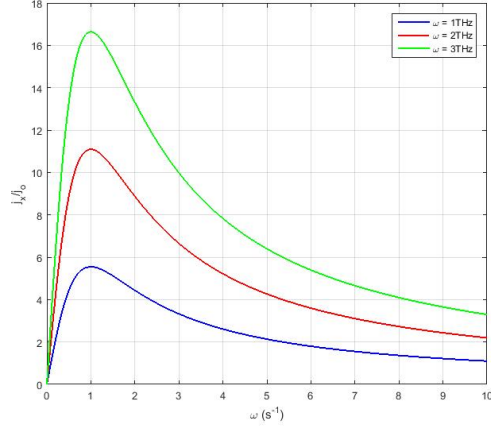


Figure 1: A plot of current density  $j_x$  as a function of  $E$  in graphene for expression (9).

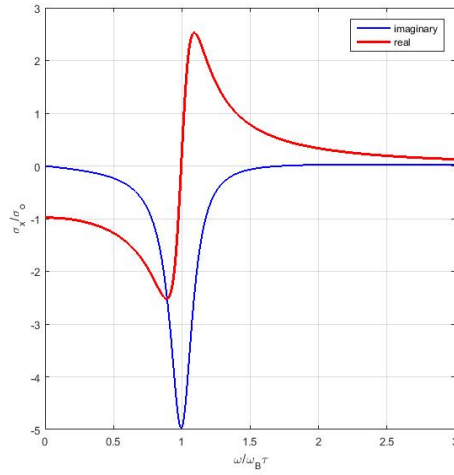


Figure 2: A plot of a normalized complex conductivity  $(\sigma_x\omega)/\sigma_0$  as a function of dimensionless frequency  $\omega/\omega_\beta$  in graphene for expression (9) when  $\omega_\beta\tau = 10$ .

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