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NONLINEAR POLARIZATION EFFECTS IN A BIREFRINGENT SINGLE MODE OPTICAL FIBER

G.C. Ishiekwene[1]
Department of Physics, College of Science and Technology, University of Liberia, Monrovia, Liberia
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy,

S.Y. Mensah
Laser and Fiber Optics Center, University of Cape Coast, Cape Coast, Ghana

and

C.S. Brown
Luxcore Networks Incorporated, Decatur, Georgia, USA.

Abstract

The nonlinear polarization effects in a birefringent single mode optical fiber is studied using Jacobi elliptic functions. We find that the polarization state of the propagating beam depends on the initial polarization as well as the intensity of the input light in a complicated way. The Stokes polarization parameters are either periodic or aperiodic depending on the value of the Jacobian modulus. Our calculations suggest that the effective beat length of the fiber can become infinite at a higher critical value of the input power when polarization dependent losses are considered.

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[1]Regular Associate of the Abdus Salam ICTP.

1 Introduction

Nonlinear effects resulting from polarization behavior have been of great research interest because they lead to various applications including pulse shaping, optical switching, intensity discriminators and all-optical logic gates [1]. In optical fiber telecommunication devices, nonlinear polarization dependent effects are also of keen interest. It has been reported that polarization dependent losses influence systems containing several elements connected by optical fibers [2]. Also, several nonlinear phenomena including optically induced birefringence [3], polarization instability [4 − 6], and solitons [7] have led to important advances from the fundamental as well as technological points of view. In addition, interest in nonlinear polarization optics is expected to develop further in view of the current emphasis on photonics-based technologies for information management. Infact, a polarization diversity detection system for distributed sensing of polarization mode coupling in high birefringence fibers has been implemented using a pump-probe architecture based on optical Kerr effect [8]. Thus, a thorough understanding of polarization and its effects are fundamental to the design and characterization of various devices that use single mode optical fibers.

In this paper, we will study the nonlinear polarization effects in a birefringent single mode optical fiber. Evolution equations involving Stokes parameters are derived and used to describe polarization changes as the beam propagates along a birefringent optical fiber. The solutions are obtained in terms of Jacobi elliptic functions. It will be shown that the output Stokes parameters depend on the Jacobian modulus which in turn depends on the input polarization of the light. We will also show that depending on the value of the Jacobian parameter, the output Stokes parameters may be periodic or aperiodic. For each case of periodicity, the effective beat length of the fiber is determined. We further observe in this work, that the effective beat length can become infinite at a higher critical input power. We will also present geometrical illustrations to enhance the visualization of these resulting effects.

The rest of this paper is organized as follows: section two deals with the theory and contains the basic equations for understanding of the concepts; section three involves a discussion on the salient features of the results; and finally, section four is devoted to the concluding remarks.

2 Theory

The evolution equations describing the polarization state of an intense light beam propagating along a birefringent single-mode optical fiber can be expressed in compact form as [5, 9]

$$\frac{d\mathbf{S}}{dz} = \left[\mathbf{\Omega}^{L}(z) + \mathbf{\Omega}^{NL}(z, \mathbf{S})\right] \times \mathbf{S} \tag{1}$$

where $\mathbf{S} = (S_1, S_2, S_3)$ is Stokes vector associated with the polarization state of the monochromatic electric fields. In one of the standard conventions [10], the Stokes parameters are defined as

$$S_\alpha = A_i^* \left(\sigma_\alpha\right)_{ij} A_j \tag{2}$$

where the indices $\alpha = 1, 2, 3$ and $i, j = 1, 2$. Here, the A_i's represent the field amplitudes along the principal axes of the fiber and σ_α are the Pauli spin matrices. The $\mathbf{\Omega}$'s with projections Ω_1, Ω_2, and Ω_2 are vectors in Stokes space related to the fiber characteristics and can be expressed explicitly as

$$\mathbf{\Omega}_\alpha^L (z) = \left(\sigma_\alpha\right)_{ji} u_{ij} \tag{3}$$

and

$$\mathbf{\Omega}_\alpha^{NL} (z, \mathbf{S}) = \frac{1}{2} w_{\alpha\beta} S_\beta \tag{4}$$

u_{ij} is a tensor of rank two in 2-dimensional space related to the fiber linear dielectric tensor ϵ_{ij} which represents the coupling in the field amplitudes due to perturbations in the fiber while $w_{\alpha\beta}$ is an expansion coefficient representing the fiber parameters responsible for nonlinearities and related to its cubic nonlinear susceptibility tensor $\chi_{ijkl}^{(3)}$.

If we assume small anisotropy along the direction of propagation and that the cubic nonlinearity is isotropic, the nonlinear evolution equations will take the following form

$$\frac{dS_1}{dz} = -\left(\frac{12\pi\omega}{nc}\chi_{1122}^{(3)}\right) S_2 S_3 \tag{5}$$

$$\frac{dS_2}{dz} = \frac{\omega}{2nc}\left(\epsilon_{11} - \epsilon_{33} + 24\pi\chi_{1122}^{(3)} S_1\right) S_3 \tag{6}$$

$$\frac{dS_3}{dz} = -\left\{\frac{\omega}{2nc}\left(\epsilon_{11} - \epsilon_{33}\right)\right\} S_2 \tag{7}$$

Following the method of Sala [11], the exact analytical solutions to this system of equations can be deduced. Using Eq.(5) and Eq.(7), S_2 can be eliminated to obtain

$$\frac{d}{dz}\left(2G_0 S_1 - G_1 S_3^2\right) = 0 \tag{8}$$

which when integrated will yield

$$2G_0\left(S_1 - S_{10}\right) = G_1\left(S_3^2 - S_{30}^2\right) \tag{9}$$

We have set $G_0 = \omega\left(\epsilon_{11} - \epsilon_{33}\right)/2nc$ and $G_1 = 12\pi\omega\chi_{1122}^{(3)}/nc$. The $S_{\alpha 0}$ denotes an input Stokes parameter, ω is the frequency of the propagating beam, c is the speed of light and n is the refractive index of the fiber.

We further assume that the beam of light is completely polarized and the total intensity is normalized so that

$$S_1^2 + S_2^2 + S_3^2 = 1 \tag{10}$$

Then, using the above equation with the help of Eq.(7) and Eq.(9), we obtain

$$\left(\frac{dS_3}{dz}\right)^2 = -D_1 S_3^4 - D_2 S_3^2 + D_3 \tag{11}$$

where

$$D_1 = \frac{1}{4} G_1^2 \tag{12}$$

$$D_2 = G_0^2 + G_1 \left(G_0 S_{10} - \frac{1}{2} G_1 S_{30}^2 \right) \tag{13}$$

and

$$D_3 = G_0^2 - \left(G_0 S_{10} - \frac{1}{2} G_1 S_{30}^2 \right)^2 \tag{14}$$

The right-hand side of Eq.(11) has roots

$$\upsilon_{1,2} = \frac{-D_2 \pm \sqrt{D_2^2 + 4D_1 D_3}}{2D_1} \tag{15}$$

which are strictly real. Then, Eq.(11) can be expressed as

$$\left(\frac{dS_3}{dz} \right)^2 = -D_1 \left(S_3^2 - \upsilon_1 \right) \left(S_3^2 - \upsilon_2 \right) \tag{16}$$

The exact analytical solution to Eq.(16) can be obtained in terms of elliptic functions as

$$S_3 = \pm \sqrt{\upsilon_1} \, \mathbf{cn} \left(\sqrt{D_1 \left[\upsilon_1 - \upsilon_2 \right]} z + C \, ; \, \mathrm{k} \right) \tag{17}$$

where C is a constant and $\mathrm{k} = \sqrt{\upsilon_1 / (\upsilon_1 - \upsilon_2)}$ is the modulus. The above expression in Eq.(17) can be written more compactly in the form

$$S_3 = \frac{2p\mathrm{k}f}{q} \mathbf{cn} \left(G_0 f z + C \, ; \, \mathrm{k} \right) \tag{18}$$

The solutions for S_2 and S_1 can be obtained similarly and expressed as follows

$$S_2 = \frac{2p\mathrm{k}f^2}{q} \left[\mathbf{sn} \left(G_0 f z + C \, ; \, \mathrm{k} \right) \right] \mathbf{dn} \left(G_0 f z + C \, ; \, \mathrm{k} \right) \tag{19}$$

and

$$S_1 = \frac{f^2}{q} \left\{ 1 - 2m \left[\mathbf{sn}^2 \left(G_0 f z + C \, ; \, \mathrm{k} \right) \right] \right\} - 1 \tag{20}$$

where

$$f = \left[\left(1 + q S_{10} \right)^2 + q^2 S_{20}^2 \right]^{\frac{1}{4}} \tag{21}$$

cn, **sn**, and **dn** are Jacobian elliptic functions and $q = G_1 / G_0$ is a relative measure of the anisotropy and the cubic nonlinear susceptibility of the fiber. In this paper, it is assumed that $q \geq 0$ since the difference in refractive indices for the eigenpolarizations is a positive quantity for a birefringent fiber. Also, $p = \pm 1 = sgn\,(S_{30})$ with the sign function defined as $sgn\,(x) = 1$ for $x \geq 0$ and $sgn\,(x) = -1$ for $x < 0$. The Jacobian modulus k is given by

$$\mathrm{k} = \left[\frac{1}{2} + \frac{1}{4f^2} \left(q^2 - 1 - f^4 \right) \right]^{\frac{1}{2}} \tag{22}$$

and

$$-\operatorname{Re}\left[\mathrm{K}\,(m) \right] \leq C \leq +\operatorname{Re}\left[\mathrm{K}\,(m) \right] \tag{23}$$

with $\mathrm{K}(m)$ denoting the Jacobian quarter-period. Here, the Jacobian parameter $m = \mathrm{k}^2$. The constant $\mathcal{C}$ can be obtained in terms of the initial conditions

$$\mathbf{cn}\,(\mathcal{C}\,;\,m) = q\frac{|S_{30}|}{2\mathrm{k}f} \tag{24}$$

where $sgn\,(\mathcal{C}) = sgn\,(pS_{20})$.

These solutions in Eq.(18) through Eq.(20) describe the characteristics of an arbitrarily intense beam propagating along a low-loss optical fiber. The equations show that the output Stokes parameters depend on the Jacobian modulus k which depend on the input Stokes parameters and the ratio q. These solutions are general since no restrictions are placed on the relative strength of the optical fields and are also exact since the set of coupled nonlinear differential equations are solved in terms of known transcendental functions.

3 Discussion of Results

With the use of Eq.(18) through Eq.(20), the propagation characteristics of the optical beam for different initial polarization states can now be described. The propagation of an intense light along the optical fiber may result in a change in the fiber's refraction, absorption and anisotropy. We will assume that the intensity-dependent behavior of the fiber is accounted for in the cubic nonlinearity. Then, the object q relates to the intensity of the beam and may be regarded as a scaling parameter as the intensity of the input light is varied. If nonlinear effects are initially neglected so that the fiber response to the propagating light wave is considered linear, the cubic nonlinearity becomes negligible and the quantity $q = 0$. Then, one deduces immediately that $m = 0$ with the help of Eq.(21) and Eq.(22). We note that when $m = 0$, the Jacobian elliptic function takes on its degenerate trigonometric counterpart $\{\sin nx, \cos nx;\; n = 0, \pm1, \pm2...\}$. Fig.(1) illustrates the effects, when $q = m = 0$, for an input light with initial polarization state $S_{10} = -0.5$, $S_{20} = 0$ and $S_{30} = 0.3$. The output Stokes parameters are periodic with period 2π and the fiber beatlength is given by $L(m = 0) = 2\pi/G_0$. This case obviously involves polarization effects of linear and circular birefringence and dichroism in the fiber.

If we now consider effects when the fiber behaves nonlinearly due to the propagation of an intense beam, the cubic nonlinear susceptibility tensor cannot be neglected and $q > 0$. Fig.(2) and Fig.(3) show, respectively, for $q = 2.3$ and $q = 4$, the variations in the Stokes parameters as a function of dimensionless distance for an input light with initial polarization state corresponding to a right elliptically polarized beam with 45° azimuthal angle and 22.5° ellipticity. We note that such an input beam has an initial polarization state $S_{10} = 0$, $S_{20} = 0.71$ and $S_{30} = 0.71$. Also, $m = 0.58$ when $q = 2.3$. This is easily deduced resorting again to Eq.(21) and Eq.(22) with knowledge of the input Stokes parameters. In Fig.(2), the Stokes parameters are observed to be doubly periodic with period $L\,(m) = 4\mathrm{K}(m)\,/fG_0$. Thus, the period of the elliptic function determines the effective beat length of a nonliner fiber. In addition, the effective beatlength depends on input intensity and input polarization. Fig.(3) shows a qualitatively different behavior from the previous illustrations. For this case $m = 1$. We observe that the output Stokes parameters are aperiodic even though the initial polarization state is the same as in Fig.(2). The polarization tends asymptotically to a final state instead of varying periodically as in the previous cases when $0 \leq m < 1$. The final state in this example of Fig.(3) corresponds to a linearly polarized optical beam.

Another set of interesting results for an input light with initial polarization state $S_{10} = 0$, $S_{20} = 0.3$ and $S_{30} = 1$ corresponding to a right elliptically polarized beam with 45° azimuth and 37° ellipticity is shown in Fig.(4) for $q = 2.3$. We deduce for this case that $m = 1.2$ when $q = 2.3$. The Stokes parameters are doubly periodic with period $L(m) = 2\mathrm{K}(1/m)/\mathrm{k}fG_0$. We note that as q increases from 2.3 to 4, the period of the Stokes parameters also increases. This implies that the output polarization state at fiber length ($z = L$) is dependent on the beam's intensity.

We also obtained results shown in Fig.(5) for the unique polarization state in which the orientation angle is 90° and the initial Stokes parameters are such that $S_{10} = -0.5$, $S_{20} = 0$ and $S_{30} = 0.3$. In this example, $q = 2$ and $m = \infty$. For this case, the fiber's effective beat length becomes infinite at a higher critical input power. As we have reported [12], this occurs because of an intensity dependent change in the ellipticity of the polarization ellipse. This is a surprising result and is of special interest since it could have concrete implications for systems that combine birefringent fibers and components with polarization dependent losses. Ordinarily, in linear optics, increase in losses leads to a decrease in power. However, our result indicates that in the nonlinear case, increase in polarization dependent loss shifts polarization instability to higher power values.

4 Conclusions

The output polarization state of an optical beam propagating along a birefringent single mode fiber has been shown to be dependent on the Jacobian modulus which in turn depends on the input polarization of the light as well as the relative measure of the nonlinear susceptibility and anisotropic dielectric tensors characterizing the fiber. We have reported that the output Stokes polarization parameters are either periodic or aperiodic depending on the value of the Jacobian modulus. We have also found that the effective beat length of a fiber can become infinite at a higher critical value of the input power when polarization dependent losses are considered. These results show that the polarization state of a beam propagating along a birefringent single mode optical fiber depends on the initial polarization as well as the intensity of the input light in a complicated way. This has far reaching effects on polarization-sensitive devices utilizing single mode optical fibers.

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References

1. B. Daino, G. Gregori, and S. Wabnitz, Opt. Lett. **11**, 42 (1986)
2. N. Ginsin, Opt. Comm. **114**, 399 (1995)
3. H. G. Winful, Appl. Phys. Lett. **47**, 213 (1985)
4. H. G. Winful, Opt. Lett. **11**, 33, (1986)
5. F. Matera and S. Wabnitz, Opt. Lett. **11**, 467, (1986)
6. S. Trillo et al., Appl. Phys. Lett. **49**, 1224, (1986)
7. S. M. Baker and J. N. Elgin, Quantum Semiclass. Opt. **10**, 251, (1998)
8. P.L.D. Julian et al., **16**(12), 2378 (1998)
9. Y. P. Svirko and N. I. Zheludev, *Polarization of Light in Nonlinear Optics*, (John Wiley and Sons, 1998)
10. M. V. Tratnik and J. E. Sipe, Phys. Rev. A **35**, 2965 (1986)
11. K. L. Sala, Phys. Rev A **29**, 1944 (1984)
12. G. C. Ishiekwene, S. Y. Mensah, and C. S. Brown, ICTP Preprint IC/2000/97

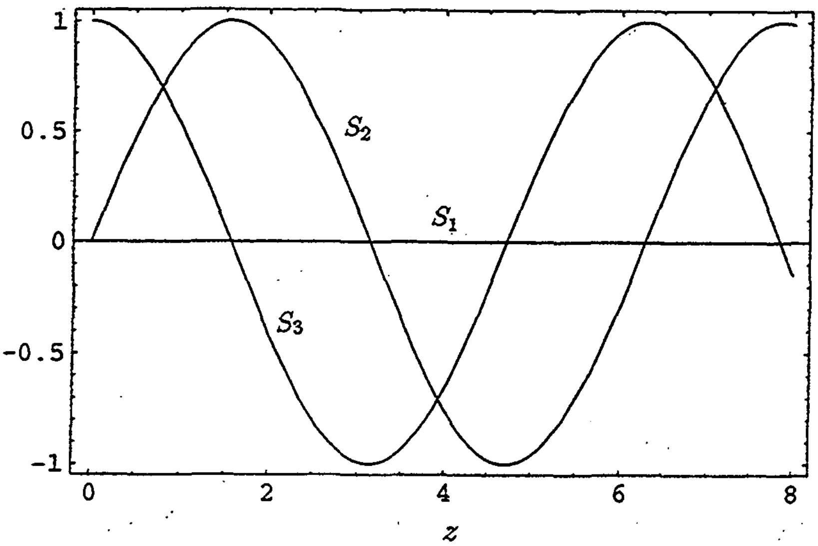

FIG. 1. Stokes parameters as a function of fiber length for initial polarization state $S_{10} = 0$, $S_{20} = 0.3$ and $S_{30} = 1$. In this figure, $q = m = 0$ and $L = 2\pi/G_0$.

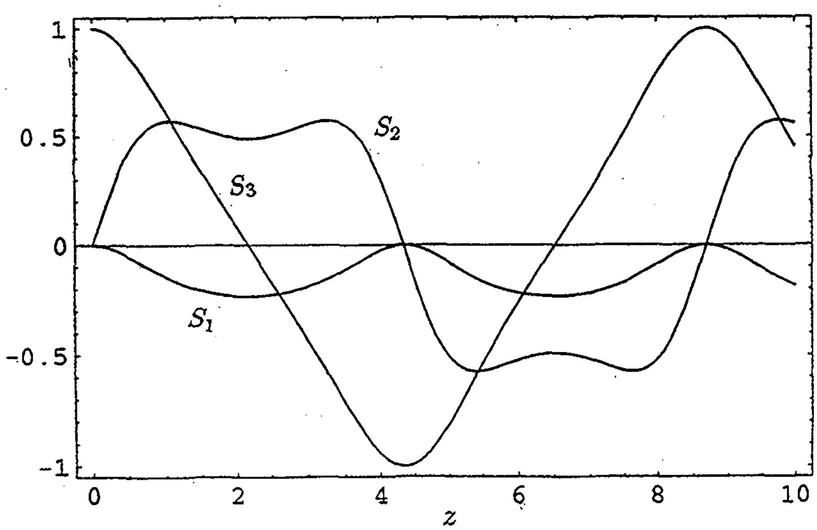

FIG. 2. Stokes parameters as a function of fiber length for initial polarization state $S_{10} = 0$, $S_{20} = 0.71$ and $S_{30} = 0.71$. Here, $q = 2.3$, $m = 0.58$ and $L = 4K(m)/fG_0$.

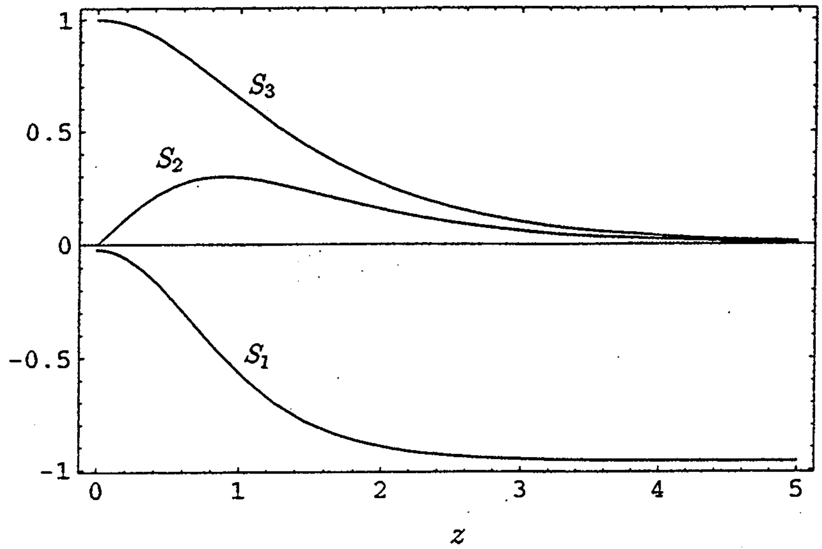

FIG. 3. Same initial polarization state as in Fig. 2 but with $q = 4$ and $m = 1$.

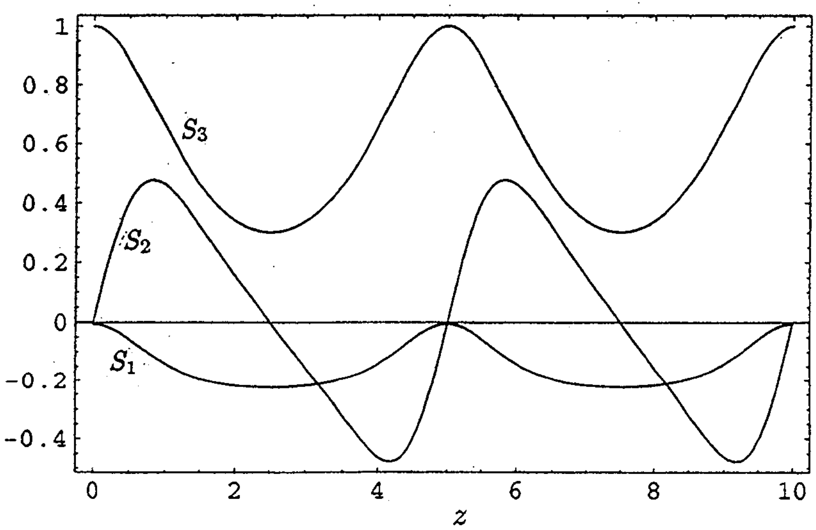

FIG. 4. Same initial polarization state as in Fig. 1 but with $q = 2.3$, $m = 1.2$ and $L = 2K\left(1/m\right)/kfG_0$.

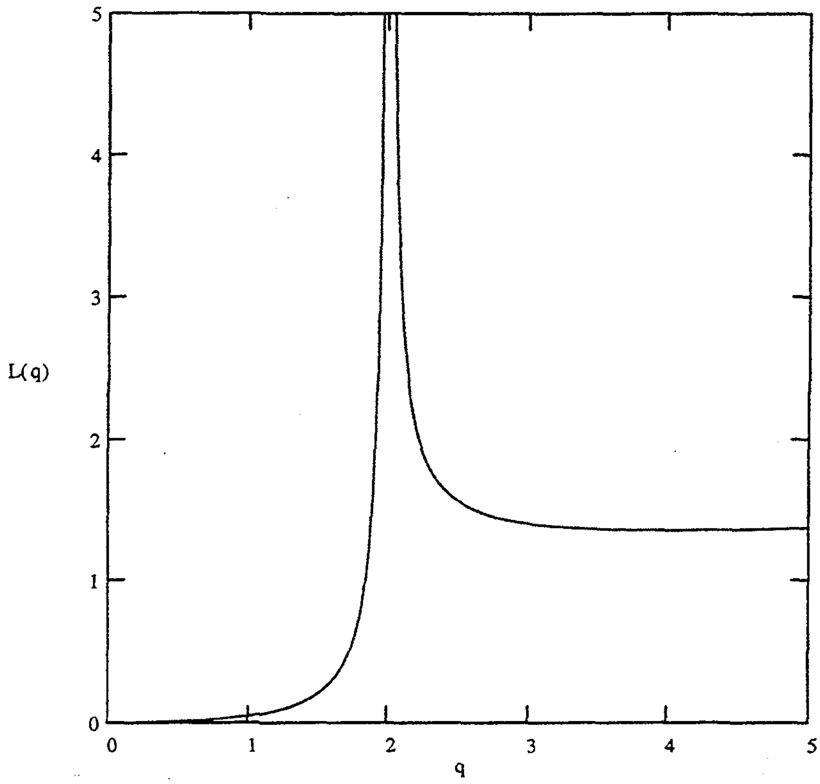

FIG. 5. Effective beatlength as a function of intensity for initial polarization state $S_{10} = -0.5$, $S_{20} = 0$ and $S_{30} = 0.3$.