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LETTER TO THE EDITOR

The negative differential effect in a semiconductor superlattice in the presence of an external electric field

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Abstract. Boltzmann's equation is solved for an electron in the lowest miniband of a superlattice semiconductor under the influence of an external electric field $E(t)$. The current-density electric field characteristic shows a negative differential conductivity when $\omega\tau \ll 1$ (quasi-static case). This occurs in the neighbourhood where the constant electric field E_0 is equal to the amplitude of the AC electric field E_1 and the peak decreases with increasing E_1.

With the development of computer-controlled molecular beam epitaxy it has become possible to grow semiconductor superlattices (SL) having a period of the order of 10^2 Å. This large lattice constant and the resultant splitting of bands into narrow minibands made it possible to observe a number of novel high-field non-ohmic behaviours which can be attributed to the interaction of electrons with optical phonons or to inter-valley scattering such as in the Gunn effect; the effect in question arises from the relatively large number of electrons near the edge of the miniband boundary. As electrons come closer to the miniband edge, their velocity decreases, and it is this decrease in velocity that accounts for the negative differential conductivity (NDC).

NDC has been observed under various conditions in SL, e.g. when the Bloch frequency $\omega_B = eEd$ ($\hbar = 1$) exceeds the increased carrier relaxation time τ^{-1} [1, 2] or when $eEd > \Delta$ [3] (Δ is the superlattice bandwidth). As indicated [3], the mechanism of NDC is based on the fact that the potential drop eEd over a lattice period may exceed the bandwidth of the miniband, and under this condition the current is dominated by the transition between localized states of adjacent wells. The current then decreases because of a decrease of the probability for these transitions, whether phonon or impurity induced. Another NDC mechanism in SL was proposed [4] that relied on a transition between Bloch miniband transport and hopping transport between Wannier–Stark quantized levels [5].

Recent work by Sibille *et al* [6, 7] has indicated that Bloch electron conduction through the superlattice miniband is responsible for this NDC over a large range of superlattice parameters. Their results excluded the mechanism of hopping transport between Wannier–Stark quantized levels [4, 5]. There have also been several Monte Carlo calculations [8–10] on the transport properties of SL. Lei *et al* [11] have also calculated the NDC using a more sophisticated theory.

In this letter we consider the same situation as in [1, 2] but with the field $E_0 + E_1 \cos \omega t$, i.e. the sample is placed in DC and AC electric fields. The problem will be

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tackled in the quasi-classical case; thus $2\Delta \gg \tau^{-1}$, eE_0d, eE_1d (ω is the frequency of the field, τ is the relaxation time of the electrons, e is the electron charge). These conditions are necessary to confine the electrons in the lowest miniband and also to allow us to use the kinetic Boltzmann equation, while bearing in mind that the electron energy is rather a complicated function of the quasi-momentum because of the periodicity of the potential seen by the electrons.

The distribution function of the electron in the lowest miniband of the SL, after solving Boltzmann's equation, with constant relaxation time, is given [12, 13] as

$$f(p,\ t) = \tau^{-1} \int_0^\infty \mathrm{e}^{-t'/\tau}\ \mathrm{d}t'\ f_0\left(p - e\int_{t-t'}^{t} (E_0 + E_1\ \cos\ \omega t'')\ \mathrm{d}t''\right). \tag{1}$$

With the help of (1) we define the current density along the SL axis as

$$j(t) = e\sum_p V_z(p) f(p,\ t). \tag{2}$$

Substituting (1) into (2), the following expression is found for the current density:

$$j(t) = e\tau^{-1}\sum_p V_z(p) \int_0^\infty \mathrm{e}^{-t'/\tau}\ \mathrm{d}t'\ f_0\left(p - e\int_{t-t'}^{t} (E_0 + E_1\ \cos\ \omega t'')\ \mathrm{d}t''\right). \tag{3}$$

Taking the energy $\mathcal{E}(p)$ of electrons in SL in the lowest miniband as

$$\mathcal{E}(p) = p_\perp^2/2m + \Delta(1 - \cos\ p_z d) \tag{4}$$

it follows that

$$V_z(p) = \Delta d\ \sin\ p_z d. \tag{5}$$

Substituting (5) into (3) and performing the following transformation:

$$p \to p - e\int_{t-t'}^{t} (E_0 + E_1\ \cos\ \omega t'')\ \mathrm{d}t''$$

the current density assumes the form

$$j(t) = ed\ \Delta\tau^{-1}\sum_p f_0(p) \int_0^\infty \mathrm{e}^{-t'/\tau}\ \mathrm{d}t'\ \sin\left(p_z - e\int_{t-t'}^{t} (E_0 + E_1\ \cos\ \omega t'')\ \mathrm{d}t''\right) \tag{6}$$

which further simplifies [14–16] to

$$j(t) = ed\ \Delta\tau^{-1}\left(\sum_p f_0(p)\ \cos\ p_z d\right) \times \int_0^\infty \mathrm{e}^{-t'/\tau}\ \sin\left(ed\int_{t-t'}^{t} (E_0 + E_1\ \cos\ \omega t'')\ \mathrm{d}t''\right) \mathrm{d}t' \tag{7}$$

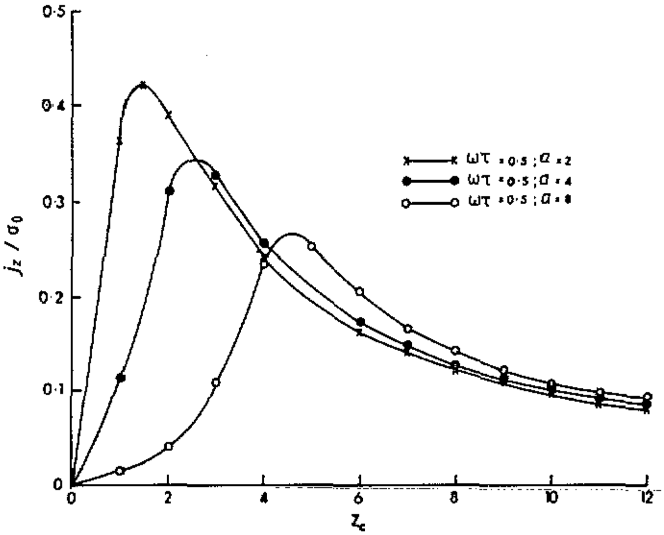

Figure 1. Expression (8); j_z/σ_0 is plotted against Z_c for: (x ——x) $\omega\tau = 0.5$, $a = 2$; (● —— ●) $\omega\tau = 0.5$, $a = 4$; (○——○) $\omega\tau = 0.5$, $a = 8$.

where $f_0(p)$ is the equilibrium distribution function.

Solving equation (7) for a non-degenerate electron gas and averaging over the AC electric field we get

$$j_z = \sigma_0 \sum_{k=-\infty}^{\infty} \frac{J_k^2(a)(k\theta + Z_c)}{1 + (k\theta + Z_c)^2} \tag{8}$$

where: $\sigma_0 = en\ \Delta d\ I_1(\Delta/kT)/I_0(\Delta/kT)$; $a = eE_1 d/\omega$; $Z_c = eE_0 d$; $\theta = \omega\tau$; $J_k(x)$ is the Bessel function; $I_k(x)$ is the modified Bessel function.

Equation (8) (see figure 1) can be expressed as

$$j_z = \sigma_0 \left(\frac{J_0^2(a)}{1 + Z_c^2} + 2 \sum_{k=1}^{\infty} \frac{J_k^2(a)[1 + Z_c^2 - (k\theta)^2]}{[1 + (Z_c - k\theta)^2][1 + (Z_c + k\theta)^2]} \right) Z_c. \tag{9}$$

When $E_1 = 0$, $J_0(a) \to 1$, $J_k(a) \to 0$ and from (9) we get [1, 2]

$$j_z = \sigma_0 Z_c / \left(1 + Z_c^2\right). \tag{10}$$

This can be expressed as

$$j_z = \sigma_0 E_0/E_c/[1 + (E_0/E_c)^2] \tag{11}$$

where $E_c = 1/ed\tau$ is the critical field of the NDC effect, $\partial j_z/\partial E_0$. Note that j_z approaches its peak value j_m at $E_0 = E_c$:

$$j_m = e\ \Delta d\ nI_1(\Delta/kT)/2I_0(\Delta/kT). \tag{12}$$

To justify the method used, we compare our work with the experimental work of Sibille *et al* [6, 7] and the results of simulations by the Monte Carlo method [8] as

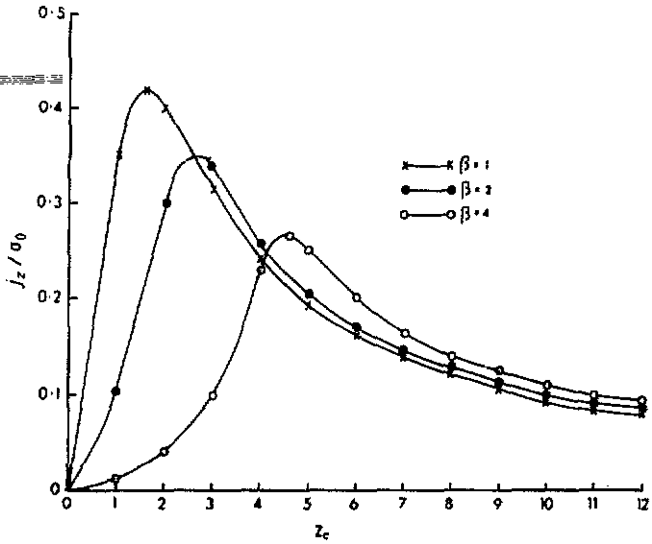

Figure 2. Expression (14); j_z/σ_0 is plotted against Z_c for: (×——×) $\beta = 1$; (●——●) $\beta = 2$; (○——○) $\beta = 4$.

presented by Ignatov *et al* [17]. To do this we transform j_m to V_m (the maximum drift velocity) and replace Δ by $\Delta/2$. The following expression is obtained:

$$V_m = \Delta d\ I_1(\Delta/2kT)/4I_0(\Delta/2kT). \tag{13}$$

As indicated [17], our results agree favourably with the experimental results and Monte Carlo simulations.

We are mainly concerned with the quasi-static case ($\theta \ll 1$). Therefore we transform expression (8) to

$$\sum_{k=-\infty}^{\infty} \frac{\alpha J_k(a) J_{k+\nu}(a)}{1+\alpha^2} = \int_0^\infty \left(\mathrm{e}^{-x}\ \sin\ Z_c x\right) \sum_{k=-\infty}^{\infty} J_k(a) J_{k+\nu}(a)\ \cos\ k\theta x\ \mathrm{d}x \tag{14}$$

where $\nu = 0$; $\alpha = k\theta + Z_c$ (see figure 2). The expression under the summation, after manipulation, becomes

$$\sum_{k=-\infty}^{\infty} J_k(a) J_{k+\nu}(a)\ \cos\ k\theta x = J_0(2a\ \sin\ \theta x/2). \tag{15}$$

Substituting (15) into (14) we get

$$j_z = \sigma_0 \int_0^\infty \mathrm{e}^{-x}\ \sin\ Z_c x\ J_0(2a\ \sin\ \theta x/2). \tag{16}$$

For $\theta \ll 1$, equation (16) gives

$$j_z = \sigma_0 \left(\frac{\frac{1}{4}\{[1+(Z_c+\beta)^2]^{1/2} + [1+(Z_c-\beta)^2]^{1/2}\}^2 - \beta^2 - 1}{[1+(Z_c+\beta)^2][1+(Z_c-\beta)^2]} \right)^{1/2} \tag{17}$$

where $\beta = eE_1 d$.

As can be seen from (17), j_z assumes its maximum value in the vicinity of $Z_c \simeq \beta$ for any given value of β, as indicated in figure 2. Thus NDC is observed where the constant field E_0 is approximately equal to the amplitude of the AC electric field E_1. Expressions (8) and (17) were computerized and the graph of j_z/σ_0 against Z_c was plotted as indicated in figures 1 and 2. Expression (17) coincides with expression (8), to a very high degree of accuracy, for $\omega\tau \ll 1$. It is worth noting that for any given value of a, $\beta = a\omega\tau$; thus for $\omega\tau \ll 1$, figure 1 can easily be compared with figure 2. It is observed that for $\beta = 1$, 2 and 4 the two graphs coincide and that the peak of the curves decreases with increase in E_1.

As established by Pamplin [18], semiconductors with n-type current–voltage characteristics reveal various instabilities which can be utilized for the amplification and generation of electromagnetic oscillations. There have been several experimental investigations in which n-type current–voltage characteristics have been observed in semiconductor SL [1, 4, 6].

We believe that the setting up and extending of such experiments could throw more light on the development of this effect.

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