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JUNIOR HIGH SCHOOL STUDENTS AND THEIR MATHEMATICS TEACHERS' UNDERSTANDING OF THE CONCEPT OF ADDITION OF FRACTIONS IN TWO SELECTED DISTRICTS IN THE CENTRAL REGION OF GHANA

BY

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VERITAS LUMEN NOBIS

Thesis submitted to the Department of Mathematics and Information Communications Technology Education of the Faculty of Science and Technology Education, College of Education Studies, University of Cape Coast, in partial fulfilment of the requirements for the award of Doctor of Philosophy degree in the Mathematics Education

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DECLARATION

Candidate's Declaration

I hereby declare that this thesis is the result of my own original research and that no part of it has been presented for another degree in this university or elsewhere.

Candidate's Signature ………………………… Date……9/3/2020……

Name: ……Ebo Amuah……………………………………

Supervisors' Declaration

We hereby declare that the preparation and presentation of the thesis were supervised in accordance with the guidelines on supervision of thesis laid down by the University of Cape Coast.

Principal Supervisor's Signature ………………………… Date……9/03/2020……

Name: ……Prof. Ernest Kofi Davis……………………

Co-Supervisor's Signature ………………………… Date ……09/03/2020……

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ABSTRACT

The study investigated students and their mathematics teachers' understanding of addition of fractions in three school contexts. The study was in three aspects: first is the understanding of addition of fractions; second is the characteristics of sentences in classroom interaction during the teaching and learning of addition of fractions; third is the understanding of the RPK in the teaching and learning of addition of fractions. A total of 616 students and their 17 mathematics teachers were sampled. The concurrent mixed method design was employed for the study. In exploring understanding of RPK and understanding of addition of fractions, categorisations (high, average, low, and NU, PU_S, PU_A, FU, respectively) of diverse levels of understanding were employed and frequency tables generated. Quantitative and qualitative approaches were used to analyse the tasks. MANOVA showed significant variations in students' level of understanding of the RPK as well as addition of fractions. Students demonstrated low understanding of the RPK and the concept of addition of fractions. Classroom interaction in high achieving schools were observed to be richer than interaction in the other school contexts. It was recommended that preservice and in-service providers should put strategies in place to help students and their mathematics teachers in low achieving schools improve on their understanding of the RPK and addition of fractions. Findings in relation to the characteristics of sentences in classroom interaction in high achieving schools should be used to improve interaction in other school contexts.

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KEY WORDS

Human elements

Mathematical object

Representation

Sentence processes

Understanding

Understanding of the concept of addition of fractions

Understanding of the specific RPK

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DEDICATION

To my late father (Mr Paul Amuah) and my children.

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CHAPTER ONE

INTRODUCTION

This chapter provides the background to the exploration of the understanding of addition of fractions among students and their mathematics teachers. The role that the understanding of addition of fraction plays in the teaching and learning of other mathematical operations is mentioned. Some difficulties faced by teachers during the teaching and learning of addition of fraction is also identified.

Background to the Study

Mathematics has continually occupied a strong position as a core subject in the education system in Ghana. From grade one up to secondary school level, it is a subject that must be passed in every high stake examination before progression onto a higher level of education. The Ghanaian syllabus has continually been improved over the years in an attempt to foster deeper level of learning (Ministry of Education, Science and Sports, 2007; Ministry of Education [MOE], 2012). In the syllabus, deeper levels of understanding have often been noted to be the desired goal to be achieved (MOE, 2010; MOE, 2012). Mereku (2010) in his analysis of the Ghanaian school mathematics textbooks over several years (1957-2010) observed that earlier textbooks consisted of mainly mathematical symbols and their associated equations. The major use of mathematical symbols and equations makes mathematics topics less practical. However, there have been recent efforts to make mathematics textbooks more practical by introducing human elements. Hence more recent textbooks showed inclusion of words and descriptions (MOE, 2012; MOE, 2012a).

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Early introduction of the teaching and learning of mathematics in early levels of education in most countries, mathematics as a subject is introduced with number systems. Introduction to number systems involves exposition to counting numbers and gradually, the introduction to rational number system. One aspect of the rational number system that most learners had experienced difficulties with, was the area of fraction (Prediger, 2008; Teppo, & van den Heuvel-Panhuizen, 2013; Bailey et al., 2015). In the early work of Wilson (2009) fraction was identified as one of the strands of the major mathematical content that comprised the syllabus of most countries.

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In the Ghanaian Junior High School mathematics curriculum, fraction and its operations is the third topic that is to be learnt in the syllabus. It is therefore a fundamental part of items to be learnt in the Junior High School level (MOE, 2012). Also at the Senior High School level, attempt is made to reconcile and connect the ideologies of fraction to other rational numbers. Accordingly, fraction is the second major topic in the first year of Senior High School mathematics syllabus (real number system). It is part of the second, third, and sixth subtopic under real number system. However, fractions addition is captured under the associative property of addition. This can be interpreted to mean that fraction and its addition forms a major base on which students learning of mathematics is built at the Junior and the Senior High School levels of education in Ghana (MOE, 2010; 2012; 2012a; 2012b).

The role that addition of fractions plays in the teaching and learning of other fraction operations is central. The four basic binary operations of addition, subtraction, multiplication, and division, form the basic connecting blocks of most mathematical generalisation (Wilson, 2009; Wu, 2013). Generally, the

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idea of addition in most basic school curriculum highlights the notion of putting things together and regrouping (especially in the whole number context). Subtraction is normally introduced to learners based on addition. Subtraction is therefore illustrated as taking away from the group (Bell et al., 2007; Vig, Murray & Star, 2014). In Bell et al (2007), the basic concept supporting learning of the concept of addition of numbers include joining, measuring, and separating. Subtraction is often considered the opposite of adding (reverse counting, as opposed to 'forward' totalling/counting). This implies that the properties of commutativity, directional effect, relationship of the numbers involved, and the operand, need to be mastered as part of understanding of addition of fractions. Accordingly, Amuah, Davis and Fletcher (2017) suggested that understanding of fractional concepts did not necessarily imply just an understanding of the algorithm (although it could be considered as part). Nor was it just a model representation, but an explanation of the operation as used in real life context.

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Thus, initial introduction to multiplication, and subsequently, division of whole numbers, normally starts from addition of whole numbers. Consequently, multiplication is conceptualised as repeated addition for a number of times, and repeated subtraction for the concept of division. Exploring understanding of fractions in the mathematics classroom does not necessarily imply showing all the properties that are exhibited when dealing with counting numbers. However, Wu (2008; 2013) has shown that it is possible to use the properties that were observed by children when teaching and learning addition of whole numbers to effectively teach the concept of addition of fractions.

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In the learning of the concept of addition of fractions, one of the
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concepts that is a necessary requirement is the basic understanding of the concept of fraction. Understanding fraction (as part of a whole), and the properties (unit fraction, equivalent fraction etc.) that are in the concept of fraction (as part of a whole), could be a major determinant of the ability to add or unite two fractions of the same type. It is important to also acknowledge that there are more complex fractions i.e., improper fractions and mixed numbers (Paas, Tuovinen, Tabbers & Gerven, 2003). The characteristic of understanding complex fractions could determine the level of understanding that the learners/students might eventually achieve during the learning process of the concept of addition of fractions (Fuchs et al., 2014; Peng et al., 2016).

To analyse the aforementioned assertion, Norton and Wilkin (2011) examined how students understanding of the various schemes of fraction resulted in the eventual performance of students in mathematics. They examined students' understanding of each construct of fraction and eventually, attempted to discover how the understanding of the various fraction constructs resulted in general understanding of fraction. Jordan, Hansen, Fuchs, Siegler, Gersten, and Micklos (2013), in a similar light, attempted to discover the predictors of learners understanding of the fractional concept itself, its operations, and general mathematical achievement. In Jordan et al. (2013), learners' attention during study, verbal skills, level of general intelligence, and the span of the working memory were assessed. The level of demonstration of the predictors mentioned was used to determine/predict the level of understanding of basic fractional concepts. Similarly, the predictors were used to predict learners' level of understanding of addition and subtraction of

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fraction. These predictors were also used to predict general mathematics
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achievement among learners. As most of the aforementioned predictors were behavioural dispositions, similar analysis was performed with mathematics content predictors (Jordan et al., 2013). The content predictors were standardised tests. These mathematics content predictors were approximate number system, number line estimation, calculation fluency, and reading fluency. Levels of performance in these batteries of standardized tests were used to estimate the level of general performance in operations of fractions addition and subtraction, and general mathematical achievement. These are but a few of the studies that considered other students' previous abilities with their ability in addition of fractions.

Indeed, teaching and learning of mathematics for thorough understanding has not been an easy task to define in mathematics education. Amongst the strands of mathematical proficiency reported in Grooves (2012), thorough understanding would involve intertwining other strands of proficiency (productive disposition, adaptive reasoning, strategic competence, and procedural fluency). Wilson (2009) was of the opinion that most mathematical generalisation were obtained by connections (established by operators, which includes addition) that were known (observed) in different concepts. These connections were then brought to harmony in the generalisation. What this means is that, despite the fact that there are concepts that have their individual and unique characteristics and ideologies, uniting these concepts with mathematical generalisations should represent a more complex and higher level of understanding for learners of concepts in mathematics (Paas et al., 2003; Fuchs et al., 2014; Peng et al., 2016). In fact, ability to demonstrate a thorough

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understanding in mathematical operations should indicate stronger grounding in understanding of the concepts used in that particular operation. Hence in addition of fractions, there is the need to be able to identify the concepts (unit fractions, equivalent fractions, etc.) that are at play, and the strength of influence that they may have on the understanding of addition of fractions (Norton & Wiklin, 2011; Namkung & Fuchs, 2016). There is therefore the need to be able to identify the level of understanding of those individual concepts, the problems that children might be experiencing in those concepts, and how they result in the understanding of the target concept (addition of fractions).

Cross (1999) opined that the business of learning was about making connection. It is also important to recall that understanding that is in-depth demonstrates connections in understanding of subsumed concepts (Groove, 2012). Cross (1999) identified four types of connections as neurological connection, cognitive connection, social connection, and experiential connection. She defined neurological connection in terms of connecting neurons in the brain and how the neurons behave when one is learning specific and general mathematical concepts. Cognitive connection in learning was described in relation to cognitive theory of learning. Consequently, in the area of fraction, it involved the formation of schemas of fraction and possible connection between the schemas in terms of their structure (Norton & Wilklin, 2011). Cognitive connection in fractions also involved the continuity between major topics (i.e., hierarchy of knowledge in mathematics). Hierarchy of knowledge in mathematics could be the connection between whole number, fractions, and addition of fractions, in increasing order of complexity (Ertmer & Newby, 2013; Fusch et al., 2014; Namkung & Fuchs, 2016; Peng et al., 2016). Learning as a

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social connection was defined in relation to socio-constructivism as a learning theory. Here the learning was observed to be a product of the communication (interaction) within a social setting (school). Therefore, communication between the mathematics teachers, students, and other learning materials that were incorporated in the teaching and learning of mathematics was characterised as learning (Davis, Bishop, & Seah, 2015; Morgan 2016). In the case of addition of fractions, this could involve the way basic fractions concepts were communicated to the learners during the teaching and learning of addition of fractions (i.e., sentence characteristics/structures in classroom interaction). It could also involve how representation, TLM, etc., were used. Words like 'hence,' 'consequently,' etc., are used as linking words in the interaction involving the mathematics teachers and students in the classroom, so that both mathematics teachers and students can make reference to other mathematical ideas in the lesson. Use of words like, 'we,' 'I,' 'it,' etc., by the teacher also has its associated relevance in the teaching and learning process. The use of such words by the teacher, encourages the students to believe that the teacher considers them (students) as knowledgeable in mathematics (Adams, 2006; Ertmer & Newby, 2013). Experiential connection was also devised from the experiential theory of learning that postulates students as being involved in an activity where the concept of addition of fractions is hosted. Consequently, the combination of two different fractional part of a wood to form the leg of a chair (whole). But the most important implicative feature here is the practical use of the fractional parts of the wood as a case for fractions addition in the practice (Kolb, 1984; Kolb & Kolb, 2008; Mughal & Zafar, 2011). This study adapted the last three of the definition of Cross (1999) to consequently describe learning

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of concepts in addition of fractions and their subsequent associated understanding.

It is obvious that, in describing connections of ideas in the teaching of the concept of addition of fractions, the ideas to be connected were mostly related and previously learnt mathematical concepts (RPK). RPK has been a major aspect of teaching and learning for many years. In fact, it is a major determinant that the teacher uses to make most of his/her decisions in channelling the processes of learning and the subsequent learning experiences that the learners are expected to encounter. These related previous knowledges could determine the strategy that the teacher would most likely adopt in the teaching and learning process (contrast or similarity teaching and learning strategy) (Clark, Ayres, & Sweller, 2005; Wessle, 2012; Graaf, 2014). As observed in the study of Rittle-Johnson, Star, and Durkin (2009) related previous knowledge is a strong determinant of the level of knowledge possessed and the flexibility of knowledge learners are able to demonstrate at the end of a lesson.

From the perspective of Cross (1999), it is implied that presenting related previous knowledge (fraction concepts, whole number addition) from the experiential teaching and learning perspective only, is not likely to lead to effective learning when the new concept (addition of fractions) being learnt is taught in purely cognitive perspective (e.g. schema form, a purely cognitive learning structure) (Norton & Wiklin 2011; Vig et al., 2014). In a similar manner, teaching and learning particular mathematical concepts (as fraction) with purely cognitive teaching perspective (schema form) would not likely foster better understanding of the concept of addition of fractions when the

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concept of addition of fractions is taught with a purely experiential teaching and learning perspective. (Clark et al., 2005; Vig. et al., 2014; Morgan, 2016).

Chick (2007) is a more focused and typical example of the importance of the form of knowledge of the RPK, to the teaching and learning of the concept of addition of fractions. Chick showed how primary school teachers who mostly adopted constructivist ideas in teaching the concept of addition of fractions exhibited mismatch between the RPK and the concept of addition of fractions. Chick (2007) in assessing pedagogical content knowledge of teachers by the use of examples, noted that a teacher called Jill used discrete models to introduce the concept of fraction. The class examined various examples of same denominator discrete models in fraction representation. The concept of fraction here served as a related previous knowledge. However, her choice of example while treating the main concept of fractions addition (12/14 and 3/14) resulted in a fractional concept that has not been dealt with by the students (improper fraction). In the classroom, this created confusion in the minds of the learners towards students modelling of the concept of fraction (Graaf, 2014; Vig. et al., 2014). In the class of Brain, addends that were mixed fractions were used to teach addition of improper fraction. Brain was expecting that the students would convert the mixed fraction to improper fraction. However, some of the students started with whole number addition and subtraction and later dealt with the fractional part. This is a typical illustration of how mathematics teachers' conception of related previous knowledge could be different from learners/students related previous knowledge.

In all the ideas noted in relation to RPK and content to be learnt, it implies that to be able to teach, and for students to be able to learn effectively,

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it is necessary for the mathematics teacher to recognise the form of knowledge of the RPK that the learners possess. This would inform the mathematics teacher to ensure that the knowledge (content) being taught will be in the same form as the RPK. Hence if the RPK is in the form of experiential or real life events (experiential theory of teaching and learning), then the content being taught will have to be in real life events. If the mathematics teacher wants to teach the content in terms of structures (cognitive teaching and learning theory), then there is the need to transform the RPK of the learner, to reflect that form of the RPK the teacher requires (cognitive theory of learning and teaching) before the teacher proceeds teaching (Kplan & Murphy, 2000; Meyer, 2004; Campbell, 2008; Everret, Harsy, Hupp & Jewell 2014; Graaf, 2014). Davis, Seah, and Bishop (2009) is an exemplary study.

Davis, Seah, and Bishop (2009) explored students' understanding of the concept of fraction in two contexts (out of school and in school). It was noted that the students exhibited difficulty in identifying fractions in the school context. Even the fractions they were able to identify in the out of school context were difficult to identify in the school context. This suggested that students understanding of the basic concept of fraction was really lacking in the school context. Davis et al (2009) recommended that teachers should explore the out of school context knowledge (experiential learning and understanding) and use it as a tool to teach fractions (addition of fractions inclusive). Findings in Amuah, Davis, and Fletcher (2017), among students in three categories (High, Average, and Low performing) of basic schools in Ghana revealed partial understanding of basic properties that needed to be fully understood to demonstrate understanding of the concept of faction. More difficulty was

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experienced when the fractions were greater than one. The least understood concept of fraction was fraction equivalence. Students in all the category of schools studied (high, average, and low performing schools), lacked understanding of the concept of equivalent fraction. The problem emanating from this finding is that the concept of equivalent fraction is adapted when dealing with some major operations on fractions (especially addition of fractions). Consequently, ability to conceptualise equivalence in fractions is related structurally and pedagogically to the ability to conceptualise addition of fractions. In line with the findings and recommendations of Davis et al (2009) and Amuah et al (2017), examining understanding of the concept of addition of fractions should be in conjunction with the RPK (fraction and addition of whole numbers).

In support of the aforesaid perspective, recent psychological research on understanding has noted that comprehensive understanding involved a knowledge re-structuring process rather than a replacement process (Larsson & Halldén, 2010; Vosniadou, 2013; Roediger III & Desoto, 2015). Knowledge reconstruction process involves knowledge reconstruction of the previous knowledge. It also involves the reconstruction of general knowledge as a result of introduction of the new concept learnt (assimilation). As an example, knowledge reconstruction could involve the revision of related previous knowledge (part-whole concept of fraction, fraction schemes, etc.) and gaining of new perspective in learning (e.g. addition of fractions). Research that normally involves the revision of related previous knowledge, normally investigates teaching and learning in the formal classroom setting (Bruce, Chang, Flynn, & Yearley, 2013; Vosniadou, 2013). This study can therefore be

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said to explore the nature of knowledge reconstructed during various stages in the teaching and learning of addition of fractions.

Statement of the Problem

The importance of fraction and fraction arithmetic in the area of mathematics cannot be over emphasized. Knowledge of fraction is key to the development of a number of topics in mathematics. In the area of probability, fraction is the means through which probability values are expressed. Percentages are often written in the form of fractions, and multiplied by a hundred. In studying triangles (geometry), cosine, tangent, and sine functions are expressed in fractional forms. Wide application of algebra requires extensive expressions in the structure of fractions (Bruce et al., 2013, Wu, 2013; Norton, Boyce, Philips, Anwyll, Ulrich, and Wilkins 2015; Kosko & Singh, 2018). Literature also suggests that fractions are widely used in everyday life of people in Ghana and all over the world (Davis, Seah, & Bishop, 2009). Fraction and for that matter arithmetic of fractions is therefore studied in the school mathematics curriculum of many countries in the world.

Despite its importance in everyday life of students and in the future study of mathematics, fractions continue to be difficult for students to learn and mathematics teachers to teach. The chief examiners' reports on the conduct of the BECE Mathematics examinations in 2013, 2014, 2015, and 2017 indicated that one of the major students' weaknesses as fraction knowledge (WAEC, 2013; 2014; 2015; 2017). Amuah, Davis, and Fletcher (2017) also found weak conception of fraction among Junior High School Three (JHS 3) public school children in Ghana. Weak conception of fraction was observed in almost all the individual concepts subsumed in fraction. In relation to the concept of

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equivalent fraction, no student could demonstrate any form of understanding. Equivalent fraction is the basic concept used for successful addition of fractions (Vig, et al., 2014).

Apart from the concept of fractions being difficult for students, literature suggests that students have difficulty with language. The Chief examiners' reports on the conduct of the BECE Mathematics examinations in 2013, 2014, and 2015, all indicated general language difficulties and specific language difficulties among students who attempted word problems involving fractional concepts and its operations (addition of fractions inclusive) (WAEC, 2013; 2014; 2015). The problem of fraction appears not to be limited to school children. The study in Ghana showed that even some prospective mathematics teachers in training programmes also had difficulty with arithmetic fraction (Davis & Ampiah, 2009).

Literature suggests that methods teachers employ in the teaching of mathematics generally and fractions specifically, appear to contribute to Ghanaian students low levels of attainment in mathematics generally and fractions specifically (Davis & Agbenyega, 2012; Agbenyega & Davis, 2015; Davis, 2018). For example, Davis (2018) observed from his study on classroom discourse in mathematics class of some selected primary school teachers in Ghana that the traditional approach to teaching was prevalent. The student is a passive participant in the learning process. Students only ask questions and respond to the teacher's question only when the teacher prompts them to do so.

In light of the weakness of the traditional approach of instruction (Agbenyega & Davis, 2015; Davis, 2018), there are other promising strategies in the teaching process that have been noted to improve students' learning and

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subsequent understanding. One of the strategies is the intentional and planned act of selecting and using the RPK effectively in the teaching and learning process (Campbell, 2008; Campbell and Campbell, 2009). The second promising strategy is related to the language of instruction. Specifically, this strategy is the conscious effort to choose the characteristic of sentences (in the langue of instruction), that is used during the teaching and learning process (O'keeffe & O'Donoghue, 2015; Morgan, 2016). Further details about these strategies will be presented in Chapter Two of the thesis.

Literature in the teaching of addition of fractions suggests that mathematics teachers do not critically reflect on their choice and use of RPK in the teaching and learning process (Meyer, 2004; Chick, 2007; Cramer, Wyberg, & Leavite, 2008). Meyer, (2004) showed how a teachers' wrong choice of RPK confused his students in the teaching and learning process. Again, the deliberate choice in the characteristics of sentences used during the teaching and learning of addition of fractions was shown in the study of O'keeffe and O'Donogue (2015) to be a distinguishing factor between high performing schools' students and low performing schools' students. Agbenyega and Davis (2015) also showed the importance of the choice of words in distinguishing a student who will eventually understand the requirement in a mathematical problem and a student who will not understand the requirements. Despite the importance of the choice of RPK and sentence characteristics (language) in the quality of teaching and learning processes, and for that matter, the quality of students' learning outcomes in mathematics generally and fractions specifically, not many studies in Ghana have looked at these variables together. This study was therefore designed to contribute to the understanding of why fractions is difficult for

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Ghanaian teachers to teach and students to learn by focusing of RPK and sentence characteristics in classroom interaction in addition of fractions.

Research Questions

The research questions were framed to explore students and their mathematics teachers' understanding of the concept of addition of fractions. However, a critical aspect that will help illuminate the variations in understanding of the concept of addition of fractions, is the related previous knowledge, and the characteristics of sentences used during the teaching and learning of addition of fractions. Consequently, the study also explored understanding of the specific RPK for teaching and learning addition of fractions. Characteristics of sentences used during classroom interaction were also investigated. The following research questions were then framed to guide the study:

1. What are students and their mathematics teachers' understanding of the related previous knowledge for the teaching and learning of the concept of addition of fractions?
2. What are students and their mathematics teachers' understanding of the related previous knowledge for the teaching and learning of the concept of addition of fractions by school context (that is, high, average and low achieving schools)?
3. What is the effect of school context on students' understanding of related previous knowledge for the teaching and learning of the concept of addition of fractions?
4. What are the characteristic of sentences used in classroom interactions during the teaching and learning of the concept of addition of fractions?

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5. What are the characteristic of sentences used in classroom interactions during the teaching and learning of the concept of addition of fractions by school context (that is, high, average, and low achieving schools)?
6. What are students and their mathematics teachers' understanding of the concept of addition of fractions?
7. What are students and their mathematics teachers' understanding of the concept of addition of fractions by school context (that is, high, average, and low achieving schools)?
8. What is the effect of school context on students' understanding of the concept of addition of fractions?

Purpose of the Study

The study sought to explore students and their mathematics teachers' understanding of addition of fractions. It endeavoured to use respondents' responses and some theoretical perspectives, to categorise students' and their mathematics teachers' understanding of the concept of addition of fractions. The study intended to use the categorisations to explore variations in respondents' understanding of addition of fractions, over the various school contexts. The significance of the differences among the school context would be investigated.

Additionally, the study sought to explore students and their mathematics teachers' understanding of the specific RPK for the teaching and learning of addition of fractions. The study aimed at categorising students and their mathematics teachers' understanding of the specific RPK (High, average, and Low understanding). Consequently, the study explored understanding of the RPK among respondents in the various school contexts. The significance of the

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difference in respondents' demonstrated understanding in the various school context, was investigated.

The study also aspired to explore classroom interaction during the teaching and learning of addition of fractions. Accordingly, the study explored the combination of processes (sentence characteristics) that could be observed in transcribed classroom interaction. Possible differences in the combination of sentence characteristics (processes) observed in the three school contexts, was also investigated.

Significance of the Study

In respect of policy making, the study is designed to explore selected characteristic (concept of addition of fractions, RPK for addition of fractions, and processes/characteristics in classroom interactions) in three school contexts (High, average, and low achieving schools). Consequently, policy makers could be guided by findings from this study to implement policies that would encourage the demonstration of characteristics observed in school contexts that showed desirable features. Policy makers would be already informed of the undesirable characteristics that were observed in some research participants. This would help them initiate policies that would help direct the transformation of mathematics teachers and students in order to help them demonstrate desirable characteristics

The study would help teacher educators and policy makers to train mathematics teachers who would be able to prepare students well enough to demonstrate a comprehensive understanding of the RPK for the teaching and learning of addition of fractions. Mathematics teacher educators would be informed to explore the strategies they could adopt to produce mathematics

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teachers who would be able to help students progress from low understanding of RPK to high understanding of RPK. For In-service training providers, the study would help inform workshops that would enable mathematics teachers in the field to guide their students develop better understanding of the concept of fraction, and addition of fractions.

The study will provide background information that would guide in-service providers in designing workshops that will help mathematics teachers overcome their weaknesses that is related to understanding of the concept of addition of fractions. Other workshops could be designed to address mathematics teachers' strategies to encourage students to be active participants during the teaching and learning of mathematics. As the study is in three school contexts, the study shows where urgent in-service interventions are needed, per the characteristics in the transcribed classroom interaction.

To education researchers and myself, there is the necessity to investigate how students' understanding of the concept of addition of fractions connect to students' understanding of other mathematical concepts at similar levels of education. Some of the recommendations in this study may be of peculiar interest to researchers.

Delimitation

The study involved only some schools within the Cape Coast metropolis, and Komenda Edna Egyafo Assamankessi (KEEA) Districts. These districts are located within the Central region of Ghana and hence they are not necessarily implicative of what happens in the other 18 districts of the central region.

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Secondly, describing understanding of addition of fractions does not necessarily indicate the kind of representation that the learners are commonly using properly. However, it focuses on the process of using the representations.

Limitations

The various aspects of the study that were examined were as follows: RPK, classroom interactions, and understanding of addition of fractions. With these aspects examined for the same class, there were no cross tabulations of evidence from these three different aspects. Consequently, the evidence was stated as per their isolated, diverse, and particular aspects of teaching and learning. The effect of findings in one aspect of the study, on a different aspect of the study, was not investigated nor stated. It is therefore important that the evidence so far is considered as such. This is part of the weaknesses that could be noted in the design that was used for the study.

Planned time for collection of the mathematics teachers' instrument for both the concept of addition of fractions and the specific RPK, was not followed. In some cases, both instruments delayed for almost a month before retrieval. This gave the teachers the opportunity to refer to other documents in responding to the questions. Therefore, creating unequal assessment practices. This implies that any difference in understanding of RPK or the concept of addition of fractions, could be as a result of other things apart from respondents' understanding.

Finally, only mathematics teachers of the pupils involved in the study were used in the study. The small number of mathematics teachers is not enough to properly determine mathematics teachers' understanding of the concept of

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addition of fractions, and the specific RPK for addition of fractions. It also didn't allow for multivariate analysis of the variables involved in the study.

Definition of Terms

Human elements: This is the presence of human agency in sentences or clauses that are used in describing mathematical concepts and procedure. Presence of human elements include the presence of nouns and pronouns, in conjunction with verbs that indicate actions of nouns. Process of eliminating human elements in mathematical discussion was described as alienation (Morgan, 2016)

Representation: Representation in mathematics includes figures and diagrams that are used to illustrate mathematical processes and concepts. These diagrams or illustrations could be two or three dimensional. Representation in this sense includes real artefacts that could be used during the description of mathematical ideas in the teaching and learning process (Singer, 2007; Vig, et al., 2014). However, representation in this text excludes mathematical models. It is important to understand that representations are not mathematical concepts on their own.

Mathematical objects: This definition of mathematical objects is specifically for the purpose of the analysis of sentence characteristics. It is a collective name used for mathematics symbols that are often used in discussion of mathematical ideas. Mathematical objects include equations, inequalities, numbers, algebraic representations, diagrams, and illustrations (O'keeffe and O'Donoghue, 2014). However, mathematical objects do not include real artefacts.

Sentence processes: It refers to categorization of sentences or phrases in interaction about mathematical concepts. These sentence categorisations were

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based on expression of the ideas that the speaker of the sentence has in his/her mind about mathematical concepts. It indirectly reflects how the reader expects the listener to view his or her mathematics. This categorisation of each sentence or clause in an interaction about a mathematical concept, is based on the structure of the sentences involved in the interaction.

Understanding of specific RPK: Understanding of RPK is a demonstration of connected thoughts in the ideas or concepts imbibed in the concept of fraction, and/or whole number and whole number addition. It also includes skills that are common to fraction, whole number, and whole number addition (Wu, 2013; Graaf, 2014; Rittle-Johnson & Schneider, 2014). Therefore, the idea of unit coordination was adapted as the concept of specific RPK for addition of fractions. Demonstration of understanding of the specific RPK does not include invention of the idea or skills in fraction, whole number, or whole number addition.

Understanding of the concept of addition of fractions: This relates to a demonstration of linked thoughts when solving addition of fractions problems given. Linked thoughts are demonstrated by correctly justifying how one aspect of the addition of fractions problem, is transformed into another in the process of solving the addition of fractions problem. Description of these transformation could be in the form of sentences, mathematical procedures, representation, or a combination of any of the forms. This led to a categorisation into four levels of understanding of addition of fractions in this research.

Understanding: Understanding of a mathematical concept involves the demonstration of comprehension of any aspect of the mathematical problem. This is because, without the aforementioned, it is impossible to solve the

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mathematical problem in the first place. Understanding also involves the linking of the knowledge used in solving the mathematical problem.

Organisation of the Study

The rest of the study involved the literature review which considered the different aspects of the study, and reviewed relevant literature that is associated with the study. In each aspect of the literature that was reviewed, there was a description of the model that was adopted for the study. The first part involved an explanation of the general idea of understanding, as it relates to relevant teaching and learning theories. The second part involved literature related to related previous knowledge (RPK). The third part involved understanding of addition of fractions, and finally, characteristics of sentences related to classroom interaction.

Chapter three involved the description of the methods that were used in the study. It included the detailed description and reasons for the research design, sample and sampling procedure, method of data analysis, and other methodological issues related to the study. The fourth chapter involved the presentation and discussion of the results of the study. Finally, the results of the study were summarised with their associated recommendations in chapter five.

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CHAPTER TWO

LITERATURE REVIEW

In examining the learning of addition of fractions in the Ghanaian classroom, the study examined the understanding of the related previous knowledge of the students and their mathematics teachers. It also examined sentence characteristics in classroom interaction when teaching addition of fractions. Finally, the study examined the understanding of addition of fractions. The study therefore drew on three theoretical frameworks to address each of these areas of learning in the mathematics classroom. Related previous knowledge (learner development model) was investigated with the unit coordination theory. Classroom interaction was examined using the Systemic Functional Linguistic theory (SFL), and the understanding of addition of fractions was examined by categorising respondents' demonstrated characteristics of understanding of addition of fractions

Conceptual Framework

Most curricula, schools, and researchers, propose deep rooted learning as the kind of learning they intend to achieve (Ben-Hur, 2006; Ministry of Education, 2012; Chi, 2013). Fostering this kind of learning has been identified to involve healthy use and selection of appropriate related previous knowledge. Mathematics contents that are taught in isolation from students' related previous knowledge have the ability to considerably reduce students' motivation for learning mathematics (Hailikari, 2009; Jitendra, Griffin & Xin, 2010; Graaf, 2014). According to Anderson (1981), related previous knowledge has been identified to be one aspect of teaching and learning that has the greatest impact on students' achievement. Hence Campbell (2008) made attempt to identify

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effective approaches in the use of related previous knowledge that had been identified from various research.

In order to be able to examine the understanding of RPK, the model of related previous knowledge adopted in this study was the Learner and development model. This model sees related previous knowledge as that which could take any form and therefore, normally needs to be synthesised (Graaf, 2014). After the synthesis, the related previous knowledge is translated into basic mental mathematics ideology (Chi, 2013; Wilkins, Norton & Boyce, 2013; National Centre on Intensive Intervention, 2014; Namkung & Fuchs, 2016; Peng, et al., 2016; Price & Fuchs, 2016).). In examining understanding of fraction, whole numbers, and whole number addition, as a related previous knowledge of the students and their mathematics teachers, the study draws on the unit coordination theory.

Unit as used in the unit coordination theory refers to any arbitrary or physical units that may be identified within mathematics learning contexts. Unit coordination therefore implies the manner and strategy that students adapt as they relate various levels of units during one form of measurement or another (Steffe, 1992; 2002). Norton, Boyce, Philips, Anwyll, Ulrich and Wilkins (2015) showed that researchers could categorise progression in students' ability to coordinate varied and similar units. This categorisation is in stages and each category is determined by the number of levels (refer to units in fraction tiles, see Figure 3) of units that the learners can relate to, at the same time. What makes the unit coordination theory so unique for this study is that no matter the form of related previous knowledge (whole numbers or fractions), it has a basis in unit coordination theory. Therefore, all concepts and skills in the unit

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coordination theory are explored in classroom interaction during the teaching and learning processes. This is because, all aspects of teaching and learning are brought together in the classroom and they are involved by means of interaction between students and teachers, and students and students. The importance of exploring classroom interaction during the teaching and learning of addition of fractions, cannot be over-emphasised.

The study adopted the theoretical framework of Systemic Functional Linguistic (Ebbelind & Segerby, 2015; O'keeffe & O'donoghue, 2015). This theory provides the framework for analysing the form of ideas (Ideational function) that could be observed in every mathematics interaction. The theory also analyses the interpersonal functions that describes the way participants in the interaction relate with each other. The important thing about this framework is that it depends on the totality of the words used in the interaction. This is the main reason for the choice of this framework. Implicitly, it evaluates the whole interaction in relation to the functions identified (Haratyan, 2011).

Meaning of Understanding

The work of Cross (1999) postulated that the whole idea of learning in mathematics is singularly about making linkages in learners understanding. Accordingly, she suggested four main general kinds of understanding that could be involved in any kind of learning. This study adapts the combination of three of Cross' (1999) definitions of understanding. These were cognitive understanding, social understanding, and experiential understanding. These forms of understanding were mostly categorised based on the various related theories of teaching and learning (Orey, 2010). The literature will therefore provide detailed explanation of each of the forms of understanding.

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Cognitive understanding

This form of understanding was based on the pure constructivists' perspective of knowledge and subsequently, learning. Learners who considered learning in the cognitive form believe that everybody has rules in their mind that govern the way they behave i.e., demonstrating understanding or not demonstrating understanding of a mathematical concept. Learners who learn towards cognitive form of understanding think about mathematical knowledge in an organised fashion. Cognitive form of understanding of mathematical knowledge considers mathematics to be universal, pure in its nature, and it does not change often. Cognitive form of understanding therefore assumes that mathematical knowledge is also learnt as rules of a pure, universal body of knowledge (Armstrong & Fukami, 2009; Ertmer & Newby, 2013). The varied mental imagery of mathematical knowledge in the mind of a learner is referred to as the schema.

The mathematics schema in the mind of the learner also have rules like mathematics itself. Since there are schemas of mathematical knowledge, understanding will mean demonstrating that the pure form of mathematical knowledge is in the learners' mind. It is easier to learn something if it fits into the rules or schema that the individual already possessed. It therefore implies that in a similar manner, learning mathematical knowledge would be easily successful if it is easily related to some of the mathematical rules that the learner already possesses in his mind. This already possessed mathematical knowledge can be considered as the related previous knowledge (Asha et al., 2012; Ertmer & Newby, 2013).

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Jitendra et al (2009) typically illustrated how cognitive understanding could be obtained. Jitendra et al (2009) described what they postulated as schema-based (or cognitive based) instruction of fractions. They noted the importance of categorising schema reasoning or cognitive learning as a domain specific formation structure. Hence, by the structure of a problem, its comparison to various mathematical domain specific schemas in the learners' mind, unravels the most appropriate action needed for a solution. Schematic or cognitive learning is a deep learning strategy that can easily evoke appropriate solution strategy. Cognitive understanding helps to vividly show the structure of relationships between various aspects of the schemas. Schema instruction that is based on the structure of the problem does not necessarily focus on the storyline of issues in a given problem but focus on the identification of salient mathematical structures that could help in solving the problem. However, it has been found that such instruction only good in cases where the students involved are low performing students (Paas, et al., 2003; Armstrong & Fukami, 2009).

Another category of schema based instruction or cognitive learning focuses on comparing solution methods during instruction. In Charalambous, Delaney, Hsu, and Mesa (2010), encouragement of the use of varied solution strategies in textbooks of high performing countries was shown to be a distinguishing feature from other countries' textbooks. What was evident in the textbook solution strategies of high performing countries was varied solution strategies for almost every question on addition of fractions. Some of the varied solution strategies for the concept of addition of fractions problems include; Unit fraction strategies, LCM strategies, whole number addition strategy

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(improper fractions alone) (Charalambous et al., 2010; O'Dwyer, Wang & Shields, 2015).

The last category of cognitive instruction focuses on metacognitive strategy. In this strategy, the students make conscious effort to monitor their own learning. Implying that every strategy that the teacher and the student decides to puts in place within the classroom, in order to monitor the individual students' thinking process, is described as a metacognitive strategy. The major metacognitive strategies that could be used in learning mathematical concepts include planning, monitoring, and self-regulation. However, one strategy that can be used in the classroom setting is the think aloud strategy. Think alound helps small group classes to monitor their thinking processes and determine if it is justified or not (Campbell, 2008; Jitendra, et al., 2009).

As an example of a theory that investigates cognitive structures in the mind of a learner, Steffe (1992; 2002) postulated a unifying theory of schemas as a technique for reasoning about fractional concepts. Steffe noted that schemas were obtained from three successive steps: reflection on previous actions, abstraction from the actions, and using the transformation or adoption of such ideas to deal with other learning. She noted that after the schema has been obtained by the learner, it becomes a structure that is segregated into three successive parts. Hence, retrieval of the schema is in the same order. The first is the recognition template: a particular set of activities that would be perceived to activate the schemes of fraction. The second is the operation. Operations was considered as a set of actions that would be carried out. The third is abstraction. Abstraction is based on reflection on the actions/operations that was carried out. Steffe defined the operations of splitting, partitioning, and iterations as the

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major activities needed for demonstration of cognitive understanding of fractions.

Norton and Wilkin (2011), in investigating the cognitive structures, showed that there were five schemes of operations in the concept of fraction. They used these five scheme of operation to predict the general achievement in mathematics. The operations were segregated as part-whole fraction scheme, partitive unit fraction scheme, partitive fraction scheme, reversible partitive fraction scheme, and Iterative fraction scheme. These operations are the only visible part in the demonstration of cognitive understanding of fraction.

The part-whole scheme considered dismembered partitions as partitions from the whole. As an example, a non-unit fraction is considered as a number of unit fraction dismembered from the whole partition. Sometimes, students' representation of the dismembered parts may not always be equal (as compared to unit fraction). When it is not equal, it is not considered a part-whole fraction scheme. However, the part-whole scheme does not consider the iteration scheme to be available to student with part-whole scheme. This is because, the student with only the part-whole fraction scheme will not be able to conceptualise a fraction greater than one. The student with the partitive unit fraction scheme is able to identify the unit fraction in an un-partitioned whole. The difference with the part-whole fraction scheme and partitive unit fraction scheme is that the whole is already marked or divided to show the partitions. However, in the partitive unit fraction scheme, the student himself have to determine the partition equivalent to the unit fraction. The partitive fractional scheme is an extension of the partitive unit fractional scheme. A student who is exhibiting possession of a partitive fraction scheme would be able to identify

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the non-unit fraction as iteration of a unit fraction. At the same time, the scheme represents iteration of a unit fraction to obtain the whole (Charalambous et al., 2010; Wilkins, Norton & Boyce, 2013). In effect, the basic idea of partitive fraction scheme is the iteration of the unit fraction towards two reasoning components; the whole and the non-unit fraction.

In the definition of the partitive fraction scheme, the definition is from the unit fraction, towards the other components of the fraction. For the reversible partitive fractional scheme, it operates in a reverse direction as the partitive fractional scheme. As the partitive fractional scheme starts from the unit fraction and operates at two levels, the reversible partitive fractional scheme starts from the non-unit fraction and operates at two levels. The student with the reversible partitive fractional scheme would have to conceive a whole beyond the fractional part. Hence it is expected that the unit fractional part would be identified in the non-unit fraction and then, used to identify the whole. But this is a reverse process of the partitive fractional scheme. Iterative fractional scheme is the only scheme which the student is able to use to comprehend a fraction greater than whole. This is because, for all the fractional scheme discussed, the whole is not lost or a fraction does not go beyond the whole. However, with iterative fractional scheme, the student would be able to iterate the fraction as greater than one (Norton & Wiklin, 2011). There are other cognitive learning theories that have tried to unite understanding of various aspects of fraction. The idea of fractional constructs will therefore be described in other parts of the literature review.

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Social understanding

Social understanding is in line with the theory of social constructivism. Social understanding is therefore associated with social teaching and learning. The postulation of social constructivism is based on the identification of the effect of the social environment on individuals' learning. Social understanding preconceives social learning as the process of acquisition of 'social' knowledge. In other words, social learning would be an attempt to get to understand the mathematical knowledge around us. This implies that mathematical knowledge, and therefore the concept of addition of fractions, is within the environment. Social understanding of the concept of addition of fractions therefore takes into consideration, the diverse contexts of mathematical knowledge (e.g. addition of fractions) in the environment. The diversity in environmental context therefore implies that the concept of addition of fractions may vary per the various contexts of the environment in which the concept of addition of fractions exists (Adams, 2006; Ertmer & Newby, 2013). Cross (1999) explanation implies two important aspects of social understanding; (1) environmental aspect, and (2) Language aspects.

Accordingly, the environmental aspect shows that mathematical knowledge is in the environment. Social understanding of the concept of addition of fractions in a farming community could be conceptualised as addition of fractions of plots of farm lands. However, in a community organised on technological advances, the concept of addition of fractions could imply addition of fractions of minutes of response time in two subsequent events. It is important to note that the understanding of addition of fractions among

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learners from the varied communities would differ, since the location of the knowledge varies.

Secondly, the role of language in the communication of mathematical knowledge within every social environment is a key part to explaining social understanding (Cross, 1999). Language serves as a means of communicating the mathematical knowledge, and communication is between people or groups of people. So to properly explain social understanding, words used in communication cannot be relegated to the background. What it implies is that the group of words used in describing a mathematical concept, has an effect on the study of the mathematics topic e.g. addition of fractions (Davis et al., 2015). It also implies that there are two communication levels of activities that are involved in social learning: intra-psychological and inter-psychological levels (Ertmer & Newby, 2013; Koole & Elbers, 2014).

Inter-psychological level of activity in learning implies that learning mathematics is involved in language between two or more people. Inter-psychological activity in the learning process is where mathematical knowledge is portrayed to be true or not. Hence within the group communicating, mathematical knowledge could be considered to be true whiles in another group, the same knowledge could be considered to be false, based on what the group seems to agree on. In a socio-constructivists classroom, what is considered to suffice as understanding of addition of fractions is dependent on what the group agrees to be correct in the social set-up. And truth to an individual is dependent on the degree to which the group seems to have a consensus that is close to the 'experts' understanding of a mathematical construct (Davis, 2018). This is why it is normally considered that learning of mathematics (e.g. addition of

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fractions) within the classroom should involve one who is considered to be more knowledgeable (the mathematics teacher).

Learning as an intra-personal communication level of activity involves the individuals' attempt to interpret the environment around him or her. Whatever interpretation is obtained on the personal level would eventually need to be subjected to what the social group understands as knowledge. This therefore serves as a way of verifying personal knowledge as perceived by the individual. This perspective of learning has an interpretation for alternative conception. Alternative conception is seen as a difference or degree of inconsistency between individual (intra-psychological) conception and social (inter-psychological) conception (Ertmer & Newby, 2013; Koole & Elbers, 2014). It is therefore important to indicate the factors that directs social learning: context and language (inter-personal or intra-personal).

The major focus for social constructivism is the intention to focus attention on the learner and the social group and not so much on the nature of mathematical knowledge, as noted with the cognitive understanding. For a socio-constructivist mathematics teacher, the emphasis would be on the ability to focus on the learner in his activities for learning addition of fractions and not on the subject matter as already stated. The teacher therefore focuses on the leaner in a bid to institute learning strategies that would help the learner to deepen the social understanding. Much focus is not on the teacher but on the learner. This implies that the learner is an active participant in the learning process. Consequently, the opportunity that would allow the students to express themselves and describe their ideas in open ended questions is what is very important in conceptualising social learning (Adams, 2006).

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The mathematics teachers' role is as an observer and a listener in the conversation of the social group. The teacher is supposed to provide contrasting ideologies of addition of fractions that are supposed to sustain the students' interest in the conversation (Adams, 2006). The teacher is supposed to acknowledge the presence of the cultural and social elements that are encouraged to be present in the conversation about addition of fractions. Shared interrogation with peers and their mathematics teacher would help the teacher comprehend the degree of social understanding that is demonstrated by the students. Peer discussion is then supposed to be allowed in an attempt to imbibe in the discussion of addition of fractions, the cultural and naïve ideologies of the learners. The aforementioned ideologies are integrated into a setting of formal knowledge.

The mathematics teachers' role is to also help guide the learners in the discussion to learn towards shared cultural and social understanding of the subject of addition of fractions. This is possible when students' ideas are guided by prompts that allow the students to reflect and abstract addition of fractions from their conversation. The mathematics teacher is therefore a facilitator that provides instances for learners to construct the concept of addition of fractions. Another role is to help students identify illogical, consequences, and causes that might be correct or incorrect in their reasoning process. The potential of learning from the immediate environment seemed to show high prospects in students learning, hence experiential learning and understanding is defined (Adams, 2006).

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Experiential understanding

Experiential learning is based on the theorising of Kolb (1984) who proposed that learning is in four cyclical stages; experience, critical reflection, abstract conceptualisation, and active experimentation. This learning theory was mostly based on theorising for adult learning and its associated understanding. The experiential learning model is driven by learners' interaction with real experience. It is a model that postulates that anything that is learnt must be relevant in a practical sense. The learning cycle is characterised by a combination of two continuums. The first continuum (experience and critical reflection stage) is concerned with acquisition of knowledge. While the second continuum (abstract conceptualisation and active experimentation stage) is concerned with the transformation of the knowledge gained for possible use elsewhere.

The knowledge acquisition continuum has its starting point on the experience aspect of the model (Cross, 1999; Orey, 2010). Experience in the learning process implies that the individual learner should have a practical feel of the mathematical knowledge in action. Knowledge in action that he/she participates in. Implication is that the learning of addition of fractions must be based on physical activities that one could manipulate. Consequently, it is not the manipulation of teaching and learning models but rather, manipulation of real items. Subsequently, there is the need for the learner to critically reflect on the experiences of manipulating real objects. Critical reflection include reflection on the mathematical knowledge (addition of fractions) and the stages in the process of experience that the mathematical knowledge evolved or transformed from. Critical reflection occurs in the identification and

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coordination of reality to characteristics in the mathematical concept (addition of fractions), that allows for sensible transformation of mathematical knowledge (addition of fractions). After critical reflection is the abstraction of the mathematical knowledge (addition of fractions) that could be found in the characteristic, and processes that were identified when the learner critically reflected on the experiences in manipulating real objects.

The second continuum can be considered a reverse of the first continuum. Abstraction will also involve the ability to represent most of the transformation observed during critical reflection and represent them in mathematical symbols (Kolb, 2008; Mughal & Zafar, 2011; Kolb, 2014). An example will be to identify the LCM in the procedure of addition of fractions and represent it as such. After abstraction, the next stage in the learning of the concept of addition of fractions will be the use of the abstracted mathematical knowledge in other or the same situation as the experience. In other words, active experimentation is tantamount to application of the abstracted mathematical knowledge in other real situations. It is important to note that experiential learning cycle does not end with active experimentation since it is expected that further critical reflection would be necessary for further refinement of concepts abstracted or possible alternative conceptions (Orey, 2010).

The reason most learners are able to conceptualise differently is that on the two opposing continuum, learners are strong at certain points of the continuum. The implication is that when learning addition of fractions, learners could be strong with reflection but not with abstraction from the reflection. Others could be strong with experience but weak in active experimentation.

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Accordingly, such learners will demonstrate experiential understanding that matches their strength in the experiential learning process. A learner who is able to demonstrate good abilities in the various stages of experiential learning has demonstrated full experiential understanding. Accordingly, no matter the learners' strength, it would be good that the learners learns to deal with the opposing continuum (Kolb, 2014).

Although experiential learning is a very innovative learning perspective, it did not seem to throw more light on the emotional aspect of learning. The motivational effect that is needed to sustain learning was not dealt with in detail. Power relation between the learner and the mathematics teacher was not also addressed (Mughal & Zafar, 2011). As resources improve and research increases, more knowledge and insight is being gained on learning and how related previous knowledge influences the extent of understanding attained.

Related Previous Knowledge Used in Teaching and Learning Processes

Related previous knowledge, also known as background knowledge could basically be described either in a simpler broad perspective or more encompassing deeper reflections. Related previous knowledge could be simply explained as the knowledge a person already possesses about the content to be taught (Norton & Wiklin, 2011; Graaf, 2014). Others include all knowledge that a person possesses that might be helpful in the learning of new content. This implies that such knowledge might not necessarily be about the new content to be learnt but can be engaged in the learners' mind to facilitate in learning a particular concept (mostly new). An example is the use of addition of whole numbers to learn addition of fractions (Meyer, 2004; Hailikari, 2009). The deeper reflection on the definition and subsequent choice of related previous

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knowledge for a mathematical topic(s) to be taught, requires understanding of specific model of RPK.

Models of Related Previous Knowledge

There are basically two identified models of related previous knowledge. The two models are considered to be on opposing extremes of their school of thought (Graaf, 2014). The Credit Exchange Model is normally in a form of assessment results that tries to illustrate a persons' readiness to move into the next level of education. It can be in the form of results of examination at the end of a term/semester/course or can be a high stake examination for the next level of education. However, what is most important in this form of related previous knowledge is that it mostly does not show the detail of what necessarily had been studied in the form of knowledge acquired. The intention of this model is not to develop new learning,. based on the old learning but to serves as evidence that learners have an idea of the old learning that may be advantageous in the next level of study. Hence documentary evidence of scores in an assessment is normally acceptable (Eng, Li & Julaihi, 2010; Vamvakoussi, Vosniadou & Doreen, 2013; Vosniadou, 2013; Chi, 2013).

The second model, called the learning and development model of RPK has its main purpose to be the advancement in a course/topic/lesson. In this model, RPK needs to be recalled, and understanding demonstrated, before progression into the lesson. It normally requires an attempt to get the learner to transform his or her previous ideas into forms that may be sufficient enough to progress in a given lesson or topic of study (Bennett, 2011; Gamlem & Munthe, 2013). Learning and development model of RPK is normally made up of different assessment portfolios that explores all possible options to identify

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forms of related previous knowledge that can be transferred to enhance progress in a particular lesson/topic. Consequently, learners and teachers are normally involved in lot of critical reflections that can throw light on the related previous knowledge of learners. This model form is normally used within the classroom setup (Ell, Hill & Grudnoff, 2012; Çaliskan, 2014). An example involves transforming fractions of equivalent forms in order to be able to learn the concept of addition of fractions. After identifying the RPK, there is the need to adopt the appropriate approach to use it successfully. Approaches in the use of RPK during teaching and learning is therefore discussed.

Approach to the use of Related Previous Knowledge in Learning

The two identified approaches in using RPK in formal education setting includes tapping or activating RPK, and building or developing new background knowledge. Tapping into students RPK could involve asking the students about what they already knew about the content to be taught. A possible line of action would be to ask the learners what they knew about key concepts that could be found in what was to be learnt. For example, in the teaching and learning of addition of fractions, the mathematics teacher could ask the students to say what they knew about general addition, different types of fraction, and the concepts found in them. It would be a possible option to ask the students to present or discuss their ideologies about key concepts in the topics being studied. Another possibility would be to mention certain vocabularies (unit fraction, equivalent fraction etc.) that were connected from the RPK to the new content area to be learnt (Chou, 2011; Ell, Hill & Grudnoff, 2012; Beny & Yunus, 2013). For researchers who believed that learning a vocabulary in a topic to be studied could be considered the same as learning the topic to be studied,

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major emphasis is placed on proper conceptualisation and usage of terminologies in the field of mathematics. Consequently, tapping into the knowledge of an area considered as RPK could simply involve the use of certain terminologies that were used when exploring the topic in previous lessons. Also exploration of links connecting terminologies in already learnt related topic and the new topic to be learnt is akin to connecting concepts between previously studied topic and the new topic to be learnt (Chou, 2011; Wessels, 2012). The mentioning of those vocabularies brings to memory the characteristic connected to such concepts. Thus, related previous knowledge that is needed for a particular learning is activated.

As an approach to the use of RPK, the building approach is the second and less defined approach in terms of its structure. In the building approach, the related previous knowledge may or may not be there. Therefore, the teachers may have to take time to construct with the learners, the specific knowledge that would be needed as RPK for the new topic to be taught. In other words, you eventually have to teach the RPK and use it as a tool to advance the topic that is to be learnt. Another technique is to locate all RPK that the learners might need. It is possible that components of RPK were learnt in isolation or some parts of it were not learnt at all. In the building approach therefore, the teacher needs to help the students connect all the knowledge that may facilitate further learning and help students comprehend it in the form that would help facilitate learning of the new concept. The arduous feature of RPK in learning is made more complex when there are varied performance levels in a given classroom (Paas et al., 2003; Oduro, Dare, Etsey, Nudzor, Bosu & Bansah, 2012). Its difficulty might be one of the reasons why teachers do not seem to attach much

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attention to learners understanding of specific RPK when teaching in the formal classrooms. Most teachers seem to assume that the students already possess the specific RPK necessary to learn a concept/topic to be studied (Conrad, 2008). They assume that the specific RPK is in the form that would support maximum achievement in the learning process. This is normally the case when the teachers are meeting the students at initial stages of entering a particular grade. However, for a class where the teachers have been teaching the children for long period within the term, what the students know in relation to the concept of the specific RPK necessary for teaching and learning of a concept/topic is sometimes a little clear to the teacher (Meyer, 2004; Hailikari, 2009; Beng & Yunus, 2013).

Meyer (2004) noted that the most difficult challenge teachers training encounter is not about acquiring skills or knowledge but about "making personal sense of constructivist teaching practice." (p. 971). Related previous knowledge is one of the major differences between constructivists and behaviourists philosophies of learning and teaching (Oduro, et al., 2012). Actually, RPK is reported by Hailikari (2009) to be the idea that introduced constructivism. This is because, the reconstruction of RPK is the constructivist idea of learning mathematics. The implication is that mathematics teachers' understanding and use of the specific RPK in the constructivist classroom is a major determinant of the teachers' teaching ability (Roediger III & DeSoto, 2015).

Meyer (2004) and Wessels (2012) noted that the understanding of the specific RPK of a learner could be considered as the single organising factor of individual thought processes during the teaching and learning of mathematical

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concept (e.g. addition of fractions). New experiences (e.g. LCM) would be connected with related previous knowledge (e.g. concept of fraction). In the aforementioned connection, RPK would establish a basis upon which learners make inferences about new and future learning of mathematics. Such understanding creates a stable and sustainable view of mathematics topics as a whole. The fact that a teacher had a good concept of related previous knowledge (necessary for a mathematics topic to be taught) does not necessarily imply that the teacher would use or demonstrate the ability to use RPK in the teaching and learning processes. The teacher should recognise the importance of such knowledge as possessed by the learner in the teaching and learning processes in order for him/her to be motivated to use it (Graaf, 2014). Conrad (2008) therefore proposed four steps that the teacher could use to assess students' understanding of RPK. The steps included reflection, selection, connection to the new content to be taught, and projection of the knowledge in a way that is acceptable in a setting. Although these processes had been elaborated for adult learners, how children respond to the same process is not yet studied.

Reflection involved the individual learner thinking of all possible experiences ranging from professional (e.g. formal concept of fraction, addition of whole numbers as described in Wu, 2013, Vig et al., 2014), informal, and even social experiences, from which the RPK or the content to be learnt, can be abstracted. This reflection normally produces a large portfolio of possible RPK that could be useful for learning. Adult learning may not be difficult as a result of the rich background or related knowledge that they have (Graaf, 2014). Selection is therefore carried out to detect the most advantageous experience from the many that could be anchored to make the learning most effective. The

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selection step is very essential because, not all RPK can be used and referred in the classroom (Clark, Ayres & Sweller, 2005). The connection stage requires the selected specific RPKs' ordered to determine sequence of usage. Projection refers to the ability to espouse the knowledge as is acceptable to a setting. Diverse settings require the projection of the knowledge differently. The projection of addition of fractions among artisans (e.g. seamstresses) would require an description of the idea of addition of fractions on the length of different sewing materials that may be needed to sew a dress. Among farmers, it may imply projection of addition of fractions as adding diverse proportion of chemicals to be used in spraying one plot of maize farmland (Pitta-Pantazi & Charalambous, 2007; Conrad, 2008). Projection includes projection of the RPK (addition of whole numbers, fraction, whole numbers) and projection of addition in a manner that is acceptable among mathematicians

Strategies for connecting related previous knowledge in learning

Unless a planned conscious effort was put in place, there is a high likelihood that RPK will be overlooked during teaching and learning. Campbell and Campbell (2009) suggested ways teachers in the classroom could consciously engage related previous knowledge. These are therefore explained with reference to the opinions in Campbell and Campbell (2009).

Known and unknown

This is normally in the form of a chart. The chart is in two columns with headings; Known, and Unknown. The mathematics teacher is supposed to know the various components of the mathematics topic he/she intends to teach. The teacher asks the students open questions to discuss what they think about the mathematics topic he/she is about to teach. Sometimes it would be better to

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delineate the mathematical concepts that may be needed in the topic or concept the mathematics teacher is to teach. Hence the discussion with the students would be in-depth in relation to mathematical concepts (addition of fractions) to be taught. Those that the students know or understand well would be put under the 'Known' column. In this study, it could be; iterations in counting, part-whole fraction concept, or addition of whole numbers (Wu, 2013, Norton, et al., 2015). The concepts that the mathematics teachers observed the students do not know in the discussions would be put under the 'Unknown' (e.g. equivalent fractions, partitive fractional concept, LCM etc.). The 'unknown' include those concepts that might have been misconceived by the students'. The teacher then lays down a plan of how to teach and connect the known to the unknown. This plan also shows the mathematics teacher where there is the need for emphasis and allows for corrections in students mathematics learning. At the end of the class, the mathematics teacher may then paste the chart on the board for the students to attempt to connect the mathematical concepts. Campbell and Campbell suggested the use of a table (see Table 1) to guide plans to consciously use RPK. However, the design of this table was specifically to help the mathematics teacher incorporate the known mathematical concepts when teaching the unknown mathematical concepts.

Table 1: Sample Guide Table for Known-Unknown use of RPK

Known	Unknown

Source: Campbell and Campbell (2009)

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Things I know, think I know, want to know

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This particular table is designed to also capture students' mathematical misconceptions distinctly from other mathematical concepts that are not understood. This is because the design of Table 2 suggested that the strategy that would be used to deal with mathematical misconceptions might be different from the strategy used in normal teaching (Chi, 2013; Vamvakoussi, Vosniadou, & Doreen, 2013). Table 2 was also designed to detect and exploit the direction of curiosity of the learners about the mathematics topic to be studied. Ideas from what students already knew may be connected to what is to be studied. Therefore, correct concepts in the RPK would then be exploited to correct mathematical misconceptions and to create a more solid understanding of the specific mathematics concept in the RPK. After these corrections, students' curiosity is then exploited to sustain interest and direction in the instruction. The structure for Table 2 is similar to the structure for Table 1. Table 2 consists of three columns. These columns are labelled respectively; Things I Know, Think I know, Want to Know.

In a similar manner as the previous strategy, the mathematics teacher throws open questions in relation to the mathematics topic about to be taught (e.g. concept of addition of fractions). So in the class discussion about the topic, the teacher writes what students already understand in the 'Things I Know' column (e.g. addition of whole numbers, part-whole concept of fraction). The mathematical ideas that were not understood correctly and misconceptions are put under the column, 'Things I Think I Know' (e.g. equivalent fraction, LCM, reverse iterative fraction scheme). Finally, things I want to know would be put under the last column to identify and understand the direction students' curiosity

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are driving them in relation to the mathematical concept that is to be taught (e.g. addition of unit fraction, units of addition of fractions, graphical representation of addition of fractions, practical application of addition of fractions). This idea could be adapted to smaller group and individuals in the classroom (Campbell & Campbell, 2009). An exemplary table guide is Table 2.

Table 2: Sample Guide Table for Things I Know – Think I Known-Need to Know

Things I Know	Think I Know	Want To Know or Need to Know

Source: Campbell and Campbell (2009)

What I know, want to know, what I learnt

The main difference between this strategy and the previous two strategies is that this strategy is used to monitor the whole mathematics learning process (starting from previous knowledge to complete end of lesson and its result), instead of serving as only a guide to the planning of the lesson. The columns involved in Table 3 are labelled; 'What I Know,' Want to Know,' and 'What I have Learnt.' So a mathematics teacher or student could document what students already know about the mathematics topic to be studied and what they want to know. These should be obtained from general class discussion or brainstorming about the mathematical topic or concept to be studied (addition of fractions). However, at the end of the lesson, the mathematics teacher is supposed to review the discussion about the mathematics topic/concept that was studied (the concept of addition of fractions) and input into the last column, 'What I learned'. Table 3 could be modified to include 'What Else I Want to Know,' 'How I Used the Information' etc., so that the lesson would also make

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provision application of the concept of addition of fractions (Campbell & Campbell, 2009). A sample is Table 3.

Table 3: Sample Guide Table for What I Know- Want to Know-What I Learned

What I Know	Want to Know	What I learned

Source: Campbell (2008)

Graphical relationship illustrations

In the original representation of Campbell and Campbell (2009) this category was made up of 'Visualising Cause and Effect,' 'Whole Pie Strategy,' 'Visual sequencing,' and 'Getting Organised Graphically.' These were simply organising models that helped one get an overview of what is studied and how it is connected to other topics or concepts. The only difference between some of the graphical illustrations was that one focused on cause and effect relationship, others focused on the relationship between general mathematically connected topics and mathematical concepts under them (Whole Pie Strategy). Mostly, their names illustrated what they were implying. A linear flow chart simply illustrates the sequence of events or concepts in the process of learning a chosen mathematical topic/concept (addition of fractions). This could be linear or cyclical. It could also involve the illustration of hierarchy in the knowledge structure as learning progressed. Thus, in addition of fractions, the model can show how unit fraction concept is connected to addition of fractions. It can equally show how the part-whole concept is connected to addition of fractions (Wu, 2013, Vig et al., 2014, Norton et al., 2015). The most important thing is that it illustrates how learning of mathematical topics is being built and

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connected to previous knowledge (see Pitta-Pantazi & Christou, 2007 for an example).

Similarities and differences in structure of concepts

Mathematical concepts that are similar in specific or general characteristics are useful tools in getting students' progress in their learning of mathematics (Rittle-Johnson & Star, 2009; Wu, 2013). Prediger (2008) observed and outlined the differences and similarities between whole numbers and fractions (see Figure 1). Figure 1 could help mathematics teachers and students to proceed smoothly in learning and avoid certain pitfalls that could really slow down learning of concept of addition of fractions. However, Prediger did not state so clearly when to use these similarities and differences observed between natural numbers and fractions. It is possible to introduce the students to Figure 1 before the lesson to be taught proceeds. It might be better to bring students' attention to necessary aspects of Figure 1 every now and then as a reminder.

In this case, the possible problem is that it might confuse the students and lead to low students sense of efficancy in relation to dealing with or learning fraction. This is because the higher the frequency of changing focuses in a mathematical lesson, the higher the working memory is likely to be tasked. Hence low performing students often resort to unfruitful learning at this stage (Paas et al., 2003; Rittle-Johnson & Star, 2009). It may serve as a form of demotivation, especially for the low achieving students. Another option is to introduce it at the end of the lesson or topic where students may even identify the similarities or differences themselves (Clark et al., 2005; Yang, 2012).

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Aspect	Natural numbers	Fractional numbers
Cardination	a number is the answer to the question "How many?"	a fraction can describe parts of a whole, quotients, ratios, proportions, ...
Symbolic representation	one number	two numbers and a line
	unique relation between number and symbolic representation	existence of many fractions representing the same fractional number
Ordering	supported by the natural numbers' sequence (counting on)	not supported by the natural numbers' sequence
	existence of a successor (discreteness)	there is no unique successor or a unique preceding number (density)
	no number between two different numbers	density: infinite many numbers between each two numbers
Operations		
Addition–Subtraction	supported by the natural numbers' sequence	not supported by the natural numbers' sequence
Multiplication	multiplication makes the number bigger	multiplication makes the number either bigger or smaller
Division	division makes the number smaller	division makes the number either smaller or bigger

Source: Prediger (2008)

Figure 1.: An illustration of similarities and differences in structure of mathematics concepts adopted

Teachers Use of Related Previous Knowledge

Schools whose teachers have been trained in proper utilisation of students' RPK in their lessons, could be transformed from low performing school to high performing schools (Hailikari, 2009; Rittle-Johnson, et al., 2009; Beng & Yumus, 2013). Meyer (2004) explored the difference between expert and novice mathematics teachers understanding and use of RPK in the teaching and learning processes. It was observed in the study that expert teachers focused on students' real life experiences as the main source of related previous knowledge. Consequently, in an attempt to assess related previous knowledge, most expert mathematics teachers used classroom discussion of students' real life experiences in their homes. Expert teachers establish connection between students' real life experiences and symbolic (or formal) mathematics

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expressions or concepts that are used for instruction. Thus, ideas explored by Davis et al (2009) connected primary school students' real life experiences of selling food items in the Ghanaian market to their understanding of measurement and fraction concepts. This practise could serve as RPK for the teaching and learning of the concept of addition of fractions.

The expert teachers interviewed in Meyer (2004) did not believe that it is possible for any student not to have any RPK in relation to the topic being dealt with in the mathematics classroom. The real life experiences of the students were used by expert teachers as connections to the formal mathematical knowledge. It was observed that the expert teachers did not directly explore symbolic/formal mathematical knowledge in the classroom as first point of call in exploring RPK. The expert teachers only explored symbolic/formal classroom RPK only to determine the extent to which the previous formal knowledge can be used by learners to explain real life phenomena. Based on that, the teacher uses the resources understood by the student in their daily lives to develop his/her lesson. Although the expert mathematics teachers focused on what the students knew as RPK, when students faced difficulties, expert teachers mostly changed their strategy of instruction. Accordingly the strategies either used real life experiences or symbolic mathematical knowledge, as the teacher deems helpful (Paas, Renkl, & Sweller, 2003; Paas, Tuovinen, Tabbers, & Gerven, 2003).

Novice teachers on the other hand, gets confused when students find it difficult to demonstrate understanding of the RPK. When interviewed by Meyer (2004), novice teachers get stranded at this point and cannot think of the possible next strategy that will help. Unlike expert teachers, novice teachers

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considered the main source of related previous knowledge as only the symbolic mathematical knowledge students obtained from their previous formal instructions in a chosen mathematics topic within the classroom setting. Novice teachers also performed pre-instruction assessment to assess what the learners do not know, instead of the focus on what students already know. The use of RPK in the context of novice teachers, was less for learning purpose. RPK served the purpose of mainly determining students' achievement at the beginning of the lesson (Meyer, 2004; Hailikari, 2009; Graaf, 2014). In an attempt to explore RPK for the concept of addition of fractionss, a novice teacher would likely give the students tasks that examined formal or symbolic mathematical understanding of the concept of fraction. However, how the knowledge should be used to guide students' learning and understanding of the concept of addition of fractionss, may be lacking. This brings to fore, the importance for novice teachers to understand the mechanism of RPK

Mechanism of related previous knowledge in learning

In examining the mechanism for RPK, the factors that are active include students' understanding of the RPK itself, and explanations of the mathematics teacher. Each factor has its own effect on students' understanding of the concept to be taught. This is also the combined effect of both factors when used together. When the effect of combining both agents lead students to demonstrate higher understanding than expected, it is described as superadditive. However, if the students' demonstrated understanding is below expectation, it is described as subadditive (Williams & Lombrozo, 2010; Graaf, 2014).

If RPK and explanation were combined, Williams and Lombrozo (2010) explained the mechanism of their interactions. They described it as a

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subsumption effect. Subsumption implied that what was to be explained in the knowledge to be learnt (concept of addition of fractionss) is subsumed in the RPK (fraction, real numbers). Some studies that have used the aforementioned structure in studying fraction and addition of fractions include Pitta-Pantazi and Charalambous (2007), Jordan et al (2013), Everret et al (2014) and Siegler and Lortie-Forgues (2014). Another explanation for subsumption is that the RPK (fraction concept) is an instance of what is to be learnt (addition of fractions). Studies that have used this structure in studying fraction and addition of fractions include Wu (2013) and Vig et al (2014). They also indicated that subsumption identifies which knowledge is deployed by the learner (real life phenomena knowledge or formal/symbolic mathematical knowledge). Subsumption could also be observed in the attitude of the learners in relation to the RPK and the attitude of the learners in relation to the content to be learnt. If attitude towards fraction concept is negative, similar attitude will be observed when learning the concept of addition of fractions. The believes and perception that the learner is supposed to have in relation to the new content (addition of fractions) to be studied is also determined by the attitude learners have towards RPK (fraction or integer concept). If RPK was considered unfriendly and with lots of difficulties, similar perception would be carried into the learning of the new mathematical content (addition of fractions). Subsumption also provides an unambiguous framework (content knowledge) under which what is to be learnt must be subsumed.

The framework for subsumption is based on the conception of RPK as knowledge to be built on (Conrad, 2008). Secondly, only explanations that are within the framework (properties in concepts) in the RPK would be considered

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by the learners as a valid form of explanation to be generalised. Explanations beyond the framework of RPK (outlining properties of a concept used as RPK) introduces a question of doubt (in the minds of the learners) as to the validity of generalisation which do not meet the specification outline of the RPK. Accordingly, the knowledge framework of RPK exerts an explanation constraint under which concepts to be learnt by students is validated. Explanation that goes beyond the outlining properties of concepts in the RPK can introduce confusion in the minds of the learners (i.e., attentional effect). Subsequently, it provides a ground for higher workload on the working memory and eventual loss of focus. In this way, explanations given by teachers in the light of RPK would be impairing learning. This situation can be described as subadditive effect (Paas, Renk & Sweller, 2003). Also, incorrect RPK (e.g. multiplication of fraction) would most likely provide the wrong framework for explanation of new concepts (addition of fractions) to be learned and thus impair learning. The mechanism for RPK during learning is supported by the cognitive load theory (CLT).

Cognitive Load Theory (CLT) suggests that presenting learners with tasks from multiple sources OR learners learning multiple concepts at the same time could have their working memory overloaded (Paas, Renkl & Sweller 2003; Vosniadou, 2013). When working memory is overloaded with information, learning is less efficient. However, when learning does not overload the working memory, the working memory is free to allow efficient processing for learning. Hence when comparing related previous knowledge (unit fraction, equivalent fraction, iterative counting scheme, etc.) with mathematical concepts being learnt (addition of fractions), a student with more

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of RPK would imply that most of the information needed for processing is already in the system of long term memory. This information is only then referred for processing and therefore reduces the tasking on the working memory. The working memory is therefore free to do more tasking towards the new content to be learnt. Consequently, more learning is made possible. The opposite is also true. Less RPK in learners implies that a lot of information is being encoded (into working memory) at the immediate time of learning. This overloads the working memory. Thus, there is less efficient learning.

The implication from the previous paragraph suggests that when teaching addition of fractions, the support given by the mathematics teacher is dependent on the students' level of understanding of the specific RPK. Much support is not needed for students with high understanding of the specific RPK for addition of fractions. On the contrary, students with low level of understanding of the specific RPK for addition of fractions will need much of teachers' support. Generally, it is worth noting that students continually progress in the level of understanding of RPK that they may possess. Therefore, teaching in a class with mixed group of learners would pose a difficulty for mathematics teachers. The solution for dealing with high and low achieving students within the same learning condition (instructional) could be found in guidance fading effect. Guidance fading effect helps all learners (high and low achieving students) achieve maximum learning of addition of fractions within the same instructional design. Guidance fading effect, according to Kalyuga, (2005; 2007) should be the learning situation where the learners' level of support given by the teacher is reduced gradually as the specific background knowledge or expertise of the learner increases. This would allow the learner to

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do a little more exploration as the learning process [illegible], and reducing the redundancy effect (Paas, Renkl & Sweller, 2003).

Specific Related Previous Knowledge for the Concept of Addition of Fractions

Bruce et al (2013) noted that comprehensive understanding requires demonstrating in-depth ability to use mathematical skills, and logical mathematical reasoning that are associated the mathematical skills. Consequently, to be able to develop the aforementioned understanding of the concept of addition of fractions, there should be solid understanding of the following areas;

1. Part whole construct of fraction
2. Quantity construct of fraction
3. Equivalence construct of fraction

In most studies that have explored understanding, the use of representation of mathematical concepts is essential. Therefore, demonstration of the above stated constructs would mostly require representations. Petite et al (2010), as cited in Bruce et al (2013) was of the view that flexible use of fraction representation contributes to deepened understanding and use of fractions addition. She noted that until students are able to intuitively use various strategies to solve addition of fractions problems, they are not ready for formal instruction on standardised procedures for the concept of addition of fractions. Epson and Levi (2011) as cited in Bruce et al (2013) noted that as students become fluent with equivalent fractions, then, they are getting better suited for considering formal instruction in the concept of addition of two unlike fractions. In a learning situation that involves only the concept of addition of fractions,

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students' ability to look at problems and detect the relationship that would be necessary in solving such problems, is normally aided when their first ability to explore such relationships initially involves a blend of mixed numbers and only proper fractions. It is therefore advised by Bruce et al (2013) not to involve equations at the initial stages of learning addition of fractions, until it is properly mastered.

Constructs of fraction

In examining the concepts that would be necessary for teaching and learning the concept of addition of fractions, at least the basic concept of fraction and its properties must be understood properly. Consequently, these could serve as related previous knowledge to be drawn on, to teach and learn the concept of fractions addition. This research therefore review literature on understanding of basic fraction concepts as developed by Behr et al (1983). The fraction concepts noted in these models conceived of fraction concept as part-whole ideology that results from partitioning. Hence based on this overarching concept, the subconstruct of ratio, operator, quotient, measure, and problem solving are defined in relation to the overarching part-whole concepts (Pitta-Pantazi & Charalambous, 2007; Davis, 2014). From the diagram (Figure 2), it could be seen that all the sub-constructs of the part-whole concepts were subsequently fed into problem solving. This implied that all subconstructs, including the overarching concept of part-whole were connected together to solve mathematical problems of fraction. But in this study, the subconstructs of ratio, quotient, and measure would be explained and used. This is because, their conception, and recent studies (see Wu, 2013) often link them to the concept of addition of fractions (Amuah, Davis & Fletcher, 2017).

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It has already been stated that using representation of fractional concepts to prove understanding is common (Singer, 2007; Teppo & van de Heuvel-Panhiuzen, 2013; Vosniadou, 2013). However, there are those researchers who believe that mathematical knowledge is in the act of doing an activity. Therefore, it is not just the representation of concepts and characteristics it demonstrates but it includes the processes that result in the concept (Davis, Seah & Bishop, 2009; Davis, 2014; Vig et al., 2014). This implies that the reasoning about fraction concepts in terms of the processes that result in them and their characteristics, is considered to be what is registered in the memory/brain of the person and not only the complete mathematical knowledge structure. The mental 'map' of this process is considered the schema (Norton & Wilklin, 2011; Roediger III & DeSoto, 2015).

Despite the fact that these two forms of reasoning were conceived and interpreted separately, the researchers in the different lines of thought achieved great strides in their attempt to make meaning of learners reasoning. Hence, despite the diversities, latter researchers (including some originators of the different lines of reasoning) are merging the two reasoning processes. Thus, where necessary, explanation would be given to the two different reasoning for fraction ideas (concepts) and how they are connected (as much as possible).

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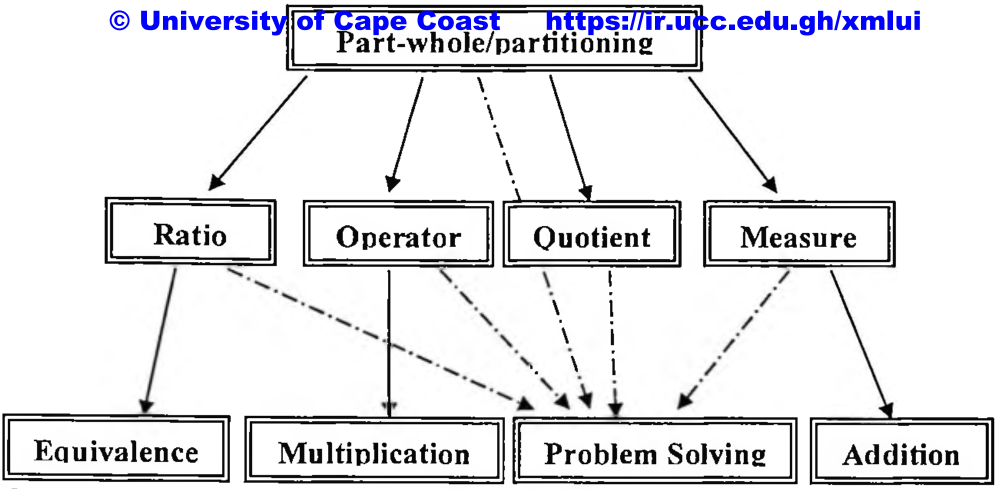

Source: Charalambous and Pitta-Pantazi (2005)

Figure 2: The theoretical model of fractions linking the five different subconstruct of fractions to operations of fraction and fraction equivalence

Part-whole fraction

Part-whole construct is the basic ideology on which fraction concept is first introduced for many countries mathematics curriculum. It involves the conception of fraction as part of a whole. Consequently, the reference point for a fraction in a part-whole construct is the whole. This implies that whichever idea or reasoning that is involved in the conception of part-whole construct, is in relation to the whole. Therefore, any reasoning that moves out of this framework is considered not to be part of the part-whole ideology of fraction (Teppo & van den Heuvel-Panhuizen, 2013). The most fundamental definition of the part-whole construct is the idea of a fraction as describing a part of an item from the complete whole. This item could be discrete or continuous in nature. Part-whole fractional construct therefore includes unit fraction and proper fractions concept. Hence there would not be much difference in conceptualising the unit fractional knowledge and the proper fractional knowledge separately in relation to the part-whole construct form of reasoning (Wu, 2008; 2013; Bailey et al., 2015).

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For researchers who reason about fractional knowledge in terms of schemas, the part-whole construct is considered differently from the partitive fraction scheme. The partitive fractional scheme involved the splitting of a whole into equal spaced units. These equal spaced units were considered to be parts of the whole. Hence, unit fractions involved the possession/acquisition of a schema that involved splitting of a whole into equal parts such that each individual part (unit fraction) is considered a result of splitting action and a part of the total number of unit parts that make the whole. The same thought process is used when reasoning about proper fractions. The implication is that proper fractional concept acquisition involved the possession/acquisition of a schema that conceives of the part as a number of groups of unit fractions taken out of the total number (groups) of unit fraction in the whole (Asha, Star, Dupuis & Rodriguez, 2012).

The difference between the schema reasoning of a concept and the construct reasoning of a concept (part-whole) is that the schema reasons in terms of the splitting processes and the number of unit fractions, whiles the construct concept is in the form of a single splitting and the comparison of the different quantities involved. The part-whole fractional scheme also involves the ability of the learner to note that the unit fraction in the part and the unit fraction in the other part that make the whole are of the same size. A person with the part-whole fractional scheme should be able to know that the two unit fractions are identical and hence are interchangeable (Amuah, Davis, & Fletcher, 2017). Although these were not part of the definition as originally postulated by Behr et al (1983), as cited in Pitta-Pantazi and Charalambous (2007), the definition of Norton and Wilkin (2011) included that. In the part that defines a fraction,

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and the part that is remaining in the whole, unit fraction is both parts are considered equal and interchangeable. However, equality and interchangeability of the unit fraction in these two parts was not mentioned in the part-whole construct. In the schematic understanding, this is a hurdle that the students must cross and is considered important enough to characterise the scheme. As a result, the scheme reasoning has five different schemes that are embedded within the part-whole sub construct of fraction (Asha, Star, Dupuis & Rodriguez, 2012).

Ratio subconstruct

The fraction subconstruct of ratio conveys the idea of comparison in the concept of fraction. Consequently, for a fraction to convey the comparison situation, it should also be understood to be a comparison within a given standard. This standardization is what Charalambous and Pitta-Pantazi (2007) refers to as the comparison index in the ratio construct. They noted a category of comparison that could exist under the ratio subconstruct. This included comparison between the same items and comparison between different items. When the comparison was between two quantities of the same types, it is called ratio. When the two items under comparison were of different types, it is called rates. It is important to note that among these comparisons (same or different items) the comparison index is constant/standardized. Hence for a given fraction, the nature of the variation in the denominator or in the numerator is constant (Suh & Moyer-Packenham, 2008). This implied that you cannot include variable rates in this sought of comparison (i.e., a situation where the change in numerator and the denominator can vary). Therefore, for the example illustrated in the question below, the multiples of change in the boys and the

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girls should be constant. If boys and girls were compared, OR, the pizza amounts compared, then we have a ratio conception of fraction. If we were talking about boys and girls, then we need to ensure that the pizza's were the same amount. As a deduction, three pizzas for the boys would also imply a multiple of 3 boys (i.e., 9) boys. Accordingly, with the same no. of pizzas (3), then the boys are greater. If we are comparing boys and the pizzas, then it is rates. But only constant rate is what qualifies to be part of the ratio subconstruct (Charlambous & Pitta-Pantazi, 2005; Wong, 2010). The question is, "Seven girls share three pizzas and three boys share one pizza. Who gets more pizza, a girl or a boy?" Pitta-Pantazi and Charalambous (2007)

In converting the two quantities (pizzas and no. of boys), it requires a recognition that there is a relationship between the two quantities such that a multiplicative effect on any of them must affect others. This is what Charalambous and Pitta-Pantazi (2005) and Pitta-Pantazi and Charalambous (2007) described as the invariant and co-variant properties of the ratio subconstruct. The covariant property referred to the fact that the two quantities that formed the fraction, change at the same time. This is the basic principle towards the formulation of equivalent fraction concept. As a property of fraction, it is implied in the procedure for the formulation of equivalence as an algorithm. It is possible to have understanding of this property developed by the learner, when helped. The invariant property depicts the idea of equality in the equivalent fractions obtained (Wong, 2010; Amuah, Davis & Fletcher, 2017). This suggests that the actual value or quantity implied in the fractional quantity does not really change. In the study of Amuah, Davis, and Fletcher (2017),

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amongst all constructs examined, the equivalent fraction was the least understood construct among junior high school students in Ghana.

As a comparison index, Chick (2007) used the act of comparing fractional quantities as the basic procedure for introducing equivalent fractions. In her study of teachers' strategy for teaching selected topics in mathematics, it was evident that the use of simple fractions in a procedure to teach equivalent fractional concept and algorithm, was very essential and beneficial to learners. However, considering this procedure (comparing of fractions, to equivalent concept of fraction) as a one directional path was going to lead to students' limited understanding of the relationship between the comparison of fractions and the equivalent fraction concept.

Basic fact about the equivalent fractional concept can be found in Wong and Evans (2007). However, the most important idea from this study is that students studying fractions must be made to know that every fraction is a member of a group (set) of fractional quantities. The equivalent fractions are also of different names. Each fraction therefore has an infinite set of names. The other property of equality of values can be found in many studies (Wu, 2013; National Centre on Intensive Intervention, 2014; Bailey et al., 2015; Peng et al., 2016; Amuah, Davis & Fletcher, 2017). It was noted by Wong and Evans (2007) that teachers should be sure of the fact that equivalent fraction is not an easy concept for learners. They noted that the symbolic notation presents a highly cognitively demanding task. It requires a maximising of four dimensions of fractional values that needs to be simultaneously coordinated when the memory is validating the results of tasks involving equivalent fractions. It coordinates the original fractional quantities which are two

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dimensional. It also coordinates the two dimensional equivalent fraction itself (Asha, Star, Dupuis & Rodriguez, 2012).

Measure subconstruct

Conception of fraction under the measure construct originated from the idea of measurement. Accordingly using the numerical value of fraction in a measurement manner is a critical characteristic of the measure construct. It involves the use of fraction representation as illustrated in something to be measured.

Normally measurement always starts from a particular point and ends at another point. It also involves the use of a predetermined standard of measurement (Charalambous & Pitta-Pantazi, 2005; Pitta-Pantazi & Charalambous, 2007). However, for a fraction conception as a standardized measure, it will depend on the fraction itself. Hence a fraction a/b could be conceived to be using the standardized unit fraction of 1/b. This is so because the division of a fraction for measurement purposes requires equal partitioning. The fraction a/b could also use multiples of 1/b for the measurement purposes. Consequently, the use of the equal partitioning as a standard for measurement is normally the convention in teaching and learning the measure construct of fraction (Amuah, Davis & Fletcher, 2017). Representation, comparison, equivalence and general discussion of fractions on the number line is a typical example of situation of the measurement construct. A theory that brings together most of the constructs of fraction in measurement activities is the unit coordination theory.

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Unit Coordination Theory

Combining the part-whole, ratio, and the measure construct in reasoning about fraction could be observed in unit coordination theory. Most of the characteristics and skill in unit coordination involve; (1) the ability to think of a fraction of interest as parts of the whole, (2) segments of a fraction of interest as being the same as or equal to other segments in the fraction of interest, or segments in the whole (especially when the same fractional value is redenominated: See Figure 3). It could also involve the use of segments in either the whole or the fraction of interest, as an iterative measurement tool in the same fractional quantity or any other quantities, etc. These characteristics involve the skill of identification of a single or multiple segmented unit(s) in a whole/part, and using it as a measurement unit in the same or another segmented unit in the whole, or part of the whole or any other separate quantity (Norton et al, 2015).

Unit coordination as the name implies is therefore the skill or ability to compare and use similar or varied types of unit to obtain mathematical solutions in mathematical problems. Learners' ability to coordinate units in mathematics problems can be characterised in stages. These stages were represented by the number of levels of units that the learner could coordinate simultaneously (See fraction tiles on Figure 3). Identification of one unit of measurement and ability to use the unit of measurement in a standardized form is considered possession of a scheme for one level of measurement. Hence it implies that the ability to use a unit of measurement as ½ (Level B) in the measurement of a whole (1), is considered equivalent to the ability to use a unit of measurement as ¼ (Level D) in the measurement of a whole (1) (Wilkins et al., 2013; Kosko & Singh, 2018).

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As could be observed from the fraction tiles, juxtaposition of some of the units suggested the idea that some units could be totally subsumed in another and hence the form of complexity in combining and using the units in mathematical problems increases (e.g $\frac{1}{2}$ and $\frac{1}{4}$). It is worthy to note that some units may not be totally subsumed but partially subsumed in another level of units, thus, a different form of complexity is observed (e.g. $1/2$ and $1/3$ such that $1/3$ is subsumed in $1/2$ one and a half times.). This later form of complexity may be greater than the former but the structures that are involved in the combination are similar. A greater form of difficulty is observed when three levels of units are combined in one mathematical problem (Pass et al., 2003; Kosko & Singh, 2018). This suggests that learners who are able to reason with one level of unit may not be able to reason with two levels of unit (Students with stage one reasoning ability). Therefore, stage one is of a lower order as compared to stage two learners, and subsequently stage 3 learners.

Stage two learners therefore are those who are able to reason with two levels of unit concurrently and with the added advantage characteristic of identifying the multiplicative relationship between the two different levels of unit (e.g. ${}^{1}/_{2}$ *and* ${}^{1}/_{3}$, ${}^{1}/_{2}$ *and* ${}^{1}/_{4}$). Stage three learners are characterised as those who are able to reason with three levels of unit (e.g. Level B, D, E, see Figure 3) concurrently and with the added ability to determine the multiplicative relationship between the levels involved. Unit coordination therefore is a way of reasoning with any kind of number units that can be used in comparison and measurement (Wilkins, Norton & Boyce, 2013).

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Steffe (1992) explored unit coordination among learners using whole numbers. The example chosen was colour coded red bar and blue bar (for a modified version, please refer to question one to three on instrument for unit coordination i.e., Appendix A). The blue bar could fit into the red bar exactly six times. These were preliminary information given to the learner. The additional information was that the orange bar fit into the blue bar exactly two times (Norton et al., 2015; Kosko & Singh, 2018). However, it was only the blue and orange bars that were made visibly available. The task therefore sought from the students, how many times the orange bar fit into the red bar. This task must be carried out only with the blue bar and maybe the red bar. The study reported the learner in this experiment was simply observed whispering the following numbers in sequence: "1,2; 3,4; 5,6; 7,8; 9,10; 11,12." This suggested that the learner in the experiment could use the single unit of two ones to count (stage one). It also suggested the possible ability to work with two units (i.e., stage two). The first unit being the use of single units of ones to represent another unit of two 1s; "1, 2", "3,4", etc. The second unit being the ability to use the amalgamated unit to determine the number of the orange bar in the red bar; "1,2 as 1", "3,4 as another 1" etc., and adds up the single 1s to obtain 6. However, the amalgamation of units only happens when the learner used physical counting to represent the single and composite units.

A student with the strong background in unit coordination would notice that 12 (one unit) divided by two (second unit) would give 6. The former use of representations as reasoning help, was used in the previous studies to determine the formation of a mathematical schema in the mind of the learner (Wu, 2008: 2013; Wilkins, Norton & Boyce, 2013). The later action

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demonstrated learners' ability to identify the multiplicative relationship between the unit levels identified in the question (stage three) (Norton et al., 2015; Kosko and Singh, 2018).

Unit coordination transcends whole numbers as the structure of reasoning is the same with fractions. Detailed description of the stages of reasoning with fractions would be exemplified in chapter three and four. Unit coordination in fractions mostly occurs when there is the need to compare two fractional values and use the results in solving a mathematical problem (see Figure 3). When the relationship between $^{1}/_{2}$ and $^{1}/_{4}$ is given, it could be required of a learner to determine the number of $^{1}/_{8}$ in $^{1}/_{2}$ when it had been given that the number of $^{1}/_{8}$ in $^{1}/_{4}$ is 2. This is a representation of one simple situation of unit coordination in fractions (Asha, Star, Dupuis & Rodriguez, 2012; Wiklin, Norton & Boyce, 2013). Recognition of one level of measurement as $^{1}/_{8}$, $^{1}/_{4}$, and $^{1}/_{2}$ as different levels of measurement is a demonstration of the learner being categorised at stage one. Subsequent ability to use and determine the multiplicative relationship between the $^{1}/_{8}$ and $^{1}/_{4}$, is evidence of stage two learner. And finally, using the relationship given to determine the multiplicative relationship that transcends all the three levels is a characteristic of stage three learners (i.e., 1x2x2=4). The requirements could be more complex when the number of one fraction in another could result in a value that could also have a fractional component. The unit coordination is similar to the partitive or reverse partitive unit fraction construct described by Charalambous and Pitta-Pantazi, (2005), Pitta-Pantazi and Charalambous (2007), Davis (2014) and Amuah, Davis, and Fletcher, (2017).

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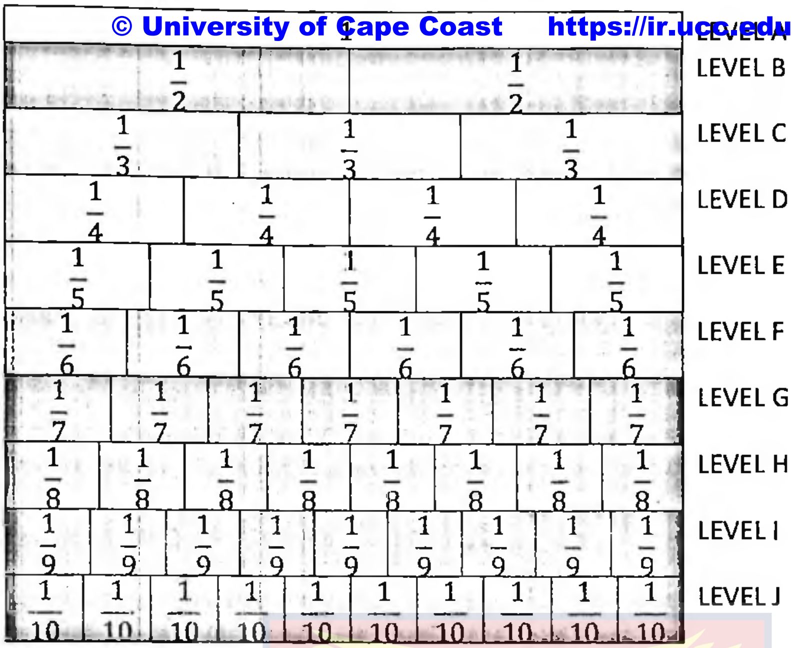

Figure 3: Fraction tiles

Literature Related to the Concept of Addition of Fractions

Strategies for adding fractions

Addition of fractions is part of the four arithmetic operations that is used in relating and manipulating numbers of the number system. In fact, it is through the concept of addition of fractions that most textbooks initiated students' journey in dealing with the four arithmetic operations or any of their combinations (Charalambous, Delaney, Hsu & Mesa, 2010; MOE, 2012; Yang, 2012; Wu, 2013;). Since addition of numbers involves dealing with two numbers (in this case fraction), it serves as a higher level of complexity in understanding of numbers and for that matter, fractional concepts (Paas, Tuovinen, Tabbers & Van Gerven, 2003). The concept of addition of fractions is therefore a topic that normally manifests linearity of reasoning, a characteristic that is mostly demonstrated by early learners in their attempt to grasp the various property of the number system. In fact, it could be said that the nature of complexity most young learners would experience when they are about to study most mathematical concepts would be easier when compared to

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the problems they would face when attempting to learn the concept of fractions addition (Prediger, 2008; Siegler & Lortie-Forgues, 2014; Price & Fuchs, 2016).

In describing the teaching and learning of fraction, the study would attempt to deal with it in a way that would reflect an integration of understanding from previously learnt fractional knowledge, and possibly in relation to whole number ideas. This would demonstrate an attempt to synchronize the smooth learning of the numbers involved in the number system (Siegler & Lortie-Forgues, 2014). Also, the points of view expressed here regarding fractions are strictly based on mathematically derived facts that can be proved by algorithms. Two imaginary fractions, $\frac{a}{b}$, and $\frac{c}{a}$, such that $a, b, c, and\ d >= 1$, then adding the two fractions should be in the form

$$\frac{a}{b} + \frac{c}{d} \dots \dots \dots . (1)$$

Although most deficiencies in students understanding of addition of fractions was from concepts of adding whole numbers, some researchers are still of the idea that it would be best for learners to approach fraction arithmetic from whole number addition (Cramer, Wyberg & Leavite, 2008; Siegler, Thompson & Schneider, 2011; Wu, 2013; Dewolf & Vosniadou, 2014). In the teaching of the concept of fraction, Wu (2013) suggested that the first introduction of putting numerical quantities of numbers together should be demonstrated geometrically with two fractional quantities without attaching relevance to the quantitative values of the fractions that are being put together.

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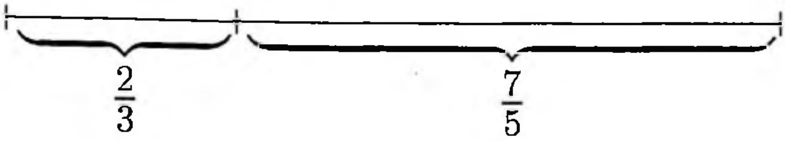

where $\frac{a}{b} = \frac{2}{3}$ and $\frac{c}{d} = \frac{7}{5}$

Source: Wu (2013)

Figure 4: Illustration of the concept of addition

The illustration in the Figure 4 suggests that if learners are introduced to the quantitative value of fraction with Figure 4, there might be confusion in the minds of the learner. Hence after discussing the concept of addition, the quantitative value of the addend would then have to be planned for in the teaching and learning of addition of fractions. It is important to note that whiles the introduction was from the concept of whole numbers, the synonymous characteristic that makes the addition possible in the fractional context would have to be identified and used in the same manner as whole numbers. In the context of whole numbers, every adjacent number is generated by addition of one (1). In a similar manner, the counting unit for a fraction should be determined. The minimum counting unit for a fraction is called unit fraction (Norton & Wiklin, 2011; Bailey et al., 2015; Siegler & Lortie-Forgues, 2014; Torbeyns, Schneider, Xin & Siegler, 2015). What makes the difference between whole numbers and fractions is that the counting unit of fraction is not static in terms of value or quantity. That is, the counting unit is the unit fraction of the fractional quantity under consideration. To illustrate, the counting unit of $\frac{2}{3}$, is $\frac{1}{3}$ as illustrated in equation 2.

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$$\frac{2}{3} = \frac{1}{3} + \frac{1}{3} \ldots\ldots\ldots\ldots\ldots (2)$$

Hence $\frac{1}{3}$ is added twice to get $\frac{2}{3}$. The counting unit for $\frac{7}{5}$ is $\frac{1}{5}$

$$\frac{7}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} \ldots\ldots (3)$$

$$\frac{7}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = 7 \times \frac{1}{5} \ldots\ldots (4)$$

$$14 = 2 + 2 + 2 + 2 + 2 + 2 + 2 = 7 \times 2 \ldots\ldots (5)$$

Hence $\frac{1}{5}$ is added seven times to obtain $\frac{7}{5}$. Equation 5 and 2 are similar to the whole number addition of 1 to any whole number (10+1=11, 11+1=12, …, 15+1=16, 16+1=17,…, 30+1=31, 31+1=32,…) a number of times to get the whole number needed. However, the major difference between the whole number addition and the fractional quantity addition is that each fractional quantity has a unique unit fraction associated with it. $\frac{1}{5}$ is the unit fraction associated with $\frac{7}{5}$. However $\frac{1}{3}$ is the unit fraction associated with $\frac{2}{3}$. Compared to whole numbers, the unit associated with 10, 15, 20, and 30 is still 1. It is important to recognise that the unit fractions' addition property (equation 4) is similar to the addition property of whole number multiplication (equation 5) (Wu, 2013). All these illustrative ideas could easily be translated into the concept of addition of fractions with similar denominator. Thus,

$$\frac{4}{5} + \frac{3}{5} = \left(\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}\right) + \left(\frac{1}{5} + \frac{1}{5} + \frac{1}{3}\right) = \frac{7}{5} \ldots\ldots (6)$$

Addition of proper fraction to a whole number needs to be also addressed. Since this kind of addition is not the same as fractions with similar denominator, a different strategy is adopted to translate all figures into fraction of the same denominator (Wu, 2008; Wu, 2013).

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$$5 + \frac{2}{3} \dots\dots (7)$$

If there is a possibility of transforming the whole number into a fractional form, it would possibly help in this situation (equation 7). Hence the need to transform 5 into fractions like $\frac{10}{2}, \frac{15}{3}, \frac{20}{4}, \frac{50}{10}$, etc., would help. Amongst all of the selected fractions, the best choice is the one with the denominator 3, so that adding it to $\frac{2}{3}$ will be a straightforward addition of numerators as previously noted in equation 6 (see equation 8).

$$5 + \frac{2}{3} = \frac{15}{3} + \frac{2}{3} = \frac{17}{3} \dots\dots (8)$$

Mixed numbers are a form of fraction that consists of whole number and fractional component (less than a whole). Mixed fractions can be considered as two numbers added together as illustrated in equation 9.

$$5\frac{2}{3} = 5 + \frac{2}{3} \dots\dots (9)$$

The strategy for dealing with such fractional quantity will therefore consist of either considering the fraction in the addition task as two different numbers put together with an addition sign (see equation 10 and 11), or converting the mixed number to improper fraction (see equation 12 and 13) (Bruce et al., 2013).

$$5\frac{2}{3} + \frac{5}{3} = 5 + \frac{2}{3} + \frac{5}{3} = 5 + \frac{7}{3} = 5\frac{7}{3} \dots\dots (10)$$

$$2\frac{3}{4} + 5\frac{2}{8} = 2 + \frac{3}{4} + 5 + \frac{2}{8} = 7 + \frac{3}{4} + \frac{2}{8} \dots\dots (11)$$

This strategy of separating the whole numbers and the fractional quantity is what is normally referred to as decomposition of numbers. However, there is a second strategy. It involves conversion of the mixed number

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to a fractional quantity (improper fraction) (Wu, 2013). This is as illustrated in equation 12 and 13.

$$\frac{45}{8}+\frac{1}{4}=\frac{40+5}{8}+\frac{1}{4}=\frac{40}{8}+\frac{5}{8}+\frac{1}{4}\ldots\ldots(12)$$

$$\frac{45}{8}+\frac{1}{4}=5+\frac{5}{8}+\frac{1}{4}\ldots\ldots(13)$$

The final strategy and possibly the most general strategy is the algorithmic form. It involves finding the lowest common multiple (LCM) of the denominators and working with any fraction in the improper form or the proper form (Wu, 2008; MOE, 2012). For a fractional quantity $\frac{a}{b}$, it could be multiplied by 1 and the value would remain the same (see equation 14). The same could also be done for another fraction $\frac{c}{d}$ (see equation 15). The value remains the same for any number when multiplied by the number 1. The eventual essence of the steps that would be taken was to make the two fractional numbers appear in a similar form (same denominator value) see equation 16 and equation 17. The forms of the two fractional quantities should also be equivalent in terms of the most basic fractional unit i.e., unit fraction (including the unit fraction) (Bruce et al., 2013; National Centre on Intensive Intervention, 2014).

$$\frac{a}{b}\times 1=\frac{a}{b}\ldots\ldots\ldots(14) \quad \text{and} \quad \frac{c}{d}\times 1=\frac{c}{d}\ldots\ldots\ldots\ldots(15)$$

The whole number 1 can be replaced with $\frac{a}{a},\frac{b}{b},\frac{c}{c}$, and $\frac{d}{d}$. However, in this instance, $\frac{d}{d}$ is chosen for (14) and $\frac{b}{b}$ is chosen for (15). The reason for these respective choices was because the resultant multiplication would ensure the denominators of the fraction (and the resultant unit fractions) would be the same.

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$$\frac{a}{b} = \frac{a}{b} \times \frac{d}{d} = \frac{ad}{bd} \ldots\ldots (16) \text{ and } \frac{c}{d} = \frac{c}{d} \times \frac{b}{b} = \frac{cb}{db} = \frac{cb}{bd} \ldots\ldots (17)$$

This implies that $\frac{a}{b} + \frac{c}{d}$ can be written as $\frac{ad}{bd} + \frac{cb}{bd}$.

Let $bd = e$ imply that e is a whole number. Then,

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{e} + \frac{cd}{e} \ldots\ldots (18)$$

Implying ad is also whole and bd, and cd is also whole (see equation 18). In order to properly understand the mathematical procedures, the underlying concepts and its characteristics needs to be well understood. Since a, b, c, d are all whole numbers. bd should represent multiplication of whole numbers.

$$\frac{a}{b} \times 1 = \frac{a}{b} \times \frac{d}{d} \ldots\ldots (19)$$

Equation 19 therefore represents multiplication of fraction by fraction and multiplication of fraction by whole numbers. Should all these (multiplication of fraction by fraction, multiplication of whole number by fraction) be taught before teaching addition of fractions? These deductions are expected if the learning of addition of fractions was to be considered schematically (Charalambous et al., 2010; Asha et al., 2012; Bruce et al., 2013).

The LCM strategy that has been represented above is what the algorithm developed illustrates. However, there are various methods of finding the LCM in the fractions addition problems. Some of such strategies when integrated into the fraction scenario produces other strategies of dealing with fraction problems.

Representation for addition of fractions

Singer (2007) opined that most of the misconceptions that learners develop in the period of their study have at its inception stages, the

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representations that were adopted during the learning processes. Singer was of the opinion that most people learn by picking their ideas from representations and not necessarily the result of the development of increasing stages in understanding of mathematical knowledge. Vig et al (2014) noted that this also means that the inability of a learner to understand or even forget a mathematical idea could be strongly mapped onto the use of representations in the teaching and learning process. The use of representations in learning has the potential to improve on the length of time learners could be fruitfully engaged in learning during the teaching and learning process.

For effective adoption of representations within the mathematics classroom, there is the need to foster the existence of certain characteristic that could promote learning and understanding. First of all, for the use of representation in mathematics classrooms, there is the need to develop and make these representations available in the classroom setting. Singer (2007) was of the opinion that developing these representations would require the representations being designed to reflect the structures in any particular mathematical domain of study (fraction is no exception). There are two basic categories of representations for teaching and learning within the mathematics classroom. These are predesigned representations and student designed representations (Jonassen, Strobel & Gottdenker, 2005; Teppo & van den Heuvel-Panhuizen, 2013). Although Vig et al (2014) considered the use of students' classroom designed representations as the most effective in the enhancement of students understanding of mathematical concepts, the representation often used within the mathematics classroom is the teacher predesigned representations.

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The focus in this literature is the predesigned representations of the concept of fraction that may be used in the mathematics classroom. Generally, the use of mathematical representations for the concept of fraction in a mathematics lesson is to make more visible the fractional concepts that most students would not have noticed/understood in conventional mathematical symbols. There were situations when learners used symbols and their associated procedures in ways that violate mathematical rules. Some of such ways that illustrate the mentioned violation is in the examples; $2.5 \times 10 = 2.50$ .... (20), and $\frac{1}{4} + \frac{2}{5} = \frac{3}{9}$ ... (21) etc. (Newton, Willard & Teufel, 2014). The use of representations is therefore a means of avoiding such pitfalls.

The use of multiple representations to enhance understanding of addition of fractions within the classroom setup would also require a development of means of ensuring the transfer from one representation to another. Especially at the lower levels of education, the use of multiple representations of mathematical concepts is seriously encouraged. In situations where multiple representation of the concept of addition of fractions is intended to help imprint the already learnt concept (fraction concept) in the learners' mind, there would be the need for the user to ensure that the second and later representations would reflect similar mathematical characteristics as the previous and already used representation. Conscious effort would have to be made by the mathematics teacher to illustrate and match the mathematical commonalities in the representations, to the learners.

In situations where the intention of using multiple representations is to advance from one mathematical concept (concept of addition of whole numbers, concept of fraction) to another, a different approach would be required to ensure

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this is effectively achieved within the mathematics classroom. This approach would require two basic characteristics. It would involve showing the similarities in the first (concept of fraction) and second (concept of addition of fractions) mathematical representations. Secondly, the approach would also involve the ability of the representation to show the differences between the old representation (concept of whole number addition) and the new representation that attempts to advance the mathematical knowledge (concept of addition of fractions) being learnt. Finally, it is important to also show how the mathematical knowledge in the mathematical representations translated into differences and advancement in the symbolic mathematical knowledge. In using a representation for identification of contrast or similarities in mathematical concepts, there should be evidence on the representation that connects to the contrast or similarities that is intended to be identified in the mathematics lesson of the concept being taught (Wong, 2010; Vig et al., 2014).

An important aspect of the use of mathematical representations in the teaching and learning of addition of fractions, shows that the use of a representation is not such a straightforward activity but involves the careful integration of the representation within the teaching and learning processes. Inability to carefully integrate representations of the concept of addition of fractions in mathematics lessons would effectively inhibit its benefits (Teppo & van den Heuvel-Panhuizen, 2013; Newton, Willard & Teufel, 2014). These representations of the concept of addition of fractions could be real artefacts, drawn models, etc. However, it is important to understand that the use of representations in the teaching and learning of addition of fractions, develops a habit among learners. Accordingly, in schools where the mathematics teachers

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do not often use representations in the teaching and learning process, they would have to be helped to develop such attitude among the teachers and curriculum designers. This is outside the domain of this study and therefore, literature in habitual change would not be considered. However, learners' attitude in reasoning in relation to representations is something that needs to be considered when using mathematical representations of the concept of addition of fractions (Vig et al., 2014). This implies that using representations in a classroom where representation were not often used to help learners develop understanding would not foster instant understanding or change in understating. This is possibly why attempt to use representations to deal with misconceptions among learners of the concept of fraction seems not to achieve sustainable and reasonable changes in conception. This is what was termed as incommensurability, a reason for not achieving results for dealing with misconceptions (Chi, 2013). Its ineffectuality is found in the fact that the habit pattern through which the understanding was developed, is different from the habitual pattern through which the understanding is being corrected. Incommensurability could be found in the fact that the learners developed the wrong conception through reference to artefacts found in their environment, however attempts to correct the wrong conception is through drawings of shapes on the board. It could also be that in this same case, the path to correction is through attempts to show constructs that in mathematics, represents the order or hierarchy of mathematical knowledge, whiles the learning path was through mathematical symbolic reasoning (Jonassen, Strobel & Gottdenker, 2005; Larsson & Halldén, 2010; Suh, 2008; Vig et al., 2014; O'Dwyer et al., 2015).

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It is important to make a distinction between changes in representation of the concept of addition of fractions and changes in conception or understanding (or commonly termed conceptual change). Representational change focused on an approach to teaching the concept of addition of fractions. It focuses on bringing the powerful representations (formal or informal) within the classroom to foster understanding of the concept of addition of fractions. The eventual end of representation is to achieve the correct abstraction of knowledge (concept of addition of fractions) from such representations. On the other hand, conceptual change focuses on the internal structures of knowledge and ensuring that learning of knowledge is successful. It focuses on the content (addition of fractions) that is taught than the representation of the content (Jonassen, Strobel & Gottdenker, 2005; Singer, 2007). This is why conceptual change can sometimes focus on misconceptions of content that were taught. Conceptual change is conceived on the awareness of misconceptions that exists in the mind of mathematics learners and how they could be avoided in the teaching and learning processes. Exposing children to conceptual change would involve first a curriculum that is legal within a particular context of study. It would involve the possibility of that curriculum being successfully implemented to bridge the gap. Finally, it would involve the use of representational changes to help address these shortfalls in conceptual changes. In sum, representational changes in the concept of addition of fractions are not the same as conceptual change but they are synonymous. Representation and representational changes could help in change in the conception of learners.

Despite the fact that representations are considered very good for the purpose of understanding of concept, there are limitations in its uses and

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capabilities to represent any given concepts in mathematics. Such limitations make the representations to ceases to be effective in illustrating that given mathematical ideologies or procedures in the teaching and learning of mathematics. Such a situation of limitations in the use of a mathematical representation was described as the model breaking point (Vig. et al., 2014).

For the teaching and learning of the concept of fractions, the use of the circle representation serves the purpose for fractions with small denominator value. However, as the denominator value increases, the representation becomes less effectual, hence the circle representations' breaking point. It would therefore be unreasonable to use the circle representation of fraction for fractional values with denominator greater than ten. Representations in mathematics are normally used for targeted problems. In other words, the problem space is limited. However, there is the possibility of attempting to modify ideas in relation to a given representation in order to extend the representations' use in other areas of mathematical learning (Cramer, Wyberg & Leavite, 2008). A typical example is the use of the chip model as a representation in teaching and learning of concepts in mathematics. The chip model as a representation, could be used to enhance students' understanding of fractions. The chip model is adapted to fractions when a chip is considered a unit fraction. It is sometimes used in counting (unit fraction iteration). It could also be used for the purpose of quantity representation or numerosity (the number of unit fraction in a given fraction). Also for addition of fractions, the chip model is still usable when the addends are converted to equivalent fraction form. The unit fraction associated with the equivalent fraction becomes the counting unit in the chip model (see Table 4). However, the chip model is only

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possible for some addition of fractions problems (see problem Type A1-A4 in Table 4). The chip model breaks for problem type A5 and A6. Consequently, to be able to extend the use of the chip model, there is the redefinition of its use. Positive unit fractions will be colour coded blue, and negative unit fractions will be colour coded green. Hence, when a green and a blue chip are put together, they cancel out to be zero. With the re-modification of the chip model, all addition of fractions problem types could be represented successfully. However, in subtraction of fraction, the extended use is only able to solve problem type S1 (see Table 5). The model breaks for the rest of the subtraction problem. What this implies is that, the use of any model will always have a breaking point such that its use cannot be extended.

Table 4: Problem Space for Addition of Fractions

Problem Type Addition, a + b	Example problem Whole number	Fractions
A-1. a and b both positive, $\lvert a\rvert \geq \lvert b\rvert$	5 + 3	$\frac{5}{6}+\frac{3}{6}$
A-2. a and b both negative, $\lvert a\rvert \geq \lvert b\rvert$	–5 + –3	$-\frac{5}{6}+-\frac{3}{6}$
A-3. a and b both positive, $\lvert a\rvert < \lvert b\rvert$	3 + 5	$\frac{3}{6}+\frac{5}{6}$
A-4. a and b both negative, $\lvert a\rvert < \lvert b\rvert$	–3 + –5	$-\frac{3}{6}+-\frac{5}{6}$
A-5. a and b different signs, a > 0	5 + –3	$\frac{5}{6}+-\frac{3}{6}$
A-6. a and b different signs, b > 0	–5 + 3	$-\frac{5}{6}+\frac{3}{6}$

Source: Vig et al (2014)

Table 5: Problem Space for Subtraction of Fractions

Problem Type Subtraction, a - b	Example problem Whole number	Fractions
S-1. a and b both positive, $\lvert a\rvert \geq \lvert b\rvert$	5 – 3	$\frac{5}{6}-\frac{3}{6}$
S-2. a and b both negative, $\lvert a\rvert \geq \lvert b\rvert$	–5 – –3	$-\frac{5}{6}--\frac{3}{6}$
S-3. a and b both positive, $\lvert a\rvert < \lvert b\rvert$	3 – 5	$\frac{3}{6}-\frac{5}{6}$
S-4. a and b both negative, $\lvert a\rvert < \lvert b\rvert$	–3 – –5	$-\frac{3}{6}--\frac{5}{6}$
S-5. a and b different signs, a > 0	5 – –3	$\frac{5}{6}--\frac{3}{6}$
S-6. a and b different signs, b > 0	–5 – 3	$-\frac{5}{6}-\frac{3}{6}$

Source: Vig et al (2014)

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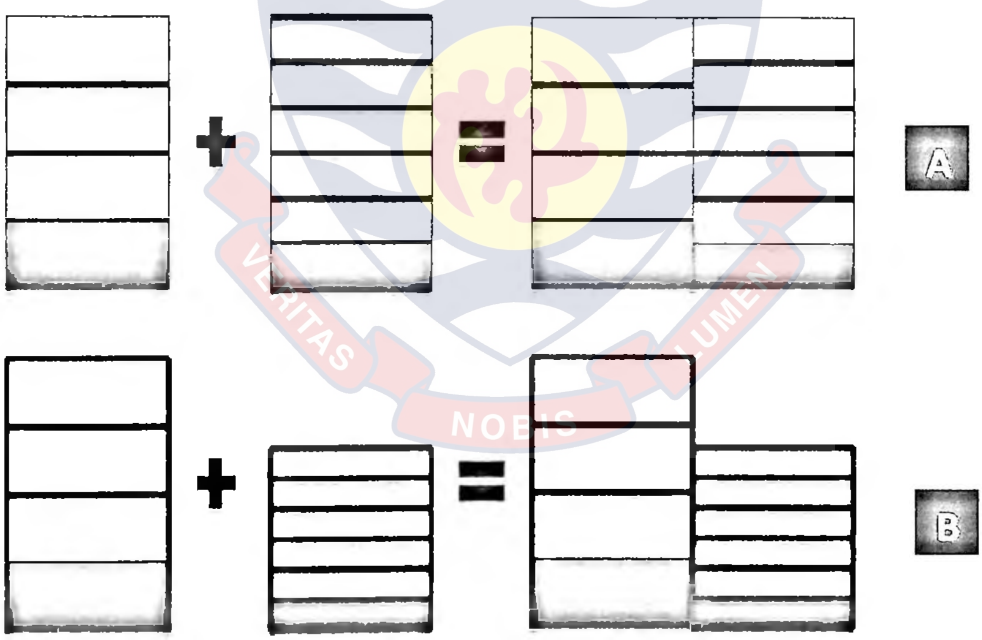
Normally, representation of the individual concept of fraction is not much of an issue for students who would be advancing into the concept of addition. Much attention is therefore not focused on mathematical representation of addition of fractions but on conditions that would be necessary to represent it (Wu, 2013, Vig et al., 2014). One of the conditions is that all the fractions involved in the concept of addition of fractions should preserve the whole as in the normal fractions' representation. The added condition is that the whole of the two fractions to be added must be equal (see area labelled A in Figure 5). Most learners whom have not encountered the mathematical representation of the concept of addition of fractions would most likely not preserve the same size of whole as noted in the second condition (see area labelled B in Figure 5) (Bruce et al., 2013; Wu, 2013).



Source: Vig et al (2014)
Figure 5: Representation for fractions addition

In previous uses of individual fraction representation, nothing actually construes the representation of fraction in relation to addition. However, most of the properties in fraction representations were rules that were put in place to

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ensure that the processes of adding fraction was going to be similar to that represented, in the case of the fractional representations used to support it. Ensuring that the areas in the mathematical representations of the two fractions to be added were equal, is synonymous to ensuring that every written fraction is in relation to a whole. Although in the mention of most fractional names, there is no reference to the whole, it is an implicit fact in reasoning about fractions (Wong, 2010; Bailey et al., 2015). Secondly, in situations where fraction comparison representations had been mentioned, fraction representation to the same whole was a necessity. Else the fractions were normally incomparable (Wong, 2010; Wu, 2013). Therefore the second condition for the representation of the concept of fractions addition to be possible is that the size of the divisions (representing unit fraction) in the whole must be the same for the two wholes' that contains the fractions to be added. This rule ensures that there is the high likelihood of reaching an accurate interpretation of the result of the fractions addition problem. When fraction representations to support the concept of addition of fractions are combined (see area labelled B in Figure 5), the likelihood of reaching wrong interpretation or confusing the user of the representation is high. This is because the learner would not know the unit fraction to use (or how to combine the various unit fractions) in formulating the resulting fraction (Charalambous & Pitta-Pantazi, 2007; Vig et al., 2014; Amuah, Davis & Fletcher, 2017).

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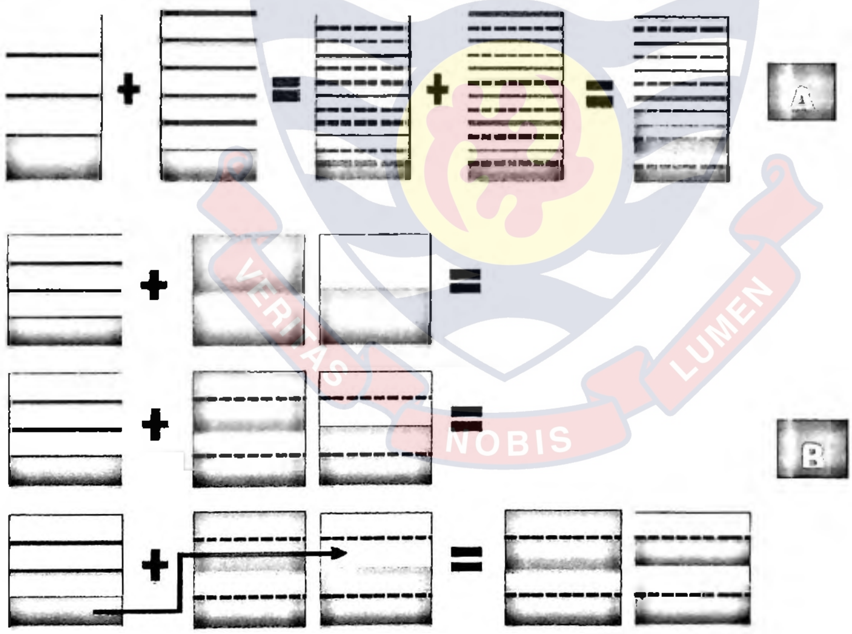
Overcoming these difficulties would involve one or both fraction representations being modified to be able to contain both unit fractions (or the same unit fraction) in their mathematical representations (see area labelled A and B in Figure 6. It is an illustration of the process of modification). This implies that there would have to be a determination of the multiplication relationship between the denominators of the fractions involved. What is implied is that the least common multiple of the denominator of the two fractions involved would be most preferable as the number of the divisions chosen for the representation increased. In effect, the representation is expected to clearly depict the two basic conditions necessary for the concept of fractions addition (Wu, 2013; Vig et al., 2014).



Source: Vig et al (2014)

Figure 6: Representation of processes for addition of fractions

The final condition for the representation to help foster learners understanding of the concept of fractions addition is that the shaded portions in one representation would have to be moved from its original whole, and joined

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to replace as much unshaded portion as possible in the other whole (see area labelled B in Figure 6). This represents the idea of combination of two quantities (Singer, 2007; Teppo & van den Heuvel-Panhuizen, 2013). What are the model/representation breaking points that could be observed in this representation of the concept of fractions addition? The issue of numerosity in representation also shows itself in the addition results where the denominator of the fraction is large.

Secondly, in determining the number of equal divisions that were needed to make apparent the two unit fractions necessary for the addition in the same representation, this was not visible in the representation. Stated in another form, the act of determining the least common multiple was outside of the representation. In the process of addition, the movement of shaded portions were not apparent in the representation. It is only an invention of users of representation to show the concept of addition. Also not illustrated in the representation is the replacement of unshaded portions in the other fractional representation. This process does not foster the idea of adding as is comparable in whole numbers. All these were identified as representation/model breaking points (see area labelled B in Figure 6). The breaking point of the representation seemed to show itself at earlier stages of the use of the representation but modification of rules in the use of the representations continued to extend their uses (Vig, et al., 2014). So far, the representation discussions were in relation to the practical addend and the sum being less than a whole. However, fractional addend and the sum can be greater than one.

For the representation of fractions addition problem, where the addend and the sum values are greater than one, more extensions in the representation

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could be identified. This is because of greater level of complexity. To ensure that the first rule of preserving the whole is maintained, there is an introduction of a second rectangular representation of the same size as the previous ones. The complexity is increased when the two fractions are greater than one. This would imply more than three area representation needed to succinctly represent the addends. This therefore introduces more complexity and difficulty in the processes involved (Paas, Renkl & Sweller, 2003; Prediger, 2008; Teppo & van den Heuvel-Panhuizen, 2013; Torbeyns, et al., 2015).

Firstly, in the situation where more than three area representations are used, this introduces confusion in the mind of the learner. This is because instead of using two area representations, which is comparable to two fractional quantities being added, there are three area representations. In the result that is supposed to represent a singular number, more than one area representation is needed. This characteristic removes the central benefit of using an active representation of the principles and procedures in the concept of addition of fractions. The major guide in observing representational breaking point is for users to note that in the processes of using the representations, when complexities of procedures and processes in using the representation exceeds the complexities of using symbolic procedures, then the symbolic procedures need to be used (Vig, et al., 2014).

The use of representations so far discussed has been within the classroom situations. However, representations could also be artefacts within the society of learners. In fact, most ethnomathematic researchers like Davis et al (2009) are of the opinion that the use of artefacts within the learners environment is the most effective representation during the teaching and

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learning processes. This is because the cultural artefacts that were being used as representations of mathematical knowledge were found and had long been part of the daily cultural lives of the learners. In situations where such cultural artefacts cannot always be brought to the classroom, representations of them could be adopted. The immediacy and the daily occurrence of such representations of cultural artefacts would more frequently be activators of the mathematics found in them.

The main opinion is that, the ability to note the actual role that representation plays in the teaching and learning of the concepts of addition of fractions is a basic idea necessary for the sustained use of a representation. Students are noted to rely on the use of fraction representations to the extent that most refuse to reason without the help of the representation (Vig, et al., 2014; O'Dwyer, et al., 2015). It is therefore important for mathematics teachers and students to understand that the role of representation is to support the learning of the concept of addition of fractions and that it does not represent the concept itself. Otherwise it results in a situation where teachers and learners focus on characteristics that would extend the use of the representation than the important mathematical concepts that is inherent in the domain.

Textbooks' approach to the concept of addition of fractions

Under normal circumstance, examining the learning of students should start by examining what the curriculum proposed to be taught to the learners. Yang (2012) noted that some curriculum writers use content to be studied, the organisation of the content to be studied, and the time spent in examining a contents' concept, as possible parameters that could tell what the learners who used such curriculum comprehend. Therefore, it is possible that in learning

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addition of fractions, determining the amount of learning is equivalent to the determination of the amount of learning that the curriculum could provide. Also, comparing learning could be considered as cross examining the content in different mathematics textbook. Comparison of content in different countries' mathematics textbook could be illuminating in improving the learning of other group of learners (countries') understanding of mathematics topics (e.g. addition of fractions). However, the illuminations that are found in textbook analysis can only theoretically explain to a degree, the difference in the amount of learning of diverse groups of students from different countries or different curriculum (Herbel-Eisenmann & Wagner, 2005; Sunday, 2014). Accordingly, the extent of influence that the curriculum has on students' learning is still not determined. Possible achievement of such a goal would be the major breakthrough in curriculum analysis (Jitendra, Griffin & Xin, 2010).

A critical analysis of sequencing of topics and how it is interwoven in different curriculums would be an interesting area of investigation. Most studies of sequencing have examined the sequencing of topics and possible organisation of topics during curriculum implementation. However, it is important to note the distinction between curriculum implementation and the curriculum itself. This difference is what actually makes attempt in examining a curriculum not definite (Yang, 2012). This is to say that a written curriculum is not necessarily the same as an implemented curriculum. The curriculum and textbooks represents a "probabilistic rather than deterministic opportunities to learn mathematics." This probabilistic nature is directly expressed in the question: what would students learn if they had solved and understood all the exercises and examples in the textbooks (Charalambous, Delaney, Hsu & Mesa, 2010;

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Yang, 2012). Hence comparison of curricula documents from various countries, examines the potential learning that ought to be realized in the various countries whose curriculum was studied. Curriculum comparison could reveal the extent that the curriculum encouraged comprehensive understanding, surface understanding, and/or procedural understanding, etc. Last but not the least, comparing curriculums from different countries could be considered learning for mathematics teachers. Mathematics Teachers' observation of how different curriculum presents the learning opportunities, and pedagogies for engaging the concept of fraction (including addition of fractions) in the classroom and the possible benefits there might be, will be educating. The aforementioned observation would be very insightful in helping teachers improve on their mathematics instructions. (Ben-Hur, 2006; Lee, 2006; Sunday, 2014; O'Dwyer et al., 2015).

There is no laid down singular strategy for examining textbooks of the curriculum involved in cross country comparison. Yang (2012) showed that whatever kind of comparison there might be, the major intention is to show the kind of attitude towards mathematics that the curriculum fosters (Herbel-Eisenmann & Wagner, 2005; Zolkower & de Freitas, 2010; Morgan, 2016). Therefore, comparison could be at the level of the goals that the various curricula intended to foster, the various way of organising the curriculum, and analysis of each particular topic within the curriculum. Yang (2012) found some differences in his comparison of the curriculum of the Common Core Standards (2010) in the United States and the Taiwanese curriculum. These are two countries that were observed to have national curriculum and hence, the comparison was found to be possible at the national level. One of the major

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findings was that fractional concepts (including addition of fractions) were introduced earlier in the Taiwanese curriculum than in the common core standards of the United States of America. Fraction was introduced in grade 2 in the Taiwanese curriculum as compared to grade 3 in the United States' Common Core Standard (2010) in mathematics. The idea of common factors, which is used in dealing with questions involving the concept of addition of fractions, is introduced in grade 5 in Taiwanese curriculum as compared to grade 6 in the USA CCSM. In relation to the similarities observed, both curricula made deliberate effort to foster review of related previous knowledge necessary for the teaching and learning of the concept of fraction (including addition of fractions). However, the Taiwanese curriculum spent more time in revising RPK and dealing with observed difficulties than the CCSSM (Meyer, 2004; Hailikari, 2009; Yang, 2012; Bruce, et al., 2013). The Taiwanese curriculum spent more time in allowing mathematics teachers to correct misconceptions and deeper learning than the CCSSM.

Specifically, in relation to the concept of addition of fractions, Yang (2012) was of the opinion that the curriculum in Taiwan dealt with LCM, as a subject matter relating to the concept of addition of fractions. This supposed that although the Taiwanese curriculum might have dealt with the LCM in previous areas, LCM was latter intertwined and learnt in the specific context of the concept of addition of fractions. However, the USA CCSSM did not exactly specify that. The Taiwanese curriculum specifically dealt with LCM in relation to the concept of equivalent fractions. Equivalent fractions necessary for addition of fractions situation/problem was generated through LCM. However, in the USA common core curriculum, LCM was introduced in relation to

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properties of rational numbers (Wu, 2013). In the Taiwanese curriculum, focus is given to local problematisation of mathematics (addition of fractions) being studied.

LCM can be introduced and dealt with in three different situations or methods. The three methods involved (1) multiples or common multiples (2) decomposing into prime numbers, and (3) short division method. These three methods of dealing with the same thing were found to be common in the Taiwanese mathematics curriculum. In this way, deep rooted learning of the concept of addition of fractions was expected to be achieved among the learners who used this particular curriculum (Rittle-Johnson & Star, 2009; Yang, 2012). In using these different methodologies, LCM was then used as a strategy to introduce the concept of equivalent fractions. It is worth noting that the concept of equivalent fractions is a transformation in fractions, just as the three methods of obtaining the LCM is a form of numerical transformation too. Accordingly, the invariant property that is observed in the equivalent fraction concept is also a form of numerical transformation. The importance of this observation, just like in Pitta-Pantazi and Charalambous (2007), is that the Taiwanese curriculum focuses on particular characteristics/concepts and gets various ideas of the same concept presented for learners' attention and connection.

Although this characteristic was not observed in the CCSM curriculum, the Taiwanese curriculum seemed to devote a separate particular time, when all the specific characteristic concepts and topics that were treated are brought together under one umbrella. For the concept of addition of fractions, this occurred as the last topic that was captured in the 7th grade of the primary school curriculum in Taiwan (Yang, 2012). While this is rare in many curricula, it

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illustrated the importance of reinforcement of students' understanding of addition of fractions. Similarly, it also reinforces the development of connections among topics (or consolidation), and forms of understanding that might be found in the learners mind (Cross, 1999; Herbel-Eisenmann & Wagner, 2005; MOE, 2012, Wu, 2013).

Charalambous et al (2010) presented an analysis of types of fractional constructs involved in the curriculum for the area of fractions addition. The countries involved in the study seemed to model a comparison between Europe (Cyprus), Asia (Taiwan), and the United Kingdom (Ireland). Similar to studies exploring comparison across national borders (Yang, 2012), Taiwan is commonly involved because of its obvious strong performance in international assessment, which mostly focuses on understanding. Charalambous, Delaney, and Mesa (2010) amongst other things investigated in its comparison, the number of pages that considered the topic of fractions addition as a way of comparing the kind of emphasis that the curriculum of the three countries involved, placed on the concept of addition of fractions. This perspective, when considered in the horizontal analysis of a singular textbook would help illustrate the emphasis sought for or priority that the curriculum gave to the topic of fraction (Wu, 2013; Sunday, 2014).

Charalambous, Delaney, and Mesa (2010) found that all three countries involved in the study explored comparable content before dealing with the concept of fractions addition. These contents comprised; (1) Concept of fractions, (2) equivalent fractional concept, (3) fraction simplification, (4) two way conversion of mixed numbers and improper fractions to each other, and (5) comparing and ordering fractions. In relation to the concept of fractions

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addition itself, the content in both Cyprus and Taiwan Curriculum seemed to focus on teaching addition of fractions with similar denominators before teaching addition of fractions with dissimilar denominators. Consequently, addition of fractions was organised, based on the kind of numbers. Ireland curriculum organised addition of fractions concept in a different manner. Hence the contents of similar and dissimilar denominators were dealt with concurrently when addition of fractions concept was presented. However, it is important to note that the teaching of fraction might not necessarily follow the order that was found in the curriculum (Paas, Tuovinen, Tabbers, & Gerven, 2003; Paas, Renkl, & Sweller, 2003).

Behr et al (1983) as cited in Pitta-Pantazi and Charalambous (2007) developed a model that could be used to examine learners understanding of fraction. Although it is possible to adapt these construct in examining the potential construct that learners would learn when a textbook question was dealt with, it should be done with caution. This is because the textbook example cannot explain all possible examples of thought patterns and pictures that the learner developed when s/he encountered the examples in the textbook (Pitta-Pantazi & Charalambous, 2007; Norton & Wilklin, 2011; Morgan, 2016). However, the finding from the study of Charalamous et al (2010) suggests that in treating addition and subtraction of fractions, the examples in the Cyprus and Irish textbooks used a lot of the part-whole construct examples. However, the Taiwanese examples used a combination of part-whole and measurement construct. It would be better to advance a line of research that sought to find how the Taiwanese curriculum would lead to students' achievement in other settings.

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Textbook Analysis: Interaction perspective

There has lately been a great deal of research that advocates a classroom interaction approach in the teaching and learning situations. In fact NCTM and the Ghanaian JHS mathematics syllabus, all advocates a mathematics classroom with rich teacher-student interaction as a way of supporting learning (MOE, 2012; Wu, 2013). In every interaction, effectiveness of the communication between the agents involved is key to successful learning. However, there could be interaction within a classroom but no learning taking place (Sfard & Kieran, 2001; Koole & Elbers, 2014). This section therefore examines literature on texts that has been analysed from the interaction perspective. Such text (written, typed, etc.) are normally discussed with strong focus on the participants (speaker and listener). The speaker could be the author or speaker of the text (in cases of reported speech). The listener is often the reader or the imagined listener. The interaction perspective of classroom interaction was chosen because transcribed classroom interaction would be analysed based on the interaction perspective.

In the opinion of Morgan (2016), it is the human agents who use and communicate the curriculum that should be the first point of call in examining learning within an education system. In examining learning, the interaction of the agents during learning was noted to be the focus. In this case, mathematics teachers, students, and parents alike interact with the curriculum in helping the learner advance in his or her learning attempt. The ability of the curriculum to be useful (properly interact) to the user was identified as a critical factor that must be considered when the curriculum is being designed (Davis, Seah & Bishop, 2009; Wu, 2013). Hence the human agent is given priority before the

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design of the curriculum. This brings up the issue of students, teachers and parents interaction with the curriculum. The curriculum can be used both at the school by the teacher, at home by the parent, and at both places by the student. It has also withstood the test of time as an indispensable aspect of teachers and students in the act of teaching and learning because of its usefulness in all the areas of teaching and learning.

Teachers had been identified to be heavily reliant on their interaction with the curriculum. This was identified to be especially true in the developing countries (Sunday, 2014; O'Keffe & O'Donoghue, 2015;). About 91-100 percent of teachers rely on their interaction with the curriculum for their teaching (Lepik, 2015). Therefore, what the curriculum communicated to the mathematics teacher in their interaction would influence the possible way that s/he would communicate the intended mathematical ideas. This is well known to be a determinant of how much students were likely to learn. In the classroom process, while the act of teachers interaction with students could have compensated for the shortfalls of the curriculum, research had identified that it does not (O'Keffe & O'Donoghue, 2015). From the aforementioned, the implication in the teaching and learning of the concept of addition of fractions is that, the nature of interaction that would be observed in the textbooks would be similar to what would be observed in the classroom.

Davis, Seah, and Bishop (2009) indicated that the interaction between students and the curriculum is the main path through which students learn both correct, incorrect, and misconceived mathematical ideas. They noted that classroom interaction is the point at which mathematics pedagogy, socio-cultural contexts, and mathematical content are interwoven in action. Morgan

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(2016) identified that the interaction between the mathematics teacher, student, and the subject matter of mathematics, is determined by the way the curriculum communicates mathematical content to each of these intersecting agents in the classroom and beyond. The words that were chosen to write the curriculum is also an influential factor in determining the way mathematics teachers and students interact with the subject of mathematics in the classroom (Davis, Bishop & Seah, 2015). These words are imbibed in a language. Hence, the importance of also considering the actual language that is used during the teaching and learning of the concept of addition of fractions in Ghana

Mathematics language analysis could involve the analysis of words and their combinations within the textbooks (O'Keffe & O'Donoghue, 2015). Secondly, it could also be involved in words and their combination within the classroom (classroom interaction). In classroom interactions, it could involve an analysis of the words and combination of words of the teachers, the words and combination of words of the learner, and the words and combination of words of the textbook. Classroom interaction normally investigates the three aforementioned contexts and attempts to find intersection that might have existed in what was spoken in the classroom among these intersecting agents (Sfard & Kieran, 2001; de Freitas & Zolkower, 2010; Koole & Elbers, 2014). The learners' words could involve spoken words within the class interactions and/or written words that could be found in their notebooks or their responses to written test. It could also involve words and their combinations in response to high stake examinations. However, with regard to such examination, it depends on the nature of the questions and their response expectations against response of the students to the expectation (de Freitas & Zolkower, 2010; Koole

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& Elbers, 2014; Morgan, 2016). The major idea that is intended here is that classroom interaction analysis can be from a lot of perspective and very complex.

In the analysis of textbooks, Haratyan (2011) and O'Keffe and O'Donoghue (2015) demonstrated the need for a detailed analysis of words within every mathematical text. They noted that vocabulary is a very crucial element in learners' development of understanding of mathematics concepts. Davis et al (2015) also demonstrated that among selected schools in Ghana, vocabulary was a determinant of the ability of group of students to understand the demand of a mathematical problem itself and subsequently, a determinant of the ability to deal with the problem appropriately. This suggested that if students do not have the vocabulary (be it specialised vocabulary in mathematics or normal English word), it would be a determining factor that prevents students from comprehending a mathematical text or concept within any classroom interaction. Therefore, a student who does not understand the meaning of equivalence (in equivalent fraction), would find it difficult to develop comprehensive understanding of addition of fractions. Also, a student that do not understand the full meaning of LCM, will be confused when a textbook uses the least common multiple in the description of the procedure for addition of fractions.

O'Keffe and O'Donoghue (2015) cited Abedi and Lord (2001) as postulating that students perform a third worse when dealing with word problems (which involves more of English words and descriptions) in mathematics as compared to a comparable mathematical problem that may involve only numerals. From the social constructivist perspective, they

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identified social interaction (generalisation and higher order thinking), and most importantly the connection between thought and words (development of higher order concept), as the aspect that supports learning. The aforementioned words and connection between the words is mainly what comprise word problems. Consequently, the need for comprehensive analysis of words in textbooks, since it may reflect in classroom interactions, and may or may not support understanding of addition of fractions. In learning, most beginning students learned spoken words or their combinations before written words or their combinations. The text language that was learned and used at schooling life was all based on spoken language. This is why the spoken words in classroom interaction during the teaching and learning of addition of fractions, is the focus of this study at the JHS level. This is especially critical in a developing country like Ghana, where language ability is very low (Davis et al., 2015).

However, when learners get older and develop the ability to translate written language into spoken language, the order could change. Hence, learners develop the ability to interpret written language into spoken language in the learning processes (Wagner, 2007; Morgan, 2016). This is normally the situation when the learners have developed the capability to initiate learning on their own. When learners read a text, there are thoughts/reasoning/words that are automatically encoded with meanings and are registered in the mind. Accordingly, in the introduction of new word and its associated connotations in the learning of mathematics, it needs to be done carefully in order not to inhibit the ideology of the old meanings that could be found within the current classroom learning discussions. More usage of old associated words and less

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integration of the new word in the learning context can put the new word in the shadows, such that there would be effective learning and associated connotation development.

A typical example in Ghana is Davis et al., (2015). In the case of Davis et al., (2015), the learners' inability to understand the text was not just about the language but about the specific words that the students were encountering during the reading of the text. In fact their study identified specific words that prevented the students from gaining understanding of the requirement of the mathematical problem. Subsequently, in some cases, when the meaning of those words was identified by some students in the focus group, the students were able to easily deal with the problem. In other situation, Davis et al., (2015) observed that some students in a focus group provided word interpretative clues to understand an illustrated mathematical problem the group could not previously solve. This action still did not directly lead to the direct ability to solve the mathematical problem. The evidence suggested that, although the students were able to obtain the meaning, fitting such a meaning within the context of the demands of the task did not seem to foster comprehension (de Freitas & Zolkower, 2010; Jitendra et al., 2010; Haratyan, 2011).

Davis et al., (2015) is a typical illustration of how meaning from new words could not foster understanding, if it were found in a mathematical textbook used in the teaching and learning of the concept of addition of fractions. However, words are found in sentences, and sentences are found in paragraphs. Thus, for a holistic perspective, sentence analysis is better.

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Analysis of Text using Systemic Functional Linguistic (SFL)

The relationship between sentences, words, the meaning and role that they represent in language is what is called Systemic Function Linguistic (SFL). Its subsequent analysis is called SFL analysis. SFL serves as a basis for the investigation of roles of words in developing textual meaning, or investigating the general meaning a sentence, phrase et cetera, portrays in a complete text. SFL is catching up in mathematics education as a tool in analysing writing or text in mathematics education (deFreitas & Zolkower, 2010; O'keeffe & O'Donoghue, 2015). The wide-ranging situation that surrounds an event is the context of that event. The interpretation of the context is constructed by individuals as they participate in practices (e.g. learning of addition of fractions) which contribute to the production of the texts. Contextual meanings are therefore properly investigated in textual analysis (Haratyan, 2011).

Social context is the place where SFL begins with specific lexico-grammatical choices, and it is constructed with stimulus from social and cultural context. Haratyan (2011) and Ebbelind and Segerby (2015) noted that meanings extracted from SFL are based on a structure that connects clauses or words in a sentence. Just as SFL may find consistencies in the structure of clauses and words in a sentence, there could also be conflicts or inconsistencies between clauses or words in a sentence that may be found in the structure of the sentences. These consistencies or inconsistencies may be extended in the interactions of social, physical, cognitive, cultural, interpersonal and situational context. A brief description of the SFL would follow in the next paragraphs but details would be in chapter three and chapter four. In chapter four, examples from the teaching and learning of addition of fractions will be presented.

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Three functions in analysing text and spoken words with SFL was identified; ideational function, interpersonal function, and textual function. The ideational function looks at a selected text or words, and attempts to identify how the author of spoken words expected the listener or reader to view the world (in this case, the subject matter of addition of fractions). In the case of this study, it investigates how the author expected the listener or reader to view his or her experience of the world when in contact with the concept of addition of fractions. Therefore, in analysing texts or words that are spoken during the teaching and learning of mathematics (e.g. addition of fractions), the sentences/clauses or words are categorised as follows: mental, material and relational (O'Keffe & O'Donoghue, 2015). These categories are described as processes. A material categorisation of a text in ideational function portrays the text or words of an actor carrying out an activity on another actor (person or object). The major characteristic of a sentence with a material categorisation was branded with action verb that shows a 'doing' or 'happening'. Mental categorisation of a mathematical text involved the spoken words or text depicting a process that goes on within a person i.e., thinking or a feeling. A relational categorisation suggested a comparison between some properties. These properties could be between mathematical attributes or between objects and a mathematical attribute. The main characteristics is that there should be a comparison between a thing, object or an act in the words, clauses or sentences (Ebbelind & Segerby, 2015).

However, a point worth noting is that this categorisation of ideational function in mathematics text or spoken words could be found in combinations within a text. Consequently, behavioural categorisation shared the combined

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presence of mental categorisation and material categorisation in one common text or spoken words under consideration. Verbal categorisation shared the combined presence of mental categorisation and relational categorisation within the same text or spoken words. And finally, existential categorisation referred to the presence of an occurrence (Haratyan, 2011).

O'Keffe and O'Donoghue (2015) found in their study that it was appropriate to do a sentence by sentence analysis of the various mathematical textbooks they compared. They compared three Irish Textbook series for the Junior Cycle Mathematics education level. Each textbook series consisted of two textbooks. These were subjected to all forms of textbooks analysis as prescribed by SFL (Ebbelind & Segerby, 2015). Findings from O'Keffe and O'Donoghue (2015) illustrated that there was little evidence of the mental category (process). This implied that there were relatively small number of sentences demonstrating its characteristics. However, the verbal category (process) was more visible within the sentences in the textbooks than the mental category (process). This was observed to be directly contrary to literature in the original Hallidays (1973) ideational analysis which suggested that among the six categorisations (which they described as processes), most of the sentences in the textbooks were categorised among the first three categories; material, mental, and relational. There were very little number of sentences with characteristics exhibited for the remaining three categories; verbal, existential, and behavioural. One of the portrayed natures for the material process was that it attempts to connect the learner to the actions and processes in the subject being studied. Whether the action is in the present or past, it is an integral part

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of this connecting to the subject matter (de Freitas & Zolkower, 2010; Morgan, 2016).

Specialised mathematical symbols, equations, diagrams, and drawings are often used during the teaching and learning of addition of fractions. In SFL, these are described as mathematical objects. Objects derived from mathematical knowledge (mostly in symbolic form) objects include factors, lengths, etc., could also be described as derived objects. Representational objects consisted of graphs and diagrams and tables (Herbel-Eisenmann & Wagner, 2005; Morgan, 2016). These representational objects were identified as communicative agents in the acts of learning interactions in mathematics (Singer, 2007; Morgan, 2016). Consequently, representation of the concept of addition of fractions is not an exception. The tenses that were used to engage representational objects also do count in helping learners engage positively with the concept of addition of fractions. Two out of the three identified textbooks studied by O'keeffe and O'donoghue (2015), had a lot of basic or derived objects under the material category (processes) of sentences. Representational objects were often associated with sentences categorised as relational. Equal sign was mostly used for basic or derived objects in sentences that were categorised as relational. This was because equal signs were observed to serve three purposes; define, shows/represents, and relate. Equal signs were used in mathematical statements that 'defined' specific equations or concepts that were used in mathematics for learners to get a holistic underpinning of the concept (e.g. addition of fractions) being taught. Equal sign 'related' by showing the relationship between items or quantities. Finally equal sign 'showed' by showing the meaning of letters in any specific scenario. Consequently, the

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presence of the equal sign exhibited three roles which could be confusing to students at times (Prediger, 2008). Linking the mathematical ideas in the processes to the sentence processes is the human agent itself. Thus, analysis of SFL will not be complete without an analysis of the human agent (interpersonal function).

In relation to the interpersonal function in a text or spoken words, O'Keffe and O'Donoghue (2015) showed that in the sentences, the use of the pronoun, 'we,' was most prevalent. Although most authors who used 'we,' wanted to involve their audience, it was important that most of them would also identify their most likely readers and whom the textbooks (and hence the author) targeted. Thus, informing the appropriateness of using the pronoun, 'we.' Academic mathematical writing that was intended for a mathematics teacher should be different from academic mathematical writing that was intended for a learner. Involvement of the teacher in the academic community seemed to consider the teacher as an expert academician. Hence the use of 'we' would be satisfactory for the teacher who is considered an expert. The same cannot be said for the learner who was very likely to be an assumed intended expert (de Freitas & Zolkower, 2010; Vogel & Huth, 2010). This is because, students may now be developing a familiarity of a particular field or mathematical concepts which would be considered quite far away from being a mathematical authority that is associated with the expert (Vogel & Huth, 2010; Morgan, 2016). The second most prevalent pronoun that was identified by O'Keffe and O'Donoghue (2015) was 'you'. The use of the pronoun 'you' was supposed to assume the involvement of the reader directly in the art of mathematics (e.g. addition of fractions). The combination of the pronoun 'we' and 'you' seemed to suggest

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that the latter was intended to get the reader closer to the seemed interested participant. It also seemed like an attempt to get the learner to be responsible for his/her own learning of the concept of addition of fractions i.e., authors attempt to share authoritative power of an expert in mathematics (Ertmer & Newby, 2013; Herbel-Eisenmann & Wagner, 2005; Wagner, 2007).

Delving deeper in the exploration of interpersonal function, Morgan (2016) focused on human agencies in the practice of mathematics itself. She focused on how the students relate or interact with mathematics as a subject matter and whether students see themselves as active future participants in the practice of mathematics. Morgan identified a wider perspective in considering mathematics classroom interactions that are independent of human involvement in mathematical practice (i.e., alienation);

(a) vocabulary and syntax – the words and the ways of combining them

(b) Visual mediators – non-verbal modes of communication, including algebraic notation, graphs, diagrams, etc.

(c) routines – repeated patterns of discursive actions (e.g. steps in solving a type of problem)

(d) endorsed narratives – those of sequences of utterances that are taken to be true.

Perspective (a) and (c) have already been described in SFL. Therefore, focus is on explaining (b) and (d). One of the most significant thing about the alienation discuss in Morgan (2016) research was her attempt to show how visual mediators (e.g. fraction tiles in the case of fractions) effected or manifested alienation in learning situations. Compound (combination of two or more forms of representation) and varied modes of representation in a

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mathematical text or classroom interactions does not always support learning and understanding in classrooms if alienation is not reduced to the barest minimum. Alienation here includes the use of specialised form of mathematical representations as algebra. Hence, the use of specialised mathematical notations as alternatives to pictorial representations is a form of alienation. In addition, the description of representation of concepts (see Figure 7), and portraying the representation as mathematical elements without human involvement is considered alienation. These same representations could be used in a way that would show the presence of human agency in mathematics.

Alienation can also be described in relation to the endorsed narratives (Sfard, 2008). Endorsed narrative describes what is considered an appropriate answer to a mathematical text question or questions in a classroom interaction. In other words, the expected answer to the question (Koole & Elbers, 2014). This brings to fore the fact that questions in mathematics could have a requirement on what the student is expected to do. Where those restrictions may occur in the text question or questions in a classroom interaction, illustrates the authors' perspective of what is considered mathematical and its associated level of human involvement (Sfard & Kieran, 2001; Morgan, 2016).

Morgan noted that alienation is a way of describing a mathematical phenomenon 'in an impersonal way, as if they were occurring on their own, and using those reified mathematical objects in the subsequent mathematical processes.' Attributing an action to a mathematical object or to representational objects (e.g. tables, graphs, or diagrams) rather than to human beings, results to alienation. Abstraction of mathematical knowledge is an activity carried out by a human being. However, abstraction is normally described as an activity that

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occurs in isolation. As this goes on in the description of a mathematical process, a new object (normally at the end of a mathematical problem) is created that was not there. The mathematical objects are viewed as isolated knowledge without human activity. Alienation is therefore considered a process and not a one tie act. Morgan (2016) explains with Figure 7 and Figure 8.

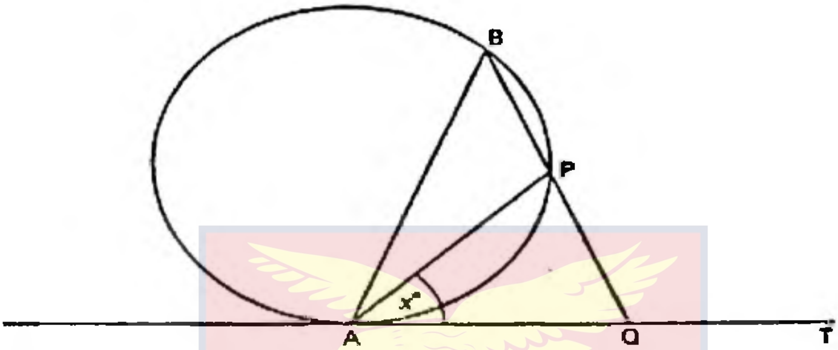

Figure 7 shows a circle with chord AB and a tangent AT touching the circle at A. The bisector of angle BAT meets the circle at P and BP produced meets AT at Q. Angle PAQ is x°.

Giving a reason in each case, write down, in terms of x°, the size of

(i) angle ABP,
(ii) angle APQ,
(iii) angle AQP.

Source: Morgan (2016)

Figure 7: Representation of alienation in questions

In Figure 6, the circle, tangent, and chord are all objects in the sentence. However, the tangent is performing an action of 'touching'. In the SFL framework, this would have been considered a desired characteristic of 'performing an action' (see O'keeffe & O'Donoghue, 2015). However in the alienation concept, these objects (cord, tangent, and chord) are atemporal (without respect to time or process) objects. In reality, how did the tangent and the chord get to its position? How did the bisector appear and who did the bisection? If BP and P 'produced' something, how did this production of something (i.e., the result of a production action) occur? The production process

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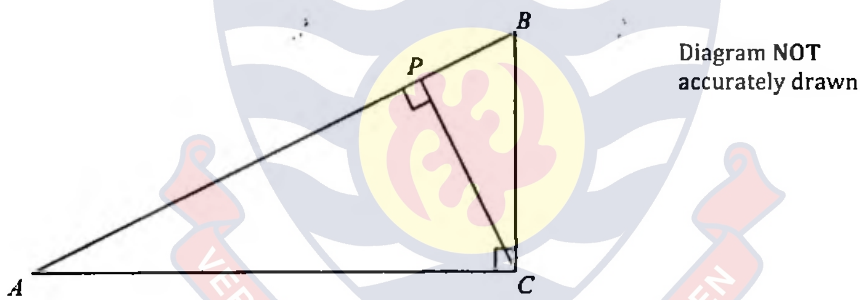
is a mathematical process (de Freitas & Zolkower, 2010; Sunday, 2014; Morgan, 2016). Hence we have objects in classroom interactions appearing out of nowhere, performing actions that should have been performed by somebody or something, engaging into a process which was not specified, and finally producing another mathematical object. This is a high degree of alienation from this text. However, Morgan (2016) suggested that the same text could be improved as, "The diagram shows a right-angled triangle ABC with the right angle formed at C. A perpendicular dropped from C meets AB at P"

The only difference the above text exhibited, that is different from (Figure 8 is that there is a mathematical process "dropped," that resulted in an object. However, the question still harboured an element of alienation and could still be improved.



In the diagram,

ABC is a triangle,
angle *ACB* = 90°,
P lies on the line *AB*,
CP is perpendicular to *AB*.

Prove that the angles of triangle *APC* are the same as the angles of triangle *CPB*.

Source: Morgan (2016)

Figure 8: Illustration of a case of alienation in mathematics questions

Morgan (2016) also illustrated ways of improving human involvement in mathematical statements. Since alienation involved obscurity, reduction of

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obscurity in mathematical object and processes identified, is a way of reducing alienation. There is the need to identify the presence of a human agent in a mathematical process. This would involve the introduction of a human activity in the presence of people. This can be in the form of a noun, pronoun, suitable proper name, or a role descriptor (Mason, Engineer, etc.). First and second person pronoun is mostly avoided. de Freitas and Zolkower (2010), in their analysis of the use of pronouns in the Connected Mathematic Project curriculum (Thinking with Mathematical Models - TMM) observed that every attempt was made to ensure that there was no sentence in the textbooks that used the first person pronoun. However, second person pronoun appeared two hundred and sixty-three times in the TMM textbook. This showed the emphasis they wanted to put on the reader as involved in performing a mathematical process (Herbel-Eisenmann & Wagner, 2005; Ebbelind & Segerby, 2015). Sentences that involved the second person pronoun, 'you' and an action verb together appeared 165 times in the textbook.

The extract question in Figure 9 is a practical example of the apparent effort to reduce alienation in mathematics teaching and learning situation. There is the presence of a proper name at the start of the sentence. In the process, there was a statement that indicated what exactly the human agent was supposed to do. This is equivalent to the mathematical process that was carried out in the diagram, although it was not stated in the form of specialised mathematical vocabulary (Vogel & Huth, 2010). However, the major thing worth identifying is the fact that the human person was identified, and how he related to the mathematical formula that was being used was also stated (i.e., his/her previous action with the formula). The mathematical action or process that needs to be executed before the answer is reached, is clearly stated. It also

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tells the reader what the answer was going to be used for or its relevance in the mathematical process the reader was encountering (Zolkower and de Freitas, 2010; Morgan, 2016). Illustration is in Figure 9.

$$F = \frac{ab}{a - b}$$

Imran uses this formula to calculate the value of F.

Imran estimates the value of F without using a calculator.

$a = 49.8$ and $b = 30.6$.

(a) (i) Write down approximate values for a and for b that Imran could use to estimate the value of F.

(ii) Work out the estimate for the value of F that these approximations give.

(iii) Use your calculator to work out the accurate value for F.
Use $a = 49.8$ and $b = 30.6$.
Write down all the figures on your calculator display.

Source: Morgan, (2016)

Figure 9: Effort to remove alienation in a question

Morgans' (2016) analysis of GCE questions from 1980 to 2011 plotted the proportion of human actor, objects as an actor, or total obscure actors in examination questions in United Kingdom. The obscurity identified here were mostly in the use of non-finite verb form or passive voice. Findings suggested that the human actors increased over the years generally. However, the proportion of objects as actors and obscure agents in the mathematical questions were observed to be generally stagnant in the trend of years studied. Similar observation was made for mathematics questions that were noted to be filled with specialised mathematical terms or words. This was observed to be possibly as a result of an increase in mathematics questions that were structured in non-mathematical form.

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Summary of literature

Literature reviewed included general concept of RPK and how to integrate it in teaching and learning. Literature in understanding of addition of fraction have also revealed structural similarities to RPK and has served as criteria for validation of RPK. Literature related to classroom interaction has shown that sentence analysis will eventually lead to results that would be obtained in other forms of analysis.

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CHAPTER THREE

RESEARCH METHODS

This chapter provides a brief description of the selected aspects of the methodology. It gives insight into the research design (mixed method) that was used for the study and the reason why it was selected. It explains how the population of the study was delineated and how the selected sampling procedure was adapted to obtain the given sample size for the study. The instrument was described based on how it was developed to obtain information relevant to the goals of the study. Issues of validity and reliability are also described. These were finally followed by the detailed and specific method(s) of data analysis that were used in this study.

Research Design

The concurrent nested design was used for this study. Concurrent nested research design (Harwell, 2011) adopts the pragmatic stance in research. The pragmatists, exploit the strength of both quantitative and qualitative data in the research designs. The combination of both aforementioned data in the research designs, is in the light of also reducing their respective weakness as much as possible. The technique of minimizing weaknesses often involves not extending conclusive findings at stages in the research, where each (quantitative data or qualitative data) design does not exhibit its strength (Cresswell, Plano-Clark, 2011). The concurrent nested design, as in this study, adapted the qualitative data perspective to determine the categories of understanding of the concept of addition of fractions. Subsequently, the study adapted the quantitative data perspective to determine the differences per the various levels of understanding in the diverse school categories.

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Concurrent nested research design is a mixed method design that required the users to collect quantitative and qualitative data. Since users of qualitive studies normally asked broad questions and users of quantitative studies normally asked narrow and sharply focused questions, the kind of questions that are used in concurrent nested research design normally reflected the ending point of the study. Therefore, in a concurrent nested research design, when the question is specific, then the data is analysed qualitative, followed by quantitative analysis. The situation is vice versa when the research question is stated in the broad form. These were the views of Harwell (2011) per his exploration of mixed method studies, but he was emphatic to note that it is not a generally defined norm.

Although there were other aspects of this study, understanding of addition of fractions was the main purpose in this study. Accordingly, the selection of the appropriate research design was therefore based on the aspect of the study that focused on the understanding of the concept of addition of fractions. Concurrent nested research design was deemed appropriate for this study because in the exploration of understanding of the concept of addition of fractions, the study adapted three ideologies from Wu (2013) and Vig et al (2014) (see Data Analysis and Chapter Four: NU, PU_S, PU_A, and FU). This was then used to explore the general ordinal levels of understanding of the concept of addition of fractions across the school types. Eventually, one data was collected and used for the dual purpose of categorization and determination of level of understanding over the three categories of schools involved in the study. Similar process was followed for the understanding of the specific RPK

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for addition of fractions. Consequently, the later was used to determine various differences in the level of understanding of the RPK per the school categories.

Population

The finding in most research is generalised towards a group of people. The group is called the population. In the case of this study, the population constitutes JHS two students and their mathematics teachers in two districts of the Central region of Ghana. The two districts are the Cape Coast metropolis (see paragraph one, p. 127) and the Komenda-Edna-Eguafo-Abirem (KEEA).

Central Region has twenty (20) educational administrative districts supervising the basic schools in their respective areas demarcated under their jurisdiction. Although there are seventeen government administrative districts and their associated directors, it is important to recognize that the demarcations for the educational districts are different from that for the political voting constituencies (GSS, 2014).

The selection of the districts in the Central Region was strategic for this study. Central region in Ghana is located close to the coast of the Gulf of Guinea and shares boundaries with Western, Ashanti, Eastern, and Greater Accra regions of Ghana. It covers 4.1% of the total land area in the country. It is one of the places that the colonial masters started most of their schools. For this reason, education expanded very fast within the region. Consequently, education has also diffused deep into the rural parts of the region. At all levels of education, there are schools that attract students from the different parts of the country. The two districts had both rural and urban communities in them. In the rural communities, farming was noted as the major source of livelihood for communities within the districts. Ghana is very well known to be a country

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with agriculture as the backbone of the economy. Cape Coast has more of professionals in their various demarcated fields of specialisation, mostly in education. Categorisation of schools into rural and urban communities was according to Ghana Statistical Services definition. This was because schools in Ghana are cited to serve specific communities and its possible environs. Communities with population less than five thousand people (5000) were classified as rural and 5000 or more were classified as urban (Ghana Statistical Services, 2014).

The population consisted of mathematics teachers and their students in public Junior High Schools within the selected districts. The distribution of public Junior High Schools within the districts as at 2016 is presented in Table 6.

Table 6: Number of JHS and number of JHS2 students per district in the study

District	Number of Schools	Number of JHS2 students
Cape Coast Metropolis	62	3573
KEEA	67	3037
Total	129	6610

GES 2014 data and EMIS 2014 data

Sample and Sampling Procedure

Sampling is the single act in research that determines the level of generalisation that could be achieved in a research. For a research finding to make the impact it is designed to make, the sampling is what would determine how definite the finding would be, and its possibility of being applied elsewhere. Information rich sample was attained after consideration had been given to dimensions as rigor of finding, statistical generalisation, time for completion of the study, and budgetary allocations (Cohen, Manion & Morrison, 2007).

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Multi-stage sampling, a probability sampling strategy was adopted as the sampling strategy that most suits the study. This strategy was adopted because the study considered two (type of community and achievement of schools) stages in the sampling. Multi-stage sampling strategy allowed all schools within the various stages of the population to have an equal chance of being selected into the study (Cohen. et al., 2007). The type of communities in which the schools were located was an important factor that could result in varied outcomes in the study. This is because, the more urban the community, the more resources that could possibly be found in the schools. During sampling, performance of the schools in the Basic Education Certificate Examination (BECE) was also considered an important factor. Therefore, the schools were selected by stratified random sampling. The type of community (urban or rural) served as the strata. Hence in each selected school within a stratum, JHS 2 students and their mathematics teachers were purposively sampled into the study.

Therefore, adoption of this sampling strategy allowed an insightful kind of method for evaluating the understanding of addition of fractions among Ghanaian students and their mathematics teachers, at the Junior High School level. Hence it was considered prudent to select the schools in a manner that would reflect the proportion of schools found in the rural and urban sectors of the Governments' educational administrative unit (District education offices) that were involved in the study (Ghana Statistical Services, 2014).

Every district education office in Ghana has an ordering of Junior High Schools within the district, according to the schools' performance in the BECE. However, this performance involved the schools' performance in all examinable

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subjects studied at the Junior High School level. In 2013, the structure of such result analysis was improved to involve number of pupils that obtained specific grades per subject. Consequently, the study used data of students achievement in BECE mathematics alone to generate an achievement index for all schools in each district involved. The formula for the achievement index obtained from that used by GES $\{Index = \sum_{i=1}^{a}(Gd_i * n_i)/\sum_{i=1}^{a}(n_i)$, where Gd_i is Grade i, n_i is number of Grade i, and $1 \leq i \leq a$ $\}$. In each district, the achievement index was used to order the schools in each district and subsequently sorted into percentiles. For the ordered schools in each district they were divided into three (i.e., 33rd, 66th, and 99th percentile). Schools within the first 33rd percentile were categorised as high achieving schools while those within the second 33rd percentile were categorised as the average achieving schools. The remaining schools in each district were categorised as the low achieving schools.

Within the categories of high, average, and low achieving schools (first stage of sampling), the schools were further categorised into rural and urban schools (second stage of sampling). A number representing 10% of the population of schools from each district, per the type of community, was obtained. Therefore, ten percent of all schools within a community, per a level of achievement was randomly selected with the table of random numbers as a representative sample (see Table 7). Students in Junior High School two were automatic candidates for the study and their mathematics teachers were automatically involved in the study. The major reason for selecting JHS 2 is because that is the class (stage) where the last topic on addition of fractions is located per the JHS syllabus. In the textbook, addition of fractions is found

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under rational number. Most mathematics teachers also teach addition of fractions at that point (MOE, 2012; 2012b).

In the sampling plan, the targeted sample is 661 students and 13 mathematics teachers. This constitutes 10% of the population of the students (see Table 6). However, the sample used in the study was 616 students and 17 mathematics teachers. The number of mathematics teachers increased because of rounding up numbers (since schools cannot be fractional numbers) after the schools were selected into their various categories (see Table 7). In one school, it was observed that there were two JHS two mathematics teachers in one class. This eventually increased the number of mathematics teachers to 17.

Table 7: Composition of selected schools according to achieving and community type

School achievement	Community type		
	Urban	Rural	Total
High	2	3	5
Average	2	3	5
Low	3	3	6
Total	7	9	16

Source: Field data (2017)

Data Collection Instruments

The core idea examined in this study was students and their mathematics teachers' understanding of the concept of addition of fractions. Phase one of this study included the understanding of the specific RPK for teaching and learning the concept of addition of fractions. The instrument and scoring rubric for phase one were adapted and adopted respectively. The second phase of this study involved the exploration of sentence characteristics in classroom interaction during the teaching and learning of the concept of addition of fractions. Notes were developed to guide the interpretation of elements in classroom interaction. The final phase of this study explored understanding of addition of fractions.

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An instrument was also developed for examining the learners understanding of addition of fractions after instruction. Its' associated scoring rubric was also developed.

Instrument for RPK

The understanding of the specific RPK involved examining students' ability in unit coordination (see page 63-67). The instrument and the scoring rubric were adapted and adopted respectively from "the written instrument for assessing students' unit coordination structures", designed by Norton et al., (2015). The instrument (see Appendix A) exploring the specific RPK had questions that covered the basic measurement skills and structure of understanding that would be necessary for teaching and learning of the concept of addition of fractions of any type (see Figure 10). Consequently, the instrument was accepted to be in alignment with the requisite RPK for the Ghanaian curriculum.

Use the following information to answer the questions about the bars shown above:

4. **Assume** that the **Medium Purple Bar** fits into the **Long Orange Bar** exactly 2 times.

   Also assume that the **Small Green Bar** fits into the **Medium Purple Bar** exactly 6 times.

   Use **all the information provided for question 4** to figure out how many times the **Small Green Bar** would fit into the **Long Orange Bar**?

   answer

   Use the space below to **draw a drawing** that will **explain how** you got your answer to question 4. Explain your drawing

Figure 10: Sample question in RPK instrument

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All of the questions involved the respondents demonstrating the ability to reason with a maximum of three levels of measurement simultaneously or otherwise (unit coordination stages). The questions were in increasing order of difficulty. In other words, it involved reasoning with structures in increasingly complicated levels (see Appendix A; see Figure 10). The RPK questions were constituted in four increasing order of complications and these were mostly obtained from earlier researchers in the field (Kosko & Singh, 2018; see Fgure 10).

In the first level of complexity, question three explored schemes or structure of reasoning (skills) that can be supported with visual illustration. Consequently, the solution to the problem also could be obtained by respondents if they followed instruction and manipulated the visual illustrations. Secondly, the fourth question also explored schemes or structures of reasoning but the visual illustrations could not be used to obtain the results since they were not drawn to scale. Questions three and four therefore required using a small measuring scale, to determine a larger measuring scale. This implied that the smaller measuring scale was multiplied a number of times to obtain the bigger measuring scale. Questions four to seven also do not have their visual illustrations drawn to scale. Thirdly, questions five and six required respondents to use reverse iteration (develop a reverse measurement scale) in order to be able to solve the question successfully. Thus, the full length of a bigger measurement scale (level) is given with a relation requiring respondents to divide in order to be able to obtain the smaller scale. This represented another order of difficulty. So far, all questions one to six resulted in whole number answers, although their underlying schemes were fractions. Fifthly, question

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seven is only different from questions five and six with respect to the answer (in improper fractional form). This represented a different order of difficulty (Paas, Renkl & Sweller, 2003; Norton. et al., 2015). Refer to Appendix A for the questions on RPK instrument.

In the scoring rubric, description of the various levels of demonstrated understanding of the specific RPK would concur with descriptions of the various levels/stages of unit coordination ability demonstrated by the respondents (students and their mathematics teachers). Respondents who demonstrated stage one, stage two, and stage three of unit coordination ability corresponds to respondents who demonstrated low, average, and high understanding of the RPK for addition of fractions. These categorisations were based on the respondents demonstrating the ability to reason logically about the relations given in the questions. Increasing levels of understanding of the RPK, demonstrated increasing levels of reasoning in the associated mathematics topics (fraction, whole numbers, whole number addition). Details of the characteristics of the aforementioned levels of understanding of the specific RPK would be evident chapter (also see Appendix B for a summary).

Interpretation notes for sentence analysis

Systemic Functional Linguistics (SFL) was adopted as the theoretical perspective for analysing interactions within the classroom. An interpretation note was developed to help identify the characteristic process in the sentence or clause (see Appendix C). The development of the interpretation note involved amalgamating characteristic of the six sentence processes from different literature i.e., Haratyan (2011), O'keeffe and O'Donoghue (2015) and Ebbelind and Segerby (2015). The interpretation note was used during analysis of

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classroom interaction. In formulating the interpretation note, for the mental process of the ideational function, the basic nature of sentences that exudes the distinguishing features of mental processes was briefly described. Similar description was done for material, verbal, existential, behavioural, and relational processes in the rubric. Detailed presentation of the characteristics of the sentences in the interpretation notes, that defines aforementioned processes, could be observed in chapter four.

Instrument for addition of fractions

In examining students' understanding of the concept of addition of fractions, an assessment instrument was developed (see Figure 11) . Since there were different types of addition of fractions that needed to be learnt by Junior High School students, the researcher considered only the kind of addition of fractions cases that would have been learnt by Junior High School students in Ghana from the Ghanaian syllabus and textbooks (MOE, 2012a; MOE, 2012b). An assessment instrument was developed with similar examples as those in the JHS2 textbook. In the various questions that were set, enough space was provided to allow the students explain each step they took and why such a procedure was important in obtaining the true result of the addition problem. Students were encouraged to use representational tools to describe their ideas. This allowed students to have multiple ways of expressing themselves (Wu, 2014; Vig et al., 2014). Hence for most questions, specific reference was made to the use of any semi-concrete illustrations and concrete explanation towards a particular ideology that was expected to be found in the examination of a sub-construct (Portides, 2008).

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The fractions in the addition of fractions instruments were of the following types, with denominators less than 10, and with different denominators; (a) unit fractions, (b) non-unit fractions or proper fraction, and (c) fractions greater than one. These types of fractions were combined in different forms as addition questions involving only two fractional values. This provided a list of possible types of questions to be used in the instruments. However, only four fractions addition questions, categorised in three types (as described in this paragraph) were used to assess addition of fractions (see Figure 11). These were carefully selected as necessary type of questions for comprehensive investigation of underlying concepts in understanding of addition of fractions (Pitta-Pantazi and Charalambous, 2002: Stronge, Ward & Grant, 2011).

Instruction: Follow the instruction below for the question below;

a. use drawing to show/represent the following addition questions. Use drawing to also show the answer. Explain how your initial drawing results in the drawing of the answer.

b. Then solve the addition questions. Explain your procedures and answers in relation to the drawing.

4. $\frac{3}{2} + \frac{5}{4}$

Figure 11: A sample addition of fractions question

Scoring rubric was also developed for scoring the addition of fractions questions. In demonstrating understanding, most researchers used visual models and description of respondents to explore the ideas respondents had in their minds. However, this study adapted the idea of Wu (2013) and Vig et al (2014). They took a step further to note that when exploring understanding of the

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concept of addition of fractions, there is the need to note how the visual clues represent the various steps or underlying ideas in the mathematical operations. The scoring rubric had the various characteristics of responses that characterised no understanding (NU), partial understanding (PU), and full understanding (FU) of the concept of addition of fractions. Partial understanding of addition of fractions included partial understanding with separated thought (PU_S) and partial understanding with associated thoughts (PU_A). The scoring rubric could be found in Appendix E. However, detailed description of responses that constituted NU, PU_S, PU_A, FU could be observed in chapter four.

Pre-Test of Instruments

The instruments were pre-tested ensure validity and reliability in their usage. Modifications in the instruments and the process of administration of the instruments were carried out in order to ensure valid and reliable data. Firstly, there was revision in some of the words used in the items for the RPK. This was done because some of the students did not seem to understanding its meaning. Secondly, in the initial stages, all instruments for the RPK were given out to the students at once. However, it was observed that the students were referring to their previous solutions to earlier items in order to answer the next question. The order of administration of the items was therefore changed. One item in the test was administered and collected before the next. This was to ensure that students demonstrated understanding that was solely constructed from their possessed knowledge and not from previous activities. The later procedure was also used in the administration of instrument for the concept of addition of fractions. Finally, the order of items in the instrument for addition of fractions was changed. The first arrangement was based on difficulty of items

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as proposed by theory (Pass et al., 2003; Vig et al., 2014). However, this was changed to reflect the order in which the item type was presented in the JHS two mathematics textbook (GES, 2012b). The change was to ensure that the order reflects contexts in Ghanaian schools.

Validity of research instruments

To ensure validity, my supervisors, studied the instrument and ensured the items examined the understanding of the specific RPK for addition of fractions. It was also evident that previous studies in the area used similar items (Steffe, 1993; Norton et al., 2015; Kosko & Singh, 2018;). Similar process was used for the instrument exploring the concept of addition of fractions

The instrument that explored the understanding of the specific RPK was given to two students to respond. It was observed that some of the words in the adapted instrument had to be changed to suit the Ghanaian context. This was because, both students could not explain the meaning of some words. Subsequently, 90 students in two classrooms of the randomly selected JHS, participated in the pilot study and the data analysed. It was observed that each question was completed in an average of 13 minutes. Accordingly the whole instrument for the specific RPK was completed in an average of 91 minutes.

To ensure validity of the instrument for the concept of addition of fractions, the fractions were selected directly from the JHS2 mathematics textbook that is recommended by the Ghana Education Service (MOE, 2012). This also ensured that the questions were matched to the ability of students at JHS2. Also, senior colleagues studied the instrument and considered it valid for exploring understanding of addition of fractions. After studying the instruments, some mathematics teachers in the schools also considered the instrument

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appropriate for examining the understanding of the concept of addition of fractions. It was observed that the students used an average of 18 minutes to answer each question. Accordingly, the instrument for the concept of addition of fractions was completed in an average of 72 minutes. This enabled planning for the time allotted in actual data collection. The scoring rubric for both instruments (RPK and addition of fractions) were critiqued by the supervisors and approved.

Reliability of research instruments

The evidence to support the reproducibility of the responses obtained in this study is very important. This is to help ensure trust in the finding of the study (Bolarinwa, 2015). Evidence of reliability was therefore computed. Cronbach alpha reliability coefficient was obtained for both (concept of RPK and concept of addition of fractions) instruments used in the study. Price et al (2015) indicated that a reliability coefficient of 0.75 can be described as a highly reliable instrument. On the contrary, reliability coefficient of 0.5 is considered low reliability. Therefore, a reliability coefficient of 0.693 for the instrument on specific RPK is considered averagely reliable. Cronbach alpha for the instrument on the concept of addition of fractions was 0.792. The aforementioned represented a highly reliable instrument (Bolarinwa, 2015).

Data Collection Procedures

Permission letter was obtained from the district education offices involved in the study (see Appendix F). Subsequently, evidence from the education office showed that two districts out of the three selected districts were actually considered one. In each school, the first visit was used to introduce the staff to the research intended. In any school (whole staff), the researcher

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discussed the goals of the study and how procedures would be organized (in collaboration with the school) to avoid distraction of academic activities in the school and ensure non-disclosure of respondents (anonymous) in the study. The students in Junior High School 2 were informed of the purpose of the research as well. Each school was given between two days and one week to prepare themselves for the research.

Before the assessment started, the students were given the first instrument (RPK) and encouraged to read, and explain aloud, what the question required of them. This was done in the presence of the mathematics teachers. The researcher called the attention of the students to the word 'assume' and its implication (not drawn to scale) on the associated visual illustrations. Six hundred and fifty students' instruments were each administered for the RPK and the concept of addition of fractions respectively. Among the students, the number of only RPK instruments that were returned and used in the study was 616 (94.7% return rate). However, 543 students attempted both the RPK and the concept of addition of fractions questions. Across both instruments used, the returned and used instruments constituted 83.5% return rate, per the sampled students. Twenty-one mathematics teachers' instruments were each given out for the RPK and the concept of addition of fractions. The number of instruments that were retried and used for the study were 17 and 16 for the RPK and the concept of addition of fractions instrument, respectively. These constituted 80.95% and 76.19% return rate for RPK and concept of addition of fractions instruments, respectively.

Students were informed that they were allowed fifteen (15) minutes for each question. They would be informed when ten minutes had elapsed and the

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time for collection. The iteration continued till the end of the whole assessment. The whole test administration took 75 minutes per school. This was the manner in which instruments for the concept of the specific RPK, was administered for all schools involved in the study. However, for the mathematics teachers, all questions were given to them at once. The instruments given to the teachers were collected after the students' assessment had been completed. It is worthy to note that most mathematics teachers did not return the instrument the same day because they claimed they were involved in other official engagements. Similar process was used for administration of instrument for the concept of addition of fractions. Time that was allowed for each question was 20mins. Thus, a total of 80 minutes was used for test administration in each school.

For the schools selected in each cell in Table 7, only one school was selected for the video sessions. This was done in the two districts involved in the study. That resulted in six schools. The videoing was done from the back of the classroom. This was in order to be able to capture the teachers writing, students and teachers gestures, and identify students position without showing their faces. All lessons for teaching addition of fractions were videoed on the days and times that the mathematics teachers scheduled for teaching addition of fractions. However, for each school studied, it was only one (first) video transcription that was selected from all videos obtained from that school. This was because, the time (90mins x 4 mathematics periods in a week x 2weeks = 720mins of transcription per school) in dealing with the volume of data from the transcribed video sessions for all the video sessions was not realistic per the period of the study. Also, similar method was used in the similar studies that used the same framework to analyse sentences in mathematics textbooks (see

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O'keeffe & O'Donoghue, 2015). Also, skimming through the videos suggested that each teacher used similar kinds of sentences throughout his/her lessons. The selected video was subsequently used for the transcription and analysis.

Administration and collection of instrument for RPK was finished in a period of two weeks. After instruction, two weeks was also used in administering the instrument on the concept of addition of fractions at all schools involved in the study. One month was used for the video sessions in all the schools involved in the study. One month was used for mop-up of all data collected and transcription. One month was used for scoring and entering data for RPK and the concept of addition of fractions.

Data Processing and Analysis

In relation to research question one, all students' and their mathematics teachers' (respondents) feedback to each item for RPK were collated. Characteristics of low, average, and high understanding were identified in the feedback for any given item. The adopted scoring rubric for unit coordination served as a guide for thorough description of low, average and high understanding of RPK in respondents' feedback (Bhattarcherjee, 2012).

Ten questionnaires were initially photocopied (2 copies). The photocopied questionnaires were shared equally among the researcher and another senior researcher. The questionnaire were scored separately by the researcher and another senior researcher. In scoring, the respondents' response for each item in the RPK instrument was matched against descriptors in the different category of understanding (see Appendix B). The match was then used to determine the kind of understanding that was demonstrated for that item. Later, scoring was compared and differences resolved. Similar process was

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repeated with another five instruments. Once scoring was consistent, the researcher continued and scored the rest of the instruments.

In answering research question two, the processing of the data in research question one was continued. For each item, all obtained data for RPK were classified as low, average, and high understanding of the RPK for addition of fractions. These levels of understanding were coded as 1, 2, and 3, respectively. Each item in the RPK was then considered a variable and created in the SPSS (version 20) software. Hence, students and their mathematics teachers' level of understanding of RPK was entered into the software. Frequency tables were therefore generated for students according to the achievement of the schools. Similar frequency tables were generated for the teachers. The frequency tables showed the percentage of students and their mathematics teachers that demonstrated a particular level of understanding (see Table 8, Table 9, Table 10, Table 11, Table 12 and Table 13) of the RPK per the items on RPK (Cohen, et al., 2007).

To answer research question three, all students' level of understanding, that were entered into SPSS (version 20), were subjected to a multivariate analysis of variance (MANOVA). Preliminary analysis was conducted on created RPK variable data to determine their suitability for MANOVA. Correlation matrix table was used to examine multicollinearity among the items. MANOVA results from SPSS was then presented to identify possible areas of differences in students' understanding of the specific RPK for addition of fractions (Huberty & Petoskey, 2000).

In order to answer research question four, the videoed lessons were used. Information obtained from the recorded videos were later played back,

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transcribed, and stored in word processing documents. The features of sentences or clauses that were used to classify them into the various sentence processes (mental, material, verbal, relational, behavioural, and existential) were described. The features in the sentences that were used to classify the human elements (general or specific), and mathematical objects (basic, relational, and representational), were described. To make the descriptions vivid, evidence from the transcriptions were shown (Bhattarcherjee, 2012).

In order to answer research question five, frequency and percentage tables (see Table 17, Table 18 and Table 19) were generated with the help of the software (Nvivo 12). These frequency and percentages were obtained after all sentences in the transcribed classroom interaction were coded per the properties (sentence processes, human elements, and mathematical objects). These tables were generated per the achievement level of the schools in the study (Cohen, et al., 2007; Bhattarcherjee, 2012).

Research question six was answered in a similar manner as research question one. All respondents' feedback was collated together per the items that examined understanding of the concept of addition of fractions. Characteristics of the feedback that described the nature of understanding (No understanding of the concept of addition of fractions {NU}, Partial Understanding of the concept of addition of fractions, with separated thoughts {PU_S}, Partial understanding of the concept of addition of fractions, with associated/connected thoughts {PU_A}, and full understanding of the concept of addition of fractions {FU}) was identified. Evidence of respondents' feedback were used to make the description vivid (Bhattarcherjee, 2012).

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Consequent of research question six, a scoring rubric was constituted. The rubric showed the item on one column and indicators/descriptors of the nature of understanding on another column. Therefore for each respondents' feedback to an item, it was matched against descriptors on the scoring rubric to determine the nature of understanding demonstrated. Initially, ten instruments retrieved from students were photocopied and shared equally among the researcher and a senior colleague. Each researcher scored his copy of the questionnaire independently. Later, scores were compared and differences resolved. The same process was repeated with five photocopied questionnaires and consistency among the scorers observed. Thus, the researcher scored the remaining instruments.

Research question seven was answered with the help of SPSS software. A variable was created, each for an item in the addition of fractions instrument. For each item, 1, 2, 3, and 4 were codes created for NU, PU_S, PU_A, and FU respectively. For each item, the students' and their mathematics teachers' demonstrated understanding were entered into the software. Frequency tables (see Table 20, Table 21 and Table 22) were generated. The aforementioned tables showed the frequency and percentage of students who demonstrated a particular level of understanding of addition of fractions for each item. The table were generated per the achievement levels of the schools. Similar tables (see Table 23, Table 24 and Table 25) were generated for teachers' demonstrated understanding of addition of fractions per the schools' achievement levels (Cohen, et al., 2007).

In response to research question eight, preliminary analysis was conducted to determine the suitability of the data for multivariate analysis of

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variance (MANOVA). Thus, a correlation matrix table was generated to test the assumption of multicollinearity. Subsequently, MANOVA was carried out to determine the effect of school type on students' understanding of the concept of addition of fractions (Huberty & Petoskey, 2000).

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CHAPTER FOUR

RESULTS AND DISCUSSION

This chapter is comprised of two main aspects: results of analysed data, and discussion of results obtained from the study. Presentation of the results were organised in the order of the research questions in this study. Hence results from understanding of related previous knowledge were presented first, nature of classroom interaction follows, and understanding of the concept of addition of fractions was finally presented. Results for students were presented first, followed by results for mathematics teachers, and finally, effect of school context on students' understanding. Discussion of the results from the study was also arranged per the research questions.

Research Question One

What are students and their mathematics teachers' understanding of the related previous knowledge for the teaching and learning of the concept of addition of fractions?

Description of the characteristics of a particular level of understanding of the RPK would be in order of the increased difficulty in the items/questions; 1-3 (illustrations drawn to scale), 4 (illustration not drawn to scale), 5 and 6 (reverse iteration), 7 (fractional results). Questions 1 and 2 are pre-ambles to question 3. Thus, reference would often be made to question 3. Also, for a given level of understanding, evidence from students and teachers was alternated with the questions. Thus, a mixture of evidence from students and teachers. The mixture is possible because it is the same questionnaire and scoring rubric administered and

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used respectively for both teachers and students. Establishing a particular level of understanding was based solely on the evidence of respondents' reasoning. Consequently, in high and average understanding, any evidence of the respondent using the given or drawn visual illustration to derive the answer does not demonstrate reasoning at the aforementioned level. On the other hand, when the respondent used the visual illustration to help expound the answer to the question, then the respondent was considered to have demonstrated understanding or reasoning at the aforementioned higher levels. The former corresponds to the use of visual illustration for learning purposes, whiles the latter corresponds to the use of visual illustration for teaching purposes.

Low understanding of the RPK for addition of fractions

Question 1-3 (illustrations drawn to scale)

All respondents who could not respond to the questions (1-3) at all were included in the category of respondents who were categorised with low understanding of the RPK. Furthermore, respondents who indicated they did not know how to answer the questions, were considered to have demonstrated low understanding. In addition to this group were those who attempted the questions (1-3) without any form of visual illustration, and still obtained an incorrect answer.

Some respondents were noted to have obtained the right answer to the questions (1-3) and should have been considered to have demonstrated average or high levels of understanding of the RPK. However, some of such respondents were observed in their response to have used the length of the small bar and matched it across the length of the longer bar. This was evidenced by small dots or marks

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across the length of the longer bar as in Figure 12. The deduction from such evidence was that the respondent could not reason with any number of given relations except with the help of a visual illustration. Such respondents demonstrated low understanding of the RPK for addition of fractions.

Use the bars shown above to answer the following three questions

1. How many times does the **Medium Yellow Bar** fit into the **Long Red Bars**?

   the medium Yellow Bar fit into the long red bar answer: 4 times

2. How many times does the **Small Blue Bar** fit into the **Medium Yellow Bar**?

   the small blue bar fit into the medium Yellow bar answer: 3 times

3. Use all the information you have now to figure out how many times the **Small Blue Bar** fits into the **Long Red Bar**?

   The small Blue Bar fits into the long Red bar answer: 12 time.

For each question above, use the space below to draw a drawing that will explain how you got your answer.

Source: Field data (2017)

Figure 12: Sample student response for low understanding of the RPK in question 1-3

Question 4 (illustration not drawn to scale)

Respondents who could not answer the question or demonstrated inability to answer the question were considered to have demonstrated low understanding of the RPK. Included among this group were respondents who attempted to obtain the answer by matching the length of the small bar in the long bar (although the drawing was not to scale). This is evidenced by the dots in the longer coloured bar that seemed to match the length of the smaller bar (see Figure 13). Some

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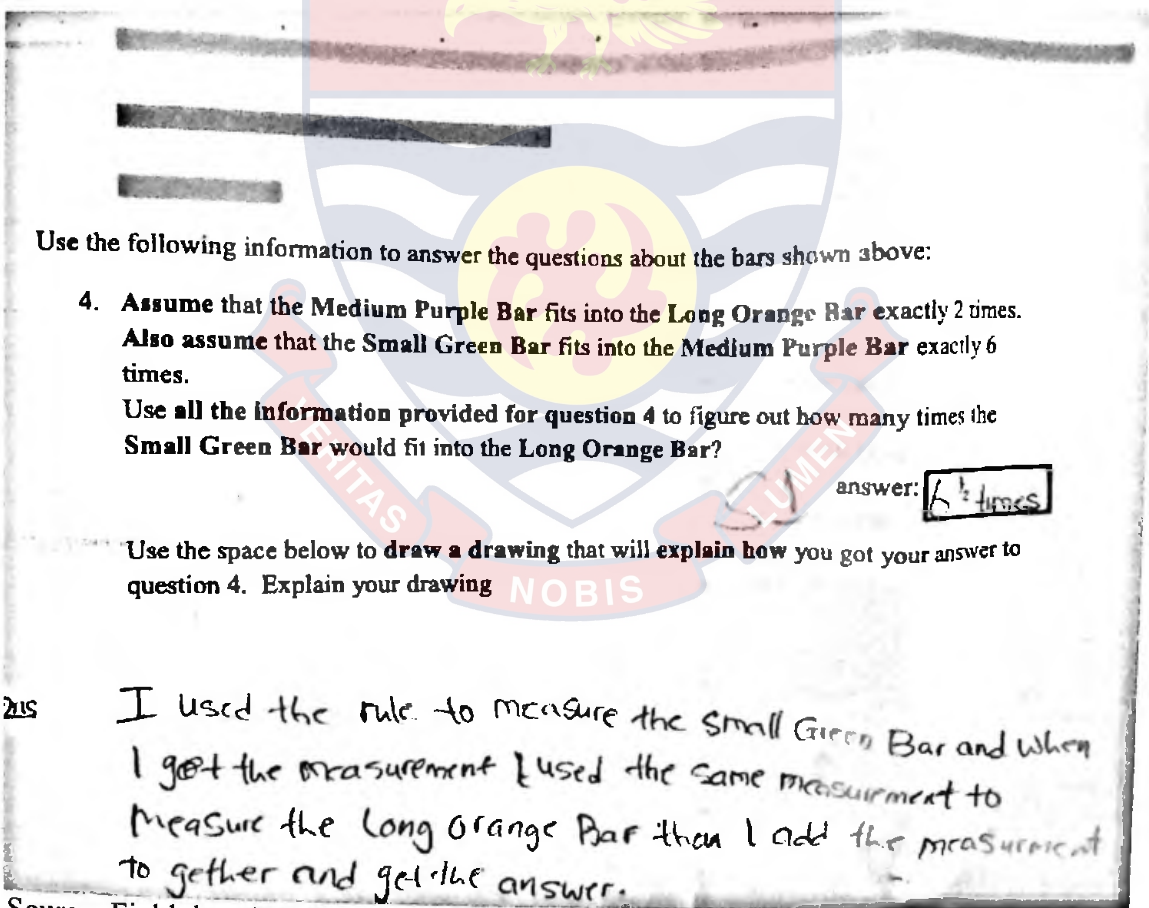
respondents also added or subtracted the numbers given in the question. Finally, some respondents redrew the description of the given relation to scale, and used the drawing to derive the answer to the question. In such circumstances, the redrawn descriptions appeared first before the derived answer. Hence in the derivation, there were evidence of the respondents marking the various lengths (similar to observation in the coloured illustration. see Figure 13) of the small bar along the length of the longer bar. Such respondents have not demonstrated the ability to reason about the specific RPK without the help of the visual illustration.



Use the following information to answer the questions about the bars shown above:

4. **Assume** that the **Medium Purple Bar** fits into the **Long Orange Bar** exactly 2 times. **Also assume** that the **Small Green Bar** fits into the **Medium Purple Bar** exactly 6 times.
Use **all the information provided for question 4** to figure out how many times the **Small Green Bar** would fit into the **Long Orange Bar**?

answer: 6 ½ times

Use the space below to **draw a drawing** that will **explain how** you got your answer to question 4. Explain your drawing

I used the rule to measure the Small Green Bar and when I get the measurement I used the same measurement to measure the Long Orange Bar then I add the measurement to gether and get the answer.

Source: Field data (2017)

Figure 13: Sample student response for low understanding of the RPK in question 4

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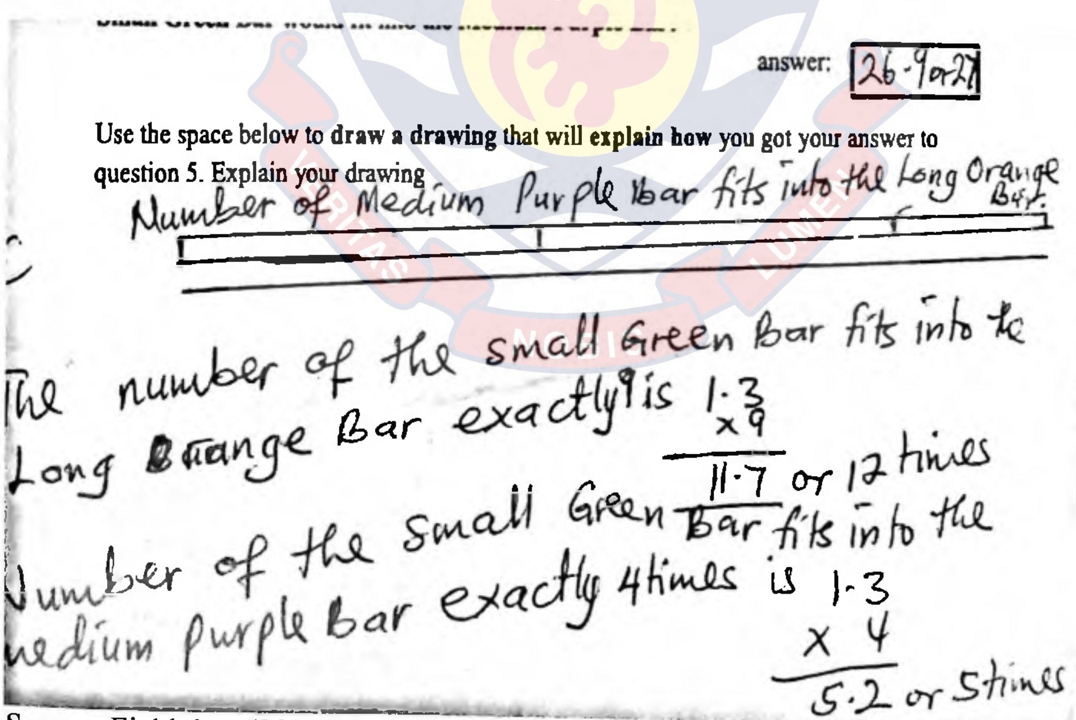
Question five and six (reverse iteration)

Low understanding in these questions included respondents who attempted to use the sizes of the bar (although not drawn to scale) to obtain the answer. Such respondents used the length of the smaller bar to mark out lengths along the long bar. Other respondents also redrew the aforementioned relation in the question and attempted to extract the right answer by measuring the length of the shorter redrawn bar across the length of the longer redrawn bar. Typical of such respondents was their presentation of the visual illustration before explanation (if any). This meant that they used the visual illustration to derive the explanation. Another implication was that the aforementioned respondents could not reason with the given relations without the help of a visual illustrations. An example of response to question five is in Figure 14. Accordingly, it showed low understanding of the RPK.



answer: 26.9 or 27

Use the space below to draw a drawing that will explain how you got your answer to question 5. Explain your drawing

Number of Medium Purple Bar fits into the Long Orange Bar.

The number of the small Green Bar fits into the Long Orange Bar exactly 9 is 1.3 x 9 = 11.7 or 12 times

Number of the small Green Bar fits into the Medium purple bar exactly 4 times is 1.3 x 4 = 5.2 or 5 times

Source: Field data (2017)

Figure 14: Sample teacher response for low understanding of the RPK in question 5

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As part of respondents who demonstrated low understanding are respondents who added or subtracted the numbers in the relations in the questions. Also, not excluded were respondents who could not attempt the questions at all or indicated they did not know the answer or strategy to answer the questions.

Question seven (fractional results)

Respondents who added or subtracted the given number relations in the question were considered to have demonstrated low understanding of the RPK. This category also included respondents who multiplied the numbers in the given relation without any supporting arguments. Evident from such respondents implied that they do not comprehend the correct strategy to address the problem. Another important characteristic of low understanding of the RPK were observed in respondents whom in their strategies, made no attempt to report on the fractional part of the results. Inference from such respondents was that they did not have the idea about the major characteristic that the question was intended to elicit. An ideal evidence in Figure 15 showed the respondents redrew the relation described in the question, but could not account for the fractional part that could be observed in the illustration. Accordingly, neither the drawing nor the answer obtained accounted for the fractional component.

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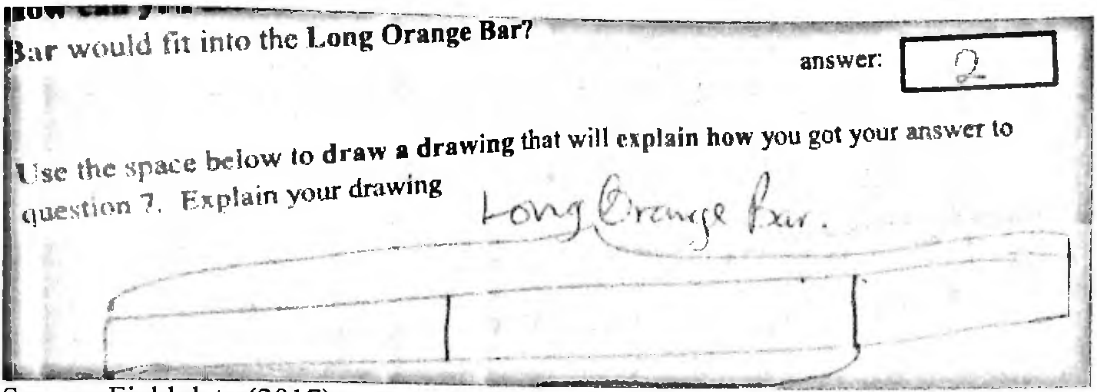

Source: Field data (2017)

Figure 15: Sample teacher response for low understanding of the RPK in question 7

Average understanding of the RPK for addition of fractions

Question 1-3 (illustrations drawn to scale)

All respondents who demonstrated the ability to reason with the given relations between any two bars without the use of any visual illustration was considered to have demonstrated average understanding. Respondents who demonstrated average understanding of the RPK were able to retain(memorise) and understand the answer to question three. However, to be able to combine and relate answer to question one, and answer to question two, the respondents had to use the visual illustrations. This was a major distinguishing characteristic of the respondents with average understanding in question three (see Figure 16).

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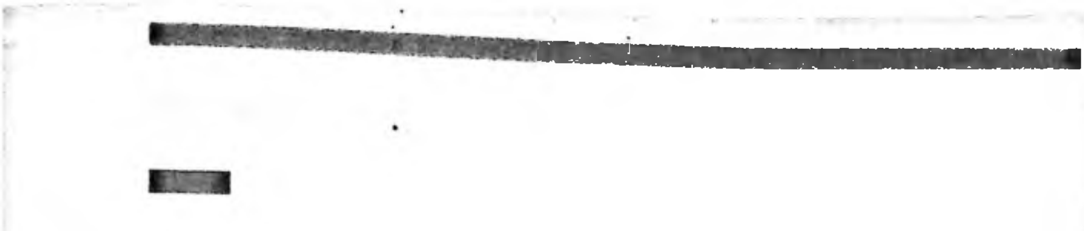

Use the bars shown above to answer the following three questions

1. How many times does the **Medium Yellow Bar** fit into the **Long Red Bars?**

   answer: ~~3½~~ 4

2. How many times does the **Small Blue Bar** fit into the **Medium Yellow Bar?**

   answer: ~~1½~~ 3

3. Use all the information you have now to figure out how many times the **Small Blue Bar** fits into the **Long Red Bar?**

   S2 answer: ~~1½~~ 12

Source: Field data (2017)

Figure 16: Sample student response for average understanding of the RPK in question 1-3

As could be observed from the visual illustration (Figure 16), the respondent did not mark the length of the small bar in the medium bar. The implication was that the relation was retained (memorized) and understood. However, it could be observed in the visual illustration that there were marked out points of the length of the medium purple bar along the length of the long red bar. Adding up the observed evidence, the respondent could imagine and iterate the length of the small bar in the medium bar. This reflected the ability to reason and coordinate two units. However, the respondent could not determine the result of multiplying the two obtained relations without the use of the visual illustration. Therefore, the respondent used the length of the medium bar along the length of the long bar while imagining that the length of the medium bar is three times the length of the small

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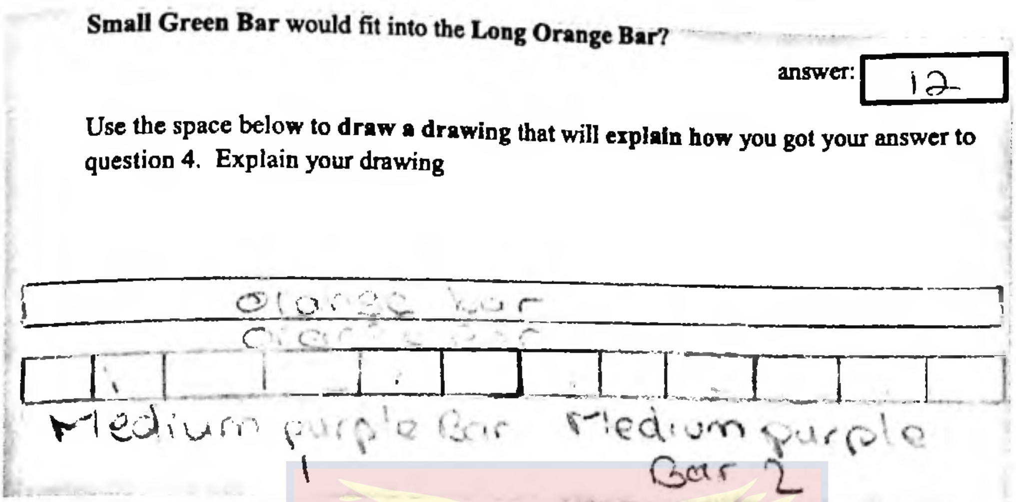

Source: Field data (2017)

Figure 17: Sample student response for average understanding of the RPK in question 4

Figure 17 represents the response of a respondent who demonstrated average understanding of RPK. Although the correct answer was obtained, the visual illustration does not represent reasoning with all two given relations simultaneously. Evidence from the illustration suggested that the relationship between the small and medium bar was considered in a composite form. That was why the medium purple bar was not divided into two (in the average bar), but showed the number of the small bar in the medium bar (composite reasoning). However, the given relation between the medium and the long bar was based on the visual representation.

Question five and six (reverse iteration)

All respondents that were observed to have used multiplication of the given relations instead of division, had demonstrated average level of understanding. The reason for such categoristation was because such respondents would have still demonstrated reasoning with the given relations which were composite in nature.

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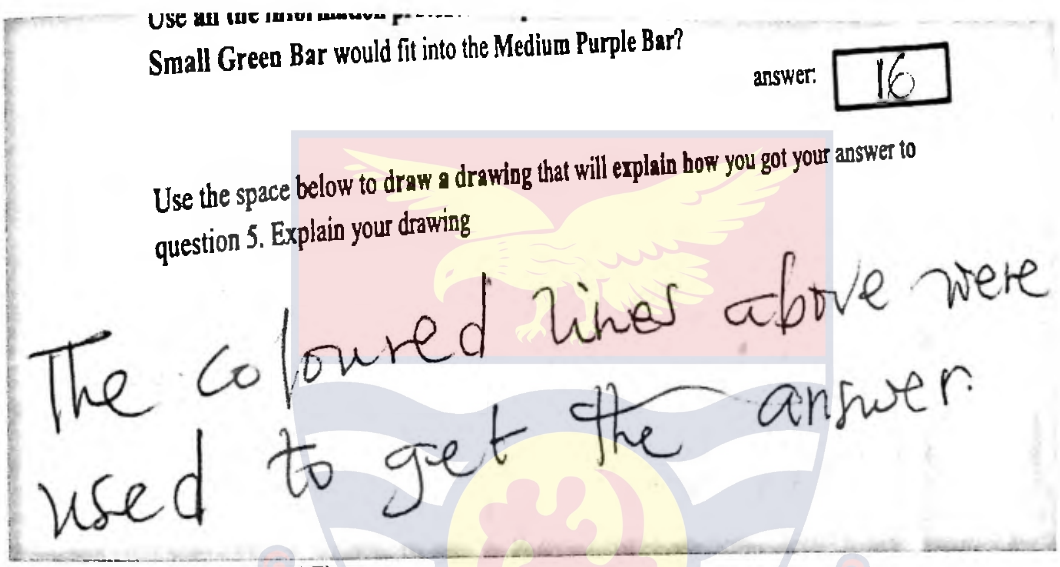
However, such respondents did not realise that the task was a case of reverse reasoning. Also considered to have demonstrated average understanding of the RPK were respondents who used division of the given relation but without visual illustrations to explain their reasoning. Such visual illustrations would normally show the use of two relations and not three.



Source: Field data (2017)

Figure 18: Sample teacher response for average understanding of the RPK in questions 5

The visual illustration in Figure 18 represents a respondents' answer to question five. The obtained answer suggested that the respondent used multiplication instead of division. Also, the explanation does not give detailed insight in the strategy of the solution to the task, except that the visual illustration in the question was a significant influence on the strategy used. The deduction was that the respondent did not read the question well, but assumed the trend of multiplication from the previous tasks. The given relations, which were composite in nature, at least involved reasoning and coordinating two units, hence average understanding of the RPK for addition of fractions.

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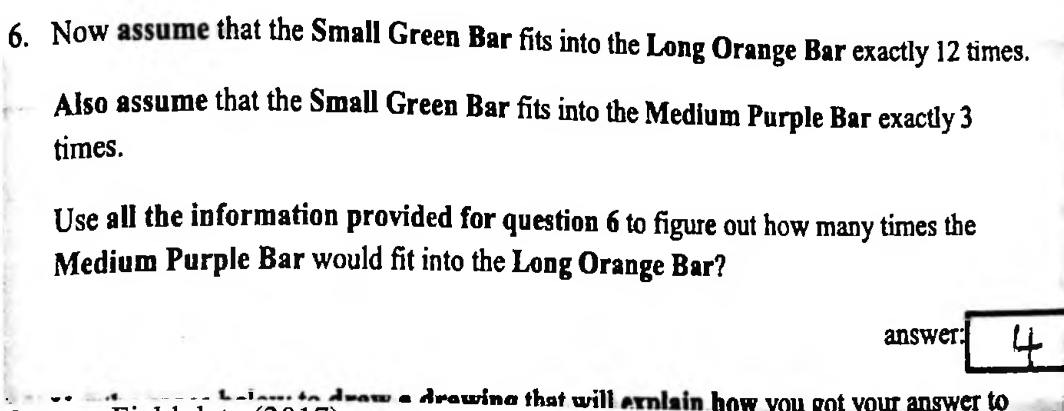

6. Now **assume** that the **Small Green Bar** fits into the **Long Orange Bar** exactly 12 times.

Also assume that the **Small Green Bar** fits into the **Medium Purple Bar** exactly 3 times.

Use **all the information provided for question 6** to figure out how many times the **Medium Purple Bar** would fit into the **Long Orange Bar?**

answer: 4

... drawing that will explain how you got your answer to

Source: Field data (2017)

Figure 19: Sample teacher response for average understanding of the RPK in question 6

Figure 19 shows a respondent who gave a correct answer to question six, but without explanation or justification of any form. Although this might suggest the use of reverse multiplicative reasoning, it was possible the respondent derived the answer from a visual illustration that redrew the relations. Given that this was the least strategy that could be used to obtain the correct answer, such strategy demonstrated the reasoning with minimum of two units simultaneously. This was because the given relation used for the aforementioned drawing was between two units. This deduction consequently corresponds to the respondent being described as demonstrating average understanding of the RPK for addition of fractions.

Question seven (fractional results)

In the visual illustration of the solution strategy, respondents who had demonstrated average understanding of the RPK used multiplication reasoning instead of division. It was observed that in Norton et al (2015), most of such respondents assumed the strategy because they did not read the question well (similar to average understanding in question 5 and 6). Also considered to reflect average understanding were responses that used division strategy but did not give

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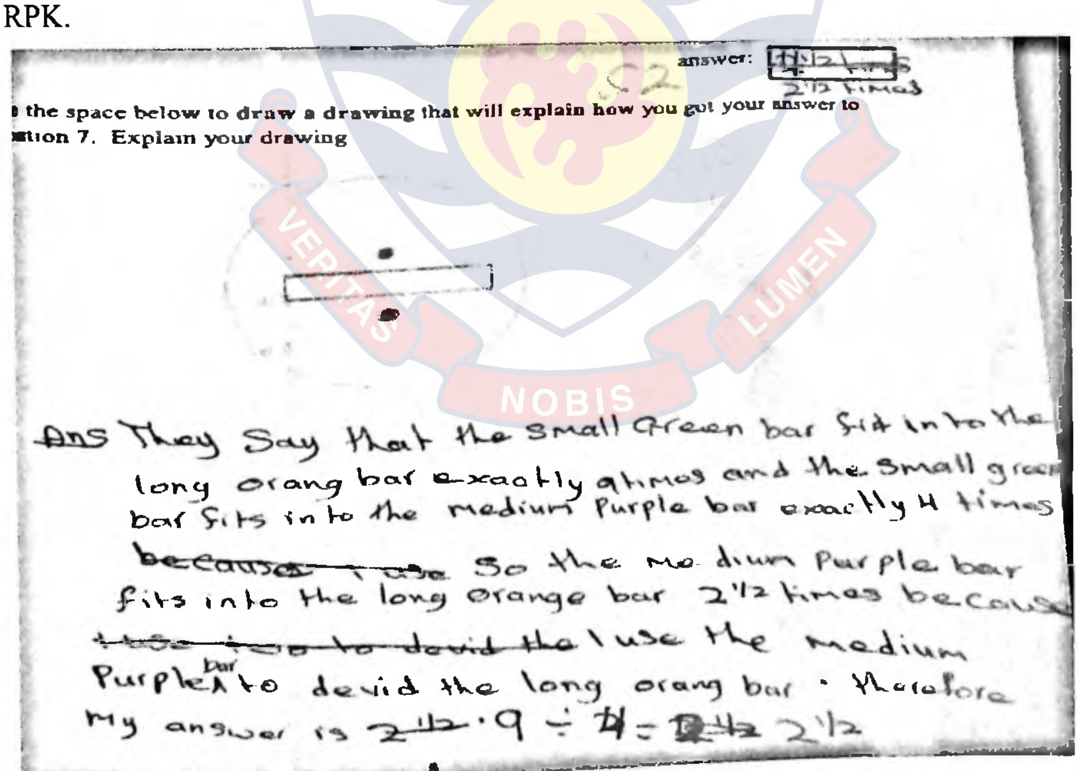
any explanation or visual justification for the strategy. In some of such strategies, the respondents did not know how to account for the fractional quantity in the correct answer. After stating the whole number value in the answer, other respondents just indicated that there was a remainder in their solution strategy. The implication was that the respondents were aware of the fractional quantity but did not know how to communicate the remainder. All these respondents were considered to have demonstrated average understanding of the RPK. Figure 20 represents a respondents' feedback on the task. It was evident from the answer and the explanation of the respondent, that the strategy involved reverse multiplicative reasoning. However, the respondent made a mistake in the fractional component. The implication was that the respondent demonstrated average understanding of the RPK.



Source: Field data (2017)

Figure 20: Sample student response for average understanding of the RPK in question 7

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High understanding of the RPK for addition of fractions

Question 1-3 (illustrations drawn to scale)

All respondent who demonstrated high level of understanding of the RPK for addition of fractions would use the relationship obtained from the first two questions. Notwithstanding that the answers to the two previous questions may be right or wrong, it was the correct use of the obtained relations that defined high understanding. Therefore, high understanding of the RPK is the multiplicative reasoning observed with the use of the two obtained relations, to derive the answer to question three. Some of the observed characteristic involved the respondent writing the correct answer (12) straight forward. Respondents who wrote an explanation would reveal the use of relational language to derive the answer. Evidence from Figure 21 shows that the respondent used relational language, linking the multiplication to the previously obtained answers. This was identified by the numbered explanation of the answers. It could be observed in last numbered explanation in the figure that the respondent, used the same form of multiplicative reasoning in obtaining the answer.

VERITAS LUMEN NOBIS

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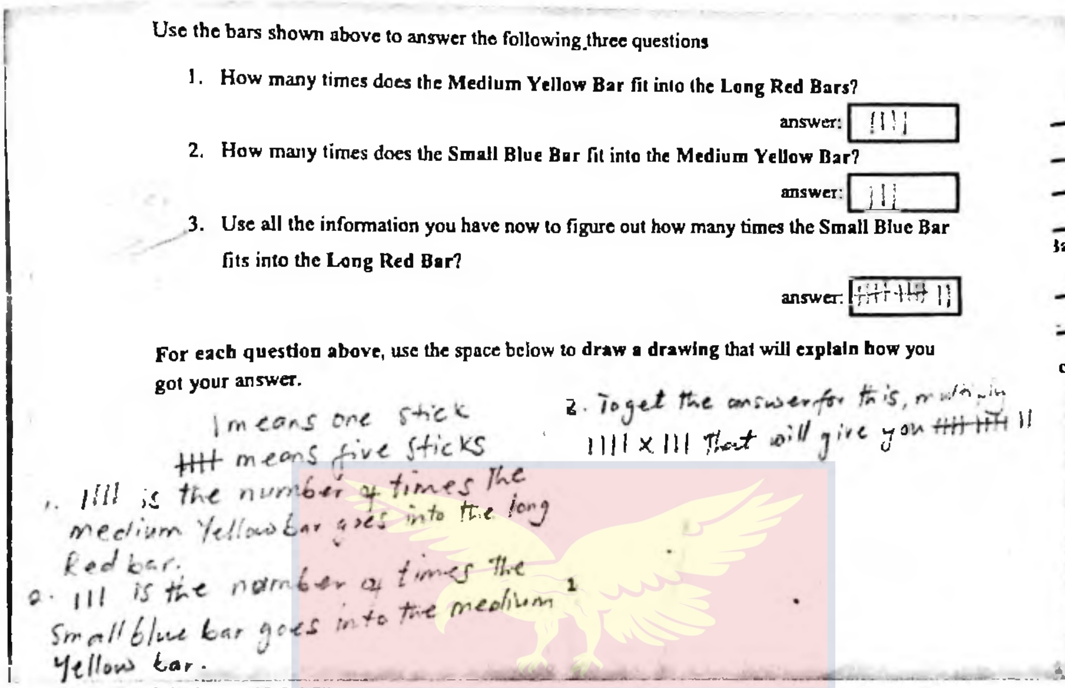

Use the bars shown above to answer the following three questions

1. How many times does the **Medium Yellow Bar** fit into the **Long Red Bars?**

answer: ||||

2. How many times does the **Small Blue Bar** fit into the **Medium Yellow Bar?**

answer: |||

3. Use all the information you have now to figure out how many times the **Small Blue Bar** fits into the **Long Red Bar?**

answer: HHT HHT ||

For each question above, use the space below to **draw a drawing** that will **explain how** you got your answer.

| means one stick
HHT means five sticks
1. |||| is the number of times the medium Yellow bar goes into the long Red bar.
2. ||| is the number of times the Small blue bar goes into the medium Yellow bar.
3. To get the answer for this, multiplying |||| x ||| That will give you HHT HHT ||

Source: Field data (2017)

Figure 21: Sample student response for high understanding of the RPK in question 1-3

Question 4 (illustration not drawn to scale)

High understanding of the RPK was demonstrated by the use of the given relation to obtain the appropriate answer to the task or question. Therefore, the respondent does not only produce an answer but has either a written explanation or a visual illustration to support his or her answer. When a diagram was used, it was ensured that the diagram was not used to derive the solution to the problem. The aforementioned characteristic was a way of ensuring that the respondents relied on the reasoning rather than the visual illustration. Supporting evidence was observed in the respondents illustrating the relations that were already given.

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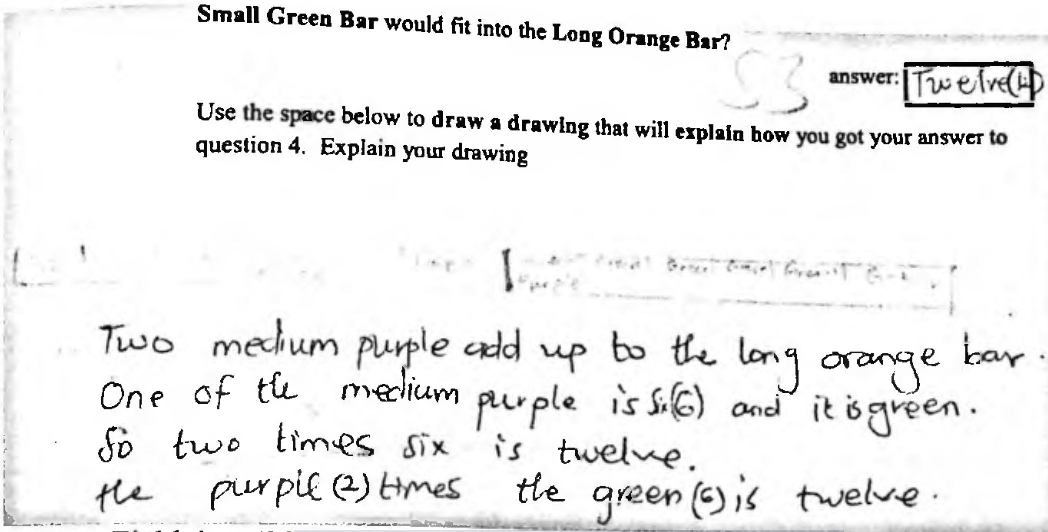
Small Green Bar would fit into the Long Orange Bar?

answer: Twelve(12)

Use the space below to draw a drawing that will explain how you got your answer to question 4. Explain your drawing

Two medium purple add up to the long orange bar.
One of the medium purple is Six(6) and it is green.
So two times six is twelve.
the purple (2) times the green (6) is twelve.

Source: Field data (2017)

Figure 22: Sample teacher response for high understanding of the RPK in question 4

In Figure 22, the respondents' explanations of the strategy showed the composition of the given relations in the final correct answer. Supporting visual illustration also showed the representation of all the given relations in the obtained answer (i.e., using one diagram). The visual illustration therefore showed the linkage between the given relations and the answer. This showed that the visual illustration was not the source of the derived answer but used for explanation purposes.

Question five and six (reverse iteration)

In these task, high understanding of the RPK was demonstrated by the evidence of reverse multiplicative reasoning in the explanation of the solution to the problems. It was also ensured that when visual illustrations were used, they only represented the correct solution and not used to derive the appropriate solution. When divisions were used in the strategy for the solutions, the visual illustration should indicate the division values (or given relations) in those division strategies.

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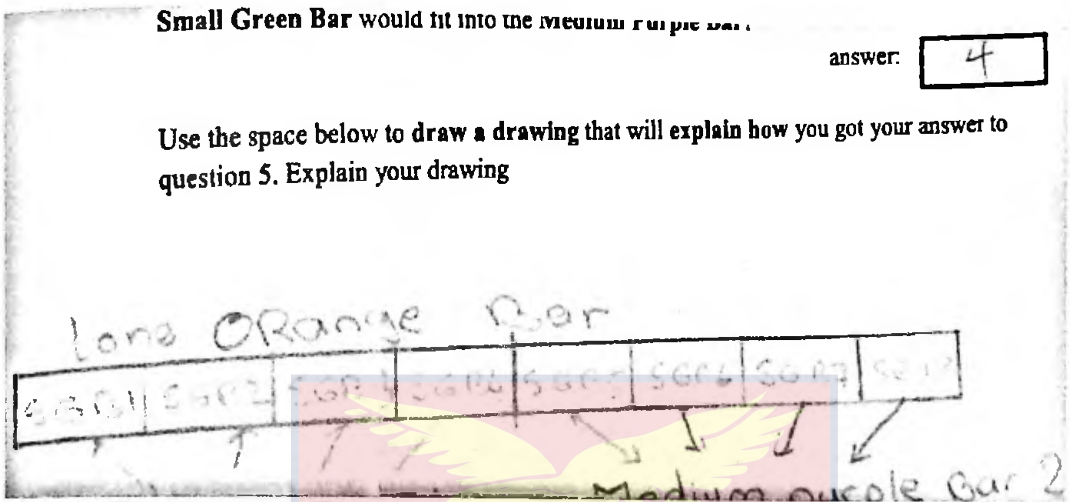
This was to ensure that the visual illustrations were consistent with the division and not used to produce the solution to the tasks.



Source: Field data (2017)

Figure 23: Sample teacher response for high understanding of the RPK in questions 5

In Figure 23, it can be observed that the respondent wrote the answer direct. Subsequently, the visual illustration showed an integration of all the given relations in one illustration. It showed how the short green bar was fitted together in the structure of the long bar and the small yellow bar. This showed that the reasoning was rather used to develop the visual illustration and not the contrary. It also showed division by the extraction of the various small green bar to make up a purple bar (i.e. division).

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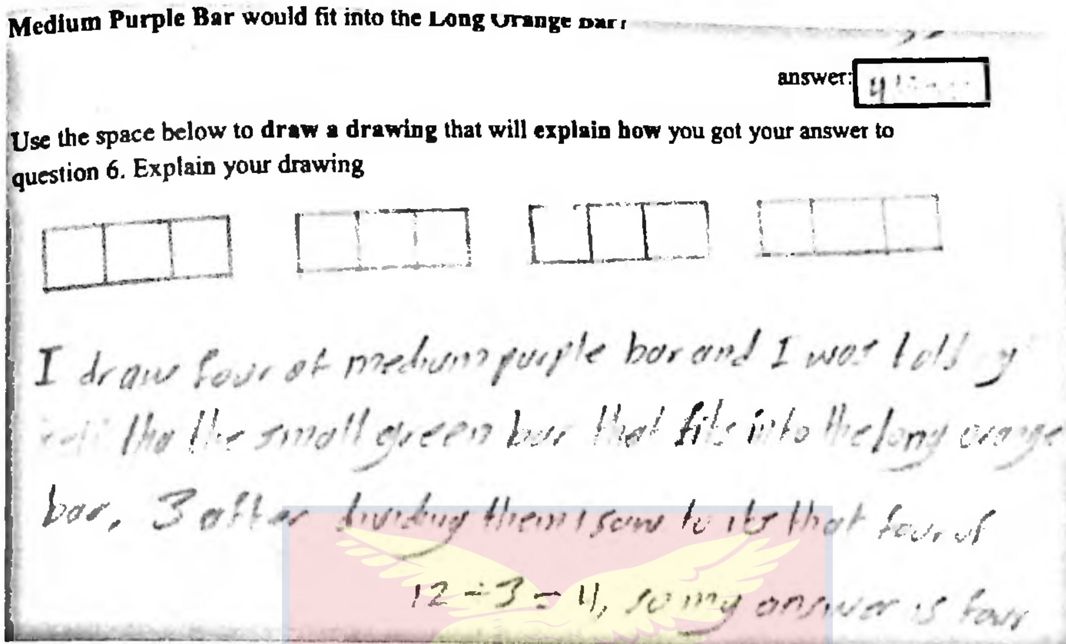

Source: Field data (2017)

Figure 24: Sample student response for high understanding of the RPK in questions 6

As observed in Figure 24, evidence showed that explanation to support the correct response were obtained from both visual illustrations and the description of the strategy. The visual illustrations showed the linkage between the obtained answer and the various given relations. In the visual illustration, the separate four boxes represented the answer. Within each of the boxes were three boxes, which represented a given relation. The total number (12) of the small boxes represented another given relation. Evidence from the written explanation also showed that the introductory statement was first, a description of the answer. This suggested that the written statement was an explanation of the solution and not a path to generate the solution. Secondly, the reverse reasoning in the explanation was evident in the division of the given relations to obtain the correct answer.

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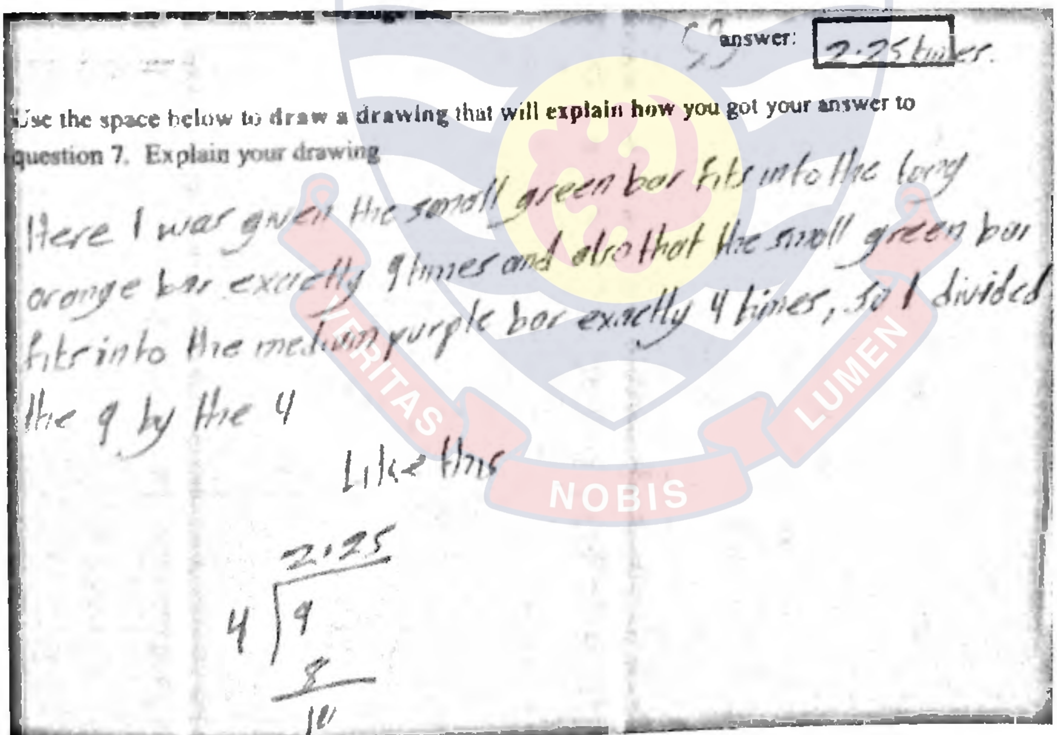
Question seven (fractional results)

To demonstrate high understanding of the RPK, respondent would have to correctly account for the fractional part in the solution. This fractional part could be described as a remainder, but it would have to be described specifically in relation to the whole. The remainder could also be described as the specific amount that is needed to make a whole. Other respondents who demonstrated high understanding of the RPK used division in their strategy. However, the division would have to be combined with an explanation or a visual illustration to demonstrate high understanding of the RPK.



Source: Field data (2017)

Figure 25: Sample student response for high understanding of the RPK in questions 7

In Figure 25, the respondent used only written explanation and the division procedure to justify his or her answer. The respondent indicated the direct linkage

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between the strategy and the given relations. This showed the respondents ability to reason with the given relation and not to depend on visual illustrations to derive the answer. In the answer, the fractional part was accounted for in decimal form.

Research Question Two

What are students and their mathematics teachers' understanding of the related previous knowledge for the teaching and learning of the concept of addition of fractions by school context (that is, high, average and low achieving schools)?

High achieving schools

Table 8 showed the percentages of students that demonstrated the various levels of understanding of the RPK, per the question types. It was revealed from the data that for all the forms of questions used in the study, there was a consistency in the trend of understanding demonstrated by the students per the three levels: low, average, and high. This suggested that there was a clear pattern in the students demonstrated understanding of the RPK necessary for the teaching and learning of addition of fractions. The average percentage of students who demonstrated low understanding of the RPK was 71.2%. Comparatively, the average percentage of students who demonstrated average (18.1%) and high (10.7%) understanding of the RPK was small. The data implied that on the average, a tenth of students in high achieving schools demonstrated the required understanding of the RPK necessary for addition of fractions. In other words, only a tenth of the students in high achieving schools were ready for instruction in addition of fractions.

Across all the question forms, the highest percentage of students demonstrated low understanding of the RPK. Relatively, smaller percentage of students demonstrated

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average and high understanding of the RPK respectively. The highest percentage (87.9%) of students who demonstrated low understanding of the RPK was observed in question three. The least percentage (58.9%) of students who demonstrated low understanding of the RPK was observed in question four. The percentage of students who demonstrated low understanding of the RPK continually increased across question four to seven. Consequently, across questions four to seven, there was a continual decrease in the percentage of students who demonstrated high and average understanding of RPK for addition of fractions.

Table 8: Understanding of the RPK by Students in High Achieving Schools

Questions	Low understanding N (%)	Average understanding N (%)	High understanding N (%)	Total N (%)
Three	167 (87.9)	21 (11.1)	2 (1.1)	190 (100.0)
Four	112 (58.9)	38 (20.0)	40 (21.1)	190 (100.0)
Five	119 (62.2)	46 (24.2)	25 (13.2)	190 (100.0)
Six	130 (68.4)	37(19.5)	23 (12.1)	190 (100.0)
Seven	149 (78.4)	30 (15.8)	11 (5.8)	190 (100.0)
Average %	(71.2)	(18.1)	(10.7)	(100.0)

Source: Field data (2017)

Average achieving schools

In average achieving schools, Table 9 showed the percentage of students who demonstrated the various levels of understanding of the RPK, per the question types. The table showed a pattern in the percentage of students who demonstrated the various levels of understanding. Across the various questions, the average percentage of students who demonstrated low understanding of the RPK was highest (68.5%). Consequently, a comparatively smaller percentage of students demonstrated average (19.0%) and low (12.4%) understanding of the RPK.

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In all the questions, there was a consistent pattern in the percentage of students who demonstrated the various types of understanding of the RPK. It was evident that in all the questions, majority of the students demonstrated low understanding of the RPK. This was always followed by the percentage of students who demonstrated average and high understanding of the RPK, respectively.

The highest percentage (89.5%) of students who demonstrated low understanding was in question three. On the other hand, the smallest percentage (55.3%) of students who demonstrated low understanding was in question four. Across questions four to seven, there was a consistent increase in the percentage of students who demonstrated low understanding. Consequently, there was a reduction in the associated percentage of students who demonstrated average and low understanding, but without any clear pattern. Among students who demonstrated average and high understanding of the RPK, there were increases to the peak (27.5% and 21.0% respectively) percentages. The later was followed by a general decline in percentages across the questions.

Table 9:Understanding of the RPK by Students in Average Achieving Schools

Questions	Low understanding N (%)	Average understanding N (%)	High understanding N (%)	Total N (%)
Three	196 (89.5)	14 (6.4)	9 (4.1)	219 (100.0)
Four	121 (55.3)	52 (23.7)	46 (21.0)	219 (100.0)
Five	135 (61.9)	60 (27.5)	23 (10.6)	218 (100.0)
Six	137 (62.6)	45 (20.5)	37 (16.9)	219 (100.0)
Seven	161 (73.5)	37 (16.9)	21 (9.6)	219 (100.0)
Average %	(68.6)	(19.0)	(12.4)	(100.0)

Source: Field data (2017)

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Low achieving schools

Table 10 presents the distribution of students' understanding of the RPK in low achieving schools. It shows frequency and percentages of students who demonstrated the various levels of understanding of the RPK, per the question types. Throughout the question types, the average percentage of students who demonstrated low understanding of the RPK was the highest (72.9%). In contrast, smaller percentages (18.2% and 8.9%) of students demonstrated average and high understanding of the RPK respectively.

Across the questions, there was a general pattern in the percentage of students who demonstrated the various levels of understanding of the RPK. In all the questions, the highest percentage of students demonstrated low understanding of the RPK. Accordingly, the percentage of students who demonstrated average understanding followed. The least percentage of students mostly demonstrated high understanding of the RPK.

Among students who demonstrated low understanding of the RPK, the highest percentage (84.9%) was observed in question three. This was followed by a decline to 66.6% in question five and another increase. Among students who demonstrated average and high understanding of the RPK, there was no easily discernible pattern in percentages.

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Table 10: Understanding of the RPK by Students in Low Achieving Schools

Questions	Low Understanding N (%)	Average Understanding N (%)	High Understanding N (%)	Total N (%)
Three	180 (84.9)	28 (13.2)	4 (1.9)	212 (100.0)
Four	146 (68.9)	39 (18.4)	27 (12.7)	212 (100.0)
Five	140 (66.0)	53 (25.0)	19 (9.0)	212 (100.0)
Six	153 (72.2)	32 (15.1)	27 (12.7)	212 (100.0)
Seven	154 (72.6)	41 (19.3)	17 (8.0)	212 (100.0)
Average %	(72.9)	(18.2)	(8.9)	(100.0)

Source: Field data (2017)

Mathematics teachers' understanding of the RPK

High achieving schools

Table 11 showed distribution of mathematics teachers' demonstration of understanding of the RPK for addition of fractions. The table consisted of frequencies and percentages of mathematics teachers who demonstrated a particular kind of understanding. The table showed a clear trend in high achieving school teachers' understanding of the RPK. Across all the questions, average percentage (23.3%) of teachers who demonstrated low understanding of the RPK was the least. This was followed by the average percentage (36.7%) of teachers that demonstrated high understanding of the RPK.

High percentage (100.0%) of teachers who demonstrated low understanding of the RPK was observed in question seven (highest difficult questions). High percentages (50.0%, 83.3%, 50.0%) of teachers who demonstrated average understanding was observed in questions four, five, and six respectively (averagely difficult questions). Similar characteristics was observed in questions three (83.3%) and four (50.0%), among teachers who demonstrated high understanding of the RPK (lowest difficult questions). There was a trend. In the easier questions (1-3),

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teachers demonstrated high understanding of the RPK. As the questions progressed in difficulty, majority of the teachers demonstrated average understanding of the RPK. Consequently, majority of teachers demonstrated low understanding when they attempted the most difficult question.

Table 11: Understanding of the RPK by Mathematics Teachers in High Achieving Schools

Questions	Low Understanding N (%)	Average Understanding N (%)	High Understanding N (%)	Total N (%)
Three	0 (0.0)	1 (16.7)	5 (83.3)	6 (100.0)
Four	0 (0.0)	3 (50.0)	3 (50.0)	6 (100.0)
Five	0 (0.0)	5 (83.3)	1 (16.7)	6 (100.0)
Six	1 (16.7)	3 (50.0)	2 (33.3)	6 (100.0)
Seven	6 (100.0)	0 (0.0)	0 (0.0)	6 (100.0)
Average %	23.3	40.0	36.7	(100.0)

Source: Field data (2017)

Average achieving schools

Table 12 showed the distribution of mathematics teachers' understanding of the RPK for addition of fractions. The table included frequencies and percentages of mathematics teachers who demonstrated a particular kind of understanding, across questions of varied difficulties. The table shows that the average percentage (50.0%) of teachers who demonstrated high understanding of the RPK was the highest. Similarly, average and low understanding were of similar percentages (25.0% each). Except for question seven, the highest percentage of teachers who demonstrated high understanding constituted the majority across all question forms. Generally, similar percentages of teachers demonstrated low and average understanding of the RPK for addition of fractions, across the various question forms.

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Table 12: Understanding of the RPK by Mathematics Teachers in Average Achieving Schools

Questions	Low Understanding N (%)	Average Understanding N (%)	High Understanding N (%)	Total N (%)
Three	1 (25.0)	0 (0.0)	3 (75.0%)	4 (100.0)
Four	1 (25.0)	1 (25.0)	2 (50.0)	4 (100.0)
Five	1 (25.0)	1 (25.0)	2 (50.0)	4 (100.0)
Six	1 (25.0)	1 (25.0)	2 (50.0)	4 (100.0)
Seven	1 (25.0)	2 (50.0)	1 (25.0)	4 (100.0)
Average %	25.0	25.0	50.0	(100.0)

Source: Field data (2017)

Low achieving school

Table 13 shows the distribution of mathematics teachers' understanding of the RPK for addition of fractions. The table constitutes frequency and percentage of mathematics teachers who demonstrated the various types of understanding, per the diverse questions. Across all the questions, the average percentage (42.8%) of teachers who demonstrated low understanding was the highest. This was followed by the average percentage (31.4%) of teachers who demonstrated high understanding.

Except for questions three and seven, there was a consistency in the trend of percentage of teachers that demonstrated a particular type of understanding of the RPK. Thus, the trend was that the highest percentage of teachers demonstrated low understanding of the RPK, across the questions. The percentage (28.6%) of teachers who demonstrated average and high understanding were the same. In question seven, the highest (57.2%) percentage of teachers demonstrated low understanding of the RPK, whiles in question three, similar percentage (57.1%) of teachers demonstrated high understanding of the RPK.

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Table 13: **Understanding of the RPK by Mathematics Teachers in Low Achieving Schools**

Questions	Low Understanding N (%)	Average Understanding N (%)	High Understanding N (%)	Total N (%)
Three	2 (28.6)	1 (14.3)	4 (57.1)	7 (100.0)
Four	3 (42.8)	2 (28.6)	2 (28.6)	7 (100.0)
Five	3 (42.8)	2 (28.6)	2 (28.6)	7 (100.0)
Six	3 (42.8)	2 (28.6)	2 (28.6)	7 (100.0)
Seven	4 (57.2)	2 (28.6)	1 (14.2)	7 (100.0)
Average %	42.8	25.7	31.4	(100.0)

Source: Field data (2017)

Research Question Three

What is the effect of school context on students' understanding of related previous knowledge for the teaching and learning of the concept of addition of fractions?

In the second paragraph, this portion of the results presents the findings on assumption of one-way Multivariate Analysis of Variance (MANOVA). This was followed by the results of MANOVA in the third paragraph. Post hoc results of One-way Analysis of Variance (ANOVA) and Bonferroni multiple comparison were presented. All analysis was in relation to the students' understanding of the RPK for addition of fractions (independent variable).

One-way Multivariate Analysis of Variance (MANOVA) was carried out to examine the mean differences in students' understanding of the RPK over five question forms, and across the three school types (achievement categories). Prior to the MANOVA, a correlation matrix table of students' understanding of the RPK in the five question forms, was generated to examine correlation among themselves (see Table 14). An assumption that needed to be satisfied for MANOVA was correlation among the independent variables (multicollinearity). A correlation

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coefficient between 0.2 and 0.6 was necessary among majority of the independent variables for multicollinearity assumption to be satisfied (Howell, 2009). Observation from Table 14 suggested that the pre-requisite for multi-collinearity was satisfied for MANOVA. Another assumption for MANOVA was that the covariance in the students' understanding of the RPK, over all question forms (independent variable), and across the three school types, must be equal. However, the Box's M of 2.607 (p=0.000<0.005) suggested that the covariances among the independent variables were not equal. In the event of violation of conditions necessary for MANOVA, it was Huberty and Petoskey's (2000) recommendation that the test statistics that is least sensitive to violations in equality of covariance condition, be adopted. Thus, Pillai's Trace was recommended because it was least sensitive to the violation of equality of covariance.

Table 14: Pearson Correlations, Means, and Standard Deviations Associated with Students Understanding of the RPK

Question	1	2	3	4	5	M	SD
1. Question Three	1					1.15	0.42
2. Question Four	0.20	1				1.57	0.78
3. Question Five	0.10	0.55	1			1.47	0.68
4. Question Six	0.07	0.47	0.59	1		1.47	0.73
5. Question Seven	0.10	0.35	0.41	0.52	1	1.33	0.62

Note: N=616; Correlation coefficient greater than 0.1 is statistically significant (p<0.01).
Source: Field data (2017)

One-way MANOVA investigated the hypothesis that there would be no significant mean differences between school types (high, average, and low achieving schools) in terms of students' understanding of the RPK in the various question forms. The statistical evidence suggested that there was a significant MANOVA effect, Pillai's Trace = 0.032, F(10, 1220)=2.004, P=0.00<0.05. This

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suggested that at least one of the independent variables was different from at least one of the other independent variables. The multivariate effect size was estimated at 0.016, which suggested that 1.6% of the variations in the canonically derived dependent variable was accounted for by the school types. In other words, school type accounted for 1.6% of variations in students' understanding of the specific RPK for addition of fractions.

Based on the fact that MANOVA effect was determined, there was the need to perform the series of ANOVA tests, that would identify the question forms where the students' understanding of the RPK was different per the various school types. Evidence from Table 15 suggested that three out of five of the question forms in the Levene's F test of homogeneity of variances, violated the assumption. Hence, with violation of assumption of equality of variance, post hoc analysis of any significant question forms, associated with students' understanding of the RPK, will have to be Bonferroni. This is because, Bonferroni post hoc analysis is resistant to violations in ANOVA assumptions (Huberty & Petoskey, 2000). Except for students' understanding of the RPK in question four, evidence from the ANOVA results in Table 15 revealed that, across the school types, students' understanding of the RPK in other question forms were not statistically significantly different at $p=0.05$ significance level.

Evidence in relation to students' understanding of the RPK in question four was a eta square of 0.014. This suggested that 1.4% of the variability in students' understanding of the RPK in question four was accounted for by variations in school types. Comparison of means in Table 15 suggested that in three (questions

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four, six and seven) out of five question forms, the highest students' understanding of the RPK was observed in average achieving schools. The remaining means did not suggest any specific patterns in students' understanding of the RPK.

Table 15: One-Way ANOVA's with Students' Understanding of the RPK and School Type as Independent Variable

Items	Levene		ANOVAs			HPS		APS		LPS	
	F(2,613)	P	F	P	η^2	M	SD	M	SD	M	SD
Question three	1.76	0.17	0.50	0.61	0.002	1.13	0.37	1.15	0.46	1.17	0.43
Question Four	7.91	0.00	4.32	0.01	0.014	1.62	0.81	1.66	0.81	1.45	0.71
Question Five	2.01	0.14	0.70	0.50	0.002	1.51	0.72	1.49	0.68	1.43	0.66
Question Six	4.0	0.02	2.11	0.12	0.007	1.44	0.70	1.54	0.77	1.41	0.71
Question Seven	4.6	0.01	1.26	0.28	0.004	1.27	0.56	1.36	0.65	1.36	0.63

Note: N=616, η^2= eta square
Source: Field data (2017)

Bonferroni post hoc was performed to compare students 'understanding of the RPK in all question forms across the three school types. Findings from the post hoc analysis was consistent with the ANOVA results. At a significant level of 0.05, a difference between average and low achieving school students' understanding of the RPK was observed in question four. In question four, the average achieving school students demonstrated the highest measure of understanding of the RPK, whiles the low achieving school students demonstrated the least measure of understanding of RPK. Other comparison showed no significant difference. The result is presented in Table 16.

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Table 16: Bonferroni Post hoc Test on Students' Understanding of the RPK

School Type	Question Three			Question Four			Question Five			Question Six			Question Seven		
	H	A	L	H	A	L	H	A	L	H	A	L	H	A	L
H															
A															
L					*										

Note: * represents point of significant difference at p=0.05

Legend

H - High Achieving School

A - Average Achieving Schools

L - Low Achieving Schools

Source: Field data (2017)

Research Question Four

What are the characteristic of sentences used in classroom interactions during the teaching and learning of the concept of addition of fractions?

Exploring classroom interaction during the teaching and learning of addition of fractions involved the description of mathematics teachers' and students' sentences (or clauses in their sentences) in the transcribed classroom interactions. The implication was that the characterisation of the sentences (or clauses in the sentences) in the transcribed classroom interaction was therefore that of both the students and their mathematics teachers'. Additionally, implication was that the analysis made no distinction of how the thoughts or the ideas of any of the participants influenced the other participants' ideas but considered them as a mixture of both. Sentences or clauses in the transcribed classroom interaction during the teaching and learning of addition of fractions, represented a means for the participants to express their ideas in relation to addition of fractions. Semantic verbs that were expressed in the sentences or clauses of the transcribed classroom interaction represented a means of classifying and analysing the ideas that were

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expressed by the participants. Semantic verbs like doing, happening, feeling, sensing, saying, behaving, and existing, can be observed in everyday events. For the purpose of analysis, semantic verbs were classified in Systemic Functional Linguistics (SFL) as processes. The processes include mental, material, relational, behavioural, verbal, and existential. Evidence from the data, representing the processes in the sentences are described.

Firstly, sentences or clauses that were categorised as mental processes were those that expressed a state of being in the transcription of classroom interaction for addition of fractions. The sentences involved issues of perception, affection, and cognition. The clause or sentence, 'but I want us to do this first,' (Appendix G, L33) and 'So assuming this is our what?, two sheets,' (Appendix H, L102), were categorised as a mental process because they expressed the wish (affection) of the speaker (mathematics teacher) in the process of teaching addition of fractions. The clauses expressed inner feeling or thinking that was in the mind of the speakers. Consequently, the presence of more of such sentences would imply that the teaching and learning of addition of fractions was mostly described imaginatively.

Secondly, sentences (or clauses within sentences) in transcribed classroom interaction that were categorised as relational processes were typically comparing ideas, expressing meaning, defining, and sometimes describing the properties of a noun or a nominal phrase. The statement by the mathematics teacher, 'Now, when we say equivalent [fractions], meaning that someone is the same as the other, whether the person is older or smaller' (Appendix G, L34) and 'Now this means that we have what?, three out of what?, five' (Appendix H, L108) were categorised

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as relational process because they directly expressed characteristics that could be used to derive meaning in the transcribed classroom interaction during the teaching and learning of addition of fractions. In Appendix H (L34), 'equivalent' was described using a synonymous object, 'someone.' The observation of large numbers of sentences (or clauses in sentences) that are characterised as relational processes during the teaching and learning of addition of fractions has consequent implication. The implication is that the teaching and learning of addition of fractions is mainly by comparison of addition of fractions to other mathematical ideas or identification of properties of other mathematical objects.

The third process is the verbal process. Sentences or clauses categorised as verbal processes during the teaching and learning of addition of fractions were sentences that were identified as reports. In sentences categorised as verbal processes, addition of fractions was described in either a direct or indirect speech form. The transcribed statements in the classroom interaction, 'Somebody is saying denominator over numerator.' (Appendix H, L5) and 'two whole number, two out of five,' (Appendix G, L14) were categorised as verbal process. This is because, they were indirect speech and direct speech, respectively. When majority of sentence or clauses in the transcribed classroom interaction are categorised as verbal processes, it is consequential. The implication is that the teaching and learning of addition of fractions is mainly described as a report of what others had said or learnt.

The fourth process is the material process. Sentences or clauses that were categorised as material process in the transcribed classroom interaction for the

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teaching and learning of addition of fractions communicates the idea of a person or object, taking an action on another person or object. Sentences or clauses categorised as material processes take the structure of animate or inanimate person or object, a verb, and an animate or inanimate object. For instance, the student's statement in the transcription, 'I multiplied four by two,' (Appendix G, L49) was categorised as a material process. This was because the subject of the sentence, 'I' carried out an action, 'multiplied or increased by an amount', on the object, 'two'. Description of components of material processes in terms of grammatical structure takes the form: noun + verb + object. Another example is the sentence, 'So you will take away one of the parts' (Appendix H, L39). When majority of sentences are categorised as material processes, the implication is that the teaching and learning of addition of fractions describes addition of fractions as an action being performed by a person or a thing.

The fifth process is the behavioural process. Behavioural processes include sentences or clauses that expressed the idea of doing something or taking an action as observed in the material process. However, the distinguishing feature of the behavioural process was that it involved one person or participant. For example, 'Student stands up…' (Appendix G, L80) and 'Redeemer continues to give his answer' (Appendix H, L4),' in the transcribed classroom interaction was categorised as behavioural process. The sentences involved the behaviour of 'standing' and 'speaking,' respectively by a single participant. Sometimes, sentences classified as behavioural processes in the transcribed classroom interaction for addition of fractions may be verbs which are normally in the present

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continuous terms (e.g. clapping, talking, laughing, mummuring, etc). When a large portion of clauses or sentences in transcribed classroom interaction are behavioural process, there is a consequential implication. The implication is that the teaching and learning of addition of fractions involves behaviours and demonstrations of certain actions.

The last process is the existential process. Sentences or clauses that were categorised as existential processes were sentences that stated the existence of a fact. Such sentences are identified by 'there is,' 'there are,' 'there were'. Existence can be animate or inanimate object. When a large proportion of sentences or clauses in the transcribed classroom interaction are categorised as existential processes, it is consequential. The implication is therefore that the teaching and learning of addition of fractions is many by statements of facts or summary of main points.

Another aspect of the results that is presented included the characteristic of the human element in the sentences or clauses of the transcribed classroom interaction during the teaching and learning of addition of fractions. The human elements in the sentences include the word(s) that identify the persona in the sentences or clauses. The human elements were in the form of noun or pronouns in the sentences or clauses observed in the transcribed classroom interaction during the teaching and learning of addition of fractions.

Some instance of noun and pronoun in the sentence include he, you, we etc., and specific names (e.g. Agnes). Nouns and pronouns in this study were therefore classified into specific and nonspecific human elements. Some specific human

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element in the sentences or clauses in the transcribed classroom interaction included he, me, Agnes, etc., Specific human elements in sentences or clauses in sentences in the transcribed classroom interaction depict a sense of specificity in who owns (authority) the knowledge of addition of fractions being described. In the sentence, 'Madam, you divide the boxes into two,' the specific human element is 'Madam' (Appendix H, L27). This is because, 'Madam' is the person/noun in the mathematical process of dividing. Hence 'Madam' and 'you' are the authority in the mathematical process. Also evident is the fact that the word 'you,' refers to 'Madam.' Therefore, when sentences in the transcribed classroom interaction exhibits specific human elements, then specific individuals were involved in activities during the teaching and learning of addition of fractions. Additional implication is that the teaching and learning of addition of fractions is conceived in relation to specific individuals in the teaching and learning process. In effect, when most sentences in such transcribed classroom interactions have specific human elements, it implied such classrooms encourage individual learning (and associated understanding) and not necessarily group learning.

Some nonspecific human element in the transcribed classroom interaction during the teaching and learning of addition of fractions include you, we, us, etc (Now we are going to use this in addition of fractions; Appendix G, L39). Therefore, when most sentences in the transcribed classroom interaction exhibits nonspecific human elements in the teaching and learning of addition of fractions, then group(s) (which is nonspecific i.e. 'us') were involved in the teaching and

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learning. In effect, classroom interaction supports group learning (and associated understanding) and not necessarily individual learning.

Common in mathematics and observed in the transcribed classroom interaction during the teaching and learning of addition of fractions was the specialised mathematical languages/symbols: equation, graphs, diagrams, inequalities etc. These specialised mathematical symbols are called mathematical objects in this study. The mathematical objects in this study were classified into three. The basic mathematical objects comprise mainly of lengths, widths, breaths, products, sum, numbers etc. Examples that could be found in the transcribed classroom interaction during the teaching and learning of addition of fractions included $\frac{2}{4}+\frac{1}{2}$ (Appendix G, L54), 4+4 (Appendix G, L61), and 18 (Appendix G, L80) etc. Nonetheless, when basic mathematical objects are involved in any form of equation or inequalities, they are categorised as relational mathematical objects. They are called relational mathematical objects because they establish the connection between mathematical objects that are found on each side of the equation or inequality. Relational mathematical object is the second category of mathematical objects and some examples in the transcribed classroom interaction for the teaching and learning of addition of fractions include; $\frac{21}{18}=\frac{7}{6}=1\frac{1}{6}$ (Appendix G, L90), $\frac{1\times4}{3\times4}+\frac{5\times2}{6\times2}=\frac{4}{12}+\frac{10}{12}=\frac{14}{12}=\frac{7}{6}=1\frac{1}{6}$, $\frac{4}{5}+\frac{2}{7}=\frac{4\times7}{5\times7}+\frac{2\times5}{7\times5}=\frac{28}{35}+\frac{10}{35}=\frac{38}{35}=\frac{13}{35}$ (Appendix G, L91), etc. Representational mathematical object is the last category of mathematical objects and comprise of diagrams, graphs, and tables (Appendix H, L102).

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The prevalence of the various categories of mathematical objects would help inform the characteristic of mathematical objects that were being referred to in the classroom interaction during the teaching and learning of addition of fractions. Undoubtedly, these mathematical objects do not occur in isolation since sentences or clauses in sentences in the transcribed classroom interaction speaks in reference to the mathematical objects. Consequently, tabular illustration of the distribution of processes of the sentences in the transcribed classroom interaction occur with their associated distribution of mathematical objects. These were organised in Table 17, Table 18, and Table 19.

Research Question Five

What are the characteristic of sentences used in classroom interactions during the teaching and learning of the concept of addition of fractions by school context (that is, high, average, and low achieving schools)?

Results of the aforementioned characteristics of sentences used in classroom interaction was presented by school context. Tabular evidence could be observed in Table 17, Table 18, and Table 18 for high achieving, average achieving, and low achieving schools respectively.

Distribution of the characteristic of sentences in classroom interaction from high achieving junior high schools

Table 17 presents data related to transcribed classroom interaction during the teaching and learning of addition of fractions in high achieving schools. It shows the frequency and percentage of the clauses or sentences categorised into the various processes. The table also shows the frequency and percentage of

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mathematical objects in their various categories. Finally, distribution of the human elements or agencies in the transcribed classroom interaction can be observed.

Evidence from Table 17 showed that 33.9% of clauses exhibited the material process characteristics. This suggested that such clauses used in the transcribed classroom interaction exhibited the idea (characteristic) of 'doing' addition of fractions. These clauses could also be 'happenings' that involved a concrete and/or external object being affected (or acted on) by the actor or object in the sentence. Secondly, 23.0% of clauses in the transcribed classroom interaction exhibited the verbal process characteristic. These aforementioned clauses, which portrayed the verbal processes characteristics exhibited the idea of mathematics being taught and learnt as a reportage of someone else's actions or ideas. This reportage may be a report of what was said by the mathematics teacher, student or other people who may not be present within the classroom. Clauses which expressed relational and mental process characteristics were of similar percentage. It was observed that 17.5% of clauses showed characteristic of relational process. These sentences involved the description of properties or relationships of an abstract phenomena. Finally, mental process characteristics were identified with clauses (16.4%) that exhibited an innate idea as a feeling, desire, perception, etc. However, an important aspect of clauses which belonged to the mental processes is that they are all internalized thoughts within the brain. The distribution of the sentences in the transcribed classroom interaction mainly included material, verbal, relational, and behavioural processes.

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Sentences characterized as material processes and with specific human agencies, constituted 51.2%. In contrast, very small percentage of clauses or sentences with specific human agencies was observed in clauses with other process characteristics. Sentences characterized as mental process and with specific human agencies represented 17.1%. Comparatively, lesser percentages (13.4% and 11.0%) of clauses or sentences with specific human agencies was observed in sentences with verbal and behavioural process characteristics, respectively.

In relation to general human agencies, 37.5% of sentences with general human agencies was observed in sentences with verbal process characteristics. Also, equal percentages (25.0% each) of clauses or sentences with general human agencies was observed in sentences with mental and behavioural process characteristics. Sentences with general human agencies that were associated with the material process was 12.5%

Concerning mathematical objects, basic mathematical object was observed to be the largest number (72) of mathematical objects in the transcribed classroom interaction. The number (54) of representational mathematical object followed. Similar percentages (26.4% and 25.0%) of basic mathematical objects were observed among clauses/sentences that exhibited the characteristic of material and verbal processes, respectively. In addition, smaller percentages (22.2% and 18.1%) of basic mathematical objects were observed in clauses/sentences that exhibited characteristic of mental and relational processes, respectively. Representational mathematical objects that was observed among clauses/sentences that demonstrated characteristic of material process comprised 57.4%. In contrast, 18.5% and 12.9%

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of representational mathematical objects were observed among clauses/sentences that demonstrated characteristic of verbal and relational processes respectively. Relational mathematical object that was observed in clauses/sentences that showed characteristic of relational process was 40.0%. This was followed by 25% of relational mathematical objects noted in clauses or sentences which showed characteristic of verbal and material processes each.

Table 17: Distribution of Sentence Processes and other Classroom Interaction Characteristics in High Achieving Schools

Item	Human		Mathematical Object			Clause/
	Specific n(%)	General n(%)	Basic n(%)	Relational n(%)	Representational n(%)	Sentences n(%)
Material	42 (51.2)	1 (12.5)	19 (26.4)	5 (25.0)	31 (57.4)	62 (33.9)
Mental	14 (17.1)	2 (25.0)	16 (22.2)	0 (0.0)	3 (5.6)	30 (16.4)
Relational	6 (7.3)	0 (0.0)	13 (18.1)	8 (40.0)	7 (12.9)	32 (17.5)
Verbal	11 (13.4)	3 (37.5)	18 (25.0)	5 (25.0)	10 (18.5)	42 (23.0)
Behavioral	9 (11.0)	2 (25.0)	6 (8.3)	2 (10.0)	3 (5.6)	17 (9.3)
Existential	0 (0)	0 (0.0)	0 (0.0)	0 (0.0)	0 (0.0)	0 (0.0)
Total	82 (100.0)	8 (100.0)	72 (100.0)	20 (100.0)	54 (100.0)	183 (100.0)

Source: Field data (2017)

Distribution of the characteristic of sentences in classroom interaction from average achieving junior high schools

Table 18 presents data obtained from transcribed classroom interaction during the teaching and learning of addition of fractions in average achieving junior high schools. It shows the frequency and percentages of the clauses or sentences categorised into their various processes. Also evident in Table 18 is the frequency and percentages of mathematical objects in the various sentence process categories. Finally, distribution of human elements per the sentences in the various processes is also presented.

Analysis of transcribed classroom interaction in average achieving schools involved a total of 188 clauses (or sentences) which were examined to determine

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the process characteristic exhibited in them. It was observed that 36.7% of sentences or clauses exhibited the relational process characteristics. This suggested that such sentences/clauses exhibited the idea of description of properties of an object. It implied that such clauses also involved the use of an object to identify another abstract mathematical idea. Closely following relational processes, 30.9% of sentences/clauses exhibited the mental process characteristics. The aforementioned sentences/clauses portrayed the ideas that suggested some form of desire, perception, affection, and/or cognition. Such sentences/clauses described mathematical thoughts as mostly innate ideas. The third frequently observed sentences/clauses, 23.4%, exhibited the behavioural process characteristics. These clauses suggested mathematics as practices that happens in a regular behavioural manner. It normally involved the subject of a clause in the sentence and an action (i.e., a verb, normally in present continuous tense).

The data revealed only four sentences/clauses in the transcribed classroom interaction during the teaching and learning of addition of fractions, had vague or general human agents. Two clauses, 50.0%, among the general human agents were found in sentences with relational process characteristics. One clause each, 25.0%, was observed in sentences with verbal and behavioural process characteristics. In relation to specific human agents, 42.0% were noticed in sentences with mental process characteristics. Comparatively smaller percentage, 25.0% and 21.0%, of sentences with specific human agents were observed in sentences with the behavioural and relational process characteristics, respectively.

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There were no representational objects used in the classroom interaction during the teaching and learning of addition of fractions. Ten sentences were associated with relational mathematical objects. Among the aforementioned, 71.5% were discovered in sentences with relational process characteristics. Eighty-seven of mathematical objects were basic. Among the aforementioned, 51.7% were noticed in sentences observed with the relational process characteristics. This was followed by 29.9% of basic mathematical objects, being observed in sentences with the mental process characteristics.

Table 18: Distribution of Sentence Processes and other Classroom Interaction Characteristics in Average Achieving Schools

Item	Human		Mathematical Object			Clause/ sentences
	Specific n(%)	General n(%)	Basic n(%)	Relational n(%)	Representational n(%)	n(%)
Material	1 (1.0)	0 (0.0)	0 (0.0)	0 (0.0)	0	1(0.5)
Mental	42 (42.0)	0 (0.0)	26 (29.9)	2 (14.3)	0	58(30.9)
Relational	21 (21.0)	2 (50.0)	45 (51.7)	10 (71.5)	0	69(36.7)
Verbal	11 (11.0)	1 (25.0)	7 (8.1)	1 (7.1)	0	16(8.5)
Behavioral	25 (25.0)	1 (25.0)	9 (10.3)	1 (7.1)	0	44(23.4)
Existential	0 (0.0)	0 (0.0)	0 (0.0)	0 (0.0)	0	0 (0.0)
Total	100 (100.0)	4 (100.0)	87 (100.0)	14 (100.0)	0	188 (100.0)

Source: Field data (2017)

Distribution of the characteristic of sentences in classroom interaction from low achieving Junior High Schools

Table 19 presents data from transcribed classroom interaction during the teaching and learning of addition of fractions in low achieving JHS. It shows the frequency and its associated percentages of clauses or sentences categorised into various processes. The table also shows the frequency and percentages of various types of mathematical objects in the processes. Lastly, the distribution of human elements in the transcribed classroom interaction can be observed.

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Analysis of classroom interaction in low achieving schools involved a sum of 106 sentences/clauses which were examined to determine the process characteristics which they exhibited. It was observed in the transcribed classroom interaction, that 41.5% of sentences/clauses exhibited verbal process characteristic. This suggested that most of the clauses used during the teaching and learning of addition of fractions were reports of other peoples' ideas of mathematics or mathematical objects. These other people in the aforementioned verbal process sentences/clauses could be people within or outside of the classrooms. Secondly 32.1% of sentences/clauses used in the transcribed classroom interactions exhibited the material process characteristics. These sentences/clauses exhibited the idea of an actor (subject of the sentence) carrying out an activity (verb) on a concrete object. Finally, 15.1% of sentences/clauses observed during the exploration of transcribed classroom interaction exhibited the behavioural process characteristics. Such clauses which exhibited the behavioural process characteristics normally expressed the notion of mathematics as an activity that is done in a regular manner.

No general human agent was observed in the transcribed classroom interactions in low achieving schools. Sentences with specific human agents that were characterised with the material processes constituted 44.8%. Comparatively smaller percentages, 25.9% and 17.2%, of the sentences with specific human agents were characterised with verbal and behavioural processes respectively.

No representational mathematical object was observed during the teaching and learning of addition of fractions. Four relational mathematical object was observed. Fifty percent of sentences with relational mathematical objects were

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observed in sentences characterised with behavioural processes. Twenty-five percent of relational mathematical objects were noted in sentences characterised with material processes. The same aforementioned was also observed with relational mathematical objects noted in sentences characterised with the verbal processes.

Table 19: Distribution of Sentence Processes and other Classroom Interaction Characteristics in Low Achieving School

Item	Human		Mathematical Object			Clause/
	Specific n(%)	General n(%)	Basic n(%)	Relational n(%)	Representational n(%)	sentences n(%)
Material	26 (44.8)	0 (0.0)	25 (35.2)	1 (25.0)	0 (0.0)	34(32.1)
Mental	2 (3.5)	0 (0.0)	1 (1.4)	0 (0.0)	0 (0.0)	4(3.8)
Relational	5 (8.6)	0 (0.0)	2 (2.8)	0 (0.0)	0 (0.0)	7(6.7)
Verbal	15 (25.9)	0 (0.0)	37 (52.1)	1 (25.0)	0 (0.0)	44(41.5)
Behavioral	10 (17.2)	0 (0.0)	6 (8.6)	2 (50.0)	0 (0.0)	16(15.1)
Existential	0 (0.0)	0 (0.0)	0 (0.0)	0 (0.0)	0 (0.0)	1(0.9)
Total	58 (100.0)	0 (100.0)	71 (100.0)	4 (100.0)	0 (0.0)	106 (100.0)

Source: Field data (2017)

Research Question Six

What are students and their mathematics teachers' understanding of the concept of addition of fractions?

In examining respondents understanding of the concept of addition of fractions, the study generally used three criteria to determine the categorization of respondents' understanding: no understanding, partial understanding, and full understanding of fractions addition. These criteria include visual illustration, written description or explanations, and numerical procedure. A combination of any two criteria was used to determine respondents' category of understanding.

The study presented descriptors for no understanding, partial understanding, and full understanding of the concept of addition of fractions. These exemplary cases were obtained from respondents' feedback that were observed during the

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study. Across the addition of fractions questions used in this study, the tabular description of requirements for no understanding, partial understanding and full understanding of the concept of addition of fractions is indicated in the scoring rubric in Appendix E. The rubric was developed in line with the two categories of fraction (fraction less than 1, and fraction greater than 1) involved in the study. In view of the fact that there were various questions used in accessing understanding of the concept of addition of fractions, within this text in chapter four, exemplary illustrations were randomly selected from all questions. However, to avoid redundancy in representations for each category of understanding, examples from two randomly selected questions were used to explain the characteristic requirement for the specified category of understanding. Only one representation was used to illustrate full understanding of addition of fractions. This was because, there was only one way of representing full understanding, regardless of the category of fraction.

No understanding of the concept of addition of fractions

Respondents who demonstrated no understanding of the concept of addition of fractions showed inability to properly represent the fractional addend with a visual illustration. No understanding also involved the ability to represent the fractional addend and an inability to represent or explain any form of transformation of the fractional addends involved in the fractions addition problem. Again, respondents who demonstrated no understanding of the concepts of addition of fractions also displayed the inability to follow the appropriate procedure in solving the addition of fractions problems (see Figure 26 and Figure 27). Some

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respondents' inappropriate procedure in approaching the addition of fractions problem included the addition of numerators and/or denominators of the addends. When only the numerators of the addends were added, the denominators were mostly obtained on the basis of LCM. Under such condition, the visual illustration of the resulting sum in the addition of fractions problem was mostly a product of the procedure. Consequently, in the visual illustration of the sum, the denominator is obtained from the LCM and the numerator of the sum (shaded portion, see Figure 26 and 25) is obtained from the addition of the numerator values of the addends. Figure 26 showed the visual illustrations and procedure of one of such respondents' attempt in dealing with question two in the addition of fractions problem. The representations from the aforementioned respondents suggested that the respondent did not have any form of understanding of the concept of addition of fractions beyond the representation of the fractional addends. The respondents' connection or association of ideas between the visual illustrations of the addend and the resulting sum, lies in the procedure for solving addition of fractions problem. The missing link here was that most of such respondents did not demonstrate the ability to comprehend the fact that the transformation in the LCM needed to be reflected in the visual illustrations of the addend. It is such transformation that would make the whole of the sum similar to the whole of the addends. The procedure for obtaining the sum in the fractions addition problem was also incorrect.

In terms of procedure for fractions addition, there were other situations of no understanding, where the numerator and the denominator of the fractional addend were just added to obtain the numerator and denominator value of the sum.

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The aforementioned respondents, in approaching the fractions addition problem, demonstrated that their understanding was supported by visual illustrations of the numerator of the addends (shaded portion) being put together to obtain the numerator value (shaded portion) of the sum. The denominator value of the sum was also obtained by means of adding the number of divisions or blocks in the two addends' visual illustrations. This is very similar to the addition of the numerator values and addition of denominator values of the addend to get the numerator and denominator of the sum. For such respondents, the idea of connection or association between the visual illustration of the addend and the sum is not in only the results of the procedure but also in the visual illustrations of the addend and the sum. The difficulty faced by such respondents was their inability to maintain the whole of the addends in the sum. Hence the denominator value of the addend and the sum varied. Although this was not always the occasion, a common visual illustration for fractions that supported such form of understanding was the fraction structure. This form of visual illustration for the addend showed a number of items separated by a division line. Consequently, the number of items on the upper part (above the division line) was the numerator of the fraction and the number of items on the lower part (below the division line) was the denominator of the fraction.

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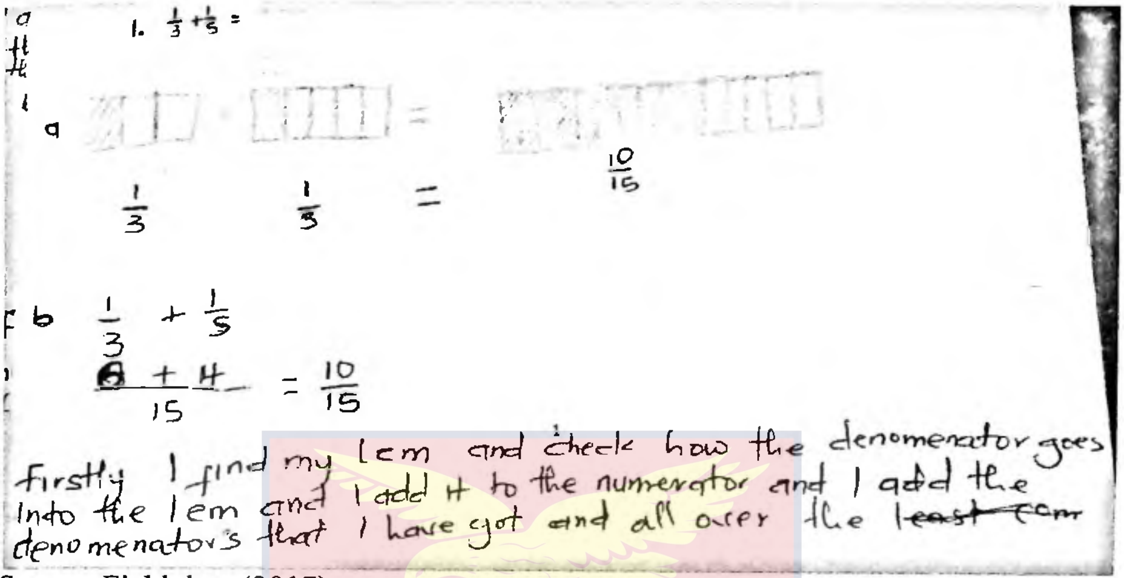
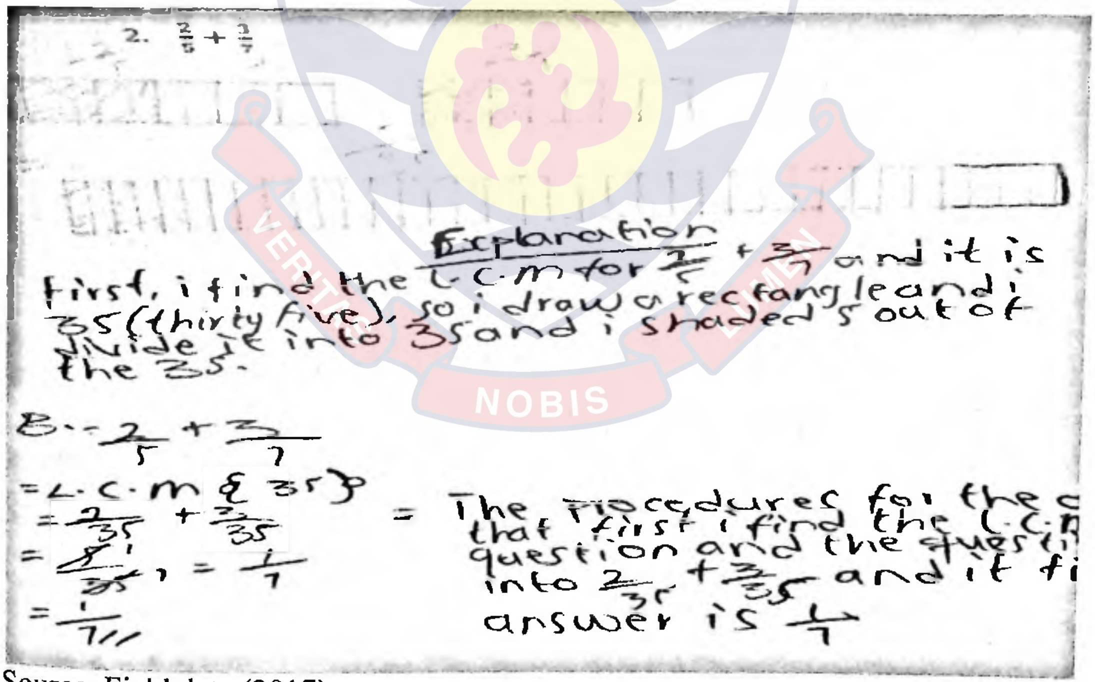


Source: Field data (2017)

Figure 26: Respondents' (student)representation categorised as no understanding of the concept of addition of fractions (NU) in question two



Source: Field data (2017)

Figure 27: Respondents' (teacher) representation categorised as no understanding of the concept of addition of fractions (NU) in question one

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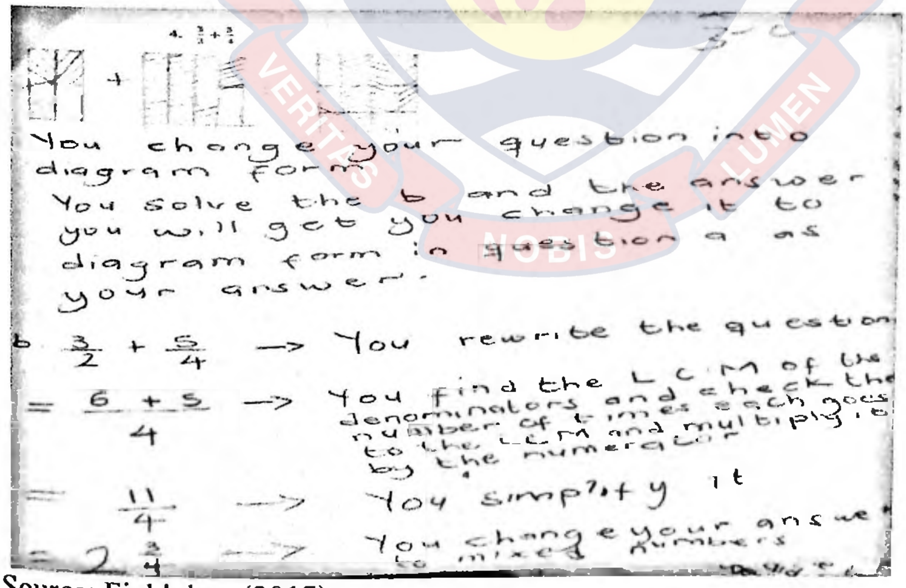
Partial understanding of the concept of addition of fractions

Most respondents who demonstrated partial understanding of the concept of addition of fractions were able to demonstrate understanding that resulted in obtaining the correct sum. The correct sum may be in different forms. However, once the correct sum was obtained, the categorization of understanding moves beyond no understanding to partial understanding. The correctness of the sum could either be observed in the visual illustration of the sum or product of the procedure. It was therefore deemed important to categorise partial understanding based on the presence of connection or association in thoughts or ideas between the visual illustration or representation, and explanation and/or procedure. The implication is the categorisation of partial understanding as, partial understanding with connected/associated thoughts (PU_A) and partial understanding without connected thoughts or with separated thoughts (PU_S).



Source: Field data (2017)

Figure 28: Respondents' (student) representation categorised as partial understanding of the concept of addition of fractions with separated thoughts (PU_S) in question four

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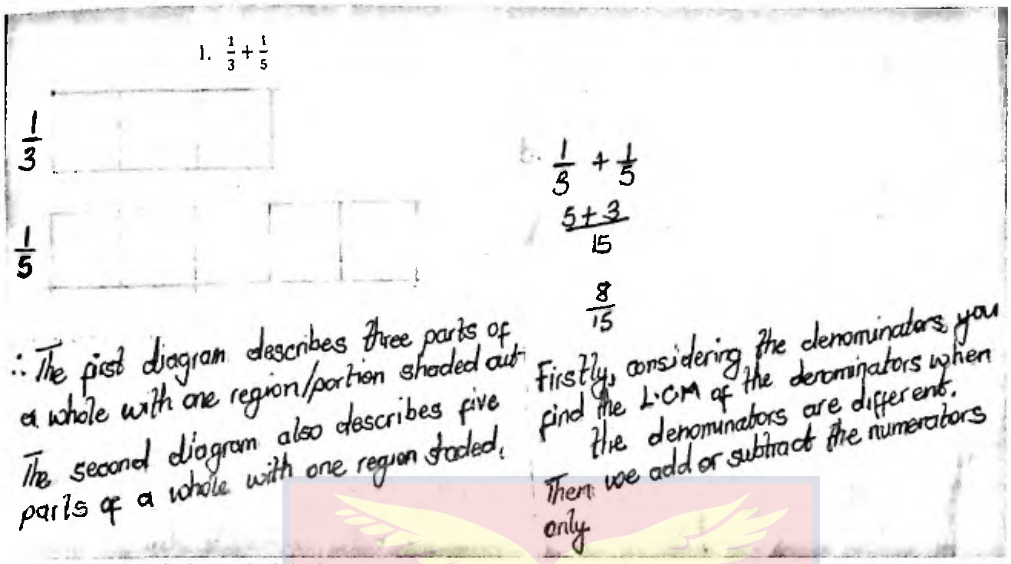

Source: Field data (2017)

Figure 29: Respondents' (teacher) representation categorised as partial understanding of the concept of addition of fractions with separated thoughts (PU_S) in question two

Respondents that demonstrated partial understanding of the concept of addition of fractions, demonstrated the ability to use the procedural method in solving the fractions addition problem. The two most common procedural methods that was observed were the box method and the LCM method. Demonstration of visual illustration of the resulting fractional sum was not much of a problem to the respondents but the ability to connect such illustrations to the sum in the procedure was what categorised the respondents into partial understanding of the concept of addition of fractions with connected or associated thoughts (PU_A) and partial understanding of the concept of addition with separated thoughts (PU_S). Hence most respondents who demonstrated partial understanding of the concept of addition of fractions with separated thoughts (PU_S) were able to use the procedural method to obtain the correct sum. However, the thoughts or ideas in the visual illustrations were considered separate and not connected to the sum obtained

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through the procedure. This mostly resulted in the respondents representing a different fractional sum in the visual illustration as compared to the sum obtained in the procedure. This suggested an inability to connect the procedure with the visual illustrations. It also showed the inability to appropriately transform the visually illustrated fractional addend (just like the procedure) into the sum. The aforesaid respondents with partial understanding of the concept of addition of fractions with separated thoughts (PU_S) could only use the procedural means to solve the fractions addition problem. A sample snapshot of respondents' script who demonstrated partial understanding of the concept of addition of fractions with separated thoughts (PU_S) is presented in the Figure 28 and Figure 29.

Figure 28 illustrates a respondents' response to question four. As can be observed from the respondents' own written explanation, the respondent solved the question using the LCM method to obtain the answer. After that, the respondent attempted to represent the answer in the visual illustration. However, the illustration was wrong. As could be observed, the respondent also found it difficult representing fractions greater than one. Similar observation could be found in Figure 29, where the respondent answered question two. The respondent demonstrated understanding of the procedure of equivalent fraction to obtain the sum. However, connecting the sum from the procedure to that of the representation was lacking. In a nutshell, partial understanding of the concept of addition of fractions with separated thoughts (PU_S) suggested that for those who could use visual illustrations, there was no demonstrated connection or link in the understanding from both procedural method and the visual illustrations.

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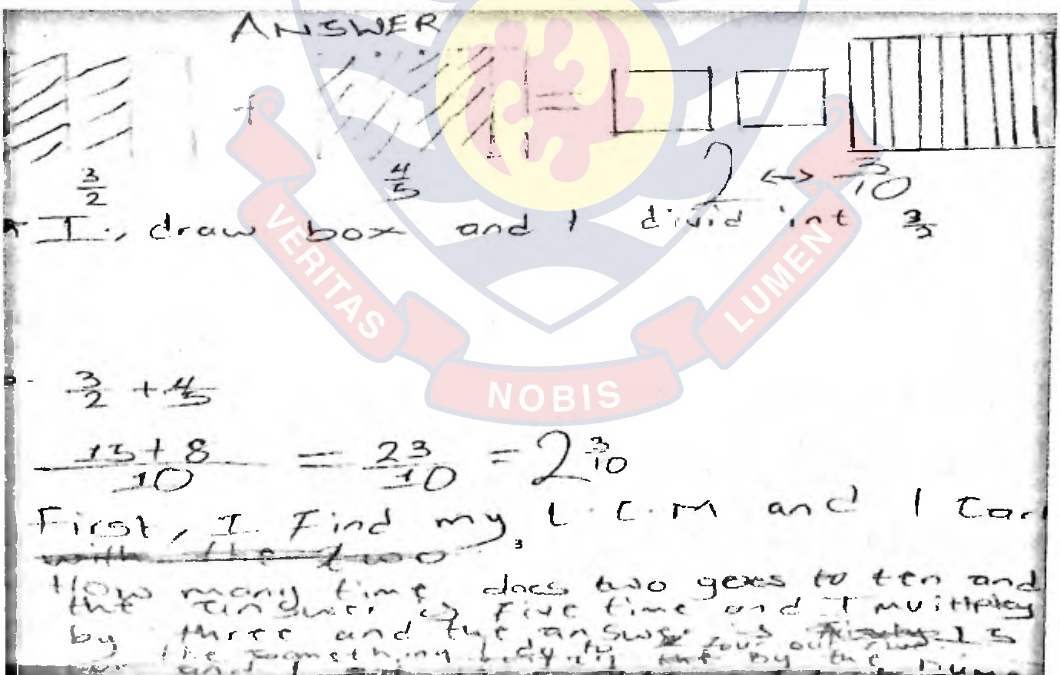
Partial Understanding of the concept of addition of fractions with connections or association (PU_A) is a step ahead of partial understanding of the concept of addition of fractions with separated thoughts (PU_S). Respondents who demonstrated partial understanding of the concept of addition of fractions with associated thoughts (PU_A) were able to use the procedural method to obtain the sum. It is easily observable that the sum and the addends were approximately the same size because all the values were still less than one (see Figure 30). The sum obtained from the procedural method was then used to generate the sum in the visual illustration. Consequently, in partial understanding with associated thoughts, the visual illustrations of the addend have no connection to the visual illustration in the sum except by means of the procedure.



Source: Field data (2017)

Figure 30: Respondents' (student) representation categorised as partial understanding of the concept of addition of fractions with associated thoughts (PU_A) in question two

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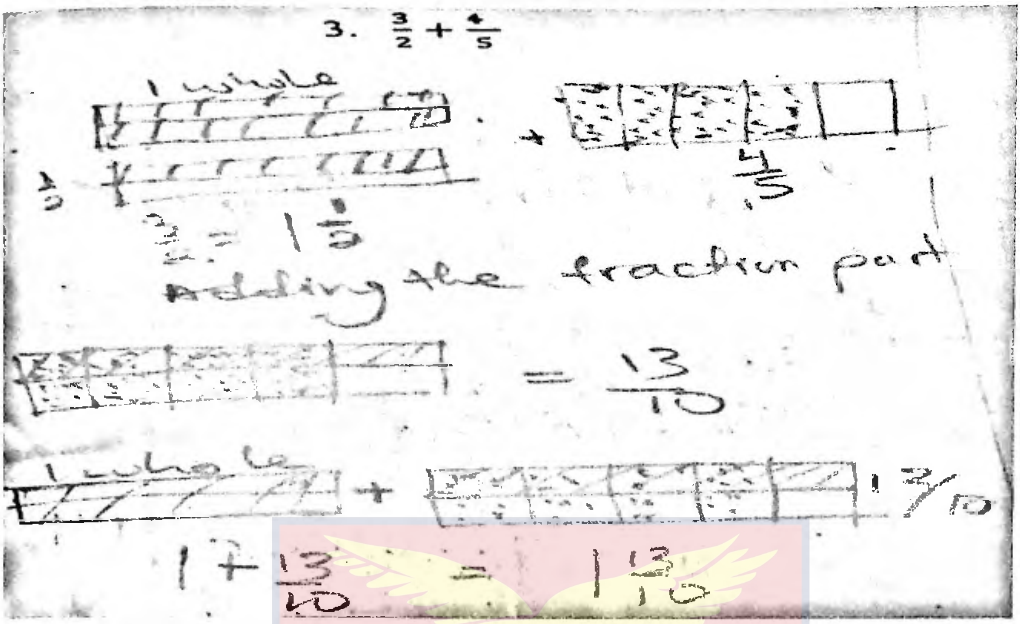

Source: Field data (2017)

Figure 31: Respondents' (teacher) representation categorised as partial understanding of the concept of addition of fractions with associated thoughts (PU_A) in question three

Respondents' illustration of partial understanding of the concept of addition of fractions with associated thoughts (PU_A) in Figure 30 followed similar but not the same description as in Figure 31. Having said that, the difference was in the fact that the sum was a mixed fractional value (see Figure 30). The visual illustration of the whole number in the sum was of different size from the visual illustration of the fractional component of the sum. To compensate, the respondent used a double ended arrow in the symbolic writing (just below the visual illustration) to show the equality of the two components (i.e., fractional component and whole number component) in the sum.

Full understanding of the concept of addition of fractions

Among respondents who demonstrated full understanding of the concept of addition of fractions, strategies were observed in the feedback. In the strategies, it

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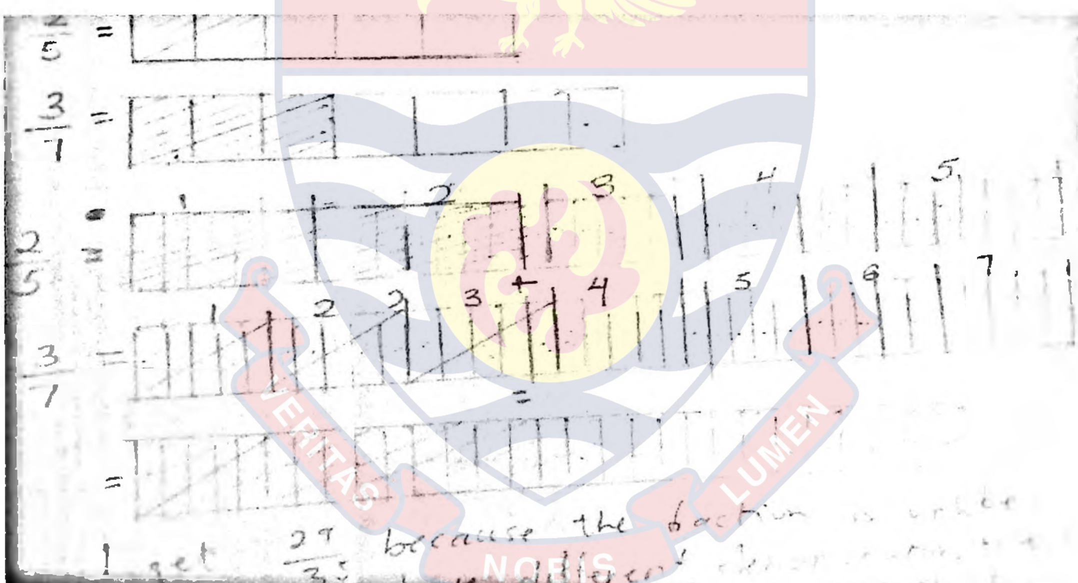
was observed that the main ideas of full understanding were demonstrated. Beyond partial understanding of the concept of addition of fractions with associated thoughts (PU_A), was the act of transforming each visual illustration of the addend from its fractional form to an equivalent fractional form. This transformation was tantamount to the procedure of LCM that generated the equivalent fractional forms in the addition of fractions problem. Another important feature with the equivalent forms of the visually illustrated fractional addend was that it should be referenced to the similar sized whole in the sum. Figure 32 shows a snapshot of sampled script that demonstrated full understanding.



Source: Field data (2017)

Figure 32: Respondents'(teacher) representation categorised as full understanding of the concept of addition of fractions (FU)

Although the same sized whole (see Figure 32) was not demonstrated in the initial visual illustration of the addends, the second visual illustration of the equivalent forms of the addend were of similar sized whole. Finally, visual illustration of the sum was of similar sized whole. Just as the use of visual

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illustration alone was accepted for demonstration of full understanding, the use of procedural method alone was also considered acceptable for demonstration of full understanding. However, in demonstrating full understanding, the procedural method would also have to show the transformation of the fractions to their equivalent forms. A major convincing characteristic in demonstrating full understanding in the procedural method was the ability of the procedure to show the same counting unit of the equivalent form of the addend (as shown in the transformed addends' visual illustration in Figure 32). In the example shown in the Figure 32, the basic counting unit is the unit fraction (1/35) of the two equivalent addend. Finally, the sum must have the same basic counting unit as the transformed or equivalent addends. In demonstration of full understanding, if the sum could be reduced to simpler fractional form, it was expected that the basic counting unit of the sum that is equivalent to the basic counting unit of the addends, would have to be indicated before the transformation to any other equivalent simpler forms (see Figure 3, Figure 5, Figure 33). Although written explanation was preferred in combination with the visual illustration and procedure, it was not a sufficient condition for demonstration of full understanding.

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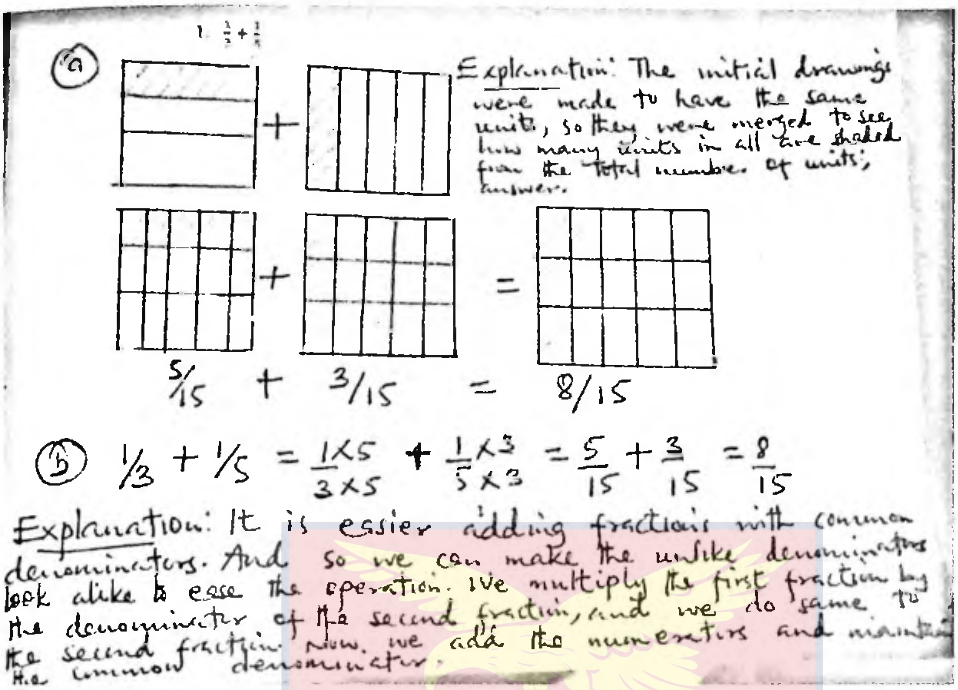

Source: Field data (2017)

Figure 33: Respondents' (student) representation categorised as full understanding of the concept of addition of fractions (FU)

Research Question Seven

What are students and their mathematics teachers' understanding of the concept of addition of fractions by school context (that is, high, average, and low achieving schools)?

Mathematics teachers and students understanding of the concept of addition of fractions is presented. Students' understanding is presented first in the various school categories and mathematics teachers' understanding is presented later. Tables include frequency and percentage of students who demonstrated the various aforementioned categories of understanding of addition of fractions across the various question forms.

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Students understanding of the concept of addition of fractions

Tables 20, 21, and 22 shows distribution of students understanding of the concept of addition of fractions across the different questions. Details of students understanding of the concept of addition of fractions is presented per the schools' categories.

High achieving school students' understanding of the concept of addition of fractions

In high achieving schools, Table 20 shows frequency distribution of the form of understanding across the diverse questions used in the study. Across all the questions, the average percentage of students who demonstrated no understanding of the concept of addition of fractions was 44.3%. Accordingly, such students in high achieving schools had no appreciation of the fundamental ideas of the concept of addition of fractions. This was closely followed by the average percentage of students who demonstrated partial understanding of the concept of addition of fractions with separated thoughts (PU_S), i.e., 40.4%. Across all the questions, a comparatively small average of 14.3% of students demonstrated partial understanding of the concept of addition of fractions with associated thoughts (PU_A). On the average, only one percent of the students in high achieving schools were able to fully understand the concept of addition of fractions. This suggested that students who were able to fully comprehend what was instructed in the classroom were just about one percent in high achieving schools.

Among students who demonstrated no understanding of the concept of addition of fractions (NU), their percentage generally increased across the

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questions. In a similar manner, the percentage of students who demonstrated partial understanding of the concept of addition of fractions with separated thoughts (PU_S) continually increased across all the questions (one to four). However, for students who demonstrated partial understanding of the concept of addition of fractions with associated thoughts (PU_A), their percentage generally dropped sharply. Among students who were able to demonstrate full understanding of the concept of addition of fractions, their percentages were relatively stagnant across all the questions. This therefore suggested that unlike students who could comprehend some of the concept of addition of fractions (PU_S and PU_A) or could not comprehend anything in relation to the concept of addition of fractions (NU), when students develop full comprehension of the concept of addition of fractions, they rarely loose such ability irrespective of the question types used in this study.

Among the percentages of students who demonstrated no understanding of the concept of addition of fractions (NU) and partial understanding of the concept of addition of fractions with separated thoughts (PU_S), there was a general progressive increase across the questions. Simultaneously, there was rather a general progressive reduction in percentages of students who demonstrated partial understanding of the concept of addition of fractions with associated thoughts (PU_A). This suggested that as the questions progressed, there was a redistribution in students' demonstrated understanding from partial understanding of the concept of addition of fractions with associated thoughts (PU_A), to partial understanding

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of the concept of addition of fractions with separated thoughts (PU_S) and no understanding of the concept of addition of fractions (NU).

However, between questions three and four, the redistribution of percentages of students demonstrating a category of understanding of addition of fractions was from students who demonstrated no understanding of the concept of addition of fractions (NU), to students who demonstrated partial understanding of the concept of addition of fractions with associated thoughts (PU_A) and partial understanding of the concept of addition of fractions with separated thought (PU_S). This observation is caused by the difference between question three and question four. The major difference between question three and question four was that question four had both addends in the addition of fractions problem to be fractions greater than one while question three had only one addend in the addition of fractions problem to be greater than one. These findings could be observed in the Table 20. Thus, students in high achieving schools better understood addition of fractions in cases where both addend have whole number and fraction parts, than problems where both added are of different types.

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Table 20: High Achieving School Students' Understanding of the Concept of Addition of Fractions

Questions	NU N (%)	PU_S N (%)	PU_A N (%)	FU N (%)	Total N (%)
One	74 (40.7)	57 (31.3)	49 (26.6)	2 (1.1)	182 (100.0)
Two	80 (44.2)	70 (38.7)	29 (16.0)	2 (1.1)	181 (100.0)
Three	87 (48.9)	80 (44.9)	10 (5.6)	1 (0.6)	178 (100.0)
Four	69 (43.4)	74 (46.5)	14 (8.8)	2 (1.3)	159 (100.0)
Average %	(44.3)	(40.4)	(14.3)	(1.0)	(100.0)

Legend

NU : No Understanding of the concept of addition of fractions

PU_S : Partial Understanding of concepts of addition of fractions with Separated Thought/Ideas

PU_A : Partial Understanding of the concept of addition of fractions with Connected/Associated Thought/Ideas

FU : Full Understanding of the concept of addition of fractions

Source: Field data (2017)

Average achieving school students' understanding of the concept of addition of fractions

In average achieving schools, Table 21 displays frequency distribution of the various students' demonstrated understanding, across the various question forms. Across the questions, the average percentage of students who demonstrated partial understanding of the concept of addition of fractions with separated thoughts (PU_S) was 46.4%. This was followed by the average percentage of students who demonstrated no understanding of concept of addition of fractions (NU) i.e., 37.7%. Across the questions, a comparatively smaller average percentage of students, 14.5%, demonstrated partial understanding of the concept of addition of fractions with associated thoughts (PU_A). Finally, across the questions, an average of 1.4% of students in average achieving schools demonstrated full understanding of the concept of addition of fractions (FU) at the Junior High School level.

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Table 21: Average Achieving School Students' Understanding of the Concept of Addition of Fractions

Questions	NU	PU_S	PU_A	FU	Total
	N (%)	N (%)	N (%)	N (%)	N (%)
One	68 (31.8)	91 (42.5)	51 (23.8)	4 (1.9)	214 (100.0)
Two	69 (32.4)	95 (44.6)	45 (21.1)	4 (1.9)	213 (100.0)
Three	98 (47.6)	93 (45.1)	14 (6.8)	1 (0.5)	206 (100.0)
Four	73 (39.0)	100(53.5)	12 (6.4)	2 (1.1)	187 (100.0)
Average %	(37.7)	(46.4)	(14.5)	(1.4)	(100.0)

Legend

NU : No Understanding of the concept of addition of fractions

PU_S : Partial Understanding of concepts of addition of fractions with Separated Thought/Ideas

PU_A : Partial Understanding of the concept of addition of fractions with Connected/Associated Thought/Ideas

FU : Full Understanding of the concept of addition of fractions

Source: Field data (2017)

The percentage of students who demonstrated no understanding of the concept of addition of fractions (NU) generally increased across the questions. There was a continual increase in the percentage of students who demonstrated partial understanding of the concept of addition of fractions with separated thoughts (PU_S) from questions one to four. However, for the percentage of students who demonstrated partial understanding of the concept of addition of fractions with associated thoughts (PU_A), there were continual decreases from question one to four. The percentage of students who demonstrated full understanding of the concept of addition of fractions didn't vary much.

There were the general increases in the percentage of students who demonstrated no understanding of the concept of addition of fractions (NU) and partial understanding of the concept of addition of fractions with separated thoughts (PU_S). Also across the questions, there were general decreases in the percentage of students who demonstrated partial understanding of the concept of addition of

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fractions with associated thoughts (PU_A). Consequently, the data suggested that across the questions, there was a progressive redistribution of students who demonstrated partial understanding of the concept of addition of fractions with associated thoughts (PU_A), to those who demonstrated no understanding of the concept of addition of fractions (NU) and partial understanding of addition of fractions with separated thoughts (PU_S). This suggested that as the questions progressed, the students who demonstrated partial understanding of the concept of addition of fractions with associated thoughts (PU-A) could no more do so but demonstrated either no understanding of the concept of addition of fractions (NU) or partial understanding of the concept of addition of fractions with separated thoughts (PU_S).

However, between question three and four, the redistribution of the percentages was from students who demonstrated no understanding of the concept of addition of fractions (NU), to students who demonstrated partial understanding of the concept of addition of fractions with separated thoughts (PU_S) and partial understanding of the concept of addition of fractions with associated thoughts (PU_A). This observation was caused by the difference between question three and question four. The major difference between question three and question four was that question four had both addends to be fractions greater than one while question three had only one addend to be greater than one. These findings could be observed in Table 21. Thus, students in average achieving schools better understood problems where both addends in the fractions addition problem are greater than one.

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Low achieving school students' understanding of the concept of addition of fractions

Table 22 displays the frequency distribution of students in low achieving schools who demonstrated the various forms of understanding across the questions. The data suggested that averagely, across the questions, 54.7% of students in low achieving schools demonstrated no understanding of the concept of addition of fractions (NU) after instruction. Averagely across the questions, 37.5% of students demonstrated partial understanding of the concept of addition of fractions with separated thoughts (PU_S). Across the questions, a comparatively smaller average percentage of students in low achieving schools, 7.8%, demonstrated partial understanding of the concept of addition of fractions with associated thoughts (PU_A). After instruction, no student in low achieving school could demonstrate full understanding of the concept of addition of fractions.

Regarding the percentages of students who demonstrated no understanding of the concept of addition of fractions (NU), there was no visible trend in progression across the various questions. Similarly, for the percentage of students who demonstrated partial understanding of the concepts of addition of fractions with separated thoughts (PU_S), there was no clearly observed trends. However, between question one and question two, there was a noticeable increase in the percentage of students who demonstrated partial understanding of the concept of addition of fractions with separated thoughts (PU_S). The data showed that for students who demonstrated partial understanding of the concept of addition of

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fractions with associated thoughts (PU_A), there was a progressive declining trend in the percentage across the questions.

Between question one and two, there was a sharp increase in the percentage of students who demonstrated partial understanding of the concept of addition of fractions with separated thoughts (PU_S). Simultaneously, there was a sharp decline in percentage of students who demonstrated partial understanding of the concept of addition of fractions with associated thoughts (PU_A). This suggested that for students in low achieving junior high schools, a good percentage of them who demonstrated partial understanding of the concept of addition of fractions with associated thoughts (PU_A), eventually demonstrated partial understanding of the concept of addition of fractions with separated thoughts (PU_S) when they encountered proper fractional addends. These could be observed from Table 22.

Table 22: Low Achieving School Students' Understanding of the Concept of Addition of Fractions

Questions	NU N (%)	PU_S N (%)	PU_A N (%)	FU N (%)	Total N (%)
One	120 (55.3)	62 (28.6)	35 (16.1)	0 (0.0)	217 (100.0)
Two	112 (52.1)	86 (40.0)	17 (7.9)	0 (0.0)	215 (100.0)
Three	116 (55.2)	86 (41.0)	8 (3.8)	0 (0.0)	210 (100.0)
Four	112 (56.3)	80 (40.2)	7 (3.5)	0 (0.0)	199 (100.0)
Average %	(54.7)	(37.5)	(7.8)	(0.0)	(100.0)

Legend

NU : No Understanding of the concept of addition of fractions

PU_S : Partial Understanding of concepts of addition of fractions with Separated Thought/Ideas

PU_A : Partial Understanding of the concept of addition of fractions with Connected/Associated Thought/Ideas

FU : Full Understanding of the concept of addition of fractions

Source: Field data (2017)

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Mathematics teachers' understanding of the concept of addition of fractions

Frequency distribution of mathematics teachers' demonstrated understanding is displayed in Tables 23, Table 24, and Table 25. Details of mathematics teachers' understanding of the concept of addition of fractions could be observed per the three school categories.

High achieving school mathematics teachers' understanding of the concept of addition of fractions

Table 23 shows the frequency distribution of mathematics teachers understanding of addition of fractions. The average percentage of teachers who demonstrated partial understanding of the concept of addition of fractions with separated thoughts (PU_S) constituted 60.0%. This was closely followed by the average percentage of teachers who demonstrated full understanding of the concept of addition of fractions (FU) i.e., 40.0%. No teacher demonstrated partial understanding with connected/associated thoughts (PU_A) and no understanding (NU). This suggested that less than half of the teachers in high achieving schools could demonstrate the required level of understanding of the concept of addition of fractions.

For teachers who demonstrated partial understanding with separated thoughts (PU_S), the highest percentage was in question two, whiles the least percentage was in question one. Consequently, they constituted 80% and 40% of teachers respectively. Among teachers who demonstrated full understanding of the concept of addition of fractions, the least percentage was noted for question two whiles the highest percentage was noted in question one. Accordingly, they constituted 20% and 60% of teachers respectively. Across question one to four, the

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general trend suggested that for teachers who demonstrated full understanding (FU), only one lost the ability to demonstrate full understanding to the advantage of the percentage of teachers who demonstrated partial understanding with separated thoughts (PU_S). Ability to demonstrate full understanding was lost when the participant encountered an addition problem where one addend was a proper fraction. Details could be viewed in the Table 23.

Table 23: High Achieving School Mathematics Teachers' Understanding of the Concept of Addition of Fractions

Questions	NU N (%)	PU_S N (%)	PU_A N (%)	FU N (%)	Total N (%)
One	0 (0.0)	2 (40.0)	0 (0.0)	3 (60.0)	5 (100.0)
Two	0 (0.0)	4 (80.0)	0 (0.0)	1 (20.0)	5 (100.0)
Three	0 (0.0)	3 (60.0)	0 (0.0)	2 (40.0)	5 (100.0)
Four	0 (0.0)	3 (60.0)	0 (0.0)	2 (40.0)	5 (100.0)
Average %	0.0%	60.0%	0.0%	40.0%	100.0%

Legend
H - High Achieving School
A - Average Achieving Schools
L - Low Achieving Schools
Source: Field data (2017)

Average achieving school mathematics teachers' understanding of the concept of addition of fractions

Table 24 displays the frequency distribution of mathematics teachers' understanding of addition of fractions. In average achieving schools, the average percentage of teachers who demonstrated full understanding (FU) of the concept of addition of fractions constituted the 56.3%. This was followed by the average percentage of teachers who demonstrated partial understanding with separated thoughts (PU_S) i.e. 37.5%.

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Among teachers who demonstrated full understanding (FU) of the concept of addition of fractions, the highest percentage was observed in question one and four whiles the smallest percentage was observed in question two. Accordingly, they constituted 75.0% and 25.0% respectively. It implied that the teachers who could not demonstrate full understanding of the concept of addition of fractions in question two and three eventually demonstrated full understanding in question four. Among teachers who demonstrated partial understanding of the concept of addition of fractions with separated thoughts (PU_S), the highest percentage was observed in questions two and three whiles the smallest percentage was observed in question one and four. Accordingly, the highest and smallest percentage constituted 50.0% and 25.0% respectively. Hence, there were no clear patterns in teachers demonstrated understanding as the teachers engaged with the added complexities across the questions. Details of the findings could be observed in the Table 24.

Table 24: Average Achieving School Mathematics Teachers' Understanding of the Concept of Addition of Fractions

Questions	NU N (%)	PU_S N (%)	PU_A N (%)	FU N (%)	Total N (%)
One	0 (0.0)	1 (25.0)	0 (0.0)	3 (75.0)	4 (100.0)
Two	0 (0.0)	2 (50.0)	0 (0.0)	2 (50.0)	4 (100.0)
Three	1 (25.0)	2 (50.0)	0 (0.0)	1 (25.0)	4 (100.0)
Four	0 (0.0)	1 (25.0)	0 (0.0)	3 (75.0)	4 (100.0)
Average %	6.2	37.5	0 (0.0)	56.3	100.0

Legend

H - High Achieving School

A - Average Achieving Schools

L - Low Achieving Schools

Source: Field data (2017)

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Low achieving school mathematics teachers' understanding of the concept of addition of fractions

Table 25 shows the frequency distribution of mathematics teachers' demonstrated understanding of addition of fractions. In low achieving schools, the average percentage of teachers who demonstrated full understanding of the concept of addition of fractions constituted 75.0%. Comparatively, the average percentage of teachers who demonstrated partial understanding of the concept of addition of fractions with separated thoughts (PU_S) was small, i.e., 14.3%. Finally, the average percentage of teachers who demonstrated partial understanding of the concept of addition of fractions with associated thoughts (PU_A) was 10.7%. No teacher demonstrated No Understanding (NU) of the concept of addition of fractions. Generally, teachers in low achieving schools demonstrated the required level of understanding of the concept of addition of fractions.

Among teachers who demonstrated full understanding of the concept of addition of fractions, the highest percentage was observed in question one whiles the least percentage was observed in the other questions. The highest and least percentages were 85.7% and 71.4% respectively. Among the teachers who demonstrated full understanding in question one, only one could not demonstrate full understanding in the remaining questions. This resulted in one (14.3%) teacher who demonstrated partial understanding with associated thoughts (PU_A). Details of the above findings could be observed in the Table 25.

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Table 25: Low Achieving School Mathematics Teachers' Understanding of the Concept of Addition of Fractions

Questions	NU N (%)	PU_S N (%)	PU_A N (%)	FU N (%)	Total N (%)
One	0 (0.0)	1 (14.3)	0 (0.0)	6 (85.7)	7 (100.0)
Two	0 (0.0)	1 (14.3)	1 (14.3)	5 (71.4)	7 (100.0)
Three	0 (0.0)	1 (14.3)	1 (14.3)	5 (71.4)	7 (100.0)
Four	0 (0.0)	1 (14.3)	1 (14.3)	5 (71.4)	7 (100.0)
Average %	0 (0.0)	14.3	10.7	75.0	100.0

Legend

H - High Achieving School

A - Average Achieving Schools

L - Low Achieving Schools

Source: Field data (2017)

Research Question Eight

What is the effect of school context on students' understanding of the concept of addition of fractions?

One-way Multivariate Analysis of Variance (MANOVA) was performed to examine the mean differences in students' understanding over four addition of fractions question forms, across the three school types. Prior to the MANOVA, a correlation matrix table of the addition of fractions question forms was generated to examine correlation among themselves. An assumption that needs to be satisfied for MANOVA is correlation among the independent variables. A correlation coefficient between 0.2 and 0.6 is necessary among majority of the independent variables for correlation assumption to be satisfied (Howell, 2009). Observation from Table 26 suggests that the pre-requisite for multi-collinearity was satisfied for MANOVA. Another assumption for MANOVA is that the covariance matrices of the addition of fractions question forms across the three school types must be equal. However, the Box's M of 59.794 ($p=0.000<0.005$) suggested that the covariances

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among the independent variables were not equal. In the event of violation of conditions necessary for MANOVA, it is recommended that the test statistics that is least sensitive to violations in equality of covariance matrices condition be adopted. Pillai's Trace is recommended because it is least sensitive to the violation of equality of covariance matrices (Huberty and Petoskey, 2000).

Table 26: Pearson correlations, Means, and standard deviations associated with concept of addition of fractions subscale

Question	1	2	3	4	M	SD
6. Question One	1				1.83	0.82
7. Question Two	0.50	1			1.75	0.74
8. Question Three	0.43	0.48	1		1.57	0.61
9. Question Four	0.23	0.28	0.31	1	1.61	0.63

Note: N=543; Correlation coefficient greater than 0.1 is statistically significant ($p<0.01$).
Source: Field data (2017)

One-way MANOVA investigated the hypothesis that there would be no mean differences between school types (high, average, and low achieving schools) and students' understanding of addition of fractions across the question forms. The statistical evidence suggested that there was a significant MANOVA effect, Pillai's Trace = 0.063, $F(8, 1076) = 4.343$, $P=0.00<0.05$ (Huberty & Petoskey, 2000). This suggested that at least one of the independent variables was different from at least one of the other variables. The multivariate effect size was estimated at 0.031, which suggested that 3.1% of the variations in the canonically derived dependent variable was accounted for by the school types.

Based on the fact that MANOVA effect was determined, there was the need to perform the series of ANOVA tests that would identify the addition of fractions question form(s) where there were differences in understanding. Evidence from Table 27 suggested that none of the addition of fractions question forms in the

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Levene's F test of homogeneity of variances violated the assumption. This implied that the variances in understanding per the addition of fractions question forms across the school types were equal. Evidence from Table 27 revealed that across the school types, understanding in each of the addition of fractions question forms were statistically significantly different at p=0.05 significance level. Also evident among the statistically significant differences in understanding of addition of fractions was the range of eta square (η^2), ranging from 0.027 (question four) to 0.053 (question one). Comparison of means in Table 27 suggested that in the understanding in all the addition of fractions question forms, students in average achieving schools demonstrated the highest level of understanding. This was followed by students in high and low achieving schools respectively.

Table 27: One-Way ANOVA; Understanding of Addition of Fractions Questions and School Type as Independent Variables

Items	Levene		ANOVAs			HPS		APS		LPS	
	F(2,540)	P	F	P	η^2	M	SD	M	SD	M	SD
Question One	2.08	0.126	8.714	0.000	0.031	1.89	0.85	1.97	0.81	1.64	0.77
Question Two	1.511	0.222	15.008	0.000	0.053	1.77	0.76	1.95	0.78	1.55	0.63
Question Three	0.985	0.374	3.294	0.038	0.012	1.59	0.63	1.65	0.64	1.49	0.57
Question Four	1.635	0.196	7.433	0.001	0.027	1.67	0.68	1.70	0.64	1.47	0.57

Note: N=543, η^2= eta square
Source: Field data (2017)

A series of post hoc was performed to compare students' understanding of addition of fractions in all four question forms across the three school types. A consistent pattern of differences was observed in addition of fractions questions one, two, and four (questions with statistically significant ANOVA results). Bonferroni statistic (p=0.05) significance level revealed that across these selected addition of fractions question forms, understanding of students' in low achieving

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schools were consistently different from understanding of students' in average and high achieving schools. However, between high and average achieving schools, there was no observed statistically significant differences in students' understanding in the various addition of fractions question forms, across all three school types. Finally, students' understanding in question three was observed to be statistically different only between average and low achieving schools. This can be observed from Table 28.

Table 28: Bonferroni post hoc Test on Concept of Addition of Fractions

	Question one			Question two			Question three			Question four		
School Type	H	A	L	H	A	L	H	A	L	H	A	L
H												
A												
L	*	*		*	*			*		*	*	

Note: * represents point of significant difference at p=0.05
Legend
H - High Achieving School
A - Average Achieving Schools
L - Low Achieving Schools
Source: Field data (2017)

Discussion

The discussion of findings was in order of the research questions. Possible reasons for observations in relation to the data were advanced.

Understanding of related previous knowledge

Findings from the study showed that majority of students in all school category demonstrated very low level of understanding of the RPK necessary for the teaching and learning of addition of fractions. Comparatively, smaller averages percentage of students demonstrated average and high understanding of the RPK for addition of fractions across all school categories (see Tables 8, Table 9, and

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Table 10). It is important to recall that the RPK here is the concept of fraction and the concept of addition of whole numbers (Wu, 2013; Norton et al, 2015). Low understanding of RPK in this study extends the the observation in Davis (2014) that grade four and six students experienced a lot of difficulties in identifying varieties of the simplest form of fraction; half. Low level of understanding of the RPK in this study also gives credibility to the low understanding of fraction among JHS3 pupils in Amuah, Davis, and Flecher (2017). Cumulatively, the aforementioned findings show that in Ghana, students progressive (Grade, 4, 6, JHS2, and JHS3) preparation towards the development of comprehensive understanding of the RPK (fraction) is seriously failing to yield the required results.

Observation of feedbacks from respondents who demonstrated low understanding showed their inability to engage mathematical strategies, procedures, or deductions without the help of the given visual illustrations (see Figures 10 and Figure 13). The aforementioned deduction is supported by the fact that in the illustrations, the respondents' attempted to represent the problem in re-drawn self-illustration, sketched to scale. Such students practice is consequently apparent when they engaged questions with higher level of complexities (see Figure 14 and Figure 15). Most importantly, when the aforesaid students confronted questions with higher difficulties, the ability to represent the relations correctly was lacking. When there exists an inability to represent a mathematical relation, then the ability to use the representation is certainly not going to result in correct procedures or answers. This is the reason most students could not develop average and high level of understanding of the RPK. This finding is also confirmed by the

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significant differences in students' understanding of RPK that was observed in question 4 between average and low achieving school students (see Table 16). Among all the questions that were not drawn to scale, question four represented the first (see Appendix A). Therefore, immediately students encountered such questions, variation in their understanding abilities will be evident.

In low achieving schools, the average percentage of mathematics teachers who demonstrated low understanding of the RPK, was highest (see Table 13). However, in high and average achieving schools, the highest average percentage of mathematics teachers demonstrated either high or average understanding of the RPK for addition of fractions (see Tables 11 and Table 12). Comparatively, this suggests that on the average, the mathematics teachers' ability to demonstrate understanding of the specific RPK could be used to determine their students' understanding of the RPK. It also supports the finding of Meyer (2004) and Hailikari (2009), that indicates that a teachers' ability to properly use RPK in his instruction could be a major determinant of the transformation of a student from low achieving to high achieving category. The highest percentage of teachers demonstrating low understanding of RPK in low achieving school supports the reason why the only significant difference between pupils understanding in RPK involves students in low achieving schools. This is true because, a teacher cannot use what he does not have. A teacher with low understanding cannot use low level of understanding effectively in the teaching and learning process to teach students to demonstrate high understanding.

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Sentence characteristics of classroom interaction

As cited in Okeffe and O'donoghue (2015), the three major sentence processes identified by Halliday (1973) were material, mental, and relational. In this study, it was observed that in high achieving schools, the main exhibited process characteristics (see Table 17) in the transcribed classroom interaction included all three major process characteristics of Halliday (1973). In the transcribed classroom interaction in average achieving schools, the main exhibited sentence process characteristics (Table 18) included two of the three major process characteristics. Finally, in the transcribed classroom interaction in low achieving schools, the main process characteristics (see Table 19) included only one of the three major process characteristics. Thus, cumulatively it seemed that the number of the major process observed in the transcribed classroom interaction is associated with the achievement levels of the schools. Accordingly, as there is an increase in the number of the three major sentence process included in the main sentence processes in the transcribed classroom interaction, the higher the achievement of the school.

In SFL, there is a total of six sentence processes. The findings in this study showed that there were four main sentence processes in the transcribed classroom interaction in high achieving school (see Table 17). However, in the transcribed classroom interaction of average and low achieving schools, there were three main sentences processes each (see Table 18, Table 19). This suggested that classroom interaction in high achieving schools were richer as compared to average and low achieving schools. Also implied from the aforementioned is that the more the

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combination of main sentence processes, the more the varieties of ways (illustrated in the verbs) that learners encounter addition of fractions (see the variety in description in all the sentence processes).

The richer classroom interaction in high achieving schools is possibly because, the mathematics teacher in high achieving school used paper folding exercise to illustrate addition of fractions. These are activities that all the students in the class were able to participate in at individual levels. Consequently, it was a way of involving the students when they explain their observations during the practical act. After students' initial wrong or right answer (after which the students would normally end their responses), the mathematics teacher always probed for further explanations. This allowed the students to express their views about addition of fractions. Also evident was that the mathematics teacher sought other students' perspective on their colleagues' explanations or feedback. Contrary, the mathematics teacher in average and low achieving school practices the all-knowing teacher led instruction (Davis,2018). According to Davis (2018) the seeking of explanation and other students view on their colleagues' explanations, could be very innovative ways of getting students to express themselves, thereby creating a rich ground for eventual profound understanding of the mathematical concept being taught. For the mathematics teacher in low achieving and average achieving schools, when a student responds to his/her questions, he continues his/her instruction.

In the study of Okeffe and O'donoghue (2015) study of three mathematics textbooks, there were three main sentences processes in the most popular textbook

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and two main sentence processes each in the other textbooks. Comparatively, this study observed four main sentence processes in the high achieving schools' transcribed classroom interaction and three main processes each in average and low achieving schools' transcribed classroom interaction (see Table 17, Table 18, and Table 19). Implication is that interactions in the classroom seemed to support richer encounter (more sentence processes) with the subject of mathematics than the textbooks. This is in contradiction to the view of Lepik (2015) that during instruction, teachers do not improve on the nature of language used in the textbooks.

Among the main sentence process characteristics observed in the three school category, the process common to all the school category is the mental process (see Tables 17, Table 18, and Table 19). Mental process sentence characteristics exhibits the concepts in addition of fractions activities that are innate in the mind. They include sentences that described the concept of addition of fractions as perceptions, affections, and cognitions. However, the percentage of sentences with verbal process characteristics in high achieving schools is comparatively lower than the percentage (highest) of sentences with verbal process characteristics in low achieving schools. Okeffe and O'donoghue (2015) and Ebellind and Segerby (2015) noted that the dominance of verbal process sentence characteristic is mathematics texts is not advisable as learners do not find the relevance in learning mathematical knowledge that have been constructed by others. Therefore, learners consider themselves as passive participants in the learning of addition of fractions. Davis (2018) also encouraged active participation of Ghanaian learners' in the classroom as a way of meeting the goals in the rational

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and objective of the mathematics curriculum. The opinion of Davis (2018) is observed in the classroom interaction of high achieving schools as the highest percentage of sentence observed exhibited the material process characteristics. Sentences with the material process characteristic exhibits the concept of addition of fractions as an activity carried out by an individual or object, on another individual or object.

Irrespective of the achievement level of the school, it was observed that the human elements were quite conspicuous, especially among the dominant processes within the classroom interaction (see Table 17, Table 18, and Table 19). Almost all the human elements were specific. This is similar to the findings of Okeffe and O'Donoghue (2015) where they observed that most of the human elements were specific. Also, Okeffe and O'Donoghue (2015) observed that the popularly used textbooks had a high percentage of human elements associated with sentences that exhibited material process characteristics. Similar observation was noted in the transcribed classroom interactions in high achieving schools (see Table 17).

Morgan (2016) was of the opinion that the use of representational objects without the considerable use of human elements in discussing the representations, does not foster understanding of mathematical concepts but rather, fosters confusion. Finding in this study showed that among all the three school categories studied, most of the mathematical objects involved were derived. It was only in high achieving schools that sentences in the transcribed classroom interaction that exhibited the material process characteristics showed a high level of representational mathematical object (see Table 17, Table 18, and Table 19). This

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could be considered a distinguishing characteristic of classroom interaction in high achieving school. This finding among high achieving schools is consistent with the finding of Okeffe and O'Donoghue (2015) that a high presence of representational objects was observed among sentences categorised as material processes because the intention is to practicalise mathematics as a real body of knowledge with relevance in our present physical world.

The opposite of human involvement is what she described as alienation. Morgan's study noted that the involvement of pronouns and specific names does not result in effective human involvement. She therefore advocated for deeper involvement of human elements as the humans doing mathematical activity in real life. The human activities serve as a way of learning and understanding mathematics.

Understanding of the concept of addition of fractions

Per the findings in this study, there was a general low level of students understanding of the concept of addition of fractions (see Tables 20, 21, 22, and 27 for PU_S and NU). Okeffe and O'donogue (2015) showed how the approach of the mathematics textbooks used in the classroom could be a determinant in students and teachers' practices in the classroom. Content of Junior High School one mathematics textbooks shows very little emphasis on the concept of addition of fractions. This is evident in the fact that the number of pages that was devoted to the concept of addition of fractions, was a little above one percent of pages in the book. The total number of pages was little below two and a half pages. Explanation of the concept of addition of fractions constituted less than half a page, whiles the

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remaining aspects of the textbooks on addition of fractions were devoted to examples and exercises (MOE, 2012a). However, it is expected that a lot of attention would be devoted to a mathematical concept widely connected to other topics.

Representation constituted a major part in extending students understanding of addition of fractions. The major weakness of respondents who demonstrated NU and PU_S is the inability to properly represent the ideas in the problem with a visual illustration (see Figures 24). Respondent in Figure 26 could not represent $^2/_5$. Another weakness is the inability to reason about the necessary condition for their visual illustration before they can solve the addition of fractions problem (see Figure 27). In Figure 27, the respondent did not represent the fractions in relation to similar sized whole. Therefore, clearly, the ability to use a representation and the ability to know the necessary condition to use a representation will automatically show low understanding. Another problem is that the respondents who are able to properly represent the problem demonstrated the inability to comprehend that the representation of the problem results in the representation of the solution (sum) (see Figure 28). They also fail to demonstrate that the representation of the problem is connected to the use of algorithmic or symbolic procedure in solving the mathematical problem (see Figure 29). The weakness of respondents, as shown in Figure 28 and Figure 29 will definitely not allow effective visualisation of connections in the various isolated concepts that can be observed in addition of fractions. Wu (2013) and Vig et al., (2014) showed how such inabilities would prevent learners' ability to comprehend addition of fractions.

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Mathematics textbook representational approach to the teaching and learning of addition of fractions is a major contributor to students' experiences during the teaching and learning of addition of fractions. In the JHS 1 mathematics textbook, there was only one visual illustration in the pages where the concept of addition of fractions was presented. It is important to recognize the point at which visual illustrations were used in any textbook. This shows the importance that the textbooks place on the use of representation in that aspects of the textbooks in the teaching and learning process.

The visual illustration in the Ghanaian JHS1 Mathematics textbook was in relation to trail exercises in the textbook and close to the last word problem exercise (MOE, 2012a). The illustration was a picture of a boy lying on a bed and listening to the radio beside the bed. Thus, listening to the word problem from the radio. This situation showed that the priority of visual illustration in the textbooks (and in the teaching and learning process) was not for the purpose of explanation of the concept of addition of fractions. This is contrary to the suggestion of Wu (2008:2013) and Vig et al., (2014) on the use of representations in the teaching and learning of addition of fractions. It also goes contrary to specific observation of Charalambous et al (2010) as they compare mathematics textbooks' approach to addition of fractions in three different countries, in three different parts of the continent. It was observed that textbooks in Taiwan (a high performing country in international assessment tests) use of a lot of visual illustrations in the presentation of the concept of addition fractions itself. This showed one of the possible reasons why Taiwanese students performed well (demonstrating high understanding) in international

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assessments. Therefore, explaining how the mathematics textbook approach in the Ghanaian JHS1 classrooms encourage low understanding of addition of fractions.

The fact that very few students demonstrated full understanding (FU) and partial understanding of addition of fractions with associated thoughts (PU_A) suggested the evidence of lack of focus on the connectedness which was observed in the textbook between representations and mathematical ideas in addition of fractions. Also evident from the JHS1 mathematics textbook was the inability of the textbook to connect the concept of addition of fractions to previously learnt concepts of fractions. There was no mention of unit fractions as a concept connected to the addition of fractions. There was only one mention of the equivalent fraction but how it connects to the concept of addition of fractions was lacking in the textbook. There was no mention of equivalent fraction of the addends (MOE, 2012a). This is often the strategy of obtaining a singular form of measurement, therefore allowing a common unit for the addition of two fractions . The implication is that the concept of connectedness of various fractional ideologies was virtually none existent among the students.

Another important recognition from the textbook was the fact that the emphasis of the textbook was on the operation of addition and not on the complexity and/or structure of the tasks selected for presentation. An exemplary situation involved the use of addition of fractions problems that involved three addends, to illustrate possible differences in methods of adding fractions (MOE, 2012a). However, in the whole textbook, all other examples and exercises comprised of only fractions addition problems with two addends. Finally, the

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textbook did not focus on explaining why and how the use of those different methods arrived at the same answer (MOE, 2012a). This really falls short of how diversity in methods of approaching a mathematical problem could be used to enhance teaching and learning (Pass et al, 2003; Charalambous et al, 2010).

Another possible reason for low students' understanding could be found in some mathematics teachers' practices during teaching and learning. The difference between home conception of fraction and school conception of fraction could be a major factor. When home conception of addition of fractions were not being utilised as a resource to enhance students understanding of addition of fractions, there is likely to be a conflict between home conception of the concept of addition of fractions and school conception of the concept of addition of fractions (Davis, Bishop & Seah, 2009).

Another possible reason for such low students' conception of addition of fractions was on the type of TLM used by the mathematics teachers or students during the teaching and learning of the concept of addition of fractions. As evidenced in Davis and Ampiah (2009), Ghanaian pre-service teachers seemed to be conversant with Cuisenaire rod as a way of using representation of concept of addition of fractions. However, when other forms were used, the pre-service teachers themselves could not explain the concept of addition of fractions. The implication is that the kind of representations that the pre-service teachers used during the teaching and learning period could determine the possible kind of understanding that the students would likely exhibit. Three out of the four sample respondents' feedback for NU and PU_S were mostly the cuisenaire rods (see

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Figure 26, 25, 26, and 27). However, two out of the three sample respondents' feedback for PU_A and FU were not cuisenaire rods (see Figure 30, 29, and 30). Therefore, teachers' and students' inability to correctly use multiple methods of representation during teaching and learning is a very likely reason.

The finding of general low understanding of addition of fractions is in line with the finding of Bailey et al (2015), Davis (2014) and Amuah et al., (2017). One of the constructs investigated in Amuah et al., (2017) is the equivalence construct of fraction. Equivalent fractions also represent one of the basic idea in concept of addition of fractions. It is a transformation that allows for easy addition of fractions when the addend is not in the same form or with the same denominator. Findings in Amuah et al., (2017) noted that all students in the various school categories could not demonstrate any form of understanding of the idea of equivalent fraction. In fact, it was the least understood construct in their findings. It is therefore consequential that low understanding of addition of fractions would be observed among students in this study (see Tables 20, 21, 22). In Bailey et al., (2015) evidence from comparison of US and Chinese students revealed that there was a wider difference in students understanding of fraction procedural knowledge (per students from the two countries) as compared to the knowledge of concept of fraction. Stated differently, students demonstrated lower understanding in fraction procedural knowledge because understanding of fraction was a determinant of students' eventual understanding of fraction procedure. The latter is consistent with the relationship identified above between Amuah et al., (2017) and the finding in this study.

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One of the findings of Bailey et al., (2015) was that the least performing students demonstrated the largest difference in understanding among the two countries involved in their study (USA and China). In the current study (see Table 28), it is the students in the low achieving schools that demonstrated significant difference from the other category of schools. Post-hoc analysis on MANOVA results showed that students' in low achieving schools demonstrated significantly lower level of understanding as compared to students in average and high achieving schools (see Table 27 & Table 28). Therefore, this study supports international findings in the area of addition of fractions.

The highest percentage of mathematics teachers in the various school categories demonstrated either partial understanding of the concept of addition of fractions with separated thoughts (PU_S) or full understanding of the concept of addition of fractions (FU) (see Tables 23, 24, and 25). Strangely, the highest percentage of mathematics teachers in high achieving schools demonstrated partial understanding of the concept of addition of fractions with separated thoughts (PU_S) while highest percentage of mathematics teachers in low achieving schools demonstrated full understanding of the concept of addition of fractions (FU). This also explains the fact that higher mathematics teachers' understanding of the concept of addition of fractions may not necessarily be associated with higher students' understanding of the concept of addition of fractions. This is because the schools are classified according to students' achievement in Basic Education Certificate Examinations (BECE) mathematics scores and not according to any form of teachers' demonstrated ability. However, it is also possible that the Ghana

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Education Service intentionally post teachers who are identified to be good to low achieving schools so that they could help the schools build up their students understanding capabilities in their various subject areas.

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CHAPTER FIVE

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

This chapter concludes the study. The chapter shows the summary of the study, key findings, conclusions, recommendations, and suggestion for future studies

Summary

The purpose of this study was to explore JHS 2 students and their mathematics teachers' understanding of addition of fractions. The study specifically investigated students and their mathematics teachers' understanding of the specific related previous knowledge for the teaching and learning of addition of fractions, their understanding of addition of fractions, and classroom interaction focusing on language use. The concurrent nested mixed method research design was used. Stratified random sampling procedure was used to select schools, whiles purposive sampling procedure was used to select JHS 2 students and their mathematics teachers. A sample of 616 JHS 2 students and their mathematics teachers (17 of them) were selected. The quantitative data collected were analysed using frequency count and MANOVA, while the qualitative data collected were analysed qualitatively. The summary of the findings from the study is presented below.

Key Findings

What are students and their mathematics teachers' understanding of the related previous knowledge for the teaching and learning of the concept of addition of fractions?

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1. Students and their mathematics teachers' (respondents) understanding of the related previous knowledge for the teaching and learning of addition of fractions were categorised into three (high, average, and low understanding of the RPK). Respondents who demonstrated low understanding of the RPK could successfully reason and use only one of the given relation in the RPK task. The aforementioned respondents can only use two or three of the given relations in the RPK task only with the help of visual illustrations. Also included in the said category of respondents were those who could not attempt the given task at all.
2. Respondents who demonstrated average understanding of the RPK could successfully reason and use two of the given relation in the RPK task. Respondents who demonstrated average understanding of the RPK only reasons with three of the given relations unless with the help of visual illustrations.
3. Finally, respondents who demonstrated high understanding of the RPK for addition of fractions can flexibly reason with all the given relations in the RPK task. The use of visual illustration among the aforesaid respondents were for the purpose of clear explanation and not to solve the given RPK task.

What are students and their mathematics teachers' understanding of the related previous knowledge for the teaching and learning of the concept of addition of fractions by school context (that is, high, average and low achieving schools)?

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1. Among all school categories, the highest percentage of students demonstrated low understanding of the specific RPK for addition of fractions. Comparatively, the percentage of students who demonstrated average and high understanding of the specific RPK were in the minority. The percentage of students who demonstrated high understanding of the RPK were always the least in all school types. Consequently, most students were not ready for the learning of addition of fractions.
2. The following summary is true across all the school types. In question three, a number of students who demonstrated low understanding of the RPK, eventually demonstrated average and high understanding of the RPK in question four. However, as the tasks progressively increased in difficulty, some students who demonstrated high understanding of the RPK, eventually demonstrated average and low understanding of the RPK. Similarly, some students who demonstrated average understanding of the RPK, eventually demonstrated low understanding of the specific RPK.
3. Across all the questions for RPK, the highest percentage of mathematics teachers in high achieving schools demonstrated average understanding of the RPK. Also, across all the questions, the highest percentage of mathematics teachers in average achieving schools demonstrated high understanding of the RPK for addition of fractions. However, in low achieving schools, across all the questions, the highest percentage of mathematics teachers demonstrated low understanding of the RPK for addition of fractions.

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What is the effect of school context on students' understanding of related previous knowledge for the teaching and learning of the concept of addition of fractions?

1. School type had a significant effect on students' understanding of the specific RPK for addition of fractions. Generally school type accounted for 1.6% of variation in students' demonstrated understanding of the specific RPK for addition of fractions.
2. In question four, school type had a significant effect on students' understanding of the specific RPK for addition of fractions. Thus, school type accounted for 1.4% of variation in students' demonstrated understanding of the specific RPK for addition of fractions. In this particular task, students in average achieving schools significantly demonstrated better understanding than students in low achieving schools. Other comparisons showed non-significant difference.

What are the characteristic of sentences used in classroom interactions during the teaching and learning of the concept of addition of fractions?

1. Five out of six sentence characteristics (sentence processes) were identified in the data. These were material, mental, verbal, relational, and behavioural processes. Sentences with the material process characteristic connotes a person (or thing) performing an action on another person or thing. Sentences with the mental process characteristics espouses the idea of perception, affection, or cognition. The aforementioned description emanated from

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actions that are innate in the mind. Verbal processes are reported speech. Relational process connotes comparisons of any form.

2. Two types of human elements were identified in the study. These were specific and general human elements. Classroom interaction in all the school types showed a high presence of specific human elements.
3. Three categories of mathematical objects were identified in the study. These include basic, relational, and representational mathematical objects.

What are the characteristic of sentences used in classroom interactions during the teaching and learning of the concept of addition of fractions by school context (that is, high, average, and low achieving schools)?

1. Characteristic of sentences used in classroom interaction during the teaching and learning of the concept of addition of fractions comprised mainly of four processes in high achieving schools. In descending order, these were material, verbal, relational, and mental processes. However, in average and low achieving schools, characteristics of classroom interaction used during the teaching and learning of addition of fractions comprised of mainly three processes each. In average achieving schools, the sentence processes comprised mainly of relational, mental, and behavioural processes. In low achieving schools, the sentence processes comprised mainly of verbal, material, and behaviour processes, in descending order.
2. In the use of mathematical objects, the use of representational objects was a distinguishing characteristic in high achieving schools.

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3. Across all the school types, specific human elements were observed to be highly present in most sentences used during classroom interaction for addition of fractions.

What are students and their mathematics teachers' understanding of the concept of addition of fractions?

1. Three major criteria that were observed in the study was used to categorise respondents' demonstrated understanding of the concept of addition of fractions. The criteria included the use of visual illustration, written descriptions, and mathematical procedures. Students and their mathematics teachers' (respondents) understanding of the concept of addition of fractions were mainly categorised into three. These included no understanding, partial understanding, and full understanding of the concept of addition of fractions. Respondents who demonstrated no understanding of the concept of addition of fractions could not demonstrate any ability beyond the understanding of the individual addends in the addition of fractions task. Consequently, most of such respondents used wrong strategies that resulted in the wrong answers.
2. Respondents who demonstrated partial understanding of the concept of addition of fractions were categorised into two sub categories i.e., partial understanding with associated thoughts (PU_S) and partial understanding with associated thoughts (PU_A). Respondents who demonstrated PU_S could solve the addition task with one criteria but could not link the second criteria to the first. Sometimes the second criteria may even lead to a wrong

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response. However, in situations where both criteria resulted in the same answer, the respondents could not connect or explain the similarities between the first criteria to the second criteria. Respondents who demonstrated PU_A could explain the similarities between the second and the first criteria

3. Respondents who demonstrated full understanding of the concept of addition of fractions could explain the linkage between every step in any of the criteria chosen.

What are students and their mathematics teachers' understanding of the concept of addition of fractions by school context (that is, high, average, and low achieving schools)?

1. Across the questions, the average percentage of students who demonstrated no understanding (NU) of the concept of addition of fractions was the highest in high and low achieving schools. This was followed by the average percentage of students who demonstrated partial understanding of the concept of addition of fractions with separated thoughts (PU_S). A comparatively smaller average percentage of students demonstrated partial understanding of the concept of addition of fractions with associated thought (PU_A) in both high and low achieving schools. No student in low achieving school demonstrated full understanding (FU) of the concept of addition of fractions.
2. In average achieving schools, the average percentage of students who demonstrated partial understanding of the concept of addition of fractions

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with separated thought (PU_S) was highest. However, in high and low achieving schools, the average percentage of students who demonstrated no understanding of addition of fractions, was highest.

3. In high and average achieving schools, there was a trend in percentage of students who demonstrated the various forms of understanding of addition of fractions across all the questions. Thus, as the addition of fractions questions increased in difficulty, some students who demonstrated PU_A (partial understanding of the concept of addition of fractions with associated thoughts), eventually demonstrated PU_S (partial understanding of the concept of addition of fractions with associated thoughts) and NU (No understanding of the concept of addition of fractions). In addition, some students who demonstrated PU_S, eventually demonstrated NU.,

4. Across the questions, and in average and low achieving schools, the average percentage of mathematics teachers who demonstrated full understanding (FU) of the concept of addition of fractions was the highest. This was followed by the average percentage of mathematics teachers who demonstrated partial understanding of the concept of addition of fractions with separated thoughts (PU_S). However, in high achieving schools, the average percentage of mathematics teachers who demonstrated partial understanding of the concept of addition of fractions with separated thoughts (PU_S) was highest. This was followed by the average percentage of mathematics teachers who demonstrated full understanding of the concept of addition of fractions (FU).

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What is the effect of school context on students' understanding of the concept of addition of fractions?

1. School type has significant effect on students' conception of addition of fractions. School type accounted for 3.1% of the variations in students' concept of addition of fractions. Multiple comparison of students' conception of addition of fractions showed a significant difference between low achieving schools, and high and average achieving schools on all fractions addition tasks except question three.

Conclusions

Students' understanding of the specific RPK for addition of fractions, was low in all the school types. The students were generally not ready for learning of addition of fractions. The highest percentage of mathematics teachers in high achieving schools demonstrated low understanding of the RPK for the teaching and learning of addition of fractions. In average achieving schools, the highest percentage of mathematics teachers demonstrated high understanding of the RPK for teaching and learning addition of fractions. On the contrary, the highest percentage of teachers in low achieving school however, demonstrated low understanding of the specific RPK. School type has significant effect on students' understanding of the RPK for the teaching and learning of addition of fractions. Specifically, school type accounted for a significant difference between low and average achieving school when the RPK task was not drawn to scale.

Characteristic of sentences used during classroom interaction in high achieving schools was markedly different from classroom interaction in average

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and low achieving schools. It was observed that the sentence characteristics in classroom interaction in high achieving schools was richer (in terms of the number of main sentence processes) as compared to average and low achieving schools. The major characteristic of sentences in high achieving schools' classroom interaction involved the description of the concept of addition of fractions, as mathematical procedures in real material terms (material processes) while classroom interaction in average achieving schools were mostly describing the concept of addition of fractions as a comparison of mathematical procedures or ideas (relational processes). Classroom interaction in low achieving schools showed that the highest percentage of sentences described the concept of addition of fractions as mathematical knowledge that was being reported (verbal processes). Also evident was the fact that the use of representational mathematical object was a distinguishing characteristic in high achieving schools.

Students' understanding of the concept of addition of fractions was generally found to be low among the various school types. A comparatively small average percentage of students in high and average achieving schools demonstrated partial understanding with associated thoughts (PU_A) and full understanding (FU) of the concept of addition of fractions. No student in low achieving schools demonstrated full understanding of the concept of addition of fractions. With an exception to students in low achieving schools, students who initially demonstrated PU_A, eventually demonstrated PU_S, and NU, as the difficulty in the addition of fractions questions increased progressively. Additionally, students who demonstrated PU_S, eventually demonstrated NU when the difficulty of the

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addition of fractions questions progressively increased. Among mathematics teachers, full understanding (FU) and partial understanding with separated thoughts (PU_S) of the concept of addition of fractions was the highest average percentage in low and high achieving schools respectively. School type has significant effect on students' understanding of the concept of addition of fractions. Except in question three, school type had effect on students' understanding of addition of fractions in all other questions.

Recommendations

The following recommendations are made for the teaching and learning based on the findings in this study;

1. Teacher educators should explore ways of including in the teacher education curriculum, ways that mathematics teacher trainees could be taught to demonstrate high level of understanding of the RPK and full understanding of the concept of addition of fractions.
2. In-service providers should organise workshops where mathematics teachers in the classroom would be trained on ways to help their pupils develop better understanding of the concept of addition of fractions and its associated RPK.
3. In an effort to close the gap between students in low achieving school on one hand, and students in average and high achieving schools on the other hand, in-service providers should train mathematics teachers of low achieving schools to focus more attention on strategically supporting students' learning with visual illustrations at various stages of learning the

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concepts used as RPK for the teaching and learning of addition of fractions. Effort should be focused on helping the students to successfully reason with visual illustrations that are not drawn to scale.

Suggestion for Future Studies

This study should be replicated in the lower stages of education in Ghana. This would help identify the starting stages at which the problem of students' low level of understanding of the RPK began. It will also identify the stage at which the high percentage of NU and PU_S began. Additionally, it would also show how the problem transforms at the various stages of the education system.

An unexpected and perplexing situation was observed among teachers. Thus, teachers in low achieving schools demonstrated low understanding of RPK for the teaching and learning of addition of fractions, but high understanding of addition of fractions. This should be investigated further.

A comparison of how the characteristics of sentences in the Ghanaian mathematics textbooks versus the characteristic of sentences used in classroom interaction during the teaching and learning of addition of fractions would be insightful. It would illustrate theoretically, how the textbooks may be influencing teachers' choices of sentences during the teaching and learning of the concept of addition of fractions, or otherwise.

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APPENDICES

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APPENDIX A

UNIT COORDINATION INSTRUMENT

Name:................. Age...... Sex:.... Date:..........

Department of Mathematics and ICT Education

University of Cape Coast

Dear student, this study is to collect information on conceptual learning of fraction and its operations. The information obtained through the process would be kept confidential and used for research purposes only. This test would therefore not form part of your assessment. However, you are encouraged to do your best. The results from this study will go a long way to help improve the teaching and learning of Mathematics at Junior High School level.

Instrument for Assessing Unit Coordination

Use the bars shown above to answer the following three questions

1. How many times does the **Medium Yellow Bar** fit into the **Long Red Bars**?

   answer:

2. How many times does the **Small Blue Bar** fit into the **Medium Yellow Bar**?

   answer:

3. Use all the information you have now to figure out how many times the **Small Blue Bar** fits into the **Long Red Bar**?

   answer:

For each question above, use the space below to **draw a drawing** that will **explain how** you got your answer.

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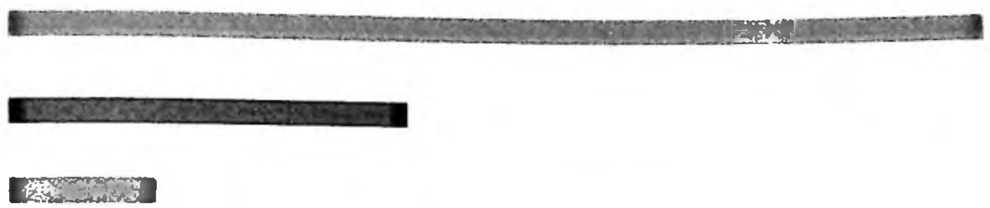
APPENDIX A - CONTINUED

Name:…………….. Age…… Sex:…. Date:………..



Use the following information to answer the questions about the bars shown above:

4. **Assume** that the **Medium Purple Bar** fits into the **Long Orange Bar** exactly 2 times.

   Also assume that the **Small Green Bar** fits into the **Medium Purple Bar** exactly 6 times.

   Use **all the information provided for question 4** to figure out how many times the **Small Green Bar** would fit into the **Long Orange Bar**?

answer

Use the space below to **draw a drawing** that will **explain how** you got your answer to question 4. Explain your drawing

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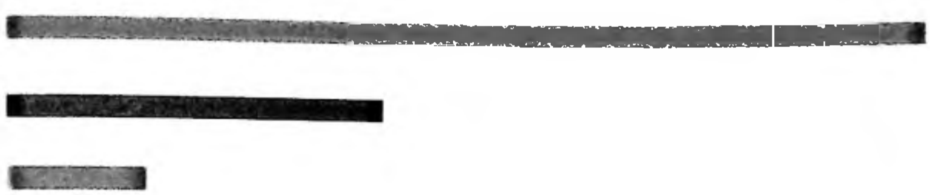
APPENDIX A - CONTINUED

Name:…………….. Age…… Sex:…. Date:……….



Use the following information to answer questions about the bars shown above:

5. **Now assume** that the **Medium Purple Bar** fits into the **Long Orange Bar** exactly 2 times.

   Also assume that the **Small Green Bar** fits into the **Long Orange Bar** exactly 8 times.

   Use **all the information provided for question 5** to figure out how many times the **Small Green Bar** would fit into the **Medium Purple Bar**?

answer

Use the space below to **draw a drawing** that will **explain how** you got your answer to question 5. Explain your drawing

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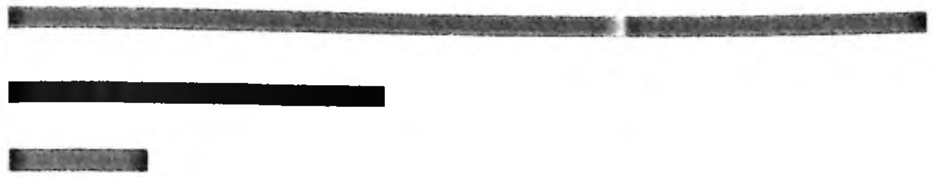
APPENDIX A - CONTINUED

Name:................ Age...... Sex:.... Date:..........



Use the following information to answer questions about the bars shown above:

6. Now **assume** that the **Small Green Bar** fits into the **Long Orange Bar** exactly 12 times.

   Also assume that the **Small Green Bar** fits into the **Medium Purple Bar** exactly 3 times.

   Use **all the information provided for question 6** to figure out how many times the **Medium Purple Bar** would fit into the **Long Orange Bar**?

   answer

   Use the space below to **draw a drawing** that will **explain how** you got your answer to question 6. Explain your drawing

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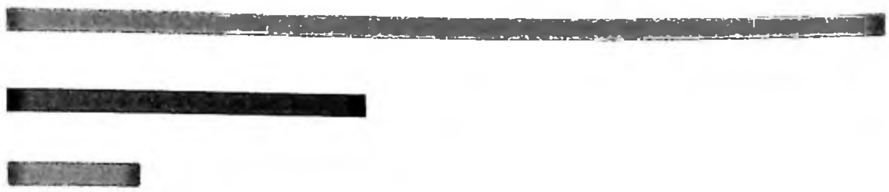
APPENDIX A - CONTINUED

Name:................ Age...... Sex:.... Date:..........



Use the following information to answer questions about the bars shown above:

7. **Now assume** that the **Small Green Bar** fits into the **Long Orange Bar** exactly 9 times.

Also assume that the **Small Green Bar** fits into the **Medium Purple Bar** exactly 4 times.

How can you use this information to figure out how many times the **Medium Purple Bar** would fit into the **Long Orange Bar**?

answer

Use the space below to **draw a drawing** that will **explain how** you got your answer to question 7. Explain your drawing

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APPENDIX B

Unit coordination scoring rubrics by Norton et al (2015)

Tasks 1-3

	Students' Unit Structures	Student Reasoning on Tasks 1-3	Written Indicators of Reasoning
Stage 1	Students can take one level of units as given, and may coordinate two levels of units in activity.	Students physically or mentally iterate the small bar in the long bar (or segment the long bar with the short bar), relying on the appearance of the bars to determine how many times it would fit.	• Student responses to Tasks 1 and 2 are off by more than 1 from the correct relation (note that students at Stages 2 and 3 might represent the relations as unit fractions). • Student responses to Task 3 indicate that they estimated the number of times small bar fit into long bar (possibly further indicated by partitioning marks), rather than taking the product of responses to Tasks 1 & 2. • Students add their solutions to Tasks 1 and 2 to solve Task 3. • Students do not respond, or otherwise indicate they do not know.
Stage 2	Students can take two levels of units (a composite unit) as given, and may coordinate three levels of units in activity.	Students mentally iterate the medium bar within the long bar four times, with each iteration representing a 3 (i.e., 3, 6, 9, 12).	• Students use relational language (e.g., "every medium bar is 3 small bars"). • Student drawings incorporate the two relations determined in Tasks 1 and 2. • Student responses justify the use of multiplication.
Stage 3	Students can take three levels of units (a composite unit of composite units) as given, and can thus flexibly switch between two and three-level structures without reliance on figurative material.	Students use the given relations to determine that there are four 3s (small bars) in the long bar.	

Task 4

	Students' Unit Structures	Student Reasoning on Task 4	Written Indicators of Reasoning
Stage 1	Students can take one level of units as given, and may coordinate two levels of units in activity.	Students rely upon the appearance of the bars without using given relations.	• Students rely upon the appearance of the bars rather than using the given relations (e.g., partitioning/segmenting the given bars). • Students add or subtract the numbers given in the relations. • Students do not respond, or otherwise indicate they do not know.
Stage 2	Students can take two levels of units (a composite unit) as given, and may coordinate three levels of units in activity.	Students use the second given relation to form a composite unit that they can iterate through activity, by the number in the first given relation.	• Students coordinate relations appropriately and with a drawing illustrating size relations, but writing indicates the drawing was the solution method (e.g., solution appears below the drawing, or erasures/corrections are present in the drawing). • Student explanations and drawings appropriately refer to multiple two-level relations, but not a single three-level relation. • Student responses indicate use of multiplication without justification or illustration (possibly with a multiplication error).
Stage 3	Students can take three levels of units (a composite unit of composite units) as given, and can thus flexibly switch between two and three-level structures without reliance on figurative material.	Students take the first given relation as a composite unit that they mentally distribute across the units given in the second relation, thus justifying the use of multiplication.	• Student drawings are used to justify or illustrate appropriate solutions rather than to produce them (e.g., drawing is integrated with or appears below an explanation). • Student explanations and drawings refer to a single three-level relation, with appropriate size relations.

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APPENDIX B - CONTINUED

Tasks 5 and 6 [record indicators present for either task]

	Students' Unit Structures	Student Reasoning on Tasks 5 and 6	Written Indicators of Reasoning
Stage 1	Students can take one level of units as given, and may coordinate two levels of units in activity.	Students mentally iterate the short (medium) bar, imagining how many times it would fit into the medium (long) bar.	• Students rely upon the appearance of the bars rather than using the given relations (e.g., partitioning/segmenting the given bars). • Students add or subtract the numbers given in the relations. • Students do not respond, or otherwise indicate they do not know.
Stage 2	Students can take two levels of units (a composite unit) as given, and may coordinate three levels of units in activity.	Students use the two given two-level relations to generate representations with which to relate them figuratively.	• Student drawings or explanations indicate multiplicative reasoning but not reverse multiplicative reasoning (leading them to multiply instead of dividing, possibly because they misread the task). • Student responses indicate use of division, but without justification or supporting illustrations. • Students rely upon their drawings of the given relations to determine the unknown relation. • Student explanations and drawings appropriately refer to multiple two-level relations, but not a single three-level relation.
Stage 3	Students can take three levels of units (a composite unit of composite units) as given, and can thus flexibly switch between two and three-level structures without reliance on figurative material.	Students assimilate the two given two-level relations into a structure for coordinating all three levels.	• Students reverse their multiplicative reasoning for both tasks. • Student drawings are used to justify or illustrate appropriate solutions rather than to produce them. • Student explanations and drawings refer to a single three-level relation, with appropriate size relations. • Students use division in ways that are consistent with drawings and explanations.

Task 7

	Students' Unit Structures	Student Reasoning on Task 7	Written Indicators of Reasoning
Stage 1	Students can take one level of units as given, and may coordinate two levels of units in activity.	Students mentally iterate the medium bar, imagining how many times it would fit into the long bar.	• Students rely upon the appearance of the bars rather than using the given relations (e.g., partitioning/segmenting the given bars). • Students add or subtract the numbers given in the relations. • Students multiply the numbers in the given relations without any explanation. • Students do not respond, or otherwise indicate they do not know. • Students make no attempt to account for the leftover part.
Stage 2	Students can take two levels of units (a composite unit) as given, and may coordinate three levels of units in activity.	Students establish a composite unit of 4 and estimate how many of these fit into a length of 9.	• Students refer to fractional part as 1/9 rather than ¼. • Students respond with 2 and a remainder. • Student responses indicate use of division, but without justification or supporting illustrations. • Student drawings or explanations indicate multiplicative reasoning but not reverse multiplicative reasoning (leading them to multiply instead of dividing, possibly because they misread the task). • Student explanations and drawings appropriately refer to multiple two-level relations, but not a single three-level relation.
Stage 3	Students can take three levels of units (a composite unit of composite units) as given, and can thus flexibly switch between two and three-level structures without reliance on figurative material.	Students coordinate 9 as two 4s with one unit left over without losing the relationship between this unit and the others.	• Students appropriately account for the left over part with a fraction or a decimal (e.g., "2 ¼"). • Student drawings are used to justify or illustrate appropriate solutions rather than to produce them. • Student explanations and drawings refer to a single three-level relation, with appropriate size relations. • Students use division in ways that are consistent with drawings and explanations.

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APPENDIX C

NOTES TO GUIDE THE INTERPRETATION OF ELEMENTS IN CLASSROOM INTERACTION

Material Processes: These involve clauses or sentences in mathematics that involve an action. It normally involves a subject acting on an object. Normally involves action verbs (cutting, folding, etc). Normally externalized from the body

Mental Processes: These involve the act of cognition, perceptions, or affections. They normally involve actions that are internal to the subject or object being discussed.

Verbal Processes: Normally in the form of reported speech. These may be direct or indirect speeches. Describes someone saying something 'sayer', what the person is saying 'verbiage' and the person to whom the content is being said to 'target'.

Behavioural Processes: I nvolves words like smiling, laughing, dreaming, etc. that depicts a persons' physiological and psychological behaviour.

Relational Processes: Statements concerned with comparison of objects, persons, concepts/ideas or any intersection of them. Hence an attribute of one thing is compared or used as an illustrative of another

Existential Processes: These are sentences that involve statements of existence. The sentences normally include there is, there was, there are.

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APPENDIX D

Instrument for Assessing Understanding of Addition of Fractions

Name:…………….. Age…… Sex:…. Date:………..

Department of Mathematics and ICT Education

University of Cape Coast

Dear student, this study is to collect information on understanding of addition of fractions. The information obtained through the process would be kept confidential and used for research purposes only. This test would therefore not form part of your assessment. However, you are encouraged to do your best. The results from this study will go a long way to help improve the teaching and learning of Mathematics at Junior High School level.

Instruction: Follow the instruction below for the question below;

a. use drawing to show/represent the following addition questions. Use drawing to also show the answer. Explain how your initial drawing results in the drawing of the answer.

b. Then solve the addition questions. Explain your procedures and answers in relation to the drawing.

1. $\frac{1}{3} + \frac{1}{5}$

Solution

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APPENDIX D - CONTINUED

Instruction: Follow the instruction below for the question below;

a. use drawing to show/represent the following addition questions. Use drawing to also show the answer. Explain how your initial drawing results in the drawing of the answer.

b. Then solve the addition questions. Explain your procedures and answers in relation to the drawing.

2. $\frac{2}{5} + \frac{3}{7}$

Solution

Instruction: Follow the instruction below for the question below;

a. use drawing to show/represent the following addition question. Use drawing to also show the answer. Explain how your initial drawing results in the drawing of the answer.

b. Then solve the addition questions. Explain your procedures and answers in relation to the drawing.

3. $\frac{3}{2} + \frac{4}{5}$

Solution

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APPENDIX D - CONTINUED

Instruction: Follow the instruction below for the question below;

a. use drawing to show/represent the following addition questions. Use drawing to also show the answer. Explain how your initial drawing results in the drawing of the answer.

b. Then solve the addition questions. Explain your procedures and answers in relation to the drawing.

4. $\frac{3}{2} + \frac{5}{4}$

Solution

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APPENDIX E

Scoring Rubric for Addition of Fractions

Question numbers	No understanding	Partial Understanding with Separated Thoughts	Partial Understanding with Associated Thoughts	Full Understanding
Question one	Uses different forms of illustration for the different fractions involved Uses representations that just shows number of items in numerator and number of items in denominator Adding denominator and numerator just like that Understanding those who add numerator and denominator wrongly in LCM	Explaining part of the LCM as what was used in the illustration Wherever respondent use the word LCM it can be replaced with other procedural methods (butterfly method). ALL IN ALL, LCM MUST BE CORRECT Students gets LCM correct but cannot illustrate it in the answer	Picks illustration of answer from result of LCM method (or illustration depicts correct answer) Representation or LCM shows unit fractions components in the addend. But results of fraction was taken from LCM. Explanation of LCM involved addends with unit fraction component but results was obtained from LCM	Illustration shows full transformation of fractions involved. **Any one** of the following is enough proof in representation I) 1. Splitting to determine equivalence 2. Merging to determine equivalence II) Describing paper folding exercise that shows transformation of fractions. And subsequent addition to the desired answer III) Answers without illustrations: Reducing fractions to unit fractions and transforming them to equivalent forms. And eventually adding successfully to determine the correct result.

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APPENDIX E - CONTINUED

Question two	Uses different forms of illustration for the different fraction forms Answer from LCM and answer from illustrations are different and part or all are wrong No writing of procedure of LCM from original question but obtained the correct answer. Attempt with unit fractions could not identify common unit	Noticing that LCM is a common denominator situation. Others are same as question one	Correct LCM and used to draw answer of representation Different but correct representation of questions. Representation of answer is still correct and from LCM Others are same as question one	Same as question one

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APPENDIX E - CONTINUED

Question three AND Question four Is seems that the conversation from proper to improper fraction is understood to be a must in handling certain questions. And this conversion is to change the position of numerators and denominators. However, they don't realize it changes the value too.	Changing the question to represent proper fraction (changes the position of the numerator and the denominator) Properly represents each fraction but not able to properly transform into equivalent forms for addition. Understanding those who find it difficult to use fractions greater than one, (DAWDA AIDOO – SCHOOL M). They also find it difficult representing fractions greater than one	Changes to mixed fractions and adds fractional parts and uses proper representation even if shading was not properly done. (But incorrect representation of answer and correct LCM results) Properly using LCM method to generate correct addition of fractions but couldn't transform answer to mixed fraction OR transformed but confused in representation	All in representation: adds fractional part and adds whole number parts too (but represents results from LCM) Representation is only fractional parts of addend. However, answer is from combination of LCM results and addition of separated whole number components. Explanation shows that fractional parts were added from unit fraction perspective but common units could not be obtained. Subsequently, answer was obtained from LCM	For those who are able to represent the whole, they seem not to be able to recognize the appropriate relationship between the size of the fractional part and the size of the whole (Esther Anoobil – School M). Recognising the whole but not being able to illustrate the size relationship. Recognizing the whole and matching to illustrate the size relationship. Those who did not shade the number greater than one because they seem confused. Answers without illustrations Reducing fractions to unit fractions and transforming them to equivalent forms. And eventually adding successfully to determine the correct result.

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APPENDIX F

Sample Permission Letter from a District

GHANA EDUCATION SERVICE

In case of reply the
Number and date of this
Letter should be quoted

METROPOLITAN EDUCATION DIRECTORATE
P. O. BOX 164
CAPE COAST

REPUBLIC OF GHANA
Tel. 03321-32514/32676
Fax 03321-32676
Email: capecoastmeo@yahoo.com
My Ref. No GES/MD/EPI.VOL.4/106
YourRef.No.

15th September 2017

CONCERNED HEADTEACHERS
(PUBLIC & PRIVATE BASIC SCHOOLS)
CAPE COAST METROPOLIS
CAPE COAST

INTRODUCTORY LETTER
MR. EBO AMUAH (U.C.C)

The bearer is this letter is a PhD (Mathematics Education) student from the Department of Mathematics and ICT Education of the Faculty of Science and Technology Education, University of Cape Coast who is conducting a study on **'Exploration of Conceptual Learning in Junior High School Mathematics Classroom in Three Selected Districts in the Central Region'**.

Permission has been granted him to undertake the study in your school as one of the selected schools for the study. However, you should ensure that the study will not interfere with normal teaching and learning activities.

Please accord him the necessary assistance to ensure a successful exercise.

Thank you.

FELIX ANSAH (MR)
DEP. DIR. SUPERVISION & MONITORING
for METRO DIRECTOR OF EDUCATION
CAPE COAST

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APPENDIX G

Selected parts of transcribed classroom interaction during the teaching and learning of addition of fractions in average achieving schools

Line	ACTION	PERSON	STATEMENT
. . .	. . .	. . .	. . .
L32	Teacher moves from the board	Teacher	so it is 2 2/5 So these are some of the things that we did yesterday. So we are going to upgrade ourselves. We are going to into the deeper form of the addition, this time we are going to mix them.
L33	Teacher writes the title Equivalent Fraction on the board	Teacher	But I want us to do this first. We will talk about (writes on the board Equivalent Fraction). But we are going to do it in the form of addition of fractions.
L34	Teacher begins to explain Equivalent fraction	Teacher	Now, when we say equivalent meaning that someone is the same as the other whether the person is older or smaller. for example e, errmm "y3 fa abusuafo ah",(consider relatives) I hope you are getting me?
L35		Students	Yes Sir
L36		Teacher	if I am for example, Priscilla is my sister meaning that "ndziyei bi wo menho a 3binso wo niho" I hope you are getting me?

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APPENDIX G - Continue

L37		Students	Yes Sir
L38		Teacher	Now let's take this example, in maths let's take this example ½ is likely to 2/4. "woy3 3nuanom", they are equivalent because if I should break this (referring to the 2/4) down. I will still get 1/2.
L39	Teacher continues to expalin equivalent fraction with further examples. Writes on the board 1/2 =2/4=3/6=4/8	Teacher	Meaning 2/4 I can write 3/6. Meaning that 3/6 "ninua ni" 2/4. You can also write 4/8 like that. Now we are going to use this in addition of fractions.
L40	Teacher continues with solving example on the board to explain issues to the students	Teacher	For example, let's take two unlikely fractions, I hope you are getting me?
. . .	. . .	. . .	. . .
L48		Teacher	"akyer3 wadwen". Yes what did you friend tell you (pointing to another student)
L49		Student	I multiplied 4 by 2
L50		Teacher	Your own you multiplied 4 by 2. So when you multiply you will get 8. "3b3y3 den na 3ha dei nso way3 8"

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APPENDIX G - CONTINUED

L51		Student	2x4
L52		Teacher	But "3y3 ni dem a, kai d3, whatever you do down here, you have to do the same thing up. Ma 3b3y3 ama maame bia,)y3d3 3y3 ma papa. I hope I'm clear?
L53		Student	Yes Sir
L54	Teacher continues to solve the question	Teacher	Meaning that "s3 3hadei mi mano two ah)y3 d3, 3hor nso m3y3 no d3n?" Multiply by two. S3 asi ha dei mi mano four ah,)y3 d3 3sor ha dei nso mi mano four". Then now I will multiply them. 2/4+1/2 2x2/4x2 + 1x4/2x4= 4/8+4/8
L55	Teacher pauses to ask a question	Teacher	Now I want you to tell me something. Something just happened right now (pointing to the board).
L56		Student	Sir, liked fraction
L57	Teacher continues with explanation ask questions as well	Teacher	Teacher: So we can say we have a likely fraction. First "na w)y3 unlikely now is what? Likely. Now when you reach here, what can you do to it?
L58		Students	Add them
L59		Teacher	Okay. 'sisia' we didn't use the LCM, have you seen that we didn't use the LCM but we used the equivalent fraction of them?. I hope I'm clear

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APPENDIX G - CONTINUED

L60		Students	Yes Sir
L61	Teacher….	Teacher	So this one the denominator you write 8. 4+4 (the numerators) too is 8. 8/8. So now what will be the answer?
L62		Students	One (1)
. . .	. . .	. . .	. . .
L79		Teacher	So 6 is the least. Meaning that all those who got a certain number, know what you are doing. Be very very careful. So now let's see something, what can we do to make the denominators the same. "Y3b3 y3 den na denominators no way3 the same thing"
L80	Student stands up to answer the question	Student	18
L81		Teacher	What did you use?
. . .	. . .	. . .	. . .

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APPENDIX G - CONTINUED

L89		Teacher	No. Let us break it down
L90		Student	7/6 21/18=7/6=1 1/6
L91		Teacher	Now, I saw someone doing something. "Mihui d3 obi bibi". The person did this 1/3+5/6. ")no ni di no oma asi ha nyina y3 the same thing". Let's see if the person is correct. 1x4/3x4+5x2/6x2= 4/12+10/12=14/12=7/6=1 1/6. This means that "s3 3 use four oo, 3 use three oo, you will get the same answer". I hope I'm clear?
L92		Students	Yes Sir
. . .	. . .	. . .	. . .

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APPENDIX H

Selected parts of transcribed classroom interaction during the teaching and learning of addition of fractions in high achieving schools

	ACTION	PERSON	STATEMENT
L1	Teacher begins class	Teacher	Somebody is saying 'you will add them' Agyekum, sit down. Sit down, Agyekum. Yes Redeemer
L2	Redeemer stands to talk.		(voice not clear)
L3	Teacher tells student to take his time	Teacher	Nya abotr3 wati (be patient, okay?). Exercise patience. Uhuh
L4	Redeemer continues to give his answer	Redeemer Hotoworshie	Denominator over numerator
L5	Teacher repeats student's response to class and calls another student to try.	Teacher	Somebody is saying denominator over numerator. uhuh
. . .	. . .	. . .	. . .
L26	Teacher pointing to the fraction on the board (½), asks students a question.	Teacher	3sid3n na y3nyaa (How did we get) one over two (½) out of this whole. 3sid3n na y3nyaa y3nyaa (How did we get) one over two (½) out of this whole? Appiah!
L27	Student answers teacher's question	Evans Appiah	Madam, you divide the boxes into two

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APPENDIX H - CONTINUED

L28	Teacher disagrees with student's answer and calls on another student to answer.	teacher	No. Sharif (voice not clear)
. . .	. . .	. . .	. . .
L38		Dadzie (Richmond Tawiah)	Madam you will shade one of the whole and divide it
L39		Teacher	Okay. So you will take away one of the parts. Is that okay?
L40		Class	Yes Madam
. . .	. . .	. . .	. . .

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APPENDIX H - CONTINUED

L101		Students	Three
L102	Teacher explains the steps to students pointing to the two rectangular figures on the board  	Teacher	Three. Okay. So let's come to the figure on the board. So assuming this is our what, two sheets. Is that okay?
L103		Students	Yes madam
L104	Teacher continues her explanation. Teacher pointing to the two rectangular figures, counts the shaded sections of the two figures together.	Teacher	The first sheet we took what, two parts and the second one we took away one. Now when we add all the parts taken from the two sheets, we have one, two, and three. Is that okay?
L105		students	Yes madam
L106		Teacher	And since our denominators are the same, we are using just what, one sheet. Is that okay?
L107		students	Yes madam

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APPENDIX H - CONTINUED

L108		Teacher	Now this means that we have what, three out of what, five. Is that okay?
L109		students	Yes madam
. . .	. . .	. . .	. . .

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