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Procedural Problem-Solving Approaches Employed by Students in Learning Extensive and Intensive Quantities of Change of State

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Abstract
The purpose of the study was to investigate the procedural problem-solving approaches students employ when solving computational problems that involve extensive and intensive quantities of change of state. A sample of 240 Form 3 science students randomly selected from five senior high schools in the Cape Coast Metropolis participated in this study. An achievement test on change of state of matter comprising of five items was used for data collection. The results showed, among other things, that students employ the structured procedural approach when solving change of state computational questions that involve extensive quantities instead of the scientific approach. The study also found that no clear procedural approach was employed by majority of the students in solving change of state computational questions that involved intensive quantities. The study further revealed that among the five problem-solving approaches, the scientific approach was the most effective in revealing students' correct conceptions of intensive quantities. These findings suggest that for students to be good problem solvers, teachers must teach concepts using the scientific approach to effectively compel learners to analyse problems based on their conceptual understanding before they proceed with computations.

Key words: Problem solving, procedural approach, extensive quantity, intensive quantity, correct conception.

Introduction

Physics is a science which engages students with hands-on and minds-on activities that require them to perform computational tasks.

Computational questions in physics usually seek verifiable answers to physical quantities like distance, mass, time, latent heat of fusion, latent heat of vaporisation, temperature and density (Ostdiek & Bord, 2013). Physical quantities are usually identified by a number, or a combination of a number and a unit, which makes them unique and easy to interpret (Duncan & Kennett, 2009). For instance, the relative density of aluminium is 2.7 and it is a typical example of the use of a number to quantify the physical quantity and on the other hand, the mass of a bag of cement is 50 kg is the use of the combination of a number and a unit. Practically, industries communicate with their partners and clients in terms of physical quantities. Water manufacturing industries, for instance, communicate to consumers using physical quantities (e.g., a volume of voltic water is 750 ml). The Ghana Highway Authority erects sign posts to communicate to road users about speed limit (e.g., the speed limit for urban driving is 50 km/h) and the air traffic controllers of the Civil Aviation Authority also communicate to pilots in terms of physical quantities (e.g., you are 500 miles away from Kotoka International Airport).

Physical quantities can be combined by employing either addition or averaging. A physical quantity that can be combined using addition is known as an extensive quantity and a physical quantity that can be combined by using averaging is known as an intensive quantity (Howe, Nunes, Bryant, Bell, & Desli, 2010). Extensive quantities such as length, mass, area, or volume can be measured directly or can be counted, whereas, intensive quantities such as speed or concentration cannot be measured directly or counted (Simon & Placa, 2012). Howe et al. (2010) further explained that an extensive quantity relies on fractional relationships. For example, 1 kg of ice block depicts an extensive quantity because it consists of the sum of the masses of the individual constituents of the ice block. Contrary, the use of averaging for a particular physical quantity produces an answer which is neither greater than nor less than that physical quantity. Thus, an intensive quantity is a constant parameter for a particular substance. For instance, when an ice cube at its melting point is divided into several pieces, each piece will have a temperature of 0 ^{o}C. With this explanations and examples in mind, Howe, Nunes and Bryant (2011) indicated that an intensive quantity establishes a proportional relationship between variables of a formula. It is important to state that when an extensive quantity divides another extensive quantity, the quotient is an intensive quantity. Therefore, intensive quantities express the relationship between two quantities which can either be intensive or extensive. For instance, density is a magnitude that predicts the strength of a relationship between the mass and volume for a particular substance at a specific temperature.

Change of state is a topic in physics that provides explanations to the phase transition of substances at melting and boiling points (Serway & Beichner, 2000; Cutnell & Johnson, 2007). At melting and boiling points, the heat energy transferred to a substance does not change the substance's temperature, rather it changes the state of the substance. For example, an ice cube at 0 ^{o}C absorbs energy in the form of heat to change its physical state to water at 0 ^{o}C. The treatment of change of state focuses on five major physical quantities (Cutnell, & Johnson, 2007; Walker, 2008), out of which two of them are intensive quantities and three are extensive quantities. Importantly, specific latent heat of fusion and specific latent heat of vaporisation are intensive quantities whereas mass, latent heat of fusion and latent heat of vaporisation are extensive quantities. The latent heat (Q) removed or supplied to a substance of mass (m) at a constant temperature is given as Q = mL. The variable L is an intensive quantity which represents the amount of heat per unit mass. Being an intensive quantity means that specific latent heat is a constant parameter for every substance that depends on proportional relation between variables and can be combined by using average. Also, specific latent heat of fusion or vaporisation is an intensive quantity because, it is a quotient of two extensive quantities. Further, the extensive quantity latent heat (latent heat of fusion or latent heat of vaporisation) varies directly with only the extensive quantity, mass, for a particular substance. This means that the greater the mass of a substance, the greater the amount of latent heat required to cause a phase transition at a constant temperature. In relation to the latter, a fractional increase in mass is equal to a fractional increase in only latent heat. For example, if 2 kg of ice require 672000 J of heat to change its state to water at a constant temperature, then doubling the mass of the ice (2 × 2 kg = 4 kg) will also double the quantity of heat (2 × 672000 J = 1344000 J) required to change its state to water without a temperature change. From this example, specific latent heat of fusion remains unaffected because it is independent of mass and latent heat of fusion. Thus, the only mathematical tool to maintain specific latent heat as a constant parameter is average. In terms of averaging, the specific latent heat of fusion of ice for the question above is $\frac{336000\,J\ +336000\,J}{2}$ = 336000 J. It is crystal clear that such analyses require high level of critical thinking and should, therefore, not be undermined in physics education.

However, in Ghana, as in other developing countries in Sub-Saharan Africa, the elective physics syllabus only emphasises the treatment of physical quantities in terms of fundamental, derived, scalar and vector quantities (Ministry of Education Science and Sports, 2010). For this reason, many high school physics textbooks and classroom instructions have neglected the treatment of physical quantities in terms of intensive and extensive quantities. Due to this lack of recognition, many students find it difficult to distinguish between intensive and extensive quantities (Howe et al., 2010; Simon & Placa, 2012). For instance, Alwan (2010) complained that the neglect of the extensive-intensive framework in many curricula has made it difficult for students to distinguish between heat (extensive quantity) and temperature (intensive quantity); heat capacity (extensive quantity and specific heat capacity (intensive quantity). Available researches with extensive quantities (Correa, Nunes & Bryant, 1998; Stavy & Tirosh, 2000; Squire & Bryant, 2003) indicate that despite the successful computations of additive problems, many students face challenges with questions that involve inverse relation variables. Additionally, literature on intensive quantity (Howe et al., 2010) suggests that students encounter challenges while solving combination and single variable problems. Lastly, Howe et al. (2010) complained that there is limited research that explicitly explores the procedures students employ while solving computational problems on intensive quantities.

One of the factors that influence the type of procedural approach students employ when solving computational problems in physics is the type of question (Walsh, Howard & Bowe, 2007). Problems in physics may either seek answers to an extensive quantity or intensive quantity. Howe et al. (2010) examined some primary school learner's reasoning with intensive quantities and found that learners face more challenges when working with intensive quantities than extensive quantities. Howe et al. reported that 11 % of the learners arrived at the correct answer when the variable, distance, directly proportional to speed, was manipulated. However, they detected that majority of the learners (81 %) arrived at the correct answer when the variable, time, inversely proportional to speed, was manipulated. The vast difference in percentage might possibly suggest that either the learners approached each problem differently or an approach used in one problem failed when applied to another problem. Alwan (2010) also showed that learners faced challenges when dealing with intensive quantities. Alwan stated that most of the learners were unable to determine the final temperature when two substances at different temperatures were mixed together. Perhaps, their participants were just adding temperatures; an approach common amongst learners. Conversely, Alwan reported that the students were successful in using the formula $Q = mc\Delta t$ (Q represents amount of heat, m represents mass of substance, c represents specific heat capacity and Δt represents temperature change) to solve for the amount of heat. This led to the conclusion that students are successful in manipulating formulae to arrive at an answer without a good understanding of the concepts that underpin the formulae. Lastly, Alwan concluded that the absence of treating physical quantities using the intensive-extensive framework poses challenges to learners.

Simon and Placa (2012) explored the possibility of getting a model to enhance reasoning about intensive quantities in relation to whole number multiplication and division problems. Based on their results, they concluded that it is impossible to develop a teaching model that will enhance learners' reasoning with intensive quantities with respect to whole number multiplication and division problems. The latter seem to be inconsistent with the effectiveness of the scientific approach as a problem-solving tool, as concluded by Walsh et al. (2007), which could also possibly be developed into a teaching model. Abrahamson (2012) probed the effectiveness of guided mediated abduction as a tool that can enhance learners' understanding of intensive quantities. The study revealed that guided mediated abduction is a tool that stimulates the central concept on which a mathematical notion hinges on and enculturates learners to be part of such a framework. Hill et al. (2014) investigated learners reasoning about concentration of sugar solution in the context of intensive quantities. Drawing samples from United Kingdom and Japan, they realised that challenges encountered by learners when reasoning with intensive quantities depend on cultural experience.

Several researches (Walsh et al., 2007; Snetinova & Koupilova, 2012; Kuo, Hull, Gupta & Elby, 2013; Zewdie, 2014) show that learners adopt different procedures when solving computational questions in physics. As a consequence, problem-solving classifications are commonly used as a way of describing the different set of procedures learners employ while solving computational problems in physics. For instance, Walsh et al. (2007) grouped learners problem-solving into four major categories. According to Walsh et al., learners' problem-solving can be categorised as scientific approach, plug and chug approach, memory-based approach and no clear approach. It must be noted that the plug and chug approach was sub-divided into structured manner and unstructured manner. The results from their research indicate that many higher level students do not approach physics problems in a planned manner. Another finding that emerged from their study was that students used different approaches to solve different questions. Snetinova and Koupilova (2012) also proposed nine procedural approaches to problem solving. The authors categorised the approaches students employed into rolodex equation matching, rational thought, listing known and unknown quantities, prior examples in text or lectures, prior experiments in lecture, sub-problems, diagrams, concept first and real situation. They further divided these categories into limiting strategy and expansive strategy. According to them, limiting strategies are successful when applied to well-structured end of chapter exercises but are ineffective when applied to complex problems. However, expansive strategies are very effective when applied to complex problems. They stated that expert problem-solvers favour this strategy.

Additionally, Hegde and Meera (2012) probed learners' approach to the mechanism of physics problem-solving by the using multiple choice questions and questions of semi-structured interview to examine students' thought processes in physics problem-solving. It was found that terms employed in a physics task compelled student to search for an equation. Based on this, they realised that a problem solver's inability to locate an equation impedes the problem-solving process. They further stated that having access to an equation does not guarantee success in arriving at an answer. This was grounded in the fact that learners do not appreciate the relationships amongst physical quantities in physics equations. Also, lack of mathematical manipulation skills was identified as an obstacle that hinders learners' problem-solving ability. Finally, their study added that a lack of conceptual understanding also hinders learners' success when guided during problem-solving. Kuo et al. (2013) in a study, on the other hand, focused on how learners combine conceptual and formal mathematical reasoning in solving mechanics problems. Through an interview, Kuo et al. tasked learners to provide an explanation to an equation and solve a mechanics problem with that equation. The results of the study indicated that some students blended mathematical operations with conceptual reasoning to solve real-life problems and they described this approach as symbolic form. They added that such learners employ a non-computational means of solving physics computational problems. Kuo et al. further reported that other students' description of the equation were more mathematical. Additionally, they stated that such students depended on only computations when solving physics problems. Inferring from the latter, it could be concluded that such students did not pay attention to the concepts that underscored the problem.

Cruz (2014) also investigated the effect of structured problem-solving strategy on the performance of 152 undergraduate students and concluded that the structured problem-solving strategy is an effective problem-solving technique that improves the learning of students. According to Polya (1957), structured problem-solving involves description, planning, implementation and checking. The description stage involves providing information and using diagrams to summarise the situation at hand. Further, the planning stage focuses on the basic relations that underscore the situation. Additionally, the implementation stage touches on computations. The final stage emphasises on checking whether the answer is right or wrong. Admittedly, this approach is similar to the scientific approach of Walsh, et al. (2007). Zewdie (2014) also employed the approaches proposed by Walsh et al. (2007) to explore some learners' procedural approach to problem solving but did not divide the plug-and-chug approach into its subcategories. In contrast to Walsh et al., Zewdie noted that none of his subjects followed the scientific approach in solving the tasks.

While studies on learners' knowledge of intensive and extensive quantities have received considerable attention by science educators (Howe et al., 2011; Hill et al., 2014), little evidence is available on studies to ascertain the procedural approaches learners employ while solving computational problems on extensive and intensive quantities. Additionally, since intensive and extensive quantities are not emphasized in the Ghanaian and many other West African countries' Senior High School (SHS) syllabi, there appear to be a gap in literature on the procedural approaches students adopt in solving problems involving extensive and intensive quantities. There is, therefore, a need to explore the procedural approaches of learners when they solve computational questions involving intensive and extensive quantities. Based on this recognition, the purpose of this study was in three-fold. First, this study explored the procedural approaches senior high school science students employ while solving computational questions that seek answers to extensive quantities. Second, the study investigated the procedural approaches senior high school science students employ when solving computational problems that seek answers to intensive quantities. Third, the study investigated the procedural approach that is most robust in unveiling students' conceptions about intensive quantities. This study attempted to answer the following research questions:

1. What procedural approaches do students employ while solving change of state computational questions that involve extensive quantities?
2. What procedural approaches do students employ while solving change of state computational questions that involve intensive quantities?
3. Which procedural approach is robust in revealing students' conceptions about intensive quantities?

Methodology

Since the primary focus of this study was to explore the procedural problem-solving approaches students employ in solving change of state computational tasks involving extensive and intensive quantities, the qualitative survey design, which is less a structured methodology, was employed to help gain an in-depth understanding of how students use procedural problem-solving approaches to solve computational tasks (Cohen, Manion, & Morrison, 2007; Creswell, 2012). This design asks open-ended questions that yield responses that are used to uncover trends in thought and probe deeper into the problem at hand. In all, 240 Form 3 students randomly sampled using the computer-generated random numbers from five out of the 10 senior high schools in the Cape Coast Metropolis, offering the General Science programme, participated in the study. An achievement test comprising of five open-ended test items was used to collect data (See Table 5 and Table 6 for the five items). Students were also asked to describe how they approached each item in a brief statement. Each item on the test fell into one of the three levels of reasoning. These levels of reasoning include: Level A, Level B and Level C as adapted from Noelting's (1980) levels of reasoning. Table 2 summarises the features of each level.

Table 2: Features of Level of Reasoning

Level of Reasoning	Feature
Level A	Question requires mere substitution into formula.
Level B	Question requires averaging to determine an intensive quantity.
Level C	Question requires manipulation of directly proportional variables.

The other features such as type of computation and quantity required of each item is also displayed in Table 3.

Table 3: Features of Each Item

Item Number	Level of Reasoning	Computation required	Quantity
1	A	Latent heat of fusion	Extensive
2	A	Mass	Extensive
3	C	Latent heat of vaporisation	Extensive
4	B	Specific latent heat of fusion	Intensive
5	A	Amount of heat per unit mass	Intensive

From Tables 2 and Table 3, Level A reasoning means a question requires mere substitution. Thus, numbers can easily be plugged into a formula to arrive at an answer. By contrast, Level B reasoning suggests that a question requires finding average to maintain a particular intensive quantity. Lastly, Level C reasoning suggests that learners must reason with how one variable varies directly with another. Table 4 provides a detailed description of each approach.

Table 4: A Description of Walsh et al. (2007) Procedural Approaches to Problem-Solving

Approaches	Description
Scientific	Begin by qualitatively describing the concept on paper. Proceed by discussing in a coherent manner. Employ an equation and conclude by evaluating the answer.
Structured manner	Identify the concepts that are involved but do not begin by qualitatively analyzing the problem on paper. Recognize the variables needed to solve the question and seek appropriate formula.
Unstructured manner	Students depend only on variables that are stated in the question to employ an equation.
Memory-based	Learners who employ this approach rely on past experiences such as remembering a method used in class and recalling procedures employed in textbooks and past questions. Learners in this category usually recall a formula and substitute the given variables into it.
No clear	Learners do not approach computational tasks in a well-defined way. Their solution and knowledge are not organized in a coherent way. Centered on the variables given, they haphazardly seek for equations that will facilitate the use of the variables stated in a question. They also change their strategy as they proceed through a solution.

The content validity of the concept test was determined by two physics educators from the Department of Science Education and two experienced physics tutors who have taught physics for more than 15 years. The data for the first two research questions, were analysed using frequencies and percentages. Since these two research questions were aimed at categorising the approaches students employed in solving computational tasks on change of state, frequencies and percentages were, therefore, reported for each problem-solving approach. The categorisation of the problem solving approaches used by students in this study was based on the previous categorisations used by Walsh et al. (2007) where students' solutions were categorised as scientific approach, plug-and-chug approach (structured manner and unstructured manner), memory-based approach and no clear approach.

The third research question was analysed based on an argument that a good problem-solving activity does not focus on just following a set of procedures but involves relating the task at hand to the concepts that underscore it. Therefore, the robust approach was selected based on the potency of an approach in revealing both correct and wrong conception a learner holds about the intensive quantities.

Results

Students' procedural approaches to tasks involving extensive quantities

Research question one sought to investigate the procedural approaches students employ in solving change of state computational questions that involve extensive quantities. As shown in Table 5, no student approached Item 1 and Item 2 in a scientific manner. However, as shown in Table 5, .4 % of the 240 students used the scientific approach while solving Item 3. Further, half of the students (50 %) used the structured manner while solving Item 1. Interestingly, Item 2 closely followed Item 1 with a percentage of 47.9 %. For Item 3, only 11.7 % of the students employed the structured manner. Comparatively, one of the least popular approaches used was the unstructured manner of the plug and chug approach. Out of 240 students, as shown in Table 5, 13.8 % and 7.9 % employed the unstructured manner while solving Item 1 and Item 2 respectively. In terms of memory-based approach, for Item 1 and Item 2, as shown in Table 3, recorded close percentages of 20.4 % and 21.3 % respectively.

However, 10 % of these students used this approach while solving Item 3. For the no clear approach, Item 1 and Item 2 recorded 15.8 % and 22.9 % respectively. However, about three-fourth of the students (77.1 %) employed the no clear approach while solving Item 3. Only .8 % of the students did not attempt Item 3.

Students' procedural approaches to tasks involving intensive quantities

Research question two was intended to investigate the procedural approaches students employ in solving change of state computational questions that involve intensive quantities. As shown in Table 6, 0.8 % and 0.4 % (of 240) students employed the scientific approach while solving Item 4 and Item 5 respectively. In relation to structured manner, only 0.8 % of the students used this approach while solving Item 4 whereas 32.9 % of the students employed this approach while solving Item 5.

Table 5: Results of Students who employed each Approach for Items 1, 2 and 3

Items		Scientific	Structured manner	Unstructured manner	Memory-based	No clear	No Attempt
1.	What heat is required to change 0.002 kg of ice at 0 ºC to water at 0 ºC? (Specific latent heat of fusion = 336000 J/kg)	0 (0)	120 (50.0)	33 (13.8)	49 (20.4)	38 (15.8)	0 (0)
2.	The amount of heat supplied to water at 100 ºC to change it to steam at 100 ºC is 90400 J. Calculate the mass of the water. Specific latent heat of vaporisation of water is 2260000 J/kg.	0 (0)	115 (47.9)	19 (7.9)	51 (21.3)	55 (22.9)	0 (0)
3.	A liquid containing x kg of water at 100 ºC required y J of heat to completely boil. If the mass of the water is tripled, how much heat is required to completely boil at 100 ºC	1 (.4)	28 (11.7)	0 (0)	24 (10.0)	185 (77.1)	2 (.8)

Numbers in brackets represent percentage

Table 6: Results of Students who employed each Approach for Items 4 and 5

Items		Scientific	Structured manner	Unstructured manner	Memory-based	No clear	No Attempt
4.	A vessel contains 2 kg of water at 0 ºC. 3 kg of water at that same temperature is later added to the water in the vessel. If specific latent heat of fusion is 336000 J/kg, calculate the specific latent heat of fusion of the water in the vessel?	2 (0.8)	2 (0.8)	0 (0)	6 (20.4)	231 (96.3)	3 (1.3)
5.	The amount of heat supplied to water at 100 ºC to change it to steam at 100 ºC is 90400 J. Calculate the mass of the water. Specific latent heat of vaporisation of water is 2260000 J/kg	1 (0.4)	7.9 (32.9)	29 (12.1)	48 (20.0)	83 (34.6)	0 (0)

Numbers in brackets represent percentages

For the unstructured manner, no student used this approach while solving Item 4. However, the unstructured manner recorded 12.1 % students for Item 5. Table 6 further displays that 2.5 % of the students used the memory-based approach in solving Item 4. However, 20 % of these students employed this approach while solving Item 5. Finally, an overwhelming majority of students' (96.3%) solutions to Item 4 was described as no clear approach. Conversely, 34.6 % of the students' solutions to Item 5 was categorised as no clear approach. Only 1.3 % of the students did not attempt Item 4.

Revealing students' conceptions about intensive quantities

Research question three sought to investigate the robust procedural approach used in revealing students' conceptions about intensive quantities. Table 7 displays findings of the robust approach which was successful in revealing the students' conceptions of intensive quantities. Robust approach, in this context, means a problem-solving procedure that has the tendency of revealing correct conceptions, alternative conceptions, correct mathematical algorithm and wrong mathematical algorithm.

Table 7: Robust procedural approaches for revealing students' conceptions of intensive quantities

Approach	Correct conceptions	Alternative conceptions	Correct mathematical algorithm	Wrong mathematical algorithm
Scientific	1	1	1	1
Structured manner	0	0	1	1
Unstructured manner	0	0	1	1
Memory-based	0	0	1	1
No clear	0	0	0	1

Keys: 1 = Yes, 0 = No

As shown in Table 7, the scientific approach is the most robust in revealing the conceptions of students about intensive quantities. This implies that structured manner, unstructured manner, memory-based approach and no clear approach are ineffective in revealing the conceptions of students about intensive quantities. Hence, there is a good reason to conclude that the scientific approach is very effective in revealing the lines of reasoning of students, when solving a computational physics problem, in addition to uncovering correct and wrong computational algorithm. Two solutions, to Item 4, that highlights the robustness of the scientific approach are displayed below.

Example 1

''The specific latent heat of fusion of a substance is the amount of energy (in joules) needed to melt a solid of 1 kg to liquid of the same mass without changing its temperature. So, if 3 kg of water at 0 °C is added to 2 kg of water at the same temperature. The specific latent heat of fusion remains the same but the mass of the water changes to 5 kg. From mathematics $\frac{336000 + 336000}{2}$

= 336000 J/kg, Since the specific latent heat of fusion is a constant value, this confirms my value''.

This student commented on his steps as follows: *''I first provided an explanation of what specific latent heat of fusion is about and after I determined the average''*. A thorough examination of the above example indicates that the accurate understanding held by the respondent about specific latent heat of fusion (as an intensive quantity) resulted in the correct answer. Thus, this example shows that an accurate understanding of specific latent heat as an intensive quantity has a higher probability of producing a correct answer.

Example 2

''Since the 3 kg mass of water is at the temperature that is 0 °C as the one in the vessel; the quantity of heat energy produced after the addition of the 3 kg mass of water will be latent heat. Therefore, the quantity of latent heat in the 2 kg mass of water (Q_1) will be the same as the quantity of latent heat in the 3 kg mass of water (Q_2) that is $Q_1 = Q_2$. Q_1 is the product of mass of water in the vessel and its specific latent heat of fusion. Also, Q_2 is equal to the product of the mass of water added and its specific latent heat of fusion.

$Q_1 = Q_2,\ m_1 l_{f1} = m_2 l_{f2}$

$m_1 = 2$ kg, $l_{f1} = ?$, $m_2 = 3$ kg and $l_{f2} = 336000$ J/kg

$l_{f1} = \frac{3 \times 336000}{2}$

= 504000 J/kg

The specific latent heat of fusion of the water in the vessel is 504000 J/kg''.

This student commented that *''I first described my understanding of the question in terms of change of state. Then I deduced a mathematical expression after I calculated to get my answer''*. An evaluation of this solution suggests that this student did not conceptualise specific latent heat of fusion as a constant parameter for water. Therefore, this solution indicates that the student arrived at a wrong answer because of an alternative view about the question.

In order to highlight the contrast between an approach which is robust and that which is not robust, there is the need to present a solution that exemplifies a non-robust approach. An example of such an approach to Item 10 is presented below.

''$Q = ml_v$

$\frac{Q}{m} = \frac{mlv}{m}$

$l_v = \frac{Q}{m}$

$l_v = \frac{4520000\,J}{2\,kg} = 2260000\ J/kg$''

The presentation of this student does not show his line of reasoning and, thus, it becomes difficult to unravel any correct or alternative conception. Evidently, a follow up question showed that the student's solution concealed his understanding of the question. The student described the steps involved in the above solution as follows: *''Heat per unit mass is like mass per unit volume in density, so I applied the way I solve mass per unit volume''.* In relation to the student's comment, there is a good reason to conclude that students sometimes relate a problem in a particular area of physics to previous solutions of problems in other areas of physics.

Discussion

The first results of this study revealed that students employ mostly the structured followed by the no clear procedural approaches when solving change of state computational questions that involve extensive quantities. The findings of this study showed that students did not use the scientific approach category in solving questions on Item 1 and Item 2. This suggests that when problems on extensive quantities involve mere substitution of numbers into a formula, students do not begin by merging verbal and diagrammatic description of the task with a formula on paper. The latter possibly means that students do not use the scientific approach when they encounter questions (on extensive quantities) that require Level A reasoning. This result confirms the assertion that many leaners do not analyse the concepts that underscore physics computational task on paper by making a diagrammatical analysis of the problem (Walsh et al., 2007; Zwedie, 2014). However, for Item 3, only one student used the scientific approach in solving the problem. This means that when a question (on extensive quantity) requires the manipulation of only a directly proportional variable, few students begin by analysing the problem qualitatively first on paper in terms of the concepts that underscore it. This finding hinges on Zwedie's (2014) assertion that few learners make an effort to express their understanding about a physics computational task on paper before solving.

Recall that about half of the students' solutions to Items 1 and 2 fell into the structured manner of the plug and chug approach. Since there are five problem-solving categories, the latter means that majority of the students prefer to create a mental picture of the question first, write down variables from the question, recognise that some variables are stated but not needed, identify variables that are not stated but needed, substitute variables into an equation and compute the variables to arrive at an answer when solving extensive quantity questions that require mere substitution into a formula. This result contradicts the findings of Zewdie's (2014) who reported that more than half of the respondents in a study preferred the memory-based approach. Thus, it is possible that the type of physical quantity and the level of reasoning a question require influences the type of problem-solving approach a student will employ. However, since close to one-eighth of the students employed the structured manner while solving Item 3, there is a possibility that many students do not think about a concept first before proceeding with an extensive quantity problem that involves the manipulation of only a directly proportional variable. The latter could possibly explain why Snetinova and Koupilova's (2012) asserted that many students do not make an effort to understand a problem before proceeding with computation. The large disparity in fraction, according to the former and latter submissions, shows that many students employ structured manner when a question requires mere substitution of variables into a formula (Level A reasoning) when compared with a question that requires the manipulation of a directly proportional variable (Level C reasoning).

A unique feature of the unstructured manner is that students whose solutions fall in this category depend solely on variables which are stated in a problem. As stated earlier, a little over one-eighth of the students employed this approach while solving Item 1 whereas a little over one-sixteenth employed this approach while solving Item 2. The disparity in fraction is quite surprising since both items require Level A reasoning. This could possibly be attributed to the nature of the two questions. Item 1 requires the use of $Q = mL_f$ while Item 2 requires the use of $m = \frac{Q}{Lf}$. Interestingly, none of the students' solutions to Item 3 fell into the unstructured manner category. Note that Item 3, which required Level C reasoning, left out an intensive quantity that could possibly play a role in computations. Thus, it just needed an understanding of manipulating a directly proportional variable (i.e., $m \propto Q$). These findings about unstructured manner category raise questions about the viability of Zewdie's (2014) results which state that many students rush to do computations when given a physics task. Additionally, the results contradict Oglive's (2009) observation that many learners search for equations based on the known variables a task employ.

Surprisingly, about one-fifth of the students relied on past experiences while solving Items 1 and 2. Such consistency in the number of students for these two items suggests that when extensive quantity questions require Level A reasoning (see Table 2), all questions will record almost the same number of students for the memory-based approach. For Item 3, the number of students whose solution fell into this approach possibly means that when a question on extensive quantity requires the manipulation of a directly proportional variable (Level C reasoning) students will adopt memory-based approach. These findings imply that the memory-based approach is not the most preferred approach amongst the students which confirms the findings of Snetinova and Koupilova's (2012). According to Snetinova and Koupilova, learners rarely rely on past examples when solving physics computational tasks. However, the results are inconsistent with Zewdie's (2014) observation that majority of learners rely on previous experiences to solve physics computational task. The absence of the no clear approach for Items 1 and 2 is not surprising since both items require Level A reasoning. In relation to Item 3, the overwhelming majority of students whose solutions fell into this category suggest that many students are inconsistent in their presentations when solving extensive quantity questions that require the manipulation of a directly proportional variable. These results are inconsistent with the findings of Zewdie (2014) who is of the view that the no clear approach is one of the least preferred approaches learners employ in solving problems.

The second result of this study revealed that students employ mostly the no clear procedural approach when solving change of state computational questions that involve intensive quantities. Recall that Items 4 and 5 recorded one of the least numbers of students who employed the scientific approach category. The latter means that for every 240 students who answer Item 4, only two of them use the scientific approach. On the other hand, for every 240 students who answer Item 5, one uses the scientific approach. Comparatively, this difference, though small, suggests that some students provide a detailed description of their solution when a question probes an intensive quantity as a constant parameter. With respect to the latter, it is possible that when students are given tasks that require Level B reasoning, they are compelled to interpret their steps in order to provide understanding on the part of examiners. Further, few students employed the scientific approach when a question on an intensive quantity requires Level A reasoning. The latter statement contradicts earlier findings of extensive quantities. Therefore, in the lens of Level A reasoning, the type of quantity which a computational task seeks could possibly influence a learner’s decision to use the scientific approach. These facts possibly explain why Howe et al. (2010) claim that learners face more challenges when working with intensive quantities than extensive quantities. Though some students used the scientific approach in the study by Walsh et al. (2007), no learner used the scientific approach in the study conducted by Zewdie’s (2014).

The study also found that a number of students employed the structured manner while solving Item 5 (32.9 %) compared to Item 4 (.8 %). Comparably, this means that many students easily construct a mental picture of the concept that underlie the question, write down variables from the question, recognise that some variables are stated but not needed, identify variables that are not stated but needed, substitute variables into an equation and compute the variables to arrive at an answer when solving questions that require Level A reasoning. Thus, the structured manner is not commonly used by learners when a question explores intensive quantity as a constant parameter (Level B reasoning). Additionally, examining these statistics indicate that structured manner is not the preferred choice when intensive quantities are considered but the most preferred choice when extensive quantities are considered. Additionally, no student employed the unstructured manner while solving Item 4. This suggests that when a task on intensive quantity requires Level B reasoning, no learner depends solely on the variables that are stated in the question. Similarly, the unstructured manner is one of the least preferred approaches by the students when Item 5 is considered. Thus, when an intensive quantity question requires Level A reasoning, only few learners depend solely on the variables that are stated in the question. This raises questions about the viability of Hegde and Meera's (2012) finding that terms employed in a physics task compels students to search for an equation.

Although the memory-based approach was one of the preferred approaches used by students in solving Items 4 and 5, the number of students who employed it were very small. Thus, few of the students relied on past experiences when solving intensive quantity questions that require Level B reasoning. These results confirm those obtained for the extensive quantities. There is, therefore, a good reason to conclude that, though the memory-based approach is one of the most preferred approaches used by students as reported in the work of Zewdie (2014), it is, however, not the case in this study. This result confirms Oglive (2009) and Snetinova and Koupilova's (2012) observations that many learners do not rely on past experiences when solving computational task in physics. Since more than 75 % of the students' solutions to Item 4 fall into the no clear approach, it suggests that majority of them are not consistent when presenting a solution to a question that seeks the constancy of an intensive quantity. This finding seems to corroborate Abrahamson's (2012) conjecture that learners have the tendency of switching their line of reasoning when dealing with intensive quantities. It also confirms the findings of Howe et al. (2010) which states that many learners are unable to solve tasks which explore an intensive quantity as a constant parameter. Thus, a problem-solver's inability to locate an equation impedes the problem-solving process (Hedge & Meera, 2012). Again, majority (34.6 %) of the students' solution to Item 5 also fall into the no clear approach. The latter results seem to be closer to the statistics quoted by Walsh et al. (2007) and Zewdie (2014) who quoted 27.3 % and 18.2 % respectively. These imply that when a question on intensive quantity requires mere substitution into a formula, few learners encounter challenges while solving such problems. A similar result was reported by Howe et al. (2010).

The third result of this study found that the scientific approach is the most robust in revealing students' correct conceptions, alternative conceptions, correct mathematical algorithm and wrong mathematical algorithm when solving change of state computational questions that involve intensive quantities. This finding confirms the study by Snetinova and Koupilova (2012) who are of the view that the use of an expansive strategy (i.e., scientific approach) is very effective for solving complex physics questions and has the capability of revealing the reasoning behind what informed the solution students present. Though the scientific approach was able to reveal students' conceptions, it also, however, showed alternatives ways students use in solving questions. These alternative views that students generate about questions presented, in many cases, lead them to provide wrong answers. These revelations agree with Walsh et al. (2007) that a scientific approach results in a wrong answer when the problem solver holds inaccurate conceptions of the concepts that underscore the problem. Further, the findings confirm that learners sometimes approach questions on intensive quantities using their knowledge of how extensive quantities are solved (Howe et al., 2011; Alwan, 2010; Simon & Placa, 2012). Additionally, the findings of this study clearly showed that some learners do not understand latent heat as an extensive quantity and specific latent heat as an intensive quantity.

Conclusions and Recommendations

The study found that majority of the students employ the structured procedural approach when solving change of state computational questions that involve extensive quantities instead of the scientific approach. This implies that there is a deficiency in the type of problem-solving approach students' use in solving computational problems. The findings of this study also confirmed existing body of literature that students' problem-solving approaches fell into scientific, structured manner, unstructured manner, memory-based and no clear.

Secondly, the results of this study showed that the no clear procedural approach was employed by majority of the students when solving change of state computational questions that involve intensive quantities. Though few of the students employed the scientific approach irrespective of the demands of a question, it showed clearly that students have difficulties dealing with computational tasks that involved intensive quantities. To help students become adept problem solvers, physics teachers should aim at providing learners with questions which involve intensive quantities and questions which require the application of variation in extensive quantities. Again, rubric to computational questions should be structured to meet the standards of scientific approach. Further, multifaceted tasks must be part of learners' assignment.

Finally, this study unveiled that among the five problem-solving approaches, the scientific approach is very effective in revealing students' correct conceptions and alternative conceptions about intensive quantities. It is, therefore, recommended that teachers should develop the scientific approach into a teaching model to enhance the understanding of intensive quantities since this will also help reveal the lines of reasoning of students.

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