UNIVERSITY OF CAPE COAST

AN INVESTIGATION INTO SENIOR HIGH SCHOOL MATHEMATICS TEACHERS' KNOWLEDGE FOR TEACHING ALGEBRA

BY

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Thesis submitted to the Department of Mathematics and I.C.T Education of the Faculty of Science and Technology Education, College of Education Studies, University of Cape Coast, in partial fulfilment of the requirements for the award of Doctor of Philosophy Degree in Mathematics Education

MARCH 2017

DECLARATION

Candidate's Declaration

I hereby declare that this thesis is the result of my own original research and that no part of it has been presented for another degree in this university or elsewhere.

Candidate's Signature: .... Date: 02/05/2018

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Supervisors' Declaration

We hereby declare that the preparation and presentation of the thesis were supervised in accordance with the guidelines on supervision of thesis laid down by the University of Cape Coast.

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ABSTRACT

Research is replete with the fact that teacher knowledge affects student performance. However, the issue of which aspects of teacher knowledge influence student achievement has been a thorny one among researchers. This is partly due to the fact that over the years, several attempts at conceptualising teacher knowledge have produced domain neutral constructs making them virtually impossible to be accurately measured. As a result, several attempts to measure teacher knowledge have relied on proxy measures such as the number of university courses taken, the type of degree the teachers' have etc. This study is posited on the fact that, instead of relying on proxy measures, there is the need for re-conceptualization of teacher knowledge in ways that is not only domain specific but also allows its components to be measured. In the early to mid-2000s, researchers of the Knowledge of Algebra for Teaching (KAT) project attempted to do exactly this by hypothesizing three types of knowledge and developing items to measure it. Using the KAT conceptualization as a framework, this study was designed to investigate whether the three types of teacher knowledge hypothesized in the KAT framework will be corroborated. Two hundred and fifty two teachers from 40 senior high schools in three regions participated in this study. The cross-sectional survey was the main design used. Factor analysis conducted on data from this study did not only corroborate the three knowledge types hypothesised in the KAT framework but also permitted a modification to be made in the framework. Furthermore, analyses of data showed that in terms of the hypothesized knowledge types, while background in education did not significantly affect the quality of teacher knowledge, teachers with ten years and above teaching experience were significantly better. Based on the findings of the study, it was recommended that further studies be conducted to corroborate the seven factors that emerged in this study from the reconceptualization of the KAT framework.

ACKNOWLEDGEMENTS

Words cannot express how indebted I am to my supervisors, Professor Eric Magnus Wilmot and Professor Ernest Kofi Davis both of the Department of Mathematics and I.C.T. Education, University of Cape Coast, who sincerely dedicated their precious time to guide me in this study, especially at a time when they were busily engaged in other important obligations. Their supervisory roles in this study revealed their unqualified commitment to completing this study.

Also, my sincerest appreciation to the University of Cape Coast, for sponsoring my study and providing research grant, which made it possible for me to successfully complete this study. My honest thanks goes to the teaching and non-teaching staff of my (Department of Mathematics and I.C.T. Education, University of Cape Coast) for their support in various ways in ensuring that this was successfully completed. his concern in edging me and seeing to it that I complete this study in time, Dr. D. D. Agyei and Mr. Eugene Adjei Johnson for their tremendous help, advice and encouragement which motivated me to finish this study on time.

In addition, my doctoral degree has been successful because of the firm support and encouragement my loving wife Georgina Doughan, my two wonderful boys Jehoash and Jeberechiah, my dear mother Grace Abekah, my in-laws Mr. and Mrs. Doughan and a special mother Mrs. Vida Wilmot. I am also thankful for the support and collaborative discussions I had with my colleagues Dr. Rosemary Twum of the Department of Mathematics and I.C.T. Education and Dr. Kofi Acheaw Owusu, Dr. Godwin Kwame Aboagye, Mr. Deodat Charles Otami and Mr. Rexford Baah all of the Department of Science. I also thank teachers and schools who were involved in this study.

DEDICATION

To my lovely wife, Georgina Doughan and my two lovely sons, Jehoash Nyameyie Doughan Yarkwah and Jeberechiah Kofi Doughan-Yarkwah

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CHAPTER ONE

INTRODUCTION

Though researchers are in agreement on the usefulness of teacher knowledge in influencing student performance (see for instance, Begle, 1972; Eisenberg, 1977; Wilmot, 2009), they are generally divided on how to effectively conceptualize it in order to show which aspect of it best predicts student performance. In addition, according to Wilmot (2008), some of these conceptualizations have presented teacher knowledge as a domain neutral construct making it virtually impossible for it to be objectively measured (for example, Shulman,1987; Grossman & Richert, 1988; & Grossman,1990). As a result, several attempts to measure teacher knowledge have relied on proxy measures such as the number of university courses taken, the type of degree the teachers' have and so on.

This study is posited on the fact that, instead of relying proxy measures, there is the need for re-conceptualization of teacher knowledge in ways that is not only domain specific but also allows its components to be measured. At the Senior High School level, researchers in the Knowledge of Algebra for Teaching (KAT) project at Michigan State University in the mid to late 2000s proposed one of such ground breaking framework of teacher knowledge (see Ferrini-Mundy, Burrill, Floden, & Sandow, 2003; Ferrini-Mundy, McCrory, Senk, & Marcus, 2005; Ferrini-Mundy, Senk, & McCrory, 2005). Furthermore, researching into the KAT framework, Wilmot (2017) has proposed a modified framework. This study therefore was designed to investigate whether the three domains of teacher knowledge hypothesized in the KAT framework will be validated. In addition, the study incorporated suggestions for further study by Wilmot (2008) in order to test whether the modified framework by Wilmot (2017) will also be corroborated.

Background to the Study

Research work is packed with the fact that teachers' knowledge greatly influences how they teach (see for example Ambrose, 2004; An, Kulm & Wu, 2004; Hill & Ball, 2004; Ross, 2003; Stipek, Givvin, Salmon, & MacGyvers, 2001). As a result of this, it is necessary to actively support the development of profound teacher knowledge so as to help them in making an operative evolution from the traditional classroom to the reform classroom, a transition that is still very much in progress or even delayed, according to 1999 TIMMS video data (Jacobs, Hiebert, Givvin, Hollingsworth, Garnier, & Wearne, 2006).

Studies conducted by Darling-Hammond (1999) and Yara (2009) claims that schools can make a tremendous transformation in students learning and a considerable portion of that difference is ascribed to teachers. This is in line with the assertion made by Clark and Peterson (1986) with regard to the fact that the teacher is the most essential factor in the effectiveness of any educational initiative as he/she interacts directly with the learner "It is the teacher's competence, ability, resourcefulness and ingenuity to efficiently utilize the appropriate language, methodology and available instructional materials to bring out the best from learners in terms of academic achievement" (Yara, 2009, p.365). Yara (2009) went further to contend that to some extent the individualities of the teachers and their experiences and behaviours in the classroom add to the learning environment of their students which in turn will have effect on their students' results. Consequently, when it comes to the teaching of mathematics it can be argued how teachers of mathematics teach the subject has a direct and momentous impact on how students learn the subject (Mullens, Murnane & Willett, 1996; Sanders & River,1996). Factors such as teacher qualification, class size, school size, (Ferguson, 1991), certification and teacher preparation (Darling-Hammond, 1999) and other school variables were established to regulate what students' learn.

In the past two decades, the subject matter knowledge of teachers of mathematics has become an object of concern. New theoretical and empirical insights into the work of teaching (Shulman, 1986a, 1987) have drawn greater attention of scholars to the role played by such knowledge in teacher training and how teaching must be done in a better if not in an excellent manner. No wonder, "mathematicians can be pleased to have at last powerful evidence that mathematical knowledge of teachers does play a vital role in mathematics learning" (Howe, 1999, p.882).

And as if to throw more light on this, Shulman (1983) pointed out that "...the teacher must remain the key. The literature on effective schools is meaningless; debates over educational policy are moot, if the primary agents of instruction are incapable of performing their functions well. No microcomputer will replace them, no television system will clone and distribute them, no scripted lessons will direct and control them, and no voucher system will bypass them (p. 504)."

One critical issue that has become a bone of contention and very problematic to address with reference to teachers' subject matter knowledge is the question of what teachers need to know about their subjects to support them stimulate powerful and flexible knowledge and understanding in students. In addition, this difficulty has further been complicated by the different conceptualizations so far given of teachers' knowledge (Kennedy, 1991). For instance, Cochran and Jones (1998) as cited in Wilmot (2008), identified the following four as the components of teacher knowledge: Content knowledge, substantive knowledge, syntactic knowledge and beliefs about the subject matter.

Thompson (1984) also in arguing that teachers' beliefs, views, and predilections about mathematics are some of the factors that play a momentous role in shaping their instructional behavior implied that the type of mathematical knowledge teachers draw upon in teaching could be prejudiced by their beliefs, views and predilections about the subject. The following year Leinhardt and Smith (1985) also suggested two types of teacher knowledge namely "lesson structure knowledge" (LSK) and "subject matter knowledge" (SMK). According to them, Lesson structure knowledge comprised smooth planning and organizing of lessons and providing clear explanations. They also hypothesized that subject matter knowledge consists of concepts, algorithmic operations, connections among different algorithms and knowledge of the types of errors students make.

When Shulman and his colleagues took centre stage on research into teacher knowledge they also presented the following seven types of knowledge as the knowledge base for teaching (see Shulman 1986b): 1) Content knowledge,

2) General pedagogical knowledge (e.g. classroom management), 3) Curriculum knowledge, 4) Pedagogical content knowledge, 5) Knowledge of learners and their characteristics, 6) Knowledge of educational contexts, and 7) Knowledge of educational ends, purposes, and values. At the heart of Shulman's (1986b) conceptualization is the idea of "Pedagogical Content Knowledge" (PCK). Shulman (1986b) argued that PCK is the type of knowledge that distinguishes the teacher as professional from those who claim to be content experts and those who simply comprehend kids. Expounding on the conceptualization of PCK and its prominence so far as the knowledge base for teaching is concerned, Shulman (1987) contended that,

> "The basic to distinguishing the knowledge base of teaching lies in the middle of content and pedagogy, in the capability of a teacher to transmute the content knowledge he or she possesses into forms that are pedagogically powerful and hitherto adaptive to the discrepancies in capability and background presented by the students". (p. 15)

Moving forward, Ma (1999) also introduced the idea of Profound Understanding of Fundamental Mathematics (PUFM). According to Ma (1999), teaching and learning which exhibits a profound understanding of fundamental mathematics is "connected, has multiple perspectives, includes the basic ideas of mathematics, and has a longitudinal coherence" (Ma, 1999, p.122). This presupposes that teachers with PUFM have the ability to explicate the why of the mathematics they teach and come up with different and flexible ways to teach the subject. Thus, to a larger extent, PUFM as a form of conceptualization put forward by Ma can be said to be associated with Shulman's PCK with the reason that they both suggest that a teacher who exhibits any of the two types of knowledge, should have a deep, broad, and thorough understanding of mathematics and the knack to teach it in flexible ways.

The main critique of Shulman's idea of PCK has been the fact it is presented as a domain neutral construct. Researchers have argued that since the PCK for teaching for instance a subject like Biology could be different from the PCK for teaching mathematics, a reconceptualization of teacher knowledge beyond Shulman's is necessary.

In a work on elementary school mathematics teachers' knowledge for teaching, Deborah Ball and her colleagues have also added to the idea of the type of knowledge required of teachers by introducing a conceptualization called the Mathematical Knowledge for Teaching [MTK] (Ball & Bass, 2000; Hill, Ball & Schilling, 2004; Hill, Rowan & Ball, 2005). Based on literature and what existing theories about teacher Knowledge are saying, Ball and her colleagues also developed and administered survey-based questions based on teaching mathematics at the elementary school level. Having done factor analyses of their data, they suggested, among other things, the idea of "specialized knowledge of content". As articulated by them,

> In addition to a general factor, specific factors representing knowledge of content in number and operations, knowledge of students. . . [there exist also] a specialized knowledge of content (SKC) made up of several items: representing numbers and operations, analyzing unusual procedures or algorithms and providing explanations for rules. (Hill, Ball & Schilling, 2004, pp 27-28) .

Another issue that has complicated things is the fact that not only have there been such different conceptualizations of the knowledge base for teaching but also a number of different instruments have been developed to measure teachers' knowledge of the content of school mathematics, as well as issues related to pedagogy.

For example in Singapore, pre-service teachers are required to pass a mathematics qualifying test by at most three attempts before their graduation from the National Institute of Education. It is believed that trainees who passed the test are considered to have adequate knowledge, both in terms of sound pedagogy and proficient content knowledge to teach mathematics at the secondary school level. On the other hand, those who fail the test (on the third attempt) are deemed to have less adequate mathematics content knowledge to teach mathematics at the secondary school level. The prime focus of the qualifying test is to ensure that these pre-service teachers have the minimum content that is essential for secondary school mathematics.

Similarly, in the United States, to be qualified to teach mathematics, different states require pre-service teachers to pass a mathematics test. A typical example of this is the PRAXIS, a teachers' licensing examination developed by Educational Testing Service (ETS), which is currently used by over 30 states in the US. All these have been done to ensure that mathematics teachers have a good knowledge of the mathematics students are required to learn in school. Upon all these, the quality of achievement of K-12 students in mathematics in the US continues to remain a national concern. Subsequently, the RAND Mathematics Study Panel (2003) made a number of recommendations which they believe would go a long way in improving teachers' mathematical knowledge for teaching. These include the need for further clarification of the knowledge demands of teaching mathematics, and a deeper understanding of ways to provide opportunities for prospective and practicing teachers to acquire this kind of knowledge. Also, the RAND Mathematics Study Panel (2003) commended the development of instruments for measuring the Mathematical Knowledge for Teaching across grade levels and mathematical domains. Amazingly, in addition, the RAND panel also isolated algebra as an essential area of focus in all these efforts. The influence of the recommendation by the RAND study panel coupled with the quest to develop a conceptualization of the knowledge base for teaching mathematics in a domain specific manner may have given the impetus to Ferrini-Mundy and her colleagues at the Michigan State University to focus their study on Algebra (see for example Ferrini-Mundy, McCrory, Senk, & Marcus, 2005; Ferrini-Mundy, Senk, & McCrory, 2005).

In addition to this, Ferrini—Mundy and her colleagues in their quest to conceptualise teachers mathematical knowledge for in the KAT project, hypothesized that subject matter knowledge in mathematics for teaching algebra is made up of three main types of knowledge. These knowledge were stated by them team as 1) knowledge of school algebra which was also labeled as "school knowledge", 2) advanced knowledge (explained to be the knowledge of the content teachers possess in other mathematics specific domains which are different from algebra) and 3) teaching knowledge. In this thesis, these three types of knowledge as put forward by the KAT project team has been explained in the conceptual framework of the second chapter. To add to the credit of Ferrini-Mundy and her colleagues is the development of items and the design of instrument to assess the algebra knowledge for teaching as possessed by teachers at the senior high school level.

Quite a number of pilots of the instruments they are developing meant to measure the knowledge for teaching algebra by the KAT project team, set out using firsthand data from pre-service and in-service mathematics teachers in different context across the US to authenticate their framework for knowledge for teaching algebra. The KAT project was a step in the right direction and a step ahead of other conceptualization because its relevance can be seen in the sense that for about close to three decades of various conceptualization of the subject matter knowledge for teaching mathematics, it's only the KAT project that came up with specific instrument and items meant to measure only one domain of mathematics at the senior high school level and that is algebra.

One other unique thing about the KAT project is the work it did towards developing instrument meant for measuring mathematical subject matter knowledge for teaching algebra. As a matter of fact, conceptualization made in the KAT project has and is a potential for serving as a guide for other scholarly works which may be concerned in conceptualizing knowledge demands for teaching other areas of mathematics particularly at the senior high school level.

Furthermore, the instrument developed by the KAT project has the edge for measuring and refining algebra knowledge needed by pre-service teachers as well as for professional development of in-service high school mathematics teachers. The project which was focused on investigating the status and discrepancies of knowledge of in-service and pre-service mathematics teachers' knowledge for teaching algebra is to a larger extent has the possibility of providing empirical data on what needs to be done nationally to improve teachers' subject matter knowledge for teaching.

As a matter of fact, this study is partially based on work done by the KAT project team, but conducted in a different settings and context. It used items on the KAT instruments, Black (2008) PhD dissertation instrument and new items that were personally developed to ascertain issues teachers' knowledge for teaching algebra among in-service and prospective senior high school mathematics teachers in the Ghanaian context. Also, in the case of this study, 74 multiple-choice type items and more extensive instrument was developed to measure the domain specific question.

The Nature of School Mathematics in Ghana

In the United States of America, junior high and senior high schools, have separate courses in algebra (e.g., Algebra I, Algebra II etc.) which are offered to students. Unfortunately in Ghana, only one integrated mathematics course is offered at the JHS (the equivalent of seventh to ninth grade) to all students. This mathematics course is a national curriculum, and is therefore, offered to all students in the public school system for the entire three years of the JHS education. The Teaching Syllabus for Junior high Schools (Ministry of Education, 2012) lists the major content areas covered on page (v) as

1. Numbers and Numerals
2. Numbers Operations and Algebra
3. Measures, Shape and Space "Geometry"
4. Collecting and Handling Data
5. Problem Solving & Application

Aside these content areas, problem solving which is not a topic in itself appear throughout the syllabus and are given much emphasis. In addition, these topics are not successively covered in the syllabus. They have been fragmented into smaller content areas, called units as well as sub-units and have been arranged in a spiral manner. The various components (units) are organized in such a manner that topics taught in the early stages of the pupils/students schooling are not covered in full but rather are returned to repetitively all over the years and advanced further, with increasing detail and deepness, as students' progress through their various stages of education.

In *The Process of Education,* Bruner (1960) as cited in Wilmot (2008) asserted that for this type of sequencing when he mentioned that, "A curriculum as it progresses should revisit the basic ideas repetitively, building upon them until the student has grasped the full formal apparatus that goes with them" (p. 13). Currently in Ghana, there are two major types of mathematics programs accessible to students at the Senior High School (SHS) level (the equivalent of grades 10 to 12 in the US). These are Core Mathematics and Elective Mathematics. The basic truth is that in these public schools as well as private senior high schools, every SHS student reads Core Mathematics for the entire three years of SHS education. However, Elective Mathematics on the other side is done by students who require further study in mathematics content preparation beyond the core mathematics coverage. For example, Elective Mathematics was a compulsory elective subject for students in the Sciences and Technical programs. Also, students reading

General Arts and Business programs are permitted to select Elective Mathematics as one of their elective subjects. Comparatively, both mathematics subjects at the SHS level presently are integrated mathematics programs with their content arranged or sequenced in a spiral way like the one at the JHS level.

Furthermore, like all other school subjects, both the Core and Elective Mathematics subjects' syllabi are centrally controlled by Ghana Education Service. Since it's a national curriculum, the content of each of these mathematics subjects is the same for all public schools in the country.

The current mathematics syllabus (both core and elective) in an attempt to help all Ghanaian young persons to attain the needed mathematical skills, insight, attitude and values that they will need to be efficacious in their chosen careers and daily lives is based on the premise that all students can learn mathematics and that all need to learn mathematics (Core Mathematics Teaching Syllabus for Senior High School, 2011, p ii). Howe (1999), mentioned that "a teacher who is blind to the coherence of mathematics cannot help students see it" p.885. For all students as a matter of fact and urgency to be able to learn mathematics, then the teaching knowledge possessed by the instructor who in this case is the teacher needs to be analytically looked at in every respect. A critical look at the Core Mathematics Teaching Syllabus for Senior High School (2011), indicates that the major content areas covered in all the Senior High School core mathematics classes are as follows:

1. Numbers and numeration
2. Plane geometry
3. Mensuration
4. Algebra
5. Statistics and Probability
6. Trigonometry
7. Vectors and transformation in a Plane.

** Problem solving and application (Ministry of Education, 2011).

As mentioned earlier on, beside these content areas, problem solving which is not a topic in itself cut across all the topics in the syllabus and are given much emphasis. The Teaching Syllabus for Senior High Schools (Ministry of Education, 2011) categorized the profile dimensions that have been specified for teaching, learning, and testing at this level into two main category namely Knowledge and Understanding-30% and Application of Knowledge-70%. Again, the Teaching Syllabus for Senior High Schools (Ministry of Education, 2011) outlines the major content areas for Senior High School Elective Mathematics as:

1. Algebra
2. Logic
3. Coordinate Geometry
4. Trigonometry
5. Calculus
6. Linear Transformation

7. Vectors
8. Mechanics
9. Statistics
10. Probability

From all indication it can be observed that algebra as a matter of fact appears to be the foundational topic in the entire mathematics teaching syllabus right from the JHS level to SHS level. In spite of the compulsory nature of algebra to all Ghanaian SHS students, various Chief Examiners report of SSSCE/WASSCE had continue to point out students' terribly poor performance in mathematics as a result of poor management of problems involving algebraic reasoning. For instance, the Chief Examiners report for 2001 identified some of the candidate's weaknesses in mathematics as "poor skills in handling algebraic expressions", "failure to use the distributive property of multiplication over addition and subtraction correctly, , and "lack of the ability to translate word problems into mathematical sentences" (p.89).

Ghana and the US comparatively have two distinct ways of offering algebra to its citizenry. While in the US specific algebra courses are offered to students, in Ghana algebra is offered as an integrated course or subject to it citizenry. In spite of these differences, there continues to be public uproar over the performance of students on algebra in both national and international assessments. In Ghana for example, students' ability to progress from SHS level to the university and other tertiary levels of the education system is solely dependent on a national examination, currently called the West Africa Senior Certificate Examinations (WASCE). In Ghana, this form of examination has been in place since 1987 when the National Educational Reform came into force. As a result of similar educational reforms in neighbouring Anglophone West Africa countries, starting May 2006, the then Senior Secondary School Certificate Examination (SSSCE) was changed into the West African School Certificate Examinations (WASCE) which is what is currently running. For one to have access to university and other tertiary education in these English speaking countries, every high school leaver is obliged to take the WASCE before. Unfortunately, since the inception of this practice in 1993, numerous Chief Examiners report of the SSSCE/WASCE has emphasized students' poor control of some of the problems on algebra. A typical example is when in 2004, the Elective Mathematics Paper 2 of the SSSCE asked students to express $3x^2 - 6x + 10$ *in the form of* $a(x - b)^2 + c$ where a, b and c are integers. Hence state the minimum value of $3x^2 - 6x + 10$ and the value of x for which it occurs (WAEC, 2004). It was emphatically acknowledged in the chief examiners' report at the time that most of the candidates attempted the question. Unfortunately, it was made clear in the report that students did poorly on the question because most of them either could not complete the square or resorted to calculus to find the minimum value, a procedure that was not acceptable.

Looking at the key role algebra plays as a foundational course, it behooves on us as educators and a nation to reverse such trends of students' poor performance in the country. In Ghana not much studies have been carried out to examine the reasons for this poor performance, globally, a number of studies conducted on students' performance in the area of mathematics have indicated that one of the prime factors that can advance students' achievement in school mathematics is the knowledge teachers possess (see for instance, Harbison & Hanushek, 1992; Mullens, Murnane & Willett, 1996; Hill, Rowan & Ball, 2005). We can only ascertain and make informed decisions on the kind of change needed in the knowledge base of mathematics teachers if and only if data about the nature of teachers' knowledge and which aspects of it best relate to student performance were available.

To ensure teacher effectiveness, it then means that a sound knowledge of the whole content of school mathematics of the integrated nature of both of the two mathematics curricula in some integrated way is very crucial. To do relatively a good work on issue relating to teacher knowledge in regard to the whole mathematics syllabus being core or elective is too over ambitious to do that in a single study. Having realized how relevant the nature of algebra in the study of school mathematics is vis-à-vis students poor performance in this area this study is restricted to algebra.

As stated earlier, considerations that the knowledge of algebra could greatly improve the achievement of students in mathematics and the several

Chief Examiners reports about students' inability to manipulate algebraic expressions give yet another focal point for focusing this study on high school teachers' knowledge for teaching algebra, their beliefs about how mathematics should be learnt as well as how it should be taught.

In addition, what makes this study exclusive is the fact that teachers were selected from different categories of schools, ranging from the most to the least resourced. According to the Ghana Education Service, Senior High Schools in the country are categorized based on available resources in these schools. According to Darling-Hammond and Hudson (1998) (as cited in Wilmot, 2008), argue that how effectively both teacher and teaching quality convert into students' outcomes largely depends on characteristics of school quality (that is administrative support, facilities, instructional resources and school climate). This could be the reason why Begle (1972) could be said to be right or having attributed the non-significant relationships between teachers' knowledge and students' achievement to the fact that all the teachers who partook in his study were from highly inspired and well-endowed schools. He Begle, consequently, argued that there is a threshold of minimum knowledge teachers must possess, below which the relationship does exist.

Currently and finally in Ghana, not much has been done to find the teachers' knowledge for teaching algebra. This study, however, intends to find out how high school prospective and in-service mathematics teachers knowledge for teaching algebra corroborate the three domains of knowledge hypothesized in the KAT framework and further ascertain whether a reconceptualization could be arrived at based on the content of algebra in the SHS syllabus.

Statement of the Problem

It was asserted by the National Center for Education Statistics (2001) that the successful competition by the United States in the global economy had solely depended on having grown-ups who are well equipped in mathematics and science. According to Ball (2003a), in order for the adult group of our society to be productive in this twenty-first century of ours then they need to be mathematically proficient. Also, National Council of Teachers of Mathematics [NCTM](2000), reiterated that the necessity for mathematics in everyday life has never been superior and will indeed continue to increase. NCTM further mentioned that the door to a productive future can be opened through mathematical capability whereas the doors to a productive future can remain closed for students lacking the appropriate mathematical competence.

In the U.S. for instance, the nation is branded as not providing its citizenry [students] with the mathematical preparation needed to be successful. Per the results of the Third International Mathematics and Science Study (TIMSS), students in the United States only achieve at average levels when compared to students in other countries (NCES, 2003a). Also, according to the National Assessment of Educational Progress (NAEP) results, less than 20% of twelfth grade students and about one-third of eighth grade students had achieved mathematical proficiency (Kehle, Wearne, Martin, Strutchens, & Warfield, 2004). A critical moment in the life of a high school student in their mathematical preparation is a solid foundation in algebra. According to Ball (2003a), algebra as a foundational course, serves as a concierge, posing varying opportunities for entry into advanced mathematics courses for groundwork for college (Pascopella, 2000, Lawton, 1997, Chevigny, 1996, Silver, 1997, Olson, 1994), and for grounding for the world of work (Silver, 1997). Olson (1994) pointed to a study by the College Board, which recommended that students who take Algebra in high school are two and one-half times more likely to further their college education than their counterparts who do not. When minority students complete Algebra in high school, the gap between the percentages of minority and non-minority students who attend college virtually disappears. Students who are not proficient in Algebra do not have access to a full range of educational and career opportunities (Ball, 2003a).

According to the NCTM (2000), Algebra should be implanted throughout the K-12 curriculum in order to provide the openings for students to advance a compacted foundation for the content. Students need to develop a deep understanding of the algebra content, and mathematics in general. Students need to develop fluency with procedures as well as conceptual understanding for why those procedures work (RAND, 2003). In addition a students' content knowledge must include the processes of problem solving, reasoning and proof, communication, making connections, and using representations (NCTM, 2000). Content knowledge provides the ability to know, understand, and have the ability to use mathematics (NCTM, 2000).

For students to be successful in future educational opportunities and careers, the role of teachers becomes crucial in ensuring that all students have the needed experiences to learn the subject mathematics (Mewborn, 2003). In this regard, as educators, we must consider the types of knowledge teachers of mathematics need to provide all students with reasonable prospects to learn algebra. Whereas we all might agree teachers need content knowledge of the subject they teach and will be teaching, there is not a common definition of content knowledge on which everyone agrees. The Content knowledge a teacher possesses has often been well-defined by the total number of university level courses taken by the individual (Even, 1993, No Child Left Behind Act [NCLB], 2001), their grade point average at the collegiate level (Even, 1993), or scores on a state authorized test (NCLB, 2001). However, this could be said to be limited.

It is an indisputable fact that teachers need a profound understanding of the mathematics they teach and will teach (CBMS, 2001). The content knowledge teachers possess like students, should not be no less than both procedural knowledge and conceptual knowledge, and understanding how this knowledge is organized and produced throughout the domain of mathematics (Shulman, 1986). If learners are estimated to develop mathematical ability and to apply mathematics in real world situations, then for the instructors no less can be expected of them (CBMS, 2001). Research is packed with the fact that teachers' content knowledge is often thin and insufficient to provide instruction for students in today's classrooms (Ball, 1988a, 2003b; Ball & Bass, 2000; Fuller, 1996; Ma, 1999; Mewborn, 2001; Stacey, et al, 2001).

However in Ghana, the integrated form of mathematics which is generally called Core mathematics is a required subject for all Senior High School students in this country. Before any student is able to gain admission into any of the tertiary institution in the country, unless the individual student passes the general (core) mathematics during the West African Senior Secondary Certificate Examination (WASSCE) at the end of their programme. From my observation, this is not a requirement only in Ghana but also in other tertiary institution in many African countries. Students underachievement in the subject that is in mathematics at this level of education (senior high school) irrespective of the compulsory nature of the subject, has taken preeminence in WAEC Chief Examiners' Reports (Chief Examiners Report of 1998, 1999, 2000, 2001, 2003, 2007, 2008, 2009, 2010, 2012, & 2014). For example, the Chief Examiner's reports of 1998, 1999, 2000, 2002 & 2004 indicated that students find it extremely challenging to handle algebraic expressions and solve algebraic problems. In a specific instance, students' flaws were seen to be in factorizing quadratic equation of the form $ax^2 + bx + c = 0$, their lack of understanding of the concept of logarithm and its applicative uses (Chief Examiners Report, 2007). In some instances common mistakes repeatedly made by students were:

$log_{10}\left(\frac{9}{7}\right) = \frac{log_{10}9}{log_{10}7}, -3(2+x) = -6+x, 35-3(2+x) = 32(2+x),$
$and\ (x+y)^2 = x^2+y^2$

,

(Chief Examiners' report, 2007, 2004, 2001, 1999, 1998). It was pointed out that "most candidates who attempted this question could not realise that $x^2-6x+7=k(2x-3)$ has equal roots when the discriminant $\{-(6+2k)\}^2 -4(1)(7+3k) = 0$" (Chief Examiners' report, 2001, p.93). Also, "most candidates in the attempt to clear fractions tended to ignore the "x" on the other side of the inequality sign. For instance, $\frac{1}{3}x - \frac{1}{5}(2+x) \geq x + \frac{7}{3}$ was simplified as $5x-3(2+x) \geq x+35$ instead of $5x-3(2+x) \geq 15x+35$".

Chief Examiners Report (2007), pointed out that collecting of like terms, problems involving expansion, factorization, and application of the distributive property for instance has become cyclical year after year. He further mentioned that most candidates lack the concept of quadratic equation and other concepts relating to algebra. In another breadth, the Chief Examiner (1999) remarking on the poor performance of students in the general mathematics, emphatically mentioned that students over the years have shown "poor skills in handling algebraic expressions….." p.117. "It looks like many candidates have problem with algebra" (Chief Examiner, 2001, p.91). All these reports point to the fact that problems faced by students in learning mathematics appeared to have linkage with their lack of conceptual knowledge and might have been resulted from the teaching experiences they might have encountered in learning algebra at the lower secondary school level.

On the international front, Ghanaian students are noted to have performed poorly on the TIMSS in 2003, 2007 and 2011, the years in which Ghana participated. This development of students' poor performance in mathematics at the high school level is alarming to the extent that research into knowledge teachers of mathematics possess, their belief about how mathematics is taught and how it is learnt is so crucial.

Literature is replete with factors such as family background, pupil-teacher ratio, socio-economic status of students that account for students' achievement (Coleman et.al., 1966), it is unblemished that in our part of the world, the knowledge teachers possess for teaching is a very crucial factor in students' achievement (Harbison & Hanushek, 1992; Hill, Rowan & Ball, 2005). This study is deliberately focused on algebra for the following reasons:

1. Algebra at the senior high school level seems to be the foundational area in both core and elective mathematics.
2. Also, algebra has applications in all the other areas of mathematics. Accordingly, teachers' good repertoire of knowledge in algebra has the potential of affecting students' achievement in mathematics.

3. Various West Africa Examination Council Chief Examiners' incessant emphasis on students' inability to perform well in algebra related tasks makes attention on teachers' knowledge for teaching algebra necessary for a study such as this.

A critical look at literature indicates that in Ghana not much work has been done in this area of teachers' knowledge for teaching algebra irrespective of the poor performance of students in mathematics at the Senior High School as a result of difficulty in algebra. A review made proposes that teachers' scores can be statistically significant prognosticators of how much students learn. It was necessary, therefore, to reconnoiter the level of knowledge mathematics teachers of Ghanaian require in the area of algebra to expedite the making of evocative decisions about the type of upgrading needed in the knowledge base of teachers and subsequently to improve students' achievement (Wilmot, 2008).

Research is replete with the fact that teacher knowledge affects student performance. For quite some time now, the issue of which aspects of teacher knowledge affect student achievement has been of great concern among researchers. This, as a matter of fact is partly due to the fact that over the years, several attempts at conceptualising teacher knowledge have produced domain neutral constructs making them nearly difficult to be accurately measured. As a result, several attempts to measure teacher knowledge have depended on alternative measures such as the number of university courses taken, the type of degree the teachers' have and so on.

This study is preised on the fact that, instead of relying on altenative measures, there is the need for re-conceptualization of teacher knowledge in ways that is not only domain specific but also allows its components to be measured. In the early to mid- 2000s, researchers of the Knowledge of Algebra for Teaching (KAT) project attempted to do exactly this by hypothesizing three types of knowledge and developing items to measure it. Using the KAT conceptualization as a framework, this study was designed to investigate whether the three types of teacher knowledge hypothesized in the KAT framework will be corroborated. In addition, it is strongly believe that adapting the KAT project instrument and instruments from other researchers, would provide a better measure of the knowledge teachers require in order to make teaching and learning better for all hence improvement in students achievement.

Purpose of the study

Instead of relying on alternative measures, there is the need for re-conceptualization of teacher knowledge in ways that is not only domain specific but also allows its components to be measured. This study, therefore, was designed to investigate whether the three domains of teacher knowledge hypothesized in the KAT framework will be corroborated and explore for further other factors if any.

Since the study was about senior high school mathematics teachers' knowledge for teaching algebra, prospective and in-service mathematics teachers knowledge for teaching was focused on. It also, compared senior high school mathematics teachers' knowledge for teaching algebra with regards to those with education background and their counterparts without education background.

The study went further to examine to what extent the Knowledge of Algebra for Teaching framework argument that there are fussy or blur boundaries among their three hypothesized knowledge types be sustained or otherwise. There was also an attempt to look at the effect of teaching experience on senior high school mathematics teachers' knowledge for teaching algebra at the senior high school level.

Research Questions

The following research questions guided the study:

1. To what extent does high school prospective and in-service mathematics teachers' knowledge for teaching algebra corroborate the three main types of knowledge hypothesized in the KAT framework?
2. What is the level of algebra knowledge possessed by SHS mathematics teachers for teaching based on the KAT framework?

Research Hypotheses

The following research hypotheses gave focus to the study:

1. There is no significant difference between the knowledge for teaching algebra of in-service and that of prospective mathematics teachers at the senior high school level.
2. There is no significant difference in the knowledge for teaching algebra between senior high school mathematics teachers with background

training in education and those of their counterparts without background training in education.

3. There is no significant difference between senior high school mathematics teachers' knowledge for teaching algebra and the years of teaching experience.

4. There is no significant difference in the knowledge for teaching algebra between mathematics teachers who teach in urban areas and their counterparts in the rural settings based on the knowledge types in the KAT framework.

Significance of the Study

In the first place, it is unclear whether the poor manipulation of algebraic expression is as a result of the knowledge of teachers or students, therefore understanding the knowledge base of teachers of mathematics could help in understanding the nature of our mathematics education program as a department hence doing the necessary restructuring.

Also, the outcome of this piece of work may help unravel whether teachers of the Senior High Schools in the three regions involved in the study have the required content knowledge for teaching algebra to influence positive revolution in students learning. The implication of this is that it would go a long way to take a critical look at mathematics teachers' professional development in the country.

In addition, the outcome of this study would go a long way to inform policy makers especially those at the university levels to implement initiatives geared towards improving students' mathematical achievement. Thus proper programs and courses would be put in place to enhance teachers' repertoire of knowledge hence improving students' achievement in mathematics.

Furthermore, the instrument used in this study was an adapted one and for that matter if it becomes successful, it would serve as a bases for assessing teachers' knowledge in other mathematically related domains.

In Ghana, with the formation of National Testing Council and forming together of the teacher educational division of the Ghana Education Service, there seems to be a general consensus that pre-service teachers be required to pass a special certification examination before being posted to the schools. To this point, the outcome of this study in a way may inform policy makers and stakeholders in education about the need to conduct such licensure examination for entry teachers before they are recruited into our educational sector.

Delimitations

Essentially, the study was designed to investigate teachers' knowledge for teaching algebra at the senior high school level in Ghana in the light of the KAT framework and its modified version. To do this, both pre-service and in-service senior high school teachers of mathematics were allowed to participate in the study.

At the pre-service level, the study only used level 400 mathematics education students. At the time of the study, this group of students had taken sufficient general education courses, mathematics content and math education courses required in their programme. In addition, this same group at the time of the study had returned from their off-campus teaching practice session (the practical component of their programme). They were, therefore, the cohort that were best ready to transition into the field of teaching. Consequently, students of the lower level cohorts (i.e., levels 100 to 300) were not used.

Also, the domain of knowledge used as the focus of the study was algebra. This means that all the items or questions on the instrument used for data collection were algebra related questions and no question from the other domains of math was used.

In addition, at the in-service level, teachers who were teaching either Core or Elective Mathematics or both Core and Elective mathematics were used. Since in general, algebra constituted the foundational domain of mathematics at the senior high school level, an assumption was made prior to the fieldwork that any teacher teaching either of the two mathematics courses had sufficient knowledge in algebra to be able to respond to the items on the instrument. No none-mathematics teacher in any of the participating senior high schools was used even if that person argued that he had prior background training in mathematics but was not teaching it. Further, even those teaching mathematics being its core or elective or both but had no background training in mathematics as well were not included in the study.

Furthermore, the study was also limited to senior high schools in only three regions of Ghana (Western, Central and Ashanti). Even in these three regions, 40 schools that availed itself for the study were used. Coincidentally, in all 15, 10 and 15 schools from the Central, Western and the Ashanti regions respectively were involved in the study. These regions were used because to a larger extent they have densely populated schools and have teachers coming from almost all over the various public universities across the nation teaching there. Quite apart from that for as many schools as possible that could help obtain a sizeable number of respondents in the study in the region that availed itself to be used were the ones involved in the study hence the 15, 10 and 15 schools respectively coming from the Central, Western, and the Ashanti regions.

Finally, the only teacher-related factors that were considered in this study were teachers' knowledge for teaching algebra, their teaching experience, and their academic qualifications were used as well as location for which these teachers are teaching. These factors were used because literature reveals that they are important determinants of what students learn.

Limitations

One of the major limitations of this study was that of small number of participating teachers. This in a way could place a limitation on the outcome of the study in that if a large number of teachers were involved it could have given different results. Also, due to financial constraints, it was not possible to include schools from all the senior high schools in the country. These could limit the generalizability of the result.

Definition of Terms

For the purpose of this study, the following terms are defined as follows:

1. Academic qualification: this is the highest level of education attained by the teacher. Teachers were classified according to whether the teacher is a degree holder with education background or without education background.
2. Teaching experience: the number of years that a particular teacher has taught mathematics regardless of his or her level of education.
3. Prospective mathematics teachers: These are final year mathematics education students who have taken sufficient general education courses, mathematics content and math education courses required in their programme. In addition, this same group are the cohort of students who had returned from their off-campus teaching practice session (the practical component of their programme).
4. In-service: These are teachers who have obtained their professional and non-professional qualification degrees and are teaching either Core or Elective Mathematics or both Core and Elective mathematics at the senior high school and have background training in mathematics.

5. Content knowledge: This includes knowledge obtained in content-specific courses.
6. Pedagogical knowledge: This includes subject matter taught in education classes.
7. Quality of Teaching Knowledge: It is the total score that a teacher obtains.

Organization of the study

Apart from the 'Introduction' chapter, there were four other chapters made up of Review of related Literature (Chapter 2), Methodology (Chapter 3), Results and Discussion (Chapter 4), and Summary, conclusion and recommendations (Chapter 5).

The literature review chapter takes a critical look at the relevant literature that is related to this research. The review is broken down into the following headings: (i) conceptual framework, (ii) teachers subject-matter knowledge, (iii) teachers' knowledge and students' achievement, (iv) teachers years of teaching experience and students' achievement, (v) teachers degree type and students' achievement.

The chapter on research methods, describes the research design and the broad paradigm under which this study falls; data collection procedure, sample and sampling technique, the instrumentation process and how data collected were analysed.

Chapter five which is the last part of this thesis ends with the summary, conclusions and recommendations where an overall summary of the research, its key findings and limitations, conclusion, recommendations and suggestions for future research are provided.

CHAPTER TWO

LITERATURE REVIEW

Literature on teachers' is replete with different conceptualizations of teacher knowledge. One significance of this line of research lies in the fact that a good conceptualization of teacher knowledge in domain specific and measurable terms could help answer the question of which aspect of teacher knowledge best influences students' performance. Unfortunately, since the time of Shulman (1986), many of such conceptualizations have been quite general (i.e., have neither been domain specific nor measurable). As a result, several proxy measures have been employed in attempts at measuring teacher knowledge. Within the last one-and-half decades, however, attempts have been made by researchers of the Knowledge for Algebra Teaching (KAT) project to re-conceptualize teacher knowledge at the high school level in domain specific and measurable terms. Wilmot (2008), attempted a corroboration of the KAT conceptualization. Though his study could not corroborate the KAT framework he made a number of recommendations for further study. Paramount among these was the need to increase the number of items on the instrument as well as the number of participants in the study.

This study is also an attempt at corroborating the KAT framework. It incorporated the aforementioned recommendations by Wilmot (2008) and, thus, increased the number of items on the instrument, as well as participants in the study.

In this chapter, the conceptual framework guiding this study has been presented. In addition, an explanation of the importance of the conceptual framework has been provided. The chapter ends with a review of literature relevant to the current study. The review covers a historical overview of research on teacher knowledge as well as review of studies that have proposed other conceptualisations of teacher knowledge.

Conceptual Framework

Through critical analyses of research literature, recommendations by professional organizations and videos of teaching, researchers in the Knowledge of Algebra for Teaching (KAT) project hypothesized that teachers' knowledge for teaching school algebra comprised three types of knowledge. These are "knowledge of school algebra" (referred to in short as "school knowledge"), "advanced knowledge of mathematics" (also referred to as "advanced knowledge"), and "teaching knowledge". The conceptual framework that guided this study is this conceptualization by the KAT project as discussed below.

Knowledge of School Algebra

"Knowledge of School Algebra" (or simply "School Knowledge") is defined as the knowledge of mathematics in the intended curriculum of middle school and high school. This is the content of school algebra that teachers are expected to help students discover or learn in their algebra classes (Wilmot, 2008). It has been stipulated that in the United States of America, ideas about knowledge such as this are described in booklets such as the National Council of Teachers of Mathematics (NCTM)'s *Principles and Standards for School Mathematics* (NCTM, 2000) while the precise grade-level algebra content is defined in the various states' standards, textbooks and other instructional resources used in the schools. According to Wilmot (2008), investigators in the KAT project restricted this type of knowledge by reviewing content standards of ten different states in the US. However in Ghana, the content of this type of knowledge base is built-in both the Core and Elective Mathematics Syllabuses which is taken by students at the SHS level. It sounds realistic, however, to conjecture that for teachers to impact students learning, the teachers themselves need to understand the content of school algebra since students at that level are expected to learn such.

Advanced Knowledge of Mathematics

Per the KAT project, *Advanced Knowledge of Mathematics* (or simply "Advanced Knowledge") was simply referred to include other mathematical knowledge, in particular college level mathematics, which gives a teacher perspective on the trajectory and growth of mathematical ideas beyond school algebra" (Ferrini- Mundy, Senk and McCrory, 2005, p.1). Areas such as number theory, abstract algebra, complex numbers, linear algebra, calculus, and mathematical modeling were listed as some of the general areas in the KAT project (see Ferrini-Mundy, McCrory, Senk, & Marcus, R, 2005). In addition, Ferrini-Mundy et al. (2005, p. 1) in the conceptualization of this advanced knowledge, saw members of the KAT project recognized that "knowing alternate definitions, extensions and generalizations of familiar theorems, and a wide diversity of uses of high school mathematics are also features of an advanced standpoint of mathematics" . Hence, it can be maintained that having an advanced viewpoint of mathematics gives teachers a deeper or profound understanding of school algebra. This kind of knowledge becomes so relevant because the possession of it could make it conceivable for a teacher to make appropriate networks across topics whereas unloading the complexity of a mathematics content to make that content more understandable. In addition, as already specified, the KAT project considered "advanced knowledge" key because they believed that possessing it affords the mathematics teacher with unfathomable or profound understanding of school algebra. Furthermore, it is hoped that any mathematics teacher who owns this type of knowledge would hold quite a respectable knowledge of the path of the content of school mathematics. One other important reason for a teacher to possess such knowledge is that it would in them help to engage in making networks across topics, eliminating difficulties while retaining integrity and unzipping of the content of school algebra to learners; practices that could be vivacious to effective teaching.

Teaching Knowledge

The last and third type of knowledge in the KAT framework is the teaching knowledge. In the framework, this type of knowledge according to Ferrini-Mundy, McCrory, Senk and Marcus (2005, p.2) is termed as "knowledge that is precise to teaching algebra that may not be taught in advanced mathematics courses. It comprises such things as what makes a particular concept problematic to learn and what misconceptions lead to precise mathematical inaccuracies. It also contains mathematics required to identify mathematical goals, within and across lessons, to choose among algebraic tasks or texts, to select what to highlight with curricular paths in mind and to ratify other tasks of teaching".

Consequently, this is the type of knowledge teachers possess which they apply in teaching the subject matter of school algebra. Furthermore, Ferrini-Mundy et al., (2005, p.1) in the KAT project mentioned that, "the knowledge been described here may fall into the kind of pedagogical content knowledge or it may be pure mathematical content applied to teaching". Also, this type of knowledge may not be taught in advanced mathematics courses, and may not essentially be accessible to mathematicians. Thus, this is the type of knowledge that could distinguish an engineer or a mathematician from an algebra teacher.

Relationships between the Three Types of Knowledge

It was hypothesized in the KAT project conceptualization that the three types of algebra knowledge for teaching, School Knowledge, Advanced Knowledge and Teaching Knowledge are not hierarchical in nature. It is also clear that they (the three types of knowledge) neither exist in continuum with well-definable boundaries. Instead, their boundaries are fuzzy in the sense that they are intertwined in many ways. A schematic illustration of this conceptualization is presented in Figure 1. In the results and discussion section (chapter 4), data gathered for this work has been used to validate or invalidate this conceptualization assertion. Figure 1 indicates the conceptual representation of the three types of Knowledge hypothesized in the KAT project. In this same conceptual framework, the variables q, r, s, t represent the other knowledge types that would be produced when two or more knowledge types intersect which has been hypothesized in this study.

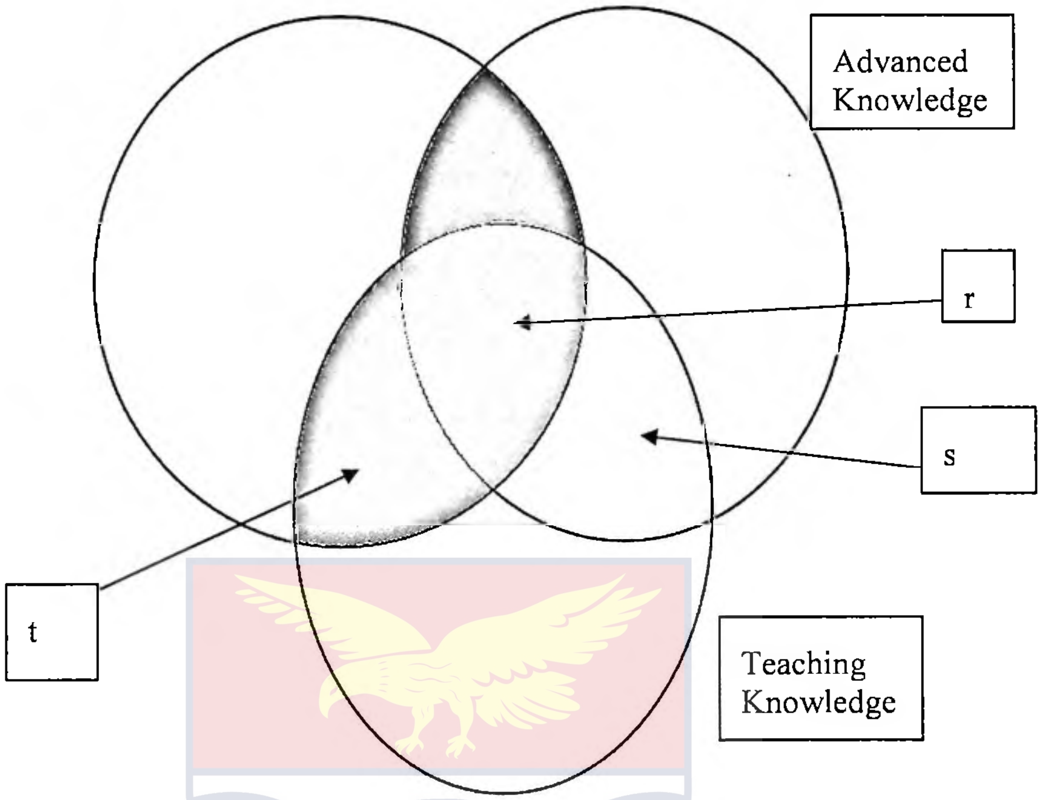


Figure 1: Conceptual Representation of the three types of Knowledge

Figure 1 indicates the theoretical framework by the KAT project team. This framework served as the conceptual framework to this current study. In this framework it has been hypothesized that the coloured regions would produce yet other knowledge types that would help the prospective and in-service mathematics teachers address crucial issues or problems that would arise in the classroom in the course of teaching. In the KAT conceptualization, only three knowledge types (*School Knowledge, Advanced Knowledge and Teaching Knowledge*) is believed to play out. However, in this study since it is domain specific, four additional knowledge types have been hypothesized which is believed would go a long way to help identify which knowledge types teachers require to be able to teach very well.

Importance of this Conceptual Framework to this Study

The primary focus of this research work is in part prejudiced by conceptualizations of content knowledge, curriculum knowledge, pedagogical knowledge, and pedagogical content knowledge suggested by Shulman and his colleagues (Shulman, 1986b; Wilson, Shulman & Richert, 1987).

It is common knowledge to know that the conceptualizations of content knowledge, curriculum and pedagogical knowledge is that anyone described as an effective mathematics teacher, specifically at the high school level, has the ability to blend these three types of knowledge into a new form of knowledge. According to Wilmot (2008), his personal intellectual work got him to think about the teacher knowledge in terms of connected or overlapping packages of knowledge (see Ma, 1999) or curriculum scripts, to use the words of Putnam (1987). In my viewpoint, these related packages of knowledge are mixture knowledge of the content of the subject matter teachers teach, knowledge of other content in the school curriculum and how they are related as well as why certain illustrations could be challenging or easy to some students. It is no surprising to me that the concept of pedagogical content knowledge which was touted by Shulman and defines it to include illustrations of specific content together with why the learning of that content is stress-free or challenging for students, have reverberated well with my personal standpoint. In the past few years until when the KAT conceptualization took off, previous researchers who have depended on Shulman's conceptualizations have only focused on teacher knowledge in a qualitative manner.

With the advent of the KAT conceptualization I have been educated in a number of ways and have contributed to my learning about teaching in a number of ways. In the first place their prominence on both advanced knowledge and school knowledge endorse for me the fact that teachers must not only know the content they are teaching, but in addition content of other areas of their subject and sometimes to a larger extent those that are connected to it. This part of assertion is solely enshrined in the KAT project's construct of "advanced knowledge" which covers the content beyond that of school algebra that which affords teachers with a deep understanding of school algebra.

Also, the squabble by Ferrini-Mundy and her team that the borderline between their three conceptualizations is blurry or fuzzy is quite debatable because I believe those overlapping regions can come together to generate yet another knowledge at the time when the teacher would be teaching would help in explaining those concepts that in a way seems complex to students.

Furthermore, efforts by the KAT project team to improve measures of the silhouette of knowledge for teaching algebra using particular school algebra curriculum and thoughts about the work involved in teaching is a complete parting from the qualitative measures of previous researchers.

Lastly, the types of knowledge as conceptualized in the KAT project has implications for one to think about the types of knowledge teachers need to possess for teaching other content related subjects that are enshrined in our high school mathematics curriculum. I strongly believe that the conceptual framework adapted in this study will not only help focus on which of these three types of knowledge as hypothesized in the KAT project validates or otherwise of Ghanaian high school mathematics teachers' knowledge for teaching algebra.

Earlier Conceptualizations of Teacher Knowledge

Duthilleul and Allen (2005) asserted that research interest on teacher knowledge was renewed in the United States following the report entitled *Equality of Educational Opportunity* by Coleman et al. (1966) "concluded that family background characteristics and community level variables accounted for more variance in student achievement than school resource variables like . . . teacher characteristics" (p.3). In other words, Coleman's committee in their report sought to conclude that schools and for that matter teachers do not influence student learning to any much extent. In the debate that ensued, different perspectives came up that interrogated as to whether schools and teachers play any significant roles in student learning. It is these debates that provided the renewed impetus on research into the knowledge base for teaching, which was spearheaded by Shulman (1986b, and 1987). As Strom (1991) puts it, "concern about the knowledge base focuses on improving the respect and status accorded teaching, thereby making it a more rewarding career" (p. 1). The central point was that for teaching to be recognized as a profession that influences learning outcomes, an argument needed to be made that it involves a specialized body of knowledge. It is at this point that Shulman (1986b) presented the idea of "pedagogical content knowledge" (PCK) as a type of knowledge, which encompasses such things as how students comprehend, and how to use resources effectively to present ideas in ways that make them more reachable to diverse categories of students.

Actually, in an attempt to categorize teachers' knowledge, Shulman (1986) outlined the following seven categories of knowledge: 1) Content knowledge, 2) General pedagogical knowledge, 3) Curriculum knowledge, 4) Pedagogical content knowledge, 5) Knowledge of learners and their characteristics, 6) Knowledge of educational context, and 7) Knowledge of educational ends, purposes and values, and their philosophical and historical grounds.

For the purposes of this study, pedagogical content knowledge is of special interest in the knowledge base of teaching because it represents the blending of content and pedagogy into an understanding of how concepts are presented to the learner. In other words this knowledge builds on other forms of professional knowledge (Shulman, 1987).

Apart from Shulman (1987), a number of researchers have put forward different conceptualisations of knowledge base for teaching (see for example; Ferrini-Mundy, Senk & McCrory 2005; Hill, Schilling & Ball, 2004; Ball & Bass 2000; Ma 1999; Leinhartdt & Smith 1985). For instance, in *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States,* Ma (1999) put forward the results and analyzes of interviews of a study conducted that involved 72 and 23 elementary schoolteachers from China and U.S. respectively. In the process of this, Ma (1999), introduced a different conceptualization of the knowledge base teachers of mathematics need for teaching mathematics, which she labeled as, "profound understanding of fundamental mathematics" (PUFM). Her PUFM encompasses a type of knowledge that goes beyond just content knowledge; it tells also about how to communicate the subject matter of school mathematics to students. Quite apart from the fact that Shulman's (1986) PCK is a universal form of knowledge (probably not circumscribed to a specific subject matter like mathematics or economics), Ma's (1999) conceptualization can be described to show some likeness to Shulman's (1986) PCK. Comparatively, both seem to encompass a multifarious amalgamation of content knowledge and pedagogical knowledge.

Earlier, Leinhartdt and Smith (1985), had proposed two types of teacher knowledge, "lesson structure knowledge" (LSK) and "subject matter knowledge" (SMK). According to them, LSK is what gives the teacher the ability to plan lessons and provide clear explanations in the course of teaching while possession of SMK by teachers gives them a clear understanding of concepts, algorithmic operations and the knowledge of the types of error students commit.

Deborah Ball and her colleagues have also researched into teacher knowledge for teaching mathematics at the elementary school level and have proposed the idea of "specialized knowledge of content" (SKC), which according to them is "made up of several items: representing numbers and operations, analyzing unusual procedures or algorithms and providing explanations for rules" (Hill, Ball & Schilling, 2004, p. 28).

Around the time that Deborah Ball and her colleagues were researching into teacher knowledge at the elementary school level, Joan Ferrini-Mundy and her colleagues on the Knowledge of Algebra for Teaching project were also studying the knowledge base for teaching algebra at the high school level (see for instance Ferrini-Mundy, Senk & McCrory 2005).

As explained earlier, this study uses the ideas Joan Ferrini-Mundy and her colleagues as the conceptual framework. Their conceptualisation is useful for this study because it learnt itself to assess both quantitatively and qualitatively; a quality that has the potential of leading to measurable types of teacher knowledge and thereby helping to eliminate the reliance on proxy measures of teacher knowledge.

Overview of Early Research on Teacher Knowledge and Teaching Practice

Various reviews of literature on teacher knowledge show that such studies started in the form of process-product research, many of which started from 1920 onwards in the US (Brophy & Gold 1986; Gage 1978; Doyle 1977). According to these reviews, the process-product studies were designed to establish a direct link between the actions of teachers in the classroom and student performance. As a result, researchers who used the process-product design coded teacher action and related them to student outcomes which were measured. As Wilmot (2008) has pointed out, "Coding teacher actions [this way] was an indirect attempt at breaking down which aspects of teachers' knowledge are transformed into their teaching practice" (p. 37).

The use of process-product design received a number of criticisms in the 1970s (see Gage & Needels, 1979). The criticisms were based on 1) the idea of causality implied in the process-product research paradigm (i.e. their over reliance on correlational methods), 2) concerns about the predictive power of the process-product design, 3) problems related to the predetermined coding categories and the need for experimental methods, and 4) the conversion of the findings of process-product researchers into rules for teaching.

As a result of these criticisms, a modification was made in the design in studies that were conducted after the process-product researchers

(Berliner, 1979; Peterson & Swing, 1982). Berliner (1979) and his colleagues in the Beginning Teacher Evaluation Study (BTES) for instance, introduced Academic Learning Time (ALT) as a variable in their modification of the process-product design. One important aspect of ALT is what the BTES program refer to as *engaged time*, the actual time students spend in task provided by the teacher in learning a particular content. Berliner and his colleagues argue that if a student is engaged on easy items over a long period, that student's academic performance will not be improved to any marked extent. On the other hand, if a student's time is spent on items that are too difficult for him/her that student will not be able to master the extra concepts, skills and operations needed for good performance at grade level. Berliner and his colleagues in the BTES programme therefore argued that their new variable ATL was important because it serves as a link between student performance and teacher behaviour and also an operational behavioural indicator of students learning. Unfortunately, the ALT constructs failed from two perspectives. It failed not only in showing the type of knowledge teachers must possess to effectively judge the right level of difficulty of tasks to give to students to improve their learning but also indicate how teachers are able to decide when to move to new materials.

Other researchers who came to the scene (see for example, Peterson and Clark, 1978; Putnam, 1987) later operated with the assumption that it was necessary to bring the mental life of the teacher to the center of research on teaching. The argument is that the knowledge of experienced teachers is organized in packages of question and explanations that make it possible for them to enhance student learning (Putnam, 1987; Shulman, 1987). Putnam (1987) refers to these packages as "curriculum scripts" and argues that teachers' agenda for teaching is shaped by the richness of their curriculum scripts. In other words, the ability of a teacher to adopt flexible and interactive approaches to teaching depends on the richness of his/her curriculum scripts. To these researchers therefore, by focusing on the mental life of the teacher, the thought process of teachers before, during and after teaching could be rightly studied in order to understand how teachers transform their knowledge into their teaching practice.

Shulman and his collaborators' work brought to the fore the newest dimension on how teacher knowledge can influence teaching (Shulman, 1986; Wilson, Shulman & Richert, 1987). It can be argued that the impetus for the renewed interest in studying teacher knowledge could be a result of Shulman and his colleagues' conceptualization of "content knowledge" and "pedagogical content knowledge" and the distinction between them (see for example, Ball, 1988; Wilson & Winneburg, 1988; Grossman, 1990).

The common thread about the aforementioned research paradigms is that they all ended up producing qualitative data on teachers' knowledge. The rationale for using the KAT conceptualization as a framework for this study stems from the fact that the approach by the KAT project was, in a number of ways, an improvement over the previous attempts. For instance, instead of continuing to conceptualize teacher knowledge as a generalize construct the

KAT project focused was domain specific (i.e., focused on algebra). In addition, by developing and validating instruments to measure their hypothesized knowledge members of the project were ensuring that their conceptualization would be measurable. The KAT project came out with two forms of instruments comprising twenty items each. To overcome the problem of too few items on any one of the forms as pointed out by Wilmot (2008), the current study merged the two instruments, adapted the items and formulated extra items for use. As a result, the adapted instrument used comprised 74 items.

Teachers Subject Matter Knowledge

Undoubtedly, research regarding teachers' knowledge is as important to scholastic reform today as it was four decades ago (Mewborn, 2001). Clarke & Peterson (1986) posited that teachers are reflective thoughtful characters and that teaching is a multifaceted, cognitively demanding process involving problem solving and decision making. In another breadth, Simon (1993) suggested that "in order for a teacher to teach very well he/she needs to know about the subject matter in both width and depth to a degree unlikely to be found amongst those beginning teacher training course" (p.9). It is a common believe that every good teacher must learn more mathematics and that the higher the level of mathematics a teacher knows the better teacher he or she becomes (American Council of Education, 1999). In a related study, Usikin (2001) harangued that "often the more mathematics courses a teacher takes, the wider the gap between the mathematics the teacher studies and the mathematics the teacher teaches. The result of the discrepancy is that teachers are often no better prepared in the content they will teach than when they were students taking that content" (p.2). Thompson & Thompson (1996) also mentioned that mathematical content knowledge is essential for effective teaching; nevertheless, study revealed that teachers require further than just a strong knowledge of the content in order to teach mathematics. Furthermore, Thompson and Thompson found that although the teacher possessed a strong understanding of rate, his understanding of the concept could not be explained in a way that would support a student develop a conceptual understanding of rate. This presupposes that there is more to learning more of mathematics courses for one to become a noble teacher.

To support Thompson and Thompson (1996) work, Mewborn (2003) opined that while teachers are said to have some level of appropriate knowledge of mathematics, unfortunately, these teachers lack a conceptual understanding of the mathematics they are to teach. It was concluded by Mewborn that mathematics teachers had a strong procedural knowledge, but lack conceptual knowledge of mathematics. For instance, Pamela et.al. (2007), examined 20 middle school mathematics teachers' knowledge of students' understanding of the equal sign and variable as well as students' success applying their understanding of these concepts. The teachers were presented with four tasks concerning students understanding of algebraic concepts. The researchers recorded the answers- correct and incorrect- that the teachers believed students at their respective grades levels would give and identified the strategies and students thinking behind each of these responses. For each work, the fraction of students' answer of each type projected by teachers was averaged across all participants at each grade level, including zeroes for teachers who did not suggest a particular response. It was established that teachers' prediction of students' understanding of the equal sign did not correspond with actual student responses. Further, it was reported that "teachers rarely identified misconceptions about either variable or the equal sign as an obstacle to solving problems that required application of these concepts" (p.249).

There is indeed some research that points to secondary teachers difficulties with concepts of the mathematics they teach. For instance, Ball (1990, 1991) compared the mathematical knowledge of pre-service elementary education majors and pre-service secondary mathematics education majors on the topic of division. It was found that the secondary majors were more successful at obtaining correct answers than the elementary majors, but they were not able to provide conceptual explanations for the procedural tasks they perform. The concluding remarks was that, secondary mathematics teachers made a few if any mistakes in their usage of mathematics procedures; but that they experienced significant difficulties to provide sound meaning and explanations of the mathematical rationales lying behind the same procedures they use. Remarking on this Mewborn (2003) discoursed that "by and large, teachers have a strong command of the procedural knowledge of mathematics, but they lack a conceptual understanding of the ideas that underpin the procedures" (p.47).

A summary of thirty studies relating teachers' subject matter knowledge to student achievement by Byrne (as cited in Darling-Hammond, 1999), confirmed that 17 show a positive relationship. However, Monk (1994) and Rowan, Correnti & Miller (2002), contended that teachers' content knowledge produces virtually no returns in terms of the impact on the achievement of their students. It was evident from their studies that the relationship becomes meaningless as the teacher's number of advance mathematics courses increases, more so beyond five courses. As if this was not enough, studies that found meaningful relationship between teachers' knowledge and students' achievements were in dilemma as to how much of the relationship was due to experience (Rowan et al., 2002).

Ball (1991) concluded that simply requiring more mathematics of prospective teachers will not increase their understanding of school mathematics, rather a different kind of mathematics is needed.

Years of Teaching Experience and Student Achievement

"Teaching is one of the few professions in which the professionals are assumed to exhibit excellence the first year on the job" (Klecker, 2002. p.4). According to Darling-Hammond (2000), one of the indicative variables for teacher competence is teachers' years of teaching experience.

Researchers in the field of Mathematics have found that teachers learn through their teaching experiences (Klecker, 2002; Rosenholtz, 1986).

Studies have also shown that "experienced teachers' knowledge about teaching is organized into packages of questions and explanations that make it possible for them to enhance student learning and overcome students misconceptions about subject matter they teach" (Putnam and Shulman, as cited in Wilmot, 2008, p. 38). Commenting on this, Leikin, (2006) said the main source of teachers' expertise is their interactions with students and learning materials. This means that through interaction with students, teachers become aware of new solutions to known problems, new properties or theorems of the mathematical objects, new questions that may be asked about mathematical objects and in this way they develop new mathematical connections. It is obvious that teachers gain more experience when they make conscious effort to know better than their students, to know the material well enough, and to predict students' possible difficulties, answers and questions (Leikin, 2006). These experiences can only be realized with time.

Studies have established that teachers with more years of teaching experience are more effective than inexperience teachers', especially those with less than three years of experience (Klecker, 2002; Rosenholtz, 1986). According to Greenwald, Hedges & Laine (1996), teaching experience had a positive and significant effect on students' achievement. Moon, Mayes and Hutchinson, (as cited in Farooq & Shalizad, 2006) in a similar study identified three main factors within teachers control that significantly influence students achievement; professional characteristics, teaching skills and classroom climate. “It is therefore certain that students would not benefit much from learning, where teachers are not competent” (Adeyemi, 2010, p 318). It means therefore that experience counts a lot in teaching because effective teaching strategies are learned on the job. No wonder a summary of a number of researches, have found teachers teaching experience to be one of the determinants of students’ achievement (Bodenhausen, 1988; Felter, 1999 & Klecker, 2002).

Klecker (2002) in a study examined the relationship between teachers’ years of teaching experience and students’ mathematics achievement. The analysis included students’ scores on eighth grade National Assessment of Education Progress (NAEP) mathematics test, and the teachers’ teaching mathematics experience measured on five categories: 2 years or less, 3-5 years, 6-10 years, 11-24 years and 25 years or more. The result indicated that students of teachers with more years of experience teaching mathematics had higher mathematics scores on the eighth grade NAEP Mathematics test. Klecker in explaining pointed out that although statistically significant differences were found between the groups, the effect sizes were small, ranging from 0.08 to 0.37. This finding is in consonance with that of Fetler, (1999), who, after controlling for students poverty rates, reported a slight positive relationship between students’ scores and teacher experience level.

Teachers' Degree Type and Students Achievement

Another research focus has been on the relationship between teacher degree type and students achievement. For instance, teachers who are able to use a broad repertoire of teaching strategies skillfully, rather than a single, rigid approach, to reach an increasingly diverse student population are most successful (Doyle, 1977; Darling-Hammond, 1999). In other words, the qualities of a good mathematics teacher involve both expertise in mathematics and an understanding of how to communicate with students.

"It is easy to be a teacher, but it is difficult to be a good teacher" (Howe, 1999. p.885). One of the roles of a good teacher is to assist students see mathematics as a coherent whole. This means that, "a teacher who is blind to the coherence of mathematics cannot help students see it" (p.885). Teacher education in general appears to promote the use of these practices.

Baturo and Nason (1996) described four kinds of mathematical knowledge; substantive knowledge- requires the knowledge of both concepts and procedures and the connections between the two, knowledge about the nature and discourse of mathematics, knowledge about mathematics in culture and society and dispositions towards mathematics. Berenson et al. (1997) also used the terms procedure-centered and concept-centered to classify extremes of knowledge in their study of trainee teachers' perceptions of teaching. Procedural knowledge is defined as the knowledge of algorithms, formulae and definitions and conceptual knowledge as the relationship of ideas. It was proposed in this study that if trainee teachers exhibit good "substantive knowledge" their understanding will tend to be conceptual. Conversely, poor "substantive knowledge" would be more likely to indicate procedural understanding. In other words, teacher knowledge that connects concepts in mathematics is more effective (Ma, 1999).

Holmes & Dougherty (2006), investigated the connection among content, pedagogy and content pedagogy as the main vehicle of teacher education programme. Their result suggests that teachers under any education programme must be exposed to both content and pedagogical content knowledge in the way they are expected to teach when they complete. Study has found that teachers with education degree are better able to use variety of instructional strategies that fit the needs of their students. Non-education degree teachers may not have studied curriculum, teaching strategies, classroom management, uses of technology or the needs of special education students (Darling-Hammond, 1999). Greenberg, Rhodes, Ye, & Stancavage (2004) affirmed that "teachers with extensive subject-area knowledge may require some extra instruction in pedagogical techniques in order to become fully effective teachers" (p.4). In other words, knowing mathematics for oneself is not the same as knowing how to teach it.

Farooq and Shalizad (2006) investigated the impact of teaching of professionally trained and untrained teachers of mathematics on the achievement of their respective students. In the study, 400 high school graduates who passed their secondary school certificate examination were conveniently selected and taught by professionally trained and untrained teachers. After analyzing the data, it was discovered that students who were taught by trained teachers perform better than their counterparts who were taught by untrained teachers. Farooq and Shalizad concluded that trained teachers of mathematics have strong influence on students' achievement in mathematics. A similar study was conducted by Moon, Mayes and Hutchinson (2004) to find out the effectiveness of trained teachers. The result confirmed that trained teachers have a wide range of pedagogical supports which they can use to influence students to perform.

In a study conducted by Wilmot (2008), analysis of variance performed on data revealed that in-service high school mathematics teachers in Ghana performed significantly better than each category of prospective mathematics teachers majoring in mathematics, mathematics education and statistics from the country's universities. It was therefore established that knowledge for teaching algebra of in-service high school mathematics teachers was significantly different from that of prospective teachers. Also Wilmot (2008) in analyzing data collected on in-service and university students in Ghana reading mathematics in different majors concluded that generally the mathematics majors performed significantly better than their counterparts majoring in statistics and mathematics education. Between the statistics and mathematics education students, the statistics majors did slightly better than the mathematics education students. However, this difference was not significance at the .05 level.

Teacher Level of Education and Students Achievement

Hanif and Saba (2000) opined that the smooth running of any education system depends on good teachers and that teachers can not in any way be substituted with any form of instructional material. Teacher quality is vital to students' achievement (Hanusshek, 1986). A number of recent studies have examined the effect of teacher qualifications on academic achievement (Darling-Hammond, 2000; Goldhaber & Brewer, 2000; Monk & King, 1994; Rivikin, Hanushek & Kain, 2005; Rowan, Correnti & Miller, 2002).

One of the most recent studies that examine the effect of teacher qualification on academic achievements is that of Easton-Brook and Divis (2009). They surveyed 544 African American and 3,874 European American students entering kindergarten in 1998 through fifth grade in 2003. On the part of the teachers, certification was used as a proxy for teacher qualification. They employed Value Added Model (VAM) techniques to examine the relationship of the teacher qualification with reading outcome for the African American and the European American students. The VAM technique is described by Rudin, Staurt and Zanutto (as cited in Easton-Brook, 2009) as "determining how much of the change in student performance over time can be attributed to differences in teachers" (p.4). According to them the approaches to the VAM depends on how predictor variables are controlled in a particular study. The study employed a two-stage process in their study: First, they studied an unconditional model, by testing whether reading scores vary significantly at kindergarten and vary significantly over time. Secondly, they introduced a conditional model, by testing whether teacher qualifications significantly influence the reading scores of the African American and European American students at different levels of poverty at kindergarten and their gains in reading over time. Easton-Brook and Davis found that teacher qualification is significantly associated with higher growth in reading for both African American and European American students. Their finding is in consonance with several other studies (Darling-Hammond, 2000; Ferguson, 1998; Goldhaber, 2002; Rivkin, Hanushek & Kain, 2005). However, Fryer & Levitt (2004) found that teacher education level accounted for virtually none of the variance in reading scores of the African American and European American students.

Darling-Hammond (1999) found that the most important factor affecting students learning is the teacher quality. To examine the relationship between teacher qualification and students achievement, she analyses data on public school teacher qualification from the School and Staffing Surveys (SASS) and data on students' achievement from the assessment in reading and mathematics conducted by the National Assessment of Education Progress (NAEP). The teacher quality variables that were used for the analysis include proportion of "well qualified" teachers (teachers holding state certificate and the equivalent of a major in the field taught), the proportion of "fully certified" teachers (teachers with standard or regular certificate and new teachers on probationary certificate but have completed all requirements) and the proportion of teachers who are "less than fully certified" (teachers with provisional certificate or temporary certificate and those without certificate) (Darling-Hammond, 1999, p.28). It was reported that even after controlling for students poverty and language background, the proportion of well-qualified teachers was the most important and consistent determinant of student achievement. For instance, the proportion of teachers with full certification and degree in the field to be taught was very significantly and positively correlated with students learning outcomes.

Other studies examining the effect of teachers' educational level (those having master's degree or not having master's degree) on students' achievement found a significant and positive relationship between these variables (Greenwald, Hedges & Laine, 1996). Goldhaber and Brewer (as cited in Greenberg, et al., 2004) investigated the achievement gains of twelfth grade students in mathematics with respect to their teachers majors and found out that students whose mathematics teachers have first degree or second degree major in mathematics have higher levels of mathematics achievement than their counterparts whose teachers majored in fields other than mathematics.

Khairani (2016) conducted a study which focused on assessing urban and rural teachers' competencies in Science, Technology, Engineering, and Mathematics (STEM) integrated Education in Malaysia. The study made the assumption that in order to accomplish the need of considerable skill workers, then the country will have to introduce STEM integration education in mainstream schools throughout the country. For this integration to be successful, like any educational reform, the study considered the teachers' readiness especially in terms of their skills and competency in executing the change. The primary purpose of the study was to assess differences between teachers' competency for STEM integration education between urban and rural teachers. The study sampled a total of 244 teachers comprising 129 urban teachers and 115 rural teachers who teach various subjects. The sample comprises 206 (84.4%) female teachers and 38 (15.65%) male teachers. The mean of their teaching experience was 13.6 years (SD = 7.3 years). These teachers came from the states of Penang, Kedah, Perak, and Selangor. The cross-sectional quantitative study survey was the main design employed in the study. In this study, the participants responses from a 18-item questionnaire were analysed using Rasch Model analysis to determine features of item that measure competency between urban and rural teachers. The study, however, revealed that both urban and rural teachers across the selected areas asserted that they have the competency in terms of appropriate academic qualification to integrate STEM education into teaching and learning. Generally, the study recorded acceptable result where 14 out of 16 items (87.5%) function similarly across both urban and rural teachers, with only two items showing otherwise. With regards to those two items urban teachers show statistically significantly more competence than their counterparts in rural settings. The aspect of the study which showed statistically significant difference between the two groups had to do with integration of incorporating ICT in integrating STEM in teaching and learning and how to organise co-curricular activities. This result I believe is not surprising since those at the urban settings are well vexed and have the necessary and needed facilities in this regard.

Summary of Literature Review

Many research works have asserted that the teacher is the most important factor that influences students' achievement (Mullens et al., 1996; Sanders & Rivers, 1996). For instance, teachers' subject matter knowledge was found to be a better predictor of students' achievement than other home based factors (Mullens et al., 1996). In another breadth, Rowan et al. (2002), asserted that teachers' years of teaching experience was found to be a better predictor of students achievement than subject matter competency. It has been further pointed out that courses taken by teachers during the period of their training is a better predictor of teacher

The review of related literature brought to fore that though researchers are in agreement on the usefulness of teacher knowledge in influencing student performance (see for instance, Begle, 1972; Eisenberg, 1977; Clark & Peterson, 1986; Wilmot, 2009), they are generally separated on how to effectively conceptualise it in order to show which aspect of it bests predicts student performance. Also, according to Wilmot (2008), some of these conceptualizations have presented teacher knowledge as a domain neutral construct making it virtually impossible for it to be objectively measured (for example, Shulman,1987; & Grossman,1990). As a result, several attempts to measure teacher knowledge have relied on proxy measures such as the number of university courses taken, the type of degree the teachers' have etc. This study is posited on the fact that, instead of relying proxy measures, there is the need for re-conceptualization of teacher knowledge in ways that is not only domain specific but also allows its components to be measured. At the Senior High School level, researchers in the Knowledge of Algebra for Teaching (KAT) project at Michigan State University in the mid to late 2000s proposed one of such ground breaking framework of teacher knowledge (see Ferrini-Mundy, Burrill, Floden, & Sandow, 2003; Ferrini-Mundy, McCrory, Senk, & Marcus, 2005; Ferrini-Mundy, Senk, & McCrory, 2005).

This study, however, is premised on the fact that, instead of relying on proxy measures, there is the need for re-conceptualization of teacher knowledge in ways that is not only domain specific but also allows its components to be measured. In the early to mid- 2000s, researchers of the Knowledge of Algebra for Teaching (KAT) project attempted to do exactly this by hypothesizing three types of knowledge and developing items to measure it. Using the KAT conceptualization as a framework, this study was designed to investigate whether the three types of teacher knowledge hypothesized in the KAT framework will be corroborated.

CHAPTER THREE

RESEARCH METHODS

This study is posited on the assumption that, instead of depending on proxy measures on teacher knowledge, there is the need for re-conceptualization of teacher knowledge in ways that is not confined to only a particular field but also allows its constituents to be measured. In the mid to the late 2000s, researchers who have worked into the Knowledge of Algebra for Teaching (KAT) project at Michigan State University which was focused on Senior High School level, proposed one of such ground breaking framework of teacher knowledge (see Ferrini-Mundy, Burrill, Floden, & Sandow, 2003; Ferrini-Mundy, McCrory, Senk, & Marcus, 2005; Ferrini-Mundy, Senk, & McCrory, 2005). In addition, researching into the KAT framework, Wilmot (2017) has proposed a modified framework. This study therefore was designed to investigate whether the three domains of teacher knowledge hypothesized in the KAT framework will be corroborated and further explore for other knowledge types if they exist.

This chapter describes the methodology that was used in the study. It focuses on the research design, population, sample and sampling procedure, instrumentation, data collection procedure and ends with issues on data analysis.

Research Design

This study aimed at investigating senior high school mathematics teachers' knowledge for teaching algrebra. To accomplish this, mathematics teachers in a sample of forty SHSs in three regions of the country were selected as participants of the study. In other words, participants for this study were selected from a cross-section of schools in the three regions. Participants were made to respond to items on a multiple-choice type of questions. The instrument was meant to measure both the content and pedagogical knowledge participants had for teaching algebra at the SHS level. Consequently, a cross-sectional survey design was used.

The cross-sectional survey was suitable for this study because it allowed for collecting data from a sample of mathematics teachers without altering their aforementioned knowledge (Nworgu, 2006; Mitchell & Jolly, 2004; Creswell; 2003; Cohen, Marion, & Morrison, 2001; Fraenkel & Wallen, 2000). Furthermore, this design was more economical because it enabled data to be collected on the sampled teachers (i.e. a snapshot of teachers in the three selected regions) at only one point in time (Mitchell & Jolley, 2004).

As a matter of fact, survey designs have been found by many researchers to have the capability to provide a prospect to reach a large sample size which in turn raises the generalization of the findings. Also, they create the room for the participants to respond to the items on the survey in a place and time convenient to them as well as producing responses that are easy to code (Gray, 2004).

In another breadth, these kind of designs are capable of providing descriptive, inferential and explanatory evidence that can be used to establish correlations and relationships between the items and themes of the survey (Cohen, Manion, & Morrison, 2007, p. 169).

There is greater anonymity associated with surveys. They also provide consistent and uniform measures and respondents are not affected by the presence and or attitudes of the researcher (Sarantakos, 2013). On the other hand, surveys also have their own deficiencies among which are the inability to ask probing questions as well as seek clarifications, inability to determine the conditions under which the respondent responded to the questionnaire items as well the ability to generate high unresponsive rate (Sarantakos, 2013).

It must, however, be noted that since data was collected at only one point in time the design could not permit the study to account for any possible changes that may occur in the knowledge and beliefs of the participants after the study.

Despite the weakness, it was considered that the strengths of gaining many teachers' responses far outweighed the weaknesses in this study; hence the cross-sectional survey was considered an appropriate design for the study.

Population

The target population was all Senior High School mathematics teachers in the Ashanti, Western and the Central Regions of Ghana including university final year education students in one of the public universities in Ghana. These three regions were selected simply because they are densely populated regions with substantial number of schools. Also, teachers found in schools in these regions are believed to be coming from across the various universities in the country hence those three regions. The accessible population from which the sample was drawn is all mathematics teachers from 40 Senior High Schools and one group of final year education students in one public university. Based on the content of algebra in both core and elective mathematics syllabi, the researcher used both teachers teaching Core and Elective Mathematics in the selected schools. The study's population comprised of 400 prospective and in-service mathematics teachers. The accessible population has a total of 127 mathematics teachers without background qualification in education but are all degree holders whereas 125 of them possess background qualification in education.

As a result of the purpose of this study, only in-service and prospective mathematics teachers teaching elective and core mathematics were used in the study. For example, one of the aims of the study was to scrutinize whether there is any difference in knowledge for teaching algebra between in-service and prospective mathematics teachers. It has been conjectured that reliant on the number of years of teaching experience, some in-service teachers will have a high level of teaching knowledge for teaching algebra which may distinguish them from their colleagues who are yet to enter into full teaching.

Sampling Procedure

All the senior high schools in the three selected regions were grouped into four categories: A, B, C and D (Ghana Education Service, 2009). Since school-type based analysis was conducted, as many schools as possible (single-sex male, single-sex female and co-educational) from each of these categories were selected for the study. Using the school-type as the basis, as many schools as possible from each of these regions were selected using the multi-stage sampling technique from each category using computer generated random numbers making a total of 40 schools from all the three regions to include teachers from all the four categories, (categories A, B, C, and D) in accordance with the Ghana Education Service (GES) classification (2009).

The rationale behind the use of these categories is based on Begle's (1972) remarks that the schools which participated in his study were all well-resourced with highly motivated teachers. In each strata, four schools each were selected for the study using the simple random sampling technique. In a region where only one each of single-sex male and single-sex female schools are found, purposive sampling technique was employed. After the schools have been selected from these categories, the census method was employed to select all mathematics teachers teaching either core mathematics or elective mathematics or both to participate in the study. Teachers who were teaching mathematics but have no background training in mathematics were not considered.

The study used only in-service and prospective mathematics teachers teaching either core or elective mathematics or both for the simple reason that one of the aims of the study is to examine the difference between in-service and prospective mathematics teachers' knowledge for teaching algebra. Since some of these in-service mathematics teachers would have been teaching for a long time, it is hypothesized that some of them might have forgotten aspects of the content of their university coursework. Secondly, it is hypothesized that, depending on the number of years of teaching experience, some in-service and prospective teachers would have a high level of teaching knowledge which may affect their knowledge about mathematics and learning mathematics.

A multi-stage sampling technique was resorted to in order to obtain the schools that participated in the study (Shaughnessis, Zechmeister & Zechmeister, 1997). With this kind of technique, it is where two or more sampling techniques are employed in a single study. In the various regions, a municipality or a district was chosen using the simple random sampling technique. There after the stratified random sampling technique was used to put the various schools into strata using the GES categorization criteria and from these categories, the schools that participated in the study were achieved through the use of simple random sampling technique. From these forty schools, five single-sex female and five single-sex male schools were selected using the purposive and convenience sampling method. The other thirty which were co-educational schools were selected using the simple random sampling technique. As already mentioned, thirty co-educational senior high schools in the three regions (Ashanti, Central, and the Western Regions) were randomly selected to participate in the study. A total of 252 teachers participated in the study which comprised 209 and 43 male and female mathematics teachers respectively.

Data Collection Instrument

The study adapted an achievement test instrument from the Knowledge of Algebra for Teaching (KAT) project and administered to subjects of the study. The original instrument for the KAT project was in two forms:

Instrument 1: Teachers' knowledge of teaching algebra consist of 17 multiple choice questions and 3 open response types. Instrument 2 was an achievement test on algebra for SHS students and consist of 10 multiple choice questions and 2 open response types. These two instruments were transformed and some adapted from other sources into multiple Choice type questions consisting of 74 items in all and spread under seven different variables under consideration for teachers. The multiple choice item format was adopted because from personal experience, people and for that matter teachers become hesitant in answering open response type of questions but relatively find comfort in answering multiple choice type questions since options are provided. The tests lasted for a total time of three-hours five minutes for the respondent teachers to measure their understanding of real number system and other algebraic concepts on the multiple choice test items. That amount of time was given to participants to complete the test so as to avoid some questions if not all unanswered. The items on the instruments were based on the content of algebra in the senior high school syllabus (both core and elective mathematics). Both the students and teachers instrument adapted from KAT were merged to form one instrument which consisted of two parts: survey items and content items which were answered by participants. To guarantee participants of the inconspicuousness and privacy of the study, the instruments did not require the use of identities such as personal names. These instruments were first adapted for use in Ghana by Wilmot (2008) in Ghanaian senior secondary schools now senior high schools. In adapting it, Wilmot (2008) included the wording of the questions in the instruments to replicate Ghanaian context. For example, items which contained US currency were changed into Ghanaian currency as well as realistic prices of items on the Ghanaian market. For example, an item that originally read, “At a storewide sale, shirts cost $8 each and pants cost $12 each. If S is the number of shirts and P is the number of pants bought, which of the following is a meaning for the expression 8S + 12P?” was adapted into, “At a storewide sale, shirts cost ¢80.00 each and a pair of trousers cost ¢120.00 each. If S is the number of shirts and P is the number of trousers bought, which of the following is a meaning for the expression 80.00S + 120.00P?”

Furthermore, names commonly used for commodities in the original items were also changed to reflect the right context in Ghana. The reliability of the instruments was found to be 0.842 (Wilmot, 2008). The new items added to the original ones in the instruments were pilot tested in one of the regions to make it more reliable. The instrument was pilot tested on 50 respondents in the Greater Accra Region which yielded a reliability coefficient of 0.786 using the KR-20 formula.

The advantage of instrument used was that it covered a wide range of areas in the syllabus for both core and elective mathematics in addition to a wide range of pedagogical and content issues. Another good thing about the instrument used was that it was spread across Bloom’s taxonomical areas and not restricted to only an aspect of it. The language as well as the terminology used in the instrument was clear and precise so as to avoid any misunderstanding on the part of any teacher. One major disadvantage of the instrument administered to participant was that of the number of items vis-à-vis the time frame to respond to the entire items on the instrument.

Validity

Content validity of the instruments was established by presenting the tests and its scheme to two mathematics education lecturers in the Department of Mathematics and I.C.T. Education, three other PhD fellows and the researcher's team of supervisors for inspection to ensure that the types of knowledge hypothesized in the KAT framework are satisfactorily covered and well structured.

The instrument was then field tested by the researcher in collaboration with one research assistant with the Department of Mathematics and I.C.T. Education and one mathematics tutor at the Komenda College of Education. This exercise enabled additional adjustments to be made to obtain the final form for the main instrument for the study.

Pilot Testing

When the instrument has been improved upon through professional advice, it was then field tested. The test was administered to senior high school prospective and in-service mathematics teachers in the Greater Accra Metropolis in order to determine its reliability and validity. 50 Prospective and in-service mathematics teachers teaching in 10 senior high schools in the Greater Accra region participated in the pilot testing exercise.

Reliability

In all 50 prospective and in-service mathematics teachers participated in the study and took them approximately three-and-half hours to complete a total of 80 items on the instrument. Total scores obtained by teachers in the pilot testing for the items ranged from 0 to 50 out of the 80 items. The reliability of the test was calculated using the KR-20 formula and the coefficient found to be .786. The discrimination and difficulty indices of the items were also determined to further check the validity of the items. Nevertheless, the reliability coefficient obtained for the final work was 0.855.

Data Collection Procedure

The primary purpose of this research was to ascertain Ghanaian high school mathematics teachers' knowledge for teaching algebra and to corroborate the three knowledge types hypothesized in the KAT project. For the purpose of confidentiality teachers' responses and names of teachers who participated in the study were not recorded in the instruments to allay their fears of being exposed. Both instruments were administered to the subjects in the schools selected to take part in this research work. Administration of these instruments was done from the month of November 2015 to July 2016 as a result of delay and unwillingness from some of the schools and teachers involved in the study. This was done during the first term and early part of the second term of the 2015/2016 academic year of the school calendar. An initial visit was paid to the forty schools which were finally involved in the research. During the visit, audience was sought from heads of the schools and the teachers who were going to be involved in the study. At the meeting, the purpose of the study, its duration, and potential benefits were explained to the heads and teachers for their consent to allow the study to take place in their schools. Also, at this meeting, time was set for the administration of the questionnaire for the study. The aforementioned were done through the acquisition of written letter from the then Department of Science and Mathematics Education of the University of Cape Coast to these schools. After this, data were collected by the researcher with assistance from research assistances by moving from one region to the other and from one school to another. In each of the schools, both in-service and prospective elective and core mathematics teachers were given copies of the instrument to respond to accordingly. The researcher was with some of the respondents when items were being responded to. The researcher, however, was at a distance while the teachers responded to the items on the instrument to alleviate influence and tension on their part. Respondents were given one-hundred and thirty-five minutes to respond to the content area questions as well as the questionnaire on their beliefs about how mathematics should be taught and how it must be learned. One major problem that was encountered was the unwillingness on the part of prospective and in-service mathematics teachers in most of the schools to respond to the items on the instrument and participate in the study for the fear of the fact that they may not perform on the test.

Data Processing and Analysis

In any research, the raw data collected from the field needs to be processed into meaningful and relevant information for decision-making. The process may include ordering and shaping of data generated from the research to produce knowledge (Howard & Sharp, 1983), as well decreasing, organizing bulky data collected, and analyzing it to produce findings (Burns & Grove, 1987: as cited in Yarkwah, 2011).

Since data analysis is aimed at answering research questions that guided a particular study and testing all the hypotheses made in a study, data analysis was done and organized according to the study's research questions and hypotheses. To clarify this, the research questions and hypotheses are re-stated here in turn and the mode of data analysis for the research questions discussed separately from those of the hypotheses. Also, the items on the achievement test were assigned either wrong or right in order to ascertain teachers' knowledge for teaching algebra.

Data from this study are both qualitative and quantitative in nature. The qualitative data is the section soliciting participants' responses to the demographic survey questions whiles the quantitative data is the part that requires participants' responses to the content items on the instruments.

Analysis of Data Related to First Research Question

The first research question that guided this study was, "To what extent does high school prospective and in-service mathematics teachers knowledge for teaching algebra corroborate the three types of knowledge hypothesized in the KAT framework?"

Data for this research question came from both in-service and prospective teachers who teach Core Mathematics or Elective Mathematics or both were used. Factor analysis was done on each item found in the instrument. The factor loading for each item was then analyzed to determine whether conclusions could be made based on the three factors that were hypothesized from the beginning of the study as stipulated by the KAT framework. Also, the nature of the loadings of the items was also to determine for to confirm the extent to which items which were originally categorized as looking at the same dimension load together. The factor loading for each item was then analyzed to determine whether conclusions could be based on three factors. The decision to look for three factors is because the theoretical framework used for study had three conceptualizations of teachers' knowledge. Another very important thing that was done was to examine to confirm the extent to which items originally categorized as assessing the same type of knowledge loaded-together on separate factors. The basic reason this was done was to determine whether the three types of knowledge hypothesized by the KAT project came out distinctively and further explore for other knowledge types if feasible.

Analysis of the Second Research Question

The second research question that guided this study was, "What is the level of algebra knowledge possessed by SHS mathematics teachers for teaching based on the KAT framework?

These research questions were answered using achievement test scores obtained from in-service mathematics teachers. Since the focus of the question was to look at the knowledge teachers possess at the SHS level, analysis basically done was the use of descriptive statistics, frequencies and percentages. In addition, bar charts were used to further explain the outcome of some of the analyses. Also, further analysis was conducted using ANOVA on the available data set. This is because the aspect of analysis that resorted to ANOVA was focused on looking at the knowledge possess by in-service and prospective mathematics teachers across the three knowledge types hypothesized in the KAT framework.

Analysis Related to the First Research Hypothesis

The first research hypothesis that guided this study was, "There is no significant difference between the knowledge for teaching algebra of in-service and that of prospective mathematics teachers at the senior high school level" Since the null hypothesis been tested basically sought to compare whether there is a difference between in-service and prospective mathematics teachers, the independent samples t-test was resorted to in analyzing data obtained from these two groups based on the combined score. Furthermore, the analysis of variance (ANOVA) was performed on the data collected to ascertain if there is any difference in the knowledge for teaching algebra between the two groups.

Analysis Related to Second Research Hypothesis

The second research hypothesis that guided this study was, "There is no significant difference in the knowledge for teaching algebra between senior high school mathematics teachers with background training in education and those of their counterparts without background training in education". To answer this research hypothesis data from both in-service and prospective mathematics teachers who participated in the study were used. Since the subjects of focus who participated in the study are independent samples, the independent samples t-test and ANOVA were conducted on the data obtained from the teachers' achievement test based on the factors that emerged from the factor analysis. This analysis, like all analyses of data from the educational research, was done at the 5% level of significance.

Analysis Related to Third Research Hypothesis

The third research question that guided this study was, "There is no significant difference between senior high school mathematics teachers' knowledge for teaching algebra and their years of teaching experience". This research hypothesis was analysed using data obtained from in-service and prospective mathematics teachers from the schools that were involved in the study. Since the teachers' knowledge is compared across more than one teaching experience, the appropriate statistical tool that was used is the analysis of variance (ANOVA). It is appropriate because it makes possible the comparison of three or more means simultaneously. ANOVA makes use of F distribution. F-test shows whether or not a difference exists among the means. However, it cannot reveal where the difference lies. In the analysis, if the F-test indicates that there is a difference among the means, a follow-up test such as Tukey test will be performed.

Analysis Related to Fourth Research Hypothesis

The fourth research hypothesis that guided this study was, "There is no significant difference in the knowledge for teaching algebra between mathematics teachers who teach in urban areas and their counterparts in the rural settings based on the knowledge types in the KAT framework." To answer this research hypothesis, an independent-samples t-test was conducted to help in comparing the mean scores on some continuous variable which in the domain of this research hypothesis happens to be test scores obtained by these two groups of subjects (teachers who teach at rural settings and their counterparts in the urban settings). Also, further analysis was conducted using multivariate analysis of variance (MANOVA) and analysis of variance (ANOVA) as a follow up test to the MANOVA to ascertain where the difference is coming from with regards to the three knowledge types hypothesized in the KAT framework. This hypothesis was tested at the 0.05 level of significance.

CHAPTER FOUR

RESULTS AND DISCUSSION

The primary purpose of this study was premised on the fact that, instead of relying on proxy measures, there is the need for re-conceptualization of teacher knowledge in ways that is not only domain specific but also allows its components to be measured. This study, therefore, was designed to investigate whether the three domains of teacher knowledge hypothesized in the KAT framework will be corroborated and explore for further other factors if any.

Since the study was about senior high school mathematics teachers' knowledge for teaching algebra, it explored prospective and in-service mathematics teachers' knowledge for teaching. It also, compared senior high school mathematics teachers' knowledge for teaching algebra with regards to those with education background and their counterparts without education background. It also examined to what extent the Knowledge of Algebra for Teaching framework argument that there are fussy boundaries among their three hypothesized knowledge types be corroborated. Effect of teaching experience on senior high school mathematics teachers' knowledge for teaching algebra at the senior high school level was ascertained.

The study used 252 prospective and in-service mathematics teachers from three regions. These teachers comprised 125 and 127 teaches with education and their counterparts without education background training in 40 schools. The constitute of the sample also consisted of 209 males and 43 female mathematics teachers.

These research questions and hypotheses are discussed with regards to the three knowledge types used in the KAT framework. Principle component analyses, descriptive statistics of percentage for the instruments that was used for the study as well as correlation analyses were calculated on all items in the instrument.

In the analysis of the results, data analyses and all results related to a particular research question or hypothesis are duly presented and discussed before focus is directed to the next research question or hypothesis.

Research Question One

The first research question that guided this study was, ""To what extent does high school prospective and in-service mathematics teachers knowledge for teaching algebra corroborate the three types of knowledge hypothesized in the KAT framework?"

To answer this question achievement test scores data from the SHS teachers (both in-service and prospective) who participated in the study were used. An exploratory and confirmatory factor analyses were performed on the items in the instrument to validate and explore respectively the knowledge types hypothesized in the KAT project conceptualization and to explore for further factors if any. Factor analysis as a statistical technique was used in this study to among other things determine items of the instrument that have the same features and which therefore go together (that is load together). In factor analysis the features of the items that load together on each factor is used to label or describe the factor or variable. Factor analysis was used in this study for three main reasons in agreement with what Bryman & Cramer (2001) have summarized. These are: 1) to try to make sense of the bewildering complexity of social behavior by reducing it to a more limited number of factors or variables, 2) to find out the extent to which items are measuring the same concept and , 3) the degree to which the number of factors can be reduced to a more limited number in order to make decision, in this case, about the dominant factors as far as senior high school teachers' knowledge for teaching algebra is concerned. Thus, in this study, factor analysis was used to derive the variables, called factors, which gave better understanding about the data collected.

In addition, factor analysis was chosen for this research question because it helps, among other things, to examine the number of variables, called factors, which could be used to either totally or to a larger extent explain the variations in responses in the data collected (Yarkwah, 2011). The KAT framework that was adopted in this study stipulated that three knowledge types (*Advanced Knowledge, School Knowledge and Teaching Knowledge*) determine teachers' knowledge for teaching algebra. Nevertheless, in this study, a case was put forward that the interlocking regions as asserted by the KAT project team to be blur or fussy is not necessarily true. In other words, it was hypothesized in this study that those knowledge for teaching algebra in the Ghanaian context go beyond the three as stipulated by the KAT project team. On the contrary, contextually, the researcher believes that in Ghana there could be more than three factors that could determine teachers' knowledge for teaching algebra. Based on this assertion, factor analysis was conducted specifically exploratory factor analysis to determine whether there was enough evidence supporting this hypothesis and if not, how many factors could statistically be supported (or which the dominant factors were). As a result of these assumptions an Exploratory Factor Analysis was done. The simple reason for this was to allow as many factors as items on the instrument measuring teachers' knowledge for teaching algebra to be extracted so that a decision could be made, based on the factor loadings, as to the number of factors that could be retained to explain the types of algebra knowledge possessed by high school mathematics teachers in Ghana. This analysis helped to answer the question of whether there is enough evidence to conclude that three or more factors could be extracted to explain the types of knowledge hypothesized in the theoretical framework or otherwise. Before the actual factor analysis was conducted to determine whether the factors as hypothesized by the KAT framework, factor analysis was performed using the ones hypothesized by the KAT framework. However, analysis performed revealed that there were so many cross loadings on some of the factors with only three or less items loading on some of the factors.

The first and foremost thing done was to examine the number of factors that were required to explain the differences in responses as well as to identify the dominant factors in knowledge of the factors that determines teachers' knowledge for teaching algebra at the SHS level.

Nevertheless, the 74 items which were meant to look at the three major categories of knowledge as hypothesized by the KAT framework were subjected to Principal Components Analysis (PCA) using SPSS. It should be noted that these items (74 items) included 27 adapted ones from the KAT project and the remaining from the other sources. Prior to performing that, (PCA) the appropriateness of data for factor analysis was considered as mentioned earlier on. Inspection of the correlation matrix that was generated by the analysis indicated the presence of many coefficients of .3 and above. The Kaiser Mayer-Oklin value was 0.654, exceeding the recommended value of 0.6 (Kaiser, 1970, 1974; as cited in Yarkwah, 2011) and the Barlett's Test of Sphericity [BTS] (Barlett, 1954; as cited in Yarkwah, 2011) reached statistical significance supporting the factorability of the correlation matrix. To corroborate or otherwise, the three knowledge types hypothesized in the KAT framework, exploratory as well as confirmatory factor analysis using the idea by Nelson (2005) which has to do with superimposing regression line on the scree plot was done. The corroboration, however, was done on the original items constructed by the KAT project team. It must be noted that exploratory factor analysis (EFA) was used to summarize data by grouping together variables that are inter-correlated whereas the confirmatory factor analysis (CFA) was done because it entails the estimation and specification of one or more hypothesized models of factor structure, each of which proposes a set of latent variables (factors) to account for covariance among set of observed variables which in this study was the three knowledge types hypothesized in the KAT framework. One critical thing that must be noted is that CFA may follow EFA and both techniques are considered complementary to each other (Munro, 2005). Table 1, shows how items loaded on the various factors and the variance explained by all possible factor loadings with regards to the three knowledge types hypothesized by the KAT project team.

Table 1: *Initial Eigenvalues and Total Variance Explained by each of the Factors on the Instrument for Corroboration of the three Knowledge types*

Component	Initial Eigenvalues			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3.398	12.585	12.585	3.288	12.179	12.179
2	1.991	7.374	19.959	1.984	7.347	19.526
3	1.788	6.623	26.582	1.905	7.056	26.582
4	1.637	6.062	32.644			
5	1.564	5.793	38.437			
6	1.287	4.765	43.202			
7	1.253	4.640	47.842			
8	1.134	4.199	52.041			
9	1.101	4.077	56.118			
10	1.071	3.965	60.083			
11	1.022	3.784	63.867			
12	.953	3.531	67.398			
13	.863	3.196	70.594			
14	.840	3.113	73.707			

Table 1 Cont'd

Component	Initial Eigenvalues			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
15	.779	2.886	76.593			
16	.725	2.684	79.277			
17	.703	2.603	81.881			
18	.647	2.397	84.278			
19	.640	2.369	86.647			
20	.591	2.190	88.838			
21	.531	1.967	90.805			
22	.496	1.836	92.640			
23	.481	1.782	94.423			
24	.429	1.589	96.011			
25	.403	1.492	97.504			
26	.345	1.277	98.780			
27	.329	1.220	100.000			

Table 1 revealed number of possible factors which could be extracted from the data to explain the variations among the item responses and their corresponding eigenvalues. The Eigenvalues as shown in the table revealed the strength and weakness of each of the factors extracted. The eigenvalues helped decide on the required number of factors that is needed for inferences to be made. Principal component analysis (PCA) revealed the presence of eleven components with eigenvalues exceeding 1. These eleven factors together explained approximately 63.867% of the variance. Nevertheless, it was hypothesized by the KAT project team that three knowledge types exist. Therefore, the eleven factors revealed by the Kaiser criterion did not make sense in this analysis. To explain the inapplicability of the eleven factors revealed in the analysis, a graphical representation called the scree plot was used for further checks so as to extract the actual factors needed to explain the teachers' knowledge for teaching algebra at the SHS level in Ghana. The scree plot was used because according to Cattel (1966) it helps reduce the number of factors from items in an instrument which is being used by the researcher.

Fundamentally, the scree plot as suggested by Cattel's (1966), are graphs of the factors on the horizontal axis against the corresponding eigenvalues on the vertical axis. Per this graph, as the number of factors increases (that is as one moves from left to right along the horizontal axis), the corresponding eigenvalues decreases (this is shown in figure 1). The change in slope of this graph sharpens as a result of decrease in the number of factors. Five factors per the nature of the scree plot can be retained. This was done because, according to Child (1970); Kim & Muellar (1978), Norasis (1990) and DeVellis (1991) the factors which are to be retained are those which lie before the point at which the corresponding eigenvalues seem to cut or lend off. However, one may argue that from the scree plot more factors could have been extracted to make a case. Nevertheless, on the other factors some have only one item loading uniquely on it which defeats the concept of factor retention hence the five factors. The graph below, Figure 2, shows the graphical representation needed for this scree-test.

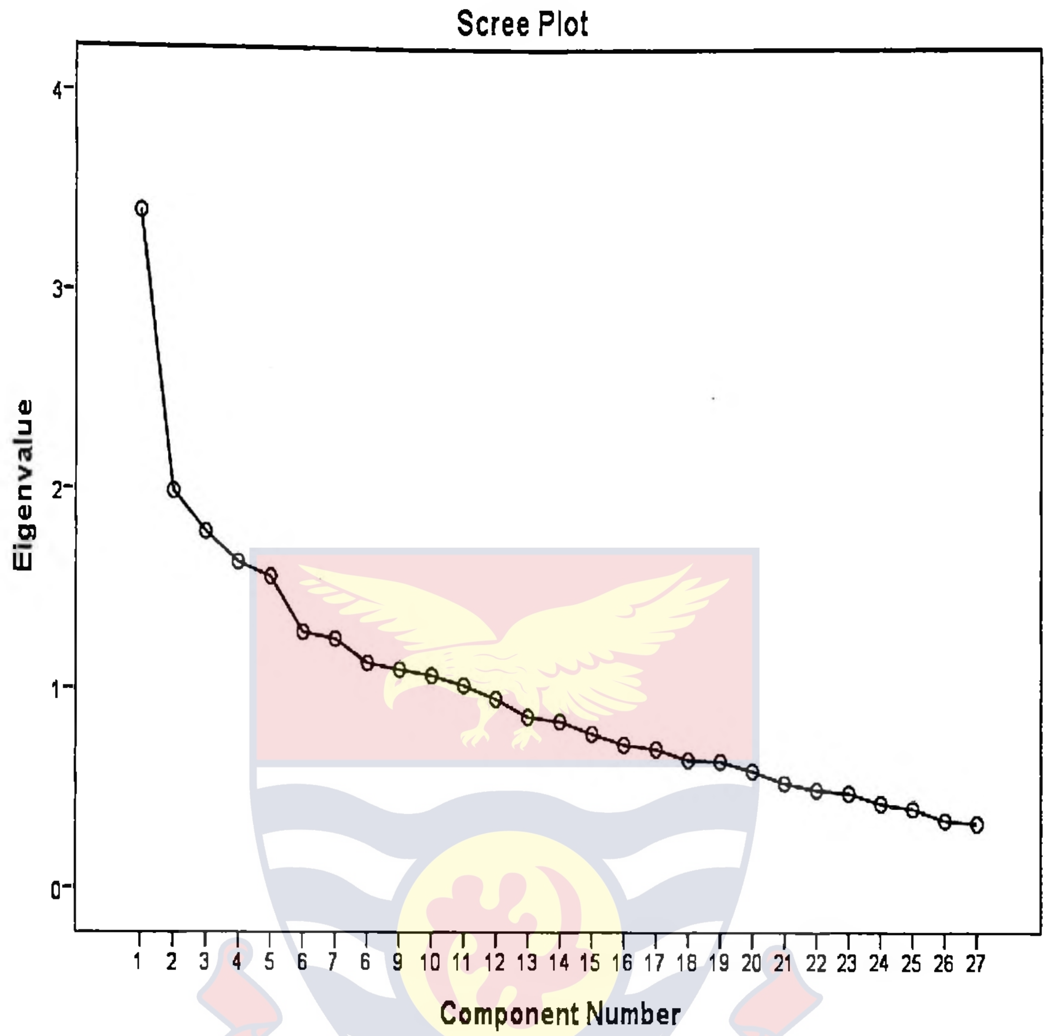


Figure 2: Scree plot showing number of factors retained

Before establishing that factors that explains teachers algebra knowledge for teaching in the Ghanaian context, a case was made to ascertain whether high school prospective and in-service mathematics teachers in Ghana's knowledge for teaching algebra corroborate the three types of knowledge hypothesized in the KAT framework, a regression line was superimposed on the scree-plot to help address issue as suggested by Nelson (2005). The main reason for doing this was that one of the problems associated with the use of the scree plot as a test to objectively identify the point where the sharp break exists (Cattel, 1996). As Allen and Scarpello (2004) stated,

> "Although the scree test may work well with strong factors, it suffers from subjectivity and ambiguity, especially where there are either no clear breaks or two or more apparent breaks. Definite breaks are less likely with smaller sample sizes and when the ratio of variables to factors is low ..." (p. 193)

Figure 3 indicates the reposition of the regression line on the scree-plot to confirm if indeed knowledge for teaching algebra in the Ghanaian context corroborate the KAT conceptualization.

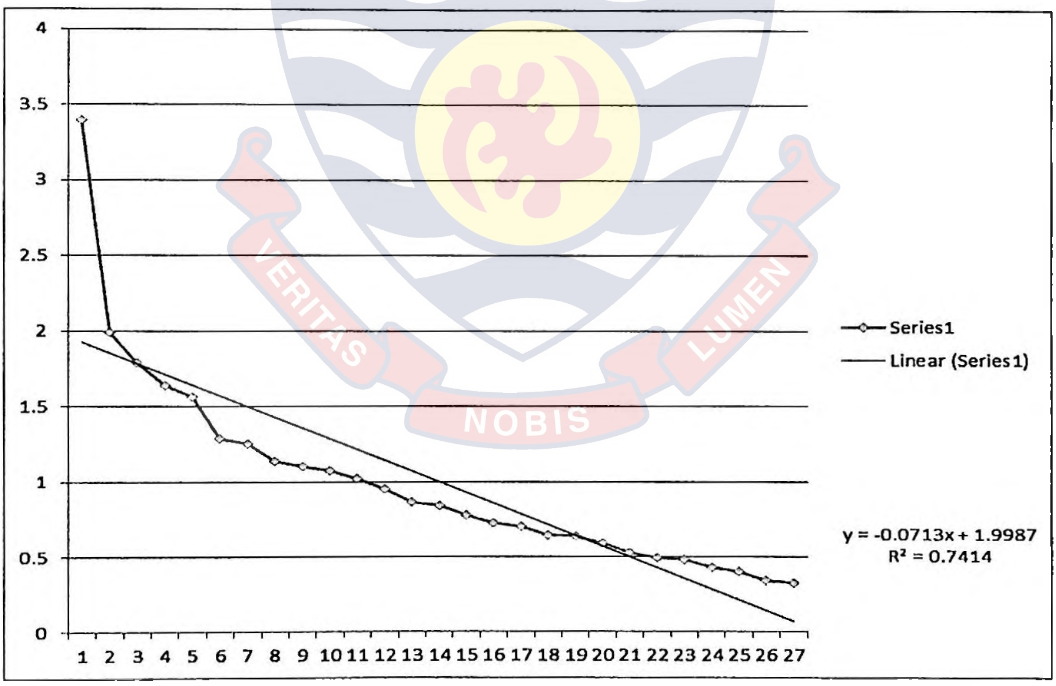


Figure 3: Scree plot with regression line showing corroboration of the three knowledge types

A cursory look at Figure 3 indicates that high school prospective and in-service mathematics teachers in Ghana's knowledge for teaching algebra corroborate the three types of knowledge hypothesized in the KAT framework. When analysis was done in respect of the KAT conceptualization, the superimposition of the regression line as well as the factor loadings after cross loadings were removed permitted the interpretation of the knowledge types hypothesized in the KAT framework. The analysis revealed that items originally categorized (see Appendix C) as measuring any of the three types of knowledge hypothesized by the KAT project team had the items loading uniquely on these retained factors. This results simply implies that data from this study corroborate the KAT framework. In other words, it could be mentioned that knowledge for teaching algebra of participating prospective and in-service high school mathematics teachers in Ghana corroborate the three types of knowledge hypothesized in the KAT framework. This result, however, contradicts Wilmot (2008) finding that items that loaded on the three factors could not explain the three types of knowledge hypothesized in the KAT framework.

Having corroborated the KAT framework in the context of Ghana, an exploratory factor analysis was conducted on the entire instrument to ascertain or otherwise the case made earlier on by the KAT project team that the interlocking regions are blur. Table 2, shows how items loaded on the various factors and the variance explained by all possible factor loadings with regards to the knowledge types.

Table 2: *Initial Eigenvalues and Total Variance Explained by Each of the Factors on the Instrument*

	Initial Eigenvalues			Rotation Sums of Squared Loadings		
Component	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	8.573	11.586	11.586	6.576	8.887	8.887
2	3.353	4.531	16.117	3.710	5.014	13.901
3	2.957	3.996	20.113	3.116	4.211	18.112
4	2.818	3.808	23.921	2.968	4.011	22.123
5	2.520	3.405	27.326	2.938	3.970	26.093
6	2.356	3.184	30.510	2.839	3.836	29.930
7	2.232	3.016	33.525	2.661	3.596	33.525
8	2.168	2.930	36.455			
9	1.977	2.671	39.126			
10	1.910	2.581	41.707			
11	1.796	2.427	44.134			
12	1.662	2.246	46.380			
13	1.591	2.151	48.531			

Table 2 Cont'd

	Initial Eigenvalues			Rotation Sums of Squared Loadings		
Component	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
14	1.569	2.120	50.650			
15	1.509	2.039	52.689			
16	1.454	1.964	54.654			
17	1.403	1.896	56.550			
18	1.356	1.833	58.382			
19	1.281	1.731	60.114			
20	1.234	1.667	61.781			
21	1.224	1.654	63.435			
22	1.143	1.544	64.979			
23	1.110	1.500	66.479			
24	1.082	1.463	67.942			
25	1.053	1.423	69.365			
26	1.041	1.407	70.771			
27	1.000	1.351	72.122			
28	.948	1.281	73.403			

Table 2 Cont'd

	Initial Eigenvalues			Rotation Sums of Squared Loadings		
Component	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
29	.930	1.256	74.659			
30	.886	1.197	75.856			
31	.867	1.171	77.027			
32	.828	1.119	78.146			
33	.805	1.088	79.234			
34	.771	1.041	80.276			
35	.740	1.001	81.276			
36	.703	.950	82.227			
37	.676	.913	83.140			
38	.668	.903	84.042			
39	.638	.863	84.905			
40	.611	.825	85.730			
41	.581	.785	86.515			
42	.568	.767	87.282			
43	.538	.727	88.008			
44	.512	.692	88.700			

Table 2 Cont'd

	Initial Eigenvalues			Rotation Sums of Squared Loadings		
Component	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
45	.481	.650	89.350			
46	.475	.642	89.992			
47	.460	.622	90.614			
48	.433	.585	91.199			
49	.411	.555	91.754			
50	.403	.544	92.299			
51	.397	.536	92.835			
52	.370	.500	93.335			
53	.351	.475	93.810			
54	.341	.461	94.271			
55	.334	.451	94.722			
56	.313	.424	95.146			
57	.297	.401	95.546			
58	.282	.381	95.927			
59	.269	.363	96.290			
60	.263	.355	96.646			

Table 2 Cont'd

Component	Initial Eigenvalues			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
61	.255	.344	96.990			
62	.236	.319	97.309			
63	.219	.295	97.604			
64	.204	.276	97.880			
65	.199	.269	98.150			
66	.192	.260	98.409			
67	.185	.250	98.659			
68	.180	.244	98.903			
69	.166	.225	99.128			
70	.143	.193	99.321			
71	.137	.185	99.506			
72	.132	.178	99.684			
73	.131	.178	99.861			
74	.103	.139	100.000			

Table 2 revealed number of possible factors which could be extracted from the data to explain the variations among the item responses and their corresponding eigenvalues. The Eigenvalues as shown in the table revealed the strength and weakness of each of the factors extracted. Principal component analysis (PCA) revealed the presence of twenty-seven components with eigenvalues exceeding 1, explaining 11.586%, 4.531%, 3.996%, 3.808%, 3.405%, 3.184%, 3.016%, 2.930%, 2.671%, 2.581%, 2.427%, 2.246%, 2.151%, 2.120%, 2.039%, 1.964%, 1.896%, 11.833%, 1.731%, 1.667%, 1.654%, 1.544%, 1.500%, 1.463%, 1.423%, 1.407% and 1.351% of the variance respectively. The initial eigenvalues in Table 2 suggested that twenty-seven main factors could be retained. These twenty-seven factors together explained approximately 72.122% of the variance. Nevertheless, it was hypothesized that seven knowledge levels were being considered. Therefore, the twenty-seven factors revealed by the Kaiser criterion did not make sense in this analysis. To explain the inapplicability of the twenty-seven factors revealed in the analysis, a graphical representation called the scree plot was used for further checks so as to extract the actual factors needed to explain the teachers' knowledge for teaching algebra at the SHS level in Ghana. The scree plot was used because according to Cattel (1966) it helps reduce the number of factors from items in an instrument which is being used by the researcher.

Seven factors per the nature of the scree plot that emerged were eventually retained for final analysis. This was done because, according to

Child (1970) as; Kim & Muellar (1978), Norasis (1990) and DeVellis (1991) the factors which are to be retained are those which lie before the point at which the corresponding eigenvalues seem to cut or lend off. However, one may argue that from the scree plot eight factors could have been extracted to make a case. Nevertheless, on the eighth factor only one item loaded uniquely on it which defeats the concept of factor retention hence the seven factors. The graph below, Figure 4, shows the graphical representation needed for this scree-test.

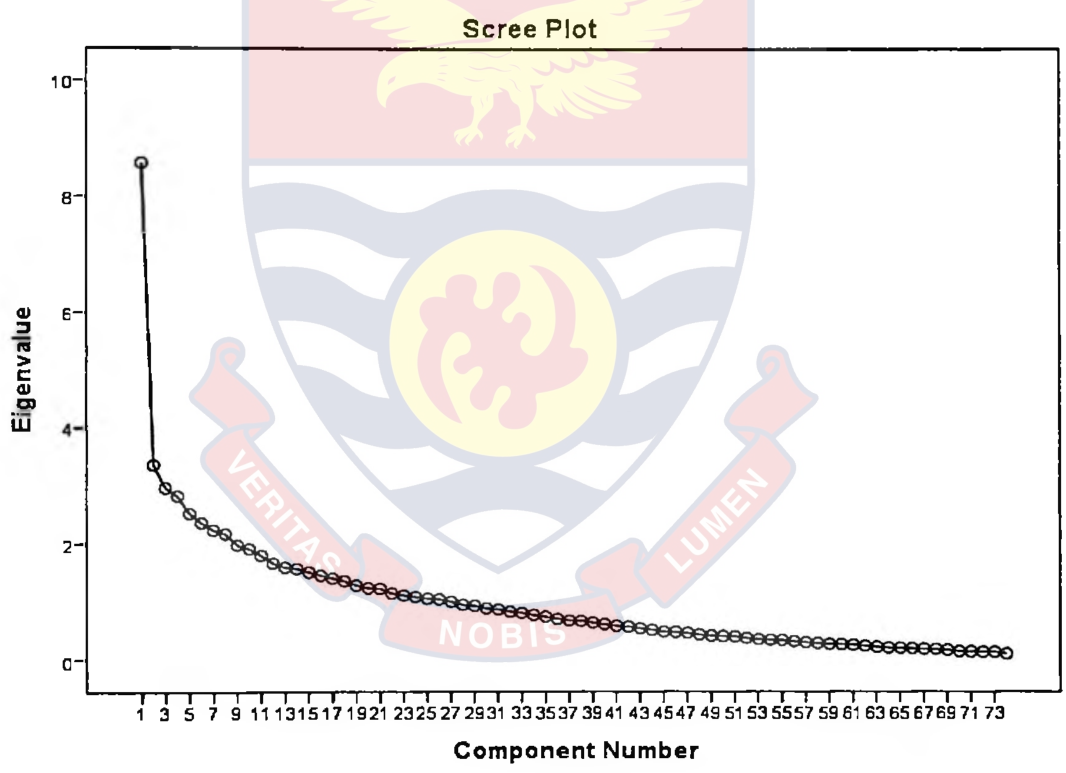


Figure 4: Scree plot showing number of factors retained

However, a critical look at the scree plot in Figure 4 reveals more than three factors as stipulated in the KAT framework. It could be realized from the scree plot once again that the number of factors needed to explain the variation in scores in the data is either seven or eight. This subjective interpretation of the scree plot, however, is what led to not using only eight factors as a way of explaining the variations. The scree plot in Figure 4 shows that after factor seven a variation in the slope of the graph begins. This is to say that there happens to be a relative variation in the steepness of the graph after the seventh factor. Hence, from the scree plot only seven factors were retained for further analysis. It was deduced from the factor analysis that only seven factors (see Figure 3) were required for the analysis. Though, the number of factors hypothesized were three per the KAT framework, this work had gone further to suggest that seven factors exist. In the KAT framework as cited by Wilmot (2008), and as indicated in Figure 3, the knowledge that existed between the intersections of the interlocking circles was hitherto described by the researchers of the KAT project as fuzzy (Wilmot, 2008, p. 51) may not be fuzzy after all. However, this work refutes the assertion and established that those intersections of the interlocking circles are not fuzzy. After a thorough examination, some colleague researchers were consulted and advised that only items which were loading on only one factor be considered for final analysis hence the seven factors used. Also, most of the items which had cross loadings happened to load weak on factor eight hence it deletion. In all 27 items were deleted (that is those with cross loadings) leaving 47 for final analysis.

The next output from the factor analysis using SPSS that was considered was the table of communalities as shown in Table 3 below. Table

3 shows how much of the variance in the item responses have been accounted for by the extracted factors. A high value in the table of communalities simply meant that the items had a lot of characteristics in common and low values would mean that the items have little in common with each other. Table 3 indicates the communalities of items on the instrument used.

Table 3: *Communalities of Items on Instrument Used*

Item No.	Initial	Extraction
q1	1.000	.673
q2	1.000	.739
q3	1.000	.646
q4	1.000	.635
q5	1.000	.782
q6	1.000	.725
q7	1.000	.725
q8	1.000	.768
q9	1.000	.795
q10	1.000	.729
q11	1.000	.662
q12	1.000	.666
q13	1.000	.685
q14	1.000	.809
q15	1.000	.760
q16	1.000	.744
q17	1.000	.756
q18	1.000	.725
q19	1.000	.653
q20	1.000	.684
q21	1.000	.741

Table 3 Cont'd

Item No.	Initial	Extraction
q22	1.000	.689
q23	1.000	.625
q24	1.000	.697
q25	1.000	.620
q26	1.000	.715
q27	1.000	.707
q28	1.000	.743
q29	1.000	.704
q30	1.000	.670
q31	1.000	.711
q32	1.000	.721
q33	1.000	.734
q34	1.000	.742
q35	1.000	.740
q36	1.000	.630
q37	1.000	.661
q38	1.000	.699
q39	1.000	.635
q40	1.000	.734
q41	1.000	.728
q42	1.000	.763

Table 3 *Cont'd*

Item No.	Initial	Extraction
q43	1.000	.704
q44	1.000	.774
q45	1.000	.728
q46	1.000	.718
q47	1.000	.658
q48	1.000	.762
q49	1.000	.670
q50	1.000	.570
q51	1.000	.742
q52	1.000	.645
q53	1.000	.600
q54	1.000	.698
q55	1.000	.742
q56	1.000	.729
q57	1.000	.747
q58	1.000	.775
q59	1.000	.599
q60	1.000	.740
q61	1.000	.749
q62	1.000	.775
q63	1.000	.721

Table 3 *Cont'd*

Item No.	Initial	Extraction
q64	1.000	.747
q65	1.000	.722
q66	1.000	.710
q67	1.000	.610
q68	1.000	.669
q69	1.000	.737
q70	1.000	.612
q71	1.000	.721
q72	1.000	.700
q73	1.000	.742
q74	1.000	.781

Extraction Method: Principal Component Analysis

From Table 3, the loadings as exhibited correspond to the correlation between the items and their factors. Each item in the table was examined for the factor on which the variable loads (that is where the correlation is greatest) by using the absolute values of the loadings. After this, the types of items or variables loading strongly on a particular factor was defined or labeled based on the characteristics. To aid in the interpretation of the seven components that were retained, Varimax rotation was performed. The rotated solution as indicated in Table 4 revealed the presence of simple structure (Thurstone, 1947) with the seven factors or components indicating a number of strong loadings. Table 4 indicates that fifteen items loaded substantially on only factor one whereas eight and five items loaded substantially on factors two and three respectively. Also, items four, five, five and five loaded on factors 4, 5, 6 and 7 respectively. These seven factors explained a total of 33.525% of the variance, with the first factor or variable contributing 11.586%, factor two contributed 4.531% whereas factor three contributed 3.996% of the variance respectively (see Table 2 for total variance explained by each factor). Table 4 below shows the item and their corresponding factors that loaded.

Table 4: *Factor Analysis showing loadings of each item*

Item	Components						
No.	1	2	3	4	5	6	7
q35	.640						
q18	.631						
*q33	.624				.324		
q20	.618						
*q28	.601					.303	
q30	.601						
q57	.551						
q39	.525						
q32	.512						
q66	.506						
q63	.485						
*q29	.432				.342		
q74	.416						
q31	.384						
*q53	.383			.357			
q23	.343						

Table 4 *Cont'd*

Item	Components						
No.	1	2	3	4	5	6	7
q7	.339						
q46	.332						
q3	.309						
q40		.595					
q24		.588					
q17		.555					
q27		.534					
q34		.515					
*q12		.502	-.300				
q59		.423					
*q58	.304	.420					
q14		.377					
q1		.341					
q52							
q45							

Table 4 *Cont'd*

Item No.	Components 1	2	3	4	5	6	7
q62			.654				
*q71			.462	-.437			
q73			.459				
q47			.418				
*q25	.313		.413				
*q19	.314		.406				
q9			.403				
q48			.328				
q56							
q51				.642			
q49				.638			
*q67	.320			.502			
q10				-.456			
q70				.311			
*q55	.303			.310			
q13							
q38					.606		
q4					.497		
q54					-.470		
*q26		.335			.380		.360

Table 4 *Cont'd*

Item	Components						
No.	1	2	3	4	5	6	7
q16					-.348		
q2					.305		
q68							
q72							
q69							
q50						.568	
q61						.531	
q36						.400	
q37						.368	
q65						.316	
q5							
q64							
q6							
q8							
q43							.579
*q41						.353	.531
*q15	.457						.475
*q21	.362					-.311	.397
q11							.386
q42							-.334

Table 4 *Cont'd*

Item	Components						
No.	1	2	3	4	5	6	7
*q44	.304						.334
q22							-.324
q60							.302

The factor Analysis showing loading of each item revealed that seven factors could be extracted after the exploratory factor analysis which explained teachers' knowledge for teaching algebra in the Ghanaian context. In all, it was noted that fifteen items which were related or have the same characteristics loaded uniquely on the first factor. Also, eight, five, four, five, five and five items respectively loaded on factors two, three, four, five, six and seven. However, these factors were labeled as School Algebra Knowledge, Advanced Algebra Knowledge, Algebra Teaching Knowledge, Algebra Knowledge, Advanced Algebra Teaching Knowledge, School Algebra Teaching Knowledge and Algebra Pedagogical Content Knowledge were used based on the characteristics of the items that loaded on each factor.

A critical examination of Table 4 revealed that there were three items that had a high loading on the first factor which has to do with School Algebra Knowledge that describes teachers' knowledge for teaching algebra. This means that the School Algebra Knowledge which is one of the factors determining teachers' knowledge for teaching algebra is quite high. Under this factor, 15 items loaded substantially on it with the highest loading being item 35 which was

"When is this statement true?

The opposite of a number is less than the original number.

A. This statement is never true.

B. This statement is always true.

C. This statement is true for positive numbers.

D. This statement is true for negative numbers.

This was followed by items 18 and 20 with factor loadings of .631 and .618 respectively. This presupposes that senior high school mathematics teachers as a matter of fact had realized how valuable and critical it is for them to possess the School Algebra Knowledge in teaching the integrated kind of mathematics in the Ghanaian context.

With regards to the second factor the highest loading is item 40 which had a factor loading of .595 and read as

'In a first year elective mathematics class, which of the following is **NOT** an appropriate way to introduce the concept of slope of a line?

A. Talk about the rate of change of a graph of a line on an interval.

B. Talk about speed as distance divided by time.

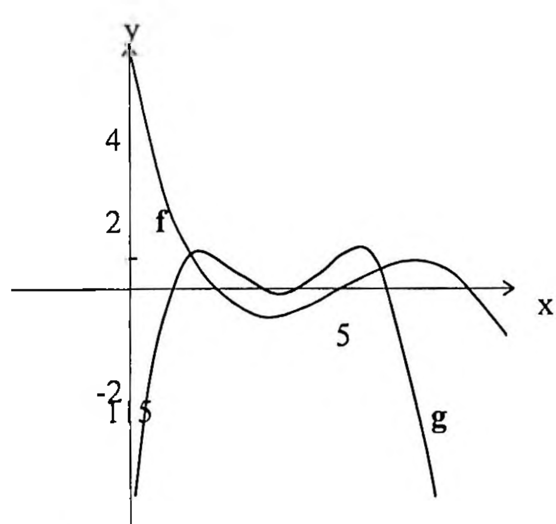
C. Toss a ball in the air and use a motion detector to graph its trajectory.

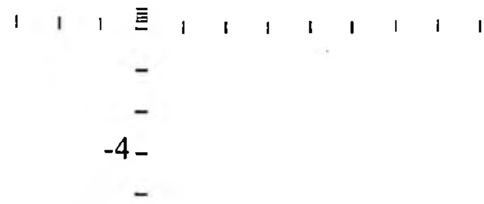
D. Apply the formula $slope = \frac{rise}{run}$ to several points in the plane.

E. Discuss the meaning of m in the graphs of several equations of the form $y = mx + b$.

This item loaded on the *Profound Algebra Knowledge* for teaching which means that in the course of teaching teachers need this kind of knowledge which occurs as a result of their ability to combine their school algebra knowledge and that of their advanced algebra knowledge to explain key concepts of bother to students during the instructional process.

On the third factor (*advanced algebra knowledge*) the item with the highest loading has to do with "The graphs of two real-valued functions, f and g, are shown below. Each mark on the axes represents one unit.





How many solutions does the equation $f^3 - 2f^2g + fg^2 = 0$ have on the interval [0, 8]?

A. 2

B. 3

C. 5

D. 6

E. 7

This item had the highest loading (0.654) which indicates that SHS mathematics teachers yet need this kind of knowledge in dispensing knowledge to students. This kind of knowledge was simply referred to include other mathematical knowledge, in particular college level mathematics, which gives a teacher perspective on the trajectory and growth of mathematical ideas beyond school algebra" (Ferrini- Mundy, Senk & McCrory, 2005, p.1). Also, it is hoped that any mathematics teacher who possesses this type of knowledge would hold quite a respectable knowledge of the path of the content of school mathematics. One other important reason for a teacher to possess such knowledge is that it would allow them to engage in making networks across topics, eliminating difficulties while retaining integrity and unzipping of the content of school algebra to learners; practices that could be vivacious to effective teaching.

Conclusion Related to Research Question One

The preceding analysis and results revealed that knowledge for teaching algebra in the Ghanaian context corroborate the three knowledge types hypothesized in the KAT framework. This results was in contrast to what Wilmot (2008) found that knowledge for teaching algebra by high school mathematics teachers in the Ghanaian settings does not corroborate the knowledge types hypothesized in the KAT framework. Also, further analysis and results revealed that knowledge for teaching algebra in the Ghanaian context is seven and not three as asserted by the KAT project team. These factors can be retained because they explain teachers' knowledge for teaching algebra by senior high school mathematics teachers hence those factors. Though, seven factors were retained based on the elbow of the scree-plot, it does not necessarily mean that these are the only factors needed to explain knowledge needed by prospective and in-service mathematics teachers in teaching algebra knowledge possessed at the SHS level. In other words, it means that there are other factors apart from those that have been retained in this research piece to be explaining teachers' knowledge for teaching algebra at the SHS level. These seven factors that were retained happen to be the factors that explained SHS teachers' knowledge for teaching algebra which explained approximately 33.525% of the variance. Furthermore, it can be concluded that data on the knowledge for teaching algebra of SHS mathematics teachers confirms that those interlocking regions that were described by researchers in the KAT project as fuzzy after all is not. Also, the regression line reposed on the scree-plot indicates that Ghanaian high school mathematics teachers' knowledge for teaching algebra corroborates the KAT framework.

This is also an indication that factors obtained from the factor analysis have items loading onto them could have measured characteristics that had some similar traits among them. The outcome of the communalities indicated that what the factors have in common are quite large (see communalities in Table 2). One would have expected that items measuring the same construct would load together on a particular factor. However, the item loadings in this study revealed that most of the items measuring the same construct loaded together except in the case of three items. For instance item 66 which loaded on Factor one (that is School Knowledge) was originally a Teaching Knowledge item. This presupposes that respondents in the study saw the item to be more of School Knowledge than Teaching Knowledge. Item 66 was structured as follows:

66. Currently, Germany has a law against creating new surnames for newborns by combining the parents' surnames with hyphens. A language expert explains why hyphenation is not a good idea for naming:

If a double-named boy grew up to marry and have children with a double-named woman, those children could have four names, and their *children could have eight, and their children could have 16… The bureaucracy shudders.*

(Excerpt from the front page of *The Wall Street Journal*, Wednesday, October 12, 2005)

For which of the following topics could the situation described by the expert be used as an introduction?

A. Direct variations

B. Linear functions

C. Quadratic functions

D. Exponential growth

Also, item 47 which was labeled as School Knowledge question turned out to load substantially on the Advanced Knowledge domain. This simply means that Ghanaian high school teachers who were involved in this study did not in any way see this item as a School Knowledge item but rather an advanced knowledge item based on their experience and knowledge. This item was phrased as follows:

47. Which of the following can be represented by areas of rectangles?

i. The equivalence of fractions and percents, e.g. $\frac{3}{5} = 60\%$

ii. The distributive property of multiplication over addition: For all real numbers a, b, and c, we have $a(b + c) = ab + ac$

iii. The expansion of the square of a binomial: $(a + b)^2 = a^2 + 2ab + b^2$

A. ii only

B. i and ii only

C. i and iii only

D. ii and iii only

E. i, ii, and iii

In addition, item 10 which was also categorized originally as a School Knowledge item ended up loading substantially as a Teaching Knowledge item. This was a bit surprising because this item was clearly a School Knowledge item because per the definition or description of the School Knowledge by the experts in the KAT project, this knowledge was defined as the knowledge of mathematics in the intended curriculum of middle school and high school and it is the content of school algebra that teachers are expected to help students discover or learn in their algebra classes (Wilmot, 2008). This kind of knowledge was purported to be in various states' standards, textbooks and other instructional resources used in the schools. In Ghana, the content of this type of knowledge base is built-in both the Core and Elective Mathematics Syllabuses which is taking by students at the SHS level. It becomes evidently clear that for teachers to impact students learning, the teachers themselves need to understand the content of school algebra since students at that level are expected to learn such. The item, however, was put in this form:

10. A carpet installer decides to replace carpets in some offices on a university campus and uses the formula Cost = 350 + 1.6A, where

"A" is the number of square feet of carpet to be replaced, to determine the cost. In how large an office can the carpet be replaced for ¢9,600.00?

A. 1666.25ft^2

B. 1656.25ft^2

C. 1656.15ft^2

D. 1656.05ft^2

E. 1606.05ft2

These confirms that knowledge for teaching algebra in the Ghanaian context may be said to be seven as revealed by this study and not three as put forward by the researchers in the KAT project.

Also, factor analysis established, by way of the seven factors that emerged that the interlocking regions (see Figure 4) as perpetuated by the KAT project team to be fuzzy is not after all so. The implication of this is that high school mathematics teachers in Ghana in the course of teaching combine two or more knowledge types to be able to respond appropriately to students' questions that may arise or come up in the course of teaching. To this far it was concluded that the new framework for algebra knowledge for teaching can be put in this form as shown in Figure 5.

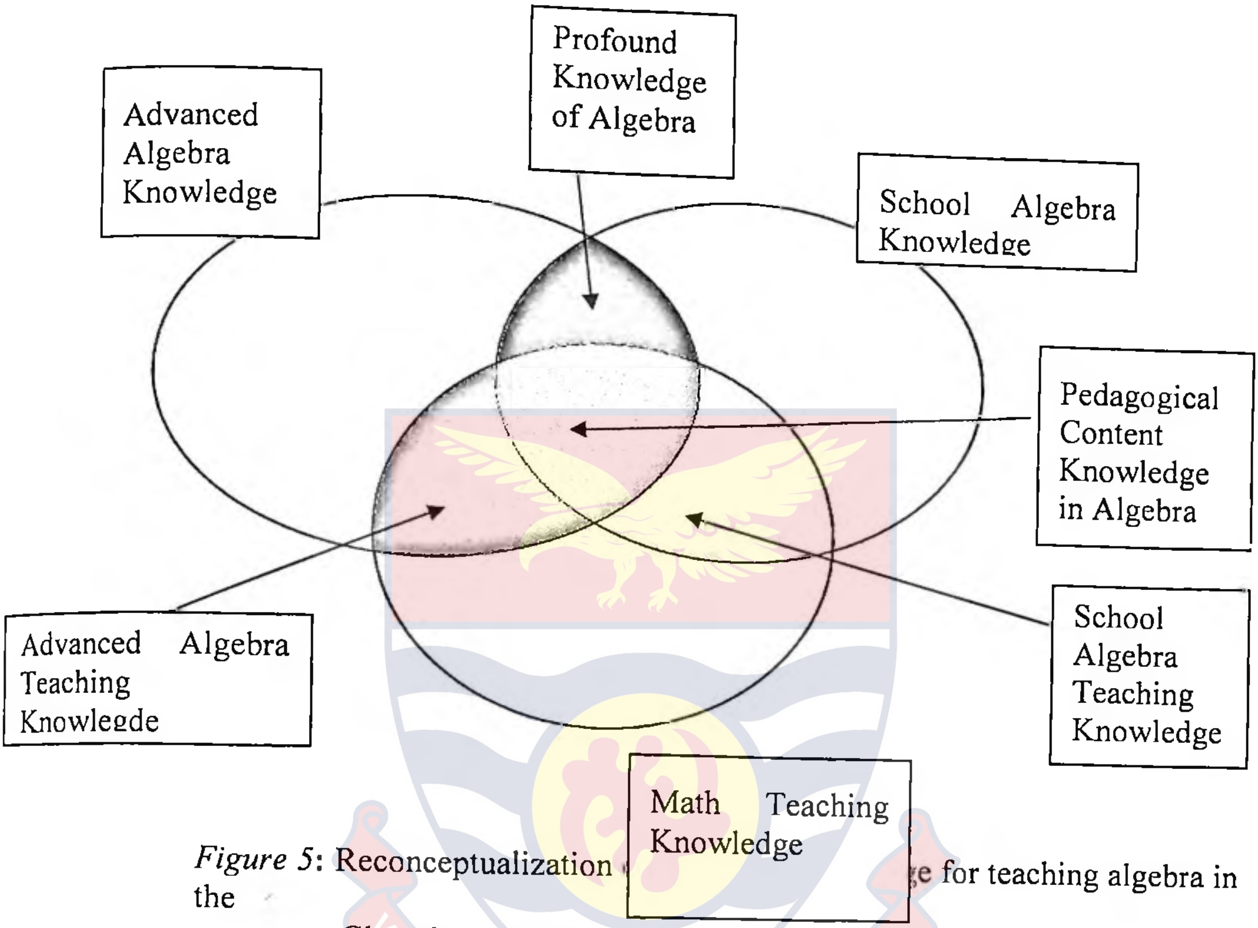


Figure 5: Reconceptualization ge for teaching algebra in the Ghanaian context developed in this study

These other knowledge types indicated in Figure 4 have been operationally defined and explained as follows:

Profound Knowledge of School Algebra

For this type of knowledge, it is basically the teachers' ability to hold alternative definitions, extensions and generalizations of familiar theorems, and application of a comprehensive range of high school algebra. It also entails content that comes before school algebra as well as those that advance it. As a matter of fact, any teacher who is said to possess this type of knowledge is seen to have a deep understanding of school algebra.

School Algebra Teaching Knowledge

In the case of this type of knowledge, teachers who possess it are said to have a good knowledge of the trajectory of school algebra. To a large extent, this type of knowledge allows instructors of mathematics to teach algebra in an unsolidified manner which in a way heighten understanding of heterogeneous groups of students. This type of knowledge help teachers in the course of teaching to make connections across the various school algebra topics; remove complication that may ensue while maintaining some level of reliability and unpack complexity to make the content being taught more comprehensive.

Advanced Algebra Teaching Knowledge

In line with the conceptualization of School Algebra Teaching Knowledge, that of the Advanced Algebra Teaching Knowledge is the type of knowledge that empowers instructors of mathematics to make appropriate connections across various topics in advanced algebra courses. In the case of this knowledge, instructors who have it are able to engage in appropriate pruning and breakdown when it becomes necessary for them to teach advanced algebra courses. It must be noted that teachers who possess this kind of knowledge only do not have a good understanding of same but also have a good understanding of advanced algebra.

Pedagogical Content Knowledge in Algebra

Shulman (1986b), initially conceptualized this type of knowledge and defined it as involving a complex combination of some form of content and pedagogy. Nevertheless, Shulman's conceptualization is a generalized form of knowledge whereas conceptualization in this study is a domain specific type of knowledge which emphasize algebra.

Research Question Two

The second research question that guided this study was, "What is the algebra knowledge teachers at the SHS possess in teaching mathematics? To answer this research question data gathered from in-service mathematics teachers were used. Only data from in-service teachers were used because with the prospective mathematics teachers they are still in school and are yet to fully get into teaching at the various designated SHS in Ghana.

The results of the analysis (as shown in Table 5) indicates that SHS in-service mathematics teachers who participated in the study from the various regions and schools involved in this study have relatively high knowledge for teaching algebra in their respective schools. The mean and standard deviation scores are shown in Table 5.

Table 5: *Descriptive statistics of in-service mathematics teachers' knowledge in algebra at SHS level*

Category of teacher	N	Mean	Std. Deviation	Std. Error Mean
In-service	153	.3739	.14314	.01157

The results in Table 5 indicated that although SHS teachers were sampled from various school-type they exhibited evidence of algebra mathematical ability at the level in which they teach. This result was not surprising because per the kind of questions they were given it was expected they perform to expectation. The overall subscale mean and standard deviation obtained are mean = .3739 and standard deviation = .1431 as presented in Table 5. On the whole, this result indicates that mathematics teachers at the SHS level have what it takes to teach the students at that level.

Though teachers showed evidence of mathematical knowledge for teaching algebra, teachers' performance on some of the items leaves much to be desired. For instance, item 27 was one of the items of interest that was commented on because this item happens to load on *school knowledge* of teachers who were involved in the study.

Here is the item:

27. One pipe can fill a tank in 20 minutes, while another takes 30 minutes to fill the same tank. How long would it take the two pipes together to fill the tank?

A. 50 min

B. 25 min

C. 15 min

D. 12 min

Amazingly, most in-service SHS mathematics teachers who answered this question had it wrong and that was a bit disturbing. Figure 6 shows the performance of teachers on this item.

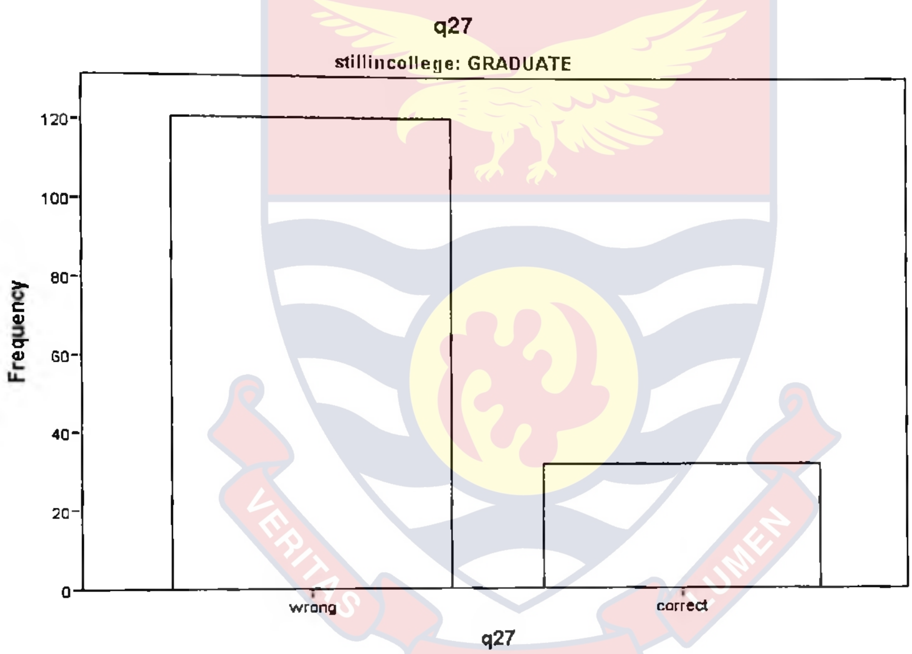


Figure 6: Responses of in-service teachers on question 26

This question as said earlier on loaded as a school knowledge item and for that matter in-service mathematics teachers were expected to perform better on it. Like it was mentioned in Chapter two, the *school knowledge* was simply seen as the knowledge of mathematics in the intended curriculum of middle school and high school and this is the content of school algebra that teachers are expected to help students discover or learn in their algebra classes (Wilmot, 2008). According to Wilmot (2008), investigators in the KAT project restricted this type of knowledge by reviewing content standards of ten different states in the US. However, in Ghana, the content of this type of knowledge base is built-in both the Core and Elective Mathematics Syllabuses which is taken by students at the SHS level. This then presupposes that for teachers to impact students learning, the teachers themselves need to understand the content of school algebra since students at that level are expected to taught such hence expectation teachers' to would have performed better on this item. Unfortunately, a cursory look at Figure 5 shows that 79.1% (121 out of 153) of in-service mathematics teachers had the item wrong.

This item as a matter of fact focused on finding out whether teachers can apply algebraic techniques to solve rate problems. There was only one correct answer to this question. Majority of in-service mathematics teachers who attempted this question had it wrong (see Figure 5). The item though quite simple, proved that in-service mathematics teachers who participated in this study lack the conceptual and procedural understanding of how to go about question involving rate problems. In a way it also shows that teachers could not do critical analysis of the question to realize that if two pipes separately filled that tank at the 20min and 30min respectively then it would take two pipes filling at the same time less time to do that. Majority of teachers selected options A and C. It could mean that teachers in representing such cases algebraically made a mistake and hence the wrong answer. From these two options chosen by most respondents, especially with those who selected option A, it clearly shows that they did not do due diligence to the question. In other jurisdiction, it can be said that conceptually majority of teachers who participated in this study do not understand or might have forgotten rate problems at the time of data collection.

Also, another item of interest which loaded on the *algebra pedagogical content knowledge* that was discussed is item 29. The item was structured as follows:

29. Four steps to derive the quadratic formula are shown below:

$i.\; x^2 + \frac{bx}{a} = \frac{-c}{a}$

$ii.\; \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$

$iii.\; x = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} - \frac{b}{2a}$

$iv.\; x^2 + \frac{bx}{a} + \left(\frac{b}{2a}\right)^2 = \frac{-c}{a} + \left(\frac{b}{2a}\right)^2$

What is the correct order for these steps?

A. I, iv, ii, iii

B. I, iii, iv, ii

C. Ii, iv, I, iii

D. Ii, iii, I, iv

Basically, this question tried to find out from teachers their procedural and conceptual understanding of quadratic formula as indicated above. Also, it was to ascertain whether in-service mathematics teachers involved in this study would be able to lay bare and identify without any problems the various steps involved in arriving at the "almighty formula".

It was worth noting that most of the in-service mathematics teachers who took part in this study proved that they have strong procedural and conceptual understanding of the said question with few of them falling apart. Figure 7 indicates the responses of in-service mathematics teachers on question 29.

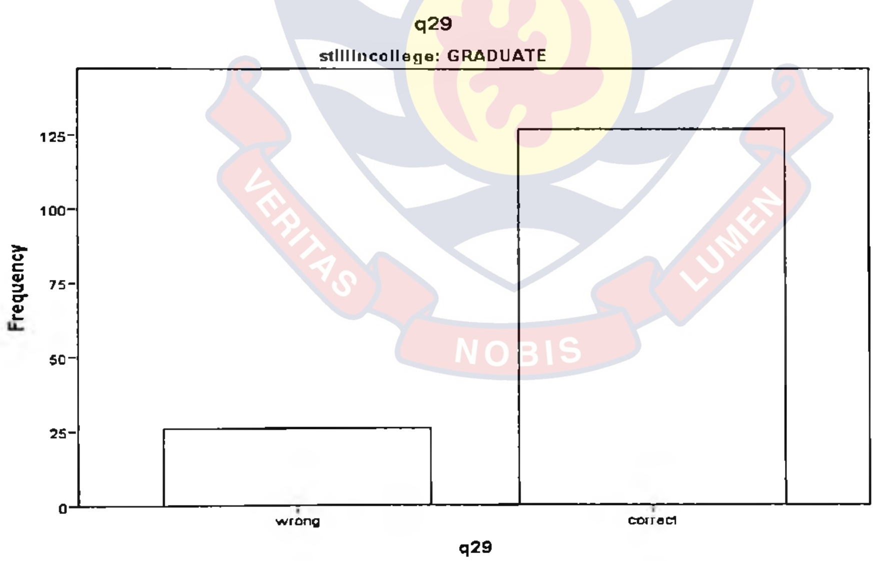


Figure 7: Responses of in-service teachers on question 29

A look at Figure 7 shows that 127 (83.0%) of in-service teachers who responded to this question had it correct with only 17% getting it wrong. This indicates that most of the teachers who responded to this question are well vexed in quadratics. This result was not astonishing because out of the 153 in-service mathematics teachers who responded to this question, 90 (72.0%) of them are teachers with education background. With this number of teachers with education background, it would have been highly unacceptable if majority of them had had the question wrong because they have gone through the requisite methodological training that seeks to empower them in this direction.

In addition, item 35 which sought to find out in-service teachers' knowledge on how to use properties of numbers to demonstrate whether a particular assertion is true or not and which loaded on the *advanced knowledge* component was discussed. The question was structured as follows:

35. When is this statement true?

The opposite of a number is less than the original number.

A. This statement is never true.

B. This statement is always true.

C. This statement is true for positive numbers.

D. This statement is true for negative numbers.

There was only one correct response to this question. Though majority of the respondent had this question correct, quite a number of them as well had it wrong. The only correct option in this question is option "C". Figure 8 indicates the rate of responses by the in-service mathematics teachers on this item.

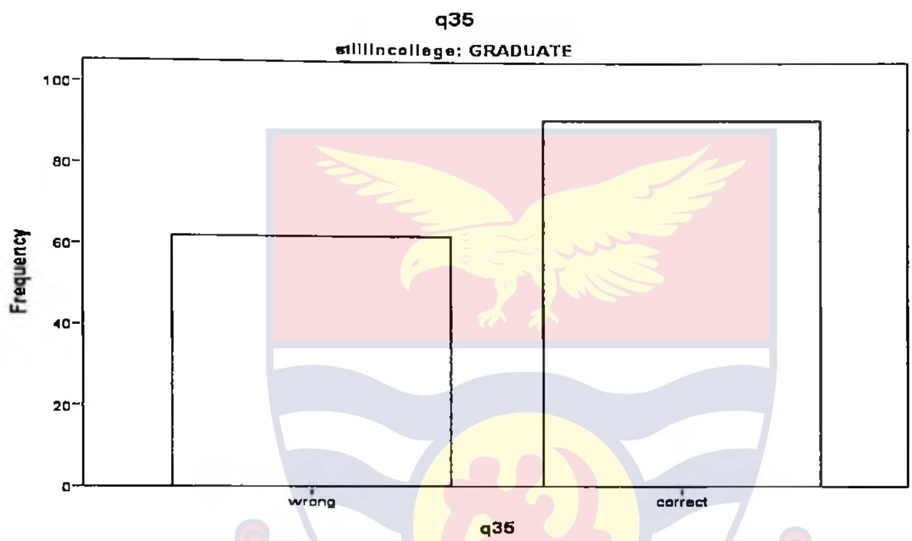


Figure 8: Responses of teachers on item 35

A cursory look at Figure 8 reveals that 91 (59.5%) out of 153 of the in-service mathematics teachers had the question correct. Though this is quite encouraging, however, almost about 50.0% of the respondents had the item wrong. This as a matter of fact is very disturbing because looking at the nature of the question which requires teachers to show through their response to the said question on how to use properties of numbers to demonstrate whether a particular assertion is true or not leaves much to be desired.

This study went further to analyze question item 39. This item loaded on the *teaching knowledge* component and it tries to find out from in-service mathematics teachers how to allay the fears of students on how a particular question is supposed to be solved. The question in a way tries to find out from teachers how to use the quadratic formula to find the roots of a second degree polynomial and to solve quadratic equations. The question was posed this way:

39. Susan was trying to solve the equation $2x^2 = 6x$.

First she divided both sides by 2.

$$x^2 = 3x$$

Then she divided both sides by x:

$$x = 3$$

Gustavo said, "You can't divide both sides by x." Susan responded, "If you can divide both sides by 2, why can't you divide by x?" They asked their teacher to explain.

Which of the following explanations is correct?

A. Since x is a variable it can vary, you may not be dividing both sides by the same number.

B. You can't cancel x because it does not represent a real number.

C. You can only divide by whole numbers when solving equations.

D. It is better to take the square root of both sides after dividing by 2, that way you won't have to worry about dividing by x.

E. If you divide both sides by x, then you might be dividing by 0, and would miss the solution $x = 0$.

As stated earlier on, this type of question loaded on the *teaching knowledge* component. This type of knowledge as explained in Chapter two per the KAT framework as a type of knowledge according to Ferrini-Mundy et al. (2005, p.2) is termed as "knowledge that is precise to teaching algebra that may not be taught in advanced mathematics courses. It comprises such things as what makes a particular concept problematic to learn and what misconceptions lead to precise mathematical inaccuracies. It also contains mathematics required to identify mathematical goals, within and across lessons, to choose among algebraic tasks or texts, to select what to highlight with curricular paths in mind and to ratify other tasks of teaching". Consequently, this is the type of knowledge teachers possess which they apply in teaching the subject matter of school algebra.

Based on this definition, in-service mathematics teachers were expected to help these two students as per the question overcome their misconception. Figure 9 indicates the responses given by in-service mathematics teachers who were involved in this study.

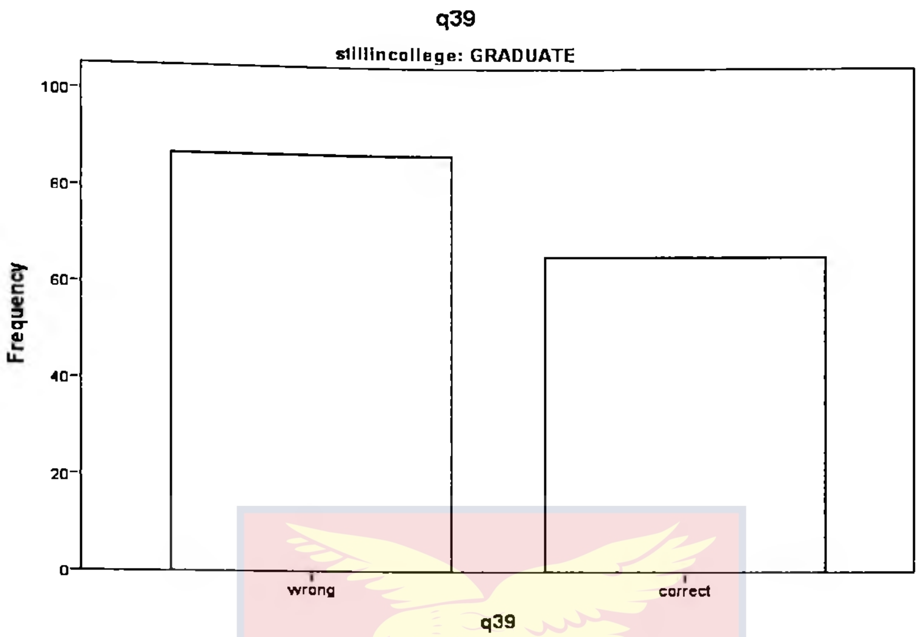


Figure 9: Responses of teachers on item 39

Figure 9 shows that majority of the in-service mathematics teachers (i.e. 87 out of 153 respondents) were unable to help these two students overcome their misconceptions and fears. This question had only one correct response but it saw some teachers picking two responses. Also most of the teachers ended choosing option "D" as their response. This means that the teaching knowledge possess by in-service mathematics teachers is quite questionable. Only 43.1% of the respondents responded accurately to the question in question.

A further analysis was conducted to ascertain the knowledge level of these teachers across the three knowledge types as hypothesized in the KAT framework. Table 6 indicates the mean and standard deviation scores across the three knowledge types.

Table 6: *Mean and Standard Deviation scores across the three knowledge types in algebra*

Category of teachers		N	Min.	Max.	Mean	Std. Dev.
In-service	SCHOOL KNOWLEDGE	153	.61	1.52	1.0688	.19002
	TEACHING KNOWLEDGE	153	.00	.88	.2369	.16232
	ADVANCED KNOWLEDGE	153	.00	.73	.3159	.17800

A careful look at Table 6 shows that in-service mathematics teachers' knowledge for teaching algebra with regards to the combined mean scores on the three types of knowledge indicates that the knowledge base of these teachers on the *school knowledge* is relatively better than that of the *teaching knowledge* and *advanced knowledge*. This revelation was quite surprising because per the KAT framework, the advanced knowledge type gives a teacher perspective on the trajectory and growth of mathematical ideas beyond school algebra" (Ferrini- Mundy, Senk & McCrory, 2005, p.1) whereas the teaching knowledge type is a kind of knowledge that is precise to teaching algebra that may not be taught in advanced mathematics courses.

It gives teachers a broader view on what makes a particular concept problematic to learn and what misconceptions lead to precise mathematical inaccuracies. Furthermore, the teaching knowledge type also contains mathematics that is required to identify mathematical goals, within and across lessons, to choose among algebraic tasks or texts, to select what to highlight with curricular paths in mind and to ratify other tasks of teaching. On this basis, it presupposes most teachers lack a substantial amount of knowledge with regards to the *teaching knowledge* and the *advanced knowledge* to be able to appropriately give the right kind of knowledge to students let alone lead them to success.

Conclusion Related to Second Research Question

Generally, in-service mathematics teachers who took part in this study showed evidence of mathematical understanding and knowledge of algebra they teach at the SHS level. This was not shocking since in-service teachers who participated have quite a number of books, exposure and are well vexed in the content at the SHS. Also, analysis of some specific items relating to *school knowledge, advanced knowledge and teaching knowledge* revealed a startling revelation. The results indicated that in-service mathematics teachers at the SHS as at the time this study was conducted have weak school knowledge and teaching knowledge. This was surprising because most of the in-service mathematics teachers have education background. It means that the procedural and conceptual knowledge possessed by these teachers are weak and for that matter at a critical moment most of these teachers may find it difficult to explain to their students what they the students are supposed to learn. This result confirms what Thompson and Thompson (1996) and Mewborn (2003) finding that teachers lack conceptual understanding of the mathematics they are to teach. In another breadth, the result of this study contradict Mewborn (2003) finding that teachers have strong procedural knowledge. Another revelation was that the combined mean scores on the three knowledge types indicated that in-service mathematics teachers' knowledge on the *advanced knowledge* and the *teaching knowledge* leaves much to be desired. The implications of this is that teachers at a point in time during the instructional period may not be able to conceptually and procedural help students in understanding the mathematics they teach.

Research Hypothesis One

The first research hypothesis that guided this study was, "There is no significant difference between the knowledge for teaching algebra of in-service and prospective mathematics teachers at the senior high school level?" To answer this question achievement test scores data from in-service and prospective mathematics teachers who participated in the study from the 40 SHSs were used. The two categories of teachers were assumed to be independent samples. Consequently, the Independent Samples t-test was performed on their total mean scores. This was chosen to test this hypothesis at 0.05 level of significance. The independent samples t-test was chosen because it aids to relate the mean scores on some continuous variable, in this case the test scores, for two different groups of subjects (in-service and prospective mathematics teachers). Also, analysis of variance (ANOVA) was conducted on the three knowledge types as hypothesized in the KAT framework between in-service and prospective mathematics teachers at the senior high school level. Nevertheless, before ANOVA was conducted on the mean scores across the three knowledge types a multivariate analysis of variance (MANOVA) was conducted to ascertain the combined effect of the difference between the two groups. Before proceeding to the interpretation of the result of the independent samples t-test, ANOVA and MANOVA results, descriptive statistics of the two groups are duly presented and discussed in Table 7.

Table 7: *Descriptive statistics of Algebra knowledge for teaching between in service and prospective mathematics teachers*

Category of teacher	N	Mean	Std. Deviation	Std. Error Mean
Prospective	99	.3591	.10466	.01052
In-service	153	.3739	.14314	.01157

A cursory look at Table 7 reveals that in general, the performance of the two groups of participants was quite encouraging. The results of the analysis (as shown in Table 7) indicates that both in-service and prospective mathematics teachers at the SHS who participated in the study as of the time when data was collected have somewhat same knowledge level for teaching algebra at the senior high school level. The mean and standard deviation scores of in-service mathematics teachers (M = .3739, SD = 0.1431) and that of prospective mathematics teachers (M = .3591, SD = 0.1047). The result, however, indicates that in-service teachers have slightly higher level of knowledge for teaching algebra than their prospective mathematics teacher counterparts. However, the overall mean and standard deviation scores (M = .3665, SD =.1239) shows that both groups in general have relatively a similar level of knowledge for teaching algebra at the senior high school level.

Table 8 indicates the independent samples t-test conducted to ascertain whether there is any significant difference in the knowledge for teaching algebra between senior high school in-service and prospective mathematics teachers.

Table 8: *Results of Independent Samples t-test of in-service and prospective mathematics teachers*

	Levene's Test for Equality of Variances		t-test for Equality of Means			
	F	Sig.	T	Df	p-value	Mean Difference
Equal variances assumed	5.368	.021	-.886	250	.376	-.01479
Equal variances not assumed			-.946	246.229	.345	-.01479

Table 8 which contains the results of the independent-samples t-test conducted to compare difference in knowledge for teaching algebra between in-service and prospective senior high school mathematics teachers revealed that there was no statistically significant difference between in-service mathematics teachers who teach at SHS level (M = .3739, SD = 0.1431) and that of prospective mathematics teachers (M = .3591, SD = 0.1047); t(250) = -.946, p = .345.

Though there was no statistically significant difference in knowledge for teaching algebra between in-service and prospective mathematics teachers' teaching at the high school level, a further analysis was conducted to find out whether any difference exist between these two groups on any of the three knowledge levels hypothesized by the KAT project team. Table 9 indicates results of the mean and standard deviation scores on the three knowledge types between the prospective and in-service mathematics teachers.

Table 9: *Mean and standard deviation scores on the three knowledge levels between prospective and in-service mathematics teachers*

Type of Knowledge	Type of teacher	Mean	Std. Deviation	N
SCHOOL KNOWLEDGE	Prospective	1.0637	.16153	99
	In-service	1.0688	.19002	153
	Total	1.0668	.17905	252
TEACHING KNOWLEDGE	Prospective	.2412	.15491	99
	In-service	.2369	.16232	153
	Total	.2386	.15915	252
ADVANCED KNOWLEDGE	Prospective	.2990	.14673	99
	In-service	.3159	.17800	153
	Total	.3093	.16632	252

A careful look at Table 9 shows that the overall mean and standard deviation scores (M = .3665, SD =.1239) generally indicates that the two groups have a relatively high level of knowledge for teaching algebra at the senior high school level. Nevertheless, analysis by analysis was made on the various knowledge types.

A critical look at Table 8 shows that on the *School Knowledge* both groups performed relatively well with a mean and standard deviation scores of (M =1.064 , SD = 0.162) and (M =1.069 , SD = 0.190) for prospective and in-service mathematics teachers respectively. This result was not surprising because in Chapter Two this kind of knowledge was defined as the knowledge of mathematics in the intended curriculum of middle school and high school. It basically the content of school algebra that teachers are expected to help students discover or teach in their algebra classes (Wilmot, 2008). Also it is not startling because these two groups know what is contained in the curriculum at that level and have also taught students and most of them still teaching at that level hence the performance.

With regards to the *teaching knowledge*, it basically focused on knowledge that is precise to teaching algebra that may not be taught the individual teacher in advanced mathematics courses. It's a type of knowledge that tend to look at things such as what makes a particular concept problematic to learn and what misconceptions lead to precise mathematical inaccuracies. The mean and standard deviation scores (M =0.241, SD = 0.155) and (M =0.27, SD = 0.162) for prospective and in-service mathematics teachers respectively leaves much to be desired. The results indicates that these two groups scored less than fifty percent on the two knowledge types. This result seem a bit problematic because it means that when it comes to algebra, teachers (both prospective and in-service) may have problem identifying students misconceptions as well as help students identify what makes a particular concept problematic to learn.

The *Advanced Knowledge* type simply focuses on what is referred to include other mathematical knowledge, in particular college level mathematics, which gives a teacher perspective on the trajectory and growth of mathematical ideas beyond school algebra" (Ferrini- Mundy et al., 2005, p.1). The mean and standard deviation scores for prospective and in-service mathematics teachers were (M =0.299, SD = 0.147) and (M =0.316, SD = 0.178) respectively. Though the results were not bad, the in-service mathematics teachers did relatively better than their colleague prospective mathematics teachers. This could mean that the content knowledge level of the prospective mathematics teachers is questionable and for that matter needs to be beefed up. For the in-service mathematics teachers, they did relatively better than their counterparts because it could be that some of them if not all keep upgrading themselves and reading more in their area of specialty that keeps them on top of their business. This results contradicts what Wilmot (2008) found that a statistically significant difference exist between in-service and pre-service mathematics teachers' knowledge for teaching algebra. It can therefore be concluded that knowledge for teaching algebra of in-service high school mathematics teachers is significantly similar to that of prospective teachers.

However, to find out whether there were differences in the knowledge for teaching algebra between these two groups, across the three knowledge types that was hypothesized in the KAT framework, One-way Multivariate Analysis of Variance (MANOVA) was conducted. This was done after a preliminary assumption test had been conducted to check for normality, linearity, univariate and multivariate outliers and homogeneity of variance-covariance matrices with no serious violation made or noted. There was no statistically significant difference between in-service and prospective mathematics teachers in the knowledge base across the three types of knowledge F (3,248) = .297, p = 0.827; Pillai's Trace = 0.004; partial eta squared = 0.004). This means that the population means scores on the three knowledge types of SHS mathematics teachers for teaching algebra are the same for both the prospective and in-service teachers.

A corresponding analysis of variance (ANOVA) with category of teachers as the independent variable was not conducted as a follow-up test to the MANOVA since there was no statistically significant difference. Table 10 indicates the results of the summary of MANOVA test conducted on the overall mean scores.

Table 10: *Summary of MANOVA test between In-service and Prospective Mathematics Teachers' knowledge for teaching algebra*

Knowledge Types		Value	F	Sig.	Partial Eta Squared
School Knowledge Teaching Knowledge Advanced Knowledge	Pillai's Trace	.004	.297[a]	.827	.004
	Wilks' Lambda	.996	.297[a]	.827	.004
	Hotelling's Trace	.004	.297[a]	.827	.004
	Roy's Largest Root	.004	.297[a]	.827	.004

A cursory look at Table 10 indicates that both prospective and in-service SHS mathematics teachers have relatively the same level of knowledge across the three knowledge types as used in this study.

As a matter of fact one expects that the in-service mathematics teachers could do better especially on the *teaching* and *advanced* knowledge being ascertained by this research. Unfortunately, it turned out after the analysis that no statistically significant difference was realized between the in-service and prospective mathematics teachers. This was a surprise because most of this in-service mathematics teachers involved in this study have been teaching for quite a number of years and for that matter expected to exhibit high level of knowledge with regards to the teaching and advanced knowledge.

This calls for concern because per the KAT project, *Advanced Knowledge of Mathematics* (or simply "Advanced Knowledge") was simply other mathematical knowledge, in particular college level mathematics, which gives the teacher at the level in question a perspective on the trajectory and growth of mathematical ideas beyond school algebra" (Ferrini- Mundy, Senk & McCrory, 2005, p.1). Areas such as number theory, abstract algebra, complex numbers, linear algebra, calculus, and mathematical modeling were listed as some of the general areas in the KAT project (see Ferrini-Mundy, McCrory, Senk, & Marcus, R, 2005).

It has become clear per Ferrini-Mundy et al. (2005) that this kind of knowledge is so relevant that the possession of it could make it possible for a teacher to make appropriate networks across topics whereas trying as much as possible to unload the complexity of a mathematics content to make that content more understandable by other people and in this case students at the SHS level. This kind of knowledge, also affords the mathematics teacher with unfathomable or profound understanding of school algebra. Furthermore, it is hoped that any mathematics teacher who owns this type of knowledge would hold quite a respectable knowledge of the path of the content of school mathematics. One other important reason for a teacher to possess such knowledge is that it would in them to engage in making networks across topics, eliminating difficulties while retaining integrity and unzipping of the content of school algebra to learners; practices that could be vivacious to effective teaching. All these as a matter of urgency should have brought about some sought of difference but unfortunately not. This means that in-service mathematics teachers are no better in making connections between and among topics let alone have the upper hand in eliminating difficulties whereas retaining integrity and unzipping the content of school algebra to learners than their novice prospective mathematics teachers.

Also, it was so surprising to note that there was no statistically significant difference between in-service and prospective mathematics teachers with regards to the *Teaching Knowledge*. It was expected that the in-service mathematics teachers perform better than their prospect mathematics teachers on this subscale. This is because this kind of knowledge in question talks about the teachers' ability to identify which particular concept is problematic to learn and what misconceptions lead to such mathematical inaccuracies. Furthermore, it is assumed to contain mathematics necessary to ascertain mathematical goals, within and across lessons, to select among algebraic tasks or texts, to select what to highlight with curricular paths in mind and to sanction other tasks of teaching.

The more worrying aspect of this is that this is a type of knowledge teachers possess which they apply in teaching the subject matter of school algebra. Furthermore, Ferrini-Mundy et al. (2005, p.1) in the KAT project mentioned that, "the knowledge been described here may fall into the kind of pedagogical content knowledge or it may be pure mathematical content applied to teaching". Also, this type of knowledge may not be taught in advanced mathematics courses, and may not essentially be accessible to mathematicians. Thus, this is the type of knowledge that could distinguish an engineer or a mathematician from an algebra teacher. In this light, it is quite surprising to realize that in-service teachers who were involved in this study and that of their prospective mathematics were seen to have same level of knowledge in this regard. With the vast number of teaching experience by these in-service teachers, it was expected that they exhibit more of this knowledge in question than their colleague prospective mathematics teachers. This presupposes that in-service mathematics teachers are not growing in their teaching knowledge and which means that it would be quite difficult at certain point in time for these in-service mathematics teachers to show a high level of teaching knowledge than the prospective teachers. It also means that their (in-service teachers) pointing out that a particular concept is problematic to learn and what misconceptions lead to such mathematical inaccuracies would be quite difficult.

Conclusion Related to Research Hypothesis One

The point that the independent-samples t-test result revealed no statistical significant difference in the mean scores of these two groups of teachers (both in-service and prospective mathematics teachers) shows that both groups of teachers displayed an evidence of mathematical understanding and knowledge of algebra they teach at the SHS level. This result was a bit confounding because it was expected that with the in-service teachers they could have done better than the prospective mathematics teachers because it is assumed that having taught the subject for quite some time, their level of mathematical knowledge would be above that of the prospective teachers but turned out to be different. This was also shocking since these in-service teachers have quite a number of books, exposure and are well vexed in the content at the SHS level than their prospective mathematics teacher counterparts. In addition, no statistically significant difference found could also mean that the prospective mathematics teachers are receiving better and higher order knowledge in this regard in their training. It could also mean that with regards to this prospective teachers, new courses have been added and programme restructured to meet current trends in teaching that has beefed up their knowledge level in the area. This result is in contrast to what Wilmot (2008) found that in-service high school mathematics teachers in

Ghana performed significantly better than each category of prospective mathematics teachers majoring in mathematics, mathematics education and statistics from the country's universities. His study went further to establish that knowledge for teaching algebra of in-service high school mathematics teachers was significantly different from that of prospective teachers.

Research Hypothesis Two

The second research hypothesis that gave focus to this study was, "There is no significant difference in the knowledge for teaching algebra between senior high school mathematics teachers with background training in education and their counterparts without education background''

To answer this research hypothesis, data collected from both in-service and pre-service mathematics teachers at the senior high schools in three regions were used. The research hypothesis basically compared scores of teachers in the schools involved in the study with education background and their colleagues without education background. To do this, an independent-samples t-test was conducted to help in comparing the mean scores on some continuous variable which in this case happens to be test scores obtained by these two groups of subjects (teachers with education and those without education background). In addition to this ANOVA was also conducted to ascertain if any difference exist between teachers with background training in education and their colleagues without background training in education. This hypothesis was tested at the 0.05 level of significance.

Also, items in the instrument were statistically analyzed to determine which of them were statistically significant. Table 11 shows the mean and standard deviation scores of the knowledge for teaching algebra between senior high school mathematics teachers with background training in education and their counterparts without education background

Table 11: *Descriptive Statistics of Teachers' Scores for teaching algebra between those with education background and those without background training in education*

Teachers' Degree Type	N	Mean Score	Standard Deviation	Standard Error Mean
EDUCATION	125	.3694	.13320	.01191
NONEDUCATION	127	.3669	.12600	.01118

The results of the analysis (as shown in Table 11) indicates that SHS teachers who participated in the study as of the time when data was collected have relatively same knowledge level for teaching algebra at the senior high school level. The mean and standard deviation scores of teachers with education background (M = .3694, SD = 0.1332) and those without education background (M = .3669, SD = 0.1260). The analysis, however, revealed that teachers without education background have slightly higher level of knowledge for teaching algebra than their counterparts with education background. This could be as a result of the fact that the difficulty level of some of the content level questions well answered by those without

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Table 12: *Results of Independent Samples t-test of teachers with education background and their counterparts without education background*

	Levene's Test for Equality of Variances		t-test for Equality of Means			
	F	Sig.	T	Df	p-value	Mean Difference
Equal variances assumed	.819	.366	.153	250	.879	.00249
Equal variances not assumed			.153	248.731	.879	.00249

Table 12 which contains the results of the independent-samples t-test conducted to compare difference in knowledge for teaching algebra between senior high school mathematics teachers with background training in education and their counterparts without education background revealed that there was no statistically significant difference between teachers with education background (M = .3694, SD = 0.1332) and their counterparts without education background (M = .3669, SD = 0.1260); t(250) = .153, p = .879. The magnitude of the difference in the means was very small [eta squared = 0.0094].

Though there was no statistically significant difference between the two groups, further analysis was conducted on the three knowledge levels hypothesized in the KAT project and which was confirmed in this study. Furthermore, analysis was conducted on the individual items to find out whether the two groups have any variation in terms of their knowledge for teaching algebra with regards to the various items. Table 13 indicates mean and standard deviation scores for the three knowledge levels confirmed from the KAT project.

Table 13: *Mean and Standard deviation scores on the three knowledge levels from the KAT project between teachers with Education and non-Education background*

Type of Knowledge	Degree Type	N	Mean	Std. Deviation	Std. Error Mean
SCHOOL	EDUCATION	125	1.0557	.18059	.01615
KNOWLEDGE	NON-EDUCATION	127	1.0760	.17809	.01580
TEACHING	EDUCATION	125	.2400	.17720	.01585
KNOWLEDGE	NON-EDUCATION	127	.2372	.13983	.01241
ADVANCED	EDUCATION	125	.3211	.18324	.01639
KNOWLEDGE	NON-EDUCATION	127	.2976	.14760	.01310

The results of the analysis (as indicated in Table 13) show that SHS teachers with education and non-education background who participated in this study as of the time when data was collected have relatively the same knowledge level for teaching algebra at the senior high school level across the three knowledge levels. The mean and standard deviation scores of teachers with education background were (M = 1.0557, SD = 0.18059), (M = 0.2400, SD = 0.1772) and (M = 0.3211, SD = 0.1832) for *School Knowledge, Teaching Knowledge and Advanced Knowledge* respectively whereas mean and standard deviation scores for those without education background are (M = 1.0760, SD = 0.1781), (M = .2372, SD = .1398), and (M = .2976 , SD =

0.1476) for the *School Knowledge, Teaching Knowledge and Advanced Knowledge* respectively. The analysis, however, revealed that teachers with education background have slightly higher level of knowledge for teaching algebra than their counterparts without education background on the *Teaching Knowledge and Advanced Knowledge*. This could be as a result of the fact that the difficulty level of some of the content level questions was well answered by them than their counterparts without education background. It could also mean that the teachers with education background had a better understanding and are well trained to handle questions as stipulated in the instrument than their counterparts without education background. This could also be as a result of their experiences, teaching and advanced knowledge in what they teach than their counterparts. This was not surprising because a couple of the teachers who were involved in the study and have education background have been teaching for a number years. Nevertheless, SHS mathematics teachers without education background performed slightly better than their counterparts with education background on the School Knowledge subscale. This may be because this group of mathematics teachers (teachers without education background) has been taught enough and tougher content than their colleague counterparts with education background. It must, however, be noted that this kind of knowledge is knowledge of mathematics in the intended curriculum of middle school and high school. This is the content of school algebra that these teachers are expected to help their students discover or learn in their mathematics classes (Wilmot, 2008). In Ghana, the content of this type of knowledge base is supposedly built-in both the Core and Elective Mathematics Syllabuses which is taken by students at the SHS level. It sounds realistic, however, to conjecture that for teachers to impact students learning, the teachers themselves need to understand the content of school algebra since students at that level are expected to learn such. And because teachers without background training in education do more content courses it was not surprising from the results of the analysis that they did better in terms of these type of knowledge than their colleague counterparts with education background. Table 8 indicates the results of the independent samples t-test on the three knowledge levels between teachers with education and non-education background.

To find out whether there were differences in the knowledge for teaching between these two groups, across the three knowledge types that was hypothesized in the KAT framework, One-way Multivariate Analysis of Variance (MANOVA) was conducted. This was done after a preliminary assumption test had been conducted to check for normality, linearity, univariate and multivariate outliers and homogeneity of variance-covariance matrices with no serious violation made or noted. The results of the analysis revealed that there was no statistically significant difference between teachers with education background and their counterparts without education background knowledge for teaching across the three knowledge types hypothesized by the KAT framework $F(3,248) = 1.334$, $p = 0.264$; Pillai's

Trace = 0.016; partial eta squared = 0.16). This means that the population means scores on the three knowledge types between teachers with education background and their counterparts without education background are the same for both groups. It also means that there no is statistically significant difference between the teachers with education background training and their counterparts without education background training per the KAT hypothesized knowledge types.

As a result of the no significant difference in the knowledge types, the corresponding analysis of variance (ANOVA) with background training in education as the independent variable was conducted (see Appendix B) but not discussed for each of the three knowledge types as a follow-up test to the MANOVA because no statistically significant difference was observed. This was because the combined effect showed no statistically significant difference between teachers with education background training and their counterparts without education background training per the KAT hypothesized knowledge types.

From the results discussed so far it can be infer that there was no statistically significant difference between teachers with education background and their counterparts without education background who teach mathematics at the senior high schools in the three regions where data was collected across the three knowledge types at the time of data collection. This indicates that these two groups of teachers possess relatively the same knowledge levels. It became necessary, however, to ascertain whether any significant difference exist between these two groups with regards to item by item analysis. Table 14 indicates the results of item by item analysis between teachers with education background and their counterparts without education background.

Table 14: *Mean and Standard deviation scores of responses by teachers with background training in education and their counterparts without education background*

	Teaching Certificate	N	Mean	Std. Deviation	p-value
q1	EDUCATION	125	.26	.438	.202
	NONEDUCATION	127	.19	.393	.202
q2	EDUCATION	125	.73	.447	.160
	NONEDUCATION	127	.65	.480	.160
q3	EDUCATION	125	.74	.443	.828
	NONEDUCATION	127	.75	.436	.828
q4	EDUCATION	125	.43	.497	.129
	NONEDUCATION	127	.34	.475	.129
q5	EDUCATION	125	.23	.424	.404
	NONEDUCATION	127	.19	.393	.404
q6	EDUCATION	125	.26	.443	.207
	NONEDUCATION	127	.20	.399	.207
q7	EDUCATION	125	.22	.413	.171
	NONEDUCATION	127	.29	.456	.171
q8	EDUCATION	125	.12	.326	.386
	NONEDUCATION	127	.09	.282	.386
q9	EDUCATION	125	.26	.438	.717
	NONEDUCATION	127	.24	.426	.717

Table 14 Cont'd

	Teaching Certificate	N	Mean	Std. Deviation	p-value
q10	EDUCATION	125	.21	.408	.097
	NONEDUCATION	127	.30	.460	.097
q11	EDUCATION	125	.59	.493	.058
	NONEDUCATION	127	.47	.501	.058
q12	EDUCATION	125	.38	.488	.722
	NONEDUCATION	127	.36	.483	.722
q13	EDUCATION	125	.17	.375	.186
	NONEDUCATION	127	.11	.314	.187
q14	EDUCATION	125	.27	.447	.734
	NONEDUCATION	127	.29	.456	.734
q15	EDUCATION	125	.50	.502	.710
	NONEDUCATION	127	.47	.501	.710
q16	EDUCATION	125	.11	.317	.480
	NONEDUCATION	127	.14	.350	.480
q17	EDUCATION	125	.34	.477	.824
	NONEDUCATION	127	.33	.472	.824
q18	EDUCATION	125	.61	.490	.721
	NONEDUCATION	127	.63	.485	.721

Table 14 *Cont'd*

	Teaching Certificate	N	Mean	Std. Deviation	p-value
q19	EDUCATION	125	.42	.496	.787
	NONEDUCATION	127	.44	.498	.787
q20	EDUCATION	125	.78	.413	.595
	NONEDUCATION	127	.81	.393	.595
q21	EDUCATION	125	.56	.498	.697
	NONEDUCATION	127	.54	.501	.697
q22	EDUCATION	125	.11	.317	.122
	NONEDUCATION	127	.18	.387	.122
q23	EDUCATION	125	.32	.468	.482
	NONEDUCATION	127	.36	.483	.482
q24	EDUCATION	125	.42	.495	.097
	NONEDUCATION	127	.31	.466	.097
q25	EDUCATION	125	.66	.477	.190
	NONEDUCATION	127	.73	.445	.190
q26	EDUCATION	125	.45	.499	.130
	NONEDUCATION	127	.35	.480	.130

Table 14 *Cont'd*

	Teaching Certificate	N	Mean	Std. Deviation	p-value
q27	EDUCATION	125	.19	.395	.825
	NONEDUCATION	127	.18	.387	.825
q28	EDUCATION	125	.53	.501	.537
	NONEDUCATION	127	.57	.497	.537
q29	EDUCATION	125	.84	.368	.223
	NONEDUCATION	127	.78	.416	.223
q30	EDUCATION	125	.85	.360	.537
	NONEDUCATION	127	.82	.387	.537
q31	EDUCATION	125	.78	.413	.489
	NONEDUCATION	127	.82	.387	.490
q32	EDUCATION	125	.70	.462	.851
	NONEDUCATION	127	.69	.466	.851
q33	EDUCATION	125	.74	.438	.610
	NONEDUCATION	127	.77	.421	.610
q34	EDUCATION	125	.49	.502	.903
	NONEDUCATION	127	.48	.502	.903
q35	EDUCATION	125	.60	.492	.627
	NONEDUCATION	127	.63	.485	.627
q36	EDUCATION	125	.19	.395	.825
	NONEDUCATION	127	.18	.387	.825

Table 14 *Cont'd*

	Teaching Certificate	N	Mean	Std. Deviation	p-value
q37	EDUCATION	125	.36	.482	.926
	NONEDUCATION	127	.35	.480	.926
q38	EDUCATION	125	.25	.434	.331
	NONEDUCATION	127	.20	.399	.331
q39	EDUCATION	125	.38	.488	.074
	NONEDUCATION	127	.50	.502	.074
q40	EDUCATION	125	.43	.497	.384
	NONEDUCATION	127	.38	.487	.384
q41	EDUCATION	125	.22	.413	.948
	NONEDUCATION	127	.21	.411	.948
q42	EDUCATION	125	.06	.231	.474
	NONEDUCATION	127	.08	.270	.473
q43	EDUCATION	125	.32	.468	.405
	NONEDUCATION	127	.37	.485	.405
q44	EDUCATION	125	.29	.455	.846
	NONEDUCATION	127	.30	.460	.846
q45	EDUCATION	125	.13	.335	.290
	NONEDUCATION	127	.09	.282	.291

Table 14 *Cont'd*

	Teaching Certificate	N	Mean	Std. Deviation	p-value
q46	EDUCATION	125	.27	.447	.719
	NONEDUCATION	127	.25	.436	.719
q47	EDUCATION	125	.16	.368	.223
	NONEDUCATION	127	.22	.416	.223
q48	EDUCATION	125	.37	.484	.586
	NONEDUCATION	127	.40	.492	.586
q49	EDUCATION	125	.33	.471	.624
	NONEDUCATION	127	.30	.460	.624
q50	EDUCATION	125	.20	.402	.692
	NONEDUCATION	127	.22	.416	.691
q51	EDUCATION	125	.50	.502	.044**
	NONEDUCATION	127	.37	.485	.044
q52	EDUCATION	125	.21	.408	.810
	NONEDUCATION	127	.22	.416	.810
q53	EDUCATION	125	.60	.492	.919
	NONEDUCATION	127	.61	.491	.919

Table 14 *Cont'd*

	Teaching Certificate	N	Mean	Std. Deviation	p-value
q54	EDUCATION	125	.32	.468	.482
	NONEDUCATION	127	.36	.483	.482
q55	EDUCATION	125	.38	.486	.378
	NONEDUCATION	127	.32	.469	.378
q56	EDUCATION	125	.38	.486	.588
	NONEDUCATION	127	.41	.494	.588
q57	EDUCATION	125	.53	.501	.022**
	NONEDUCATION	127	.67	.472	.022
q58	EDUCATION	125	.43	.497	.458
	NONEDUCATION	127	.39	.489	.458
q59	EDUCATION	125	.36	.482	.004**
	NONEDUCATION	127	.20	.399	.004
q60	EDUCATION	125	.30	.462	.474
	NONEDUCATION	127	.35	.478	.474
q61	EDUCATION	125	.18	.382	.563
	NONEDUCATION	127	.20	.405	.563
q62	EDUCATION	125	.13	.335	.140
	NONEDUCATION	127	.20	.399	.139
q63	EDUCATION	125	.36	.482	.071
	NONEDUCATION	127	.47	.501	.071

Table 14 *Cont'd*

	Teaching Certificate	N	Mean	Std. Deviation	p-value
q64	EDUCATION	125	.17	.375	.691
	NONEDUCATION	127	.15	.358	.691
q65	EDUCATION	125	.38	.486	.066
	NONEDUCATION	127	.27	.445	.066
q66	EDUCATION	125	.38	.488	.244
	NONEDUCATION	127	.46	.500	.244
q67	EDUCATION	125	.54	.500	.909
	NONEDUCATION	127	.55	.499	.909
q68	EDUCATION	125	.14	.344	.304
	NONEDUCATION	127	.09	.294	.304
q69	EDUCATION	125	.08	.272	.971
	NONEDUCATION	127	.08	.270	.971
q70	EDUCATION	125	.22	.419	.399
	NONEDUCATION	127	.18	.387	.399
q71	EDUCATION	125	.10	.306	.279
	NONEDUCATION	127	.15	.358	.278
q72	EDUCATION	125	.17	.375	.186
	NONEDUCATION	127	.11	.314	.187
q73	EDUCATION	125	.39	.490	.510
	NONEDUCATION	127	.43	.497	.510
q74	EDUCATION	125	.28	.451	.128
	NONEDUCATION	127	.37	.485	.128

A cursory look at Table 14 indicates that three items (that is items 51, 57 and 59) showed statistically significant difference between teachers with education background and their counterparts without education background. These three items were some of the items of interest in this study looking at the nature of it.

For instance, item 51 which was one of the items to ascertain teachers' knowledge for teaching algebra happens to be like this:

51. Mr. Nkrumah asked his algebra students to divide $x^2 - 4$ by $x + 2$. Eric said, "I have an easy method, Mr. Nkrumah. I just divide the x^2 by x and the 4 by the 2. I get $x - 2$, which is correct." Mr. Nkrumah is not surprised by this as he had seen students do this before. What did he know? (Mark one answer.)

A. He knew that Eric's method was wrong, even though he happened to get the right answer for this problem.

B. He knew that Eric's answer was actually wrong.

C. He knew that Eric's method was right, but that for many algebraic fraction division problems this would produce a messy answer.

D. He knew that Eric's method only works for some algebraic fractions.

E. I'm not sure.

This item as a matter of fact focused on addressing simplifying of algebraic expressions, specifically the simplification of the rational expression (x2 – 4)/(x + 2). There was only one correct answer to the question. However, in some cases response in option D may work out. In other words, there are two responses that one could consider correct to this particular question or situation. If the sole focus of the question was about the method for correctness in all situations, only option "A" would be considered correct. Nevertheless, the wording of options given in the question with regards to "D" shows that there are certain situations in which Eric's method would work. This particular technique, however, works only in a very few special circumstances. Instances can be given for which the technique would work for Eric, but there are many counterexamples for which that particular technique may not work. Also, further analysis was done to ascertain how many teachers with education background had this question right or wrong as well as how many teachers without education background had same question right or wrong. Table 15 shows the distribution of responses on this item.

Table 15: *Distribution of responses by education and non-education teachers' knowledge for teaching algebra on item 51*

Certificate	Response	Frequency	Percent
EDUCATION	Wrong	63	50.4
	correct	62	49.6
	Total	125	100.0
NONEDUCATION	Wrong	80	63.0
	correct	47	37.0
	Total	127	100.0

A critical look at Table 15 indicates that out of the 125 teachers with education background 50.4% had the item wrong which was a bit worrying because it was envisage that with their background they should have been able to get the correctly. Also, 63.0% of teachers with non-education background had the item wrong which was far worse than their counterparts with education background. The item though quite simple, proved that teachers who participated in this study lack the conceptual understanding of how to go about this question. In a way it also shows that teachers could not critical analysis the question to realize that there are several counterexamples to the solution of Eric.

In all, majority of the teachers who participated in the study could not simplify the algebraic fraction using the appropriate procedural method. In addition, teachers could not in their minds indicate that Eric and for that matter students do not know the correct procedure for the simplification of this given algebraic expression. In a nutshell, majority of these participants at critical moment may not be able to give explanations that would help students conceptually understand why the method they employ as student would or would not work.

Also, item 57 on the same instrument measuring teachers' knowledge for teaching algebra indicated a statistically significant difference between teachers with education background and those without education background. This item of interest was captured like this:

57. If $f(x) = ax^3 + bx^2 + cx + d$, what is the slope of the line tangent to this curve at $x = 2$?

A. $8a + 4b + 2c$

B. $8a + 4b + 2c + d$

C. $12a + 4b + c$

D. $12a + 4b + c + d$

This item was basically structured to measure teachers' Advanced Knowledge in Algebra. Relatively teachers who participated in this the study exhibited a great sense of content knowledge for teaching algebra with those without education background doing better than their counterpart with education background. This result as a matter of fact wasn't surprising since content wise the former is more grounded than the previous. Table 16 indicates the distribution of responses by teachers with education background and their counterparts without education background.

Table 16: *Distribution of responses by education and non-education teachers knowledge for teaching algebra on item 57*

Certificate	Response	Frequency	Percent
EDUCATION	Wrong	59	47.2
	correct	66	52.8
	Total	125	100.0
NONEDUCATION	Wrong	42	33.1
	correct	85	66.9
	Total	127	100.0

A look at Table 16 indicates that generally both group of teachers did relatively well on this item which loaded on the advanced knowledge factor. This results indicate that to a larger extent both teachers are well prepared when it comes to the content they teach students at the level for which they operate to assist them understand the needed content. It is also envisaged that those without education background are likely to do this better than their counterparts with education.

Another item that showed statistical significance was item 59. This item loaded on the *Algebra Teaching Knowledge*. In the framework, this type of knowledge according to Ferrini-Mundy et al (2005, p.2) is termed as "knowledge that is precise to teaching algebra that may not be taught in advanced mathematics courses. It comprises such things as what makes a particular concept problematic to learn and what misconceptions lead to precise mathematical inaccuracies. It also contains mathematics required to identify mathematical goals, within and across lessons, to choose among algebraic tasks or texts, to select what to highlight with curricular paths in mind and to ratify other tasks of teaching".

Consequently, this is the type of knowledge teachers possess which they apply in teaching the subject matter of school algebra. The question in this context tried to ascertain from teachers involved in the study how they could help students who are faced with expanding a polynomial involving three variables of degree two.

The question reads like this:

59. Which of the following (taken by itself) would give substantial help to a student who wants to expand $(x+y+z)^2$?

i. See what happens in an example, such as $(3+4+5)^2$.

ii. Use $(x+y+z)^2 = ((x+y)+z)^2$ and the expansion of $(a+b)^2$.

iii. Use the geometric model shown below.

x	y	z
x^2	xy	xz
xy	y^2	yz
xz	yz	z^2

A. ii only

B. iii only

C. i and ii only

D. ii and iii only

E. i, ii and iii

The question as a matter of urgency focused on addressing expansion of polynomial algebraic expression in three variables, specifically the expansion of degree two of the form $(x+y+z)^2$. There was only one correct answer to the question. It was quite alarming to know that majority (182 out of 252) of the teachers who took part in this study had the item wrong with barely 70 getting it correct. Table 17 shows the distribution of responses of teachers who responded to this item

Table 17: *Distribution of responses by education and non-education teachers knowledge for teaching algebra on item 59*

Certificate	Response	Frequency	Percent
EDUCATION	Wrong	80	64.0
	correct	45	36.0
	Total	125	100.0
NONEDUCATION	Wrong	102	80.3
	correct	25	19.7
	Total	127	100.0

A careful look at Table 17 indicates that most of the teachers performed poorly on this item with especially those without education background. A second look at the table indicates that only 36.0% (45) and 19.7% (25) teachers with education and those without education background respectively had the question right. This result is quite discouraging because once again it shows that most of the teachers at the SHS lack good knowledge in problem solving as well as what it takes for them to identify that a particular concept is problematic to learn and what misconceptions lead to those precise mathematical inaccuracies. It also means that most of the teachers who participated in this study lack the requisite knowledge that contains mathematics required to identify mathematical goals, within and across lessons, to choose among algebraic tasks or texts, to select what to highlight with curricular paths in mind and to ratify other tasks of teaching. It also means that the type of knowledge teachers need to possess which they apply in teaching the subject matter of school algebra is questionable.

Conclusion Related to Hypothesis Two

The fact that the independent-samples t-test, MANOVA and ANOVA results showed no significance in the mean scores of these two groups of teachers (those with education background and their counterparts without education background) shows that both groups of teachers displayed an evidence of mathematical understanding and knowledge of algebra they teach at the second cycle level. This result was not startling as both groups were estimated to perform similarly on the test having been teaching at the same level and same content to students at that level. Nevertheless, literature is replete with the fact that for effective teaching to occur specifically in mathematics both conceptual and procedural knowledge are essential, for that matter, those tutors with training background in education should accomplish or do better superiorly. It might happen that those with background training in education felt buoyant in their knowledge and have been restricted to what they previously know. On the contrary, it is likely that those without background training in education sensed the need to upgrading their knowledge by way of reading or learning other courses to bring their knowledge to a level that had posh their performance confidently.

Also, analysis of some specific items that showed statistical significant difference revealed that for most of the teachers the type of knowledge they needed to possess which they apply in teaching the subject matter of school algebra is questionable. In addition, majority of these participants it came to light that at critical moment may not be able to give explanations that would help students conceptually understand why the method they employ as student would or would not work.

Research Hypothesis Three

The third research hypothesis that guided this study was, "There is no significant difference between knowledge possess by teachers with different teaching experience for teaching algebra at the senior high school level". To correctly answer this research hypothesis, data from both in-service and prospective mathematics teachers who teach mathematics at the SHS level were used. The research hypothesis specifically compared scores of teachers in the schools involved in the study who teach mathematics at the time of the study. To accomplish this, Analysis of Variance was conducted to help in comparing the mean scores on some continuous variable which in the domain of this research hypothesis happens to be test scores obtained by these group of teachers with varying teaching experience. This hypothesis was tested at the 0.05 level of significance. Table 18 shows the mean and standard deviation scores of the knowledge for teaching algebra possessed by teachers with different teaching experience for teaching algebra at the senior high school level.

Table 18: *Mean and standard deviation scores of teachers with varying teaching experience*

Years of teaching experience		N	Mean	Std. Deviation	Std. Error
0-5yrs		181	.3513	.12203	.00907
6-10yrs		37	.3944	.13974	.02297
more than 10yrs		34	.4288	.13656	.02342
Total		252	.3681	.12937	.00815
Model	Fixed Effects			.12673	.00798
	Random Effects				.03011

A cursory look at Table 18 indicates that all the teachers with the varying teaching experience possess quite a substantial amount of algebra knowledge to teach at the level in which they are currently teaching. It was revealed in the analysis that teachers with 6-10 years and 10 years and above teaching experience possess higher and better knowledge than their colleagues with a lower teaching experience. Analysis conducted revealed that inexperience, moderately experienced and experienced teachers involved in the study were 181, 37 and 34 respectively. The mean and standard deviation scores recorded by these three groups are Mean= .3513, Sd = .1220 , Mean= .3944, Sd = .1397, and Mean = .4288, Sd = .1366 for inexperience, moderately experienced and experienced teachers respectively. A critical look at the mean scores indicates that the mean scores were different for each category of teachers. A look at the mean scores of the various categories reveals that algebra knowledge possessed by these teachers increase from inexperience to experience teachers. This assertion is evident in the mean plot of the category of experience displayed in Figure 9.

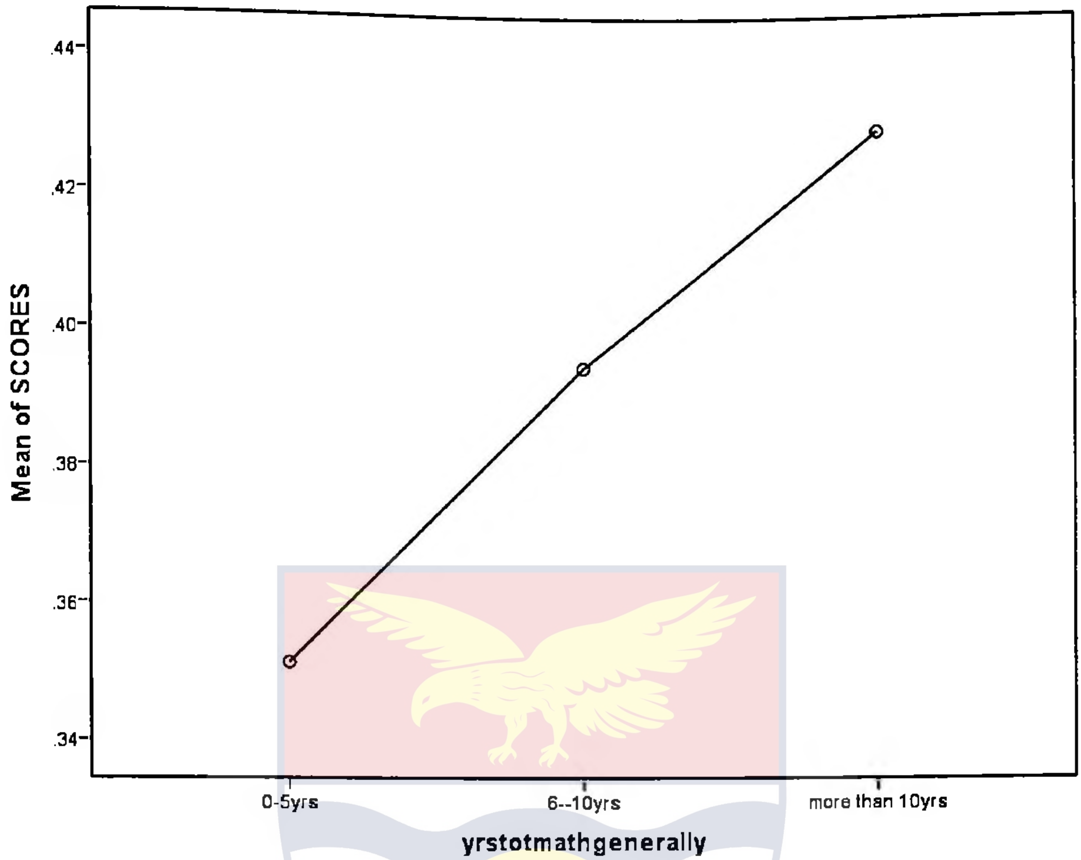

Figure 10: Mean plot of teachers Scores in terms of teaching experience

Preliminary analyses were first conducted to ascertain the appropriateness of the statistical test used. First, an analysis of variance (ANOVA) test was conducted to find out whether there was any statistical significant difference between the three categories of teachers teaching experience at 0.05 levels. This preliminary test was to provide the basis for a meaningful statistical test using the three categories of teaching experience as units. Thus combining all the teachers with similar teaching experience across the various schools and using the mean score to compare each other. The means scores of teachers with different levels of teaching experience are presented in Table 18.

One of the assumptions that must be met before an ANOVA test is conducted is that the variability of scores for each group is similar. This assumption was tested using the Levene test for equality of variance. Generally, several books mention that if the sig. value in the output table of the Levene test is less than 0.05 then an inference is made that the group variances are significantly different at 0.05 level (that is the group variances are not equal), and therefore the assumption of homogeneity of variance is violated. Alternatively, if the sig. value obtained is greater than 0.05, it suggests that variances for the two groups are equal. The result of the Levene test on the total scores is presented in Table 19.

Table 19: *Test of Homogeneity of Variance of Teachers Scores*

Levene Statistic	df1	df2	Sig.
.181	2	249	.834

From the outcome of the homogeneity test, it was observed that the sig. value was greater than 0.05. This suggests that the group variances were not significantly different (thus the group variances were approximately equal). The assumption of equality of variance was therefore satisfied. To investigate whether the group mean scores of the various teaching experiences were significantly different, the ANOVA test was then applied. The results of the ANOVA test are presented in Table 20.

Table 20: *ANOVA Table for Mean Differences in Categories of teaching experience*

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	.202	2	.101	6.287	.002
Within Groups	3.999	249	.016		
Total	4.201	251			

It can be observed that the sig. value (0.002) is less than 0.05 level of significance indicating that there is a significant difference across the categories of teaching experience as alluded to earlier. It was therefore concluded that there seems to be adequate evidence that there is a significant difference in the mean scores for at least one pair of category of teaching experience of teachers who teach at the SHS with regards to the algebra knowledge possessed. However, the ANOVA (see Appendix C) test does not reveal exactly where the differences among the groups occur. In view of that, Tukey's HSD Test was conducted to examine which pairs of groups have significant difference between groups mean scores at the 0.05 level of significance. Tukey' HSD in this case was appropriate because it presents results of multiple comparisons among all the different pairings, in this instance, of the three different categories of teaching experience. In Table 21, the result of the multiple comparisons is presented.

Table 21: *Multiple Comparisons of Differences in Teachers Teaching Experience*

(I) years of teaching	(J) years of teaching	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	Upper Bound
0-5yrs	6--10yrs	-.04312	.02286	.145	-.0970	.0108
	more than 10yrs	**-.07749***	.02369	.003	-.1333	-.0216
6--10yrs	0-5yrs	.04312	.02286	.145	-.0108	.0970
	more than 10yrs	-.03437	.03011	.489	-.1054	.0366
more than 10yrs	0-5yrs	**.07749***	.02369	.003	.0216	.1333
	6--10yrs	.03437	.03011	.489	-.0366	.1054

*. The mean difference is significant at the 0.05 level.

The post hoc results in Table 21 shows that the mean scores for teachers with teaching experience from 0-5 years [inexperience teachers] (M= .3513, SD = .1220) was significantly lower than the mean scores of teachers with teaching experience ranging from 6-10 years [moderately experienced teachers] (M= .3944, SD = .1397) and teachers with more than 10 years teaching experience [experienced teachers] (M = .4288, SD = .1366). Also, the mean score of teachers with teaching experience from 6-10 years (M= .3944, SD = .1397) was significantly higher than the mean score of teachers with teaching experience from 0-5 years (M= .3513, SD = .1220) but lower than teachers with more than 10 years teaching experience. Furthermore, the mean score of teachers with more than 10 years teaching experience (M = .4288, SD = .1366) was significantly higher than the mean score of teachers with teaching experience ranging from 0-5 years and 6-10 years respectively.

A further and final test was conducted by lamping together the mean scores of teachers with teaching experience ranging from 0-5 years and that of 6-10 years and compared to the mean scores of teachers with more than 10 years teaching experience to ascertain whether there would be any difference. This analysis follows from the result of the multiple comparisons in Table 19. An independent t-test was conducted to authenticate the variance between the two groups. Table 22 indicates the result of the t-test performed.

Table 22: *Test of differences in Teachers Mean score between two groups of teaching experience in algebra*

	Levene's Test for Equality of Variances		t-test for Equality of Means				
	F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference
Equal variances assumed	.060	.806	-3.062	249	.002	-.07293	.02381
Equal variances not assumed			-2.866	40.517	.007	-.07293	.02544

A cursory look at Table 22 indicates that the sig. value under the Levene test is greater than 0.05 signifying that the variances for the two groups are roughly equal. The p-value for the t-test was less than 0.005 (p= 0.002). This result presupposes that the mean score of the teachers with more than 10 years teaching experience was significantly higher than the mean score of teachers with teaching experience which ranges from 0-5 years and 6-10 years teaching experience combined. This means that teachers with over 10 years teaching experience did better on the achievement test in algebra than their counterparts with less teaching experience. In practice, if the saying that 'experience is the best teacher' holds, then one would have expected a similar result. But when the effect size was calculated to have a sense of the magnitude of the difference, it became apparent that the difference in achievement between the two groups of teachers was very small (eta squared 0.036).

Conclusion Related to Research Hypothesis Three

It is a common knowledge that in most professional jurisdiction, staff years of experience is considered as a germane factor in human resource policies, including compensation systems, benefits packages, and promotion decision. The main reason is that experience gained over time as a matter of fact enhances the knowledge, skills, and productivity of staff or workers. According to Ladd (2008), teachers with over 20 years teaching experience are more effective than their counterparts with no or less teaching experience. He further stated that these teachers, however, are not much more effective than those with 5 years of teaching experience. Though the results indicates a significant difference between these categories of teachers (that is inexperience, less experience and experienced teachers) the magnitude of the difference in mean was very small (eta squared 0.036). Though experience matters, however, more does not always mean better. This presupposes that after a certain point in time, the teachers' years of teaching experiences deteriorate in what is termed as diminishing returns. In other words, teachers turned to relax at the peak of their teaching. It could be deduced that this non-significant finding in terms of the magnitude of results, was as a result of the assertion that teachers grow tired of learning and that the benefit of experience begins to level off after about five years (Rosenholz, 1986). It is clear from the analysis that the success of many teachers as well as students seems to be reliant on factors other than the teachers' years of teaching.

However, another conclusion could be drawn in the sense that teachers with more teaching experience are better than their counterparts with less teaching experience when it comes to algebra knowledge for teaching. This conclusion is, however, in agreement with a wide range of findings on the relationship between years of teaching experience and students' achievement. The findings have been that teachers with more years of teaching experience are more effective than inexperienced teachers (Klecker, 2002; Rowan, 2002; Rosenholz, 1986). In precise terms, Greenwald, Hedges, and Laine, (1996) found a strong positive relationship between teacher experience and students outcomes. Rowan (2002) also found a significant effect of teaching experience on reading and mathematics outcomes in elementary school. "It is therefore certain that students would not benefit much from learning, where teachers are not competent" (Adeyemi, 2010, p 318). It means therefore that experience counts a lot in teaching because effective teaching strategies are learned on the job. No wonder a summary of a number of researches, have found teachers teaching experience to be one of the determinants of students' achievement (Bodenhausen, 1988; Felter, 1999 & Klecker, 2002). All these findings are in agreement to the finding that emerged in this study regarding teaching experience.

Research Hypothesis Four

The fourth research hypothesis that guided this study was, "There is no significant difference in the knowledge for teaching algebra between mathematics teachers who teach in urban areas and their counterparts in the rural settings based on the knowledge types in the KAT framework". To appropriately respond to this research hypothesis, data from both in-service and pre-service mathematics teachers who teach at these areas were used. The research hypothesis specifically compared scores of teachers in the schools involved in the study who teach at both rural and urban settings. To accomplish this, an independent-samples t-test was conducted to help in comparing the mean scores on some continuous variable which in the domain of this research hypothesis happens to be test scores obtained by these two groups of subjects (teachers who teach at rural settings and their counterparts in the urban settings). Also, further analysis was conducted using multivariate analysis of variance (MANOVA) and analysis of variance (ANOVA) as a follow up test to the MANOVA to ascertain where the difference is coming from with regards to the three knowledge types. This hypothesis was tested at the 0.05 level of significance.

Also, items in the instrument were statistically analyzed to determine which of them were statistically significant. Table 23 shows the mean and standard deviation scores of the knowledge for teaching algebra between senior high school mathematics teachers who teach at rural settings and their counterparts at the urban settings.

Table 23: *Mean and Standard Deviation scores of mathematics achievement test in algebra for teachers teaching at urban and rural settings*

Setting	N	Mean	Std. Deviation	Std. Error Mean
Urban	135	.3760	.14670	.01263
Rural	45	.3510	.11783	.01756

The results of the analysis (as shown in Table 23) reveals that SHS teachers who participated in the study as of the time when data was collected and teaches at rural and urban settings have comparatively same knowledge level for teaching algebra at the senior high schools they teach. The mean and standard deviation scores of teachers at the urban settings (M = .3760, SD = 0.1467) and those rural settings (M = .3510, SD = 0.1178). The analysis, nonetheless, revealed that teachers who teach at the urban settings have slightly higher level of knowledge for teaching algebra than their counterpart's who teach at rural settings. However, the overall mean and standard deviation scores (M = .3635, SD =.1323) shows that both groups in general have a relatively high level of knowledge for teaching algebra at their various settings.

Table 24 indicates the independent samples t-test conducted to ascertain whether there is any significant difference in the knowledge for teaching algebra between senior high school mathematics teachers who teach at urban settings and their counterparts at the rural settings.

Table 24: *Results of Independent Samples t-test of teachers who teach at urban settings and their counterparts who teach at rural settings*

	Levene's Test for Equality of Variances		t-test for Equality of Means			
	F	Sig.	T	Df	p-value	Mean Difference
Equal variances assumed	2.937	.088	1.037	178	.301	.0250
Equal variances not assumed			1.157	93.06	.250	.0250

Table 24 which contains the results of the independent-samples t-test conducted to compare difference in knowledge for teaching algebra between senior high school mathematics teachers who teach at urban settings and their counterparts who teach at rural settings revealed that there was no statistically significant difference between teachers who teach at urban settings (M = .3760, SD = 0.1467) and their counterparts at the rural settings (M = .3510, SD = 0.1178); t(178) = 1.037, p = .301.

In addition, to find out whether there was any difference in the knowledge for teaching between these two groups, across the three knowledge types that was hypothesized in the KAT framework, One-way Multivariate Analysis of Variance (MANOVA) was conducted. This was done after a preliminary assumption test had been conducted to check for normality, linearity, univariate and multivariate outliers and homogeneity of variance-covariance matrices with no serious violation made or noted. The results of the analysis revealed that there was no statistically significant difference between teachers who teach at the rural setting and their counterparts at the urban settings across the three knowledge types hypothesized by the KAT framework $F(3,248) = 0.593$, $p = 0.736$; Pillai's Trace = 0.014; partial eta squared = 0.007). This means that the population means scores on the three knowledge types between teachers who teach at the rural setting and their counterparts at the urban settings across the three knowledge types are relatively similar.

As a result of the no significant difference in the knowledge types, the corresponding analysis of variance (ANOVA) with background training in education as the independent variable was conducted (see Appendix C) but not discussed for each of the three knowledge types as a follow-up test to the MANOVA because no statistically significant difference was observed. This was because the combined effect showed no statistically significant difference between the two groups per the KAT hypothesized knowledge types.

From the results discussed so far it can be infer that there was no statistically significant difference between teachers who teach at the rural areas and those at the urban settings who teach mathematics at the senior high schools in the three regions where data was collected across the three knowledge types at the time of data collection. This indicates that these two groups of teachers possess relatively the same knowledge levels.

Conclusion Related to Research Hypothesis Four

The fact that the independent-samples t-test and MANOVA result showed no statistically significant difference in the mean scores of these two groups of teachers (those teaching at urban settings and their counterparts teaching at the rural settings) shows that both groups of teachers displayed an evidence of mathematical understanding and knowledge of algebra they teach at the second cycle level. This result was not staggering as both groups were estimated to perform similarly on the test having been teaching at the same level and same content to students at that level. However, it was found that there was a slight difference in mean performance indicating that those at the urban settings did slightly better than their rural setting counterparts. This was also not astounding since those at the urban settings have better facilities and opportunities than their colleagues at the rural settings. Also, it might happen that those at the urban settings receive more in-service training than their counterparts. In addition, it is possible that urban setting mathematics teachers exposed to technology and other pamphlets or textbooks that helps them in the discharge of their work than their rural counterparts teaching mathematics. This finding was in contrast to the claim made by Khairani (2016) that urban teachers performed statistically significantly more competent than their counterparts in rural settings. The aspect of the study which showed statistically significant difference between the two groups had to do with integration of incorporating ICT in integrating STEM in teaching and learning and how to organise co-curricular activities. The implication of this finding is that both category of teachers have the requisite knowledge to be able to impact their students. It also means that both category of teachers are able to influence their students irrespective of the situation and location.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Summary

Overview of the study

To become a dynamic member of this modern society, one needs to be proficient in mathematics in this 21st century (Ball, 2003a), and the requirement for mathematics in our day to day life continues to grow (NCTM, 2000). For instance in the U.S.A., it is said that they are not preparing their students for the hassles to be mathematically proficient (Black, 2007). It is indisputable fact that algebra serves as a janitor course, contributing varying prospects for entry into advanced mathematics courses (Ball, 2003a), for groundwork for college studies (Pascopella, 2000, Lawton, 1997, Chevigny, 1996, Silver, 1997, Olson, 1994), and for groundwork to zoom into the world of work (Silver, 1997).

According to Mewborn (2003), teachers play significant role in safeguarding that all students have the prospects and know-hows needed to learn mathematics. Consequently, as educators we must weigh the types of knowledge needed by mathematics teachers to provide all learners with reasonable opportunities to learn algebra. Research is replete with the fact that teacher's subject matter knowledge for teaching is often squeaky and insufficient to provide the instructional prospects needed for students to efficaciously learn mathematics (Ball, 1998a, 2003b, Ball & Bass, 2000; Fuller, 1996, Ma, 1999, 257

This study was premised on the assumption that the mathematics teachers' knowledge for teaching is one of the most central factors that goes a long way to influence learners performance, especially in our part of the world (see for example, Mullens et al., 1996; Sanders and Rivers, 1996; Jordan, Mendro, and Weerasinghe, 1997; Wright, Horn, & Sanders, 1997). This study also acknowledges the fact that literature on teaching and student performance is packed with evidence that supports this proposition; there is extensive discrepancy among researchers about the precise nature of the connection between teachers' subject matter knowledge and student performance. The discrepancy in agreement of various findings of, for instance, Monk (1994) and Rowan, et al. (2002), on one side, and Harbison and Hanushek (1992) and Mullens et al. (1996), on another side, epitomizes the discrepancy. This study as a matter of fact, recognizes the necessity for re-conceptualization of teacher knowledge in means that it is not only area specific, even within purview of school mathematics, but in a way that lends itself to some form of direct measurement instead of by proxy in the face of the voluminous different general conceptualizations of teacher knowledge. This is because it is blurred whether the knowledge needed by teachers for teaching algebra for example, would be the same as that needed to teach a different domain like statistics. Also, this study acknowledges the need for enhanced ways of measuring the diverse constituents or types of subject matter knowledge needed by mathematics teachers that could result from such re-conceptualizations as a result of the various different representation measures that have so far been used by various researchers.

Looking back on Chapter two of this thesis, these deliberations triggered the key role of the work of the Knowledge of Algebra for Teaching (KAT) project at Michigan State University to this study. In the KAT project, researchers have hypothesized that the knowledge used by teachers in teaching school algebra are in three folds: "knowledge of school algebra" (referred to in simple terms as "school knowledge"), "advanced knowledge of mathematics" (also referred to as "advanced knowledge"), and "teaching knowledge".

This dissertation used the KAT conceptualization as the structure for studying Ghanaian high school mathematics teachers' knowledge for teaching algebra. The study also adapted the instruments developed by the KAT project as well as some items used by Black (2007) in a PhD. work for collecting data in the Ghanaian context instead of relying on proxy measures of teacher knowledge.

Literature is replete with factors such as family background, pupil-teacher ratio, socio-economic status of students that account for students' achievement (Coleman et.al., 1966), it is unblemished that in our part of the world, the knowledge teachers possess for teaching is a very crucial factor in students' achievement (Harbison & Harnushek, 1992; Hill et al., 2005). This study is purposefully focused on algebra for the following reasons:

1. Algebra at the senior high school level seems to be the foundational area in both core and elective mathematics.
2. Also, algebra has applications in all the other areas of mathematics. Accordingly, teachers' good repertoire of knowledge in algebra has the potential of affecting students' achievement in mathematics.
3. Various West Africa Examination Council Chief Examiners' incessant emphasis on students' inability to perform well in algebra related tasks makes attention on teachers' knowledge for teaching algebra necessary for a study such as this.
4. Also, this study was an attempt at finding out if the KAT conceptualization could be used to corroborate high school prospective and in-service mathematics teachers' knowledge for teaching algebra with regards to the three domains of knowledge hypothesized in the KAT framework in the Ghanaian context. A critical look at literature indicates that in Ghana not much work has been done in this area of teachers' knowledge for teaching algebra irrespective of the poor performance of students in mathematics at the Senior High School level as a result of difficulty in algebra.

The study resorted to the use of the cross sectional survey design for collecting data. Instead of relying on alternative measures, there is the need for re-conceptualization of teacher knowledge in ways that is not only domain specific but also allows its components to be measured. This study, therefore, was designed to investigate whether the three domains of teacher knowledge hypothesized in the KAT framework will be corroborated and explore for further other factors if any. The study also compared senior high school mathematics teachers' knowledge for teaching algebra with regards to those with background training in education and their counterparts without education background. It further examined the algebra knowledge possessed by high school mathematics teachers based on the three hypothesized knowledge in the KAT. There was also an attempt to look at difference in knowledge based on the hypothesized knowledge with regards to teaching experience.

The study used in-service and prospective mathematics teachers from three regions specifically Western, Central and Ashanti regions. A total of 252 prospective and in-service senior high school mathematics teachers from these three regions participated in the study. Out of the 252 participants, 125 and 127 were teachers with background training in education and those without background training in education respectively. The study also involved 99 and 153 prospective and in-service mathematics teachers respectively. For better understanding, specific findings and conclusions that emerged from the analyses of the data have been discussed as well as a conclusion of the findings from the study.

Based on the research questions and hypothesis that guided the study, the findings of this work have been divided into various sections, each relating to the research objectives. The first part has to do with the corroboration of the three types of knowledge hypothesized in the KAT framework and the reconceptualization of same with regards to the knowledge for teaching algebra at the SHS level. Also, the second part looks at the algebra knowledge possessed by SHS mathematics teachers for teaching based on the KAT framework whereas the first, second, third and fourth aspect has to do with the difference between the knowledge for teaching algebra by in-service and prospective mathematics teachers at the senior high school level, difference in the knowledge for teaching algebra between mathematics teachers with background training in education and their counterparts without education background, differences in knowledge between teachers with varying teaching experience and difference in the knowledge for teaching algebra between mathematics teachers who teach in urban areas and their counterparts in the rural settings.

Two forms of instruments were adapted merged to form one aspect of the instrument from Wilmot (2008) and others adapted from Black (2007). The complete instrument consisted of two sections. The first section of the teachers' instrument administered to both in-service and prospective mathematics teachers contained seven survey questions designed to elicit background information about the teacher. The first part of the instrument sought information on teachers' qualification, level of education, algebra courses taught so far at the time of the study, current area of teaching, number of years of teaching mathematics and the type of degree earned. The second part of the instrument required teachers to respond to 74 multiple-choice types of items that were categorized into the various knowledge level hypothesized in the KAT project. These items were based on the content of algebra in the senior high school core and elective mathematics syllabus as well as items based on the tasks of teaching. Each of the multiple choice items has one correct answer and three or four destructors. Teachers used a maximum of two and half hours to answer items on the instrument.

The instrument for data collection did not require the use of any form of identity that could be traced to any particular subject that was involved in the study. All subjects before administration of the instrument were also assured of the anonymity and the confidentiality of the whole exercise. In addition, all analysis and discussions were barren of names of participating schools, rather, codes which were not traceable to subjects involved and participating schools were used.

The data gathered were subjected to various kinds of analysis based on the objectives of the study. Descriptive statistics was used throughout the entire work. ANOVA table was used to find out whether scores of teachers who are inexperienced, less experienced and experienced differ significantly. Before the ANOVA test was conducted, a preliminary analysis was performed at the school-type level. This preliminary analysis was to ensure that the various school-types were homogenous before the ANOVA test was conducted. The independent samples t-test was used to determine whether or not differences exist between the knowledge for teaching algebra by in-service and prospective mathematics teachers at the senior high school level; whether or not differences exist in the knowledge for teaching algebra between mathematics teachers with background training in education and their counterparts without education background; and whether or not differences exist in the knowledge for teaching algebra between mathematics teachers who teach in urban areas and their counterparts in the rural settings.

Every human endeavour seems to have some imperfection attached to it and the domain of research is not an exception. This study also had its limitations. The use of the survey though advantageous to gather a large amount of data but characteristically, was not able to afford answers to in-depth or probing questions nor could this survey pursue explanations and determine the conditions or contexts related to how the participants responded to the multiple-choice items (Sarantakos, 2013). One other thing about survey research is that they are not able to give account for under or over self-estimations of capabilities.

One other limitation of this study was that of the sample size in this case the number of participating teachers. This in a way could place a limitation on the outcome of the study in that if a large number of teachers were involved it could have given different results. Also, some teachers' refusal to respond to certain questions on the instrument could also place a limitation on the outcome of the results of the analysis.

Also, the next limitation was about some mathematics teachers who participated in the study. Since participation in the study was voluntary, some teachers who the researcher believe could have added to the beauty and given vital information to the study for one reason or the other refused to participate in the study. Also, some first class and rural SHSs who were randomly selected to be part of the study turned the offer down at the very day of the data collection which in a way affected the sample size accordingly affected generalizability.

Another limitation of this study was that it could have used more schools hence more teachers but due to financial constraints, it was not possible to include schools from all the senior high schools across the nation. Schools and teachers that participated in the study were selected from only three regions and 40 public senior high schools across the three regions.

Key Findings

1. Extent to which high school prospective and in-service mathematics teachers' knowledge for teaching algebra corroborates the three main types of knowledge hypothesized in the KAT framework.

   It was revealed that senior high school mathematics teachers' knowledge for teaching algebra corroborates the three knowledge types as hypothesized in the KAT framework. Also, the study re-conceptualized the KAT framework into seven different knowledge types and this was done through factor analysis of data obtained.

2. Algebra knowledge possessed by SHS mathematics teachers for teaching based on the KAT framework.

   The study revealed in-service mathematics teachers who participated in this study showed evidence of mathematical understanding and knowledge of algebra they teach at the SHS level based on the KAT framework. Also, analysis of some specific items relating to *school knowledge, advance knowledge and teaching* revealed a startling revelation. The results indicated that in-service mathematics teachers at the SHS as at the time this study was conducted have weak *school knowledge* and *teaching knowledge.*

3. Difference between the knowledge for teaching algebra of in-service and that of prospective mathematics teachers at the senior high school level.

   Results of the study indicated that both in-service and prospective mathematics teachers at the SHS who participated in the study as of the time when data was collected have somewhat same knowledge level for teaching algebra at the senior high school level based on the three knowledge types. The mean and standard deviation scores of in-service mathematics teachers (M = .3739, SD = 0.1431) and that of prospective mathematics teachers (M = .3591, SD = 0.1047). Nevertheless, it was revealed that in-service teachers have slightly higher level of knowledge for teaching algebra than their prospective mathematics teacher counterparts. However, the overall mean and standard deviation scores (M = .3665, SD =.1239) shows that both groups in general have a relatively high level of knowledge for teaching algebra.

4. Difference in the knowledge for teaching algebra between senior high school mathematics teachers with background training in education and those of their counterparts without background training in education.

The study revealed no statistically significant difference between teachers with background training in education (M = .3694, SD = 0.1332) and their counterparts without education background (M = .3669, SD = 0.1260); t(250) = .153, p = .879. It was also revealed that for most of the teachers the type of knowledge they needed to possess which they apply in teaching the subject matter of school algebra is questionable. In addition, majority of these participants it came to light that at critical moment may not be able to give explanations that would help students conceptually and procedurally understand why the method they employ as student would or would not work

5. Difference between senior high school mathematics teachers' knowledge for teaching algebra and their years of teaching experience

   Analyses of data showed that in terms of the hypothesized knowledge types, while background in education did not significantly affect the quality of teacher knowledge, teachers with ten years and above teaching experience were significantly better

6. Difference in knowledge for teaching algebra between mathematics teachers who teach in urban areas and their counterparts in the rural settings.

   Results of the analysis indicated no statistically significant difference between those teaching at urban settings and their counterparts teaching at the rural settings with regards to their mean scores. Though no statistically significant difference was realized, it was found that there was a slight difference in mean performance indicating that those at the urban settings did slightly better than their rural setting counterparts.

Conclusions

Even though data for this study were collected from 40 senior high schools across three regions, the findings of this study may have implications for planning concerning improving teachers' knowledge for teaching algebra in the schools that participated in the study. The following conclusions were drawn as a result of the findings:

Related to the first research question, factor analysis was initially done to ascertain whether senior high school in-service and prospective mathematics teachers' knowledge for teaching algebra would validate the KAT framework by way of reposing a regression line on the scree-plot as suggested by Nelson (2005) since it's a useful approach to determining where an eigenvalue scree breaks. Before doing this, the possibilities of selecting seven or eight as elbows of the scree-plot was done and the analysis was repeated using other components. The reason for doing all these was to use the nature of the factor loadings to determine which of these analyses best explained factors that were eventually retained and therefore, provided indication of the extent to which data on the in-service and pre-service mathematics teachers' performance validated the three-dimensional conceptualization and possibly ascertain whether there are more than three of knowledge for teaching algebra as pined in the theoretical framework.

However, after superimposing regression line on the scree-plot (see Figure 2), it was evident that in-service and prospective mathematics teachers' knowledge for teaching algebra corroborated the KAT framework with an R square value of 0.7414 which fits the scree-plot quite well.

Since these three knowledge types are not domain specific, there was the need for reconceptualization of the KAT framework and this led to the use of eight and later seven components of the reconceptualization. However, when the eight-component factor analysis was done it revealed a lot of cross loadings with eighth component having only one item left after the cross loadings were taken out. Therefore, using the Costello and Osborne (2005) recommendation, it became practically impossible to interpret using eight factors when less than three items were uniquely loaded hence the eight-component factor analysis was not pursued simply because no significant interpretation could be drawn with the consequential small unique factor loadings. Moving forward, when seven-component factor analysis was done, the seven factors could explain 33.53%.

On the seven-component factor analysis, fifteen items loaded substantially on only factor one whereas eight and five items loaded substantially on factors two and three respectively. Also, items four, five, five and five loaded on factors 4, 5, 6 and 7 respectively. This, however, led to the retention of seven factors for reconceptualization.

As a result of these, the three knowledge types used in the KAT project have been re-conceptualized into *School Algebra Knowledge,* *Advanced Algebra Knowledge, Algebra Teaching Knowledge, Algebra Knowledge, Advanced Algebra Teaching Knowledge, School Algebra Teaching Knowledge and Pedagogical Content Knowledge in Algebra.*

Nevertheless, a critical examination of the scree-plot led to extraction of seven factors (see Figure 11). This extraction presupposes that the intersections which were hitherto labeled by researchers in the KAT project as been fuzzy (Wilmot, 2008) may not be fuzzy after all. Using the idea that teachers agenda for teaching is shaped by the curriculum scripts they possess (see for example Putnam, 1987). An argument is made that looking at the intersections of the three knowledge hypothesized to be fuzzy they may be part of the packages of knowledge of teachers that enable them adopt flexible and interactive approaches to teaching (Wilmot, 2008).

For example, it is possible during the course of teaching, an advanced knowledge possess by the teacher may help the teacher to explain the school knowledge in an advance form because at the time of teaching those two knowledge may come together to facilitate the teacher direct the thought of the students. In like manner, a teacher teaching students may also fall on his/her advance knowledge possess at the time of teaching to enable him/her blend with the school knowledge in order to better explain the concepts being thought to students which may end up in yet another knowledge. On the basis of the above conclusions, the KAT framework which guided this study was modified as below.

PKA

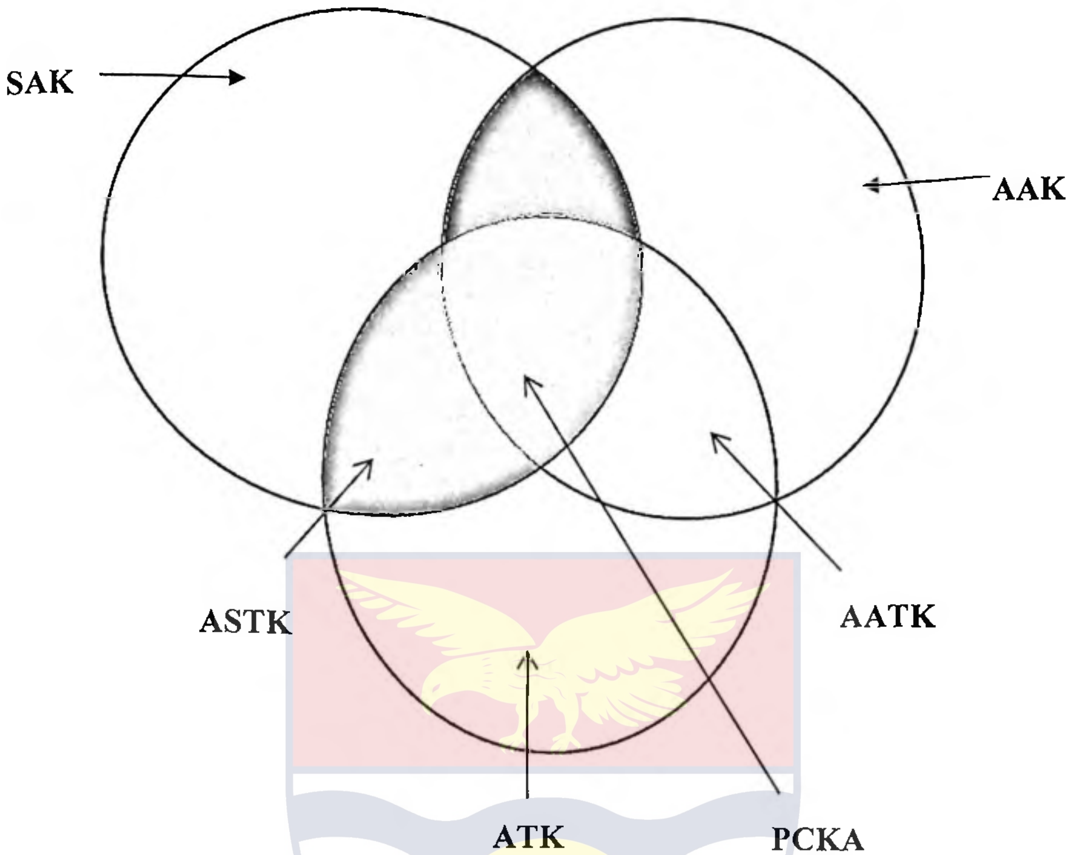

Figure 11: Reconceptualization of the KAT framework

Below are the full text of the meanings of the various abbreviations in Figure 11

PKA: Profound Knowledge of Algebra
AAK: Advance Algebra Knowledge
APCK: Algebra Pedagogical Content Knowledge
AATK: Advance Algebra Teaching Knowledge
ASTK: Algebra School Teaching Knowledge
SAK: School Algebra Knowledge
ATK: Algebra Teaching Knowledge

These results indicate that data gathered for this study corroborated the KAT framework and even went further to do a reconceptualization.

Differently put, the profile of knowledge of the participating in-service and prospective senior high school mathematics teachers in Ghana did corroborate the three categories of knowledge, *knowledge of school algebra, advanced knowledge and teaching knowledge*, as hypothesized in the KAT framework and went further to re-conceptualize it as shown in Figure 3 as *School Algebra Knowledge, Advanced Algebra Knowledge, Algebra Teaching Knowledge, Algebra Knowledge, Advanced Algebra Teaching Knowledge, School Algebra Teaching Knowledge and Algebra Pedagogical Content Knowledge.*

The implication of this is that in order to conceptualize knowledge for teaching algebra by senior high school mathematics teachers, it is necessary to take into account contextual differences. Also, it is important to reconsider how certain courses and programmes are mounted in our universities to help prepare our teachers for the tasks of teaching in our senior high schools. In addition, the corroboration and re-conceptualisation of the KAT framework is consistent with

what Sherin (2002) put forward that in the bid to teaching new curriculum, especially from a reform oriented curricula, prospective and in-service mathematics teachers adapt their own knowledge and in the course develop a fresh content and pedagogical content knowledge in order to cope with demands of a new curriculum. This presupposes that to be a successful teacher in the Ghanaian context, teachers at the senior high school would have to adapt their knowledge to be able to help students to be successful.

It was generally evident that in-service mathematics teachers who took part in this study possess some sort of mathematical understanding and knowledge of algebra they teach at the SHS level. This was not shocking since in-service teachers who participated have quite a number of books, exposure and are well vexed in the content at the SHS. Nevertheless, analysis of some specific items relating to *school knowledge, advance knowledge and teaching knowledge* revealed shocking results. The conclusion is that in-service mathematics teachers at the SHS as at the time of this study have weak *school knowledge and teaching knowledge*. The Implication of this is that the procedural and conceptual knowledge possessed by these teachers are weak and for that matter at a critical point in time in the course of their teaching, they may find it difficult to explain to their students what they the students are supposed to learn. This finding is supported by what Mewborn (2003) and Thompson and Thompson (1996) put forward that teachers lack conceptual knowledge and understanding of the mathematics they teach. Nevertheless, this same finding contradicts what Mewborn (2003) said that teachers have a strong command of the procedural knowledge of mathematics they teach.

This implies that teachers as well as higher educational institutions have to take a second look at mathematics and mathematics related programmes meant for training teachers. This also presupposes that higher educational institutions such as the universities and polytechnics should mount courses that would help address knowledge in these areas.

For research hypothesis one, result revealed no statistical significant difference in the mean scores of these two groups of teachers (both in-service and prospective mathematics teachers). This indicates that both groups of teachers displayed an evidence of mathematical understanding and knowledge of algebra they teach at the SHS level. This was a bit perplexing because it was expected that the in-service teachers could have done better than the prospective mathematics teachers having taught the subject for quite some time, their level of mathematical knowledge should have been above that of the prospective teachers. This was also shocking because these in-service teachers is assumed have quite a number of books, exposure as well as vexed in the content at the SHS level than their prospective mathematics teacher counterparts. This outcome was in contrast to what Wilmot (2008) and Sherin (2002) opined. The implication of this is that the threshold of knowledge possessed by in-service mathematics teachers remains constant as asserted by Berliner (1979) that at a point in time in their profession. In addition, this implies that teachers tend to teach the same things with the same note or information without any revision or innovations.

In reference to research hypothesis two, it was established that the performance of teachers with education background training was not different from their counterparts without education background. With the background of these two groups, one would have expected without any reservation that since the teachers with education background had high professional knowledge or qualification they would, as a matter of fact, perform better. It was also concluded that those with background training in education might have felt buoyant in their knowledge and have been restricted to what they previously know hence the result. Also, it is likely that those without background training in education sensed the need to upgrading their knowledge by way of reading or learning other courses or materials to bring their knowledge to a level that had posh their performance confidently. In addition, analysis of some specific items that showed statistical significant difference revealed that for most of the teachers the type of knowledge they needed to possess which they apply in teaching the subject matter of school algebra is questionable. It was also clear that at critical moment these teachers may not be able to give explanations that would help students conceptually understand why certain methods they employ as student would or would not work. This implies that in the higher learning institutions where our teachers are trained certain things are not being done right to address the conceptual and procedural anomalies in trainee teachers' thinking.

With regards to research hypothesis three, it was concluded that there was a statistically significant difference in the mean scores of the mathematical knowledge of algebra of teachers who possess more than 10 years teaching experience and their counterparts with 0-5 years and 6-10 years teaching experience at the senior high school level. In many professional jurisdiction, staff years of experience is considered as a relevant factor in human resource policies, including compensation systems, benefits packages, and promotion decision. The main reason is that experience gained over time as a matter of fact enhances the knowledge, skills, and productivity of staff or workers. According to Ladd (2008), teachers with over 20 years teaching experience are more effective than their counterparts with no teaching experience. He further stated that these teachers, however, are not much more effective than those with 5 years of teaching experience. Though the results indicates a significant difference between these categories of teachers (that is inexperience, less experience and experienced teachers) effect size calculated indicated that the difference was very small. This could be as a result of the fact that most of these experienced teachers are coming from "first class" schools. Also, it might happen that these experienced teachers as a result of their contact with students and trends in the area they are teaching are able to manage issues relating to teaching than their inexperienced and less experienced counterparts.

In relation to research hypothesis four, it was concluded that no statistically significant difference in the mean scores of the mathematical understanding and knowledge of algebra of teachers who teach at urban settings and their counterparts at the rural settings at the senior high school level. This result was not staggering as both groups were estimated to perform similarly on the test having been teaching at the same level and same content to students at that level. This could be as a result of the fact that mathematics teachers at the urban settings have better facilities and opportunities than their colleagues at the rural settings. Also, it might happen that those at the urban settings receive more in-service training than their counterparts at the rural settings. In addition, it is possible that urban setting mathematics teachers have access to technology and other pamphlets or textbooks that help them in the discharge of their responsibility better than their rural counterparts teaching mathematics.

Recommendations

Recommendations for policy and practice

Based on the findings the following recommendations have been made for educational policy and practice in the knowledge for teaching algebra:

1. Research should be conducted to corroborate senior high school mathematics teachers' knowledge for teaching algebra based on the new conceptualization proposed in this study. It is, however, recommended that further research be conducted to establish and verify other factors that contribute to senior high school teachers' knowledge for teaching algebra since the seven factors retained only contributed 33.53%.
2. Training institutions should look at developing course geared towards helping teachers explore the seven knowledge types needed for teaching algebra.
3. Tertiary institutions must restructure their programmes to help increase the content knowledge of those with education background.
4. Also, the diploma in education as it used to be, must be revisited to help increase the teaching knowledge of teachers without educational background

5. Furthermore, a comparable research should be conducted to include senior high schools from the remaining seven regions in Ghana. A study of that kind would provide additional information for modification and upgrading in teachers' knowledge for teaching algebra. This would go a long way to help restructuring of university programs as well as the integrated mathematics curriculum of Ghana. This current study used only public senior high schools in three regions of Ghana. A study which could include senior high schools in the remaining seven regions would be helpful in drawing a more generalize conclusion on this subject.
6. It is also recommended that in-service training on current issues in the area of these teachers should be organized for both category of teachers (teachers who teach at urban settings and their counterparts at the rural settings).
7. Mentorship should be instituted and monitored in our schools to bridge the knowledge type gap in teaching algebra between experienced and inexperienced mathematics teachers.

Suggestions for Further Research

1. Further research needs to be conducted using the new framework (Figure 8) in this study to ascertain whether it corroborate teachers' knowledge for teaching algebra based on the Ghanaian context.
2. Also, further research be conducted to include teachers with varying background since that was not considered in this study. In addition,

the number of teachers (sample size) be increased to give a proper picture of what is happening in this area of research.

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Appendices

APPENDIX A

Data Collection Instrument

An investigation into senior high school mathematics teachers' knowledge for teaching algebra

A PhD degree Dissertation Study

By

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PART I : BACKGROUND QUESTIONNAIRE

1. If you are still in college, what year are you in your college/university preparation?

☐ 1st year

☐ 2nd year

☐ 3rd year

☐ 4th year

☐ Graduate student (specify level & major) ____________________

☐ Other (specify)____________________

2. What is/was your bachelor's degree in?

☐ Mathematics

☐ Mathematics Education

☐ Other (specify) ____________________

3. What is/was your minor in college/university?

☐ Mathematics

☐ Other (specify) ____________________

4. If you have a master's degree, in what area was it?

☐ Mathematics

☐ Mathematics Education

☐ Other (specify) ____________________

☐ I do not have a master's degree

5. Including the current semester, which of the following *types* of courses have you taken?

Check all that apply.

Mathematics Courses

☐ Calculus

☐ Linear Algebra (e.g., vector spaces, matrices, dimensions, eigenvalues, eigenvectors)

☐ Abstract Algebra (e.g., group theory, field theory, ring theory; structuring integers, ideals)

☐ Advanced Geometry and/or Topology

☐ Real and/or Complex Analysis

☐ Number Theory and/or Discrete Mathematics

☐ Differential Equations and/or Multivariate Calculus

Mathematics Education Courses

☐ Methods of teaching mathematics (planning mathematics lessons, using curriculum materials and manipulatives, organizing and delivering mathematics lessons, etc.)

☐ Psychology of learning mathematics (how students learn, common student errors or misconceptions, cognitive processes, etc.)

☐ Assessment in mathematics education (developing and using tests and other assessments)

6. Are you currently a teacher?

☐ Yes. Go to question 7

☐ No. Have you taught in the senior secondary school in the last five years?

☐ Yes. Go to question 7

☐ No. Skip all the questions on the next page and move to Part II:

Assessment Questions

7. Which of the following algebra courses have you taught in the last five years? Check all that apply.

☐ Core Mathematics in SSS 1

☐ Core Mathematics in SSS 2

☐ Core Mathematics in SSS 3

☐ Elective Mathematics in SSS 1

☐ Elective Mathematics in SSS 2

☐ Elective Mathematics in SSS 3

☐ Other (please specify) ____________________

8. Which area are you currently teaching?

☐ Urban area

☐ Rural area

☐ Other (please specify)

9. Which category of students do you teach?

☐ Science

☐ Arts

☐ Home Economics

☐ Business

☐ Other (please specify)

10. For how many years have you taught core mathematics at the SHS level?

☐ 0

☐ 1-2 years

☐ 3-6 years

☐ 7-10 years

☐ More than 10 years

11. For how many years have you taught elective mathematics at the SHS level?

☐ 0

☐ 1-2 years

☐ 3-6 years

☐ 7-10 years

☐ More than 10 years

12. For how many years have you taught mathematics in general at the SHS level?

☐ 0

☐ 1-2 years

☐ 3-6 years

☐ 7-10 years

☐ More than 10 years

13. Indicate the type of teaching certification you have at the SHS level.

☐ B. Ed (Math)

☐ Diploma in education

☐ Postgraduate diploma/certificate in education

☐Other (please specify): ....

14. Which area/province have you been teaching in the last five years?

☐ Urban area

☐ Rural area

☐ Other (please specify)..

15. Gender

☐Male

☐Female

☐Other

(please specify)...

13. Which school type do you teach currently?

☐Single sex male

☐Single sex female

☐Co-educational

PART II: ASSESSMENT QUESTIONS

Instructions

This instrument contains 74 multiple-choice questions about knowledge for teaching algebra. You have 150 minutes to answer these questions. You may use a calculator if you choose.

In this booklet, each multiple-choice question has only one right answer. Please circle the correct answer for the multiple-choice questions, and write all your responses to the free-response questions.

1. A restaurant has a dinner combo plate. For the plate, you can choose two entries from six different choices. Then you can choose between baked yam, rice, mashed yam, or coleslaw. Last, you choose between stew and salad. How many possible dinner combo plates are available?

   A. 120

   B. 48

   C. 240

   D. 12

   E. None of these

2. Timothy's age in 15 years will be twice what it was 5 years ago. If t represents Timothy's age now, write the equation that models this situation.

Response:

3. Find the number that must divide each term in the equation $5x^2+2x = 20$ so that the equation can be solved by completing the square.

Response:

4. A small company invested ¢2,000.00 by putting part of it into a municipal bond fund that earned 4.5% annual simple interest and the remainder in a corporate bond fund that earned 9.5% annual simple interest. If the company earned ¢1,500.00 annually from the investments, how much was in the municipal bond fund?

A. ¢8,000.00

B. ¢10,000.00

C. ¢9,000.00

D. ¢7,000.00

E. None of these

5. A gramophone record rotates 220 times during the performance of a certain tune, the smallest and largest radii of the record being 6.25cm and 12.75cm respectively, the circular paths being equidistant. Calculate the total distance traversed on the record by the needle.

   A. 131.3m

   B. 130.3m

   C. 220.3m

   D. 220.0m

6. A circular ball is thrown downward with an initial velocity 5 feet per second from a bridge located in a water which is 220 feet above the water. How long will it take for the ball to hit the water? Round your answer to the nearest 0.01 second.

   A. 3.87 seconds

   B. 3.80 seconds

   C. 3.78 seconds

   D. 3.65 seconds

   E. 3.56 seconds

7. Given a set D whose elements are the odd integers, positive and negative (zero is not an odd integer). Which of the following operations when applied to any pair of elements will yield only elements of D?

i. Addition

ii. Multiplication

iii. Division

iv. Finding the arithmetic mean

The correct answer is

A. i and ii only

B. ii and iv only

C. ii, iii, and iv only

D. ii and iii only

E. ii only

8. A particle moves in a straight line with uniform acceleration. At time 2 s, the particle is 10m from its starting point and at 4s the particle is 40m from its starting point. Find the velocity of the particle when it is 160m from its starting point.

A. 5 m/s

B. 25 m/s

C. 40 m/s

D. 45 m/s

9. A and B begin work together. A's initial salary is GH¢200.00 a year and he has an annual increment of GH¢20.00. B is paid at first at the rate of GH¢80.00 a year and has an increment of GH¢8.00 every half-year. At the end of how many years will B have received more money than A?

   A. 5 years

   B. 5.5 years

   C. 6 years

   D. 6.5 years

10. A carpet installer decides to replace carpets in some offices on a university campus and uses the formula Cost = 350 + 1.6A,where a is the number of square feet of carpet to be replaced, to determine the cost. In how large an office can the carpet be replaced for ¢9,600.00?

   A. 1666.25ft^2

   B. 1656.25ft^2

   C. 1656.15ft^2

   D. 1656.05ft^2

   E. 1606.05ft^2

11. Which of the following is a false statement?

A. 2, 3, 9/2, 27/4… $2(3/2)^{n-1}$… is a geometric sequence with common ratio 3/2.

B. 5, 2, -1… -3n+5… is an arithmetic sequence with common difference 5.

C. If $\{a_n\}$ is a sequence, then $S_n =$ …. Is the nth partial sum of the sequence.

D. Two terms of a sequence can be equal.

E. None of these

12. A rectangular piece of cardboard measures 35inches by 30inches. An open box is formed by cutting four squares that measure x inches on a side from the corners of the cardboard and then folding up the sides. Determine the volume of the box in terms of x.

A. $4x^3 - 130x^2 + 1050x$

B. $4x^3 + 130x^2 + 1050x$

C. $4x^3 - 130x^2 + 1050$

D. $4x^2 - 130x + 1050$

13. Ice forms on a refrigerator ice-box at the rate of (4-0.6t)g per minute after t minutes. If initially, there is no ice on the box, find the mass of ice formed in 5 minutes.

A. 5g

B. 17g

C. 26g

D. 35g

14. Find the number of terms of the sequence $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$ that must be taken so that the difference between the sum and 2 is less than 10^{-3}.

A. 10

B. 11

C. 12

D. 13

15. The major sectorial angle of a circle with radius 14cm is 270^0.If the sector is folded to form a cone, find the surface area of the cone.

A. $460.0cm^2$

B. $460.8cm^2$

C. $461.0cm^2$

D. $461.8cm^2$

16. In how many ways can the fraction $\frac{1}{2}$ be written as a sum of two positive fractions with numerator equal to 1 and denominator a natural number?

A. 0

B. 1

C. 2

D. 4

E. More than 4

17. A desktop screen measures 60inches diagonally and its aspect ratio is 16 to 9. This means that the ratio of the width of the screen to the height of the screen is 16 to 9. Find the width and height of the screen. Round to the nearest tenth of an inch.

A. height = 3.27inches, width = 29.4inches

B. height = 3.27inches, width = 52.3inches

C. height = 29.4inches, width = 52.3inches

D. height = 52.3inches, width = 29.4inches

E. height = 52.0inches, width = 29.0inches

18. If p:q and r:s are two equal ratios and (q≠0, s≠0) then

A. p=r and q=s

B. pr = qs

C. p+r = q+s

D. p-r = q-s

E. ps = qr

19. A cup of hot tea is heated to 180^0F and placed in a room that maintains a temperature of 60^0F. The temperature of the tea after t minutes is given by $T(t) = 60+120e^{-0.038t}$. Find the temperature, to the nearest degree, of the tea 5 minutes after it is placed in the room.

A. 637.6^0F

B. 159.2^0F

C. 72.2^0F

D. 60.1^0F

E. None of these

20. Solve algebraically: $\log_3 (x-4) = 2$

A. x = 10

B. x = 18

C. x = 729

D. x = 13

E. None of these

21. The major sectorial angle of a circle with radius 14cm is 270^0.If the sector is folded to form a cone, find the surface area of the cone.

A. $460.0cm^2$

B. $460.8cm^2$

C. $461.0cm^2$

D. $461.8cm^2$

22. Which of the following is a true statement?

A. The solution of the matrix equation AX = B, is $X = A^{-1}B$, provided A^{-1} exists.

B. $\begin{bmatrix} p & q \\ r & s \end{bmatrix}$ and $\begin{bmatrix} p & q \\ r & s \end{bmatrix}$ are inverses.

C. A singular matrix is a matrix that has a multiplicative inverse.

D. All matrices have an inverse.

E. None of these

23. A farmer wishes to make a rectangular hen-run of area $50m^2$ against a wall which is to serve as one of the boundaries. Find the smallest length of wire netting required for the other three sides.

A. 5m

B. 10m

C. 11m

D. 20m

24. Given that a + b = c where a, b, and c are integers and a is positive, which one of the following statements is true?

A. a is always greater than c

B. a is always less than c

C. b is always less than c

D. c is never zero

E. c – a is always positive.

25. What is the conclusion of this statement : *If* $x^2 = 4$, *then* $x = -2$ *or* $x = 2$.

A. $x^2 = 4$

B. $x = 2$

C. $x = -2$

D. $x = -2$ *or* $x = 2$

26. Kwame's average driving speed for a 4-hour trip was 45 miles per hour. During the first 3 hours he drove 40 miles per hour. What was his average speed for the last hour of his trip?

A. 50 miles per hour

B. 60 miles per hour

C. 65 miles per hour

D. 70 miles per hour

27. One pipe can fill a tank in 20 minutes, while another takes 30 minutes to fill the same tank. How long would it take the two pipes together to fill the tank?

A. 50 min

B. 25 min

C. 15 min

D. 12 min

28. Which statement best explains why there is no real solution to the quadratic equation $2x^2 + x + 7 = 0$?

A. The value of $1^2 - 4.2.7$ is positive.

B. The value of $1^2 - 4.2.7$ is equal to 0.

C. The value of $1^2 - 4.2.7$ is negative.

D. The value of $1^2 - 4.2.7$ is not a perfect square.

29. Four steps to derive the quadratic formula are shown below:

$$i.\ x^2 + \frac{bx}{a} = \frac{-c}{a}$$

$$ii.\ \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$iii.\ x = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} - \frac{b}{2a}$$

$$iv.\ x^2 + \frac{bx}{a} + \left(\frac{b}{2a}\right)^2 = \frac{-c}{a} + \left(\frac{b}{2a}\right)^2$$

What is the correct order for these steps?

E. I, iv, ii, iii

F. I, iii, iv, ii

G. Ii, iv, I, iii

H. Ii, iii, I, iv

30. Kofi's solution to an equation is shown below:

Given: $n + 8(n + 20) = 110$

Step 1: $n + 8n + 20 = 110$

Step 2: $9n + 20 = 110$

Step 3: $9n = 110 - 20$

Step 4: $9n = 90$

Step 5: $\frac{9n}{9} = \frac{90}{9}$

Step 6: $n = 10$

Which statement about Kofi's solution is true?

A. Kofi's solution is correct

B. Kofi made a mistake in step 1

C. Kofi made a mistake in step 3

D. Kofi made a mistake in step 5

31. Araba Atta correctly solved the equation $x^2 + 4x = 6$ by completing the square. Which equation is part of her solution?

A. $(x + 2)^2 = 8$

B. $(x + 2)^2 = 10$

C. $(x + 4)^2 = 10$

D. $(x + 4)^2 = 22$

32. Which of the following is a valid conclusion to the statement ''If a student is a high school band member, then the student is a good musician''?

A. All good musicians are high school band members.

B. A student is a high school member band member.

C. All students are good musicians

D. All high school band members are good musicians.

33. The equation of line l is $6x + 5y = 3$, and the equation of the line q is $5x - 6y = 0$. Which statement about the two lines is true?

A. Lines l and q have the same y-intercept

B. Lines l and q are parallel

C. Lines l and q have the same x-intercept

D. Lines l and q are perpendicular

34. John's solution to an equation is shown below:

Given: $x^2 + 5x + 6 = 0$

Step 1: $(x + 2)(x + 3) = 0$

Step 2: $x + 2 = 0$ *or* $x + 3 = 0$

Step 3: $x = -2$ *or* $x = -3$

Which property of real numbers did John use for Step 2:

A. multiplication property of equality

B. zero product property of multiplication

C. commutative property of multiplication

D. distributive property of multiplication over addition

35. When is this statement true?

The opposite of a number is less than the original number.

E. This statement is never true.

F. This statement is always true.

G. This statement is true for positive numbers.

H. This statement is true for negative numbers.

36. Kwame solved the equation $\frac{1}{x-5} = \frac{5}{12x-60}$.

Step 1: He factored the denominator in the expression on the right side of the equation and obtained $\frac{1}{x-5} = \frac{5}{12(x-5)}$.

Step 2: He multiplied both sides by $x - 5$ and obtained $1 = \frac{5}{12}$.

Conclusion: The solution set is the empty set.

A. The conclusion is correct.

B. The conclusion is wrong because we cannot multiply both sides by $x - 5$.

C. The conclusion is wrong because another procedure produces a conclusion different from the one obtained.

D. The conclusion is wrong because if we 'cross multiply' by the common denominator we obtain a different solution.

E. There is some other reason why the solution is wrong

37. Students in Mr. Carson's class were learning to verify the equivalence of expressions. He asked his class to explain why the expressions a – (b + c) and a – b – c are equivalent. Some of the answers given by students are listed below.

Which of the following statements comes closest to explaining why

a – (b + c) and a – b – c are equivalent? (Mark ONE answer.)

A. They're the same because we know that a – (b + c) doesn't equal a – b + c, so it must equal a – b – c.

B. They're equivalent because if you substitute in numbers, like a=10, b=2, and c=5, then you get 3 for both expressions.

C. They're equal because of the associative property. We know that a – (b + c) equals (a – b) – c which equals a – b – c.

D. They're equivalent because what you do to one side you must always do to the other.

38. The set of nonnegative rational numbers with the operations of addition and multiplication has one of the following characteristics:

A. It is not closed under one of these operations

B. More than one of its elements does not have an inverse for the operation of multiplication.

C. Zero is not a member of this set

D. The distributive law of multiplication over addition does not hold

E. None of the above is a characteristic of the given set

39. Susan was trying to solve the equation $2x^2 = 6x$.

First she divided both sides by 2.

$x^2 = 3x$

Then she divided both sides by x:

$x = 3$

Gustavo said, "You can't divide both sides by x." Susan responded, "If you can divide both sides by 2, why can't you divide by x?" They asked their teacher to explain.

Which of the following explanations is correct?

A. Since x is a variable it can vary, you may not be dividing both sides by the same number.

B. You can't cancel x because it does not represent a real number.

C. You can only divide by whole numbers when solving equations.

D. It is better to take the square root of both sides after dividing by 2, that way you won't have to worry about dividing by x.

E. If you divide both sides by x, then you might be dividing by 0, and would miss the solution $x = 0$.

40. In a first year elective mathematics class, which of the following is **NOT** an appropriate way to introduce the concept of slope of a line?

A. Talk about the rate of change of a graph of a line on an interval.

B. Talk about speed as distance divided by time.

C. Toss a ball in the air and use a motion detector to graph its trajectory.

D. Apply the formula $slope = \frac{rise}{run}$ to several points in the plane.

E. Discuss the meaning of m in the graphs of several equations of the form $y = mx + b$.

41. Consider the statement below.

For all a, b □ S, if ab = 0, then either a = 0 or b = 0.

For which of the following sets S is the above statement true?

i. the set of real numbers

ii. the set of complex numbers

iii. the set of 2x2 matrices with real number entries

A. i only

B. ii only

C. iii only

D. i and ii only

E. i, ii and iii

42. Some students were asked to prove that the following statement is true:

When you multiply any 3 consecutive whole numbers, your answer is always a multiple of 6.

Below are proofs offered by three of them.

Kate's answer
A multiple of 6 must have factors of 3 and 2.
If you have three consecutive numbers, one will be a multiple of 3 as every third number is in the three times table.
Also, at least one number will be even and all even numbers are multiples of 2.
If you multiply the three consecutive numbers together the answer must have at least one factor of 3 and one

factor of 2.

Leon's answer

$1 \times 2 \times 3 = 6$

$2 \times 3 \times 4 = 24 = 6 \times 4$

$4 \times 5 \times 6 = 120 = 6 \times 20$

$6 \times 7 \times 8 = 336 = 6 \times 56$

Maria's answer

n is any whole number

$n \times (n+1) \times (n+2) = (n^2 + n) \times (n+2)$

$= n^3 + n^2 + 2n^2 + 2n$

Canceling the n's gives $1 + 1 + 2 + 2 = 6$

Which are valid proofs?

A. Kate's only

B. Maria's only

C. Kate's and Leon's

D. Leon's and Maria's

E. Kate's and Maria's

43. The statement 'For all whole numbers, if to the product of two consecutive whole numbers we add the larger number, the result is equal to the square of the larger number' can be expressed symbolically as: For all whole numbers n

A. $n^2 + 1 = n(n-1) + n + 1$

B. $(n+1)^2 = n^2 + 2n + 1$

C. $n^2 = n(n-1) + n$

D. $(n+1)n = n^2 + n$

E. $(n-1)^2 + 2n = n^2 + 1$

44. The polynomial $p^2 - p - 6$ can be factored into $(p-3)(p+2)$. If natural numbers are substituted in place of p, which one of the following statements is true about the set of numbers obtained?

A. Some numbers will be odd

B. The number zero does not appear

C. None of the numbers will be prime

D. All of the numbers will be less than 100

E. None of the above statements is correct

45. Let $f(x) = \log_2 x^2$. Which of the following functions have the same graph as $y = f(x)$?

i. $y = 2\log_2 x$

ii. $y = 2\log_2 |x|$

iii. $y = 2|\log_2 x|$

A. i only

B. ii only

C. iii only

D. i and ii only

E. i, ii, and iii

46. Students are given the following problem:

Find the number of the real roots of the equation $9^x - 3^x - 6 = 0$

Peter denotes $y = 3^x$ and gets the equation $y^2 - y - 6 = 0$, which has 2 different roots. He concludes that the given equation also has 2 different roots.

Which of the following is true about Peter's solution?

A. Peter's conclusion and his arguments are correct.

B. Peter's original approach to the problem (substitution of $y = 3^x$) is not correct.

C. Peter factors wrong.

D. The quadratic equation $y^2 - y - 6 = 0$ does not have 2 different roots.

E. Peter does not take into account the range of the function $y = 3^x$.

47. Which of the following can be represented by areas of rectangles?

i. The equivalence of fractions and percents, e.g. $\frac{3}{5} = 60\%$

ii. The distributive property of multiplication over addition: For all real numbers a, b, and c, we have $a(b + c) = ab + ac$

iii. The expansion of the square of a binomial: $(a + b)^2 = a^2 + 2ab + b^2$

A. ii only

B. i and ii only

C. i and iii only

D. ii and iii only

E. i, ii, and iii

48. A student is asked to give an example of a graph of a function $y = f(x)$ that passes through the points A and B (see Figure 1). The student gives the answer shown in Figure 2. When asked if there is another answer the student says: "No, this is the only function."

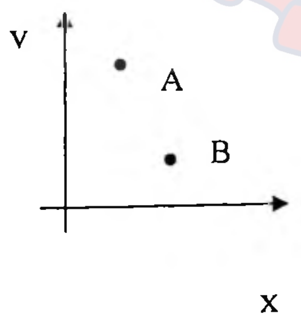


Figure 1

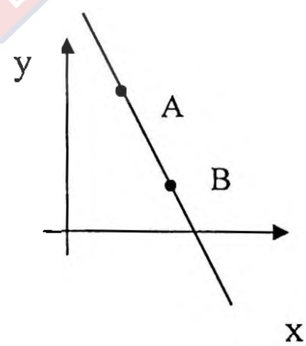


Figure 2

Which of the following best evaluates the student's answer of "No" to the second question?

A. The student is right, because that is the only way a line will pass through both points.

B. The student is right, because this function is of the form $f(x) = mx + b$.

C. The student is right, because his graph passes the vertical line test.

D. The student is wrong, because graphing is not an appropriate way to solve this problem.

E. The student is wrong, because there are infinitely many functions that pass through points A and B.

49. A textbook contains the following theorem:

If line l_1 has slope m_1 and line l_2 has slope m_2 then $l_1 \perp l_2$ if and only if $m_1 \cdot m_2 = -1$ (i.e. "slopes of perpendicular lines are negative reciprocals"). (McDougal Littell, Algebra 2)

Three teachers were discussing whether or not this statement generalizes to all lines in the Cartesian plane.

Mrs. Allen: The statement of the theorem is incomplete: it doesn't provide for the pair of lines where one is horizontal and one is vertical. Such lines are perpendicular.

Mr. Brown: The statement is fine: a horizontal line has slope 0 and a vertical line has slope ∞ and it's OK to think of 0 times ∞ as -1.

Ms. Corelli: The statement is fine; horizontal and vertical lines are not perpendicular.

Whose comments are correct?

A. Mrs. Allen only

B. Mr. Brown only

C. Ms. Corelli only

D. Mr. Brown and Ms. Corelli.

E. None are correct.

50. Consider the statement below.

For all $a, b \in S$, *if* $ab = 0$, *then either* $a = 0$ *or* $b = 0$.

For which of the following sets S is the above statement true?

i. the set of real numbers

ii. the set of complex numbers

iii. the set of 2x2 matrices with real number entries

A. i only

B. ii only

C. iii only

D. i and ii only

E. i, ii and iii

51. Mr. Nkrumah asked his algebra students to divide $x^2 - 4$ by $x + 2$. Eric said, "I have an easy method, Mr. Nkrumah. I just divide the x^2 by x and the 4 by the 2. I get $x - 2$, which is correct." Mr. Nkrumah is not surprised by this as he had seen students do this before. What did he know? (Mark one answer.)

A. He knew that Eric's method was wrong, even though he happened to get the right answer for this problem.

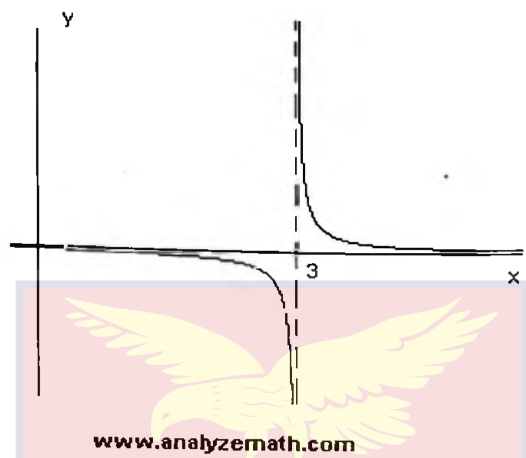
B. He knew that Eric's answer was actually wrong.

C. He knew that Eric's method was right, but that for many algebraic fraction division problems this would produce a messy answer.

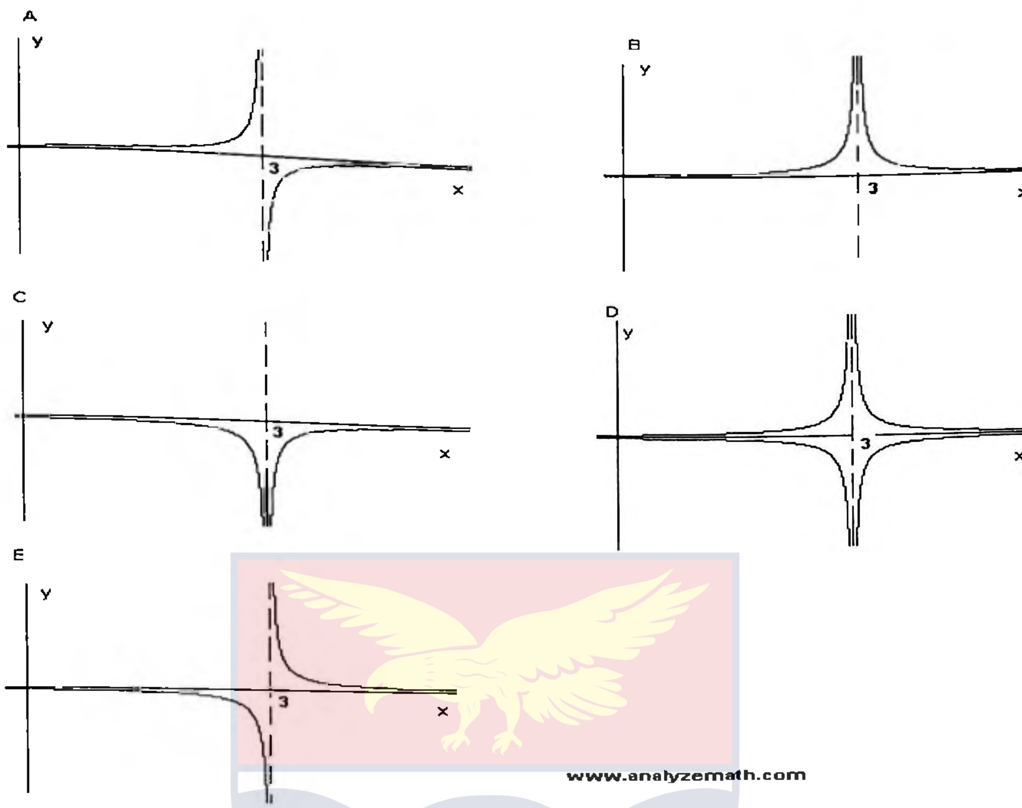
D. He knew that Eric's method only works for some algebraic fractions.

E. I'm not sure.

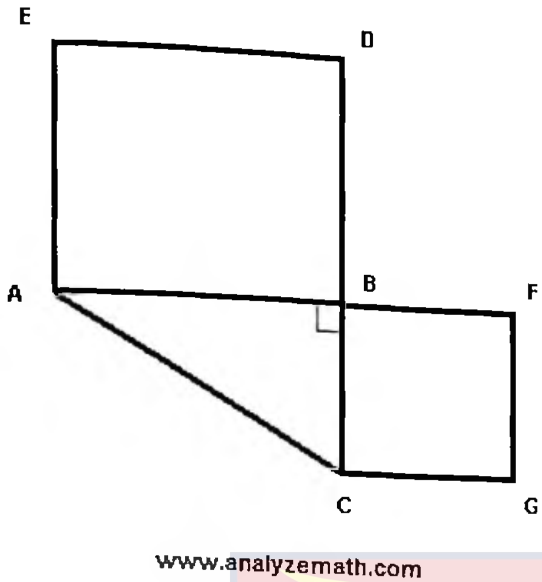
52. The graph of y = 2/(x - 3) is shown below



Among the following, which is the best possible graphical representation of y = -2/|x - 3|



53. In the figure below ABC is a right triangle. ABDE is a square of area 200 square inches and BCGF is a square of 100 square inches. What is the length, in inches, of AC?



A) $10\sqrt{3}$

B) $10\sqrt{2}$

C) 300

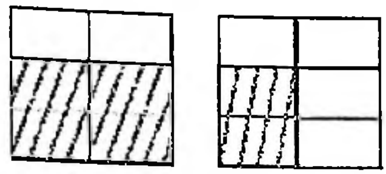
D) 10

E) 15

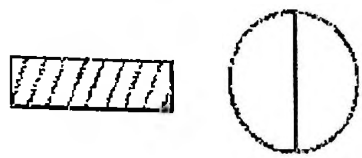
54. At a professional development workshop, teachers were learning about different ways to represent multiplication of fractions problems. The leader also helped them to become aware of examples that do not represent multiplication of fractions appropriately. Which model below cannot be used to show that $1\frac{1}{2} \times \frac{2}{3} = 1$? (Mark ONE answer.)

A)

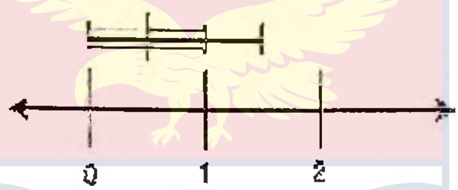
B)



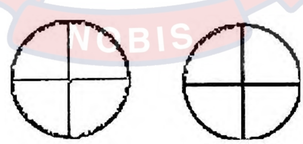
C)



D)



55. Mrs. Johnson thinks it is important to vary the whole when she teaches fractions. For example, she might use five dollars to be the whole, or ten students, or a single rectangle. On one particular day, she uses as the whole a picture of two pizzas. What fraction of the two pizzas is she illustrating below? (Mark ONE answer.)



A. $\frac{5}{4}$

B. $\frac{5}{3}$

C. $\frac{5}{8}$

D. $\frac{1}{4}$

56. Mr. Fitzgerald has been helping his students learn how to compare decimals. He is trying to devise an assignment that shows him whether his students know how to correctly put a list of decimals in order of size. Which of the following sets of numbers will best suit that purpose?

A. .5 7 .01 11.4

B. .60 2.53 3.14 .45

C. .6 4.25 .565 2.5

D. Any of these would work well for this purpose. They all require the students to read and interpret decimals.

57. If $f(x) = ax^3 + bx^2 + cx + d$, what is the slope of the line tangent to this curve at $x = 2$?

A. $8a + 4b + 2c$

B. $8a + 4b + 2c + d$

C. $12a + 4b + c$

D. $12a + 4b + c + d$

58. In a first year elective mathematics class, which of the following is **NOT** an appropriate way to introduce the concept of slope of a line?

A. Talk about the rate of change of a graph of a line on an interval.

B. Talk about speed as distance divided by time.

C. Toss a ball in the air and use a motion detector to graph its trajectory.

D. Apply the formula $slope = \frac{rise}{run}$ to several points in the plane.

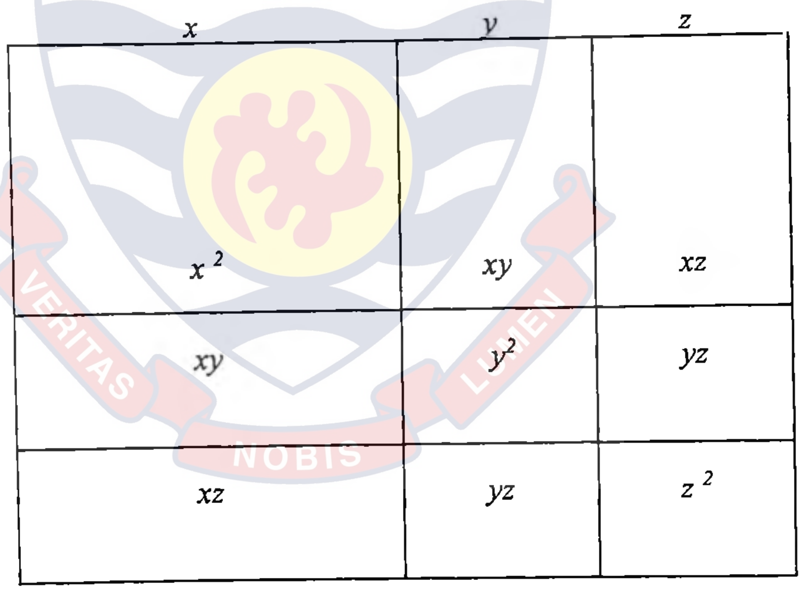
E. Discuss the meaning of m in the graphs of several equations of the form $y = mx + b$.

59. Which of the following (taken by itself) would give substantial help to a student who wants to expand $(x+ y + z)^2$?

iv. See what happens in an example, such as $(3 + 4 + 5)^2$.

v. Use $(x + y + z)^2 = ((x+ y) + z)^2$ and the expansion of $(a + b)^2$.

vi. Use the geometric model shown below.



A. ii only

B. iii only

C. i and ii only

D. ii and iii only

E. i, ii and iii

60. Which relation is a function?

A. $\{(-1,3),(-2,6),(0,0),(-2,2)\}$

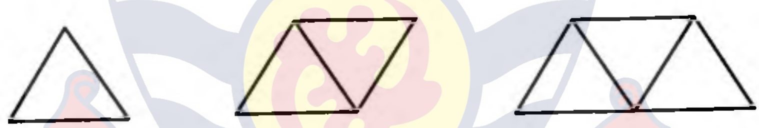
B. $\{(-2,-2),(0,0),(1,1),,(2,2)\}$

C. $\{(4,0),(4,1),(4,2),(4,3)\}$

D. $\{(7,4),(8,8),(10,8),(10,10)\}$

E. $\{(7,-4),(8,-8),(-10,8),(10,-10)\}$

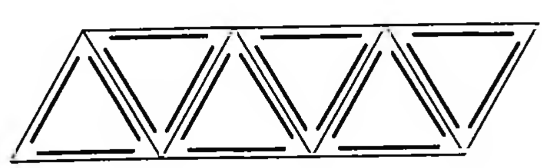
61. Amy is building a sequence of geometric figures with toothpicks, by following a specific pattern (making triangles up and down alternatively). Below are the pictures of the first three figures she builds. Variable *t* denotes the position of a figure in the sequence.



t=1 t=2 t=3

In finding a mathematical description of the pattern, Amy explains her thinking by saying:

"First, I use three sticks for each triangle:



But then I see that I am counting one stick twice for each of the triangles except the last one, so I have to take those away."

If f represents the total number of toothpicks used in a picture, which of the following equivalent formulas most closely matches Amy's explanation?

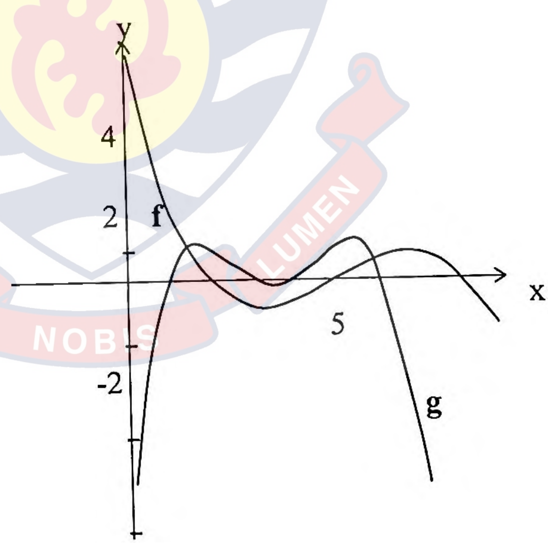
A. $f = 2t + 1$

B. $f = 2(t + 1) - 1$

C. $f = 3t - (t - 1)$

D. $f = 3t + 1 - t$

62. The graphs of two real-valued functions, f and g, are shown below. Each mark on the axes represents one unit.



How many solutions does the equation $f^3 - 2f^2g + fg^2 = 0$ have on
-4
the interval [0, 8]?

A. 2

B. 3

C. 5

D. 6

E. 7

63. Students were asked to solve the following problem.

Is it possible to have a polynomial of degree 10 of the form

$P(x) = x^{10} + a_9x^9 + \ldots + a_1x + 6$ *with 10 distinct integer roots?*

Which of the following is the most acceptable response to the question?

A. Yes, because every polynomial of degree *n* has *n* roots.

B. Yes, $P(x) = (x+1)^6(x-1)^2(x-2)(x+3)$.

C. Yes,

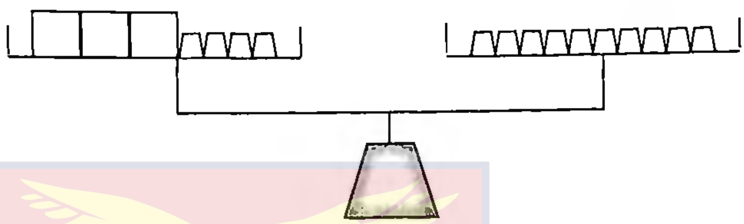
$P(x) = (x-1)^2(x+1)(x-2)(x+2)(x-3)(x+3)^2(x-6)(x+6)$.

D. No, because the only possible integer solutions to $P(x) = 0$ are $\pm 1, \pm 2, \pm 3, \pm 6$ (i.e. there are only eight factors of 6).

E. No, because $x^{10} + 6 = 0$ has some solutions that are not integers.

64. Some textbooks suggest that teachers use a pan balance to represent mathematical sentences. For instance, if B represents the weight of each box pictured below (in ounces), and represents a one-kilogram weight, the balance pictured below represents the equation

3B + 4 = 10



Ms. Clarke is preparing to teach a unit on solving linear sentences. If X represents the weight of a given box, which of the following sentences can **NOT** be represented by a pan balance?

A. 13 = 4X + 5

B. 3X + 10 = 4

C. 3X + 3 = 2X + 15

D. 9 + 6X < 21

65. Currently, Germany has a law against creating new surnames for newborns by combining the parents' surnames with hyphens. A language expert explains why hyphenation is not a good idea for naming:

If a double-named boy grew up to marry and have children with a double-named woman, those children could have four names, and *their children could have eight, and their children could have 16... The bureaucracy shudders.*

(Excerpt from the front page of *The Wall Street Journal*, Wednesday, October 12, 2005)

For which of the following topics could the situation described by the expert be used as an introduction?

A. Direct variations

B. Linear functions

C. Quadratic functions

D. Exponential growth

66. Consider the following mathematical topics:

i. Composition of functions

ii. One-to-one functions

iii. Inverse functions

iv. Domain and range of functions

67. Which of the following orders could be used to teach these topics in a rigorous advanced algebra class?

A. ii, i, iii, iv

B. ii, iii, iv, i

C. iv, ii, iii, i

D. They can be taught in any order.

68. Mr. Matheson asked students to solve the following system of equations:

$$\begin{cases} 2x + y = 3 \\ 4x + 2y = 6 \end{cases}$$

Orlando wrote:

$(-2)(2x + y) = 3(-2)$

So $-4x - 2y = -6$

$$4x + 2y = 6$$

$$0 = 0$$

This system doesn't have a solution.

Which of the following is true about Orlando's response?

A. Orlando's solution and reasoning are correct.

B. Orlando made an arithmetic error.

C. You cannot add equations.

D. Orlando drew the wrong conclusion from $0 = 0$.

E. None of the above

69. Which of the following questions that involve the equation $2x^2 - 3 = x + 1$ can be answered by graphing?

i. Determine how many real solutions the equation $2x^2 - 3 = x + 1$ has.

ii. Find the exact coordinates of the point(s) where the functions $f(x) = 2x^2 - 3$ and $g(x) = x + 1$ intersect.

iii. Determine the exact values of the solutions to the equation $2x^2 - 3 = x + 1$.

A. i only

B. ii only

C. iii only

D. i and ii

E. i, ii, and iii

70. When both sides of an equation reduce to the same number for certain values of the unknown number, the equation is said to be

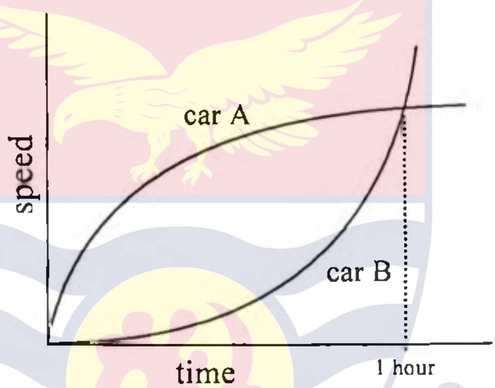
A. literal

B. satisfied

C. substituted

D. transitive

E. unsatisfied



71. The given graph represents speed vs. time for two cars. (Assume the cars start from the same position and are traveling in the same direction.) Use this information and the graph below to answer the question that follows.

What is the relationship between the ***position*** of car A and car B at $t =$ 1 hour?

A. The cars are at the same position.

B. Car A is ahead of car B.

C. Car B is passing car A.

D. Car A and car B are colliding.

E. The cars are at the same position and car B is passing car A.

72. Kwamena is taking medications for a recent illness. Every 6 hours he takes an antibiotic, every 4 hours he takes a pain reliever, and every 3 hours he drinks a glass of water. If he starts this regime at 10 am, at what time will he be taking both medicines and a glass of water?

A. 12:00 noon

B. 4:00 pm

C. 6:00 pm

D. 10:00pm

E. None of these

73. As a teacher, how would you view student errors and misconceptions? I would see student replies that reveal a misconception as [choose one]

A. more important than correct ones, as they provide an opportunity to extend learning for that student and for others in the class who may share the same misconception.

B. to be avoided at all cost.

C. needing to be immediately countered by the teacher's intervention about what the correct solution is.

D. useful for assessing student ability.

74. The Assistant headmaster who teaches mathematics students are working on the following problem: Is 371 a prime number?

As he walks around the room looking at their papers, he sees many different ways to solve this problem. Which solution method is correct? (Mark ONE answer.).

A. Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.

B. Break 371 into 3 and 71; they are both prime, so 371 must also be prime.

C. Check to see whether 371 is divisible by any prime number less than 20.

D. Break 371 into 37 and 1; they are both prime, so 371 must also be prime

APPENDIX B

ANOVA Table for teachers with background training in education and their colleagues without education background

		Sum of Squares	df	Mean Square	F	Sig.
SCHOOL KNOWLEDGE	Between Groups	.056	2	.028	.878	.417
	Within Groups	7.990	249	.032		
	Total	8.047	251			
TEACHING KNOWLEDGE	Between Groups	.011	2	.006	.223	.801
	Within Groups	6.346	249	.025		
	Total	6.358	251			
ADVANCED KNOWLEDGE	Between Groups	.064	2	.032	1.164	.314
	Within Groups	6.879	249	.028		
	Total	6.943	251			

APPENDIX C

ANOVA Table for effect of years of teaching experience on knowledge for teaching algebra

		Sum of Squares	df	Mean Square	F	Sig.
SCHOOL	Between Groups	.302	2	.151	4.862	.008*
KNOWLEDGE	Within Groups	7.744	249	.031		
	Total	8.047	251			
TEACHING	Between Groups	.078	2	.039	1.544	.216
KNOWLEDGE	Within Groups	6.280	249	.025		
	Total	6.358	251			
ADVANCED	Between Groups	.190	2	.095	3.498	.032*
KNOWLEDGE	Within Groups	6.753	249	.027		
	Total	6.943	251			

E. *significance at $p<0.05$

APPENDIX D

Rotated Component Matrix[a]

Item numbers	Component 1	Component 2	Component 3
q58	.653		
q40	.591		
q57	.589		
q66	.540		
q46	.524		
q39	.493		
q67	.437		
q44	.415		
q45	.351		
q65		.625	
q70		.559	
q41		.367	
q62			.463
q36			.461
q56			.377

APPENDIX D Cont'd

Rotated Component Matrix[a]

Item numbers	Component 1	2	3
q22			.372
q61			.371
q47			.362
q68			-.324

Extraction Method: Principal Component Analysis.

Rotation Method: Varimax with Kaiser Normalization.

a. Rotation converged in 6 iterations.