An Investigation of Junior High School students' ideology of fraction in the Cape Coast Metropolis of Ghana

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Abstract

This paper reports of a study that investigated Ghanaian JHS students' understanding of selected concept of fraction. In all, nine JHS 3 students comprising, three each from high, average and low performing Junior High schools were purposely selected. An interview guide was developed and used to interview the student participants. The data collected from the respondents was analyzed qualitatively and presented as per the concepts examined. The study revealed among others that the students had a better understanding of the part-whole concept. The least understood concept was the equivalence concept.

Introduction

UNESCO chose "No Child Left Behind" as a theme because the guiding principle for most educational systems in member countries is that all children should have equal educational and life opportunities in future. In this regard, per any given country, the elementary school curriculum is mostly the same. It is therefore expected that the core thoughts in the basic school curriculum must be mastered by all children in the basic schools (Wilson, 2009). Mathematics is considered a very important subject among the curriculum of many countries (Ghana is no exception) because progress in mathematics implies higher chances of educational and social advancement especially in the light of modern world advancement in information and communication technology (Mereku, 2010).

Wilson (2009) noted the five major basic building block of elementary school mathematics curriculum to be numbers, place value system, whole number operations, fractions and decimals, and problem solving. He noted that the importance of a strengthened elementary school mathematics foundation can never be more necessary than now. To this regard, the duty of the Ghanaian basic school teacher has never been so important and demanding (Mereku, 2010; TIMSS, 2003).

Ghanaian students performed least in the area of fraction in formal examinations, hence the reason other researchers have focused their attention on cultural effects on conception of fraction (Anamuah-Mensah and Mereku, 2005; Davis, Bishop, and TiongSeah 2010). However, there are not much studies in the Ghanaian context that focused on the students' deep rooted understanding of the concept of fraction. Also, fractions as a major basic building block in the Ghanaian basic school curriculum are the reason for conducting this study in the area of fraction. This study therefore sets out to be a model study that would provide the option for teachers to examine deep rooted ideologies of fraction among their students and unravel the students' associated in-debt difficulties. It uses the different findings of the different classification of schools to illustrate related findings that may be observed by teachers and the reasons for such different observations.

Conceptual Framework

The study adapts the conceptual model of fraction knowledge as proposed by Behr, Lesh, Post and Silver (1983) and cited in (Charalambous & Pitta-Pantazi, 2005). This conceptual model of fraction captures fractional knowledge as a hierarchical construct which can be sub-divided into smaller sub-constructs. Each construct/sub-construct has its associated properties that need to be demonstrated and linked by a person who is said to know fractions. Its hierarchical nature suggests that some basic fractional knowledge, with its properties, needs to be learnt before other properties of either the same construct or other sub-constructs are learnt. This framework of fraction knowledge is such that all other constructs are built on the part-whole and partitive fraction construct. Hence every other construct has a direct or indirect link to the part-whole construct (Norton & Wilkin, 2009). Other sub-constructs are Ratio, Operator, Quotient, Measure, and Problem solving. These are also linked to sub-constructs defined as operation of operators on properties of the previous constructs. It is important to understand that the concept being explored amongst the students is already formed. This conceptual framework therefore provides a basis for examining the presence or otherwise of the properties found in the constructs. Onwuegbuzie and Leech (2007) and Leech and Onwuebuzie (2007) strongly advised qualitative researchers not to compare a lot of attributes but to rather focus on 'thick description.' This study therefore used only a part of the conceptual framework of fraction knowledge (i) part-whole/partitive construct (ii) equivalence sub construct, and (ii) measure sub construct. These constructs were focused on because most of the other constructs extend from these basic concepts. Hence literature was reviewed on this line.

Part-Whole and Partitive Construct

Fractions can be considered as subset of the rational number scheme (Wu, 2008; Streefland, 1993, NCTM, 2002). It is important to understand that the part-whole and the partitive constructs are different. One of the basic properties in this construct is that fractional quantity is considered in relation to a bigger amount. The fraction quantity remaining is also conceptualized in relation to the whole such that when the two fraction quantities are put together, should exhaust the whole. Hence this construct recognizes that fractions do not exist in isolation but in relation to a bigger whole. Also, the remaining fraction of the whole (after a given fraction of a whole has been selected) is also considered in relation to the whole (Cramer & Wyberg, 2009, Peled & Awawdy-Shahbari, 2009).

The partitive construct is very similar to the part-whole construct. However, the difference lies in the ability of an individual to recognize a fraction of interest as made of a number of unit fractions. To be able to do this, one should be able to conceptualize any given whole as a number of unit fractions in the whole. Therefore considering a fraction in relation to the whole involves considering the fraction itself and the remaining fraction (when referenced to the whole) as being made of a number of unit fractions. Charalambous and Pitta-Pantazi (2005) and Steffe (2002) described this property as embededness/inclusive property.

In this description, for any given fraction, it is considered as many pieces in the fraction as related to many pieces in the whole. However, the many pieces in the fraction of interest are considered as a unit. The same ideology is applied to the remaining fraction quantity. In a similar manner, the many pieces in the whole are considered as a unit on its own (Steffe, 2002). Hence the relationship is that the many pieces found in the fraction of interest are parts obtained from the many pieces in the whole. Pitta-Pantazi and Charalambous (2007) were of the opinion that a student with this kind of understanding of fraction would be able to describe a fraction without double counting of the divisions when identifying the numerator and the denominator aspects in a representation. The explanation so far in relation to the part-whole/partitive fraction construct is what this paper adapted.

With this in mind, it is important to recognize the opinion of different scholars and researchers who seemed to believe that the opinion presented above is a limited view of the partitive construct (Norton and Wilkins, 2009; Baturo 2004). Norton and Wilkins (2009) were of the opinion that an important aspect of the partitive construct includes the ability to identify a fraction of interest as a number of *successive additions* of unit fractions to form the fraction of interest. Hence the whole would also be successive addition of unit fractions to form a whole. However, in the concept of fraction as postulated by in Behr et al, this perspective is part of the properties conceptualized in the measure sub construct of fraction knowledge.

The measurement sub construct is developed on a sound understanding of the part-whole/partitive construct, but does not imply that a well-grounded understanding of the part-whole/partitive construct can be translated to the understanding of other sub-constructs (Mazzocco & Devlin, 2008, pp.628).

Fraction ideology as part of a whole is the basic way of introducing fraction in most countries curriculum the world over (Wu, 2008). Understanding the partitive fraction construct involves (i) Conceptualizing the whole as made up of equal sized unit fractions, (ii) Conceptualizing the a fraction of interest as being made of a number of equal sized unit fractions, and (iii) conceptualizing the number of unit fractions in a fractional quantity as being part of the unit fractions in the whole. This was what was described by Tillema and Hackenberg (2011) as three level reasoning involved in reasoning about the partitive construct. According to Norton and Wilkins (2009), the part-whole fractional scheme, although idealized as different from partitive fraction, researchers' interaction with children seems to suggest that children conceive and use both concepts in the same manner. A strong base in the part whole ideology of fraction is necessary for progression in other sub-constructs. However, part-whole ideology on its own does not allow for easy progression. This gap is mostly filled by the partitive fractional ideology (Tillema & Hackenberg, 2011).

Fraction Equivalence

Equivalent fractions portray the idea of relative amount (Pitta-Pantazi & Charalambous, 2007) whether the items being compared are the same or not, this was observed to be trivial among children (Lammon, 1999; Pitta-Pantazi & Christou, 2010; Adjiage & Pluvinage, 2007). However, the comparison as examined in the equivalence sub construct requires the quantity to be of the same type. Two properties are required in the illustration of the equivalence sub construct; the covariance and the invariant property. The co-variance property of the equivalence sub construct comes from the property that when the numerator value of the original fraction is changing, the denominator value of the same fraction is changing in the same manner. In visual illustration, it means the changing of the divisions in the fraction illustration. This affects both the illustration of the fraction of interest and the whole (i.e. covariance). It is also important to recognize the fact that any fraction and its equivalence are of the same quantitative value (invariant property) (Charalambous & Pitta-Pantazi, 2005; Pitta-Pantazi & Charalambous, 2007). The consequence of the covariance property of equivalent fraction is what results in the invariant property. The covariance and invariant properties are not the product of multiplication or division of any fraction/quantity or its representation. Equivalent fractions are as a result of regrouping or re-arrangement (Wu, 2008). In any illustration of the idea of equivalent fraction, the co-variance and the invariance properties are the major distinction of equivalent fraction from the other fraction sub-constructs presented in this study.

Measure Ideology

Bell et al: (2007) noted that measurement is the basic and most common way of introducing fraction representation to most children but still, fractions remains a problem area for most children. The measure construct of fraction knowledge conceives of fraction for the purpose of measurement. Detailed and extensive work was done on this construct of fraction by Smith, Solomon, and Carey (2005). For every measurement, one requirement is that there should be a quantity to be measured (distance, area, or even volume). Hence fraction is conceived as a quantity on its own. Another basic requirement in measurement is the standardization of the unit of measurement. Hence for any fractional quantity, there should be a standard unit of measurement which should be a fraction. It therefore requires that to properly conceptualize fraction in the measurement construct, the quantity to be measured could represent the fraction of interest, $\frac{a}{b}$,whiles the unit of measurement could be considered as a unit fraction,$\frac{1}{b}$(Lamon,1999;Wu,2008). Starting from the origin of a measurement, a replication of the unit of measurement ($\frac{1}{b}$), 'a' number of times would give $\frac{a}{b}$. To properly grasp the measurement construct, one needs to use other unit fractions other than half as a unit of measurement (Vamvakoussi & Vosniadou, 2004; Lamon, 1999).

The Ghanaian primary and JHS mathematics syllabus starts formal teaching and representation of fraction in class two and continues consistently to JHS three. The rationale is to help young learners reason mathematically and communicate effectively with people using accurate mathematical data. The syllabus encouraged the use of different and varied methods of representation. Practical approach and problem solving were encouraged approach/method to be used in the classroom for instruction (MOESS, 2007a; MOESS, 2007b). The concepts examined are those that should have been studied by the students' concerned (three concepts).

Despite some difficulties among researchers in describing mathematical reasoning, its basic nature is that it builds on the structure of mathematics. To understand a mathematical concept, a student must absorb the mathematical reasoning that develops that concept. Hence per this study, demonstrated understanding of a particular concept, implies demonstrating the reasoning behind that mathematical concept (Wilson, 2009; Simon, McClintock, Akar, Watanabe, and Zembat 2010). However, this study focuses on the conceptual understanding of fraction among students.

Research Question

There was one major question that guided the study; What is the nature of Junior High School students understanding of the concept of fraction in the Cape Coast metropolis? The specific research questions are as follows:

i. What are Junior High School students understanding of the part-whole/partitive construct of fraction
ii. What are Junior High School students understanding of the equivalence sub-construct of fraction
iii. What are Junior High School students understanding of the measure sub-construct of fraction

Methodology

An attempt to obtain an in-depth understanding of students' conception of fraction a qualitative approach was used because there would be analysis of statements of explanation and possible illustrations. The case study design was adopted for the study because the intention was not towards generalization but to help reveal deep understanding issues (Creswell, 2008; Polkinghorne, 2005) that are not mostly available in most Ghanaian literature that examined fractional understanding.

Permission was obtained from the Metropolitan Director of Education, seeking her consent to allow the researchers conduct the study in the schools selected. The consent of the parents of the students was also sought through letters written to their parents. This was followed by a focus group interview which was videoed and later, transcribed. This allowed the researcher to replay the videoed interviews and captured students' illustrations that manifested students' understanding. A colleague videoed the interview sessions. This allowed the interviewer the concentration to re-examine issues in students' understanding (Boeije, 2010, Patton, 2003). A semi-structured interview guide was used in the study. All examples in the interview guide were adapted from the JHS3 syllabus (MOESS, 2007b).

Sampling

Three schools were selected from the list of all basic schools (66) in the Cape Coast Metropolitan Assembly. Maximum variation strategy was adopted for this study in order to meet the diverse school categories observed in the school population (Onwuegbuzie, and Leech, 2007; Leech & Onwuegbuzie, 2007; CREATE, 2007). Selection was based on the performance of the schools according to GES ranking of schools per their BECE results (GES, 2011). The purposive sampling technique was used to select the schools concerned. Since Ghanaian schools have been observed to have different levels of performance, the participating schools were purposely selected to reflect high, average and low performing schools (CREATE, 2007). Hence one school was selected each from high, average and low performing school group. Each school was selected from the middle of the 33rd percentile of each performance group (schools in each group were arranged according to performance). In each school, six students were selected. Among the six students, two each were selected as high, average, and low performing students. Junior High School three was chosen as the class of focus because by this level, students would have had the opportunity to learn all fractional concepts that the Ghana basic school curriculum expects them to possess as a basic education requirement (MOESS, 2008).

Analysis

Analysis was done at the school level. The common understanding that was observed in the students of a particular school is what is presented in this study. The interviews were studied to determine understanding of the properties of each sub construct. Hence, we present the understanding per students of high, average, low performing schools and compared. To demonstrate full understanding, it was required that students should be able to explain his/her ideology in real life terms and connect it logically to his/her chosen semi-concrete illustration. This helped to ensure that students had full understanding of what he/she was describing. No understanding indicates a total disconnection in ideological presentation and concepts being examined (Bell, et al, 2007). Presenting the findings was structured under the heading of the different construct/sub construct of fraction knowledge (part-whole, equivalence, and the measure). Presenting the findings for the part-whole construct was further divided into the three subheadings reflecting the school types. This was because of the differences in the way the students conceptualized fractions for the part-whole construct. Findings under the other sub-constructs of fraction were put together (per sub construct) for all the three school category because there was no significant difference in the nature of the students' conceptual understanding per the category of schools.

Findings

Understanding of Part-Whole/Partitive Construct

High Performing School

Among students of high performing school, evidence from the study suggested the presence of a mental framework that considered fractions as part of a whole (some number of objects put together). This framework had modelled fraction as a sharing. The sharing considered a number of selected parts together as the fraction. It also considered the remaining parts together as another fraction. The ability to add these two groups of divisions (items) in the fraction, to result in the whole (total number of objects when put together) was also evident. Also noted from the study was the fact that the students could not demonstrate understanding of the unit fractions to be equal. The students' mental framework of fraction seemed not to comprehend the similarity in the following two ideologies; unit fraction as a division of a singular item into many equal parts, and unit fraction as the division of a number of items (as a whole) into many equal parts (whether less than a whole or not). When considering items (as a group), students' conceptualized fraction as a number of items to be divided by a number of people (interview except below). However because the number of items to be shared is normally smaller than the number of people sharing, each person would definitely get less than one. Hence, because the sharing will result in each person obtaining less than a whole, reasoning about fraction switched from each share being considered as a fraction of the total number of objects to each share being considered as part of a singular whole in the total number of objects. However, despite the fact that students conceptualize equality of the unit fraction (property of partitive construct), they could not express it in terms of unit fraction but rather, decimals. This is illustrated in the interview except below. This seemed to suggest that any fraction in reference to a single whole is considered decimals.

Interviewer: how much orange will each person get? [Students had previously illustrated and explained ¾ to be three whole oranges to be shared amongst four persons]

Akua: 0.75 [other students' nods in agreement]

Interviewer: In fraction how much is it?

Akua: When it is that way and you convert it to decimals, you can't convert it back again.

Interviewer: Do you all agree with her?

All students: [They nod in agreement]

Interviewer: So, show me how the division will be done? Will each person get an equal amount?

Akua continues: Yes. The first person will get the division first 0.75 division of the first orange. The remaining division will be added to the next until it reaches 0.75. The process will continue to the last division.

Interviewer: I can see the last division is smaller than the rest. Do I then say that not everybody will get the same quantity?

Akua: Everybody will get the same quantity except the last person.

Average Performing School

Students' conceptual understanding of fraction suggested a mental framework that conceived fraction as one singular item that has been divided into smaller parts. Hence the students' conceptualized fraction as a group of small parts in the whole. One important characteristic of the framework was that these small parts or their groups were always described in relation to the whole. The same manner of conceptual understanding was per the remaining fraction. The two fractions when put together were understood to conserve the whole. Per this form of reasoning, equality of the unit fractions and the ability to conceptualize the non-unit fractions as being made of a number of unit fraction (embededness) was well established. However, there was a disconnection of the naming of the fractions from the ideas. All unit fractions less than a half was named as quarter. This can be found from the interview except below.

Interviewer:	In the drawing for the 5/8, what is the name of each small division?
Abena:	It is called a quarter
Interviewer:	Where is the quarter?
Abena:	She points to one small division (a 1/8).
Interviewer:	Write a quarter let's see.
Abena:	¾.
Interviewer:	If you add all the small quarters what will you get?
Abena:	You have to get one because the whole object is one item divided into 8.

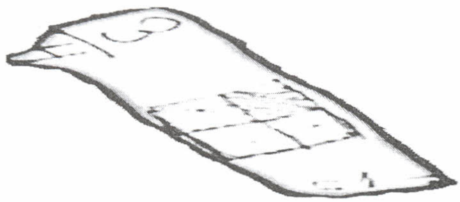
This suggests that although the naming has a problem, the concept was very correct and clear to them. However, students found it difficult to reason with fractions as a number of items (not subdivisions) in a group. When an attempt was made to investigate students' conceptualization in that regard, the students continued to do meaningless long division. Hence inability to demonstrate the different properties examined.

Low Performing School

For students of low performing school, evidence from the study suggested inability to understand most of the properties related to part-whole or partitive construct of fraction. The students were unable to conceive of fraction in relation to one singular whole item. Hence any property of fraction conceived as a singular whole was not existent. The framework for fraction that was evident was only in relation to a number of items; the value above the fraction represented the number of items to be shared among the denominator value (the number of people to share the items). Their conceptual understanding suggests a mental framework that does not support a sharing where an individual obtains a share less than one. Hence, when the items are shared, some will get a whole and some will not get a whole. This leads to a dichotomous unit fractional framework. The first unit fractional framework conceptualizes a unit fraction as an individual who obtains a share. The second unit fractional framework conceptualizes a unit fraction to be an individual who would not obtain a share in the sharing (since the denominator is bigger than the numerator). The fraction of interest was considered a part of the total number of people (since the fraction was conceptualized on the base of the number of people sharing). The remaining fraction was conceived as people who would not get a share. Although the conservation property was demonstrated in their mental framework, it was based on the number of people sharing and not the items to be shared. This can be observed in the interview excerpt below.

Interviewer: Can you show me with the aid of a diagram? [The students' had described a fraction in terms of people sharing and those who would and would not get shares]

All students: The four is the four divisions and the three people sharing it are the shaded part. The part



that is left is the person that will not get some among the four people doing the sharing (see fig 1).

Figure 1: Snapshot of LPS students' illustration of the concept of $\frac{3}{4}$

In students' ability to name the unit fractions and acknowledge its equality, conceptualize a fraction in relation to the whole, acknowledge conservation, it was evident in this regard that although the naming of the fractions were very correct, the framework was wrong. The understanding illustrated did not also support the reasoning about fraction in relation to a single whole.

Understanding of Equivalence Sub-construct

Students from high performing school were of the opinion that they do not understand the concept of equivalent fraction. However, they knew that it involved a number of fractions which bore some similar property but did not know the exact nature of such similarity nor the process that led to it.

Average performing school students' conception considered any fraction (with a numerator and a denominator that can be divided by the same factor) and divide (the numerator and the denominator, separately) with the common factor to result in another fraction. In these students' conception, 1/8 and 1/16 were considered equivalent because it was the same item that was divided further and hence the same number of division has to be selected from the new divisions. However, there was no consideration given to the size of the small division. This could be observed in the interview except below.

Interviewer: Why did you divide? Why did you not multiply or add or subtract? [Student had indicated that 1/8 when divided by 2 gives 1/16, hence they are equivalent].

Araba: From the diagram, when you draw 1/8 (as below). If it is divided into two, you will get 1/16. This implies that 1/8 is the same as 1/16. If you multiply, you will get more than.

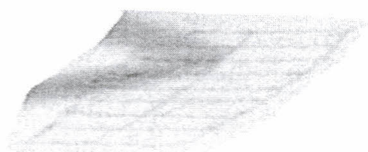


Figure 2: An illustration of APS students' conception of equivalent fraction

Equivalent fraction for low performing students was conceived as a procedure from one fraction to another. There was no meaning attached to it and the denominator and numerator values of any fraction was considered separately. Hence there was partial understanding of equality ideology among high performing students and the rest (average and low performing students) did not. The students' did not also understand the covariance property of equivalent fraction ideology.

Understanding of Measurement Sub-Construct

Students of high performing school were able to conceive a fraction as quantities to be measured and a unit of measurement. There was also the presence of a mental framework that allowed the ability to use the unit of measurement to successfully measure a fractional quantity of interest. However, this mental framework was only existent for simple common unit fractions being used as measurement unit/standard for common fractions. The framework seemed not to extend in the measurement of fractions greater than one when using common unit fractions as measurement standards. Students' conception could uniquely identify the measuring unit and the measurement standard. Conversely the ability to use the measurement unit to measure the fractional quantity proved to be non-existent. The possible reason for such was the lack of the ability to conceptualize a fractional quantity as greater than one. Hence all effort was focused on the numerator of the fractional quantity to be measured by the numerator of the unit fraction used as the measurement unit.

Average school students relied on algorithms which sometimes helped achieve the final conclusion but without any understanding. Low performing students were similar to average performing school but without arriving at the right conclusion and any understanding of the algorithms used.

Discussion of Results

High performing school students' conception of fraction was in relation to items in a group. This is consistent with most research findings as the basic way of introducing fractions in the light of students' earlier introduction to whole number division (Wu, 2008). Lamon (1999) opined that most students do not understand the partitive construct because most teachers and students focus on the part-whole construct. Findings from this study did not fully support Lamon since there were some properties of partitive construct that were demonstrated by the students (at least the unit fraction was understood to a degree). However, what was different was the reference point of the whole or the kind of mental framework that was observed from the interview. Students of the high performing school seemed to focus only on the whole as a group of items whiles students of average performing school focused on a singular item being divided into many parts. However, the findings showed that although students understood the part-whole/partitive construct, the kind of representation associated with it determines the possible limitations of their understanding (Cramer and Wyberg, 2009; Dey and Dey, 2010). It is also important to recognize that the two kinds of representation are not easily extended (understanding in one particular frame of mind does not necessarily imply understanding of the other frame of mind) into each other in students' mind (average performing school). This is because, representation of fraction as a group of items does not foster conception of fraction as less than a whole. Hence, its association to whole number sharing and the fact that introduction to whole number sharing in schools normally result in whole numbers, restricts logical knowledge transfer. However, concept of decimals starts with values less than a whole/one. Therefore reasoning about fraction as less than a whole then becomes decimals which cannot be converted back to fraction. On the other hand, representation of fraction in respect to a singular whole divided into parts does not foster the ideology of fractions being considered in relation to many items (group of items), hence when the reasoning shifts to a group of items, connection is made to whole number division.. Izsak (2008; 2009) and Keijzer & Terwel (2003) also noted that the kind of representation instructors use to represent certain fraction concepts should have logical connections that are depicted in the representation. The conceptions of low performing school students represent the possible difficulty students' face in crossing the bridge between whole number division and fractional knowledge. Hence students face difficulty in the ability to reason about sharing where an individual obtains less than a whole (in reference to sharing in a group). It is also important to observe that students of high performing school relied more on conceptual illustration of explanations, whiles average and low performing students demonstrated a tendency of using algorithm in most explanations. Most of such explanations were not correct in the case of students of low performing school. The form of difficulties faced by the students suggest students' high tendency for linearity in their thinking process, hence, over generalization (Smith, Solomon, & Carey, 2005). Despite all these, most students expressed a good understanding of the various properties of the part-whole/partitive fraction except students in the low performing schools. Observation was that these low performing students demonstrated misunderstanding of almost all the properties of the concept of fraction observed.

All students in the various school types did not understand the concept of equivalent fraction. Average performing school students imagined the two equivalent fractions to be separate fractions on their own as was also noted in Pitta-Pantazi and Charalambous (2007). Students of the low performing school had a framework that suggested equivalent fractions to be a procedure from one fractional value to another; an illustration of heavy reliance on algorithms. Apart from students' of high performing school who noted that equivalent fractions had an idea of equality attached to it, none of the students had an idea about the co-variance property. Hence equivalent construct was the least understood construct among the students.

The measurement construct was difficult for low and average performing school students but students' of high performing school were an exception. For all students (high, average and low), there was the lack of the ability to conceptualize fraction as being equal or greater than a whole. Conceptualizing fraction as greater than whole normally contradicts initial students' introduction to fraction as magnitude less than whole. This also suggested that the idea of fraction being considered as a value in the numbering system was likely to be absent (Smith, Solomon, & Carey, 2005). Lee and Orill (2009) noted that researchers should also examine students' conception of fraction beyond common fractions as half, a third, three-fourth etc. Fractions that are not commonly used as 5/7, 7/6/ 9/13, 14/11 should be used in the process of advancing understanding of fraction. This is consistent with the finding of this study where high and average performing school students were able to demonstrate the concept of the measure sub construct of fraction but only in the case of simple and common fractions. Fraction serves the purpose of number utility where whole numbers are not capable. Hence numbers below one and between two whole numbers represents areas where fraction fills-in in improving the utility of number in our world (Smith, Solomon, & Carey, 2005; Lee & Orill, 2009).

Results from this study supports the model of Pitta-Pantazi and Charalambous (2007) that the part-whole/partitive construct serves as the basis for the development of other sub-constructs of fraction. Also inability to have a well-established understanding of the part-whole/partitive construct would affect the level of understanding of the other sub-constructs. This is supported by the fact that high performing students were the only group that could demonstrate understanding of some of the properties of the equivalence and the measurement sub construct of fraction. Students of average performing school, although demonstrated some understanding of properties of the part-whole/partitive construct could not demonstrate understanding of the equivalence and the measurement sub construct of fraction. Low performing students did not understand the part-whole/partitive construct and could not understand the remaining sub-constructs.

Conclusions and Implication

The best understood (sub)construct among the students studied was the part-whole. This was followed by the measurement sub-construct. The least understood was the equivalence sub-construct. Even though the study was conducted on a small scale, it therefore could not be generalized. The findings points to what may be happening in other schools. It is therefore necessary to conduct the study on a larger scale with more schools.

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