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Dynamics of Maize Price in Ghana: Linear versus Nonlinear Cobweb Models

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Abstract
This paper intends to study the price dynamics of maize in Ghana using cobweb models derived from linear demand and nonlinear supply function and then compare with that from linear demand and supply functions which are constructed from real economic price and production data of maize. Comparative analysis of stability conditions of the equilibrium was discussed.
The results from the linear cobweb model provided unstable equilibrium state of prices towards the zero equilibrium price as well as the supply. Thus the system is unstable and no equilibrium price is achieved towards the equilibrium point $P_e = 0$ which is also not realistic because of producers' sensitivity to price.
However, the nonlinear cobweb model provided two equilibria of which one is also stable at the zero equilibrium price and the other unstable at non-zero equilibrium price which is realistic and a reflection of maize price system due to inflation and insufficiency of food supply at the markets in Ghana.
It is deduced that linear model though provides an acceptable estimate of most problems researchers come across but in real economic situations, there is the need to use nonlinear approach as it is shown in this study to avoid under or overestimation and make better predictions.
Keywords: maize, price dynamics, cobweb models and difference equation

1. Introduction
Maize is the most important cereal crop produced in Ghana and also the most widely consumed staple food in Ghana with increasing production since 1965 (FAO, 2008; Morris *et al*., 1999).
The per capital consumption of maize in Ghana in 2000 was estimated at 42.5 kg (MoFA, 2000) and an estimated national consumption of 943000 Mt in 2006 (SRID, 2007).
White maize consumption is projected to increase due to population growth and increasing per capita income. Based on the most recent domestic production data, the shortfall between domestic production and domestic consumption would reach 267 000 Mt by 2015 in case there is no productivity improvement (MOFA, 2011).
In sub-Saharan Africa, maize is a staple food for an estimated 50 % of the population and provides 50 % of the basic calories. It is an important source of carbohydrate, protein, iron, vitamin B, and minerals. Africans consume maize as a starchy base in a wide variety of porridges, pastes, grits, and beer. Green maize (fresh on the cob) is eaten parched, baked, roasted or boiled and plays an important role in filling the hunger gap after the dry season. Maize grains have great nutritional value as they contain 72 % starch, 10 % protein, 4.8 % oil, 8.5 % fibre, 3.0 % sugar and 1.7 % ash (Chaudhary, 1983).
Maize is cultivated worldwide and represents a staple food for a significant proportion of the world's population (International Food Biotechnology Council, 1990) and in Ghana; it is produced predominantly by smallholder resource poor farmers under rain-fed conditions (SARI, 1996).
In every market-oriented economy with perfect information, key variable in the food system is the price of the commodity. Prices determine revenues which in turn provide incentives to participants through rewards (profits) and penalties (losses). Prices therefore, serve as an efficient means for seeking out production possibilities and potential, as well as allocating scarce resources within an economy. In view of this, changes of food price are becoming increasingly relevant to producers and consumers in the new competitive food markets. (Gortz and Weber, 1986; Kuwornu et. al., 2011).
According to ISSER (2008), high food prices have radically different effects across countries and population groups. Countries that are net food exporters benefit from improved terms of trade while net food importing countries however, struggle to meet domestic food demand. Given that almost all countries in Africa are net importers of cereals, they will be hard hit by rising prices. Higher food prices lead poor people to limit their food consumption and shift to even less-balanced diets, with harmful effects on health in the short and long run: for the fact that the market is the main arbiter of how the available food is distributed both within and between countries. Sanyang and Jones (2008) also observed that, higher food prices result in people eating less frequently and in lesser quantities, as well as cheaper and less nutritious food.
Despite the global effort and control measures taken over the years, there has been a steady upward trend in

world food prices at a modest rate. A major contributor has been increase in crude oil price since a large increase in crude oil prices stands out among numerous factors to explain most of the jump in food prices over the last decade. Indeed, as we found in a recent World Bank study, oil prices were more important to food prices than several other long-term price drivers, including exchange rates, interest rates and income (Baffes, 2013). Oil price increases exert both direct and indirect upward pressures on aggregate prices (ISSER, 2008).

In Ghana, food prices for maize, rice and other cereals increased by 20 to 30 percent between the last few months of 2007 and beginning of 2008 (Wodon *et al.,* 2008). This raised concerns about future food prices and their effect on food security in Ghana. Research has shown that foodstuff prices exhibit high volatility with continual increasing prices in recent years with out-of-sample forecast revealing that maize increased by 23% (Kuwornu et. al., 2011).

In agriculture, planting decisions such as quantity to supply are taken on the basis of expected prices at harvest; hence having knowledge of the dynamics of food price would give farmers the opportunity to take informed decisions regarding planting in the future (White and Dawson, 2005).

In this study, we intend to study the price dynamics of maize in Ghana using cobweb model with linear demand and nonlinear supply function of price and then compare with linear demand and supply function to find out the best fit model for real economic data of maize price and production in Ashanti Region, Ghana. This we believe would assist stakeholders to come out with schemes to stabilize prices and ensure food availability.

The cobweb model or cobweb theory is an economic model that explains why prices might be subject to periodic fluctuations in certain types of markets. It describes cyclical supply and demand in a market where the amount produced must be chosen before prices are observed (Web Definition, 2014).

Cobweb models therefore describe the price dynamics in a market of a non-storable good that takes one time unit to produce (Brock and Hommes, 1997). In economic modeling, many examples of cobweb with different form of functions (linear or nonlinear) have been demonstrated. Finkenstadt (1995) applied linear supply and nonlinear demand functions. Hommes (1991) and Jensen and Urban (1984) used linear demand functions with nonlinear supply equations. Junhai and Lingling (2007) established a nonlinear real estate model based on cobweb theory, where the demand function and supply function are quadratic. These findings indicate that the nonlinear cobweb model may explain various irregular fluctuations observed in real economic data.

Supply and demand models are effective means of modeling how market forces determine the price, the quantity supplied of by producers and the quantity demanded of by consumers of a good or service. At the market-clearing equilibrium, we have the condition that demand equals the supply.

In more realistic economic models the supply and demand curves will be non-linear. If the supply and demand curves are monotonic and single-valued, the behavior remains qualitatively the same as in the linear case. Only the three classes of behavior, convergence to a fixed point, period-2 cycles, and instability, will be observed. However, for economic systems with backward bending supply curves or multiple-valued demand curves the dynamics described by the non-linear at equilibrium (market clearing equation-MCE) can exhibit a broad spectrum of complex behavior (Jensen and Urban, 1984).

2. Method

In order to study the dynamics of maize price in Ashanti Region-Ghana, records of price and production of maize from seventeen major commercial centres were selected. This paper uses secondary data of maize that span a period of 19 years, from the Ministry of Food and Agriculture Statistical Directorate Kumasi- Ghana as shown in table 1. Linear and nonlinear difference equations that constitute cobweb models are used for the study of price dynamics.

Table 1: Price and Production of Maize in Ashanti Region

Year	Prices of Maize (Bag of 100 kg) in GH¢				Production of Maize (Metric Tons)				
	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter	Total
1994	1.23	1.78	1.49	1.67	10198.13	14830.01	12403.34	13878.19	51309.67
1995	2.69	3.53	1.64	2.22	15947.12	20917.51	9730.17	13171.87	59766.67
1996	2.80	3.41	4.61	4.45	10152.87	12357.34	16710.75	16125.71	55346.67
1997	6.51	8.73	6.74	-	17268.13	23180.01	17886.86	-	58335.00
1998	5.06	5.62	4.43	4.26	19354.45	21514.8	16930.53	16287.52	74087.30
1999	4.64	4.94	5.76	4.60	15016.85	15978.78	18646.42	14875.28	64517.33
2000	9.32	11.05	9.90	10.39	14663.71	17381.97	15579.11	16342.87	63967.66
2001	13.87	18.76	12.98	13.04	13403.83	18127.28	12539.71	12595.85	56666.67
2002	14.83	15.38	10.97	11.36	25347.56	26300.98	18760.61	19417.51	89826.66
2003	14.03	17.06	16.32	14.49	14649.34	17818.76	17039.9	15132	64640.00
2004	17.80	21.67	23.46	23.06	12629.82	15372.77	16646.07	16362.01	61010.67
2005	29.67	45.02	34.25	28.56	11638.64	17660.19	13436.17	11203.67	53938.67
2006	25.91	27.39	20.98	18.87	15226.02	16097.1	12328.32	11090.56	54742.00
2007	25.85	32.34	26.41	26.06	13190.26	16500.28	13474.43	13296.04	56461.01
2008	32.34	56.66	56.72	49.39	10103.38	17700.44	17717.1	15428.41	60949.33
2009	60.79	71.77	55.13	52.21	15780.74	18631.95	14311.43	13552.55	62276.67
2010	53.29	55.51	52.76	46.82	21597.84	22497.63	21385.72	18976.81	84458.00
2011	55.10	75.41	81.34	89.95	10572.89	14470.08	15607.96	17260.74	57911.67
2012	110.69	124.44	90.97	76.00	18169.81	20426.27	14932.32	12475.61	66004.01
2013	77.09	76.24	74.86	81.67	17036.49	16847.17	16542.22	18047.12	68473.00

The table 1 contains the quarterly price and production of maize data at the right and left sections of the table respectively.

2.1 Linear cobweb model

We consider an economic system with a single good. The cobweb model uses the demand and supply functions of price curves to determine the price P_k at k time from the price P_{k-1} at time k-1. The market clearing equation (MCE) is the first order non-homogeneous difference equation which describes this process (Nicholson, 1978). Given an initial price P_{k-1}, the market responds at time k with a quantity D_k determined by the supply curve S_k. therefore the market demand determines the current price P_k.

If the demand and supply curves are linear given as:

$$D_k = \alpha - \beta P_k \quad (1)$$

$$S_k = \delta + \gamma P_{k-1} \quad (2)$$

where β represents the slope while α represents intercept for the demand function, and also γ and δ respectively represent the slope and intercept of the supply function (Ezekiel, 1938; Goldberg, 1986).

At the MCE (1) = (2) ie $D_k = S_k$

$$P_k = AP_{k-1} + B \quad \text{where} \quad A = -\gamma/\beta \quad \text{and} \quad B = (\alpha - \delta)/\beta \quad \text{for (k=1, 2, 3, …)} \quad (3)$$

The solution of difference equation (3) is given as

$$P_k = A^k P_0 + B\left(1 - A^k \big/ 1 - A\right) \quad \text{where A} \neq 1 \text{ ,and k=0, 1, 2, …} \quad (4)$$

The homogenous part of the solution is also given by: $Ph_k = A^k P_0$

By the fact a constant is not changed by the application of the lag operator (Kirchgässner and Wolters, 2007; Anokye and Oduro, 2013), applying the lag operator in the equation (3) hence gives;

$$P_k = B/(1-A) = (\alpha - \delta)/(\beta + \gamma) \quad \text{where } A = -\gamma/\beta \text{ and } B = (\alpha - \gamma)/\beta$$

If we let P_e denote equilibrium price then in this case the system is in equilibrium when the price remains constant for all time periods, i.e

$$P_k = P_{k-1} = ... = P_e = (\alpha - \delta)/(\beta + \gamma) \text{ where if } \alpha \geq \delta \quad (5)$$

Convergence conditions

For $|\gamma/\beta| < 1$ then the price converges to the equilibrium price

However, for $|\gamma/\beta| > 1$, the system is unstable and no equilibrium price is achieved and for $|\gamma/\beta| = 1$ the price oscillates in a period-2 cycle between P_{k-1} and P_k .

2.2 Nonlinear cobweb model

If the demand function is linear and supply function or curve is backward bending non-linear given as

$$D_k = \alpha - \beta P_k \quad (6)$$

$$S_k = \delta + \gamma P_{k-1} - \rho P_{k-1}^2 \quad (7)$$

MCE at (12) = (13), the following nonlinear model is obtained;

$$P_k = C - BP_{k-1} + AP_{k-1}^2 \text{ where } C = (\alpha - \delta)/\beta \text{ , } B = \gamma/\beta \text{ and } A = \rho/\beta \quad (8)$$

This is quadratic equation having two fixed points, $P_k = \frac{(1+B) \pm \sqrt{(1+B)^2 - 4AC}}{2A}$ provided $(1+B) - 4AC \geq 0$

- if $\frac{(1+B)^2}{4A^2} - \frac{C}{A} > 0$ we have two equilibrium points
- if $\frac{(1+B)^2}{4A^2} - \frac{C}{A} = 0$ only one equilibrium point exists
- if $\frac{(1+B)^2}{4A^2} - \frac{C}{A} < 0$ then there are no equilibrium points

This implies that bifurcation occurs when $\frac{(1+B)^2}{4A^2} - \frac{C}{A} = 0$ or equivalently bifurcation values are $(1+B) = \pm 2\sqrt{AC}$ which gives $P_k = \frac{(1+B)}{2A}$

Bifurcation Discussion

- At the left of $(1+B) = 2\sqrt{AC}$:no equilibrium
- At $(1+B) = 2\sqrt{AC}$: we have a (node) up, i.e attractive from below and repelling from above.

At the right of $(1+B) = 2\sqrt{AC}$: we have two equilibria, the smaller one is a "sink", while the bigger one is a "source" which explains the node behaviour of $P_k = \frac{(1+B)}{2A}$.

Note that similar conclusions hold for the other value $(1+B) = -2\sqrt{AC}$.

3. Results and Discussion

The objective of the analysis is to study the price dynamics of maize in Ghana using cobweb model with linear demand and nonlinear supply function of price and then compare with linear demand and supply function to find out the best fit model for real economic data of maize price and production in Ashanti Region, Ghana. The modeling of the functions and estimate of the model parameters in this study are done using SPSS.

3.1 Demand and Supply Functions of Price

Preliminary data analysis was performed to ensure that the price and production data are stationary before regression analysis was employed to formulate the various demand and supply functions. The following table contains the coefficients of the parameters estimated from data analyzed.

Table 2: Coefficients of demand and supply functions

Model	Unstandardized Coefficients		Standardized Coefficients			
	B	Std. Error	Beta	t	sig.	
2. Price	-96.162	20.257	-0.488	-4.747	0.000	Demand Function
a. Dependent Variable: Production			b. Linear Regression through the Origin			
3. Price	354.28	36.887	1.715	9.604	0.000	Supply Function
Price**2	-2.782	0.453	-1.096	-6.14	0.000	
4. Price	167.994	43,229	0.419	3.886	0.000	
a. Dependent Variable: Production_Lag			b. Linear Regression through the Origin			

The table above contains parameter estimates of demand and supply functions with their p-values showing significant level of the respective parameters in the function.

3.1 Demand Function of Price

The demand function from the table 2 is given below having checked that its parameter estimate is significant.

$$D_k = -96.162P_k \quad (9)$$

3.2 Supply Functions of Price

$$S_k = 167.994P_{k-1} \quad (10)$$

$$S_k = 354.28P_{k-1} - 2.782P_{k-1}^2 \quad (11)$$

MCE for (9 & 10) provide the linear cobweb model given as follows;

$$P_k = -1.75P_{k-1} \quad (12)$$

Since $|\gamma/\beta| = |1.75| > 1$ the system is unstable and no equilibrium price would be achieved. Thus prices would never converge towards the equilibrium point, $P_e = 0$, which is also not realistic because means producers are more sensitive to price.

MCE for (9 & 11) provide the following nonlinear cobweb model;

$$P_k = -3.684P_{k-1} + 0.029P_{k-1}^2 \quad (13)$$

This is quadratic equation having two fixed points, $P_k = \dfrac{(4.684) \pm \sqrt{(4.684)^2}}{2(0.029)}$ i.e. P_k=161.52 as "source" or P_k = 0 as "sink" ,bifurcation occurs when P_k=80.75 and it's very attractive from below and repelling from above.

The nonlinear cobweb also shows that price will be unstable at P_k =161.52 and therefore fluctuating scope would be bigger and bigger but stable at P_k = 0 since $P_k^{'} < 1$ for stability to prevail. In Ghana prices of commodities are driven by inflation and insufficiency of food supply at the market. In reality price of maize cannot be expected to be stable.

4. Conclusion

A nonlinear model for real foodstuff market has been presented with backward bending supply function of price and linear demand function of price based on the cobweb theory. It is a simple dynamic model with two fixed points. It was compared with linear model which provided unstable equilibrium of prices towards the zero equilibrium price as well as the supply. Thus prices would never converge to the equilibrium price $P_e = 0$ which is also not realistic because of producers' sensitivity to price.

However, the nonlinear model provided two equilibria of which one is also stable at the zero equilibrium price and the other unstable at non-zero equilibrium price which is realistic and true reflection of maize price in Ghana due to inflation (prices of petrol are reviewed every two weeks and it has counter effects on food prices) and insufficiency of food supply at the market.

It is deduced that linear model though provides an acceptable approximation of most problems researchers encounter but in real economic situations, there is the need to use nonlinear approach as it is shown in this study to avoid under or overestimation and make better predictions.

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