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AMPLIFICATION OF ACOUSTIC PHONONS IN A DEGENERATE SEMICONDUCTOR SUPERLATTICES

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Abstract

Amplification of acoustic phonons propagating along the axis of a semiconductor superlattice (SL) is investigated using the microscopic theory. A non-quantizing electric field is applied to the SL to produce a drift velocity v_D of the charge carriers and whenever v_D exceeds v_s (sound velocity) amplification occurs. The threshold field E_0 at which absorption switches over to amplification depends on the SL parameters and v_s. It is noted that E_0 in SL is by far lower than that in bulk semiconductor and that there exists the possibility of finding a field E^* such that $-\Gamma(E^*) \geq \Gamma(-E^*)$. This allows in principle the use of SL as hypersound generator.

1. Introduction

The possibility of controlling the properties of semiconductor superlattice (SL) either by varying the SL parameters or by external fields makes it suitable for opto- and acousto-electronic devices. In view of that a lot of information has been reported on SL [1-10].

The interaction of acoustic waves with conduction electrons in homogeneous materials has been considered on several occasions [11-15]. For instance, the study of ultrasonic amplification and non-Ohmic behaviour of II - VI and III - V compounds such as GaAs, GaSb and InSb [16-19]. Apart from that there are several reviews on this subject [20-23]. More recently, this subject is being studied in two-dimensional and nanostructures [24-26].

With SL because of its novel properties much work is also being done. In [27] Shmelev et al studied the hypersound amplification in a non-quantised electric field. They noted that in principle SL can be used as hypersound generator which is impossible to construct in the case of homogeneous semiconductors. Kryuchkov [28] reported of oscillations of the attenuation coefficient of ultrasound in quantising electric field. Mensah et al [29] considered the effect of a high-frequency electric field on the hypersound and observed that the field modulates the amplification coefficient. Photostimulated attenuation has also been reported [30,31].

Very recent paper on generation of high-frequency coherent acoustic phonons in SL is reported in [32] where it is emphasized that the effect can be employed for electric generation of high-frequency coherent acoustic phonons.

In this paper, we consider the amplification of hypersound in a degenerate semiconductor SL in the presence of constant electric field. The motivation is due to the numerous applications that can be derived from such study, e.g., phonon spectrometer, generation of high-frequency electric oscillations, tetrahertz modulation of light, non-destructive testing of microstructures and acoustic scanning.

The organization of the paper is as follows. In section 2, a kinetic equation based on the linear approximation is set up for the phonon distribution function. In section 3 we derive the growth rate of the phonon distribution function at low temperatures. In section 4 we calculate the amplification coefficient and finally in section 5 we give brief discussion and conclusions.

2. Kinetic Equation

In developing the kinetic equation we follow the work of Lee and Tzoar [33]. We consider an electron phonon system where the electrons are assumed to be drifting relative to the ion lattice because of an external field. In this process we ignore the electron-electron interactions because firstly, we assume that the wavelength of the phonon is short compared with the screening length for the electrons. Secondly it would only produce higher order corrections to the phonon distribution function. The electron-phonon interaction C_q is assumed to be weak and treated as a perturbation. The unperturbed electron distribution function is given by the shifted Fermi-Dirac function:

$$\tilde{f}_p = \left[\exp \beta\varepsilon\left(p - mv_D\right) - \beta\mu\right]^{-1} \tag{1}$$

where μ is the chemical potential, p is momentum of the electron, $\beta = \frac{1}{kT}$, k is the Boltzmann constant and v_D is the net drift velocity relative to the ion lattice site.

According to perturbation theory, the transition probability per unit time from the initial state $|p\rangle$, which consists of one electron having a momentum p, to the final state $|p'\rangle$, $(p' = p - q)$ which consists of an electron with momentum p' and a phonon with a wave vector q is obtained as follows (emission rate)

$$p^{+} = 2\pi \left|C_{-q}\right|^{2} (N_q + 1)\, \delta_{p,p'} \delta\left(\varepsilon_{p'} - \varepsilon_p + \omega_q\right) \tilde{f}_p \left(1 - \tilde{f}_{p'}\right) \tag{2}$$

here $\hbar = 1$ (energy unit), where $|C_{-q}|$ is the quantum-mechanical matrix element describing the electron-phonon coupling. Similarly, the probability of absorbing a phonon q by a p' electron is given as

$$p^{-} = 2\pi \left|C_q\right|^{2} N_q \tilde{f}_{p'} \left(1 - \tilde{f}_p\right) \delta_{p',p} \delta\left(\varepsilon_{p'} - \varepsilon_p + \omega_q\right) \tag{3}$$

The factor $N_q + 1$ accounts for the presence of N_q phonons in the system when the additional phonon is emitted. The $\tilde{f}_p \left(1 - \tilde{f}_{p'}\right)$ represents the probability that the initial p state is occupied and the final electron state p' is empty. Similarly, the terms in Eq(3) arises from the absorption process and the factors $N_q \tilde{f}_{p'} \left(1 - \tilde{f}_p\right)$ again take care of the boson and fermion statistics. The total rate of absorption and emission of phonon can be obtained by the summation of Eq(2) and Eq(3) over all the initial and final electron states

$$P^{\pm} = \sum_{p,p'} p^{\pm} \tag{4}$$

The summation p' can be computed using the known δ symbol. If the phonon state q is highly excited (intense acoustic wave), $|C_{-q}|^2 \sim |C_q|^2$ and we can write the kinetic equation for the phonon distribution as

$$\frac{\partial N_q(t)}{\partial t} = 2\pi \sum_{p} \left|C_q\right|^2 \left\{ \left[N_q(t) + 1\right] \tilde{f}_p \left(1 - \tilde{f}_{p'}\right) \delta\left(\varepsilon_{p'} - \varepsilon_p + \omega_q\right) \right.$$
$$\left. - N_q(t) f_{p'} \left(1 - f_p\right) \left(\varepsilon_{p'} - \varepsilon_p + \omega_q\right) \right\} - \gamma N_q(t) \tag{5}$$

where $N_q(t)$ represents the number of phonons with a wave vector q at time t and γ is the phonon losses. These losses can include phonon scattering or phonon absorption due to non-electronic mechanisms, phonon decay due to anharmonicity of the lattice etc. In this paper we neglected γ.

It is important to note that the electron distribution functions in Eq(5) should be the exact, time-dependent functions $F_p(t)$ and $F_{p'}(t)$ instead of the unperturbed $\tilde{f}_p$ and $\tilde{f}_{p'}$. However, to the lowest order in the electron-phonon coupling, one can approximate F_p by $\tilde{f}_p$.

We rewrite Eq(5) in a more transparent and convenient form after some algebraic manipulations as

$$\frac{\partial N_q(t)}{\partial t} = 2\pi |C_q|^2 \left[\frac{N_q(t)+1}{1-\exp\beta(\omega_q - \mathbf{q}\cdot\mathbf{v}_D)} + \frac{N_q}{1-\exp\beta(\mathbf{q}\cdot\mathbf{v}_D - \omega_q)} \right]$$

$$\times \sum_p \left(\tilde{f}_p - \tilde{f}_{p'}\right) \delta\left(\varepsilon_{p'} - \varepsilon_p + \omega_q\right)$$

$$= 2\pi |C_q|^2 \left[N_q(t) - \frac{1}{\exp\beta(\omega_q - \mathbf{q}\cdot\mathbf{v}_D) - 1} \right] \sum_p \left(\tilde{f}_p - \tilde{f}_{p'}\right) \delta\left(\varepsilon_p - \varepsilon_{p'} - (\omega_p - \mathbf{q}\cdot\mathbf{v}_D)\right)$$

$$= 2|C_q|^2 \left[N_q(t) - \frac{1}{\exp(\omega_q - \mathbf{q}\cdot\mathbf{v}_D) - 1} \right] \operatorname{Im} Q\left(q, \omega_q - \mathbf{q}\cdot\mathbf{v}_D\right) \tag{6}$$

where

$$\tilde{f}_p = \left[\exp\beta(\varepsilon_p - \mu) + 1\right]^{-1} \tag{7}$$

is the Fermi-Dirac equilibrium function and

$$Q = \sum_p \frac{f_p - f_{p'}}{\varepsilon_p - \varepsilon_{p'} - \omega - i\delta} \tag{8}$$

3. Phonon Instability

From Eq(8) we see that the phonon generation rate is given as

$$\Gamma_q = 2|C_q|^2 \operatorname{Im} Q\left(q, \omega_q - \mathbf{q}\cdot\mathbf{v}_D\right) \tag{9}$$

$$= 2\pi \left|C_q\right|^2 \sum_p \left(\tilde{f}_p - \tilde{f}_{p'}\right) \delta\left(\varepsilon_p - \varepsilon_{p'} - \left(\omega_q - \mathbf{q}\cdot\mathbf{v}_D\right)\right)$$

By recalling that $\tilde{f}\left(\varepsilon_p\right) \geq \tilde{f}\left(\varepsilon_{p'}\right)$ when $\varepsilon_p < \varepsilon_{p'}$, it is easy to see that

$$\Gamma_q \gtrless 0 \, for \, \omega_q - \mathbf{q}\cdot\mathbf{v}_D \gtrless 0 \tag{10}$$

It follows from Eqs(6) and (10) that for $\omega_q - \mathbf{q}\cdot\mathbf{v}_D > 0$, the system when perturbed would always return to its equilibrium configuration,

$$N_q^0 = \left[\exp\beta\left(\omega_q - \mathbf{q}\cdot\mathbf{v}_D\right) - 1\right]^{-1} \tag{11}$$

since $\frac{\partial N_q}{\partial t} \lessgtr 0$ for $N_q \gtrless N_q^0$. Note that the particular case $v_D = 0$ is included since the phonon frequency ω_q is, by definition, a positive quantity.However, for $\omega_q - \mathbf{q}\cdot\mathbf{v}_D < 0$ there is no stable equilibrium configuration since $\frac{\partial N_q(t)}{\partial t} \geq 0$ for all t according to Eqs(6) and (10). The number of phonons would then generally grow exponentially at a rate given by Eq(9). Therefore, if we set $\gamma = 0$ for a particular phonon wave vector q, the criterion for the onset of phonon instability is just the Cerenkov condition

$$\omega_q - \mathbf{q}\cdot\mathbf{v}_D < 0. \tag{12}$$

For finite $\gamma > 0$, the instability criterion becomes

$$\Gamma_q > \gamma$$

It is clear that in addition to a drift velocity threshold whose component in the q direction still has to exceed the phonon phase velocity there is also a threshold that the phonon rate should become greater than the linear losses γ.

4. Calculation of the Amplification Coefficient

In this section, we need to calculate the time rate of change of the phonon distribution function. We hereby ignore the phonon decay rate γ as stated earlier; since it can be taken back into accounts trivially at the end. The time rate is then given by Γ_q, Eq(9). To evaluate Γ_q, we need to know the quantity $|C_q|$. From [34] $|C_q|$ has been given as

$$|C_q| = \begin{cases} \left(\frac{\Lambda^2 q}{2\rho v_s}\right)^{\frac{1}{2}} & \text{acoustic phonons} \\ \left[\left(\frac{2\pi^2 e\omega_0}{q^2}\right)\left(k_\infty^{-1} - k_0^{-1}\right)\right]^{\frac{1}{2}} & \text{optical phonons} \end{cases} \tag{13}$$

where Λ is the deformation potential conatant, ρ is the crystal density, v_s is the sound velocity, ω_0 is the frequency of an optical phonon, k_0 and k_∞ are, respectively the low frequency and optical permeabilities of the crystal.

It is important to state that in [34] C_q is calculated without taking into accounts screening effects. However, we know that if the electron concentration is finite, with the electron-phonon interaction that we are considering be it deformation potential or piezoelectric coupling, there will be a spatial redistribution of electrons and thereby leads to screening effects. This modification can be accounted for in the framework of the standard linear response approach. Taking accounts of the finite electron concentration leads to a modification of the interaction $H_{int} \rightarrow H_{int}^{scr}$ as well as

$$|C_q|^2 \rightarrow |C_q^{scr}|^2 = \frac{|C_q|^2}{|\varkappa^{(el)}(q)|^2} \tag{14}$$

where $\varkappa^{(el)}(q)$ is the electron permittivity. For details of such calculations see [25].

From Eq(9) using cylindrical coordinates, we change the summation to integral through the following transformation

$$\sum_p \rightarrow \frac{1}{(2\pi)^3} \iiint p_\perp dp_\perp d\phi dp_z \tag{15}$$

and obtain

$$\Gamma_q = \frac{|C_q|^2}{(2\pi)^2} \iiint p_\perp dp_\perp d\phi dp_z \left(f\left(\varepsilon_p\right) - f\left(\varepsilon_{p'}\right)\right) \delta\left\{\varepsilon_p - \varepsilon_{p-q} - \left(\omega_q - \mathbf{q}\cdot\mathbf{v}_D\right)\right\} \tag{16}$$

For a strongly degenerate semiconductor SL

$$f(\varepsilon) \sim \phi\left(\varepsilon(p) - \mu\right) \tag{17}$$

where μ is the fermi energy of the quasi-two-dimensional electron gas.

The dispersion relation of the SL is given in the usual form as form as

$$\varepsilon(p) = \frac{p_\perp}{2m} + \Delta\left(1 - \cos p_z d\right) \tag{18}$$

where Δ is the width of the lowest-energy miniband, m is the transverse effective electron mass (in the x-y plane), $p_\perp$ and p_z are the quasi-momentum components across and along the SL axis respectively and d is the period of the SL. It should be emphasized here that the electric field is non-quantised i.e., $eEd << 2\Delta$. The phonon and the electric field are directed along the SL axis.

Substituting Eqs(17) and (18) into (16) and integrating first with respect to $d\phi$ we obtain

$$\Gamma_q = \frac{|C_q|^2}{2\pi} \iint p_\perp dp_\perp dp_z \left(f\left(\varepsilon_p\right) - f\left(\varepsilon_{p'}\right) \delta\left(\varepsilon_p - \varepsilon_{p-q}\right) - \left(\omega_q - q_z v_D\right)\right) \tag{19}$$

We then integrate with respect to dz. After some cumbersome calculations we obtain

$$\Gamma_q = \frac{|C|^2}{4\pi\Delta d \sin\frac{qd}{2}\sqrt{1-b^2}} \int p_\perp dp_\perp \theta\left(\frac{p_\perp^2}{2m} - \left\{\varepsilon_F - \Delta\left[1 - \sqrt{1-b^2}\cos\frac{qd}{2} - \frac{\omega_q}{2\Delta}\left(1 - \frac{v_d}{v_s}\right)\right]\right\}\right)$$
$$-\theta\left(\frac{p_\perp}{2m} - \left\{\varepsilon_F - \Delta\left[1 - \sqrt{1-b^2}\cos\frac{qd}{2} + \frac{\omega_q}{2\Delta}\left(1 - \frac{v_d}{v_s}\right)\right]\right\}\right) \tag{20}$$

where

$$b = \frac{\omega_q}{2\Delta \sin\frac{qd}{2}}\left(1 - \frac{v_d}{v_s}\right) \tag{21}$$

Finally, integrating with respect to $dp_\perp$ we obtain

$$\Gamma_q = \frac{m\left|C_q\right|^2 \omega_q\left(1-\frac{v_d}{v_s}\right)}{4\pi\Delta d \sin\frac{qd}{2}\sqrt{1-b^2}} \tag{22}$$

To find the exact form of v_d we solved the Boltzmann kinetic equation in the τ approximatoin[27,29] and then calculated v_D as

$$v_D = \sum_p v_z \int_0^\infty f_0\left(p - eEt\right) e^{-\frac{t}{\tau}} \frac{dt}{\tau} \tag{23}$$

where $v_z = \Delta d \sin p_z d$ and obtained

$$v_d = \frac{1}{2}\frac{\Delta d^2 eE\tau}{\left(1+\left(eEd\tau\right)^2\right)} \tag{24}$$

here $\hbar = 1$.

5. Discussion and Conclusions

It is observed from Eq(22) that Γ_q depends on $\frac{v_d}{v_s}$ in a complicated form and that there exists a transparency "window" when $\omega_q \gg \frac{2\Delta \sin qd}{1-\frac{v_d}{v_s}}$. However, when $2\Delta \gg \omega_q$ the expression becomes

$$\Gamma_q = \Gamma_0\left(1-\frac{v_d}{v_s}\right) \tag{25}$$

where $\Gamma_0 = \frac{m|C_q|^2\omega_q}{4\pi\Delta d \sin\frac{qd}{2}}$ and so whenever $v_d > v_s$, Γ_q changes sign and amplification occurs. This is due to Cerenkov effect. We write Eq(25) in terms of the electric field E as

$$\Gamma_q = \Gamma_0\left(1-\frac{\Delta d}{2v_s}\frac{eEd\tau}{1+\left(eEd\tau\right)^2}\right) \tag{26}$$

and determine the threshold field E_0 for which amplification occurs. We consider a linear approximation on E, i.e., $eEd\tau \ll 1$. From Eq(26) we obtain

$$\Gamma_q = \Gamma_o\left(1-\frac{\Delta d}{2v_s}eEd\tau\right) \tag{27}$$

which gives the threshold field E_0 as

$$E > E_0 = \frac{2v_s\hbar^2}{\Delta d^2 e\tau} \tag{28}$$

where $\hbar$ has been reintroduced for the purpose of estimation. We noted that E_0 is a function of the SL parameters (d, Δ). We present for the purpose of comparism the relation of the threshold fields for the SL and homogeneous semiconductors in the form

$$\frac{E_0^{\text{hom}}}{E_0^{SL}} = \frac{m}{2}\frac{v_s^{\prime}}{v_s}\frac{\Delta d^2}{\hbar^2} \tag{29}$$

where $v_s^{\prime}$ is the velocity of sound in bulk semiconductor. Note that we have taken the same value of τ for both materials. For numerical estimates, we assume the following parameters: $m = 0.2m_e$; $\tau = 10^{-12}s$; $v_s^{\prime} = 5 \times 10^3 ms^{-1}$; $v_s = 4.7 \times 10^3 ms^{-1}$; $\Delta = 0.1eV$; $d = 100A^{\circ}$ and obtain for $E_0^{SL} \approx 4.16 \times 10^2 Vm^{-1}$ whiles in the usual semiconductor $E_0^{\text{hom}} \simeq 5.0 \times 10^3 Vm^{-1}$. Hence amplification of phonons in SL could occur at a lower field than in homogeneous semiconductors. We present a 3-dimensional plot of this relation (see fig.1).

Proceeding further, we plotted Eq(26) graphically (see figs.2 and 3). It is observed that the dependence of Γ on E is quite different from that of the homogeneous semiconductor. The principal difference in the amplification of short wave sound in SL from a homogeneous semiconductor is the possibility of the existence of E^* such that $E^* > E^0$ where $-\Gamma(E^*) \geq \Gamma(-E^*)$ [27]. This situation permits the use of SL as a hypersound generator in a similar way as the long-wave sound generator operating on the homogeneous semiconductor. It is interesting to note that when $qd \ll 1$ and $\frac{1}{2}\Delta d^2 = \frac{1}{m^*}$, m^* is effective mass of the electron along the SL axis [34] and Eq(22) reduces to the expression for the bulk material [20].

Finally, we want to indicate that our calculations do not take into accounts phonon losses, nonetheless, we believe that the phonon losses may not considerably affect the results.

In conclusion, we have studied the amplification of hypersound in a degenerate semicinductor SL. Analytical expressions have been obtained for the amplification coefficient Γ_q. It is shown that the threshold field E_0 for amplification is by far lower than that of the homogeneous semiconductor and that there exists a nonlinear dependence of Γ on E which enables the use of SL as hypersound generator.

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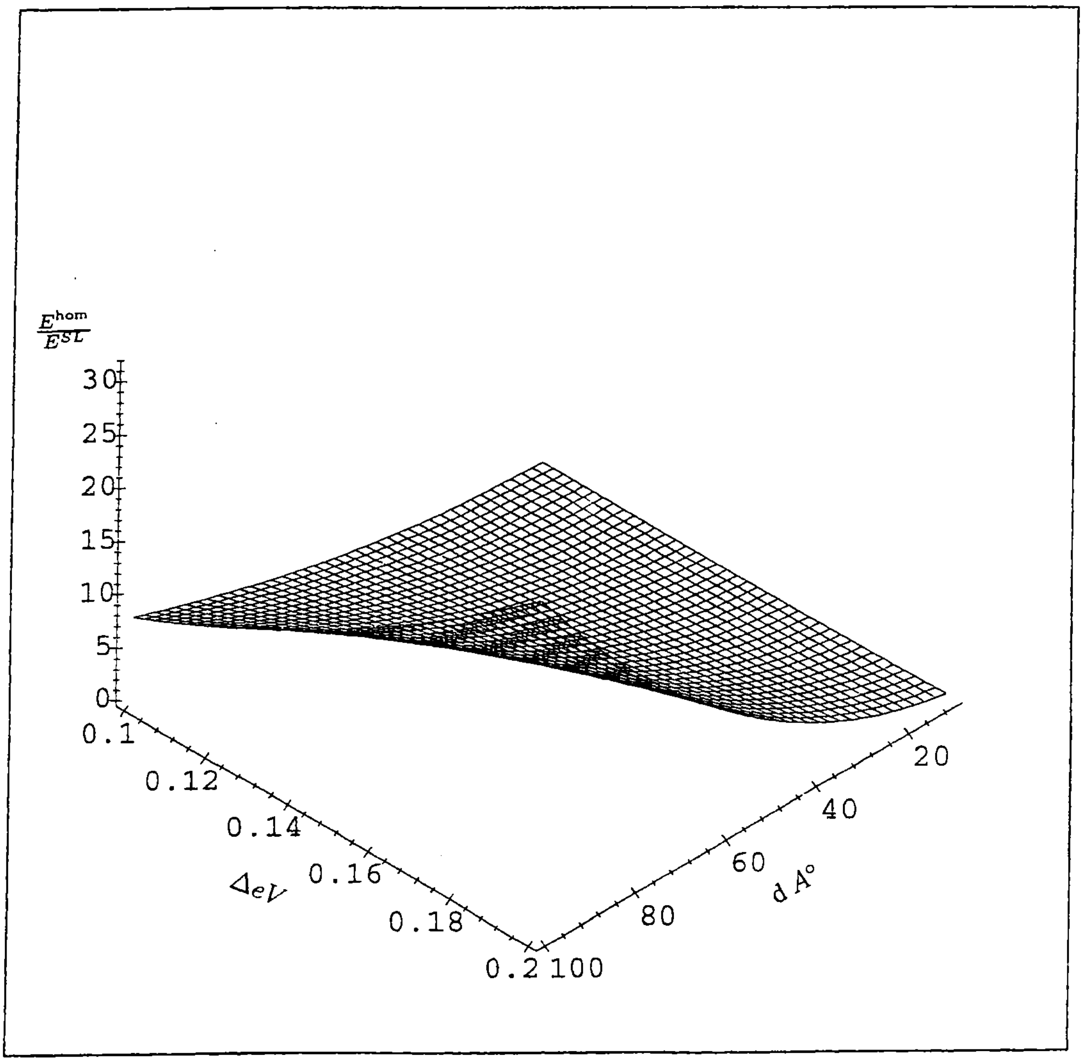
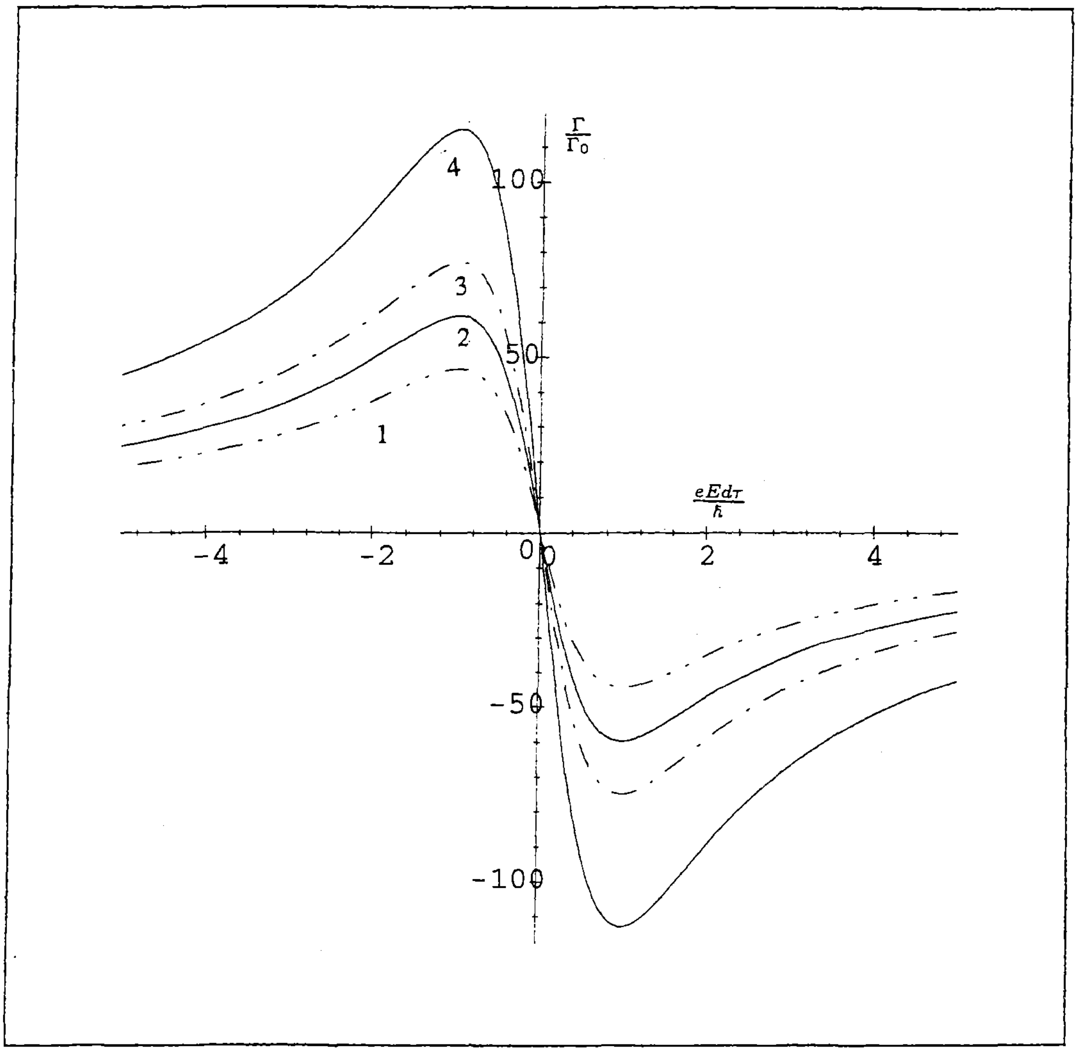


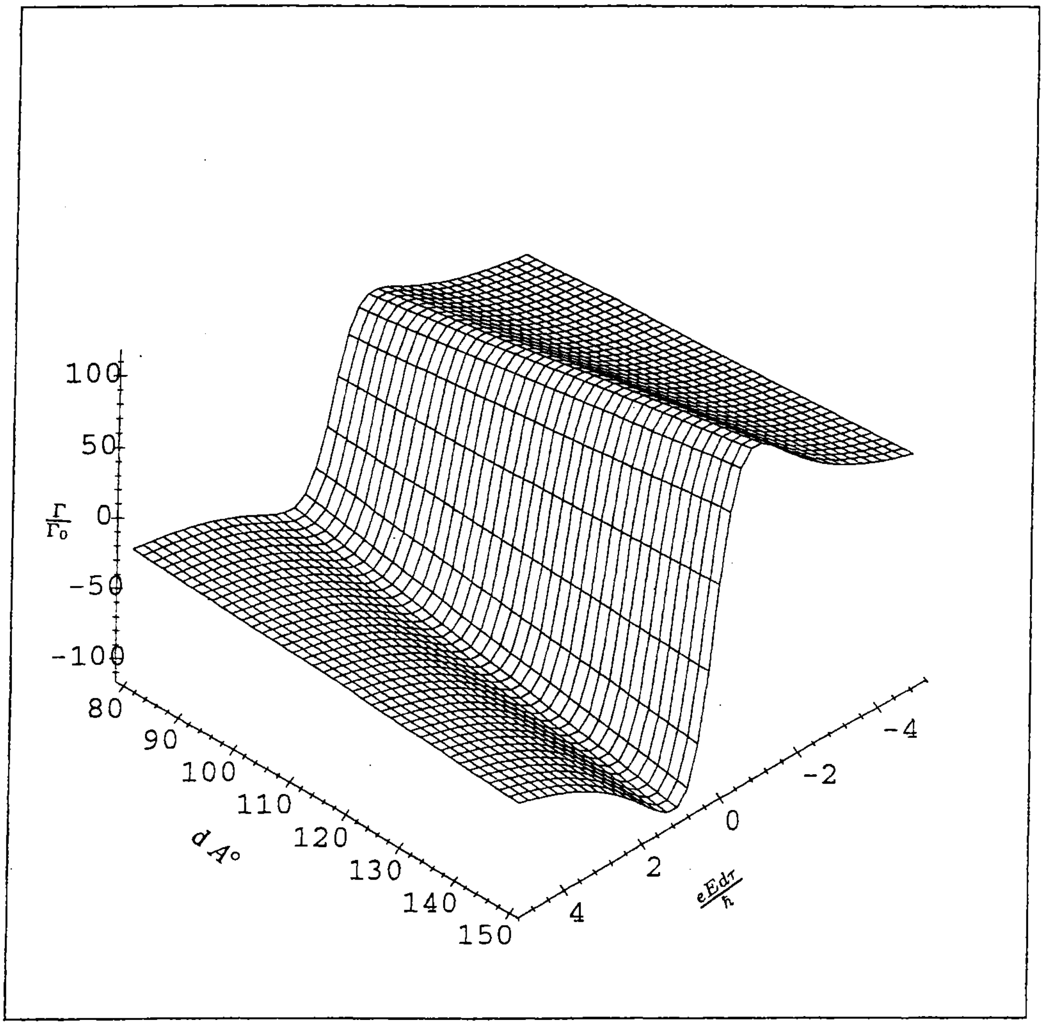
Plot of $\frac{E^{hom}}{E^{SL}}$ against d and Δ.

fig.1



Plot of $\frac{\Gamma}{\Gamma_0}$ against $\frac{eEd\tau}{\hbar}$ for: (1) $d = 60A^{\circ}$, (2) $d = 80A^{\circ}$, (3) $d = 100A^{\circ}$ and (4) $d = 150A^{\circ}$.

fig.2



Plot of $\frac{\Gamma}{\Gamma_0}$ against $\frac{eEd\tau}{\hbar}$ and d.

fig.3