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LETTER TO THE EDITOR

Amplification of acoustic waves due to an external temperature gradient in superlattices

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Abstract. Amplification of short-wave sound propagating along the axis of a semiconductor superlattice by electrons of the lowest miniband in an external temperature gradient is investigated theoretically. The mechanism of absorption of phonons is due to Landau damping. Unlike a homogeneous material, the threshold field at which absorption turns over to amplification depends on the superlattice parameters and the wavelength of the sound wave.

The possibility of amplifying acoustic waves in a semiconductor in the presence of external temperature gradient was first proposed by Gulaev and Epshtein (1966). A thorough theory describing this effect was developed by Sharma and Singh (1974). In their paper, they considered the hydrodynamic case, which is valid for $ql \ll 1$ (where q is the acoustic wave number and l is the electronic mean free path), and assumed ionized impurity scattering as the dominant scattering mechanism of electrons. They suggested that amplification of sound is possible if the two faces of a semiconductor slab of thickness 0.8 mm are maintained at 77 K and 194 K, respectively. Later Epshtein (1975) considered the converse case $ql \gg 1$. He suggested that amplification of sound is possible only given a large temperature gradient, since, at $ql \gg 1$, interaction between acoustic waves and electrons can be considered as absorption and emission of phonons by separate electrons, and the probability of these processes depends on the energy of the electrons.

In this letter, we study the above-mentioned phenomenon in superlattice (SL) semiconductors. We indicate that the threshold value of $(\nabla T)_0^{\mathrm{SL}}$ depends on the parameters of the SL and on the wavelength of the sound. Here the sound is considered in the short-wave region $ql \gg 1$. The energy spectrum of the conduction electrons in the SL is given in the single-miniband approximation. We consider that $\mathrm{d}\nabla_x T \ll 2\Delta$ (d is the SL period, 2Δ is the width of the lowest-energy miniband) and $2\Delta \gg \tau^{-1}$, ($\hbar = 1$) are conditions allowing quasi-classical solution of the problem, i.e., by means of Boltzmann's equation, where the collision integral is taken in the τ approximation. Furthermore, we suppose τ to be constant. It is worth noting, however, that this condition is not only a means of escaping unnecessary mathematical difficulties but in some cases it accurately describes the real situation. For example, the energy independence of τ during the scattering of carriers by optical phonons at low temperatures (Shmelev *et al* 1982, Bass and Teterov 1986) or by acoustic phonons (Palmier and Chomette 1982).

The sound absorption coefficient is determined by the formula

$$\Gamma = \frac{|\Lambda|^2 q^2}{4\pi^2 \rho s \omega_q} \int [f(\varepsilon_p) - f(\varepsilon_{p+q})]\delta(\varepsilon_{p+q} - \varepsilon_p - \omega_q)\, \mathrm{d}^3 p \tag{1}$$

where Λ is the constant of the deformation potential, ρ is the density of the SL sample, s is the velocity of sound, $f(\varepsilon_p)$ is the distribution function, p is the momentum of the carrier and ω_q is the phonon frequency.

The distribution function of electrons in the lowest miniband of SL, $f(p, r, t)$ is found by solving the kinetic equation. The solution is found in the linear approximation of the ∇T. In this approximation, the distribution function is independent of the sound flux and is of the form

$$f(p) = f_0(p) + f_1(p) \tag{2}$$

where

$$f_0(p) = [\mathrm{d}n/mTI_0(\Delta/T)] \exp(-\varepsilon_p/T) \qquad f_1(p) = \tau \left[(\varepsilon(p) - \xi)\frac{\nabla T}{T}\right] \frac{\partial f_\delta(p)}{\partial \varepsilon}\, v(p).$$

Here $f_\delta(p)$ is the equilibrium distribution function, T is the temperature in units of energy, $I_0(x)$ is the modified Bessel function, ξ is the chemical potential and $v(p)$ is the electron velocity.

The energy of the electrons in the lowest miniband is

$$\varepsilon(p) = p_\perp^2/2m + \Delta(1 - \cos p_x d) \tag{3}$$

where $p_\perp$ is the electron momentum in the plane forming the SL layer and m is the transverse effective electron mass (in the YOZ-plane). Considering sound to be propagated along the SL axis and substituting (2) and (3) into (1), after some transformations we obtain the following expression for the absorption coefficient of sound by a non-degenerate electron gas:

$$\Gamma = \Gamma_0 \left[\!\left[1 - \{[(\Delta - \xi) + T]\sqrt{1 - b^2} - \Delta \cos(qd/2)(1 - 2b^2)\}\frac{\tau d}{b} \right.\right.$$
$$\left.\left. \times \frac{\nabla_x t}{t} \tanh(\Delta/T\sqrt{1 - b^2} \cos qd/2) \right]\!\right] \tag{4}$$

where

$$\Gamma_0 = \frac{|\Lambda|^2 q^2 nb\Theta(1 - b^2)}{\rho s \omega_q^2 \sqrt{1 - b^2}} \sinh\left(\frac{\omega_q}{2T}\right) \cos\left(\frac{\Delta}{T}\cos\frac{qd}{2}\sqrt{1 - b^2}\right)$$
$$b = \tfrac{1}{2}\omega_q\, \Delta \sin(\tfrac{1}{2}qd) \tag{5}$$

It is worth noting from (4) that as in the case of constant electric field (Shmelev *et al* 1988) we observe the appearance of a 'transparency window' when $\omega_q \gg 2\Delta \sin(\frac{1}{2}qd)$.

In this situation $\Gamma = 0$. This is a consequence of the laws of conservation. From (4) it follows that

$$\nabla_x T > (\nabla_x T)_0 = \{[(\Delta - \xi) + T]\sqrt{1-b^2} - \Delta\cos(qd/2)(1-2b)^2\}^{-1} \times \frac{bT}{\tau d}\coth\left(\frac{\Delta}{T}\cos\left(\frac{qd}{2}\right)\sqrt{1-b^2}\right) \quad (6)$$

and for $\xi < T$; $b^2 \ll 1$ and $qd \ll 1$ we get

$$\nabla_x T > (\nabla_x T)_0 = \frac{\omega_q}{\Delta d^2 q\tau}\coth(\Delta/T). \quad (7)$$

Thus the sound absorption switches over to amplification, i.e., Γ becomes negative.

The value $(\nabla_x T)_0$ has meaning as the threshold gradient. We present the relation of the threshold gradient for the SL and homogeneous semiconductor in the form

$$\frac{(\nabla_x T)_0^{\text{hom}}}{(\nabla_x T)_0^{\text{SL}}} = \frac{ms\Delta qd^2}{\omega_q}\tanh(\Delta/T). \quad (8)$$

Hence it is seen that at $qd \ll 1$, the threshold gradient in an SL could be much smaller than in a homogeneous material.

For example, at $\Delta = 0.01$ eV, $d = 1.0 \times 10^{-6}$ cm, $\omega_q = 10^{10}\,\text{s}^{-1}$, $s = 2.3 \times 10^5\,\text{cm s}^{-1}$ and $T = 77$ K, $(\nabla_x T)_0^{\text{hom}} = 2.6\,(\nabla_x T)_0^{\text{SL}}$ (at the same value of τ and with effective mass $m = 0.2\,m_e$) and for $\Delta = 0.1$ eV; $(\nabla_x T)_0^{\text{hom}} = 26\,(\nabla_x T)_0^{\text{SL}}$. The threshold gradient calculated by Epshtein (1975) using n-InSb ($s = 2.3 \times 10^5\,\text{cm s}^{-1}$, $\mu = 8 \times 10^5\,\text{cm}^2\,\text{V}^{-1}\,\text{s}^{-1}$ at 77 K) was $(\nabla_x T)_0^{\text{hom}} \approx 10^3\,\text{K cm}^{-1}$. This shows that for an SL with $\Delta = 0.01$ eV and at 77 K, $(\nabla_x T)_0^{\text{SL}} \approx 384\,\text{K cm}^{-1}$ and for one with $\Delta = 0.1$ eV, $(\nabla_x T)_0^{\text{SL}} \approx 38.4\,\text{K cm}^{-1}$.

In conclusion, note that at $\Delta \gg T$, and ξ from (7) we obtain the usual condition of sound amplification

$$(\nabla_x T) > (\nabla_x T)_0^{\text{hom}} = \frac{se}{\mu}.$$

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