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LETTER TO THE EDITOR

Hypersound amplification by a superlattice in a non-quantised electric field

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Abstract. Short-wave sound absorption propagating along the axis of a semiconductor superlattice by electrons of the lowest miniband in a non-quantised electric field is investigated theoretically. The mechanism of the attenuation of phonons is due to Landau damping. The relationship discovered for the absorption coefficient $\Gamma = \Gamma(E)$ is essentially different from that for homogeneous materials, and, additionally the threshold field at which the absorption switches over to amplification depends on the superlattice parameters and the length of the sound wave. The more important difference between short-wave sound amplification by a superlattice and amplification in the homogeneous materials is the possibility of finding a field E^* such that $-\Gamma(E^*) \geq \Gamma(-E^*)$. This situation allows in principle the use of a superlattice as a hypersound generator similar to a generator of long-wave sound. The above generator is impossible to construct in the case of homogeneous semiconductors.

One of the most useful properties of semiconductor superlattices (SL) is the possibility of controlling their characteristics (either by variation of the SL parameters or by external field). A lot of information has been reported on SL (see, for example, Shik 1974, Silin 1985, Bass and Tetervov 1986) but unfortunately only a few of them report on the acousto-electronic properties. Meanwhile it seems to us that SL could find an important place in acousto-electronic devices.

Among the works devoted to the interaction of sound with conduction electrons of the SL we shall note those related to this Letter. The spectrum of elementary excitations in the electron–phonon system was found and the value of phonon attenuation propagating along the SL axis was determined by Shmelev *et al* (1977). The absorption coefficient and renormalisation of short-wave sound velocity were calculated by Shmelev and Zung (1977). The hypersound absorption in a SL placed in quantised electric field was considered by Azizyan (1974). The possibility of hypersound amplification in such a field has been taken into account by Kryuchkov and Yakovlev (1977) and Epshtein (1979). The amplification of sound in a SL placed in non-quantised electric field has been studied by Dubin (1979). The amplification is due to Cherenkov emission of phonons by charged carriers whose drift velocity exceeds that of the phase velocity of sound (Dubin 1979). The value of the threshold electric field E_0 (Dubin 1979) is determined in the same manner as in the case of homogeneous semiconductors with the square law of dispersion (Bonch-Bruevich and Kalashnikov 1977).

In this Letter we indicate (unlike Dubin (1979)) that the value of E_0 depends on the parameters of the SL and on the wavelength of sound. The latter, in our opinion, is very important, since for a given value of applied electric field the SL can, while amplifying, filter waves of definite frequencies.

Here the sound is considered in the short-wave region $ql \gg 1$ (q is the acoustic wavenumber, l is the electronic mean free path). As is known (see Bonch-Bruevich and Kalashnikov 1977), such sound could be considered as a flow of monochromatic phonons (of frequency ω_q). The energy spectrum of the conduction electrons in the SL is given in the single-miniband approximation. The electric field applied along the SL axis (OX axis) is supposed to be non-quantised: $eEd \ll 2\Delta$ (d is the SL period, 2Δ is the width of the lowest-energy miniband), $2\Delta \gg \tau^{-1}$ ($\hbar = 1$) is considered to be a condition allowing quasi-classical solution of the problem, i.e. by means of Boltzmann's equation where the collision integral is taken in the τ-approximation. Further, we suppose τ to be constant; it is worth noting, however, that this condition is not only a means of escaping unnecessary mathematical difficulties but in some cases it reflects the real situation, for example the energy independence of τ during scattering of carriers by optical phonons at low temperatures (Shmelev *et al* 1982, Bass and Tetervov 1986) or by acoustic phonons (Palmier and Chomette 1982).

The sound absorption coefficient is determined by the formula (Abrikosov 1987)

$$\Gamma = \frac{|\Lambda|^2 q^2}{8\pi^2 \rho s \omega_q} \int [f(\varepsilon_p) - f(\varepsilon_{p+q})] \delta(\varepsilon_{p+q} - \varepsilon_p - \omega_q) \, d^3\boldsymbol{p} \tag{1}$$

where Λ is the constant of deformation potential, ρ is the density of the sample (SL), s is the velocity of sound, $f(\varepsilon_p)$ is the distribution function, and $\boldsymbol{p}$ is the momentum of the carrier.

As is shown by estimates, the attenuation length of the hypersound in the sample (SL) is much more than the wavelength of sound. This allows us to neglect the term containing the derivative of the distribution function on coordinates in the kinetic equation. Then the solution of the equation concerned will be (Ignatov and Romanov 1978)

$$f(\boldsymbol{p}) = \int_0^\infty \frac{1}{\tau} \exp(-t/\tau) f_0(\boldsymbol{p} - e\boldsymbol{E}d) \, dt \tag{2}$$

where

$$f_0(\boldsymbol{p}) = (dn/mTI_0(\Delta/T)) \exp(-\varepsilon_p/T)$$

is the equilibrium distribution function, T is the temperature in units of energy, and $I_0(x)$ is the modified Bessel function.

The energy of electrons in the lowest miniband is

$$\varepsilon(\boldsymbol{p}) = p_\perp^2/2m + \Delta(1 - \cos p_x d) \tag{3}$$

where $\boldsymbol{p}_\perp$ is the electron momentum in the plane forming the SL layer, and m is the transverse effective electron mass (in the YOZ plane).

Considering that the sound is propagated along the SL axis, and substituting (2) and (3) into (1), after some transformations we obtain the following expression for the absorption coefficient of sound by non-degenerated electron gas

$$\Gamma = \frac{|\Lambda|^2 q^2 n \Theta(1-b^2)}{2\rho s \omega_q \sin(qd/2)\sqrt{1-b^2}} \int_0^\infty \exp(-t/\tau)\,\mathrm{d}t \frac{1}{\tau}\Big[I_0(z)$$

$$+ 2\sum_{k=1} (-1)^k I_{2k}(z)\cos(2keEdt)\sinh\left(\frac{\omega_q}{2T}\cos(eEdt)\right)$$

$$\times \cosh\left(\frac{\Delta}{T}\cos(qd/2)\sqrt{1-b^2}\cos(eEdt)\right)$$

$$- 2\sum_{k=0} (-1)^k I_{2k+1}(z)\sin[(2k+1)eEdt]\sinh\left(\frac{\Delta}{T}\cos(qd/2)\right.$$

$$\left.\times\sqrt{1-b^2}\cos(eEdt)\right)\cosh\left(\frac{\omega_q}{2T}\cos(eEdt)\right)\Big] \tag{4}$$

where

$$z = (\Delta/T)\sin(qd/2)\sqrt{1-b^2} \qquad b = \omega_q/(2\Delta\sin(qd/2))$$

and Θ is the Heaviside step function.

We shall consider the following particular cases of formula (4).

(i) At $E = 0$ from (4) we obtain the absorption coefficient of phonons (Shmelev and Zung 1977)

$$\Gamma_0 = \frac{|\Lambda|^2 q^2 n b \Theta(1-b^2)}{\rho s \omega_q^2 \sqrt{1-b^2}} \sinh\left(\frac{\omega_q}{2T}\right)\cos\left(\frac{\Delta}{T}\cos\frac{qd}{2}\sqrt{1-b^2}\right). \tag{5}$$

Let us note the appearance of a 'transparency window' when $\omega_q \gg 2\Delta\sin(qd/2)$; in this situation $\Gamma = 0$. This is a consequence of the laws of conservation.

(ii) In a linear approximation on E and under the condition that $\omega_q \ll T$, from (4) we have

$$\Gamma = \Gamma_0\left[1 - eEd\tau\sqrt{1/b^2 - 1}\tanh\left(\frac{\Delta}{T}\cos\frac{qd}{2}\sqrt{1-b^2}\right)\right]. \tag{6}$$

From (6) it follows that in a field with strength

$$E > E_0 = \frac{b}{ed\tau\sqrt{1-b^2}}\coth\left(\frac{\Delta}{T}\cos\frac{qd}{2}\sqrt{1-b^2}\right) \tag{7}$$

the sound absorption switches over to amplification, i.e. Γ becomes negative. The value E_0 has the sense of threshold field. Note that E_0 is a function of the SL parameters (d, Δ), temperature, frequency and acoustic wavenumber. The fact is that Γ_0 can be quite large when ω_q is reasonably close to $2\Delta\sin(qd/2)$. Near resonance $\omega_q = 2\Delta\sin(qd/2)$ the absorption coefficient becomes so large that the initial criteria are broken down. Therefore the situation when $(2\Delta\sin(qd/2))^2 - \omega_q^2 \ll \omega_q^2$ is not considered here.

We present the relation of the threshold fields for the SL and homogeneous semiconductors in the form

$$E_0^{\mathrm{hom}}/E_0^{\mathrm{SL}} = msd\sqrt{1/b^2 - 1}\tanh[(\Delta/T)\cos(qd/2)\sqrt{1-b^2}]. \tag{8}$$

Hence it is seen that at $qd \ll 1$ and $\Delta/T \gg 1$ the threshold field in the SL could be much smaller than in the homogeneous material. For example, at $T = 300$ K, $\Delta = 0.1$ eV, $d = 5\times10^{-7}$ cm, $s = 5\times10^{5}$ cm s^{-1}, $\tau = 10^{-12}$ s, $\omega_q = 10^{10}$ s^{-1} the value $E_0^{\mathrm{SL}} \simeq 10$ V cm^{-1}, while in the 'usual' semiconductor $E_0^{\mathrm{hom}} \simeq 60$ V cm^{-1} (at the same value of τ and with effective mass $m = 0.2\,m_e$). This amplification of sound in SL could occur at a smaller field than in the homogeneous semiconductor.

(iii) At $T \gg \Delta$, ω_q the expression (4) can be integrated to give the result

$$\Gamma = \frac{\Gamma_0}{1+(eEd\tau)^2}\left(1 - \frac{\Delta^2 \sin(qd)}{\omega_q T}(1-b^2)eEd\tau\,\frac{1+(eEd\tau)^2}{1+(2eEd\tau)^2}\right). \tag{9}$$

When $eEd\tau \gg 1$ we obtain from (9)

$$E_0^{\mathrm{SL}} \simeq 4\omega_q T/ed\tau\Delta^2 \sin(qd) \tag{10}$$

and with $\Delta = 0.01$ eV, $T = 300$ K, $d = 5\times10^{-7}$ cm, $\omega_q = 10^{10}$ s^{-1}, and $\tau = 10^{-12}$ s the value of $E_0^{\mathrm{SL}} \simeq 90$ V cm^{-1}.

The expression (4) is presented graphically with the help of a computer (see figure 1). The dependence of Γ on E is quite different from that of a homogeneous semiconductor. The principal difference in the amplification of short-wave sound in SL

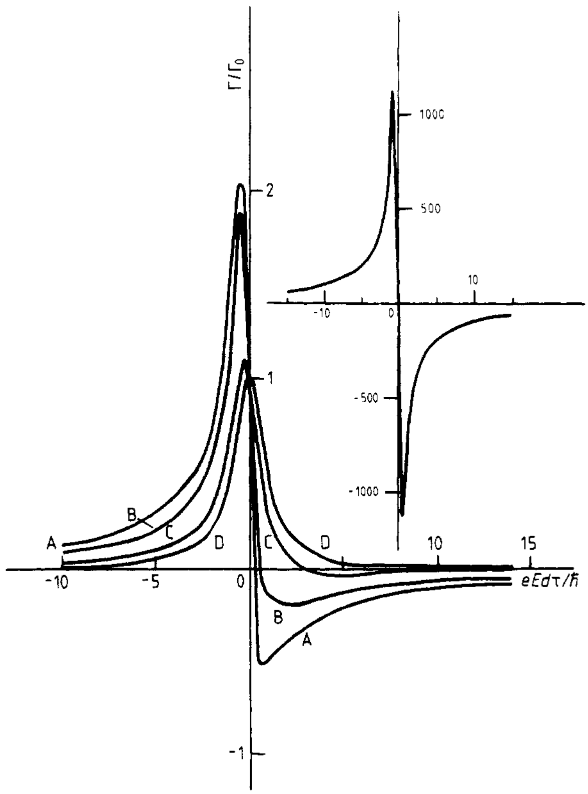

Figure 1. The dependence of Γ/Γ_0 on E. The z-values are: curve A, 0.001; curve B, 0.015; curve C, 0.026; curve D, 0.032. For the inset, $z = 0.9$.

from a homogeneous semiconductor is the possibility of existence of E^* such that $E^* > E_0$ where $-\Gamma(E^*) \geqslant \Gamma(-E^*)$. This situation permits using SL as a hypersound generator in a similar way as the long-wave sound generator operating on homogeneous semiconductors (White and Wang 1966, Baibakov 1968).

In conclusion, note that at $\Delta \gg T$ and $\omega_q \ll T$, from (4) we obtain the usual condition of sound amplification $E > E_0^{\text{hom}} = (sm/e\tau)$.

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References

Abrikosov A A 1987 *Osnovi Teorii Metallov* (Moscow: Nauka)
Azizyan A O 1974 *Izv. Akad. Nauk Arm. SSR Fiz.* **9** 208–10
Baibakov V I 1968 *Fiz. Tverd. Tela* **10** 678–82
Bass F G and Tetervov A P 1986 *Phys. Rep.* **140** 237–322
Bonch-Bruevich V L and Kalashnikov 1977 *Fizika Poluprovodnikov* (Moscow: Nauka)
Dubin D Y 1979 *Fiz. Tekh. Poluprov.* **13** 413–4
Epshtein E M 1979 *Fiz. Tekh. Poluprov.* **13** 611–2
Ignatov A A and Romanov Yu A 1978 *Izv. Vyssh. Uchebn. Zaved. Radiofiz.* **21** 132–7
Kryuchkov S V and Yakovlev V A 1977 *Fiz. Tverd. Tela* **11** 590–3
Palmier J P and Chomette A 1982 *J. Physique* **43** 381–91
Silin A P 1985 *Usp. Fiz. Nauk* **147** 485–521
Shik A Y 1974 *Fiz. Tekh. Poluprov.* **8** 1841–64
Shmelev G M, Chaikovskii I A, Pavlovich V V and Epshtein E M 1977 *Phys. Status Solidi* b **80** 697–701
Shmelev G M and Zung Huang 1977 *Fizicheskie Proceci v Poluprovodnikov* (Kishinev: Shtiinca)
Shmelev G M, Enaki N A and Tsurkan G I 1982 *Ukr. Fiz. Zh.* **27** 458–60
White D L and Wang Wen-Chung 1966 *Phys. Rev.* **149** 628–30