

Stark-cyclotron Resonance in an Array of Carbon Nanotubes

S. S. Abukari^{a,*}, S. Y. Mensah^a, R. Musah^c,
N. G. Mensah^b, K. A. Dompok^a

^a*Department of Physics, College of Agriculture and Natural Sciences, U.C.C, Ghana.*

^b*Department of Mathematics, College of Agriculture and Natural Sciences, U.C.C, Ghana*

^c*Department of Applied Physics, University for Development Studies, Navorongo, Ghana*

Abstract

Using the kinetic approach based on the semiclassical Boltzmanns transport equation with constant relaxation time, we theoretically studied the Stark-cyclotron resonance in an array of carbon nanotubes. Exact expression for the current density was obtained. We noted that Stark-cyclotron resonance occurs when the Larmor frequency coincides with the Stark frequency.

Introduction

Carbon nanotubes (CNs) are allotropes of carbon with a nanometers in diameter and have a length-to-diameter ratio of the order 10⁷. In 1952, Radushkevich and Lukyanovich reported clear images of 50 nanometer diameter tubes of carbon [1] and using the vapor-growth technique, Orberlin et al [2] published hollow carbon fibers with nanometers in diameter. See ref [3] on additional reports on the observation of carbon nanotubes. However, the credit of the discovery of carbon nanotubes (CNs) goes to S. Iijima [4]. This

*Corresponding author
Email: sulemana70@gmail.com

one-atom thick sheet of graphene rolled up to a seamless cylinder with a diameter of the order of a nanometer has since attracted a great deal of interest mainly due to their novel and unique thermal [5, 6, 7, 8], chemical and physical properties [9, 10]. These properties depend on the fundamental indices (n, m) of the CNs. The indices (n, m) determine the diameter and the chiral angle of the *CNs*. As n and m vary, the conduction ranges from metallic to semiconducting [11], with an inverse diameter dependent band gap of $\leq 1eV$ [11]. Electron transport properties in CNs have been the subject of intense research [10, 11, 12, 13, 14, 15, 16, 17, 18]. More recently, electronic transport in an array of CNs is the subject of many theoretical papers [19, 20, 21, 22, 23, 24, 25, 26, 27, 28]. Nevertheless, the electrodynamic properties of an array of CNs is worth further studying because it is the basis for developing carbon-based devices. Using the kinetic transport equation, we shall in this work study the effect of Stark cyclotron resonance in an array of CNs in the presence of both constant electric and magnetic fields by following the approaches of [29, 30].

Theory

Proceeding as in references [29, 30], we consider the motion of an electron in the presence of both constant electric field \vec{E} and magnetic field \vec{H} . The electric field \vec{E} is directed along the axis of the array of the CNs and magnetic field \vec{H} directed at angle to the nanotubes axis. The CNs are arranged such that the distance between the neighboring CNs is larger than the CNs diameter and the interaction between the nanotubes is neglected [19, 20, 21, 22, 23, 24, 25, 26, 27, 28]. We shall take the CNs axes to be parallel to the x axis. The conductivity is derived using the Boltzmann kinetic equations

describing electron transport in an array of CNs for the distribution functions in the relaxation time approximation as follows:

$$e\vec{E} + \frac{e}{c}[V, \vec{H}] \frac{\partial f}{\partial p} = \frac{F - f}{\tau} \quad (1)$$

where, e is the electron charge. The quasi-momentum is represented as $p = (p_x, s)$, where p_x is the component of the electron dynamical momentum along the nanotube axis, while $s = 1, 2, \dots, m$ is the number characterizing the quantization of momentum along the perimeter of the nanotube crosssection, F and f are the equilibrium and nonequilibrium distribution functions respectively. See [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27]. Taking into account the hexagonal crystalline structure of a unrolled graphene and using the tight binding approximation, the dispersion relation for the conduction electrons in the metallic zigzag CNs is given by [9, 17, 18].

$$\varepsilon(p_x, s) = \pm \gamma_0 \sqrt{1 + 4\cos(ap_x)\cos(\pi \frac{s}{m}) + 4\cos^2(\pi \frac{s}{m})} \quad (2)$$

where $\gamma_0 \approx 2.7eV$, $a = \frac{3b}{2\hbar}$, $b = 0.142nm$ is the distance between the neighbouring carbon atoms in the CNs. + and - signs are related to the conduction and valence bands respectively. The energy $\varepsilon(p_x, p_y)$ in Eq. (1) can be represented as a Fourier series [5, 13, 14]:

$$\varepsilon(p_x, s) = \sum_{r=-\infty}^{\infty} \varepsilon_{rs} \expi[rp_x]a \quad (3)$$

where $\hbar = 1$, and similarly, the expression for the distribution function in Fourier series are

$$F(p_x, s) = \sum_{r=-\infty}^{\infty} F_{rs} \expi[rp_x]a \quad (4)$$

where F_{rs} and ε_{rs} . We define current density for the array of CNs are [19, 20, 21, 22, 23, 24, 25, 26, 27, 28] as

$$j_x = \frac{e}{\tau} \sum_{s=1}^n \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} F(p_x) dp_x \int_0^\infty e^{-\frac{t}{\tau}} v(p_x, s) dt \quad (5)$$

and the quasiclassical velocity $v(p_x)$ of an electron moving along the CNs axis ie x -component can be expressed as

$$v(p_x, s) = \frac{\partial \varepsilon(p_x, s)}{\partial p_x} = i\gamma_0 a \sum_{r=-\infty}^{\infty} r \varepsilon_{rs} \expi[rp_x] a \quad (6)$$

The non relativistic equation motion of an electron in the presence of electric and magnetic fields which are constant in time and spatially uniform is given by [29]

$$\frac{dp}{dt} = (e\vec{E} + \frac{e}{c}[v(p_x), \vec{H}]) \quad (7)$$

The components of Eq.(7) are given by [28, 29, 30]

$$\frac{dp_x(t)}{dt} = e\vec{E} - \omega_\perp p_y(t) \quad (8)$$

$$\frac{dp_y(t)}{dt} = \omega_\perp p_x(t) - \omega_\parallel p_z(t) \quad (9)$$

$$\frac{dp_z(t)}{dt} = \omega_\parallel p_y(t) \quad (10)$$

where $\omega_\parallel = eB\cos\theta/m^*$ are the parallel and perpendicular cyclotron frequencies. Asuming that $|\omega_\parallel| \gg |\omega_\perp|$, and solving Eqs.(8)-(10) we obtain

$$p_x(t) = p_x + \omega_B t - \frac{\omega_\perp}{\omega_\parallel} p_\perp \sin(\omega_\parallel t + \theta) + \frac{\omega_\perp}{\omega_\parallel} p_\perp \sin(\theta) \quad (11)$$

$$p_y(t) = p_{\perp} \cos(\omega_{\parallel} t + \theta) \quad (12)$$

$$p_z(t) = p_{\perp} \sin(\omega_{\parallel} t + \theta) \quad (13)$$

here $p_{\perp} = \sqrt{p_y^2 + p_z^2}$ and $\tan\theta = p_z/p_y$ see [28, 29, 30]. Substituting Eq.(6) into Eq.(5) and using Eqn.(11), we obtain the current density as

$$j_x = j_0 \sum_{r=1}^{\infty} \sum_{m=-\infty}^{\infty} \sum_{s=1}^n r \varepsilon_{rs} F_{rs} j_m^2(\beta) \left[\frac{(r\Omega - k\omega_{\parallel})\tau}{1 + ((r\Omega - k\omega_{\parallel})\tau)^2} \right] \quad (14)$$

where $\Omega = eaE_{\parallel}$, $j_0 = 4e\gamma_0\sqrt{3}/\hbar^2$, j_m is the Bessel function of m^{th} order and $\beta = \frac{\omega_{\perp} 3b}{\omega_{\parallel} \hbar} p_{\perp}$.

Results, Discussion and Conclusion

We present the results of a kinetic equation approach of a $2D$ array of zigzag CNs subject to both constant electric field \vec{E} and magnetic field \vec{H} . Exact expression for the direct current density was obtained in eq. (14). The nonlinearity is analyzed using the dependence of the normalized direct current density j_x/j_0 as a function of $\omega_c\tau$. A plot of a normalized current density j_x/j_0 as a function of $\omega_c\tau$ in metallic zigzag CNs array for expression (8) when $\Omega\tau = 4, 6$ and 8 is shown in Fig. 1. We observed that resonance occurs when the ratio of the Stark and the cyclotron frequencies is an integer (i.e $k\omega_c\tau = \Omega\tau$). The peak current density corresponding to the Stark-cyclotron resonance also shifts towards large $\omega_c\tau$ values with increasing $\Omega\tau$ values. In conclusion, we considered the nonlinear properties in impure graphene subject to electric and magnetic fields. The results indicate Stark-cyclotron

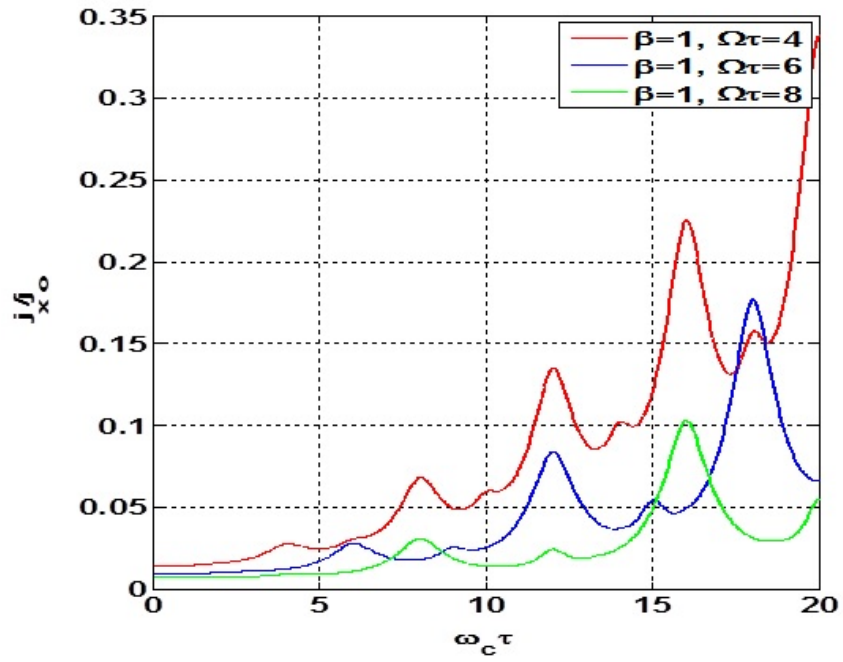


Figure 1: A plot of a normalized current density j_x/j_0 as a function of $\omega_c\tau$ in a An Array of CNs for expression (8) when $\Omega\tau = 5, 10,$ and $15\dots$

resonance which is in essence due to nonparabolicity of the energy spectrum of the CNs.

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