UNIVERSITY OF CAPE COAST

AN INVESTIGATION INTO SENIOR HIGH SCHOOL TEACHERS' KNOWLEDGE FOR TEACHING ALGEBRA

MARY DONKOR

2020

UNIVERSITY OF CAPE COAST

AN INVESTIGATION INTO SENIOR HIGH SCHOOL TEACHERS' KNOWLEDGE FOR TEACHING ALGEBRA

BY

MARY DONKOR

Thesis submitted to the Department of Mathematics and I.C.T. Education of the Faculty of Science and Technology Education, College of Education Studies, University of Cape Coast, in partial fulfilment of the requirements for the award of Master of Philosophy Degree in Mathematics Education

APRIL 2021

DECLARATION

Candidate's Declaration

I hereby declare that this thesis is the result of my own original research and that no part of it has been presented for another degree in this university or elsewhere.

Candidate's Signature: …………………………… Date: ……………........

Name: Mary Donkor

Supervisor's Declaration

I hereby declare that the preparation and presentation of the thesis was supervised in accordance with the guidelines on supervision of thesis laid down by the University of Cape Coast.

Supervisor's Signature: …………………………… Date: ………………....

Name: Dr. Forster D. Ntow

ABSTRACT

The performance of students has silently found its way into the hearts of various researchers and policy makers of which they have accepted the importance and the role of the teacher to play a crucial part in how students perform. As a result of that, several studies are filled with the fact that teacher knowledge affects student performance. However, the issue of which aspects of teacher knowledge influence student achievement has been a bone of contention among researchers. As a result, several attempts to measure teacher knowledge have relied on proxy measures such as the number of university courses taken, the type of degree the teachers' have and so on. Using the expanded KAT conceptualization framework, this study was designed to investigate whether the seven types of teachers' knowledge hypothesised will be corroborated. Two hundred and seventy-eight teachers from 16 senior high schools in the Eastern region and one public university in the Central Region of Ghana participated in this study. The cross-sectional survey was the main design used. Confirmatory factor analysis conducted on data from this study did not corroborate the seven knowledge types as hypothesised in the expanded KAT framework but rather three. Furthermore, analyses of data showed that preservice teachers exhibited high algebra knowledge for teaching than their in-service counterparts. Also, the study revealed that mathematics teachers with professional background qualification are relatively better than their counterparts without professional background qualification for teaching mathematics. It was recommended that in-service training on current issues should be organized especially for those in-service mathematics teachers to whip up their knowledge based in the area.

KEY WORDS

Conceptualization of Teacher Knowledge

School Algebra Teaching Knowledge

Advanced Algebra Teaching Knowledge

Mathematics Teaching Knowledge

Profound Knowledge of School Algebra

Advanced Algebra Teaching Knowledge

School Algebra Teaching Knowledge

Pedagogical Content Knowledge in Algebra

Preservice mathematics teachers

In-service mathematics teachers

Professional background qualification

Non-professional background qualification

ACKNOWLEDGEMENTS

I would like to express my sincere and profound gratitude to the Almighty God for how far he has brought me. I also thank my supervisor, Dr. Foster Ntow of the Department of Basic Education, University of Cape Coast, for his professional guidance, advice, encouragement and goodwill with which he guided this work. I really appreciate his selfless support.

I am also grateful to all members of the Department of Mathematics and I.C.T. Education, University of Cape Coast, for their generous contributions to make this work better. Again, I would want to thank Dr Christopher Yarkwah, Department of Mathematics and I.C.T. Education, University of Cape Coast, for his tremendous help, advice and encouragement which motivated me to finish this study on time.

Finally, my master's degree has been successful because of the firm support and encouragement by my loving husband Emmanuel Obeng Appiah, my two wonderful kids Abdiel and Astrid, my dear mother Doris Arthur and dad Daniel Donkor, my in-laws Mr. and Mrs. Obeng and my special siblings Sandra Donkor and Kate Donkor. I also thank teachers and schools who were involved in this study.

DEDICATION

To my family and my late brother Daniel Donkor

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER ONE

INTRODUCTION

Mathematics is one of the most intriguing and useful subjects in the world, yet many people have misconceptions about it. Mathematics is sometimes perceived incorrectly as a combination of rigid rules and procedures that seem to be unrelated to each other. Some people have the notion that mathematics learned in school is irrelevant, unnecessary and does not relate to the issues we encounter in our professional and personal lives (Hatfield, Edwards, & Bitter, 1997). These misconceptions have blinded many in not appreciating meaning and roles of mathematics in our daily lives. As a result of that, many students do not attach the necessary seriousness to the learning of mathematics. Though there are lots of factors that determine the performance of students, the teacher is not exempted.

The performance of students in mathematics continues to engage the minds of various researchers and policy makers of which they have accepted the importance and the function of the instructor to play a crucial part in how the learners perform (see for instance Begle, 1972; Eisenberg, 1977; Wilmot, 2009; Yarkwah, 2017). Yet, there is a bone of contention as to the way forward on how to efficiently theorise teachers' knowledge to depict which part best forecasts learner performance. Wilmot (2009), revealed that some of these conceptualisations made it difficult to objectively measure teachers' knowledge as they exhibited instructor knowledge as a domain neutral construct (Shulman, 1987).

Background to the Study

There is no disputing the fact about how teachers' knowledge influences how they teach (Ambrose, 2004; An, Kulm & Wu, 2004; Hill & Ball, 2004; Stipek, Givvin, Salmon, & MacGyvers, 2001). This brings to the fore the need to vigorously support the advancement of profound instructor knowledge to assist them make a changeover from the conservative schoolroom to a more transformed classroom. Teachers have been awarded with the credit that the tremendous remodeling of students learning in schools has been due to their efforts as claimed by Darling-Hammond (2003) and Yara (2009). Yara (2009) went ahead to affirm that student's studies are affected by the kind of environment they find themselves, and some major contributing factor is teachers' experience, their personality and behaviours in the classroom. Nevertheless, literature is filled with the fact that for effective teaching to occur specifically in mathematics both conceptual and procedural knowledge are essential. For that matter, it is expected that individuals with formal preparation in education should accomplish or do better than their counterparts without education background. Abe (2014) accentuated the positive aspects of the need to allow qualified and professional teachers only to teach secondary school mathematics since his findings revealed a momentous variance between the learners' performance of professional teachers and their non-professional colleagues. And this bring to the confirmation of what Shulman (1983) said that "… the teacher must remain the key. The literature on effective schools is meaningless; debates over educational policy are moot, if the primary agents of instruction are incapable of performing their functions well. No microcomputer will replace them, no television system will clone and distribute them, no scripted lessons will direct and control them, no voucher system will bypass them (p. 504)." There is a huge bone of contention with reference to teachers' subject matter knowledge which raised the question about what knowledge base instructors have about their respective area of specialisation in order to support them stimulate commanding and malleable knowledge and understanding in learners. In this regard, Schwerdtfeger (2017) in her study that examined variances in data of mathematical modeling that exist between elementary preservice instructors and a set of elementary in-service instructors indicated that no arithmetical momentous difference existed in elementary preservice instructors and elementary in-service instructors' knowledge of mathematical modeling. Schwerdtfeger in the same study tried to find out if any variables had the capability to forecast the preservice or in-service instructors' knowledge of mathematical modeling. In this case, the researcher resorted to using multiple regression to find out the variables of eons of teaching experience, grade level currently taught, or type of school in which instruction occurs did not have any forecaster features of knowledge of mathematical modeling. She also concluded in same study that no relationship existed between preservice and in-service instructors' apparent knowledge of mathematical modeling and their definite knowledge of mathematical modeling.

Kennedy (1991) asserted that there have been several difficulties by various conceptualisations on the issue of instructors' knowledge for instruction. Close to three decades of numerous conceptualisations of subject matter knowledge for teaching mathematics, only the Knowledge of Algebra for Teaching [KAT] framework has made progress in the sense that, it's only the KAT project that came up with specific instrument and items meant to measure only one domain of mathematics at the senior high school level and that is algebra. At this point it is imperative to note that researchers in the KAT project advanced specific knowledge types because it is generally held that teachers' mathematical knowledge is important and is connected to students' learning. It has also been noted that research linking teacher knowledge to student achievement has produced ambiguous and sometimes contradictory results. Privation of clarity about the impact of teacher knowledge results from inadequate theory about particular mathematical knowledge or kinds of knowledge that might be important for teaching. The researchers argue that since teachers' knowledge relates to students' performance there is the need to ascertain which knowledge type teachers need in order to influence students' performance positively.

This study was based on the fact that, instead of relying on those factors, there is the need to reconceptualise teachers' knowledge in ways which can be measured as in Knowledge of Algebra for Teaching (KAT) project by Ferrini-Mundy, McCrory, Senk, and Marcus (2005). In their study, they hypothesized that, the subject matter knowledge in mathematics for teaching algebra is made up of three main types of knowledge. This knowledge was stated by the team as 1) knowledge of school algebra which was also labeled as "school knowledge", 2) advanced knowledge (explained to be the knowledge of the content teachers possess in other mathematics specific domains which are different from algebra) and 3) teaching knowledge (which comprise the pedagogical knowledge teachers use in teaching mathematics). However, their findings revealed that the interlocking regions were blurry meaning knowledge in those regions were not clearly defined. In other words, no knowledge type can be clearly defined in those regions. Considering this, Wilmot (2017) proposed a modified framework which served as a basis for the expanded KAT framework to be developed. In the new expanded KAT framework, those blurry regions as stipulated in the KAT framework had been clearly defined.

Through that study, the expanded KAT framework which Wilmot, Yarkwah and Abreh (2018) worked on revealed the uniqueness of measuring teachers' knowledge which unfolded the hidden types of knowledge as mentioned in the original KAT framework. It was revealed that those regions that were considered fuzzy were not so but rather produced another knowledge type which have be explained in the chapter two.

The Structure of School Mathematics in Ghana

A critical look at the United States of America mathematics curriculum indicates that Junior High and Senior High Schools have distinct courses in algebra (Take for example, Algebra I, Algebra II etc.) which are offered to students at these levels. Nevertheless, in our part of the world (Ghana), unfortunately, a combined mathematics option is presented to our students at the Junior High School (JHS) level (the equivalent of 7^{th} – 9^{th} grade). At the JHS level, the mathematics that is offered to students is a nationwide one, and is therefore, offered to students in government schools for the three-year period of education at the JHS. The main content areas concealed in the Teaching Syllabus for Junior High Schools (Ministry of Education, 2012) are covered in (v) as follows:

1. Numbers and Numerals
2. Numbers Operations and Algebra
3. Measures, Shape and Space “Geometry”
4. Collecting and Handling Data
5. Problem Solving & Application

Quite apart from these content areas stipulated in the curriculum, problem solving which is not a topic in itself cut-across the major content areas stipulated in the teaching syllabus and are given considerable prominence. In addition, these topics are not consecutively covered in the syllabus. As a matter of fact, in the curriculum meant for the identified level, they have been divided into smaller content areas, usually called units as well as sub-units and have been put in a spiral way. Here what happens is that the various parts (units) have been structured in such a way that topics taught in the early stages of the pupils’ education are not covered in full but rather are in bit such that it is done repeatedly all over the schooling years of the individual students and advanced further, with increasing detail and deepness, as they (students) progress through their various levels of education.

At the moment in Ghana, two major types of mathematics programmes exist and are accessible to students at the Senior High School (SHS) level. These are Core Mathematics and Elective Mathematics. The issue is that in our public schools as well as private senior high schools, every single student is made to read Core

Mathematics for a period of three years of his/her SHS education. However, Elective Mathematics as a subject is meant for students who intend to pursue further study in mathematics content preparation beyond the core mathematics coverage. Elective Mathematics for example happens to be a compulsory elective subject for students reading the Sciences and Technical programs. In some cases, students reading General Arts and Business programs are allowed to read Elective Mathematics as one of their elective subjects as a result of their subject combination and what they intend doing at the tertiary level. Relatively, both the core and elective mathematics subjects at the SHS level presently is an integrated mathematics programs with their content organized in a spiral manner like the one at the JHS level.

In addition, like all other school subjects, the Ghana Education Service is the body designated to administer both the Core and Elective Mathematics subjects' syllabi to all schools across the country. One unique thing about this is that since it's a national curriculum, the content administered to students in each of these mathematics subjects is the same for all across the public SHSs in the country.

It is imperative to note that the current mathematics syllabus (both core and elective) in an effort to support all Ghanaian young persons to attain the needed mathematical skills, insight, attitude and values that they will need to be successful in their chosen careers and daily lives is based on the proposition that all students can learn mathematics and that all need to learn mathematics (Core Mathematics Teaching Syllabus for Senior High School, 2011, p ii).

Howe (1999), had posited that "a teacher who is blind to the coherence of mathematics cannot help students see it" p.885. In an attempt to ensure that students are be able to learn mathematics, then it presupposes that the teaching knowledge possessed by the person at the center of affairs which in this case is the mathematics teacher needs to be diagnostically looked at in every respect. An examination of the Core Mathematics Teaching Syllabus for Senior High School (2011), indicates that the major content areas covered in all the Senior High School core mathematics classes are as follows:

1. Numbers and numeration
2. Plane geometry
3. Mensuration
4. Algebra
5. Statistics and Probability
6. Trigonometry
7. Vectors and transformation in a Plane.

* Problem solving and application (Ministry of Education, 2011).

However, as mentioned regarding the JHS mathematics syllabus, beside these content areas, problem solving once again which is not a topic in itself cut across all the topics in the syllabus and are given much importance. The Mathematics Teaching Syllabus for Senior High Schools (Ministry of Education, 2011) classified the profile dimensions that have been detailed for teaching, learning, and testing at this level into two main categories namely Knowledge and Understanding-30% and Application of Knowledge-70%. Again, the Mathematics

Teaching Syllabus for Senior High Schools (Ministry of Education, 2011) outlines the major content areas for Senior High School Elective Mathematics as:

1. Algebra
2. Logic
3. Coordinate Geometry
4. Trigonometry
5. Calculus
6. Linear Transformation
7. Vectors
8. Mechanics
9. Statistics
10. Probability

A critical analysis of the major content areas shows that algebra as a matter of fact appears to be the foundational topic in the entire mathematics teaching syllabus right from the JHS level to SHS level. Despite the essential nature of algebra to all Ghanaian SHS students, various Chief Examiners for SSSCE/WASSCE had emphatically pointed out students' appalling poor performance in mathematics as a result of poor handling of problems involving algebraic reasoning. For example, it was emphatically reported in 2001 Chief Examiners report that some of the candidate's weaknesses in mathematics was as a result of "poor skills in handling algebraic expressions", "failure to use the distributive property of multiplication over addition and subtraction correctly", and “lack of the ability to translate word problems into mathematical sentences” (p. 89).

Ghana and the US comparatively have two distinct ways of offering algebra to its citizenry. While in the US specific algebra courses are offered to students, in Ghana algebra is offered as an integrated course or subject to it citizenry. In spite of these differences, there continues to be public uproar over the performance of students’ in algebra in both national and international assessments. In Ghana for example, students’ ability to progress from SHS level to the university and other tertiary levels of the education system is solely dependent on a national examination, currently called the West Africa Senior Certificate Examinations (WASSCE). In Ghana, this form of examination has been in place since 1987 when the National Educational Reform came into force. As a result of similar educational reforms in neighbouring Anglophone West Africa countries, starting May 2006, the then Senior Secondary School Certificate Examination (SSSCE) was changed into the West African School Certificate Examinations (WASSCE) which is what is currently running. For one to have access to university and other tertiary education in these English speaking countries, every high school leaver is obliged to take the WASSCE before. Unfortunately, since the inception of this practice in 1993, numerous Chief Examiners report of the SSSCE/WASSCE has emphasized students’ poor control of some of the problems on algebra. A typical example is when in 2004, the Elective Mathematics Paper 2 of the SSSCE asked students to express $3x^2 - 6x + 10\, in\ the\ form\ of\ a(x - b)^2 + c$ where a, b and c are integers. Hence state the minimum value of $3x^2 - 6x + 10$ and the value of x for which it occurs (WAEC, 2004). It was emphatically acknowledged in the chief examiners' report at the time that most of the candidates attempted the question. Unfortunately, it was made clear in the report that students did poorly on the question because most of them either could not complete the square or resorted to calculus to find the minimum value, a procedure that was not acceptable.

Looking at the key role algebra plays as a foundational course, it behooves on us as educators and a nation to reverse such trends of students' poor performance in the country. In Ghana not much studies have been carried out to examine the reasons for this poor performance, globally, a number of studies conducted on students' performance in the area of mathematics have indicated that one of the prime factors that can advance students' achievement in school mathematics is the knowledge teachers possess (see for instance, Harbison & Hanushek, 1992; Hill, Rowan & Ball, 2005; Mullens, Murnane & Willett, 1996). We can only ascertain and make informed decisions on the kind of change needed in the knowledge base of mathematics teachers if and only if data about the nature of teachers' knowledge and which aspects of it best relate to student performance were available.

Statement of the Problem

Issues on student's performance have been problematic to handle of late. Various researchers (See for example; Harbison & Hanushek, 1992; Mullens, Murnane & Willett; 1996 Wilmot et al, 2018; Yarkwah, 2017;) coupled with stakeholders in education have combed through all possible factors that affects student's performance. One of those factors is the teachers' knowledge in their subject area of which mathematics is no exception.

As Ball (2003a) asserted, for productivity to increase among the individuals in this twenty-first century, then there is the need for them to be mathematics proficient. Also, the National Council of Teachers of Mathematics [NCTM] (2000) mentioned that, the gateway to a productive future can be assessed by mathematical capabilities whereas the vice versa is true for students lacking the appropriate mathematical competence. Algebra serves as a building block for a firm foundation in a student's mathematical advancement, thus making it an important topic in the life of a high school student.

Ball (2003a) stated again that students who are well equipped in algebra have access to wider range of educational and career opportunities, which also confirms what (Mewborn, 2003) said that the role of teachers becomes crucial in ensuring that students have the requisite knowledge and skills to learn mathematics so they can excel in their future educational opportunities and careers. This makes the teacher an important asset in the development of the student career life, and all this can be worthwhile when the teacher possesses a great knowledge in the subject area.

Students' abysmal performance of mathematics despite its compulsory nature has taken predominance in the Chief Examiner's report of West Africa Examination Council (WAEC) (2008; 2009;2010;2012;2014;2015;2016;2017, 2018). Due to this, West Africa Examination Council Chief Examiners' incessant emphasis on students' inability to perform well in algebra related tasks makes attention on teachers' knowledge for teaching algebra necessary for a study such as this. Some of the reports indicated that students find it extremely challenging to handle algebraic expressions and solve algebraic problems. In specific instance, students' flaws were seen in an attempt to clear fractions tended to ignore the "x"on the other side of the inequality sign. For instance, $\frac{1}{3}x - \frac{1}{5}(2 + x) \geq x + \frac{7}{3}$ was simplified as $5x - 3(2 + x) \geq x + 35$ instead of $5x - 3(2 + x) \geq 15x + 35$" .

This coupled with others point to the fact that problems faced by students in learning mathematics appears to have some connection with their lack of conceptual knowledge and might have been as a result from the teaching experiences they might have encountered in learning algebra at the lower secondary school level (See for example; Bodenhausen,1988; Darling-Harmmond;2000; Farooq & Shalizad, 2006; Klecker, 2002;). Based on this, it was therefore necessary to find out the level of knowledge mathematics teachers in Ghana require in the area of algebra to accelerate the process of making important decisions about the type of upgrading needed in the knowledge base of teachers and subsequently to improve students' achievement (Wilmot, 2009).

A study conducted by Knuth, Stephens, Blanton, and Gardiner (2016) revealed that students who start an algebra curriculum in the early stages of their education take to the subject better in secondary school. However, a study by Mewborn (2003) indicated that while teachers are said to have some level of appropriate knowledge of mathematics, unfortunately, these teachers deficient in the conceptual understanding of the subject they profess. In addition, Mewborn said that, mathematics teachers have a strong procedural knowledge, but lack conceptual knowledge of mathematics. In other words, Mewborn communicated that, a large number of mathematics instructors possess a robust expertise of the routine knowledge but lack a conceptual understanding of the thoughts that support the processes. This implies that, a number of mathematics teachers find it difficult to conceptualize the procedures for students understanding in the mathematics classrooms. Again, Mathematics teachers who have strong foundation in subject matter knowledge of mathematics specifically algebra are able to solve problems using a variety of methods, adapting to different contexts (see for instance, Black 2008). They are also able to identify errors and misconceptions of students on the mathematical concepts in question. This in a way presupposes that the knowledge teachers possess greatly influence students' performance (Eisenberg, 1977; Wilmot, 2009; Wilmot, Yarkwah, & Abreh, 2018; Yarkwah, 2018,). Research is packed with the fact that teachers' content knowledge is often thin and insufficient to provide instruction for students in today's classrooms (Ball, 1988a, 2003b; Ball & Bass, 2000; Ma, 1999; Mewborn, 2003; Stacey, Helme, Steinle, Baturo, Irwin, & Bana, 2001). This generally affects the overall performances of students in these subjects where these problems exist and mathematics is no exception.

This study was based on the assumption that, instead of relying on proxy measures on teacher knowledge, it has become imperative for researchers to re-conceptualise knowledge possess by instructors in a manner primarily not limited to only a particular field but permits the various constituents to be sedate. In the mid to the late 2000s, a team of researchers worked into the Knowledge of Algebra for Teaching (KAT) project at Michigan State University which focused on Senior High School level, came out with a classic framework that sought to measure teachers' knowledge in domain specific terms (see Ferrini-Mundy, Burrill, Floden,

& Sandow, 2003; Ferrini-Mundy, McCrory, Senk, & Marcus, 2005; Ferrini-Mundy, Senk, & McCrory, 2005) as cited in Yarkwah (2017). Moving forward, researching into the KAT framework, Wilmot, Yarkwah and Abreh (2018) proposed a modified framework. To this end no research has been carried out to either confirm or debunk the claim by Wilmot et all (2018) on this new framework.

Purpose of the study

Unarguably, knowing the consequence of teachers' knowledge on students in the classroom and how it affects their performance in mathematics and also the significant role algebra plays in building the mathematical foundation of students becomes crucial at this point in time. This study was designed to investigate if the expanded knowledge framework for teaching algebra developed by Wilmot et al (2018) could be corroborated in other settings in Ghana to confirm its validity and reliability. Also, this study inquired about the knowledge that was possessed by mathematics teachers with professional background qualification and their counterparts without professional background qualification for teaching mathematics to ascertain whether algebra knowledge possessed by these groups of teachers differ. Since eventually these preservice and in-service teachers either with or without education background would find themselves in the classroom teaching. Further, the study sought to investigate the general difference between the knowledge that preservice and in-service mathematics teachers possess in teaching algebra.

Research question

The research question which served as a guide to this study was:

To what extent does high school preservice and in-service mathematics teachers' knowledge for teaching algebra corroborate the seven knowledge types as indicated in the expanded KAT framework?

Hypotheses

The following research hypotheses gave focus to the study:

$H_0 1$. There is no significant difference in the knowledge for teaching algebra between senior high school mathematics teachers with professional background qualification and their counterparts without professional background qualification.

$H_0 2$. There is no significant difference between preservice and in-service senior high school mathematics teachers' knowledge for teaching algebra.

Significance of the study

Since teachers' knowledge for teaching still is a part of factors that contribute students' performance, then there is the need to critically look at the knowledge base of teachers which will help us as a department to understand the nature of our mathematics education program hence modifying it to suit the needs of today's era.

Further, the outcome of this study will help expose whether teachers of the Senior High Schools in the selected region and the preservice teachers involved in the study possess the required content knowledge for teaching algebra to make a tremendous change in students learning. The implication of this is that, this study will help take a keen look at mathematics teachers' professional development in the country at large.

In addition, the result of this study may inform policy makers and implementers in the university to modify the programs and courses to enhance teachers' repertoire of knowledge hence improving students' achievement in mathematics.

Furthermore, the instrument used in this study was an adopted one and for which it was successful in other regions in Ghana, using it in another region would strengthen its reliability and validity to serve as a basis for assessing teachers' knowledge in other mathematically related domains.

Delimitation

Fundamentally, this study was intended to focus on teachers' knowledge for teaching algebra at the senior high school, and for that matter both preservice and in-service senior high school teachers of mathematics were allowed to participate in the study.

For the preservice students, the study was limited to the final year students (that is the level 400 students) since this group of students at the time of the study had taken enough general education courses, mathematics content and mathematics education courses required in their programme. In addition, this same group had gone out for their off-campus teaching practice session which makes them the right cohort of students to use for the study. Also, participants were drawn from only one public university since those universities admit students across the country and they are the pioneers who run variety of mathematics related programmes which their products eventually would find themselves in the senior high schools teaching.

In addition, at the in-service level, teachers who are teaching Core or Elective mathematics or both were used. Since at the senior high school level, mathematics taught is that kind of amalgamated type of mathematics in the Ghanaian context, algebra make up the basis of mathematics at the senior high school level, an assertion was made prior to the fieldwork that any teacher teaching either of the two mathematics courses had enough knowledge in algebra to be able to respond to the items on the instrument.

Furthermore, participants for the in-service in this study were limited to only one Region in Ghana that is Eastern Region. Participants from this Region were not included in the initial study of Wilmot et al (2018) study yet they are of the same characteristics as the ones who participated. Not only that, this Region was selected because it is densely populated with secondary schools and all caliber of teachers from various Universities would be found in these schools.

To add to that, this study only choose teachers as participants since the main factor under study was teachers' knowledge, no other except teachers and for that reason those who were teaching mathematics only were asked to participate.

Finally, since the domain under study was algebra, this means that all the items or questions on the instrument which were used for data collection was algebra related questions and no question from the other domains of mathematics were considered. In this regard, the instrument that was used in the study of Wilmot et al (2018) was adopted.

Limitations

The greatest limitation faced in the study was in connection to the use of achievement test in assessing the algebra teaching knowledge of senior high school mathematics teachers. Teachers naturally do not want to be examined, thus may not bring up their maximum effort to reflect their actual algebra knowledge as expected, especially when they are being assessed by another teacher. This may have affected the outcome of the study.

Further, small number of teachers participated in the study, and this to a large extent could hinder the conclusion of the research because if the sample size of respondents involved were relatively large it could have given different outcome. One other challenge that was faced using the adopted instrument was that some teachers were not at post because of the corona virus pandemic, so getting these teachers to sit and answer the items on the instrument was not easy. In addition, one other major problem that was encountered was the unwillingness on the part of some preservice and in-service mathematics teachers in most of the schools to answer questions on the instrument given in the research for the fear that they may not perform on the test. Also, due to financial constraints, it was not possible to include schools from all the senior high schools in the selected region and from the entire country. These could limit the generalisability of the result.

Definition of Terms

For the aim of this study, the following terms are defined to facilitate easy comprehension:

1. Academic qualification: this is the highest level of education attained by the teacher.
2. Professional and non-professional teachers: Teachers were classified according to whether the teacher is a degree holder with education background or without education background.
3. Content knowledge: This includes knowledge obtained in content-specific courses.
4. Pedagogical knowledge: This includes subject matter taught in education classes.
5. Pedagogical Content Knowledge: the combination of Content knowledge and Pedagogical Knowledge
6. Quality of Teaching Knowledge: It is the total score that a teacher obtains.

Organisation of the Study

The entire study is made up of five chapters.

The Introduction (Chapter 1) comprises of the Background of the Study, Statement of Problem, Purpose of the Study, Research questions and Hypotheses, Significance of the Study, Delimitations, Limitations, Definition of Terms and the Organisation of the Study.

The Second Chapter (Literature Review) takes a critical look at the literature relevant to the study. The review is broken down into the following sub-headings;

i. Conceptual framework
ii. Knowledge of school algebra
iii. Advanced knowledge of algebra

iv. Mathematics teaching knowledge

v. Profound knowledge of school algebra

vi. School algebra teaching knowledge

vii. Advance algebra teaching knowledge

viii. Pedagogical content knowledge in algebra

ix. Relationship between the seven types of knowledge

x. Overview of early research on teacher knowledge and teaching practice

xi. Earlier conceptualisations of teacher knowledge

xii. Teachers subject matter knowledge

xiii. Teacher qualification and their student performance

Chapter Three (Research Methods) looks at the Research Design, Population, Sampling Procedure, Data Collection Instruments, Data Collection Procedures and lastly Data Processing and Analysis.

CHAPTER TWO

LITERATURE REVIEW

Research on teachers' knowledge for teaching is abounding with different conceptualizations. In this line of research, one must not lose sight of the fact that a good conceptualization of teacher knowledge in domain specific and measurable terms would go a long way to help answer the question of which aspect of knowledge required by teachers' best influences students' performance. Most of these conceptualizations since the time of Shulman (1986) have been quite general. This means that these conceptualizations have neither been domain specific nor measurable. Based on this, a lot of proxy measures have been put forward in an attempt to measuring teacher knowledge. Almost in about two decades now, attempts were made by researchers of the Knowledge for Algebra Teaching (KAT) project to re-conceptualize teacher knowledge at the high school level in domain specific and measurable terms (Wilmot et al, 2018; Yarkwah,2017). Wilmot (2009) attempted a corroboration of the KAT conceptualization. Though his study could not validate the KAT framework he made a number of recommendations for further study. However, Yarkwah (2017) and Wilmot et al (2018) corroborated and reconceptualized the KAT conceptualization, which yielded the new types of knowledge making it seven knowledge Therefore, this study is an attempt at corroborating Wilmot et al (2018) conceptualization which came up with seven knowledge types.

Conceptual framework

Detailed analyses of research literature on the recommendations of researchers in the Knowledge of Algebra for Teaching (KAT) project which hypothesized that teachers' knowledge for teaching school algebra consist of three types of knowledge. These are "knowledge of school algebra" (referred to as "school knowledge"), "advanced knowledge of mathematics" (also known as "advanced knowledge"), and "teaching knowledge". A further study by Wilmot et al (2018) which was able to corroborate the original KAT framework revealed the remaining knowledge types which was considered blurry and these knowledge types are; Profound Knowledge of School Algebra, Advanced Algebra Teaching Knowledge, School Algebra Teaching Knowledge and Pedagogical Content Knowledge in Algebra. These knowledge types conceptualized by the KAT framework coupled with that of the Expanded KAT framework will serve as the conceptual framework that will guide this study.

Knowledge of school algebra

"Knowledge of School Algebra" (also known as "School Knowledge") as hypothesized in the KAT framework was defined as the knowledge of mathematics which is enshrined in the curriculum for middle school and high school. This simply means that it is content of school algebra that instructors are estimated to assist students discover in their algebra classes (Wilmot, 2007). In the US, the ideas concerning knowledge such as this are described in booklets such as the National Council of Teachers of Mathematics (NCTM)'s *Principles and Standards for School Mathematics* (NCTM, 2000) while the precise grade-level algebra content is defined in the various states' standards, textbooks and other instructional resources used in the schools. However, in US the researchers of the KAT project restricted this knowledge type by reviewing content standards of ten different states as mentioned by (Wilmot, 2008). Nevertheless, in Ghana the knowledge base of this content is embedded in both the Core and Elective Mathematics Syllabuses which is taken by students at the SHS level. Due to this, for great impact to be made in students learning, teachers must exhibit high understanding of content of school algebra since students at that level will learn from them, and they can only pass on what they know and nothing more.

Advanced knowledge of algebra

From the KAT project, *Advanced Knowledge of Mathematics* (or "Advanced Knowledge") was simply referred to include other "mathematical knowledge, in particular college level mathematics, which gives a teacher perspective on the trajectory and growth of mathematical ideas beyond school algebra" (Ferrini- Mundy, Senk and McCrory, 2005, p.1) as cited in Yarkwah (2018). Areas such as number theory, abstract algebra, complex numbers, linear algebra, calculus, and mathematical modeling were listed as some of the general areas in the KAT project (see Ferrini-Mundy, McCrory, Senk, & Marcus, 2005). Further, Ferrini-Mundy et al. (2005, p. 1) in the conceptualization of this advanced knowledge, recognized that "knowing alternate definitions, extensions and generalizations of familiar theorems, and a wide diversity of uses of high school mathematics are also features of an advanced standpoint of mathematics". This type of knowledge is also referred to as the applied algebra. It is the Application of the algebra contents in other topics. Hence, it can be established that having an advanced viewpoint of mathematics gives teachers an in-depth or profound understanding of school algebra. This kind of knowledge becomes helpful as possession of it can guide a teacher to make appropriate networks across topics whereas unloading the complexity of a mathematics content to make that content more understandable. Moreover, it is believed that if a teacher possesses such knowledge, he or she would hold quite a respectable knowledge of the path of the content of school mathematics. One important reason why every teacher has to be endowed with such knowledge is because, it enables teachers to make connections across topics to eradicate misconceptions as well as difficulties students face whilst disseminating the content of school algebra to learners.

Mathematics teaching knowledge

The last knowledge type hypothesized by the KAT project is "Teaching knowledge". This type of knowledge in the framework according to Ferrini-Mundy, McCrory, Senk and Marcus (2005, p.2) is termed as "knowledge that is precise to teaching algebra that may not be taught in advanced mathematics courses. It comprises such things as what makes a particular concept problematic to learn and what misconceptions lead to precise mathematical inaccuracies. It also contains mathematics required to identify mathematical goals, within and across lessons, to choose among algebraic tasks or texts, to select what to highlight with curricular paths in mind and to ratify other tasks of teaching".

However, this type of knowledge that is possessed by teachers as the fall back on and apply when they are teaching algebra. Moreover, the KAT project acknowledged that "the knowledge been described here may fall into the kind of pedagogical content knowledge or it may be pure mathematical content applied to teaching" Ferrini-Mundy et al., (2005, p.1). Thus, this is the type of knowledge that could distinguish an engineer or a mathematician from an algebra teacher.

Profound knowledge of school algebra

This knowledge type was produced as a result the intersection of school algebra knowledge and advanced algebra knowledge types. This aspect of knowledge provides the teacher with an outstanding compression of algebra which guides them to explain key concepts which bother the students during the instructional process. Teachers with such knowledge type operate at a higher level than their peers who just teach with the school or advanced knowledge. It is an advanced form of these knowledge types which means possession of such knowledge places the teacher at an advantageous point. Thus, it provides "alternate definitions, extensions and generalizations of familiar theorems, and a wide variety of applications of high school algebra" (Wilmot et al 2018, p. 35).

School algebra teaching Knowledge

This type of knowledge emerged from the intersection of school algebra knowledge and mathematics teaching knowledge. The possession of this knowledge type gives the teacher a wider range of the school knowledge coupled with variety of ways to communicate complex issues to students for easy comprehension. Thus, a teacher with such knowledge can combine various teaching methods for smooth instruction of algebra contents to eradicate any challenges that might rise up. Since this knowledge type is an advanced form of the two types of knowledge, it does provide teachers with enough skills to reach out to different kind of learners in the same classroom. They can employ series teaching techniques which will appeal to all learning styles without making students feeling dejected.

Advance algebra teaching knowledge

This concoct of knowledge type emanated from the intersection of the advanced algebra teaching knowledge and that of mathematics teaching knowledge. The acquisition of such knowledge type as asserted by Wilmot et al (2018) states "a teacher's ability to bridge, trim and decompose algebraic concepts even at a stage more advanced that school algebra is the evidence of the possession of this type of knowledge".

Consequently, such knowledge type places the teacher at an advantageous point since such person can teach any advanced or complex algebra when the need arises. Also, teachers of this knowledge type have the ability to make connections within various contents of mathematics being it at the same level of the student or above, whereas employing diverse methods of communicating such concept to student which at the end will be able to cater for all individual differences.

Pedagogical content knowledge in algebra

This is the final piece of the puzzle. This knowledge type was formed from the intersection of all the three types of knowledge which is located at the center of the framework. Hence, it is produced from interfusing the three knowledge types which are (school algebra knowledge, mathematics teaching knowledge and the advanced algebra knowledge). Persons of such knowledge can handle higher order tasks of algebra but not only that but can also combine several methods to teach difficult algebra contents for students to understand.

Relationship between the seven types of knowledge

The three types of knowledge (School Knowledge, Advanced Knowledge and Teaching Knowledge) as hypothesized in the KAT framework project revealed that none of the knowledge types were hierarchical in nature. Thus, they (the three types of knowledge) neither exist in a continuous manner and either of them has a well-defined boundary. The KAT framework project brought to bear that the intersected boundaries of these knowledge types were fuzzy. This claim by the team (KAT project researchers) was examined by Wilmot et al (2018) and they discovered that the boundaries of these knowledge types as hypothesized by the KAT researchers weren't fuzzy after all, but those boundaries produced another type of knowledge on its own. A nice pattern was discovered from these intersections which unmasked the blurry knowledge types that were hidden. Instead of the knowledge types having a clearly defined boundaries with fuzzy intersections, it was rather discovered that school algebra knowledge and advanced algebra knowledge produced profound knowledge of school algebra, whilst school algebra knowledge and mathematics teaching knowledge produced school algebra teaching knowledge. Further, mathematics teaching knowledge and advanced algebra knowledge merged to produce advanced algebra teaching knowledge and lastly, all the three unique knowledge types combined to produce the final knowledge type at the center which was known as the pedagogical content knowledge in algebra. Data that will be gathered for this work will be used to validate or invalidate this expanded KAT framework.

Below is the framework that served as a foundation for the study:

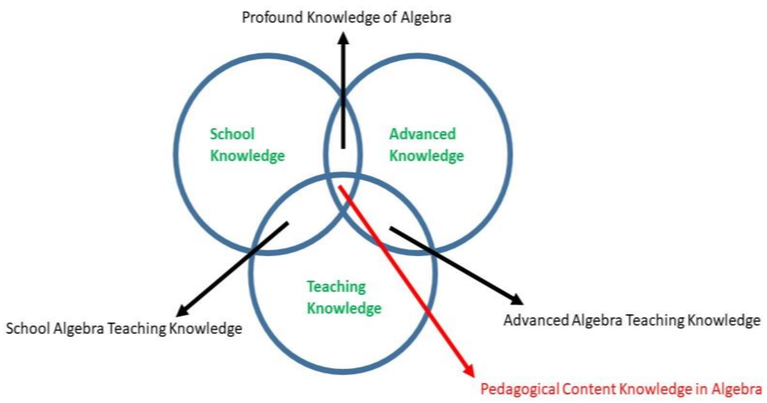


Figure 1: Expanded KAT framework of domain specific teacher knowledge for teaching algebra (Willmot, Yarkwah & Abreh, 2018).

Relevance of the Conceptual Framework to the Study

The foundation of this study is hinged on the conceptualizations of content knowledge, curriculum knowledge, pedagogical knowledge, and pedagogical content knowledge put forward by Shulman and his colleagues (Shulman, 1986b; Wilson, Shulman & Richert, 1987), and the recent discoveries by Yarkwah (2017) which are profound knowledge of school algebra, school algebra teaching knowledge, advance algebra teaching knowledge and pedagogical content knowledge. Every teacher at the senior high school has the skill to conceptualise content knowledge, curriculum and pedagogical knowledge as an effective teacher. A study by Wilmot (2008) exposed his thought about teachers' knowledge concerning the overlapping packages of knowledge (see Ma, 1999) or Putnam (1987) curriculum scripts. My disposition on these related packages of knowledge according to Ma is the combination of knowledge on the content of the subject matter teachers teach, knowledge of other content in the school curriculum and how they are related as well as why certain illustrations could be challenging or easy to some students. The concept of pedagogical content knowledge as promoted by Shulman resonate well with my view which include illustrations of specific content together with why the learning of that content is stress-free or challenging for some students.

Before the KAT conceptualisation was introduced, most previous researchers who depended on Shulman's conceptualizations have only focused on teacher knowledge in a qualitative manner. The introduction of the KAT conceptualisation has enlightened me about learning about teaching in a number of ways. To begin with, advanced knowledge and school knowledge has brought to light reasons for which instructors need not only familiarise themselves with the content which they are giving to students, but also, they must have some additional knowledge about areas of their subject and other fields that are connected to it. This claim is embedded in the KAT project's construct of "advanced knowledge" which covers the higher knowledge teachers should possess in understanding school algebra.

The assertion of Ferrini-Mundy and her team that the meeting points of their three conceptualizations or the knowledge that existed between the intersections of the interlocking circles is blurry may not be fuzzy after all as discovered by Yarkwah (2017). However, his study refutes that assertion and established that those intersections of the interlocking circles are not blurry. The study corroborated the three forms of knowledge put forward by Ferrini-Mundy and her group and in addition revealed that the interlocking sections of the three initial hypothesized knowledge types produced yet another knowledge namely; *Profound knowledge of school algebra, Advanced Algebra Teaching Knowledge, School Algebra Teaching Knowledge and Algebra Pedagogical Content Knowledge,* making the seven knowledge types in total. The divulgence of these new knowledge types will help teachers to explain those concepts that seems complex to students.

Furthermore, the expanded KAT framework has enlightened lots of teachers about the different types of knowledge a teacher must fall on in teaching algebra and to help them explain topics which are complex and interrelated in the curriculum for better understanding of learners. The researcher strongly believe adopting this conceptual framework will not only place emphasis on the seven knowledge types as hypothesized by the expanded KAT framework, but it will also enable validate SHS mathematics instructors' knowledge for dispensing algebra in their various schools settings.

Overview of Early Research on Teacher Knowledge

Several reviews on teacher knowledge in literature depict such research started in the 1920 in the form of process-product research in the US (Brophy & Gold 1986; Gage 1978; Doyle 1977). These reviews suggested that, the process-product studies were designed to create a bridge regarding the activities of instructors in their respective schoolroom and learner performance. Because of this, researchers who used the process-product design measured student's outcomes and related them to the actions of the teacher. Wilmot (2009) opined that, "Coding teacher actions [this way] was an indirect attempt at breaking down which aspects of teachers' knowledge are transformed into their teaching practice" (p. 37).

Several criticisms were discovered in the process-product in the 1970's (see Gage & Needels, 1979). The following criticisms were highlighted 1) the inkling of causation disguised in the process-product research paradigm (i.e. their over dependence on relational approaches), 2) concerns about the predictive power of the process-product design, 3) problems related to the predetermined coding categories and the need for experimental methods, and 4) the conversion of the findings of process-product researchers into rules for teaching.

Because of these criticisms, adjustments were made in the design in studies that were conducted after the process-product researchers (Berliner, 1979; Peterson & Swing, 1982). Berliner (1979) and his team in the Beginning Teacher Evaluation Study (BTES) for example, presented a variable known as the Academic Learning Time (ALT) in their alteration of the process-product design. One crucial aspect of ALT is what the BTES program refers to as *engaged time*, the definite period learners devote in task delivered by the instructor in learning a specific content. Berliner and his colleagues made an argument that, there wouldn't be any significant improvement in student's academic performance if they are introduced to easier tasks always. On the other hand, if too difficult items are what the student spends his/her time on then the learner at a point of not being able to major the additional ideas, skills and procedures required for decent performance at where he/she finds him/herself on the educational ladder. Berliner and his colleagues made a case that their new variable ALT served as a mediator between learner's output and their instructor's behaviour and also an operational behavioural pointer of learners learning. Regrettably, the ALT concepts couldn't reach its goal because of these two reasons; It failed not only in showing the kind of knowledge instructors must retain to effectually point out the accurate level of struggle of works to give to learners to advance their erudition but also indicate how teachers are able to decide when to move to new materials.

Later (Peterson and Clark, 1978; Putnam, 1987) also came into the limelight and opined that it was important to bring the mental life of the teacher to the center of research on teaching. Their argument was that, knowledge of expert instructors is systematized in bundles of inquiry and enlightenments that make it conceivable for them to improve learner learning (Putnam, 1987; Shulman, 1987). Putnam (1987) states these bundles as "curriculum scripts" and made a case that instructors' agenda for coaching is fashioned by the richness of their curriculum scripts. Thus, the capability of a coach to flexibly use interactive strategies to teaching depends on the quality of his/her curriculum scripts. The researchers of this study suggested that, "by focusing on the mental life of the teacher, the thought process of teachers before, during and after teaching could be rightly studied in order to understand how teachers transform their knowledge into their teaching practice" Yarkwah (2017, pg. 43).

Shulman and his teams' work brought to light the current scope on how teacher knowledge can affect teaching (Shulman, 1986; Wilson, Shulman & Richert, 1987). It can be said that the driving force for the revived interest in studying teacher knowledge could be a result of Shulman and his colleagues' conceptualization of "content knowledge" and "pedagogical content knowledge" and the distinction between them (see Ball, 1988; Wilson & Winneburg, 1981;

Grossman, 1990). The familiar outcome about the initially mentioned research studies is that they all ended up producing qualitative data on teachers' knowledge, which stems up the reason for employing the Expanded KAT framework for this study. The rationale for choosing this framework is also hinged on the using of the KAT approach as their foundational framework which in diverse ways was an improvement over the previous attempts. For example, rather than continuing to generalize teacher knowledge as a construct, the KAT project conceptualized it and made the focus domain specific (i.e., focused on algebra). Further, the Expanded KAT framework developed and validated an instrument which made the knowledge based of teachers measurable.

Earlier Conceptualisations of Teacher Knowledge

Duthilleul and Allen (2005) purported that in the United States, intriguing research on teachers knowledge revived following the report entitled *Equality of Educational Opportunity* by Coleman et al. (1966) "concluded that domestic circumstantial features and communal level variables were the basis for more discrepancy in learner accomplishment than school resource variables like . . . instructor features" (p.3). In the nut shell, the report from Coleman's committee made a conclusion that teachers and school do not affect student learning in any way. Following this, a debate was held on the issue on how teachers and schools influence students learning. It was through these debates which was spearheaded by Shulman (1986b, and 1987) that revived a new driving force into the research of knowledge base for teaching. According to Storm (1991), "concern about the knowledge base emphases on refining the respect and position afforded teaching, thereby making it a more rewarding career" (p. 1). The underlining factor was that, for teaching to be acknowledged as a profession which influence learning outcomes, then a case needed to be made that a specialized body of knowledge is involved in reaching that desired goal. Going forward, Shulman (1986b) established the idea of "pedagogical content knowledge" as a kind of knowledge which entails details on how students understand and the appropriate ways to effectively use resources to present ideas in ways that make them more reachable to different categories of students.

Shulman (1986) conceptualized teachers' knowledge in seven categorization: 1) Content knowledge, 2) General pedagogical knowledge, 3) Curriculum knowledge, 4) Pedagogical content knowledge, 5) Knowledge of learners and their characteristics, 6) Knowledge of educational context, and 7) Knowledge of educational ends, purposes and values, and their philosophical and historical grounds.

In the context of the current research, the pedagogical content knowledge is of greater importance in the knowledge base for teaching since it depicts the blending of content and pedagogy for comprehension of how concepts can be presented to the learner.

Several projects have been held aside Shulman (1987) on conceptualization of teachers' knowledge by different researchers (see for instance, Ferrini-Mundy, Senk & McCrory 2005; Hill, Ball & Schilling 2004; Ball & Bass 2000; Ma 1999; Leinhartdt & Smith 1985). Also, in the study *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and* *the United States,* Ma (1999) discussed the results and analyzes of interviews of a study conducted that comprised of 72 and 23 elementary schoolteachers from China and U.S. respectively. Through this study, Ma (1999) introduced another conceptualization of knowledge that will become handy to teachers in the teaching of mathematics, which she named as, "profound understanding of fundamental mathematics" (PUFM). The PUFM contained a knowledge type that surpasses content knowledge; it provides information on how to communicate the subject matter of school mathematics to students. Apart from the fact that Shulman's (1986) PCK is a universal form of knowledge (thus, it doesn't lend itself to a particular subject), then Ma (1999) PUFM and Shulman (1986) PCK has something in common since both seem to encompass a multifarious amalgamation of content and pedagogical knowledge.

In another breath, Leinhartdt & Smith (1985) propounded other types of knowledge which were "lesson structure knowledge" (LSK) and "subject matter knowledge" (SMK). Per their explanation, LSK provides teachers with skills to plan lessons and provide vivid explanations in the course of teaching whereas SMK knowledge type gives teachers a clear understanding of concepts, algorithmic operations and the knowledge of the types of error students commit.

Another project on the knowledge base instructors possess for coaching at the elementary school by Debora Ball and co proposed a knowledge type called "specialized knowledge of content" (SKC), per them is "a constituent of several items: signifying numbers and operations, analyzing unusual procedures or algorithms and providing explanations for rules" (Hill, Ball & Schilling, 2004, p.

28). Whereas Debora Ball and her colleagues were working on teachers' knowledge base in delivering mathematics at the elementary school, Joan Ferrini-Mundy and her colleagues on the Knowledge of Algebra for Teaching project were also studying the knowledge base for delivering algebra at the high school level (see for instance Ferrini-Mundy, Senk & McCrory 2005) around the same time.

As established earlier, the conceptual framework which underpins this study is the KAT framework by Joan Ferrini-Mundy and her colleagues. Their framework is useful for this study since it lends itself to assess both qualitatively and quantitatively; a quality that has the potential of leading to measurable types of teacher knowledge and thereby helping to eliminate the reliance on proxy measures of teacher knowledge.

Teachers Subject Matter knowledge

Before an effective teaching and learning will commence, one important factor that enable the task to be successful is the subject matter knowledge of the teacher. Many remarkable intellectuals, arithmeticians, and policy architects decide that an instructor's mathematical knowledge is a crucial constituent of his or her success as an instructor (see for instance Askey, 1999; Ball & Bass, 2000b; CBMS, 2001; Hill, Rowan, & Ball, 2005; Milgram, 2004). Subject matter refers to what one knows about what he or she teaches (Cochran, DeRuiter & King, 1993). In light of this study, and the focal point of this research, subject matter knowledge of the mathematics teacher is what he or she knows (knowledge possessed) about algebra (Algebra content). Teachers' subject matter knowledge has been analyzed and approached more qualitatively, emphasizing knowledge and understanding of facts, concepts, and principles and the ways in which they are taught as well as knowledge about the discipline (Ball, 1988, 1991; Even, 1990; Kennedy, 1991; Leinhardt & Smith, Shulman, 1986; Tamir, 1987; Wilson et al., 1987). As a matter of fact, the subject matter knowledge is the knowledge about what (Content) the teacher communicates for students to grasp or retain in their minds.

Discussion on teachers' content knowledge (TCK) and teacher's pedagogy knowledge (TPK) of late has attracted increasing attention from several agents of change in education industry. Research regarding teachers' knowledge is as important to scholastic reform today as it was four decades ago (Ball, Lubienski & Mewborn, 2003). Teachers must have requisite knowledge in the content areas for which they are expected to teach. This must include a deep understanding of the mathematics they are teaching (NCTM, 2000). For a teacher to teach very well he or she needs to know about the subject matter in a comprehensive way which surpasses those beginning teacher training course (Simon, 1993). Teachers can only teach within the knowledge they have; thus, they cannot communicate knowledge they do not possess. It is an undisputable fact that every good teacher must learn more mathematics and that the higher the level of mathematics a teacher knows the better teacher he/she becomes in its execution (American Council of Education, 1999). Teachers are metacognitive characters and that teaching is a multifaceted, cognitively demanding process involving problem solving and decision making (Clarke & Peterson, 1986). Admittedly, the subject matter knowledge of teachers is very essential, however, Thompson and Thompson (1996) stated that, mathematical content knowledge is essential for effective teaching; nevertheless, study revealed that teachers require further than just a strong knowledge of the content in order to teach mathematics. This suggests that, there is more to acquiring just the concepts and the subject matter. Mewborn (2003) stated that, as much as teachers are expected to have some level of comprehensive knowledge of mathematics, subsequently, these teachers do not conceptually understand the mathematics they are to dispense to students. He further said that, mathematics teachers have a robust routine knowledge, but deficient in conceptual knowledge of mathematics. Which can also be explained that, a large number of mathematics teachers have a strong command of the procedural knowledge but lack a conceptual understanding of the ideas that reinforce the processes.

Further, mathematics teachers who have strong and an in-depth understanding of the subject matter knowledge of mathematics specifically algebra are able to solve problems using varying methods and adapting to suit different contexts (Black 2008). They are also able to identify errors and misconceptions of students on the mathematical concepts and help them overcome that challenge.

In summary, secondary school mathematics teachers need to have adequate knowledge of the subject matter with respect to algebra both in depth and in breadth to enable them to effectively communicate algebra knowledge to students. The in-depth Content knowledge expected from mathematics teachers cannot be less than whatever knowledge their students possess. The Knowledge of secondary school mathematics teachers should be deeper than what students are to grasp, in effect, teachers are to demonstrate control over the content knowledge than their students. That is what puts the teacher above his or her students with respect to teaching and learning. Teachers are supposed to demonstrate high level of content knowledge of what they teach. Although, having a deep control over the subject matter or content of a particular mathematical subject is not sufficient to effectively teach mathematics, however, it forms the basis to enable a mathematics teacher teach effectively if he or she has an adequate knowledge of pedagogy.

Teacher qualification and their student performance

Every country's educational success can be hinged on the caliber of teachers within the educational setting. As it has been known over years that a country's development is largely dependent on the quality and quantity of her qualified teachers. When seasoned teachers are employed, it goes a long way to affect the overall performance of students (Abe & Adu, 2013). (Abe, 2014) opined that, there is a significant relationship between student's performance and their teachers who taught them. On the contrary, there have been other studies which findings revealed an opposing thing, for instance Igwe (1990) study on "*the influence of teachers qualification on academic performance of students in science subjects*" revealed that there was no relationship between teacher's qualification and their student's performance. Not only that, other works such as Adeniji (1999), Osokoya (1999), and Oladele (1999) all brought to light how insignificantly teacher's qualification barely affect student performance and further stated that there could be other contributing factors that warrant student poor performance as cited in Abe (2014). In contrary to the above issues, a lot of research is replete with the findings that student's performance can be attributed to the kind of qualifications the teacher holds and that was confirmed by what Lussa (1985) said that, no teacher can give what he or she does not have. Thus, a teacher or an instructor can only pass on knowledge that he/she possess. Moving on to other studies, Adesina (1982) and Fafunwa (1985) asserted that, a number of teachers without good qualification in mathematics will encounter challenges when teaching secondary school mathematics. Seweje and Jegede (2005) also added their voice that, a teacher's teaching skill and ability is not obtained from their academic background only but also the pedagogical skill they possess. Which signifies having enough and strong content doesn't guarantee the smooth delivery of lesson. These pedagogical skills are usually acquired through professional training of the teacher, yet most schools contain teachers with non-professional training which its consequential effect on the students' performance is tragic. This has been confirmed by Owolabi and Adedayo (2012) by their findings revealing that professional teachers have positive impact on students' performance than the unprofessional teachers. With all that been said, other studies took it a little further to ascertain if there is a difference between the knowledge that is possessed by these professional and non-professional teachers. In as much as most works agree it is better to have professional mathematics teachers in the various schools, Yarkwah (2017) study refuted that claim by reporting that there is absolutely no difference between the knowledge that is possessed by these two individual groups. Which suggest either of these calibers of teachers are capable and have the required mathematical knowledge to teach mathematics at the said level.

Procedural and Conceptual Knowledge of Algebra

For some many years some teachers have taught without consideration to procedural and conceptual knowledge acquisition. It must be noted that in recent times, the teaching and learning process has metamorphosed in ensuring that focus is shifted towards a equilibrium between procedural and conceptual understanding of mathematics. According to Hope (2006), in procedural mathematics understanding, it is simply the knowledge that emphases on skills and step-by-step procedures without overt reference to mathematical ideas. In another breadth, Anderson (1989) explains procedural knowledge as "organization of conceptual knowledge into action units" (p. 24), without conceptual knowledge, this description of procedural knowledge is inoperable. With reference to Anderson's definition of procedural knowledge, it is imperative to know that without conceptual knowledge, procedural knowledge is useless and meaningless. The underlying reason is that it would be executed without understanding and mathematizing would be like a puzzle of numbers and operations without understanding.

With procedural knowledge, one tries to answer the question "How?"; the procedure to be followed without necessarily understanding "Why?" the conceptual understanding of what is being done. In doing so, what then happens is that ordinary procedural skills frequently nosedive to provide readily applicable methods to solve mathematics problems. Again, according to Hope (2006), conceptual mathematics understanding is simply the knowledge that involves a comprehensive understanding of fundamental and foundational concepts behind the algorithms completed in mathematics. This aspect of knowledge acquisition tries to answer the question of “WHY?”; the details of whatsoever is being done. Consequently, it encompasses a condition where students are able to reconstruct formulas and proofs without the rote process. Furthermore, students are permitted to make choices and apply their understanding through vigorous engagement (Boaler, 2000). To this end, Wilkins (2000) posits that students must have an understanding of both if they are to understand mathematics in depth. Also, according to Ghazali and Zakaria (2011), the major problem students learning a topic like algebra and algebra related topics encounter are primarily concepts and not with those involving algorithms and procedures..

According to Mary and Heather (2006), to successfully complete an algebra problem, individual students must improve both procedural and conceptual understanding. In conclusion, the balance between procedural and conceptual knowledge of algebra is a necessity for teachers and their students at all levels of the education ladder. To this end, it must be noted that students at the SHS level need to develop both the conceptual and procedural understanding of algebra to gain firm and robust foundation for future mathematics. When the objective of studying mathematics is solely to pass examinations, one may tend to focus more on procedural knowledge at the expense of conceptual knowledge of algebra and mathematics as a whole. This as a matter of fact does not in any way help in leading students to build a strong foundation in mathematics at the SHS level which affects their general performance in mathematics and future pursuance of mathematics at the tertiary level.

Lim (2002) asserts that procedural understanding can support in acquisition control over conceptual understanding. This will only be possible if SHS mathematics teachers would go the extra mile to let their students know 'why' after knowing 'how'. Until today, experts in mathematics education have been researching to understand the balance between the two understandings. Some researchers are of the opinion that both are noteworthy and that incorporating both of them is imperative to raise learnners' understanding (Mary & Heather, 2006). It is very difficult and sometimes impossible for teachers who do not understand the concepts themselves to assist their students gain conceptual understanding and even sometimes procedural understanding. To this point, it is imperative to find out if senior high school mathematics teachers have mastery or control over algebra content conceptually and procedurally to be able to help their students overcome difficulties during the instructional process.

Summary of Literature Review

Algebra has a significant role in mathematics since it forms the foundation for a strong mathematical background. Students who had a strong grasp of the foundational concepts of algebra, performed substantially well. Teachers plays keen role in students learning and in the forming of a strong mathematical basics for the future. Teachers whose foundation in algebra is weak would find it difficult or challenging to communicate well algebra contents to students for them to comprehend. This would produce students with poor general performance in mathematics since their algebra foundation is not adequate enough.

Teachers as professed by many studies that has been done (see Mullens, 1996; Sanders & Rivers, 1996), stated that there is a constructive relationship that exist between instructors and the achievements of the students. Mullens also exclaimed instructors' knowledge on topics they dispense was found to be a superior forecaster of learners' accomplishment than other contributing elements.

The review of these related literature brought to the notice that although in as much researchers were in agreement on the expediency of instructors' knowledge in inducing learner performance, they subjectively did not effectively conceptualise it so as to point out which domain of it favourably forecast learner performance. (see for instance, Begle, 1972; Eisenberg, 1977; Clark & Peterson, 1986; Wilmot, 2009; Wilmot et al 2018). In light of that, this research was conducted to ascertain the role teachers' knowledge play in the performance of their students. Thus, this study is hinged firmly on discovering the knowledge types teachers possess in the teaching of algebra in the senior high schools.

CHAPTER THREE

RESEARCH METHODS

The study was to find out if the knowledge that the study participants possess corroborate the re-conceptualized KAT framework by Wilmot et al. (2018) and further examine any difference between in-service and preservice mathematics teachers' knowledge for teaching algebra.

This chapter highlights the research methods that were employed in this research. It focused on the research design, population, sample and sampling procedure, instrumentation, data collection procedure and ends with issues on data analysis.

Research Design

This study was focused on investigating preservice and in-service senior high school mathematics teachers' knowledge for teaching algebra in an attempt to possibly corroborate the algebra knowledge framework proposed by Wilmot et al. (2018). Participants were requested to respond to items on a multiple-choice type of questions. The instrument for this study was meant to measure seven knowledge types prospective teachers are expected to have for teaching algebra at the SHS level. As a result, a cross-sectional survey design was employed.

The cross-sectional survey was appropriate in the context of this research in that it permits gathering data from a sample of mathematics instructors without modifying their previous knowledge (Creswell; 2003; Cohen, Marion, & Morrison, 2000; Mitchell & Jolly,2004;Nworgu, 2006). In addition, this design was more cost-effective because it enabled information to be gathered on the preservice and in-service mathematics teachers (i.e. a snapshot of instructors in the selected demographics) at only one point in time (Mitchell & Jolley, 2004).

Survey designs have been established by many researchers to have the potential to provide an exposure to reach a large sample size which in turn raises the generalisation of the findings. Also, they also provide opportunity for the respondents to react to the items on the survey in a place and time suitable and proper to them as well as producing responses that are easy to code (Gay, 2011). In another breadth, these kinds of designs have the potency of providing descriptive, inferential and explanatory proof that can be used to create correlations and associations between the items and themes of the survey (Cohen, Manion, & Morrison, 2007, p. 169).

Sarantakos (2013) opined that cross sectional surveys also provide a dependable and unvarying procedures and participants are not affected by the existence and or attitudes of the investigator. In as much as there are positive sides of this research design, there are a number of disadvantages that also come with it. According to Sarantakos, some shortfalls of this survey design is the failure to enquire penetrating questions as well as pursue illuminations from respondent. Further, it does not have the ability to ascertain the circumstances under which the participants responded to items on the instrument as well the capability to produce extraordinary impassive rate.

Since data was collected at only one point in time the design could not permit the study to account for any possible changes that may occur in the knowledge of the participants after the study. Notwithstanding the weakness, it was considered that the strengths of gaining many teachers' responses made the cross-sectional survey the appropriate design for the study.

Population

The target population for this study was all pre-service teachers and in-service mathematics teachers in Ghana. The accessible population of the in-service teachers was made up of all mathematics teachers from 16 Senior High Schools in the Eastern Region and final year mathematics education students from one public university in the Central Region of Ghana. Based on the content of algebra in both core and elective mathematics syllabi, the researcher used teachers either teaching core mathematics or elective mathematics or both Core and Elective Mathematics in the selected schools. In all, there were 278 respondents. For the purpose of this study, only in-service and pre-service mathematics teachers teaching elective and core mathematics were used in the study. The pre-service teachers sampled for this study were from a public university in the Central Region of Ghana. Participants were also sampled from this group because the study sought to find out the knowledge these people possess as they prepare to teach the subject in the various senior high schools. It must be noted that it is imperative to compare in-service and pre-service mathematics teachers' knowledge for teaching algebra in this case because for the in-service teachers these are teachers who have been teaching mathematics for quite a number of years and are therefore expected to possess rich pedagogical skills. On the other hand, these pre-service teachers are novice teachers who have just returned from the field on macro teaching practice and about completing their university education. For instance, one of the aims of the study was to find the difference between preservice and in-service senior high school mathematics teachers' knowledge for teaching algebra. In that case, it was necessary to include samples of the preservice teachers to enable that purpose fully carried through.

Sampling Procedure

All senior high schools that participated in the study were selected from the Eastern Region of Ghana. A computer-generated random number was used to select 16 schools from which the samples were drawn.

Multi-stage sampling procedure was used in the selection of the participants. With this category of technique, two or more sampling techniques are employed in a single study, and also, this technique makes room for sampling to be carried out in several stages. Specifically, purposive, convenience, census and the simple random sampling techniques were used.

The simple random sampling procedure was used in the selection of various schools in the Eastern Region which participated in the study since not all the schools were included in the study. The computer random generating number was used to randomly sample the schools which were included in the study. For the Convenience sampling technique, it was used in picking the public university involved in the study because it's a university with quite a number of students reading Bachelor of Education in Mathematics who have gone through the requisite training deserving to be involved in this study. In addition, it was the university where the researcher happens to school and for that matter collecting data from its students was a bit easier.

Purposive sampling was used to select the participants of the study because the major purpose of the study was to find out the knowledge that was possessed by the mathematics teachers in teaching algebra so no other teachers except those who teach and would be teaching mathematics qualified as a participant in the study. In other words, the study sought to focus only on mathematics teachers because they were the needed cohort of people for the study. Additionally, the researcher purposively selected the Eastern Region because it is one of the regions that is highly populated with senior high schools aside Ashanti Region and also, Wilmot, et al. (2018) did not include participants from that region. Hence, it was prudent to include participants from that Region since they have the same characteristics as those who took part in the first study conducted by Wilmot and his team on the reconceptualisation of KAT project. The census technique was adopted since all mathematics teachers in selected schools were included in the study. It was the sole aim of the study to involve all mathematics teachers teaching within all selected schools since the focus was to investigate the knowledge that these teachers possess. Thus, it was appropriate to adopt the census method in order to obtain the needed participants.

The sample size comprised of 177 in-service and 101 pre-service mathematics teachers making 278 participants in all. The 177 in-service mathematics teachers were from 16 senior high schools in the Eastern Region whereas the 101 preservice teachers were selected from one public university in Central Region of Ghana. Also, the sample comprised of 71 and 207 mathematics teachers with non-professional background qualification and professional background qualification respectively.

Data Collection Instrument

The study adopted an achievement test instrument from Wilmot et al (2018) study that re-conceptualized teacher knowledge for teaching at the senior high school level. The instrument adopted was made up of 46 multiple choice type of questions covering the seven knowledge types. This study adopted the instrument containing 46 test items that loaded uniquely on the new framework put forward by Wilmot et al (2018). The multiple-choice item format was employed because from personal experience, people and for that matter teachers become hesitant in answering open response type of questions but find it relatively comforting in answering multiple choice type questions since options are provided.

There are numerous advantages of using this adopted instrument. One key advantage of employing the instrument in carrying out the study was that, it covered a wide range of areas in the syllabus for both core and elective mathematics in addition to pedagogical and content issues which basically covered the theorized knowledge types in the expanded framework. Another good thing about the instrument used was that it was spread across Bloom's taxonomical areas and not limited to only an aspect, which at the end was able to assess teachers holistically without been skewed towards one direction. The language in addition to the terminology used in the instrument was clear and accurate so as to prevent any misinterpretation on the part of any teacher. This to a large extent helped in unraveling knowledge for teaching algebra at the high school level and assisted ascertain if really other knowledge types existed.

Validity

The content validity of the instrument was substantiated by showing the tests and its scheme to experts in the area of mathematics education and fellow colleagues as well as the supervisor to ensure that the types of knowledge hypothesised in the new framework are satisfactorily covered and well structured. No further adjustments were made since this study was to confirm their findings using the same instruments used by Wilmot et al. (2018).

Reliability

The reliability coefficient for the instrument from the initial work which the instrument was adopted was calculated (see for example Yarkwah, 2017) using the KR-20 formula and was found to be 0.855. This finding coincides with Vaske's (2008) suggestion that reliability coefficients in the 0.65 – 0.80 range are 'adequate' and acceptable, but higher than that has strong reliability which is very good. Since the reliability coefficient is 0.855, it is deemed to be trustworthy and dependable. However, the reliability coefficient obtained before and after data collection was recorded to be 0.75 and 0.830 respectively which indicates that the instrument is reliable enough to produce dependable results over a period of time.

Data Collection Procedure

Before the entire study began, a research visit letter was obtained from the supervisor and the Department of Mathematics and I.C.T. Education to request for ethical clearance from the Institutional Review Board of the University of Cape

Coast to enable the data collection start. The schools which were finally involved in the study were initially visited for the distribution of letters from the department. Throughout the real visit, permission was sought from the head teachers, heads of the departments and the preservice and in-service teachers who were involved in the study. It must be stated that data from the preservice teachers were obtained when they had returned from macro teaching practice. The researcher explained to them the purpose of the study, the duration involved in answering the items on the questionnaire, the measures to ensure privacy of the data collected from them and the potential benefits of partaking in the study for their consent. The instruments used for the study were administered to respondents in their respective schools. The administration of the questionnaire (achievement test on knowledge for teaching algebra) was done in the months of March and was continued from the month of June to August 2020. The duration of the distribution of the instruments delayed as a result of the closure of Secondary Schools in the country and after their resumption, it was difficult getting access to participants since various protocols were to be met before meeting the respondents.

There were few mathematics teachers who decided not to partake in the study and they were allowed to opt out. Some respondents agreed to answer the questionnaires right after the meeting, others also scheduled different times for the administration of the test. For the purpose of confidentiality teachers' responses and names of teachers who participated as well as schools these teachers teach were not recorded in the instruments to assuage their fears of being exposed. The instruments for the study were administered to the subjects in their various natural settings. The researcher collected data from 16 various Secondary Schools in the Eastern Region. The researcher used the list of the sampled Secondary schools in the Eastern Region to visit their various schools for the administration of the test instrument. Throughout the data collection process, the researcher was present with each of the respondents answering the items on the teacher made achievement test. In order not to influence and also to reduce tension on the part of the respondents, the researcher stayed a distant away from where the respondents were answering the items. Respondents were given a maximum time of 150 minutes to respond to all the items on the instrument. In all, 350 instruments were given out to participants with a return rate of approximately 79%.

Data Processing and Analysis

In any research work, raw data collected from the field needs to undergo some sort of scrutiny before it is processed into meaningful and relevant information for decision-making. Howard and Sharp (1983) asserted that the process may include ordering and shaping of data generated from the research to produce knowledge. In another breadth, Burns and Grove (1987) mentioned that this same process helps in decreasing, organizing bulky data collected, and analysing it to produce findings. The items on the achievement test were assigned either wrong or right in order to ascertain teachers' knowledge for teaching algebra. Data from this study were purely quantitative in nature. This is because participants were expected to respond to the content items on the instruments.

Analysis of data related to research question

The first research question that guided this study was, "To what extent does high school preservice mathematics teachers' knowledge for teaching algebra corroborate the seven types of knowledge hypothesized in the expanded KAT framework by Wilmot et al (2018)?"

Information meant for this research question was collected from preservice and in-service teachers who were going to teach or were teaching Core Mathematics or Elective Mathematics or both at the senior high school level. Factor analysis specifically Confirmatory Factor Analysis (CFA) was used in this case. This was necessary because CFA is mostly employed during the course of scale development to ascertain the latent arrangement of a test instrument (in this research the achievement test questionnaire on knowledge for teaching algebra at the SHS level). In this situation, CFA basically was employed to ascertain the number of fundamental proportions of the instrument (factors) and the array of item–factor relationships (*factor loadings*). Further, CFA supports the context of how a particular test item must be graded. When the latent arrangement has at least two factors , the pattern of factor loadings supported by CFA will designate how a test may be scored by using subscales; that is, the number of factors is indicative of the number of subscales, and the pattern of item–factor relationships (which items load on which factors) indicates how the subscales should be scored hence it usage in this study. After the CFA, the factor loading for each item was then analysed for conclusions to be made based on the seven factors that have been hypothesised from the beginning of the study as stipulated by the expanded KAT framework.

Also, the nature of the loadings of the items were also determined to confirm the extent to which items which were originally categorised as looking at the same dimension load together. Finally, a regression line was imposed on the scree plot to confirm or otherwise the knowledge types hypothesized in the expanded KAT framework.

Analysis of data related to first research hypothesis

The first research hypothesis that guided this study was, "There is no significant difference in the knowledge for teaching algebra between senior high school mathematics teachers with professional background qualification and their counterparts without professional background qualification". To answer this research hypothesis, data from preservice and in-service mathematics teachers who have background training in mathematics and their counterparts without background training in education were used. Since the subjects of focus who participated in the study were independent samples, the independent samples t-test was used in analysing data that was obtained from these two groups based on the combined score. Also, further analysis was conducted using multivariate analysis of variance (MANOVA) and analysis of variance (ANOVA) as a follow up test to the MANOVA to ascertain where the difference was or were coming from with regards to the seven knowledge types hypothesized in the expanded KAT framework. The ANOVA was resorted to in this case because two independent groups (that is teachers with professional background qualification and their counterparts without professional background qualification) as against the seven knowledge types hypothesized in the framework. This hypothesis was tested at the 0.05 level of significance.

Analysis of data related to the second hypothesis

The second hypothesis that served as a guide this study was, “There is no significant difference between preservice and in-service senior high school mathematics teachers’ knowledge for teaching algebra”.

This hypothesis was answered using scores obtained from the preservice and in-service senior high school mathematics teachers. Since the focus of the question was to look at the knowledge these preservice and in-service mathematics teachers possess at the SHS level for teaching algebra, analysis basically was focused on the use of both descriptive, frequencies and percentages, and inferential statistics (Independent Samples t-test). In addition, item by item analysis was done in some cases to further explain the outcome of some of the analyses. This analysis, like all analyses of data from the educational research, was done at the 5% level of significance.

CHAPTER FOUR

RESULTS AND DISCUSSION

In this chapter, presentation of statistical analyses of survey data collected from sixteen schools in the Eastern Region and one public university in the Central Region of Ghana have been carried out. These analyses helped address the following research question and null hypothesis which were formulated to guide the focus of the study:

1. To what extent does high school preservice and in-service mathematics teachers' knowledge for teaching algebra corroborate the seven knowledge types hypothesised in the expanded KAT framework by Wilmot et al (2018)?
2. There is no significant difference in the knowledge for teaching algebra between senior high school mathematics teachers with professional background qualification and their counterparts without professional background qualification.
3. There is no significant difference between preservice and in-service senior high school mathematics teachers' knowledge for teaching algebra.

These research question and hypotheses are discussed in relation to the various sub-constructs used in the instrument. In addition, in the analysis of the results, data, analyses and all results related to a particular research question or hypothesis are duly presented and discussed.

Research question one

The research question that gave focus to the study was "To what extent does high school preservice and in-service mathematics teachers' knowledge for teaching algebra corroborate the seven knowledge types hypothesized in the expanded KAT framework by Wilmot et al (2018)?

To answer this question, data from the SHS teachers (both in-service and preservice) who participated in the study were used. Confirmatory factor analysis was performed on the items in the instrument, since the underlining purpose of this question was to validate the seven knowledge types which have been hypothesized by Wilmot et al (2018). This type of analysis is usually performed to test whether measures of a construct are consistent with researchers' purported factors.

The Expanded KAT framework that was adopted in this study stipulated that seven knowledge types (Advanced Knowledge, School Knowledge and Teaching Knowledge, Profound knowledge of school algebra, School algebra teaching knowledge, Advance algebra teaching knowledge, Pedagogical content knowledge in algebra) determine teachers' knowledge for teaching algebra.

Appendix A showed a number of attainable factors which could be extracted from the data to give meaning among the varying item responses and their corresponding eigenvalues. The eigenvalues helped to validate each of the factors hypothesised for inferences to be made. Principal component analysis (PCA) came out with eighteen components with eigenvalues exceeding 1 as shown in Appendix A. These eighteen factors altogether explained approximately 65.043% of the variance. Nevertheless, it was hypothesized that seven knowledge levels were being considered. Thus, the eighteen factors revealed by the Kaiser criterion were not practical in this analysis. To make known the irrelevant nature of the eighteen factors revealed in the analysis, a graphical representation known as the scree plot was imposed to further verify the actual factors needed to corroborate the hypothesized teachers' knowledge for teaching algebra at the SHS level in Ghana as hypothesized in the expanded KAT framework by Wilmot et al. (2018).

Subsequently, the scree plot as put forward by Cattel (1966) which focuses on the graphs of the factors on the horizontal axis against the corresponding eigenvalues on the vertical axis was resorted to. Per this graph, as the number of factors increases (that is as one moves from left to right along the horizontal axis), the corresponding eigenvalues decreases (this is shown in Figure 2). It must be noted that variation in slope of this graph refines as a result of decrease in the number of factors. Per the nature of the scree plot it can be said that the seven factors as hypothesized in the expanded KAT framework are confirmed as the very factors that take into account senior high school teachers' knowledge for teaching algebra. This was resorted to in that Child (1970); Kim & Muellar (1978), Norasis (1990) and De Vellis (1991) asserts that the factors which are to be retained are the very ones that lie before the point at which the corresponding eigenvalues seem to cut or lend off.

With reference to the scree plot, one may argue that from the scree plot eight factors could have been extracted to make a case. Nevertheless, on the eighth factor only one item loaded uniquely on it which defeats the concept of factor retention hence the seven factors. Figure 2, shows the graphical representation for this scree-test.

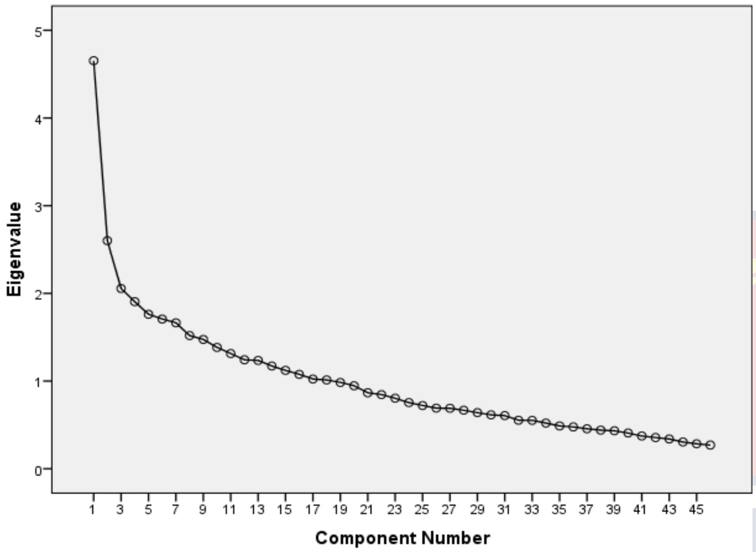


Figure 2: Scree plot showing number of factors retained.
Source: Field Survey (2020)

A look at the scree plot indicates that after the seventh factor the graph changes course which presupposes that it is only possible per the nature of the graph to retain only the first seven factors. The graph, however, confirms the fact that high school preservice and in-service mathematics teachers in Ghana's knowledge for teaching algebra corroborate the seven types of knowledge hypothesized in the expanded KAT framework by Wilmot et al. (2018).

However, since the use of the scree plot is subjective, a Confirmatory Factor Analysis was conducted using the item loadings on each factor. This Confirmatory Factor Analysis conducted (See Table 1) also revealed seven factors based on the item loadings. To make this analysis more authentic, some senior members and colleagues who have expertise in this area were consulted and advised that only items loading uniquely on only one factor be considered for final analysis hence the seven factors used (See for example Wilmot et al (2018). Also, most of the items which had cross loadings happened to load weak on the other factors which were eventually deleted. In all, 29 out of 46 items loaded uniquely on the seven knowledge types as indicated in the instrument for the expanded KAT framework. In order to determine which items to retain on each of the factors, a cutoff point of 0.3 was used.

Table 1 indicates the results of the Confirmatory Factor Analysis performed to confirm the seven knowledge types as hypothesized in the expanded KAT framework with the item loadings.

Table 1: Confirmatory Factor Analysis showing Loadings of each Item

Item	Component						
	PKSA	SATK	AATK	SK	TK	PCKA	AK
SK1	.548						
SK15	.544						
AK45	.542						
SK14	.462						
AK4	.441						
SK32	.434						
SK9		.637					
TK51		.618					
TK3		.545					
SK8		.517					
AK38			.418				
TK39			.572				
SK17				.585			
SK19				.571			
SK18				.552			
SK10				.459			
SK2				.459			
TK16					.608		

Table 1 cont'd

Item	PKSA	SATK	AATK	SK	TK	PCKA	AK
TK35					-.572		
TK40					-.566		
AK26						.567	
AK29			.438				
SK5						.471	
AK37						-.464	
TK24						-.451	
AK23							.723
AK36							-.569
AK30							-.421
AK11							.407

Source: Field Survey (2020)

Legend: SK: School Knowledge; AK: Advanced Knowledge;

TK: Teaching Knowledge;

PKSA: Profound Knowledge of School Algebra;

SATK: School Algebra Teaching Knowledge;

AATK: Advanced Algebra Teaching Knowledge;

PCKA: Pedagogical Content Knowledge in Algebra

Table 1 indicates the Confirmatory Factor Analysis (CFA) showing loading of items on each of the factors that were considered. Results per the analysis that was conducted revealed that the seven factors (the seven knowledge types) were extracted after the Confirmatory Factor Analysis which explained teachers' knowledge for teaching algebra in the Ghanaian context. A cursory look at Table 1 shows that in all, 29 out of 46 items loaded uniquely on the seven knowledge types hypothesized in the expanded KAT framework. It was noted that six items which were related or have the same characteristics loaded uniquely on the first factor. In addition, four, three, five, three, four and four items respectively loaded on factors two, three, four, five, six and seven (see for example Hair, Black, Babin, & Anderson, 2010 on number of items to retain). These factors however were labeled as *Profound Knowledge for School Algebra, Pedagogical Content Knowledge for teaching Algebra, Advanced Algebra Teaching Knowledge, School Algebra Knowledge, Teaching Knowledge, School Algebra Teaching Knowledge and Advanced Knowledge* based on the characteristics they exhibited. A look at the item loadings indicated that six items loaded negatively under three of the factors that determined teachers' knowledge for teaching algebra at the SHS level. The negative loadings simply means that the items answered by these respondents might have been difficult for them or the understanding was not there or the nature of the sample might have contributed to it. However, it must be noted that though these six items loaded negatively it still met the condition for factor loadings (See for example Hair, Black, Babin, Anderson, & Tatham, 2006).

The results revealed three items that had high loadings on the *Profound Knowledge for Teaching Algebra* factor which happens to be the first factor. In addition, the other three items that loaded under this same factor were coming from items measuring School Knowledge and Teaching Knowledge. This presupposes that in-service and preservice teachers who participated in this study saw those items more of items focusing on *Profound Knowledge for Teaching Algebra* than any of the other factors. Item loadings on the *Profound Knowledge for Teaching*

Algebra presupposes that this kind of knowledge plays a key role in determining senior high school teachers' knowledge for teaching algebra in that it has most of the items loading under it. The item with the highest loading under this knowledge type was item 1 which reads as below:

1. A restaurant has a dinner plate. For the plate, you can choose two entries from six different choices. Then you can choose between baked yam, rice, mashed yam, or coleslaw. Last, you choose between stew and salad. How many possible dinner combo plates are available?
   A. 120
   B. 48
   C. 240
   D. 12
   E. None of these

This item was followed by items 15 and 45 with factor loadings of 0.544 and 0.542 respectively. This indicates that preservice and in-service senior high school mathematics teachers as a matter of fact have realized how treasured and critical it is for them to possess the *Profound Knowledge for Teaching Algebra* in teaching the integrated kind of mathematics in the Ghanaian context and presupposes that in the course of teaching SHS mathematics teachers need this kind of knowledge which occurs as a result of their ability to combine their school algebra knowledge and that of their advanced algebra knowledge to explain key concepts of bother to students during the instructional process.

Regarding factor two, the item with the highest loading was item 9 followed by item 44 which had factor loadings of 0.637 and 0.618.

The first item that loaded on this factor (*Pedagogical Content Knowledge for teaching Algebra)* and read as:

9. If $p\!:\!q$ and $r\!:\!s$ are two equal ratios and ($q \neq 0, s \neq 0$) then ……………………..
   A. $p = r \text{ and } q = s$
   B. $pr = qs$
   C. $p + r = q + s$
   D. $p - r = q - s$
   E. $ps = qr$

This item which loaded on the second factor also sends a signal to SHS mathematics teachers that it is crucial to be well informed on how to go about employing appropriate method in teaching the various contents in mathematics for better understanding of students.

However, among all the factors, it was factor seven *(Advanced Knowledge)* that had the item with the highest loading of 0.723. The item reads as follows:

23. Students in Mr. Carson's class were learning to verify the equivalence of expressions. He asked his class to explain why the expressions $a - (b + c)$ and $a - b - c$ are equivalent. Some of the answers given by students are listed below.

    Which of the following statements comes closest to explaining why $a - (b + c)$ and $a - b - c$ are equivalent? (Mark ONE answer.)

    A. They're the same because we know that $a - (b + c)$ doesn't equal $a - b + c$, so it must equal $a - b - c$.
    B. They're equivalent because if you substitute in numbers, like $a = 10, b = 2, and\ c = 5$, then you get 3 for both expressions.

C. They're equal because of the associative property. We know that $a - (b + c)$ $equals$ $(a - b) - c$ $which$ $equals$ $a - b - c$.

D. They're equivalent because what you do to one side you must always do to the other.

This item had the highest loading (0.723) shows that SHS mathematics teachers yet need this kind of knowledge in dispensing knowledge to students. This kind of knowledge was simply referred to include other mathematical knowledge, in particular college level mathematics, which gives a teacher perspective on the trajectory and growth of mathematical ideas beyond school algebra" (Ferrini-Mundy, Senk & McCrory, 2005, p.1). Also, it is hoped that any mathematics teacher who possesses this type of knowledge would hold quite a respectable knowledge of the path of the content of school mathematics. In addition, SHS mathematics teachers must possess this kind of knowledge simply because this is what would allow them to engage in making networks across topics, eliminating difficulties while retaining integrity and unzipping of the content of school algebra to learners; practices that could be vibrant to effective teaching.

Once again, a more robust analysis was conducted to finally ascertain whether indeed these seven factors are what determine in-service and pre-service mathematics teachers' knowledge for teaching algebra. To do this a Confirmatory Factor Analysis was conducted by way of imposing a regression line on the scree plot to ascertain this fact. From Figure 2, it will be observed that the elbow of the graph, or the sharp break as indicated by Cattel (1993), can be seen to exist at either factor number 7or 8. However, since such interpretations from scree plots are based on visual observation of the elbow of the graph, it is generally subjective (Hayton, Allen & Scarpello, 2004), making it impossible for factor 7 or 8 to be settled on depending on the individual researcher doing the analysis. To remove this subjectivity, the study resorted to applying the suggestion by Nelson (2005) of superimposing the regression line on the scree plot. The resultant graph is presented in Figure 3.

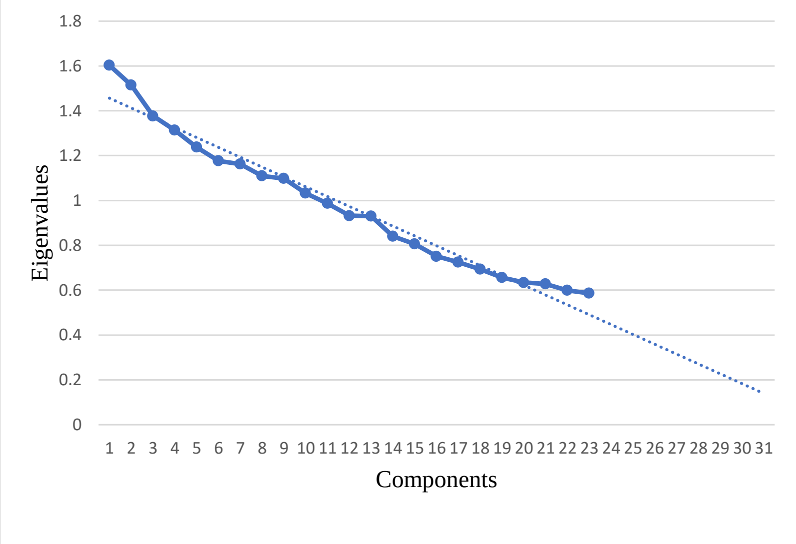


Figure 3: Scree plot with regression line imposed on it

Source: Field Survey 2020

Figure 3 shows that high school preservice and in-service mathematics teachers' knowledge for teaching algebra in Ghana corroborate the three types of knowledge hypothesized in the original KAT framework and not the expanded KAT framework by Wilmot et al (2018).

Conclusion related to research question one

It can be concluded from the preceding analysis and results that seven variables or factors as hypothesized in the expanded KAT framework was not confirmed as the number of factors that defines senior high school mathematics teachers' knowledge for teaching algebra based on the scree plot with the regression line imposed extracted only three factors. Though, the scree plot with regression line imposed on it in Figure 3 confirms that three factors can be retained it does not necessarily mean that these are the only factors needed to explain knowledge needed by preservice and in-service mathematics teachers for teaching algebra at the SHS level until a further research confirming those factors are done. The aspect of not only three knowledge types presupposes that there are other knowledge types apart from those that have been confirmed in this research piece to be explaining teachers' knowledge for teaching algebra at the SHS level. These three factors that have been confirmed to be explaining preservice and in-service SHS mathematics teachers' knowledge for teaching algebra approximately explained 17.301% of the variance. The results also indicates that data on the knowledge for teaching algebra of SHS mathematics teachers confirms that those interlocking regions that were described by researchers in the expanded KAT framework project as not being fuzzy is after all not the case. This is also an indication that factors obtained from the factor analysis have items loading onto them could have measured characteristics that had some similar traits among them. To this end, one would have expected that items measuring the same construct would load together on a particular factor. Nevertheless, the item loadings in this study revealed that most of the items measuring the same construct loaded together except in the case of six items. Take for example, item 14 which loaded on Factor one (that is *Profound Knowledge of School Algebra*) was originally a School Knowledge item. This shows that respondents in the study saw the item to be more of *Profound Knowledge of School Algebra* than School Knowledge. Item 2 was structured as follows:

2. What is the conclusion of this statement: $If\ x^2 = 4, then\ x = -2\ or\ x = 2.$
   A. $x^2 = 4$
   B. $x = 2$
   C. $x = -2$
   D. $x = -2\ or\ x = 2$

Also, another item that had a cross loading on two different factors but happens to be a School Knowledge item was item number 20. This item loaded on both the *Pedagogical Content Knowledge in Algebra* as well as the *School Knowledge*. The item loading for *Pedagogical Content Knowledge in Algebra* and *School Knowledge* was 0.487 and 0.418 respectively. This item loadings indicates that senior high school mathematics who were involved in this study saw this item to be more of a *Pedagogical Content Knowledge in Algebra* question than a *School Knowledge* question. The item in question was as follows:

20. Which of the following is a valid conclusion to the statement ''If a student is a high school band member, then the student is a good musician''?
   A. All good musicians are high school band members.
   B. A student is a high school member band member.

C. All students are good musicians

D. All high school band members are good musicians.

The item as indicated above is as a matter of fact a *School Knowledge* as is a question that mathematics teachers at the said level are expected to teach their students. In other perspective, this type of item is said to be aspect of the content enshrined in the mathematics curriculum for which mathematics teachers at the SHS level are supposed to teach their students. Unfortunately, the item loaded on both the *Pedagogical Content Knowledge in Algebra* question than a *School Knowledge* which eventually was deleted. These findings as revealed in this study confirms the finding by Yarkwah (2017) and Wilmot et al. (2018).

Research hypothesis one

The first research hypothesis that guided this study was "There is no significant difference in the knowledge for teaching algebra between senior high school mathematics teachers with professional background qualification and their counterparts without professional background qualification."

To answer this research hypothesis, data collected from both in-service and pre-service mathematics teachers with professional and non-professional background at the senior high schools in the Central and Eastern Regions respectively were used. The focus of this research hypothesis primarily was to compare scores of preservice and in-service mathematics teachers meant to teach core or elective mathematics or both at the SHS level with professional and non-professional qualification on the algebra knowledge for teaching. To do this, an independent-samples t-test was conducted to help in comparing the mean scores on some continuous variable which in this study happens to be test scores obtained by these two groups of subjects (teachers with professional qualification and those without professional qualification). In addition to this, ANOVA was also performed to establish if any difference exists between teachers with professional and non-professional qualification. To do this, the hypothesis was tested at the 5% level of significance. In addition, items in the instrument were statistically analyzed to determine which of them were statistically significant.

Table 2 shows the mean and standard deviation scores of the knowledge for teaching algebra between senior high school mathematics teachers with professional and non-professional qualification.

Table 2: Descriptive Statistics of Preservice and In-service Mathematics Teachers with Professional and Non-Professional Qualification

Qualification	N	Range	Min.	Max.	Mean		Std. Dev.
	Stat.	Stat.	Stat.	Stat.	Stat.	Std. Error	Statistic
Non-Professionals	71	.65	.00	.65	.4522	.01324	.11155
Professionals	207	.57	.17	.74	.5063	.00763	.10973

Source: Field survey (2020)

A look at the results as indicated in Table 2 shows that relatively mathematics teachers with professional background qualification performed better than their counterparts without professional background qualification on the algebra knowledge for teaching at the senior high school level.

The mean and standard deviation scores of teachers with professional background qualification (M = 0.5063, SD = 0.00763) whereas those without professional background qualification (M = 0.4522, SD = 0.01324). The results of the analysis, however, showed that teachers without professional background qualification did slightly better on the level of knowledge for teaching algebra than their counterparts without professional background qualification. This result could mean that some relatively difficult questions meant for these teachers at the said level were well answered by those without professional background qualification. Also, it could mean that mathematics teachers with professional qualification were able to handle very well questions bothering on the various knowledge types hypothesized in the expanded KAT framework. Again, it can be realized from the means and standard deviations for the two groups that those with professional qualification scored a little above average corresponding to 0.51 whereas those without professional qualification scored below average corresponding to 0.45. Also, it was observed that mathematics teachers without professional background qualification as at the time of data collection had scores ranging from 0% - 65% whereas their counterparts with professional qualification had scores ranging from 17% - 74%. This presupposes that teachers with professional qualification are slightly better than their colleagues without professional qualification when it comes to teaching mathematics at the senior high school level. This result could mean that because those with professional qualification are trained purposely to teach at the said level it then purports that their programme structure was tailored to address content issues at that level and to a large extent a little above what they teach at same level placing them in a better position. However, with their counterparts without professional qualification, since their programme structure is not necessarily tailed towards the said level it was not surprising that they could not match those with professional qualification in terms of algebra teaching knowledge. Another thing that could have resulted in this observation is the fact that those with professional qualification have gone through courses that help them address students' problem when it comes to methods to employ in the teaching and learning of the subject mathematics unlike their colleagues with non-professional qualification.

In order to ascertain whether there is a statistically significant difference between the performances of the two categories teachers of mathematics, an independent samples t-test was conducted. The summary statistics are indicated in

Table 3: Results of Independent Samples t-test on test scores of mathematics with and without Professional Qualification

	Levene's Test for Equality of Variances	t-test for Equality of Means				
	F	T	Df	Sig. (2-tailed)	Mean Diff.	Std. Error Diff.
Equal variances assumed	.710	-3.567	276	.001	-.05407	.01516
Equal variances not assumed		-3.539	119.693	.001	-.05407	.01528

Source: Field survey (2020)

A look Table 3 indicates the results of the independent samples t-test conducted to compare the mean scores of professional and non-professional mathematics teachers who teach at the senior high school level. The test was conducted to ascertain whether there is a statistically significant difference between the algebra teaching knowledge of professional and non-professional mathematics teachers. The results as contained in Table 2 conducted to compare difference in knowledge for teaching algebra between senior high school mathematics teachers with professional background qualification and their counterparts without professional background qualification revealed that there was a statistically significant difference between teachers with professional background qualification (M = 0.5063, SD = 0.00763) and their counterparts without professional qualification (M = 0.4522, SD = 0.01324); t (276) = -3.567, p = .001. The magnitude of the difference in the means was medium [eta squared = 0.0441] corresponding to approximately 4.4%.

As a result of the statistically significant difference between the two groups, further analysis was conducted on the seven knowledge levels hypothesized in the expanded KAT framework by Wilmot et al (2018) study. Analysis was conducted on the seven factors to also ascertain whether any difference exist in any of the seven knowledge levels that were generated from the factor analysis in that study. Furthermore, analysis was conducted on the individual items to find out whether the two groups have any variation in terms of their knowledge for teaching algebra with regards to the various items. Table 4: indicates mean and standard deviation scores for the seven knowledge levels as confirmed in this study.

Table 4: Mean and Standard deviation scores on the seven knowledge levels from the expanded KAT framework between teachers with Professional Background Qualification and their counterparts without Professional Background Qualification

Background Qualification		SK	AK	TK	PK	SATK	AATK	PCKA
	Mean	.6796	.2394	.3296	.3850	.1662	.4592	.3239
	N	71	71	71	71	71	71	71
Non-Professionals	Std. Deviation	.16398	.26537	.19153	.18859	.17562	.23759	.22518
	Minimum	.00	.00	.00	.00	.00	.00	.00
	Maximum	.88	1.00	.80	.78	.60	1.00	1.00
	Mean	.7156	.2963	.4348	.5282	.1498	.4647	.3172
	N	207	207	207	207	207	207	207
Professionals	Std. Deviation	.13260	.21841	.21666	.23127	.15450	.22266	.22218
	Minimum	.19	.00	.00	.00	.00	.00	.00
	Maximum	.94	1.00	.80	1.00	.60	1.00	1.00

Source: Field survey (2020)

Table 4 Cont'd.

Background	Qualification	SK	AK	TK	PK	SATK	AATK	PCKA
	Mean	.7064	.2818	.4079	.4916	.1540	.4633	.3189
	N	278	278	278	278	278	278	278
Total	Std. Deviation	.14184	.23214	.21515	.22951	.15999	.22614	.22256
	Minimum	.00	.00	.00	.00	.00	.00	.00
	Maximum	.94	1.00	.80	1.00	.60	1.00	1.00

Source: Field survey (2020)

Commented [DN1]: reduce the width of the table because it is difficult to read

Legend: SK: School Knowledge; AK: Advanced Knowledge; TK: Teaching Knowledge; PK: Profound Knowledge;

SATK: School Algebra Teaching Knowledge; AATK: Advanced Algebra Teaching Knowledge;

PCKA: Pedagogical Content Knowledge in Algebra

A critical examination of Table 4 indicates that SHS teachers with professional qualification and their counterparts without professional qualification who participated in this study as of the time when data was collected have relatively the same knowledge level for teaching algebra at the senior high school level across four out of the seven knowledge levels (that is, *Advanced Knowledge, School Algebra Teaching Knowledge, Advanced Algebra Teaching Knowledge, and Pedagogical Content Knowledge in Algebra*). However, the other knowledge types for which these two categories of teachers differed in knowledge regarding the knowledge for teaching algebra were *School Knowledge, Teaching Knowledge, and Profound Knowledge of School Algebra*. In this regard, SHS mathematics teachers with professional qualification performed relatively better than their counterparts without professional qualification.

The mean and standard deviation scores of mathematics teachers with professional qualification are (M = 0.7156, SD = 0.1326), (M = 0.4348, SD = 0.2167) and (M = 0.5282, SD = 0.2313) for *School Knowledge, Teaching Knowledge and Profound Knowledge of School Algebra* respectively whereas mean and standard deviation scores for those without professional qualification for teaching mathematics are (M = 0.6796, SD = 0.1640), (M = 0.3296, SD = 0.1915), and (M = 0.3850 , SD = 0.1886) for the *School Knowledge, Teaching Knowledge and Profound Knowledge of School Algebra* respectively. The analysis, however, revealed that teachers with professional background qualification have slightly higher level of knowledge for teaching algebra than their counterparts without professional background qualification for teaching algebra on the three knowledge levels highlighted above. The difference in mean performance could also imply that the teachers with professional background qualification had a better understanding and are well trained to handle questions as specified in the instrument than their counterparts without professional background qualification. In addition, the difference in mean performance could also be attributed to the fact that most of these mathematics teachers with professional qualification have had enough experience and exposure to what they are supposed to be teaching at the supposed level than their counterparts without professional qualification.

A critical look at these knowledge types for which there were differences in their mean performance shows that for example with the *School Knowledge*, it is the knowledge of mathematics in the intended curriculum of middle school and high school. It is the content of school algebra that teachers at the Senior High School level are expected to help students discover or learn in their algebra classes (Wilmot, 2009). This type of knowledge is the one enshrined in the syllabuses of the Core and Elective Mathematics and for that matter for great impact to be made in students learning, teachers must exhibit high understanding of content of school algebra since students at that level will learn from them, and they can only pass on what they know and nothing more. An interaction with some respondents and examination of the courses taken by teachers with professional background qualification indicates that while training in the university as mathematics educators, they were exposed to a course titled "*EMA 312: Implementing Secondary School Mathematics Curriculum*" which basically examines the content of the syllabuses that teachers are supposed to use in teaching Core and Elective

Mathematics at the said level. As a result, the performance of mathematics teachers with professional background qualification however was not too surprising since they are conversant with what they are supposed to be teaching their students.

Also, regarding the *Teaching Knowledge* for which teachers with professional qualification performed slightly better than their colleagues without professional qualification, it was realized that it is the type of knowledge in the framework according to Ferrini-Mundy, McCrory, Senk and Marcus (2005, p.2) termed as "knowledge that is precise to teaching algebra that may not be taught in advanced mathematics courses. It takes a look at what makes a particular concept problematic to learn and what misconceptions lead to precise mathematical inaccuracies. Furthermore, it contains mathematics required to identify mathematical goals, within and across lessons, to choose among algebraic tasks or texts, to select what to highlight with curricular paths in mind and to ratify other tasks of teaching".

However, this type of knowledge that is possessed by teachers as they fall back on and applies when they are teaching algebra. Moreover, the KAT project acknowledged that "the knowledge been described here may fall into the kind of pedagogical content knowledge or it may be pure mathematical content applied to teaching" Ferrini-Mundy et al., (2005, p.1). Thus, this is the type of knowledge that could distinguish an engineer or a mathematician from an algebra teacher. This however tells why teachers with professional qualification performed slightly better than their colleagues without professional qualification since they (those with professional qualification) have been exposed to.

The third knowledge type for which difference in mean was realized was the *Profound Knowledge of School Algebra.* This type of algebra knowledge was produced as a result the intersection of *School Algebra Knowledge and Advanced Algebra Knowledge* types. It provides the mathematics teacher with an outstanding understanding of algebra which guides them to explain key concepts which inconvenience the students during the instructional period. It is assumed that teachers with such knowledge type function at a higher level than their colleagues who just teach with the *School or Advanced* knowledge. This to a large extent tells why teachers with professional qualification performed better than their peers without professional qualification.

To find out whether there were differences in the knowledge for teaching algebra between these two groups of mathematics teachers across the seven knowledge types confirmed in this study as hypothesized in the expanded KAT framework, One-way Multivariate Analysis of Variance (MANOVA) (See Appendix C) was conducted. This was done after a preliminary assumption test had been conducted to check for normality, linearity, univariate and multivariate outliers and homogeneity of variance-covariance matrices with no serious violation made or noted. The results of the analysis revealed that there was a statistically significant difference between teachers with professional background qualification and their counterparts without professional background qualification in the knowledge for teaching algebra across the seven knowledge types hypothesized in the expanded KAT framework $F(7,270) = 4.320$, $p = 0.001$; Pillai's Trace = 0.101; partial eta squared = 0.101). This means that the population means scores on the seven knowledge types between teachers with professional background qualification and their counterparts without professional background qualification are different for both groups. It also means that there is a statistically significant difference between the teachers with professional background qualification and their counterparts without professional background qualification per the expanded KAT framework hypothesized knowledge types by Wilmot et al (2018).

As a result of the statistically significant difference in the algebra knowledge for teaching between these two groups, the corresponding analysis of variance (ANOVA) with teaching qualification as the independent variable was conducted for each of the seven knowledge types as a follow-up test to the MANOVA. Here teaching qualification was coded as 1 and 2 for teachers with professional background qualification and those without professional background qualification respectively. This was to find out the knowledge type(s) which was or were contributing to the differences between teachers with professional background qualification and their counterparts without professional background qualifications in terms of the seven knowledge types. The results of one-way ANOVA as a follow-up test to MANOVA on the seven knowledge types of teachers with professional background qualification and their counterparts without professional background qualifications are shown in Table 5.

Table 5: Results of ANOVA on the seven knowledge levels from the expanded KAT framework between teachers with Professional Background Qualification and their counterparts without Professional Background Qualification

Knowledge Category		Partial Eta Squared	Df	Mean Square	F	Sig.
	Between Groups	.078	1	.069	3.436	0.065
School Knowledge	Within Groups		276	.020		
	Total		277			
	Between Groups	.005	1	.171	3.197	0.075
Advanced Knowledge	Within Groups		276	.053		
	Total		277			
	Between Groups	.126	1	.585	13.197	0.001**
Teaching Knowledge	Within Groups		276	.044		
	Total		277			

Source: Field survey (2020)

Table 5 Cont'd.

Profound Knowledge Of School Algebra	Between Groups	.422	1	1.084	22.153	0.001**
	Within Groups		276	.049		
	Total		277			
School Algebra Teaching Knowledge	Between Groups	.000	1	.014	.557	.456
	Within Groups		276	.026		
	Total		277			
Advanced Algebra Teaching Knowledge	Between Groups	.064	1	.002	.032	.858
	Within Groups		276	.051		
	Total		277			
Pedagogical Content Knowledge In Algebra	Between Groups	0.014	1	.002	.048	.827
	Within Groups		276	.050		
	Total		277			

Source: Field Survey (2020)

A one-way between-groups analysis of variance was conducted to ascertain whether there is any difference between teachers with professional qualification and their counterparts without professional qualification on the seven knowledge types as hypothesized in the expanded KAT framework. Interestingly, two knowledge types out of the seven knowledge types were statistically significant using a Tukey alpha level of 0.05 were *Teaching Knowledge* : $F (1, 276) = 13.197$, $p = 0.001$; partial eta squared = 0.126 and *Profound Knowledge of School Algebra*: $F (1, 442) = 22.153$, $p = 0.001$; partial eta squared = 0.422.

A cursory look Table 5 indicates that the partial eta squared for both the *Teaching Knowledge* and the *Profound Knowledge of School Algebra* that showed statistically significant differences out of the seven knowledge types for teaching algebra at the senior high school level accounted for the variation that existed between teachers with professional qualification and their counterparts without professional qualification was caused by these two variables.

Regarding the Teaching Knowledge that showed statistically significant difference between the two groups of teachers, it was not surprising because when it comes to this kind of knowledge according to Ferrini-Mundy, McCrory, Senk and Marcus (2005, p.2) it is the kind of knowledge that is specific to teaching algebra that may not be taught in advanced mathematics courses. Here, what one does is to deal with what makes a particular concept problematic to learn and what misconceptions lead to precise mathematical inaccuracies. It also contains mathematics required to identify mathematical goals, within and across lessons, to choose among algebraic tasks or texts, to select what to highlight with curricular paths in mind and to ratify other tasks of teaching. This kind of knowledge is what distinguishes an engineer or a mathematician from an algebra teacher. Therefore, teachers with professional qualification having gone through such training were expected that they do better than their colleague counterparts without professional background qualification for teaching mathematics at the said level.

Also, regarding the *Profound Knowledge of School Algebra*, it is imperative to note that it is produced as a result of the intersection of school algebra knowledge and advanced algebra knowledge types. It basically provides the teacher with an exceptional comprehension of algebra which guides him/her to explain key concepts which bother the students during instructional process. With this kind of knowledge, the individual mathematics teacher is able to operate at a higher level than their peers who just teach with the school or advanced knowledge. It is an advanced form of these knowledge types which means possession of such knowledge places the teacher at an advantageous point. Thus, it provides "alternate definitions, extensions and generalizations of familiar theorems, and a wide variety of applications of high school algebra" (Wilmot et al 2018, p. 35). This presupposes that teachers without professional background qualification were expected to have performed better than their counterparts with professional background qualification. However, the results of the analysis rather showed the opposite (See Table 5). This is because for the *Advanced Knowledge*, teachers without professional background qualification are more exposed to it and are expected to do better in that regard whereas when it comes to the *School Algebra Knowledge* both teachers know what is enshrined in the curriculum for which they are to expose students to. However, because teachers with professional background qualification are exposed in details regarding *School Algebra Knowledge* while going through their university education, they are expected to do better than their counterparts without professional background qualification hence the difference. This result presupposes that teachers with professional background qualification are well prepared and ready to impart knowledge at the senior high school level than their counterparts without professional background qualification.

Having realized the statistical significant difference, a further analysis was done on the various items that loaded under these two knowledge types that revealed the difference. Table 6 indicates mean and standard deviation scores on the various items that loaded on *Profound Knowledge of School Algebra*.

Table 6: Mean and Standard Deviation Scores of teachers with Professional Background Qualification and their counterparts without Professional Background Qualification

Item No.	Qualification	N	Mean	Std. Deviation	Std. Error Mean
1	Non-Professional	71	.34	.476	.057
	Professional	207	.41	.492	.034
15	Non-Professional	71	.54	.502	.060
	Professional	207	.72	.450	.031
45	Non-Professional	71	.34	.476	.057
	Professional	207	.60	.491	.034

Source: Field Survey (2020)

Table 6 cont'd

Item No.	Qualification	N	Mean	Std. Deviation	Std. Error Mean
14	Non-Professional	71	.73	.446	.053
	Professional	207	.93	.252	.017
4	Non-Professional	71	.25	.438	.052
	Professional	207	.54	.500	.035
32	Non-Professional	71	.62	.489	.058
	Professional	207	.77	.423	.029

Source: Field Survey (2020)

A look at Table 6 shows that for all the items that loaded on the *Profound Knowledge of School Algebra* teachers with professional background qualification outperformed their colleagues without professional background qualification and this attest to the statistical significant difference between the two groups in favour of teachers with professional background qualification.

Again, item by item analysis was done pertaining these items that loaded on the *Profound Knowledge of School Algebra* to ascertain how many respondents under each of the two categories of teachers are getting the item right and how many are getting it wrong. Before the item by item analysis is presented, it must be noted that focus was placed on items 4, 14, 15, 32, and 45 since these items showed statistically significant difference between these two groups.

For item 4, the question read as follows:

4. Given a set D whose elements are the odd integers, positive and negative (zero is not an odd integer). Which of the following operations when applied to any pair of elements will yield only elements of D?

    i. Addition

    ii. Multiplication

    iii. Division

    iv. Finding the arithmetic mean

    The correct answer is

    A. i and ii only

    B. ii and iv only

    C. ii, iii, and iv only

    D. ii and iii only

    E. ii only

The question basically focused on the teachers' combined knowledge on *School Algebra Knowledge and Advanced Knowledge* addressing the teachers' ability to handle students' challenges in the said area. There was only one correct answer to the question (that is E). It was quite alarming to know that more than 50% (149 out of 278) of the teachers who took part in this study had the item wrong with 129 getting it correct. Table 7 indicates frequency and percentages of respondents in the two categories who had the item right likewise those who had it wrong.

Table 7: Frequency and Percentage of responses on item 4 between professional and non-professional teachers

Qualification	Response	Frequency	Percent	Valid Percent	Cumulative Percent
Non-Professionals	Wrong	53	74.6	74.6	74.6
	Correct	18	25.4	25.4	100.0
	Total	71	100.0	100.0	
Professionals	Wrong	96	46.4	46.4	46.4
	Correct	111	53.6	53.6	100.0
	Total	207	100.0	100.0	

Source: Field data (2020)

A look at Table 7 indicates that out of the 207 teachers with professional qualification for teaching mathematics at the senior high school level, 53.6% had the item right. Though not bad, the number of respondents within the same category who had it wrong was disturbing looking at the nature of the question given. This is simply because it was expected that with their background qualification they should have performed relatively better than what we are seeing. However, when it comes to teachers without professional background qualification for teaching mathematics, 74.6% of them had the item wrong which was far worse than their counterparts with professional background qualification for teaching mathematics. The item though quite simple, proved that teachers who participated in this study lack the conceptual understanding of how to go about this question. In a way it also shows that teachers could not do critical analysis of the question to understand what odd integers are.

Another item of interest that revealed significant difference between

14. What is the conclusion of this statement $If\ x^2 = 4, then\ x = -2\ or\ x = 2$.

A. $x^2 = 4$

B. $x = 2$

C. $x = -2$

D. $x = -2\ or\ x = 2$

The question above primarily focused on the teachers' combined knowledge on *School Algebra Knowledge and Advanced Knowledge* addressing the teachers' ability to handle students' challenges in the said area. The question tried to find out from teachers with professional qualification and their counterparts without professional qualification for teaching mathematics their procedural and conceptual understanding of quadratics as indicated in the question. Also, it was to ascertain whether these two groups of teachers involved in the study would be able to lay bare and identify without any problems how to assist their students use the right approach in arriving at the required answer.

It was worth noting that both groups of teachers who took part in this study proved that they have strong procedural and conceptual understanding of the said question with few of them falling apart. Table 8 indicates the responses of both groups of teachers on question 14.

Table 8: Frequency and Percentage of responses by teachers with professional background qualification and their counterparts without professional background qualification

Qualification	Response	Frequency	Percent	Valid Percent	Cumulative Percent
	Wrong	19	26.8	26.8	26.8
Non-Professional	Correct	52	73.2	73.2	100.0
	Total	71	100.0	100.0	
	Wrong	14	6.8	6.8	6.8
Professional	Correct	193	93.2	93.2	100.0
	Total	207	100.0	100.0	

Source: Field Survey (2020)

A careful look at Table 8 indicates that both groups of mathematics teachers' knowledge for teaching algebra with regards to the mean scores (See Table 5) on the type of knowledge being considered indicates that the knowledge base of these teachers on the *Profound knowledge of school algebra* is relatively better. It can be deduced from Table 8 that the out of 71 mathematics teachers with non-professional background qualification for teaching mathematics 52 (73.2%) had the said item right whereas 19 (26.8%) had the same item wrong. On the side of the teachers with professional background qualification for teaching mathematics, 193 (93.2) of them had the item right whereas 14 (6.8%) had the item wrong. It is refreshing to know that majority of these teachers who were involved in this study had the item right with few getting wrong. However, it can be said that though both groups performed well on the item teachers with professional background qualification did better than their counterparts without professional background qualification for teaching mathematics. Though majority performed well on the item in question it was also disturbing to know that over 30 of these same teachers had the item wrong.

Another item of interest that showed statistically significant difference between the two groups and have been discussed was item 15.

The item read like this:

15. Kwame's average driving speed for a 4-hour trip was 45 miles per hour. During the first 3 hours he drove 40 miles per hour. What was his average speed for the last hour of his trip?
    A. 50 miles per hour
    B. 60 miles per hour
    C. 65 miles per hour
    D. 70 miles per hour

This item had only one correct answer and the two groups of teachers who were involved in this study were expected to be able to demystify the correct answer. In this question, the respondents were expected to calculate the average speed of Kwame for the last hour on his trip. It must be noted that the average speed of an object is the total distance covered by the time taken. When rewriting this formula in terms of average speed, we have $average\ speed = total\ distance/total\ time$. Therefore, to determine average speed, the individual respondent had to find the distance for both 4 hours and 3 hours trip and determine the differences between these two distances in order to get the average speed of the last one hour. Table 9 indicates the responses of both groups of teachers on question 15.

Table 9: Frequency and Percentage of responses by teachers with professional background qualification and their counterparts without professional background qualification

Qualification	Response	Frequency	Percent	Valid Percent	Cumulative Percent
	Wrong	33	46.5	46.5	46.5
Non-Professional	Correct	38	53.5	53.5	100.0
	Total	71	100.0	100.0	
	Wrong	58	28.0	28.0	28.0
Professional	Correct	149	72.0	72.0	100.0
	Total	207	100.0	100.0	

Source: Field survey (2020)

A look at Table 9 indicates that on this item, for teachers with professional background qualification for teaching mathematics at the senior high school, 149(72%) had the item right with 58(28%) getting it wrong. This shows that most of these teachers with professional background qualification have what it takes to address students' conceptual and procedural knowledge looking at their performance regarding this question. This also presuppose that most of these class of teachers really did understand and have what it takes to help students over their difficulty when it comes the problems relating to rate of change. Also, for teachers without professional background qualification for teaching mathematics at the senior high school level, 38(53.5%) had the item correct whereas 33(46.4%) had the item wrong. This results also indicates that majority of these teachers with such background have what it takes to help their students overcome rate related problems. However, it must be noted that almost 50% of these teachers involved in the study had the item wrong. This in a way is very disturbing in that these are group of teachers with high background content in mathematics looking at their background training and therefore were expected to have performed better than their counterparts with supposedly low mathematics content from the university. This means that these teachers possess relatively high mathematics content, some of them in a way still lack the conceptual and procedural understanding of rate related problems. It also means that at a very critical point in time during instructional delivery some of these group of teachers would find it difficult to assist students with rate related problems hence some students' difficulty in solving such problems.

The above revelations are of great concern looking at the fact that students' performance is greatly influenced by teachers' knowledge. This is because teachers are supposed to exhibit adequate control over the contents they teach to enable them to communicate effectively the mathematical contents and concepts required to their students to positively enhance their performance. Research is abounding with the fact that the teacher is the most essential factor that influences students' achievement (see for instance, Begle, 1972; Hanushek, 1972; Eisenberg, 1977; Harbison & Hanushek, 1992; Shulman & Quinlan, 1996; Mullens, Murnane & Willett, 1996; Rowan, Chiang & Miller, 1997; Wilmot, 2009; Yara, 2009). In this regard, mathematics teachers who have difficulty in some contents they teach may contribute in building weak algebra foundation for their students.

However, for the knowledge being considered, it is a kind of knowledge that was produced as a result the intersection of school algebra knowledge and advanced algebra knowledge types. Teachers with such knowledge type operate at a higher level than their peers who just teach with the school or advanced knowledge. It is an advanced form of these knowledge types which means possession of such knowledge places the teacher at an advantageous point. This presupposes that majority of these group of teachers are capable of handling the content enshrined in the SHS mathematics curriculum to their students without much problem.

Conclusion related to research hypothesis one

Generally, both groups of teachers who took part in this study showed evidence of mathematical understanding and knowledge of algebra they teach at the SHS level. These results were not surprising because teachers who participated have quite a number of books, exposure and are familiar with the content enshrined in the SHS mathematics syllabus.

It must also be noted that teachers with professional background qualification at any point in time stands at an advantageous point in handling content at the said level than their counterparts without professional background qualification for teaching algebra. Results also revealed that at some point in time teachers in these two categories of teachers may not be able to handle questions regarding *Profound Knowledge of School Algebra* to their students.

Research hypothesis two

The second research hypothesis that guided this study was, "There is no significant difference between preservice and in-service senior high school mathematics teachers' knowledge for teaching algebra?" To answer this research hypothesis, data was obtained from in-service and pre-service mathematics teachers from sixteen SHSs and public university for that purpose. These two groups of teachers were assumed to be independent samples. Subsequently, the Independent Samples t-test per the nature of the data was carried out on their total mean scores. In this case, the Independent Samples t-test was resorted to because it helps to relate the mean scores on some continuous variable which in this case was the test scores for the two major groups of teachers in this study, that is, in-service and pre-service teachers. This analysis was done at the 0.05 level of significance. Descriptive statistics of the two groups have been duly presented before moving onto the interpretation of the result of the ANOVA and MANOVA.

Table 10 - Descriptive statistics of in-service and preservice mathematics teachers

Teacher Category	N	Mean	Std. Deviation	Std. Error Mean
Pre-service	101	.5721	.07543	.00751
In-service	177	.4471	.10476	.00787

Source: Field survey (2020)

A look at Table 10 indicates that out of the 278 respondents who participated in the study, 101 and 177 are pre-service and in-service mathematics teachers respectively. A superficial look at the results shows that algebra knowledge of pre-service mathematics who participated in this study was relatively better than their in-service counterparts. The mean score of the pre-service mathematics teachers was 0.5721 which corresponds to approximately 57.2% with a standard deviation of 0.07543 whereas that of the in-service mathematics teachers was 0.4471 corresponding to approximately 44.7% with a standard deviation of 0.10476. Even though a cursory look at the mean scores indicates that there is a difference in the knowledge of algebra for teaching between the two major groups of teachers, ideally the independent samples t-test will indicate the statistical significant difference. However, the results in Table 10 indicates that even though the in-service teachers who have been teaching for quite some time now were expected to have performed better than their pre-service mathematics teacher counterparts the performance however was the reverse. To ascertain this revelation, the independent samples t-test was conducted.

Table 11 indicates the independent samples t-test performed to find out whether there is any significant difference in the knowledge for teaching algebra between senior high school in-service and pre-service mathematics teachers.

Table 11: Results of Independent Samples t-test of pre-service and in-service mathematics teachers

	Levene's Test for Equality of Variances		t-test for Equality of Means			
	F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference
Equal variances assumed	8.207	0.004	10.534	276	0.000	0.12504
Equal variances not assumed			11.494	261.379	0.000	0.12504

Source: Field survey (2020)

Table 11 indicates the results of the independent-samples t-test carried out to compare the difference in knowledge for teaching algebra between preservice and in-service senior high school mathematics teachers. The results revealed that there was a statistically significant difference between preservice mathematics teachers who teach at the SHS level ($M = 0.5721$, $SD = 0.1431$) and that of in-service mathematics teachers ($M = 0.4471$, $SD = 0.10476$); $t(276) = 11.494$, $p = 0.001$. As a result of the statistically significant difference between the preservice and in-service mathematics teachers, the effect size was calculated. The magnitude of the difference in the means indicates that the difference between the two main groups (preservice and in-service) was large [eta squared = 0.3237] which is approximately 32.4%.

The result, however, shows that preservice mathematics teachers have slightly higher algebra teaching knowledge than their in-service counterparts. However, the overall mean and standard deviation scores ($M = .4925$, $SD =.1125$). The overall mean and standard deviation scores indicates that the performance of these teachers on the algebra for teaching achievement test was below average and this leaves much to be desired.

As a result of the statistically significant difference between preservice and in-service mathematics teachers teaching at the high school level, a further analysis was conducted to find out whether any difference exist between these two groups on any of the seven knowledge types in the expanded KAT framework.

Table 12 indicates the results of the mean and standard deviation scores on the seven knowledge types between the pre-service and in-service mathematics teachers.

Table 12: Mean and standard deviation scores on the seven knowledge types between pre-service and in-service mathematics teachers

TEACHER CATEGORY		SK	AK	TK	PKSA	SATK	AATK	PCKA
Pre-service	Mean	.7587	.3036	.5089	.6887	.1545	.5386	.3531
	N	101	101	101	101	101	101	101
	Std. Deviation	.09643	.19493	.19499	.14991	.15200	.22404	.21511
In-service	Mean	.6766	.2693	.3503	.3792	.1537	.4203	.2994
	N	177	177	177	177	177	177	177
	Std. Deviation	.15465	.25059	.20508	.18744	.16480	.21646	.22497
Total	Mean	.7064	.2818	.4079	.4916	.1540	.4633	.3189
	N	278	278	278	278	278	278	278
	Std. Deviation	.14184	.23214	.21515	.22951	.15999	.22614	.22256

Source: Field survey (2020)

Legend: SK: School Knowledge; AK: Advanced Knowledge; TK: Teaching Knowledge; PKSA: Profound Knowledge of School Algebra; SATK: School Algebra Teaching Knowledge; AATK: Advanced Algebra Teaching Knowledge and PCKA: Pedagogical Content Knowledge of Algebra

A look at Table 12 shows that out of the seven knowledge types put forward by Wilmot et al (2018) preservice mathematics teachers performed relatively better than their in-service counterparts with the exception of the *School Algebra Teaching Knowledge*. This outcome was not surprising because this type of knowledge happens to be the intersection of school algebra knowledge and mathematics teaching knowledge. The possession of this knowledge type gives the teacher a wider range of the school knowledge coupled with variety of ways to communicate complex issues to students for easy comprehension. This presupposes that a teacher with such knowledge is able to exhibit the ability to combine various teaching methods for smooth instruction of algebra contents to eliminate any challenges that might rise up in the course of teaching. Furthermore, since both groups of teachers have been exposed extensively to the *School Algebra Knowledge and Teaching Knowledge*, it was not surprising that both groups were able to perform at the same level on the combined knowledge type. This means that both teachers at any point in time are able employ various methods to address complex issues that might arise during instructional delivery.

Another knowledge type of interest was the *School Knowledge* for which preservice mathematics teachers were seen to have performed slightly better than the in-service mathematics teachers. A cursory look at Table 12 shows that the mean and standard deviation scores of these two groups are (Mean = 0.7587 and St. Dev. = 0.09643) and (Mean = 0.6766, St. Dev. = 0.15465) respectively for preservice and in-service mathematics teachers. This result was a bit surprising looking at the experience of these two groups of teachers. The knowledge type in question is basically knowledge of mathematics in the intended curriculum of middle school and high school. This is the content of school algebra that teachers are expected to help students discover or learn in their algebra classes (Wilmot, 2007). For example in the US, knowledge of this kind have been enshrined in booklets such as the National Council of Teachers of Mathematics (NCTM)'s *Principles and Standards for School Mathematics* (NCTM, 2000) while the precise grade-level algebra content is defined in the various states' standards, textbooks and other instructional resources used in the schools. However, in Ghana the knowledge base of this content is enshrined in both the Core and Elective Mathematics Syllabuses which is taken by students at the SHS level. Due to this, for great impact to be made in students learning, teachers must exhibit high understanding of content of school algebra since students at that level will learn from them, and they can only pass on what they know and nothing more. This sends a certain signal to us as mathematics educators looking at the fact that in-service teachers though performed above average their preservice counterparts performed relatively better than they did. It then means that the preservice teachers are well conversant with what is enshrined in the core and elective mathematics syllabuses than their seniors (i.e., the in-service mathematics teachers). This is a bit amazing since those who teach and are teaching at the supposed level are supposed to know better.

The other knowledge of interest that was considered for discussion was the Profound Knowledge of School Algebra. Interestingly, for this knowledge type, once again preservice mathematics teachers performed better than their in-service counterparts. The mean and standard deviation scores on this knowledge for preservice and in-service mathematics teachers are (Mean = 0.6887, Std. Dev. = .14991) and (Mean = 0.3792, Std. Dev. = 0.18744) respectively. A cursory look at the mean and standard deviation scores in this instance indicates that the preservice mathematics teachers performed above average corresponding to approximately 68.9% whereas their in-service counterparts performed below average corresponding to 37.9%. This performance by the in-service mathematics teachers is quite worrisome since they are practicing teachers. It is also disturbing because this type of knowledge provides the teacher with an outstanding comprehension of algebra that facilitates their explanation of key concepts which creates problem for the students during the instructional process. In addition, another reason which makes this performance disturbing is the fact that teachers who possess such knowledge type function at a higher level than their colleagues who just teach with the school or advanced knowledge type. Appropriate possession of this knowledge type as a matter of fact places the teacher at an advantageous position. This is to say that it provides "alternate definitions, extensions and generalizations of familiar theorems, and a wide variety of applications of high school algebra" (Wilmot et al 2018, p. 35). Hence the teachers' inability to exhibit adequate knowledge in this area places the students at disadvantage because the master of the subject (instructor) is unable to expose them to a wide range of applications of high school algebra and this in a way make the students stereotype.

To further ascertain the contributing factor to the difference in knowledge between the preservice and in-service mathematics teachers, a critical look at the courses taken by these two groups of teachers was resorted to. It was revealed per the data that these two groups of teachers did almost the same courses while receiving their undergraduate teacher training. For example, some of the major algebra courses that was seen done by both groups of teachers were *Calculus, Linear Algebra, Abstract Algebra, Differential Equations, Vector Algebra, Algebra and Trigonometry*, *Methods of teaching mathematics, Psychology of learning mathematics and Assessment in mathematics education*. These courses to a large extent are able to equip anyone who intends to teach at the senior high school to impart the needed and required algebra knowledge to students at the said level. An interaction with some of the respondents regarding these courses taken revealed that for the preservice mathematics teachers there has been modification in terms of the content of the courses to meet current demands at the senior high school level. This in a way could be one of the contributing factors regarding the difference in knowledge between these two groups. The statistically significant difference revealed between preservice and in-service mathematics teachers regarding their algebra teaching knowledge implies that at any point in time in dealing with algebra related issues as well as general mathematics problem in the classroom, preservice mathematics teachers involved in the study are in better position to do that than their in-service mathematics counterparts. The outcome of this results is in contradiction to what Yarkwah (2017) found in his work that there was no statistically significant difference between the algebra knowledge possess by preservice and in-service mathematics teachers who teach at the SHS level.

Conclusion related to research hypothesis two

The fact that the independent-samples t-test result indicated a statistically significant difference in the mean scores of these two groups of teachers (that is, preservice and in-service mathematics teachers) in favour of the preservice mathematics teachers indicates that at any point in time in the course of teaching, they are in a better position to address students challenges than their in-service mathematics counterparts. It also means that the current courses been run by universities that train mathematics teachers for the country's high schools had undergone some modifications as revealed by some respondents hence the statistically significant difference in algebra knowledge between the two groups. The result was a somehow surprising in that it was expected that the in-service mathematics teachers could have done better than the preservice mathematics teachers because it is assumed that having taught the subject for quite some time, their level of mathematical knowledge would be above that of the preservice teachers but turned out to be the opposite. It was again surprising because these in-service mathematics teachers have quite a number of books, exposure and are well vexed in the content at the SHS level than their preservice mathematics teacher counterparts.

CHAPTER 5

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Summary

According to Ball (2003a), for one to become a dynamic member of today's modern society, then there is the need to be proficient in mathematics, and the obligation for mathematics in our day to day life continues to grow (NCTM, 2000). Taking the U.S.A. for example, Black (2007) mentioned that they are not preparing their students for the hassles to be mathematically proficient. For anyone to make a lasting impact on society and for that matter people, it is worth noting that having a strong foundation in algebra is the way to go since it contributes varying prospects for entry into advanced mathematics courses (Ball, 2003a), for underpinning of college studies (Pascopella, 2000, Lawton, 1997, Chevigny, 1996, Silver, 1997, Olson, 1994), and for underpinning to skyrocket into the world of work (Silver, 1997).

This study was basically hinged on the assumption that senior high school teachers' knowledge for teaching algebra goes a long way to affect the algebra knowledge of students at the said level, therefore affecting their general performance in mathematics. As a result of this, the algebra teaching knowledge levels of teachers who teach mathematics at the SHS level should be ascertained so as to improve positively students' algebra knowledge therefore their general performance in mathematics.

The study primarily focused on two main categories of teachers who teach mathematics at the senior high school level namely in-service and preservice mathematics teachers. These teachers were further categorized into teachers with professional and non-professional qualification for teaching mathematics at the said level.

Primarily the focus of this study was in three folds and these were:

1. To ascertain if any difference exist between preservice and in-service mathematics teachers' knowledge for teaching algebra at the SHS level.
2. To find out if there exist any difference in the knowledge for teaching algebra between mathematics teachers with professional background qualification and their counterparts without professional background qualification at the senior high school level.
3. To determine to what extent high school preservice and in-service mathematics teachers' knowledge for teaching algebra corroborate the seven knowledge types in the expanded KAT framework.

A look at literature reveals that in Ghana not much work has been done in this area of teachers' knowledge for teaching algebra irrespective of the poor performance of students in mathematics at the Senior High School level as a result of their inability to handle algebra and algebra related questions.

The main design used in this study was the cross sectional survey design for collecting data. Instead of relying on alternative measures, there is the need for re-conceptualization of teacher knowledge in ways that is not only domain specific but also allows its components to be measured. Primarily, this study was intended to investigate whether the seven domains of teacher knowledge hypothesized in the expanded KAT framework by Wilmot et al. (2018) will be corroborated. The study also compared senior high school mathematics teachers' knowledge for teaching algebra with regards to those with professional qualification for teaching mathematics and their counterparts without professional background training for teaching mathematics at the said level. It further examined the algebra teaching knowledge between preservice and in-service mathematics teachers at the Senior High School level based on the seven knowledge types hypothesized in the expanded KAT framework.

In all, the study used 177 in-service mathematics teachers from sixteen schools in the Eastern Region whereas it sampled 101 pre-service mathematics teachers from one public university who read Bachelor of Education in Mathematics from the Central Region of Ghana. Therefore, in all there were 278 in-service and pre-service mathematics teachers involved in this study. Furthermore, the study used an achievement test questionnaire in collecting data from the participants in this study. The data collected from these two categories of teachers were dichotomously scored since there was either right or wrong answers to select from.

Based on the research question and hypotheses that guided the study, the findings of this work have been divided into various sections, each relating to the research objectives. The first part of this study had to do with the corroboration of the seven types of knowledge hypothesized in the expanded KAT framework at the SHS level. Also, the second part of the study looks at the difference in algebra knowledge possessed by SHS mathematics teachers with professional background qualification and their counterparts without professional background qualification for teaching mathematics based on the expanded KAT framework. Also, the third part focused on finding out the preservice and in-service mathematics teachers' knowledge for teaching algebra at the senior high school level.

The instrument used was an adopted one from Wilmot et al. (2018) that contains 46 items. The instrument sought information on teachers' qualification, level of education, algebra courses taught so far at the time of the study, current area of teaching, number of years of teaching mathematics and the type of degree earned.

The data gathered were subjected to various kinds of analysis based on the objectives of the study. Descriptive statistics was used throughout the entire work. Both MANOVA and ANOVA tables were used to find out whether any difference exist in the scores of teachers who possess professional background qualification for teaching mathematics and their counterparts without professional background qualification as well as in-service and preservice mathematics teachers. The independent samples t-test was used to determine whether or not differences exist between the knowledge for teaching algebra between in-service and preservice mathematics teachers at the senior high school level; and whether or not differences exist in the knowledge for teaching algebra between mathematics teachers professional background qualification for teaching mathematics and their counterparts without professional background qualification.

Every human endeavour seems to have some imperfection attached to it and the domain of research is not an exception. This study also had its limitations. The use of the survey though advantageous to gather a large amount of data but characteristically, was not able to afford answers to in-depth or probing questions nor could this survey pursue explanations and determine the conditions or contexts related to how the participants responded to the multiple-choice items (Sarantakos, 2013). This is because respondents were not asked to explain their choices of response to the various items in the test administered which could have given a better insight to the thinking of respondents.

One other limitation of this study was that of the sample size in this case the number of participating teachers. This in a way could place a limitation on the outcome of the study in that if a large number of teachers were involved it could have given different results. Also, some teachers' refusal to respond to certain questions on the instrument could also place a limitation on the outcome of the results of the analysis.

Also, the next limitation was about some mathematics teachers who participated in the study. Since participation in the study was voluntary, some teachers who the researcher believed could have added to the findings and given vital information to the study for one reason or the other refused to participate in the study. This however affected the generalisability of the study's findings because the responses from these other teachers could have given a different picture regarding the item leadings and performance on the various items in general. In addition, responses from these teachers could in a way have influenced the number of items that loaded uniquely. Also, some first class and rural SHSs who were randomly selected to be part of the study turned the offer down at the very day of the data collection which in a way affected the sample size accordingly affecting the generalizability.

Key findings

1. Extent to which high school preservice and in-service mathematics teachers' knowledge for teaching algebra corroborate the seven main types of knowledge hypothesized in the expanded KAT framework.

   Results revealed that senior high school mathematics teachers' knowledge for teaching algebra did not corroborate the seven knowledge types as hypothesized in the expanded KAT framework. Below is how the framework would currently look like:

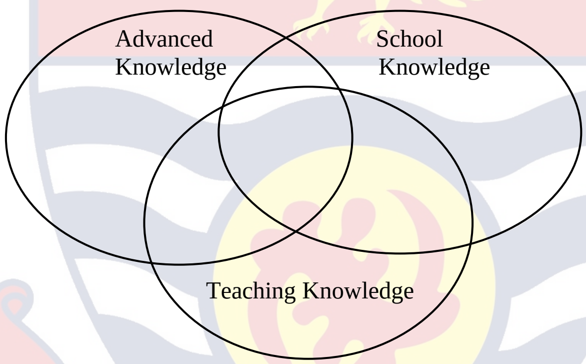


Figure 4: Retained knowledge types in the expanded KAT framework by Wilmot et al (2018) in this study

Source: Field Survey (2020)

2. Based on research hypothesis one, it was revealed that teachers with professional background qualification for teaching mathematics who participated in this study showed evidence of mathematical understanding and knowledge of algebra they teach at the SHS than their counterpart without professional background qualification for teaching mathematics based on the expanded KAT framework.

Also, a statistically significant difference in knowledge for teaching algebra was observed on *School Knowledge, Teaching Knowledge, and Profound Knowledge of School Algebra*. In this regard, SHS mathematics teachers with professional background qualification performed relatively better than their counterparts without professional qualification.

3. Results based on research hypothesis two indicated a statistically significant difference in the mean scores of preservice and in-service mathematics teachers' knowledge for teaching algebra at the senior high school level in favour of the preservice mathematics teachers.

Conclusions

Even though data for this study were collected from 16 senior high schools across one region and one university in another region, the findings of this study may have implications for planning concerning improving teachers' knowledge for teaching algebra at the senior high school level. The following conclusions were drawn as a result of the findings:

1. With regards to the research question that guided this study, factor analysis was initially done to ascertain whether senior high school in-service and preservice mathematics teachers' knowledge for teaching algebra would corroborate the expanded KAT framework by Wilmot et al. (2018). A number of conclusions can be drawn from the finding for research question one.
    a. From the Confirmatory Factor Analysis, it can be concluded that the seven knowledge types as hypothesized in the expanded KAT framework was not confirmed to be the types of knowledge senior high school

mathematics teachers require in order to teacher algebra at the said level rather it confirmed three knowledge types required by these teachers which are *School Algebra, Teaching Knowledge and Advanced Knowledge*.

b. It can also be concluded that the interlocking regions as asserted by the original KAT project team to be blurry after all is the case per the outcome of this study. It also debunks the assertion made by Wilmot et al. (2018) that those interlocking regions are not fuzzy.

2. With reference to research hypothesis one, it can be concluded that there was a statistically significant difference in the mean scores of the mathematical knowledge of algebra of teachers who have professional background qualification for teaching mathematics and their counterparts without professional background qualification for teaching mathematics at the said level. This presupposes that at any point in time, teachers with professional background qualification for teaching mathematics at the senior high school level stand at an advantageous position of dealing with both conceptual and procedural knowledge regarding algebra at the said level. The implication of this is that higher learning institutions are helping address some of the numerous challenges regarding conceptual and procedural anomalies in teachers. This presupposes that at any point in time in the course of teaching, preservice mathematics are in a better position to address students' challenges than their in-service mathematics counterparts. It also means that the current courses been run by universities that train mathematics teachers for the country's high

schools had undergone some modifications as revealed by some respondents hence the statistically significant difference in algebra knowledge between the two groups.

3. Results indicated a statistically significant difference in the mean scores of preservice and in-service mathematics teachers' knowledge for teaching algebra at the senior high school level in favour of the preservice mathematics teachers. This result indicates that preservice mathematics teachers are in a better position at any point in time in the course of teaching to address students' challenges than their in-service mathematics counterparts.

Recommendations

Based on the findings of the study, the following recommendations have been made for educational policy and practice in the knowledge of algebra for teaching at the senior high school level.

1. It is recommended based on research question one that research should be conducted to further corroborate senior high school mathematics teachers' knowledge for teaching algebra based on the expanded KAT conceptual framework which was not confirmed in this study. It is also recommended that further research be conducted to establish and verify other factors that contribute to senior high school teachers' knowledge for teaching algebra since the three knowledge types could only explain 17.301% of the factors retained to have confirmed the knowledge types required by senior high school teachers.

2. It is also recommended that SHS mathematics teachers form teams to assist each other in their respective areas of difficulty so as to improve on their algebra teaching knowledge levels based on their background qualification.
3. Also, it is recommended that teachers without professional background qualification should be given regular in-service training to help them improve on their pedagogical content knowledge in algebra and their general knowledge base so as to be at the same level of their counterparts with professional background qualification.
4. A study involving both public and private senior high schools across the country be conducted. A study of that nature would provide additional information for modification and upgrading in teachers' knowledge for teaching algebra. This as a matter of fact would help tertiary institutions that train mathematics teachers restructure their programmes as well as the integrated mathematics curriculum of Ghana. For more generalize information on teachers' knowledge for teaching algebra, it would be ideal to consider all categories of SHSs across the country.
5. It is also recommended that in-service training on be organized current issues or changes in the subject area especially for those in-service mathematics teachers to whip up their knowledge based in the area. In addition, mentorship training must be enforced in the SHS to help bridge the gap between in-service and preservice mathematics teachers at the SHS level.

Suggestions for Further Research

1. It is suggested that further research be conducted in all the sixteen regions using the expanded KAT framework to ascertain whether it corroborate teachers' knowledge for teaching algebra based on the Ghanaian context.
2. In addition, research be conducted to comprise teachers with varying background qualification since that was not fully considered in this study. Also, number of SHS mathematics teachers involved in such a study be increased to give a proper picture of what is happening in this area of research.

REFERENCES

Ambrose, R. (2004). Integrating change in preservice elementary school teachers' orientations to mathematics teaching by building on beliefs. *Journal of Mathematics Teacher Education*, 7, 91-119.

Abe, T. (2014). The effect of teachers' qualifications on students' performance in mathematics. *Sky Journal of Educational Research*, *2*(1), 10-14.

Adeniji, I. A. (1999). *A path analytic study of some teachers characteristics and teacher job performance in Secondary Schools in Ogun State, Nigeria* Unpublished doctoral thesis. University of Ibadan.

Adesina, S. (1982). *Planning and Educational development in Nigeria*. Ibadan, Nigeria: Board Publications.

Adieze, L. C. (1986, May, 22). Killing the teaching profession. *Daily Times*, p. 4

American Council of Education. (1999). To touch the future: Transforming the way teachers are taught. *An action agenda for college and university presidents*. Washington, DC. Retrieved from http//www.acenet.edu/resources/present/report.cfm.

An, S., Kulm, G., & Wu, G. (2004). The pedagogical content knowledge of Middle school mathematics teachers in China and the U.S. *Journal of Mathematics Teacher Education*, *7*(2), 145-172.

Anderson, J. R. (1989). A theory of the origin of human knowledge. *Artificial Intelligence*, *40*, 313-351.

Askey, R. (1999). Knowing and teaching elementary mathematics. *American Educator*, *23*(3), 1–8.

Ball, D. L. (1988a). *Research on teaching mathematics: Making subject matter knowledge part of the equation*. East Lansing, MI: National Center for Research on Teacher Education.

Ball, D. L. (1991). Research on teaching mathematics: Making subject matter knowledge part of the equation. In J. Brophy (Eds), *Advances in research on teaching:* (vol 2). *Teacher's knowledge of subject matter as it relates to their teaching practice,* Greenwich, CT: JAI press.

Ball, D. L. (2003a). *Mathematical proficiency for all students: Toward a strategic research and developmental program in mathematics education*. Santa Monica, CA: RAND.

Ball, D. L. (2003b, February 6). *What mathematical knowledge is needed for teaching mathematics?* Secretary's Summit on Mathematics; U. S. Department of Education, Washington, DC.

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning.* (pp. 83-104). Westport, CT: Ablex.

Ball, D. L., Lubienski, S. T. & Mewborn D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching (4th ed.)* (pp. 433-456). New York, NY: Macmillian.

Begle, E. G. (1972). *Teacher knowledge and student achievement in algebra*. SMSG Reports, No. 9. Stanford: School Mathematics Study Group.

Black, D. J. B. (2008). *The relationship of teachers' content knowledge and pedagogical content knowledge in algebra and changes in both types of knowledge as a result of professional development.* Unpublished doctoral thesis, Aubum University. Alabama, USA.

Bodenhausen, J. (1988). Does the academic background of teachers affect the performance of their students? *Paper presented at the Annual Meeting of the American Educational Research Association*, New Orleans, LA. (ERIC Document Reproduction No. ED293836).

Brophy, J., & Good, T. L. (1986). Teacher behavior and student achievement. In M.C. Wittrock (Ed.), *Handbook of Research on Teaching*, (3^{rd} ed.). (pp. 328-375). New York: Simon and Schuster McMillan.

Cattell, R. B. (1966). The meaning and the strategic use of factor analysis. In R.B. Cattell (ED). *Handbook of Multivariate Experimental Psychology*, Chicago: RandoMcNally.

Chevigny, B. G. (1996). Mississippi learning: Algebra as political curriculum. *The Nation*, 16-21.

Chief Examiners Report. (2008, 2009, 2010, 2012, 2014, 2015, 2016, 2017, 2018). West African Examination Council, Accra: Ghana.

Child, D. (1970). *The Essentials of factor Analysis*. London, Holt, Rinehart & Winston.

Clarke, C. M., & Peterson, P. L. (1986). Teachers' thought processes. In M. C. Wittrock (Ed), *Handbook of research on teaching,* (2nd ed.). New York, USA: Macmillan.

Cochran, K., DeRuiter, J., & King, R. (1993). Pedagogical content knowing: An integrative model for teacher preparation, *Journal of Teacher Education, 2*, 55-69

Cohen, L., Manion, L., & Morrison, K. (2000). *Research methods in education,* (5th ed). London: Routledge Falmer Publishers.

Cohen, L., Manion, L., & Morrison, K. (2007). *Research methods in education* Concept in Upper Basic Technology in Ekiti State, April J. Sci. Technol.

Coleman, J. S., Campbell, E.Q., Hobson, C; J., McPartland, J., Mood, A .M., Weinfeld, F.D., York, R. L. (1966). Equality of educational opportunity. Washington, DC: US Government Printing Office.

Creswell, J. S. (2003). *Research Design: Qualitative and Quantitative, and Mixed Methods.* Thousand Qaks, Calif.: Sege.

Darling-Hammond, L. (2003). Keeping good teachers: Why it matters, what leaders can do. *Educational Leadership, 60* (8), 6-14.

Darling-Hammond, L. (2000). *Teacher quality and students' achievement: A review of state policy evidence*. Working paper, Center for the study of teaching and policy, University of Washington.

De Vellis, R. F. (1991). *Scale Development: Theory and Applications.* London: Sage.

Doyle, W. (1977a). Learning the classroom environment: An ecological analysis. *Journal of Teacher Education, 28*(6), 51-55.

Duthilleul, Y. & Allen, R. (2005, September 28-30). Which teachers make a difference? Implications for Policy Makers in SACMEQ Countries. *Paper presented at the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ)'s International Invitational Educational Policy research conference in Paris*. (pp. 172-187). Paris, University of France.

Eisenberg, T. A. (1977). Begle revisited: Teacher knowledge and student achievement in algebra. *Journal for Research in Mathematics Education, 8*(30), 216-222.

Even, R. (1993). Subject-matter knowledge and pedagogical content knowledge: Preservice secondary teachers and the function concept. *Journal for Research in Mathematics Education*, *24*(2), 94-116

Fafunwa F.A. (1985). History of Education. Ibadan, NPS Educational Publishers Limited.

Farooq, M. S., & Shalizad, N. (2006) Effect of Teachers Professional Education on Students Achievement in Mathematics. *Bulletin of Education & Research*, *28*(1), pp. 47-55.

Ferrini-Mundy, J., Burrill, G., Floden, R., & Sandow, D. (2003). Teacher knowledge for teaching school algebra: Challenges in developing an analytical framework. Paper presented at the meeting of the American Educational Research Association, Chicago, IL.

Ferrini-Mundy, J., McCrory, R., Senk, S., & Marcus, R (2005). Knowledge for Algebra Teaching. A paper presented to the American Educational Research Association (AERA) Annual Meeting in Montreal, Canada, on April 14, 2005.

Ferrini-Mundy, J., Senk, S. & McCrory, R. (2005). *Measuring secondary school mathematics teachers' knowledge of mathematics for teaching: Issues of conceptualization and design*. Paper presented to the ICMI Study Conference in Águas de Lindóia , Brazil in May 2005.

Furrugia C. (1987). The Challenges of Scale educational development in Small States of the Commonwealth. London, Paul Mau.

Gage, N. (1978). The scientific basis of the art of teaching. New York: Teachers College Press.

Gay, L. R. (1987). *Educational Research.Competencies for Analysis and Application*. Merril Publishing Company Columbus, Ohio 43216.

Ghazah, N. H. C., & Zakaria, E. (2011). Students' procedural and Conceptual understanding of mathematics. *Australian Journal of Basic & Applied Sciences*. *5*(7). 684-691.

Grossman, P. (1990). The making of a teacher: Teacher knowledge and teacher education. New York, NY: Teachers College Press. Hampshire, U.K.: Gower Publishing.

Hair, J., Black, W. C., Babin, B. J., & Anderson, R. E. (2010). *Multivariate data analysis* (7th ed.). Upper saddle River, New Jersey: Pearson Education International.

Harbinson, R. W., & Hanushek, E. A. (1992). Educational performance for the poor: Lessons from rural northeast Brazil. Oxford, England: Oxford University Press.

Hayton, J. C., Allen, D. G. & Scarpello, V. (2004). Factor retention decisions in exploratory factor analysis; A tutorial on parallel analysis. *Organizational Research Methods, 7*(2), 191-205.

Hill, H., & Ball, D. (2004). Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for Research in Mathematics Education*, 35(5), 330-351.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, *42,* 371–406.

Hope, M. (2006). *Preservice teacher procedural and conceptual understanding of fractions and the effects of inquiry-based learning on this understanding*. Unpublished doctoral dissertation, Clemson University.

Howard, K., & Sharp, J. A. (1983). *The management of a student research project.*

Howe, R. (1999). *Knowing and Teaching Mathematics, 46 (8).* pp. 881-887. Ibadan: Wisdom Publishers Limited.

Igwe, D.O. (1990). Science teacher qualification and students' performance in *Journal of Mathematics Teacher Education, 4*(3), 205-225.

Kennedy, M. (1991). A survey of recent literature on teachers' subject matter knowledge. A paper repared for ERIC Clearinghouse. Retrieved from knowledge. *In V, Richardson (Ed.), Handbook of research on teaching* (4th ed.) (pp.433-456). New York, NY: Macmillan. http://ncrtl.msu.edu/http/ipapers/html/pdf/ip903.pdf.

Klecker, B. M. (2002). *The Relationship between Teachers' Years of Teaching Experience and Students' Mathematical Achievement.* Paper presented at the Annual Meeting of the Mid-South Educational Research Association, Chattanooga, TN.

Kim, J. O., & Mueller, C.W. (1978). *Factor Analysis: Statistical Methods and Practical Issues*. London: Sage.

Knuth, E., Stepehns, A., Blanton, M., & Gardiner, A. (2016). Build an early foundation for algebra success. *Kappanmagazine.org, 97*(6)

Lassa, P.N. (1985). *Teaching Mathematics in Nigerian Primary Schools*, Jos: University Press.

Lawton, M. (1997). Riley hauls out less-than-fresh math findings in education blitz. *Education Week, 17*(9), 24.

Leinhardt, G., & Smith, D. A. (1985). Expertise in mathematics instruction: subject matter knowledge. *Journal of Educational Psychology, 77*(3), 247-271.

Lim, C., S., (2002). Practice Make Perfect? An Insight into The Culture of Mathematics Learning in Two Chinese Primary Schools. *Proceedings of Mathematics Education National Seminar*. (pp: 163-171). Tokyo, Japan. University of Japan

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and United States*. Mahwah, NJ: Lawrence Erlbaum Associates.

Mary, M. C., & Heather, J. (2006). Algebraic equations: Can middle-school students meaningfully translate from words to mathematical symbols? *Reading Psychology, 27,* 147-164.

Mewborn, D. (2003). Teaching, teachers; knowledge and their professional development. In J. Kilpatrik, W.G. Matin, & D. Schifter (Eds). *A research companion to the Principles and Standards for School Mathematics* (pp.45-52). Reston, VA: National Council of Teachers of Mathematics.

Milgram, R. J. (2004). The mathematics pre-service teachers need to know. *Stanford, CA: Stanford University*. Monica, CA: RAND.

Ministry of Education. (2011). *Teaching syllabus for core mathematics (senior high school)*. Curriculum research and Development division, Accra: Ghana.

Mitchell, M. L., & Jolley, J. M. (2004). *Research design explained* (5th ed.). Burr Ridge, IL: Irwin.

Mullens, J. E., Murnane, R. J., & Willett, J. B. (1996). "The contribution of training and subject matter knowledge to teaching effectiveness: A multilevel analysis of longitudinal evidence from Belize". *Comparative Education Review*, *40*(2) 139-157.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics.* Reston, VA.: NCTM. ND students in Kaduna Polytechnics. Unpublished postgraduate diploma dissertation. University of Ibadan, Ibadan.

Nelson, L. R. (2005). Some observations on the scree test, and on coefficient alpha. *Journal of Educational Research and Measurement, 3*(1), 220-248.

Norasis, M. (1990). *SPSS/PC+ Statistics 4.0*, Chicago, IL, SPSS.

Nworgu, B. G. (2006). *Educational Research: Basic issues and methodology.* Ibadan: Wisdom Publishers Limited.

Oladele, J. O. (1999). *The effect of early qualification on the achievement of Pre-ND students in Kaduna Polytechnics*. Unpublished postgraduate diploma dissertation. University of Ibadan, Ibadan.

Olson, L. (1994). Algebra focus of trend to raise academic stakes. *Education Week*, *13*(34), 1-2.

Osokoya M. M. (1999). *Some determinants of secondary school students' academic achievement in Chemistry in Oyo State*. Unpublished doctoral thesis. University of Ibadan, Ibadan.

Owolabi, O. T., & Adedayo, J. O. (2012). Effect of Teacher's Qualification on the Performance of Senior Secondary School Physics Students: Implication on Technology in Nigeria. *Canadian Center of Science and Education, 5*(6), 74-77.

Pascopella, A. (2000). Algebra I for everyone. *Curriculum Development, 36*(4), 69-71.

Sanders, W., & Rivers, J. (1966). *Cumulative and residual effects of teachers on future student academic achievement*. Knoxville, TN: University of Tennesssee Value-added Research and Assessment Center.

Sarantakos, S. (2013). Social research (4th ed.). London: Palgrave Macmillan.

Seweje, R. O., & Jegede, S. A. (2005). Science Education and Science Teaching Methods. Lagos: Atlantic Association.

Shulman, L. S. (1983). Autonomy and obligation: The remote control of teaching. L. S. Shulman and Sykes, G. (Eds), *Handbook of teaching and practice*. (pp.484-504). New York, NY: Longman.

Shulman, L. S. (1986a). Paradigms and research programs in the study of teaching: A contemporary perspective. In M. C. Wittrock (Ed.), *Handbook of research on teaching* (3rd ed., pp. 3-36). New York: Macmillan.

Shulman, L. S. (1986b). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4-14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57, 1-22.

Silver, E. A. (1997). "Algebra for all" – Increasing student's access to algebraic ideas, not just algebra courses. *Mathematics Teaching in the Middle School*, 2(4), 204-207.

Simon, M. A. (1993). Preservice elementary teacher's knowledge of division. *Journal for Research in Mathematics Education 24*(3), 233-254.

Stacey, K., Helme, S., Steinle, V., Baturo, A., Irwin, K., & Bana, J. (2001). Preservice teachers' knowledge of difficulties in decimal numeration. *Journal of Mathematics Teacher Education, 4*(3), 205-225.

Stipek, D., Givvin, K., Salmon, J., & MacGyvers, V. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education.* 17, 213 – 226.

Strom, S. (1991). The knowledge base for teaching. ERIC Digest, ERIC Clearinghouse on Teacher Education Washington DC. Retrieved, from http://www.ericdigests.org/pre-9219/base.htm

Tamir, P. (1987). *Subject matter and related pedagogical knowledge in teacher education. Paper presented at the annual meeting of the American Association for Educational Research,* Washington, DC., USA: Sage.

Thompson, A. G., & Thompson, P. W. (1996). Talking about rates conceptually, Part 2: Mathematical knowledge for teaching. *Journal for Research in Mathematics Education*, 27, 2-24.

Vaske, J. J. (2008). Survey research and analysis: Applications in parks, recreation and human dimensions. State College, PA: Venture.

West African Examinations Council. (2004). Chief Examiner's Report. Accra: WAEC Press.

West African Examinations Council. (2004). Senior Secondary School Certificate Examinations: Elective Mathematics Paper 2. Accra: WAEC Press

Wilkins, J. L. M. (2000). Preparing for the 21 st century: The status of quantitative literacy in the United States. *School Science and Mathematics*, *100*(8), 405-418.

Wilmot, E. M. (2009). *An investigation into the profile of Ghanaian high School mathematics teachers' knowledge for Teaching Algebra and its relationship with student performance*, Unpublished doctorial thesis, Michigan State University, Michigan, USA.

Wilmot, E. M. (2017). *Re-conceptualisation of the Knowledge of Algebra for Teaching framework*. Unpublished manuscript.

Wilmot, E. M., Yarkwah, C. & Abreh, M. K. (2018). Conceptualizing teacher knowledge in domain specific and measurable terms: validation of the expanded KAT framework, *British Journal of Education* 6(7), 31-48.

Wilson, S. M., & Winneburg, S. (1981). Peering at history through different lenses: The role of disciplinary perspectives in teaching history. *Teachers College Record*, *89*, 525-539.

Wilson, S. M., Shulman, L. S., & Richert, A. (1987). '150 different ways' of knowing: Representations of knowledge in teaching. In J. Calderhead (Ed.). *Exploring teachers' thinking* (pp. 104-124). Sussex, England: Holt, Rinehart and Winston.

Yara, P. O. (2009). Students Attitude towards Mathematics and Academic Achievement in Some Selected Secondary Schools in Southwestern Nigeria. *European Journal of Scientific Research*, *36*(3), 336-341.

Yarkwah, C. (2017). *An investigation into senior high school mathematics teachers' knowledge for teaching algebra.* Unpublished doctoral dissertation, University of Cape Coast, Cape Coast, Ghana.

Appendices

APPENDIX A

Initial Eigenvalues and Total Variance Explained by each of the Factors on the Instrument

Component	Initial Eigenvalues			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	4.653	10.116	10.116	2.840	6.173	6.173
2	2.601	5.654	15.769	2.660	5.782	11.955
3	2.056	4.469	20.238	2.459	5.346	17.301
4	1.904	4.140	24.378	2.307	5.016	22.317
5	1.762	3.830	28.207	2.109	4.584	26.901
6	1.707	3.711	31.918	2.056	4.469	31.369
7	1.664	3.618	35.536	1.917	4.167	35.536
8	1.518	3.301	38.837			
9	1.474	3.205	42.043			
10	1.384	3.008	45.051			
11	1.313	2.855	47.906			
12	1.243	2.701	50.607			
13	1.235	2.685	53.293			
14	1.172	2.547	55.840			

Appendix A Cont'd.

Component	Initial Eigenvalues			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
15	1.122	2.438	58.278			
16	1.076	2.340	60.618			
17	1.023	2.224	62.841			
18	1.013	2.202	65.043			
19	.984	2.139	67.182			
20	.946	2.057	69.240			
21	.866	1.883	71.123			
22	.846	1.839	72.961			
23	.804	1.747	74.708			
24	.754	1.640	76.348			
25	.721	1.567	77.915			
26	.691	1.503	79.418			
27	.689	1.498	80.917			
28	.667	1.450	82.366			
29	.640	1.390	83.757			
30	.614	1.336	85.092			

Appendix A Cont'd.

Component	Initial Eigenvalues			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
31	.605	1.316	86.408			
32	.553	1.203	87.611			
33	.552	1.199	88.810			
34	.521	1.132	89.942			
35	.488	1.061	91.003			
36	.477	1.036	92.040			
37	.455	.990	93.030			
38	.440	.957	93.987			
39	.433	.941	94.928			
40	.407	.885	95.813			
41	.373	.811	96.624			
42	.355	.772	97.396			
43	.339	.738	98.133			
44	.305	.663	98.796			
45	.284	.618	99.414			
46	.270	.586	100.000			

Source: Field Survey (2020)

APPENDIX B

AN INVESTIGATION INTO SENIOR HIGH SCHOOL TEACHERS'

KNOWLEDGE FOR TEACHING ALGEBRA

An MPhil Thesis Study

By

Mary Donkor

University of Cape Coast

I am a postgraduate student of Department of Mathematics and ICT Education of the University of Cape Coast pursuing an MPhil Degree in Mathematics Education. I am conducting a study on the topic, "An investigation into senior high school teachers' knowledge for teaching algebra". I am most appreciative for making part of your time to respond to the items in this instrument. Please be assured that all information you will provide will be treated as confidential and will only be used for academic purposes only.

CODE PART I: BACKGROUND QUESTIONNAIRE

1. Sex

   □ Male

   □ Female

   □ Other (please specify) ....

2. What was your bachelor's degree in?

   □ B.Sc. Mathematics

   □ Mathematics Education

   □ Other (specify) ___________________

3. What was your minor in college/university?

   □ Mathematics

   □ physics

   □ statistics

   □ Other (specify) ___________________

4. Indicate the *types* of courses you took during your programme(s) of study? Tick all that apply.

   Mathematics Courses

   ☐ Calculus

   ☐ Linear Algebra (e.g., vector spaces, matrices, dimensions, eigenvalues, eigenvectors)

   ☐ Abstract Algebra (e.g., group theory, field theory, ring theory; structuring integers, ideals)

   ☐ Advanced Geometry and/or Topology

   ☐ Real and/or Complex Analysis

   ☐ Number Theory and/or Discrete Mathematics

   ☐ Differential Equations and/or Multivariate Calculus

   Mathematics Education Courses

   ☐ Methods of teaching mathematics (planning mathematics lessons, using curriculum materials and manipulatives, organizing and delivering mathematics lessons, etc.)

   ☐ Psychology of learning mathematics (how students learn, common student errors or misconceptions, cognitive processes, etc.)

   ☐ Assessment in mathematics education (developing and using tests and other assessments)

5. If you have a master's degree, in what area was it?

   ☐ Mathematics

   ☐ Mathematics Education

   ☐ Other (specify) ____________________

   ☐ I do not have a master's degree

6. Which of the following algebra courses have you taught in the last five years? Tick all that apply.

   ☐ Core Mathematics in SHS 1

   ☐ Core Mathematics in SHS 2

   ☐ Core Mathematics in SHS 3

   ☐ Elective Mathematics in SHS 1

   ☐ Elective Mathematics in SHS 2

- Elective Mathematics in SHS 3
- Other (please specify) ___________________

7. Which programme of students do you teach?
   - Science
   - General Arts
   - Visual Arts
   - Home Economics
   - Business
   - Other (please specify) ……………….
8. Are you a professional teacher?
   - Yes
   - No

PART II: ASSESSMENT QUESTIONS

Instructions

This instrument contains 72 multiple-choice questions about knowledge for teaching algebra. You have 150 minutes to answer these questions. You may use a calculator if you choose.

In this booklet, each multiple-choice question has only one right answer. Please circle the correct answer for the multiple-choice questions, and write all your responses to the free-response questions.

1. A restaurant has a dinner combo plate. For the plate, you can choose two entries from six different choices. Then you can choose between baked yam, rice, mashed yam, or coleslaw. Last, you choose between stew and salad. How many possible dinner combo plates are available?
   A. 120
   B. 48
   C. 240
   D. 12
   E. None of these

2. Find the number that must divide each term in the equation $5x^2+2x = 20$ so that the equation can be solved by completing the square.

Response:

A small company invested ¢2,000.00 by putting part of it into a municipal bond fund that earned 4.5% annual simple interest and the remainder in a corporate bond fund that earned 9.5% annual simple interest. If the company earned ¢1,500.00 annually from the investments, how much was in the municipal bond fund?

A. ¢8,000.00
B. ¢10,000.00
C. ¢9,000.00
D. ¢7,000.00
E. None of these

3. Given a set D whose elements are the odd integers, positive and negative (zero is not an odd integer). Which of the following operations when applied to any pair of elements will yield only elements of D?

v. Addition
vi. Multiplication
vii. Division
viii. Finding the arithmetic mean

The correct answer is

F. i and ii only
G. ii and iv only
H. ii, iii, and iv only
I. ii and iii only
J. ii only

4. A and B begin work together. A's initial salary is GH¢200.00 a year and he has an annual increment of GH¢20.00. B is paid at first at the rate of GH¢80.00 a year and has an increment of GH¢8.00 every half-year. At the end of how many years will B have received more money than A?

   A. 5 years
   B. 5.5 years
   C. 6 years
   D. 6.5 years

5. Which of the following is a false statement?
   A. 2, 3, 9/2, 27/4… $2(3/2)^{n-1}$… is a geometric sequence with common ratio 3/2.
   B. 5, 2, -1… -3n+5… is an arithmetic sequence with common difference 5.
   C. If $\{a_n\}$ is a sequence, then $S_{n=\ldots}$ Is the nth partial sum of the sequence.
   D. Two terms of a sequence can be equal.
   E. None of these

6. A rectangular piece of cardboard measures 35inches by 30inches. An open box is formed by cutting four squares that measure x inches on a side from the corners of the cardboard and then folding up the sides. Determine the volume of the box in terms of x.

   A. $4x^3 - 130x^2 + 1050x$
   B. $4x^3 + 130x^2 + 1050x$
   C. $4x^3 - 130x^2 + 1050$
   D. $4x^2 - 130x + 1050$

7. In how many ways can the fraction $\frac{1}{2}$ be written as a sum of two positive fractions with numerator equal to 1 and denominator a natural number?
   A. 0
   B. 1
   C. 2
   D. 4
   E. More than 4

8. If p:q and r:s are two equal ratios and ($q \neq 0$, $s \neq 0$) then
   A. p=r and q=s
   B. pr = qs
   C. p+r = q+s
   D. p-r = q-s
   E. ps = qr

9. Solve algebraically: $\log_3(x-4) = 2$
   A. x = 10
   B. x = 18
   C. x = 729
   D. x = 13
   E. None of these

10. Which of the following is a true statement?
    A. The solution of the matrix equation AX = B, is $X = A^{-1}B$, provided A^{-1} exists.
    B. $\begin{bmatrix} p & q \\ r & s \end{bmatrix}$ and $\begin{bmatrix} p & q \\ r & s \end{bmatrix}$ are inverses.
    C. A singular matrix is a matrix that has a multiplicative inverse.
    D. All matrices have an inverse.
    E. None of these

11. A farmer wishes to make a rectangular hen-run of area $50m^2$ against a wall which is to serve as one of the boundaries. Find the smallest length of wire netting required for the other three sides.

    A. 5m
    B. 10m
    C. 11m
    D. 20m

12. Given that a + b = c where a, b, and c are integers and a is positive, which one of the following statements is true?

    A. a is always greater than c
    B. a is always less than c
    C. b is always less than c
    D. c is never zero
    E. c – a is always positive.

13. What is the conclusion of this statement : $If\ x^2 = 4, then\ x = -2\ or\ x = 2$.

    A. $x^2 = 4$
    B. $x = 2$
    C. $x = -2$
    D. $x = -2\ or\ x = 2$

14. Kwame's average driving speed for a 4-hour trip was 45 miles per hour. During the first 3 hours he drove 40 miles per hour. What was his average speed for the last hour of his trip?

    A. 50 miles per hour
    B. 60 miles per hour
    C. 65 miles per hour
    D. 70 miles per hour

15. One pipe can fill a tank in 20 minutes, while another takes 30 minutes to fill the same tank. How long would it take the two pipes together to fill the tank?

A. 50 min
B. 25 min
C. 15 min
D. 12 min

16. Four steps to derive the quadratic formula are shown below:

$i. x^2 + \frac{bx}{a} = \frac{-c}{a}$

$ii. \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$

$iii. x = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} - \frac{b}{2a}$

$iv. x^2 + \frac{bx}{a} + \left(\frac{b}{2a}\right)^2 = \frac{-c}{a} + \left(\frac{b}{2a}\right)^2$

What is the correct order for these steps?

A. I, iv, ii, iii
B. I, iii, iv, ii
C. Ii, iv, I, iii
D. Ii, iii, I, iv

17. Kofi's solution to an equation is shown below:

Given: $n + 8(n + 20) = 110$

Step 1: $n + 8n + 20 = 110$

Step 2: $9n + 20 = 110$

Step 3: $9n = 110 - 20$

Step 4: $9n = 90$

Step 5: $\frac{9n}{9} = \frac{90}{9}$

Step 6: $n = 10$

Which statement about Kofi's solution is true?

A. Kofi's solution is correct
B. Kofi made a mistake in step 1
C. Kofi made a mistake in step 3
D. Kofi made a mistake in step 5

18. Araba Atta correctly solved the equation $x^2 + 4x = 6$ by completing the square. Which equation is part of her solution?

A. $(x + 2)^2 = 8$
B. $(x + 2)^2 = 10$
C. $(x + 4)^2 = 10$
D. $(x + 4)^2 = 22$

19. Which of the following is a valid conclusion to the statement ''If a student is a high school band member, then the student is a good musician''?

E. All good musicians are high school band members.
F. A student is a high school member band member.
G. All students are good musicians
H. All high school band members are good musicians.

20. When is this statement true?

The opposite of a number is less than the original number.

A. This statement is never true.
B. This statement is always true.
C. This statement is true for positive numbers.
D. This statement is true for negative numbers.

21. Kwame solved the equation $\frac{1}{x-5} = \frac{5}{12x-60}$.

Step 1: He factored the denominator in the expression on the right side of the equation and obtained $\frac{1}{x-5} = \frac{5}{12(x-5)}$.

Step 2: He multiplied both sides by $x - 5$ and obtained $1 = \frac{5}{12}$.

Conclusion: The solution set is the empty set.

A. The conclusion is correct.
B. The conclusion is wrong because we cannot multiply both sides by $x - 5$.
C. The conclusion is wrong because another procedure produces a conclusion different from the one obtained.
D. The conclusion is wrong because if we ‘cross multiply’ by the common denominator we obtain a different solution.
E. There is some other reason why the solution is wrong.

22. Students in Mr. Carson’s class were learning to verify the equivalence of expressions. He asked his class to explain why the expressions a – (b + c) and a – b – c are equivalent. Some of the answers given by students are listed below.

Which of the following statements comes closest to explaining why a – (b + c) and a – b – c are equivalent? (Mark ONE answer.)

A. They’re the same because we know that a – (b + c) doesn’t equal a – b + c, so it must equal a – b – c.
B. They’re equivalent because if you substitute in numbers, like a=10, b=2, and c=5, then you get 3 for both expressions.

C. They're equal because of the associative property. We know that a – (b + c) equals (a – b) – c which equals a – b – c.

D. They're equivalent because what you do to one side you must always do to the other.

23. The set of nonnegative rational numbers with the operations of addition and multiplication has one of the following characteristics:

A. It is not closed under one of these operations
B. More than one of its elements does not have an inverse for the operation of multiplication.
C. Zero is not a member of this set
D. The distributive law of multiplication over addition does not hold
E. None of the above is a characteristic of the given set

24. Susan was trying to solve the equation $2x^2 = 6x$.

First she divided both sides by 2.

$x^2 = 3x$

Then she divided both sides by x:

$x = 3$

Gustavo said, "You can't divide both sides by x." Susan responded, "If you can divide both sides by 2, why can't you divide by x?" They asked their teacher to explain.

Which of the following explanations is correct?

A. Since x is a variable it can vary, you may not be dividing both sides by the same number.

B. You can't cancel x because it does not represent a real number.

C. You can only divide by whole numbers when solving equations.

D. It is better to take the square root of both sides after dividing by

2, that way you won't have to worry about dividing by *x*.

E.If you divide both sides by *x*, then you might be dividing by 0, and would miss the solution $x = 0$.

25. The statement 'For all whole numbers, if to the product of two consecutive whole numbers we add the larger number, the result is equal to the square of the larger number' can be expressed symbolically as: For all whole numbers n

A. $n^2 + 1 = n(n-1) + n + 1$

B. $(n+1)^2 = n^2 + 2n + 1$

C. $n^2 = n(n-1) + n$

D. $(n+1)n = n^2 + n$

E. $(n-1)^2 + 2n = n^2 + 1$

26. Let $f(x) = \log_2 x^2$. Which of the following functions have the same graph as $y = f(x)$?

i. $y = 2\log_2 x$

ii. $y = 2\log_2 |x|$

iii. $y = 2|\log_2 x|$

A. i only

B. ii only

C. iii only

D. i and ii only

E. i, ii, and iii

27. Students are given the following problem:

Find the number of the real roots of the equation $9^x - 3^x - 6 = 0$

Peter denotes $y = 3^x$ and gets the equation $y^2 - y - 6 = 0$, which has 2 different roots. He concludes that the given equation also has 2 different roots.

Which of the following is true about Peter's solution?

A. Peter's conclusion and his arguments are correct.

B. Peter's original approach to the problem (substitution of $y = 3^x$) is not correct.

C. Peter factors wrong.

D. The quadratic equation $y^2 - y - 6 = 0$ does not have 2 different roots.

E. Peter does not take into account the range of the function $y = 3^x$.

28. A textbook contains the following theorem:

If line l_1 has slope m_1 and line l_2 has slope m_2 then $l_1 \perp l_2$ if and only if $m_1 \cdot m_2 = -1$ (i.e. " slopes of perpendicular lines are negative reciprocals").

(McDougal Littell, Algebra 2)

Three teachers were discussing whether or not this statement generalizes to all lines in the Cartesian plane.

Mrs. Allen: The statement of the theorem is incomplete: it doesn't provide for the pair of lines where one is horizontal and one is vertical. Such lines are perpendicular.

Mr. Brown: The statement is fine: a horizontal line has slope 0 and a vertical line has slope ∞ and it's OK to think of 0 times ∞ as –1.

Ms. Corelli: The statement is fine; horizontal and vertical lines are not perpendicular.

Whose comments are correct?

A. Mrs. Allen only
B. Mr. Brown only
C. Ms. Corelli only
D. Mr. Brown and Ms. Corelli.
E. None are correct.

29. Consider the statement below.

For all a, b ∈ S, if ab = 0, then either a = 0 or b = 0.

For which of the following sets S is the above statement true?

i. the set of real numbers
ii. the set of complex numbers
iii. the set of 2x2 matrices with real number entries

A. i only
B. ii only
C. iii only
D. i and ii only
E. i, ii and iii

30. Mr. Nkrumah asked his algebra students to divide $x^2 - 4$ by $x + 2$. Eric said, "I have an easy method, Mr. Nkrumah. I just divide the x^2 by x and the 4 by the 2. I get $x - 2$, which is correct." Mr. Nkrumah is not surprised by this as he had seen students do this before. What did he know? (Mark one answer.)

A. He knew that Eric's method was wrong, even though he happened to get the right answer for this problem.
B. He knew that Eric's answer was actually wrong.
C. He knew that Eric's method was right, but that for many algebraic fraction division problems this would produce a messy answer.
D. He knew that Eric's method only works for some algebraic fractions.
E. I'm not sure.

31. Mr. Fitzgerald has been helping his students learn how to compare decimals. He is trying to devise an assignment that shows him whether his students know how to correctly put a list of decimals in order of size. Which of the following sets of numbers will best suit that purpose?

A. .5 7 .01 11.4

B. .60 2.53 3.14 .45

C. .6 4.25 .565 2.5

D. Any of these would work well for this purpose. They all require the students to read and interpret decimals.

32. If $f(x) = ax^3 + bx^2 + cx + d$, what is the slope of the line tangent to this curve at $x = 2$?

A. $8a + 4b + 2c$

B. $8a + 4b + 2c + d$

C. $12a + 4b + c$

D. $12a + 4b + c + d$

33. Which of the following (taken by itself) would give substantial help to a student who wants to expand $(x + y + z)^2$?

i. See what happens in an example, such as $(3 + 4 + 5)^2$.

ii. Use $(x + y + z)^2 = ((x + y) + z)^2$ and the expansion of $(a + b)^2$.

iii. Use the geometric model shown below.

	x	y	z
x	x^2	xy	xz
y	xy	y^2	yz
z	xz	yz	z^2

A. ii only
B. iii only
C. i and ii only
D. ii and iii only
E. i, ii and iii

34. Which relation is a function?

A. $\{(-1,3),(-2,6),(0,0),(-2,2)\}$

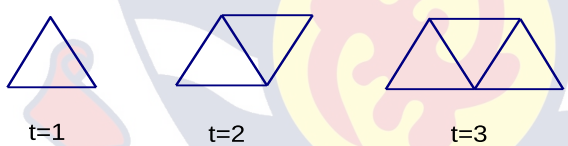
B. $\{(-2,-2),(0,0),(1,1),,(2,2)\}$

C. $\{(4,0),(4,1),(4,2),(4,3)\}$

D. $\{(7,4),(8,8),(10,8),(10,10)\}$

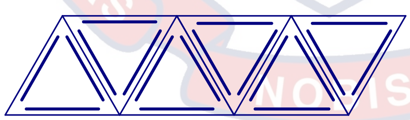
E. $\{(7,-4),(8,-8),(-10,8),(10,-10)\}$

35. Amy is building a sequence of geometric figures with toothpicks, by following a specific pattern (making triangles up and down alternatively). Below are the pictures of the first three figures she builds. Variable *t* denotes the position of a figure in the sequence.



In finding a mathematical description of the pattern, Amy explains her thinking by saying:

"First, I use three sticks for each triangle:



But then I see that I am counting one stick twice for each of the triangles except the last one, so I have to take those away."

If *f* represents the total number of toothpicks used in a picture, which of the following equivalent formulas most closely matches Amy's explanation?

A. $f = 2t + 1$
B. $f = 2(t + 1) - 1$
C. $f = 3t - (t - 1)$
D. $f = 3t + 1 - t$

36. Students were asked to solve the following problem.

Is it possible to have a polynomial of degree 10 of the form

$P(x) = x^{10} + a_9x^9 + \ldots + a_1x + 6$ *with 10 distinct integer roots?*

Which of the following is the most acceptable response to the question?

A. Yes, because every polynomial of degree n has n roots.

B. Yes, $P(x) = (x+1)^6(x-1)^2(x-2)(x+3)$.

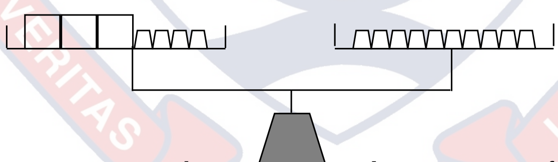
C. Yes, $P(x) = (x-1)^2(x+1)(x-2)(x+2)(x-3)(x+3)^2(x-6)(x+6)$.

D. No, because the only possible integer solutions to $P(x) = 0$ are ±1, ±2, ±3, ±6 (i.e. there are only eight factors of 6).

E. No, because $x^{10} + 6 = 0$ has some solutions that are not integers.

37. Some textbooks suggest that teachers use a pan balance to represent mathematical sentences. For instance, if B represents the weight of each box pictured below (in ounces), and represents a one-kilogram weight, the balance pictured below represents the equation

3B + 4 = 10



Ms. Clarke is preparing to teach a unit on solving linear sentences. If X represents the weight of a given box, which of the following sentences can **NOT** be represented by a pan balance?

A. 13 = 4X + 5

B. 3X + 10 = 4

C. 3X + 3 = 2X + 15

D. 9 + 6X < 21

38. Currently, Germany has a law against creating new surnames for newborns by combining the parents' surnames with hyphens. A language expert explains why hyphenation is not a good idea for naming:

If a double-named boy grew up to marry and have children with a double-named woman, those children could have four names, and their children could have eight, and their children could have 16… The bureaucracy shudders.

(Excerpt from the front page of *The Wall Street Journal*, Wednesday, October 12, 2005)

For which of the following topics could the situation described by the expert be used as an introduction?

A. Direct variations
B. Linear functions
C. Quadratic functions
D. Exponential growth

39. Consider the following mathematical topics:

i.	Composition of functions
ii.	One-to-one functions
iii.	Inverse functions
iv.	Domain and range of functions

Which of the following orders could be used to teach these topics in a rigorous advanced algebra class?

A. ii, i, iii, iv
B. ii, iii, iv, i
C. iv, ii, iii, i
D. They can be taught in any order.

40. Mr. Matheson asked students to solve the following system of equations:

$$\begin{cases} 2x + y = 3 \\ 4x + 2y = 6 \end{cases}$$

So Orlando wrote:

(–2)(2*x* + *y*) = 3 (–2)

$$\begin{array}{r} -4x - 2y = -6 \\ 4x + 2y = 6 \\ \hline 0 = 0 \end{array}$$

This system doesn't have a solution.

Which of the following is true about Orlando's response?

A. Orlando's solution and reasoning are correct.

B. Orlando made an arithmetic error.

C. You cannot add equations.

D. Orlando drew the wrong conclusion from $0 = 0$.

E. None of the above

41. When both sides of an equation reduce to the same number for certain values of the unknown number, the equation is said to be

A. literal

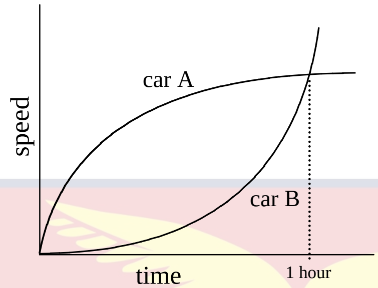
B. satisfied

C. substituted

D. transitive

E. unsatisfied

42. The given graph represents speed vs. time for two cars. (Assume the cars



start from the same position and are traveling in the same direction.) Use this information and the graph below to answer the question that follows.

What is the relationship between the ***position*** of car A and car B at $t = 1$ hour?

A. The cars are at the same position.

B. Car A is ahead of car B.

C. Car B is passing car A.

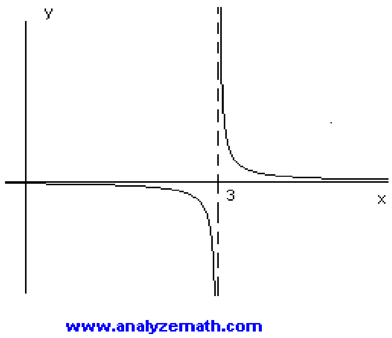
D. Car A and car B are colliding.

E. The cars are at the same position and car B is passing car A.

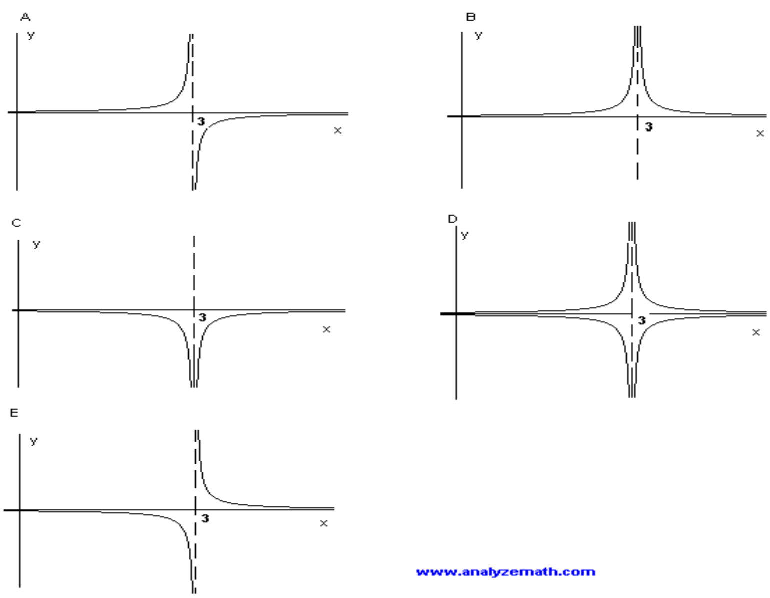
43. Kwamena is taking medications for a recent illness. Every 6 hours he takes an antibiotic, every 4 hours he takes a pain reliever, and every 3 hours he drinks a glass of water. If he starts this regime at 10 am, at what time will he be taking both medicines and a glass of water?

A. 12:00 noon
B. 4:00 pm
C. 6:00 pm
D. 10:00pm
E. None of these

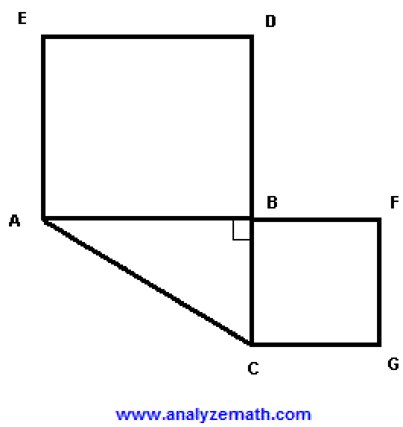
44. The graph of y = 2/(x - 3) is shown below



45. Among the following, which is the best possible graphical representation of y = -2/|x - 3|



46. In the figure below ABC is a right triangle. ABDE is a square of area 200 squ are inches and BCGF is a square of 100 square inches. What is the length, in inches, of AC?



A) 10√3

B) 10√2

C) 300

D) 10

E) 15

APPENDIX C

Results of MANOVA on teachers with Professional Background qualification and their counter without Professional Background qualification

Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[c]
Intercept	Pillai's Trace	.967	1314.570[b]	6.000	271.000	.000	.967	7887.422	1.000
	Wilks' Lambda	.033	1314.570[b]	6.000	271.000	.000	.967	7887.422	1.000
	Hotelling's Trace	29.105	1314.570[b]	6.000	271.000	.000	.967	7887.422	1.000
	Roy's Largest Root	29.105	1314.570[b]	6.000	271.000	.000	.967	7887.422	1.000
Teacher Category	Pillai's Trace	.450	36.986[b]	6.000	271.000	.000	.450	221.914	1.000
	Wilks' Lambda	.550	36.986[b]	6.000	271.000	.000	.450	221.914	1.000
	Hotelling's Trace	.819	36.986[b]	6.000	271.000	.000	.450	221.914	1.000
	Roy's Largest Root	.819	36.986[b]	6.000	271.000	.000	.450	221.914	1.000

MANOVA Table Cont'd

Tests of Between-Subjects Effects

Source	Dependent Variable	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[g]
Corrected Model	SK	.434[a]	1	.434	23.284	.000	.078	23.284	.998
	AK	.076[b]	1	.076	1.408	.236	.005	1.408	.219
	TK	1.618[c]	1	1.618	39.859	.000	.126	39.859	1.000
	PKSA	6.160[d]	1	6.160	201.658	.000	.422	201.658	1.000
	SATK	3.944E-005[e]	1	3.944E-005	.002	.969	.000	.002	.050
	AATK	.900[f]	1	.900	18.715	.000	.064	18.715	.991
Intercept	SK	132.460	1	132.460	7113.778	.000	.963	7113.778	1.000
	AK	21.109	1	21.109	392.271	.000	.587	392.271	1.000
	TK	47.471	1	47.471	1169.364	.000	.809	1169.364	1.000
	PKSA	73.325	1	73.325	2400.317	.000	.897	2400.317	1.000
	SATK	6.105	1	6.105	237.650	.000	.463	237.650	1.000
	AATK	59.135	1	59.135	1230.290	.000	.817	1230.290	1.000
Trcategory	SK	.434	1	.434	23.284	.000	.078	23.284	.998
	AK	.076	1	.076	1.408	.236	.005	1.408	.219
	TK	1.618	1	1.618	39.859	.000	.126	39.859	1.000
	PKSA	6.160	1	6.160	201.658	.000	.422	201.658	1.000
	SATK	3.944E-005	1	3.944E-005	.002	.969	.000	.002	.050

	AATK	.900	1	.900	18.715	.000	.064	18.715	.991
Error	SK	5.139	276	.019					
	AK	14.852	276	.054					
	TK	11.204	276	.041					
	PKSA	8.431	276	.031					
	SATK	7.091	276	.026					
	AATK	13.266	276	.048					
Total	SK	144.289	278						
	AK	37.000	278						
	TK	59.080	278						
	PKSA	81.778	278						
	SATK	13.680	278						
	AATK	73.840	278						
Corrected Total	SK	5.573	277						
	AK	14.928	277						
	TK	12.823	277						
	PKSA	14.592	277						
	SATK	7.091	277						
	AATK	14.166	277						

APPENDIX D

Levene's Test of Equality of Error Variances

	F	df1	df2	Sig.
SCHOOL KNOWLEDGE	8.685	1	276	.003
ADVANCED KNOWLEDGE	12.368	1	276	.001
TEACHING KNOWLEDGE	.354	1	276	.552
PROFOUND KNOWLEDGE OF SCHOOL ALGEBRA	9.936	1	276	.002
SCHOOL ALGEBRA TEACHING KNOWLEDGE	.793	1	276	.374
ADVANCED ALGEBRA TEACHING KNOWLEDGE	2.761	1	276	.098